(1) Refaça detalhadamente os cálculos para modos TM de guias de onda de seção tranversal constante.

Semelhante ao calculo de TE, onde consideramos $E_z = 0$, na direção da guia de onda, vamos considerar $B_z = 0$ para os modos transversais magnéticos (TM).

Da lei de Ampére - Maxwell

$$\vec{\nabla} \times \vec{B} = \frac{\mu\epsilon}{c}\frac{\partial \vec{E}}{\partial t} = -i\mu\epsilon \frac{w}{c}\vec{E}$$

logo, com $B_z = 0$:

$$\begin{cases} \dfrac{\partial B_z}{\partial y} - \dfrac{\partial B_y}{\partial z} = -i\mu\epsilon \dfrac{w}{c} E_x \\ \dfrac{\partial B_x}{\partial z} - \dfrac{\partial B_z}{\partial x} = -i\mu\epsilon \dfrac{w}{c} E_y \\ \dfrac{\partial B_x}{\partial y} - \dfrac{\partial B_y}{\partial x} = -i\mu\epsilon \dfrac{w}{c} E_z \end{cases} \Rightarrow \begin{cases} \dfrac{\partial B_y}{\partial z} = i\mu\epsilon \dfrac{w}{c} E_x \\ \dfrac{\partial B_x}{\partial z} = -i\mu\epsilon \dfrac{w}{c} E_y \\ \dfrac{\partial B_x}{\partial y} - \dfrac{\partial B_y}{\partial x} = -i\mu\epsilon \dfrac{w}{c} E_z \end{cases}$$

E tomando a Lei de Indução de Faraday:

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} = i\frac{w}{c}\vec{B} \qquad E_y \propto \exp(iK_z z)$$

$$\begin{cases} \dfrac{\partial E_z}{\partial y} - \dfrac{\partial E_y}{\partial z} = i\dfrac{w}{c} B_x \\ \dfrac{\partial E_x}{\partial z} - \dfrac{\partial E_z}{\partial x} = i\dfrac{w}{c} B_y \\ \dfrac{\partial E_x}{\partial y} - \dfrac{\partial E_y}{\partial x} = i\dfrac{w}{c} B_z \end{cases} \Rightarrow \begin{cases} \dfrac{\partial E_z}{\partial y} - iK_z E_y = i\dfrac{w}{c} B_x & (i) \\ iK_z E_x - \dfrac{\partial E_z}{\partial x} = i\dfrac{w}{c} B_y & (ii) \\ \dfrac{\partial E_x}{\partial y} - \dfrac{\partial E_y}{\partial x} = i\dfrac{w}{c} B_z \end{cases}$$

A partir de (i) podemos calcular E_y

$$\frac{\partial B_x}{\partial z} = iK_z B_x = -i\mu\epsilon\frac{w}{c}E_y \Rightarrow B_x = -\mu\epsilon\frac{w}{cK_z}E_y$$

$$\frac{\partial E_z}{\partial x} - iK_z E_y = i\frac{w}{c}\left[-\mu\epsilon\frac{w}{cK_z}E_y\right] = -i\mu\epsilon\frac{w^2}{K_z c^2}E_y$$

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$$\frac{\partial \mathcal{E}_z}{\partial y} = i\mathcal{E}_y\left(K_z - \mu\epsilon\frac{w^2}{K_z c^2}\right) = i\mathcal{E}_y\left(\frac{K_z^2 - \mu\epsilon\frac{w^2}{c^2}}{K_z}\right)$$

$$\mathcal{E}_y = \frac{-iK_z}{K_z^2 - \mu\epsilon\, w^2/c^2}\;\frac{\partial \mathcal{E}_z}{\partial y}$$

