Lista de problemas 3 Eletromagnetismo.

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9.0 / 10

a) Queremos encontrar a solução de

$$\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\mu\epsilon}{c^2}\frac{\partial^2 u(x,t)}{\partial t^2} = 0$$

Para resolver vamos trabalhar por meio da transformada de Fourier uma vez que podemos operar no espaço de frequencias por meio de:

$$u(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} d\omega \; e^{-i\omega t} f(x,\omega)$$

e sua inversa

$$f(x,\omega) = \int_{-\infty}^{\infty} dt \; e^{i\omega t} u(x,t)$$

onde então temos:

$$\frac{\partial^2}{\partial x^2}u(x,t) - \frac{\mu\epsilon}{c^2}\frac{\partial^2}{\partial t^2}u(x,t) = 0$$

$$\left(\frac{\partial^2}{\partial x^2} - \frac{\mu\epsilon}{c^2}\frac{\partial^2}{\partial t^2}\right)u(x,t) = 0 \Rightarrow \left(\frac{\partial^2}{\partial x^2} - \frac{\mu\epsilon}{c^2}\right) \equiv \text{operador}$$

temos então, no espaço de freq.:

$$\left(\frac{\partial^2}{\partial x^2} - \frac{\mu\epsilon}{c^2}\frac{\partial^2}{\partial t^2}\right)\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} d\omega \; e^{-i\omega t} f(x,\omega) = 0$$

que nos dá

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} d\omega \; e^{-i\omega t} \; \frac{\partial^2}{\partial x^2} f(x,\omega) + \frac{\omega^2 \mu\epsilon}{c^2} f(x,\omega) = 0$$

de solução :

$$f(x,\omega) = A(\omega)\, e^{i\left(\omega \frac{\sqrt{\mu\epsilon}}{c}\right)x} + B(\omega)\, e^{-i\omega \frac{\sqrt{\mu\epsilon}}{c} x}$$

com $\sqrt{\mu\epsilon} \equiv n(\omega)$.

Substituindo na transformada de fourier temos então

$$u(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} d\omega \; e^{-i\omega t}\left[A(\omega)\, e^{i\frac{\omega}{c} n(\omega) x} + B(\omega)\, e^{-i\frac{\omega}{c} n(\omega) x}\right]$$

b) $v(x,t) \in \mathbb{R} \implies u(x,t) - u^*(x,t) = 0$

Temos portanto

$$u(x,t) - u^*(x,t) = 0$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega \left\{ e^{-i\omega t} \left[A(\omega) e^{i\frac{\omega}{c} n(\omega) x} + B(\omega) e^{-i\frac{\omega}{c} n(\omega) x} \right] - e^{i\omega t} \left[A(\omega) e^{-i\frac{\omega}{c} n^*(\omega) x} + B(\omega) e^{i\frac{\omega}{c} n^*(\omega) x} \right] \right\}.$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega \; e^{-i\omega t} \left\{ \left[A(\omega) e^{i\frac{\omega}{c} n(\omega) x} + B(\omega) e^{-i\frac{\omega}{c} n(\omega) x} \right] - \left[A(\omega) e^{i\frac{\omega}{c} n^*(\omega) x} + B(\omega) e^{-i\frac{\omega}{c} n^*(\omega) x} \right] \right\}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega \; e^{-i\omega t} \left\{ A(\omega) e^{i\frac{\omega}{c} x} \left[e^{n(\omega)} - e^{n^*(\omega)} \right] + B(\omega) e^{-i\frac{\omega}{c} x} \left[e^{n(\omega)} - e^{n^*(\omega)} \right] \right\}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega \; e^{-i\omega t} \left\{ \left[A(\omega) e^{i\frac{\omega}{c} x} + B(\omega) e^{-i\frac{\omega}{c} x} \right] \left(e^{n(\omega)} - e^{n^*(\omega)} \right) \right\} = 0$$

$$\therefore \; e^{n(\omega)} - e^{n^*(\omega)} = 0 \implies n(\omega) = n^*(\omega)$$

? Não é isso!

e) Temos

$$u(0,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega \; e^{-i\omega t} \left[A(\omega) + B(\omega) \right]$$

$$\therefore \; A(\omega) + B(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt \; e^{i\omega t} \, u(0,t)$$

na transformada inversa de Fourier.

$$\frac{\partial u(0,t)}{\partial t} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega \; e^{-i\omega t} \left[i \frac{\omega}{c} n(\omega) A(\omega) e^{i\frac{\omega}{c} n(\omega) x} - \right.$$

$$- i \frac{w}{c} n(w) B(w) e^{-i \frac{w}{c} n(w) x} \Big] \Big|_{x=0}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dw \, e^{-iwt} \, i \frac{w}{c} n(w) \left[A(w) - B(w) \right]$$

Temos, finalmente

$$A(w) - B(w) = -\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt \, e^{+iwt} \cdot \frac{ic}{n(w) w} \frac{\partial u(0,t)}{\partial x}$$

Logo

$$\begin{cases} A(w) + B(w) \\ A(w) - B(w) \end{cases}$$

$$2A(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt \, e^{iwt} \, u(0,t) - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt \, e^{iwt} \frac{ic}{w(n(w))} \frac{\partial u(0,t)}{\partial x}$$

$$A(w) = \frac{1}{2} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt \, e^{iwt} \left[u(0,t) - \frac{ic}{w\, n(w)} \frac{\partial u(0,t)}{\partial x} \right]$$

$$2B(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt \, e^{iwt} \, u(0,t) + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt \, e^{iwt} \frac{ic}{w\, n(w)} \frac{\partial u(0,t)}{\partial x}$$

$$B(w) = \frac{1}{2} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt \, e^{iwt} \left[u(0,t) + \frac{ic}{w(n(w))} \frac{\partial u(0,t)}{\partial x} \right]$$

Finalmente

$$\begin{Bmatrix} A(w) \\ B(w) \end{Bmatrix} = \frac{1}{2} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt \, e^{iwt} \left[u(0,t) \mp \frac{ic}{w n(w)} \frac{\partial u(0,t)}{\partial x} \right]$$