IFSC USP ①

Aluno: Eduardo Fonseca Rabelo N° Usp: 11272697

Questão 1:

a) Vamos partir da definição dos campos elétrico e magnético:

1)(a) 2.5
(b) 2.5
2)(a) 2.5
(b) 2.5
10.0

$$\vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t} \qquad \vec{B} = \vec{\nabla} \times \vec{A}$$

Como estamos em coordenadas polares esféricas temos:

$$\vec{\nabla} \times \vec{A} = \left[\frac{1}{r\sin\theta}\frac{\partial(\sin\theta A_\varphi)}{\partial\theta} - \frac{1}{r\sin\theta}\frac{\partial A_\theta}{\partial\varphi}\right]\hat{r} + \left[\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{1}{r}\frac{\partial(rA_\varphi)}{\partial r}\right]\hat{\theta} + \left[\frac{1}{r}\frac{\partial(rA_\theta)}{\partial r} - \frac{1}{r}\frac{\partial A_r}{\partial\theta}\right]\hat{\varphi}$$

i) O termo em $\hat{r}$ se anula pois $\partial A_\theta/\partial\varphi = 0$ e $A_\varphi = 0$

ii) O termo em $\hat{\theta}$ se anula pois $\partial A_r/\partial\varphi = 0$ e $A_\varphi = 0$

iii) Só temos campo em $\hat{\varphi}$:

$$\frac{\partial(rA_\theta)}{\partial r} = \frac{\partial}{\partial r}\left(r\sin\theta \frac{1}{cr} Lqw\sin\left[w\left(t-\frac{r}{c}\right)\right]\right) =$$

$$= \sin\theta \frac{1}{c} Lqw \frac{\partial}{\partial r}\sin\left[w\left(t-\frac{r}{c}\right)\right] = \sin\theta\frac{1}{c}Lqw\cos\left[w\left(t-\frac{r}{c}\right)\right]\left(-\frac{w}{c}\right.$$

$$= \sin\theta\left(\frac{-w^2}{c^2}\right)Lq\cos\left[w\left(t-\frac{r}{c}\right)\right]$$

$$\frac{\partial(A_r)}{\partial\theta} = \frac{\partial}{\partial\theta}\left(-\cos\theta\frac{1}{cr}Lq\sin\left[w\left(t-\frac{r}{c}\right)\right]\right) =$$

$$\sin\theta\frac{1}{cr}Lqw\sin\left[w\left(t-\frac{r}{c}\right)\right]$$

$$\frac{1}{r}\sin\theta\frac{(-w^2)}{c^2}Lq\cos\left[w\left(t-\frac{r}{c}\right)\right] - \frac{1}{r}\sin\theta\frac{Lqw}{cr}\sin\left[w\left(t-\frac{r}{c}\right)\right]$$

②

$$\vec{B} = \vec{\nabla} \times \vec{A} = \left\{ -\sin\theta \, \frac{w^2 Lq}{rc^2} \cos\left[w\left(t - \frac{r}{c}\right)\right] - \sin\theta \, \frac{Lqw}{cr^2} \sin\left[w\left(t - \frac{r}{c}\right)\right] \right\} \hat{\varphi}$$

Consideramos o termo de radiação apenas o termo proporcional a r^{-1}. Por isso temos

$$\vec{B}_{rad} = -\sin\theta \, \frac{w^2 Lq}{rc^2} \cos\left[w\left(t - \frac{r}{c}\right)\right] \hat{\varphi}$$

Calculando então o campo elétrico:

$$\vec{\nabla}\phi = \frac{\partial\phi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\phi}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial\phi}{\partial\varphi}\hat{\varphi}$$

$$\hat{r}: \frac{\partial\phi}{\partial r} = \frac{\partial}{\partial r}\left[Lq \frac{\cos\theta}{r^2} \cos\left[w\left(t - \frac{r}{c}\right)\right] - Lq\cos\theta \frac{w}{rc} \sin\left[w\left(t - \frac{r}{c}\right)\right] \right]$$

$$= Lq\cos\theta \frac{\partial}{\partial r}\left(\frac{1}{r^2}\cos\left[w\left(t - \frac{r}{c}\right)\right] - \frac{w}{rc}\sin\left[w\left(t - \frac{r}{c}\right)\right] \right)$$

$$= Lq\cos\theta \left\{ \left[(-2)\frac{1}{r^3}\cos\left[w\left(t - \frac{r}{c}\right)\right] + \frac{1}{r^2}\cdot\frac{w}{c}\sin\left[w\left(t - \frac{r}{c}\right)\right] \right] \right.$$

$$\left. - \left[\frac{w}{c}(-1)\frac{1}{r^2}\sin\left[w\left(t - \frac{r}{c}\right)\right] - \underbrace{\frac{w}{c}\frac{1}{r}\frac{w}{c}\cos\left[w\left(t - \frac{r}{c}\right)\right]}_{\propto r^{-1}} \right] \right\}$$