Analogamente para $\mathcal{E}_x$:

$$iK_z\mathcal{E}_x - \frac{\partial \mathcal{E}_z}{\partial x} = i\frac{w}{c}\mathcal{B}_y = \frac{iw}{c}\left[\mu\epsilon\frac{w}{cK_z}\mathcal{E}_x\right] = i\mu\epsilon\frac{w^2}{K_z c^2}\mathcal{E}_x$$

$$i\mathcal{E}_x\left(K_z - \mu\epsilon\frac{w^2}{K_z c^2}\right) = \frac{\partial \mathcal{E}_z}{\partial x}$$

$$\mathcal{E}_x = \frac{-iK_z}{K_z^2 - \mu\epsilon\, w^2/c^2}\;\frac{\partial \mathcal{E}_z}{\partial x}$$

finalmente,

$$\vec{\mathcal{E}}_t = \frac{-iK_z}{K_z^2 - \mu\epsilon\frac{w^2}{c^2}}\vec{\nabla}_t\mathcal{E}_z \quad ; \quad \vec{\nabla}_t = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial z}$$

$$\vec{\mathcal{B}}_t = \mu\epsilon\frac{w}{K_z C}\hat{z}\times\vec{\mathcal{E}}_t = \mu\epsilon\frac{w}{K_z c}\hat{z}\times\vec{\mathcal{E}}$$

Basta encontrar $\mathcal{E}_z$:

$$\nabla^2\mathcal{E}_z + \mu\epsilon\left(\frac{w^2}{c^2}\right)\mathcal{E}_z = 0$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial y^2}\right)\mathcal{E}_z + \left(\mu\epsilon\frac{w^2}{c^2} - K_z^2\right)\mathcal{E}_z = 0$$

Seguimos nas condições de contorno do problema que:

$$\hat{n}\times\mathcal{E}\big|_s = 0$$

$$(n_x\hat{x} + n_y\hat{y})\times(\hat{x}\mathcal{E}_x + \hat{y}\mathcal{E}_y + \hat{z}\mathcal{E}_z)\big|_s = 0$$

$$\hat{z}(n_x\mathcal{E}_y - n_y\mathcal{E}_x) - \hat{y}n_x\mathcal{E}_z + \hat{x}n_y\mathcal{E}_z\big|_s = 0$$

$$n_x\mathcal{E}_y - n_y\mathcal{E}_x\big|_s = 0$$

$$n_x\mathcal{E}_z\big|_s = 0 \qquad n_y\mathcal{E}_z\big|_s = 0$$

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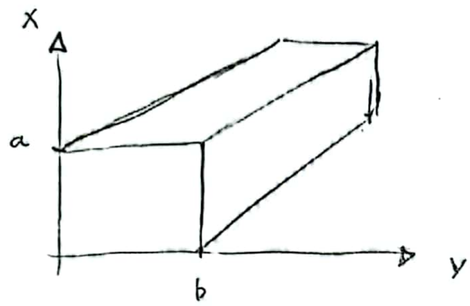
$$n_x^2 + n_y^2 = 1$$

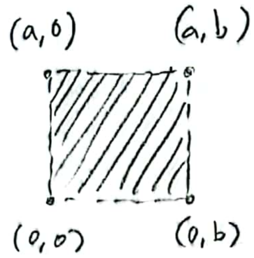
Por termos apenas componentes n_x e n_y. Portanto, temos a condição de contorno para os modos TM

$$E_z\Big|_S = 0$$

(2) Refaça detalhadamente, os cálculos para modos TE em um guia de onda de seção transversal retangular constante.

