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Numerical Integration Techniques

Overview of Code

The primary goal of this project is to write R code that will implement algorithms to will approximate definite integrals. Specifically, we look at the Improved Trapezoid Rule, and the General Simpson's Rule for numerical integration.

For simplicity, I have included all of the functions that I used in a single file, **Functions.R**. You can think of this as importing a library into your programs, as you will need to run this script in its entirety in order to use the methods described below.

Afterwards, you should be able to run these functions directly in the console, or as an R function in another script.

trapRule(fun, a, b, traps)

This function is used to approximate the integral of a function using the Improved Trapezoid Rule. The function takes in four parameters:

fun : The name of a continuous function that we want to integrate

a : Lower Limit of Integration

b: Upper Limit of Integration

traps: The Number of Trapezoids used to Approximate (traps $\in \mathbb{N}$)

To demonstrate how to use trapRule(), consider the following simple example. Suppose we want to approximate $\int_0^4 (-x^2 + 4x)dx$ using 10,000 trapezoids! First, we hard-code $-x^2 + 4x$ as a function of x in R. That is,

```
f1 <- function(x){
  -(x^2) + 4*x
}
```

Now, we simply call the trapRule function as followed,

```
trapRule(f1, 0, 4, 10000)
```

Note that $\int_0^4 (-x^2 + 4x)dx = \frac{32}{3} \approx 10.66667$. So the approximation is reasonably close.

simpRule(fun, a, b, intervals)

This function is used to approximate the integral of a function using the General Simpson's Rule. The function takes in four parameters:

fun : The name of a continuous function that we want to integrate

a : Lower Limit of Integration

b: Upper Limit of Integration

intervals: The Number of Intervals to perform Simpson's Rule on (Should be Even)

To demonstrate how to use simpRule(), consider this (not so trivial) example. Suppose we want to approximate $\int_0^1 \frac{1}{\sqrt{1+x^4}} dx$ using 4 intervals for Simpson's Rule approximation! First, we hard-code $\frac{1}{\sqrt{1+x^4}}$ as a function of x in R. That is,

```
f2 <- function(x){
  1 / ((1 + x^4)^(1/2))
}
```

Then, we call the simpRule() function as followed,

```
simpRule(f2, 0, 1, 4)
```

We then get that $\int_0^1 \frac{1}{\sqrt{1+x^4}} dx \approx 0.9271587$.

Other Functions

If you look at the source code of Functions.R, you will notice that there are two other functions: simpleTrap() and simpleSimp(). The main purpose of these two helper-functions is to make the the code easier to read in the more general implementations of Simpson's Rule and Trapezoidal Rule.

Note that simpleTrap(fun, a, b) computes and returns the area of a single trapezoid.

Also note that simpleSimp(fun, a, b) computes and return the area of Simpson's Rule on a single interval $[a, b]$.

Fun Problem

Let Y represent the exact amount of rainfall tomorrow in inches. And suppose that the probability distribution curve for the amount of rainfall tomorrow is given by $f(x) = \frac{1}{\sqrt{\pi}}e^{-(x-2)^2}$

a.) What is the Probability that we get EXACTLY 2 inches of rain tomorrow?

$$\mathbb{P}(Y = 2) = \int_2^2 f(x)dx = 0$$

b.) What is the Probability that we get approximately 2 inches of rain tomorrow?

$$\begin{aligned}
&\mathbb{P}(|Y-2| < \varepsilon) \\
&= \mathbb{P}(|Y-2| < 0.1) \\
&= \mathbb{P}(1.9 < Y < 2.1) \\
&= \int_{1.9}^{2.1} f(x)dx \\
&= \int_{1.9}^{2.1} \frac{1}{\sqrt{\pi}}e^{-(x-2)^2}dx
\end{aligned}$$

Note that $\int_{1.9}^{2.1} \frac{1}{\sqrt{\pi}}e^{-(x-2)^2}dx$ is a very difficult integral to try to compute by hand. This is an example of where our numerical integration techniques can be used to give us a reasonable approximation.

As seen previously, we first hard-code $\frac{1}{\sqrt{\pi}}e^{-(x-2)^2}$ as a function of x in R. That is,

```
f3 <- function(x){
  1/((pi)^(1/2)) * exp(-(x-2)^2)
}
```

We can then call either trapRule() or simpRule() with appropriate parameters to approximate the area and thus, the probability. For example,

```
trapRule(f3, 1.9, 2.1, 10000)

simpRule(f3, 1.9, 2.1, 10000)
```

We get $\int_{1.9}^{2.1} \frac{1}{\sqrt{\pi}}e^{-(x-2)^2}dx \approx 0.1124$.

So there is roughly an 11.24% chance that we get between 1.9 and 2.1 inches of rain tomorrow.

References

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Husch, Lawrence. "Trapezoidal Rule ." Visual Calculus - Trapezoidal Rule - 2, 2001, archives.math.utk.edu/visual.calculus/4/trapezoid.2/index.html.

Weisstein, Eric W. "Simpson's Rule." From MathWorld–A Wolfram Web Resource.
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