Problema 1: 28/08

With the same assumptions as in Problem 6.10 discuss the conservation of angular momentum. Show that the differential and integral forms of the conservation law are

$$\frac{\partial}{\partial t}(\mathscr{L}_{mech} + \mathscr{L}_{held}) + \nabla \cdot \overleftrightarrow{\mathbf{M}} = 0$$

and

$$\frac{d}{dt}\int_V d^3x \quad (\mathscr{L}_{mech} + \mathscr{L}_{held}) + \int_S da \quad \hat{\mathbf{n}} \cdot \overleftrightarrow{\mathbf{M}} = 0$$

where the field angular-momentum density is

$$\mathscr{L}_{field} = \mathbf{x} \times \mathbf{g} = \frac{\mu\epsilon}{4\pi c} x \times (\mathbf{E} \times \mathbf{H})$$

and the flux of angular momentum is described by the tensor

$$\overleftrightarrow{\mathbf{M}} = \overleftrightarrow{\mathbf{T}} \times x$$

Note: Here we have used the diadic notation for $\mathbf{M}_{ij}$ and $\mathbf{T}_{ij}$. The double-headed arrow conveys a fairly obvious meaning. For example, $\hat{n} \cdot \overleftrightarrow{M}$ is a vector whose jth component is $\Sigma_i n_i M_{ij}$. The second-rank $\overleftrightarrow{M}$ can be written as a third-rank tensor $M_{ij} = T_{ij}x_k - T_{ik}x_j$. But the indices j and k is antisymmetric and so has only three independent elements. Including the index i, $M_i j$ therefore has nine components and can be written as a pseudo tensor of a second rank, as above.

Teste