Vamos considerar um guia de onda que segue as seguintes configurações:





A solução para B_z da equação de onda é:

$$B_z = \exp\left(iK_z z - i\omega t\right)\left[\lambda_1 \cos(K_x x) + \lambda_2 \sin(K_x x)\right]\left[\lambda_3 \cos(K_y y) + \lambda_4 \sin(K_y y\right.$$

onde K_x, K_y e K_z satisfazem:

$$K_x^2 + K_y^2 + K_z^2 = \mu\epsilon \frac{\omega^2}{c^2}$$

Vamos considerar como hipótese para (TE) $E_z \equiv 0$

Tomando a lei de indução de Faraday

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} = \frac{i\omega}{c}\vec{B}$$

$$\begin{cases} \dfrac{\partial E_z}{\partial y} - \dfrac{\partial E_y}{\partial z} = i\dfrac{\omega}{c}B_y \\ \dfrac{\partial E_x}{\partial z} - \dfrac{\partial E_z}{\partial x} = i\dfrac{\omega}{c}B_y \\ \dfrac{\partial E_x}{\partial y} - \dfrac{\partial E_y}{\partial x} = i\dfrac{\omega}{c}B_z \end{cases} \Longrightarrow \begin{cases} -\dfrac{\partial E_y}{\partial z} = i\dfrac{\omega}{c}B_x \\ \dfrac{\partial E_x}{\partial z} = i\dfrac{\omega}{c}B_y \\ \dfrac{\partial E_x}{\partial y} - \dfrac{\partial E_y}{\partial x} = i\dfrac{\omega}{c}B_z \end{cases}$$

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$$\frac{\partial \mathcal{E}_x}{\partial z} = i\frac{w}{c}\beta_y \Longrightarrow \int dz\, \beta = \frac{1}{ik_z}\beta \quad ; \quad \beta \propto \exp(ik_z z)$$

$$\mathcal{E}_x = i\frac{w}{c}\cdot\frac{1}{ik_z}\beta_y = \frac{w}{ck_z}\beta_y$$

Analogamente,

$$-\frac{\partial \mathcal{E}_y}{\partial z} = i\frac{w}{c}\beta_x \Rightarrow \mathcal{E}_y = -\frac{w}{ck_z}\beta_x$$

Na lei de Ampere, temos

$$\vec{\nabla}\times\vec{\beta} = \frac{\mu\epsilon}{c}\frac{\partial \mathcal{E}}{\partial t} = -i\mu\epsilon\frac{w}{c}\mathcal{E}$$

Portanto,

$$\frac{\partial \beta_z}{\partial y} - \frac{\partial \beta_y}{\partial z} = -i\mu\epsilon\frac{w}{c}\mathcal{E}_x$$

$$= -i\mu\epsilon\frac{w}{c}\Bigg[\overbrace{\frac{w}{ck_z}\beta_y}^{\mathcal{E}_x}\Bigg] = -i\mu\epsilon\frac{w^2}{k_z c^2}\beta_y$$

$$\frac{\partial \beta_x}{\partial z} - \frac{\partial \beta_z}{\partial x} = -i\mu\epsilon\frac{w}{c}\mathcal{E}_y$$

$$= -i\mu\epsilon\frac{w}{c}\Bigg[\overbrace{-\frac{w}{ck_z}\beta_x}^{\mathcal{E}_y}\Bigg] = i\mu\epsilon\frac{w^2}{k_z c^2}\beta_x$$

$$\frac{\partial \beta_y}{\partial x} - \frac{\partial \beta_x}{\partial y} = -i\mu\epsilon\frac{w}{c}\mathcal{E}_z = 0$$

Das dependencias de z, temos

$$\frac{\partial \beta_y}{\partial z} = ik_z\beta_y \quad e \quad \frac{\partial \beta_x}{\partial z} = ik_z\beta_x$$

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$$\frac{\partial \beta_z}{\partial y} - \frac{\partial \beta_y}{\partial z} = \frac{\partial \beta_z}{\partial y} - iK_z\beta_y = -i\mu\epsilon\frac{w^2}{K_z c^2}\beta_y$$

$$\frac{\partial \beta_z}{\partial y} = \left(K_z - \mu\epsilon\frac{w^2}{K_z c^2}\right)i\beta_y = \left(\frac{K_z^2c^2 - \mu\epsilon w^2}{K_z c^2}\right)i\beta_y$$

$$\beta_y = -i\left(\frac{K_z c^2}{K_z^2c^2 - \mu\epsilon w^2}\right)\frac{\partial \beta_z}{\partial y} = \frac{-iK_z}{K_z - \mu\epsilon\frac{w^2}{c^2}}\left(\frac{\partial \beta_z}{\partial y}\right)$$

$$\beta_y = \frac{K_z c}{w}\epsilon_x$$

Analogamente,

$$\frac{\partial \beta_x}{\partial z} - \frac{\partial \beta_z}{\partial x} = iK_z\beta_x - \frac{\partial \beta_z}{\partial x} = i\mu\epsilon\frac{w^2}{K_z c^2}\beta_x$$

$$\frac{\partial \beta_z}{\partial x} = i\beta_x\left(K_z - \mu\epsilon\frac{w^2}{K_z c^2}\right) = \left(\frac{K_z^2c^2 - \mu\epsilon w^2}{K_z c^2}\right)i\beta_x$$

$$\beta_x = \frac{-iK_z}{K_z^2 - \mu\epsilon\frac{w^2}{c^2}}\left(\frac{\partial \beta_z}{\partial x}\right) = -\frac{K_z c}{w}\epsilon_y$$

logo,

$$\epsilon_y = \frac{iw}{c\left(K_z^2 - \mu\epsilon\frac{w^2}{c^2}\right)}\left(\frac{\partial \beta_z}{\partial x}\right)$$

$$\epsilon_x = \frac{-iw}{c\left(K_z^2 - \mu\epsilon\frac{w^2}{c^2}\right)}\left(\frac{\partial \beta_z}{\partial y}\right)$$

Nas condições de contorno temos para ϵ_y:

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$$\frac{\partial B_z}{\partial x} = \exp(ik_z z - i\omega t)\left[\lambda_1 K_x(-\sin(K_x \cdot x)) + \lambda_2 K_x \cos(K_x \cdot x)\right]$$
$$\left[\lambda_3 \cos(K_y y) + \lambda_4 \sin(K_y y)\right]$$

em $x=0$

$$\left.\frac{\partial B_z}{\partial x}\right|_{x=0} = \exp(ik_z z - i\omega t)(\lambda_2 K_x)\left[\lambda_3 \cos(K_y y) + \lambda_4 \sin(K_y y)\right]$$

em $x=a$

$$\left.\frac{\partial B_z}{\partial x}\right|_{x=a} = \exp(ik_z z - i\omega t)\left[\lambda_1 K_x(-\sin(K_x \cdot a)) + \lambda_2 K_x \cos(K_x a)\right]$$
$$\left[\lambda_3 \cos(K_y y) + \lambda_4 \sin(K_y \cdot y)\right]$$

Para que satisfaçamos a condiçõo de que:

$$\left.\frac{\partial B_z}{\partial x}\right|_{x=0,a} = 0 \Rightarrow \lambda_2 = 0 \quad e \quad \sin(K_x \cdot a) = 0$$

$$\sin(K_x \cdot a) = 0 \Rightarrow K_x \cdot a = n_x \pi \Rightarrow K_x = \frac{n_x \pi}{a}$$

para E_x:

$$\frac{\partial B_z}{\partial x} = \exp(ik_z z - i\omega t)\left[\lambda_1 \cos(K_x \cdot x) + \lambda_2 \sin(K_x \cdot x)\right]$$
$$\left[-\lambda_3 K_y \sin(K_y \cdot y) + \lambda_4 K_y \cos(K_y \cdot y)\right]$$

$$\left.\frac{\partial B_z}{\partial y}\right|_{y=0} = \exp(ik_z \cdot z - i\omega t)\left[\lambda_1 \cos(K_x \cdot x) + \lambda_2 \sin(K_x \cdot x)\right]$$
$$\left[\lambda_4 K_y\right]$$

$$\left.\frac{\partial B_z}{\partial y}\right|_{y=b} = \exp(ik_z \cdot z - i\omega t)\left[\lambda_1 \cos(K_x \cdot x) + \lambda_2 \sin(K_x \cdot x)\right]$$
$$\left[-\lambda_3 K_y \sin(K_y \cdot b) + \lambda_4 K_y \cos(K_y \cdot b)\right]$$

$$\left.\frac{\partial B_z}{\partial y}\right|_{y=0,b} = 0 \Rightarrow \lambda_4 = 0 \quad e \quad K_y = \frac{n_y \cdot \pi}{b}$$

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logo

$$B_z = \exp(ik_z z - i\omega t)\left[\lambda_1 \cos(k_x . x) + \lambda_2 \sin(k_x . x)\right]$$

$$\left[\lambda_3 \cos(k_y . y) + \lambda_4 \sin(k_y y)\right]$$

$$= \exp(ik_z . z - i\omega t)\left[\lambda_1 \cos\left(\frac{n_x \pi}{a} . x\right)\right]\left[\lambda_3 \cos\left(\frac{n_y \pi}{b} . y\right)\right]$$

$$= \lambda_1 \lambda_3 \exp(ik_z z - i\omega t) \cos\left(\frac{n_x \pi}{a} x\right) \cos\left(\frac{n_y \pi}{b} . y\right)$$

$$= B_0 \exp(ik_z z - i\omega t) \cos\left(\frac{n_x \pi}{a} x\right) \cos\left(\frac{n_y \pi}{b} y\right)$$

$$B_0 \equiv \lambda_1 \lambda_3$$

Vamos supor $n_x = n_y = 0$

$$B_z = B_0 \exp(ik_z z - i\omega t)$$

temos

$$E_x = -\frac{i\omega}{c\left(k_z^2 - \mu\epsilon \frac{\omega^2}{c^2}\right)}\left(\frac{\partial B_z}{\partial y}\right) = 0 = \frac{\omega}{k_z c} B_y .$$

$$E_y = \frac{i\omega}{c\left(k_z^2 - \mu\epsilon \frac{\omega^2}{c^2}\right)}\left(\frac{\partial B_z}{\partial x}\right) = 0 = -\frac{\omega}{k_z c} B_x .$$

finalmente

$$B_z = B_0 \exp(ik_z z - i\omega t) \cos\left(\frac{n_x \pi}{a} x\right) \cos\left(\frac{n_y \pi}{b} y\right)$$

só não é trivial quando $n_x, n_y = 0, 1, 2, \ldots$ e $n_x^2 + n_y^2 \neq 0$.

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(3) Refaça detalhadamente, os cálculos para modos TM de cavidades ressonantes.

Vamos retomar os valores calculados para TM no item (1) na Lei de Ampére - Maxwell.

$$\frac{\partial B_y}{\partial z} = i\mu\epsilon \frac{w}{c} E_x \qquad \frac{\partial B_x}{\partial z} = -i\mu\epsilon \frac{w}{c} E_y$$

no caso das cavidades ressonantes, as componentes de E tangenciam as tampas condutoras, e temos como condição de contorno

$$z = 0 \quad e \quad z = d$$

Devemos ter, para modos TM

$$\vec{E_t} = \sin(K_z z)\, \vec{E_t}'$$

com

$$K_z = \frac{p\pi}{d}, \quad p \in \mathbb{Z}$$

Com $p = 0 \Rightarrow E_t = 0$. Entretanto, não garante $E_z = 0$. para modos TM.

$$\frac{\partial B_y}{\partial z} = i\mu\epsilon \frac{w}{c} \sin(K_z z)\, E_x' \qquad (i)$$

$$\frac{\partial B_x}{\partial z} = -i\mu\epsilon \frac{w}{c} \sin(K_z z)\, E_y' \qquad (ii)$$

uma vez que

$$\vec{B_t} = \cos(K_z z)\, \vec{B_t}'$$

em (i)

$$K_z \sin(K_z z) B_y' = -i\mu\epsilon \frac{w}{c} \sin(K_z z)\, E_x' \Rightarrow B_y' = \frac{-i\mu\epsilon}{K_z} E_x'$$

e (ii)

$$K_z \sin(K_z z) B_x' = -i\mu\epsilon \frac{w}{c} \sin(K_z z)\, E_y' \Rightarrow B_x' = \frac{-i\mu\epsilon}{K_z} E_y'$$

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Na lei de indução de Faraday:

$$\begin{cases} \dfrac{\partial E_z}{\partial y} - \dfrac{\partial E_y}{\partial z} = i\dfrac{w}{c}\beta_x \\ \dfrac{\partial E_x}{\partial z} - \dfrac{\partial E_z}{\partial x} = i\dfrac{w}{c}\beta_y \\ \dfrac{\partial E_x}{\partial y} - \dfrac{\partial E_y}{\partial x} = i\dfrac{w}{c}\beta_z \end{cases} \Longrightarrow \begin{cases} \dfrac{\partial E_z}{\partial y} - K_z\cos(K_z)E_y' = i\dfrac{w}{c}(\cos K_z z)\beta_x' \\ K_z\cos(K_z z)E_x' - \dfrac{\partial E_z}{\partial x} = i\dfrac{w}{c}\cos(K_z z)\beta_y' \\ \dots \end{cases}$$

$$= 0$$

logo

$$\frac{\partial E_z}{\partial y} = i\frac{w}{c}\cos(K_z\cdot z)\beta_x' + K_z\cos(K_z\cdot z)E_y'$$

$$= K_z\cos(K_z\cdot z)E_y' + i\frac{w}{c}\cos(K_z\cdot z)\,i\mu\epsilon\frac{w}{K_z\cdot c}E_y'$$

$$= K_z\cos(K_z\cdot z)E_y' - \mu\epsilon\frac{w^2}{K_z c^2}\cos(K_z\cdot z)E_y'$$

$$= E_y'\cos(K_z\cdot z)\left[K_z - \mu\epsilon\frac{w^2}{K_z c^2}\right]$$

$$\therefore \cos(K_z\cdot z)E_y' = \frac{K_z}{K_z^2 - \mu\epsilon\frac{w^2}{c^2}}\left(\frac{\partial E_z}{\partial y}\right)$$

Analogamente,

$$\cos(K_z\cdot z)E_x' = \frac{K_z}{K_z^2 - \mu\epsilon\frac{w^2}{c^2}}\left(\frac{\partial E_z}{\partial x}\right)$$

podemos ver que

$$E_z = \cos(K_z z)E_z'$$

logo

$$\vec{E}_t = \frac{K_z\sin(K_z\cdot z)}{K_z^2 - \mu\epsilon\frac{w^2}{c^2}}\vec{\nabla}_t E_z'$$

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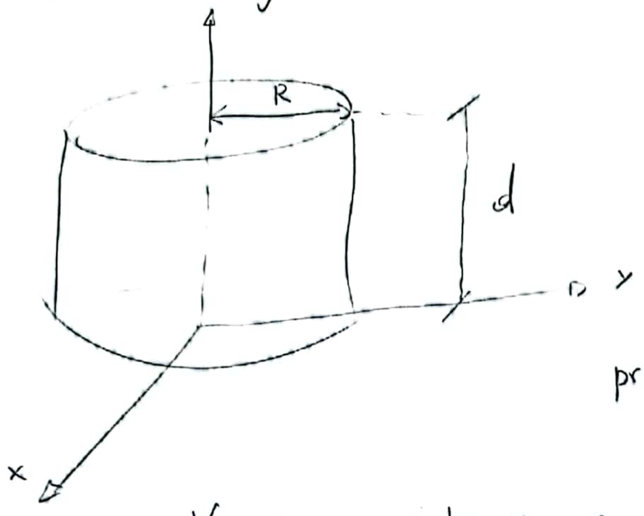
(4) Refaça, detalhadamente, os cálculos apresentados no livro J. D. Jackson para a cavidade de seção reta transversal circular.

Vamos tomar um guia de onda com um condutor plano em ambos os lados.



$$E, B \sim A \sin kz + B \cos kz$$

$$k = p\frac{\pi}{d}, \quad p \in \mathbb{Z}$$

As condições de contorno do problema:

$$\hat{n} \times \vec{E} = 0 \quad e \quad \hat{n} \cdot \vec{B} = 0$$

Vamos considerar os campos satisfazendo

$$E_t = \psi(x,y) \cos\left(\frac{p\pi}{d} z\right) \qquad B_t = \psi(x,y) \sin\left(\frac{p\pi}{d} z\right)$$

$$p = 0, 1, \ldots \qquad\qquad p = 1, 2, 3 \ldots$$

$$\vec{\nabla} \cdot \vec{E} = \underbrace{\vec{\nabla}_t \cdot \vec{E}_t}_{=0 \text{ considerando as condições impostas}} + \frac{\partial E_z}{\partial z} = 0$$

logo

$$\left.\frac{\partial E_z}{\partial z}\right|_{z=0,d} = 0$$

Analogamente

$$\left.\frac{\partial \vec{B}_t}{\partial n}\right|_{z=0,d} = 0$$

Assumindo $E_z = E_{z0} \cos kz$

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e

$$\vec{\mathcal{B}}_t = -i\mu\epsilon \frac{\omega}{k_z c} \cos(k_z \cdot z)\, \hat{z} \times \vec{\mathcal{E}}_t'$$

$$= -i\mu\epsilon \frac{\omega}{c} \frac{\cos(k_z \cdot z)}{k_z^2 - \mu\epsilon \frac{\omega^2}{c^2}}\, \hat{z} \times \vec{\nabla}_t \mathcal{E}_z'$$

substituindo na equação de onda,

$$\nabla^2 \mathcal{E}_z + \mu\epsilon \frac{\omega^2}{c^2} \mathcal{E}_z = 0$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \mu\epsilon \frac{\omega^2}{c^2} - k_z^2 \right) \mathcal{E}_z' = 0$$

Como a componente tangencial do campo elétrico à superfície lateral da cavidade ressonante deve ser nula,

$$\hat{n} \times \vec{\mathcal{E}} \Big|_S = 0$$

$$(n_x \hat{x} + n_y \hat{y}) \times (\hat{x}\mathcal{E}_x + \hat{y}\mathcal{E}_y + \hat{z}\mathcal{E}_z) \Big|_S = 0$$

$$\hat{z}(n_x \mathcal{E}_y - n_y \mathcal{E}_x) - \hat{y} n_x \mathcal{E}_z + \hat{x} n_y \mathcal{E}_z \Big|_S = 0$$

finalmente

$$n_x \mathcal{E}_y - n_y \mathcal{E}_x \Big|_S = 0$$

$$n_x \mathcal{E}_z \Big|_S = 0 \qquad \text{e} \qquad n_y \mathcal{E}_z \Big|_S = 0$$

A normal tem apenas componentes n_x e n_y

$$n_x^2 + n_y^2 = 1$$

que, como não podem ser ambas nulas, é fácil verificar que

$$\mathcal{E}_z \Big|_S = 0 \implies \cos(k_z z)\, \mathcal{E}_z' \Big|_S = 0$$

Portanto, deve valer $\forall$ z entre 0 e d

$$\mathcal{E}_z' \Big|_S = 0$$

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É possível então demonstrar as demais componentes dos campos

Para TM:

$$\vec{E}_t = -\frac{p\pi}{d\gamma^2} \sin\left(\frac{p\pi z}{d}\right) \vec{\nabla}_t \psi$$

$$\vec{B}_t = \frac{i\epsilon\omega}{\gamma^2} \cos\left(\frac{p\pi}{d} z\right) \hat{z} \times \vec{\nabla}_t \psi$$

Para TE:

$$\vec{E}_t = -\frac{i\omega\epsilon}{\gamma^2} \sin\left(\frac{p\pi}{d} z\right) \hat{z} \times \vec{\nabla} \psi$$

$$\vec{B}_t = \frac{p\pi}{d\gamma^2} \cos\left(\frac{p\pi}{d} z\right) \nabla_t \psi$$

uma vez que

$$\gamma^2 = \mu\epsilon\omega^2 - \left(\frac{p\pi}{d}\right)^2$$

deve satisfazer o problema dos autovalores

$$\begin{cases} \left(\nabla_t^2 + \gamma^2\right)\psi = 0 \\ \psi\big|_s = 0 \, , \quad \left.\frac{\partial\psi}{\partial n}\right|_s = 0 \end{cases}$$

Para cada valor de p, o autovalor γ_λ deve determinar a autofrequência $\omega_{\lambda p}$

$$\omega_{\lambda p}^2 = \frac{1}{\mu\epsilon}\left[\gamma_\lambda^2 + \left(\frac{p\pi}{d}\right)^2\right]$$

No modo TM para o caso do cilindro circular, a equação de onda transversal é dada por

$$\psi = E_z$$

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com condições de contorno

$$E_z = 0 \quad \text{em} \quad \rho = R$$

temos a solução ; portanto

$$\psi(\rho,\phi) = E_0 J_m(\gamma_{mn}\rho)\, e^{\pm im\phi} \; ; \; \gamma_{mn} = \frac{x_{mn}}{\rho}$$

com x_{mn} a n-ésima raíz da equação $J_m(x) = 0$

As frequencias de ressonancias podem ser calculados por

$$\omega_{mnp} = \frac{1}{\sqrt{\mu\epsilon}} \sqrt{\frac{x_{mn}^2}{R^2} + \frac{p^2\pi^2}{d^2}}$$

Com modos TM de menor ordem, $m=0$, $n=1$ e $p=0$.

$$\omega_{010} = \frac{2.405}{\sqrt{\mu\epsilon}\, R}$$

e expressão dos campos

$$E_z = E_0 J_0\left(\frac{2{,}405}{R}\right) \exp(-i\omega t)$$

$$B_\phi = -i\sqrt{\frac{\epsilon}{\mu}}\, E_0 J_1\left(\frac{2{,}405}{R}\right) \exp(-i\omega t)$$

Nos modos TE, também consideramos

$$\psi(\rho, \phi) = E_0 J_m(\gamma_{mn}\rho) \exp(\pm im\phi)$$

considerando as condições de contorno em B_z

$$\left.\frac{\partial\psi}{\partial\rho}\right|_R = 0 \implies \gamma_{mn} = \frac{x'_{mn}}{R}$$

com x'_{mn} a n-ésima raíz de $J'_m(x) = 0$. Os valores dessa raíz são tabelados

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Com frequências de ressonância calculadas por

$$\omega_{mnp} = \frac{1}{\sqrt{\mu\epsilon}} \left(\frac{x'^2_{mn}}{R^2} + \frac{p^2\pi^2}{d^2} \right)^{\frac{1}{2}} \quad , \quad m = 0, 1, 2, \ldots$$

O menor modo TE tem $m = n = p = 1$, TE_{111}, com frequência de ressonância

$$\omega_{111} = \frac{1.841}{\sqrt{\mu\epsilon}\, R} \left(1 + 2.912 \frac{R^2}{d^2} \right)^{\frac{1}{2}}$$

com campos deriváveis em

$$\psi = B_z = B_0 J_1 \left(\frac{1,841 \rho}{R} \right) \cos\phi \sin\left(\frac{\pi z}{d} \right) \exp(-i\omega t)$$

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