$$\hat{\theta}: \frac{1}{r}\frac{\partial\phi}{\partial\theta} = \frac{Lq}{r^2}\frac{\partial}{\partial\theta}(\cos\theta)\left\{ \frac{1}{r}\cos\left[w\left(t - \frac{r}{c}\right)\right] - \frac{w}{c}\sin\left[w\left(t - \frac{r}{c}\right)\right] \right\}$$

$$= \underbrace{\frac{-Lq}{r}\left(\frac{\sin\theta}{r}\right)\left\{ \frac{1}{r}\cos\left[w\left(t - \frac{r}{c}\right)\right]\right.}_{\propto r^{-3}} - \underbrace{\left.\frac{w}{c}\sin\left[w\left(t - \frac{r}{c}\right)\right] \right\}}_{\propto r^{-2}}$$

$$\hat{\varphi}: \frac{\partial\phi}{\partial\varphi} = 0$$

b) No caso do dipolo magnético temos:

$$\vec{B}_{DM} = \vec{\nabla} \times \vec{A}_{DM}$$

$$\vec{A}_{DM} = ik \frac{\exp(ikr)}{r} \hat{r} \times m_c \hat{z} = ik \frac{\exp(ikr)}{r} m_c (\hat{r} \times \hat{z})$$

$$= ik \frac{\exp(ikr)}{r} m_c \left[\hat{r} \times (\hat{r} \cos\theta - \hat{\theta} \sin\theta)\right]$$

$$= ik \frac{\exp(ikr)}{r} m_c \sin\theta \, \hat{\varphi}$$

Logo temos,

$$\vec{B}_{DM} = \hat{r} \left(\frac{1}{r\sin\theta} \frac{\partial}{\partial\theta} (\sin\theta A_\varphi) - \cancel{\frac{1}{r\sin\theta} \frac{\partial A_\theta}{\partial\varphi}}^{\,0} \right)$$

$$+ \hat{\theta} \left(\cancel{\frac{1}{r\sin\theta} \frac{\partial A_r}{\partial\varphi}}^{\,0} - \frac{1}{r} \frac{\partial}{\partial r} A_\varphi \right)$$

$$= \hat{r} \left(\frac{1}{r \sin\theta} \frac{ik}{r} \exp(ikr) m_c \cdot 2\sin\theta \cos\theta \right)$$

$$- \hat{\theta} \left(\frac{ik m_c \sin\theta}{r} \left(\frac{(-1)}{r^2} \exp(ikr) - ik \frac{\exp(ikr)}{r} \right) \right)$$

$$= \hat{r} \frac{ik}{r^2} \exp(ikr) m_c \cos\theta + \frac{ik}{r^2} m_c \sin\theta \exp(ikr) \hat{\theta}$$

$$- \frac{k^2}{r^2} m_c \sin\theta \exp(ikr) \hat{\theta}$$

$$= \frac{ik}{r^2} \exp(ikr) m_c (\hat{r} \cos\theta + \hat{\theta} \sin\theta) - \frac{k^2}{r^2} m_c \sin\theta \exp(ikr) \hat{\theta}$$

(8)

Calculando o fluxo

$$\oint d\vec{a}\ \hat{r}' \cdot \vec{B}_{DM}$$ Apenas em $\hat{r}$:

$$= \int_0^{2\pi} d\varphi \int_0^{\theta_0} d\theta\ r^2 \sin\theta\ \frac{ik}{r^2} m_c \cos\theta\ \exp(ikr)$$

$$= 2\pi\ ik\ m_c \exp(ikr) \int_0^{\theta_0} \sin\theta \cos\theta\ d\theta$$

$u = \sin\theta \qquad du = \cos\theta\ d\theta$

$$\int u\,du = \left(\frac{u^2}{2} + C\right) \Rightarrow \frac{\sin^2\theta_0}{2}$$

$\therefore$ Fluxo do B_{DM}:

$$\boxed{= \cancel{2}\pi\ ik\ m_c \exp(ikr) \sin^2\theta_0}$$

Aluno: Eduardo Fonseca Rabelo N° usp: 11272697 (3)

Questão 2:

a) Vamos começar calculando o campo de indução magnética. Considerando o potencial vetorial complexo na zona de radiação:

$$\vec{B}_{DE} = \vec{\nabla} \times \vec{A}_{DE} = -ik\, \vec{\nabla} \times \left[\frac{\exp(ikr)}{r} \vec{p_c} \right]$$

$$= -ik\, \vec{\nabla} \times \left[\frac{\exp(ikr)}{r} p_c \hat{z} \right]$$

Trabalhando em coordenadas esféricas.

$$\hat{z} = \hat{z}\cdot\hat{r}\ \hat{r} + \hat{z}\cdot\hat{\theta}\ \hat{\theta} + \hat{z}\cdot\hat{\varphi}\ \hat{\varphi}$$

$$\hat{z} = \cos\theta\ \hat{r} - \sin\theta\ \hat{\theta}$$

Logo

$$\vec{B}_{DE} = -ik\, \vec{\nabla} \times \frac{\exp(ikr)}{r} p_c \left(\cos\theta\ \hat{r} - \sin\theta\ \hat{\theta} \right)$$

i) em $\hat{r}$ é nulo pois $A_\varphi = 0$ e $\partial A_\theta / \partial \varphi = 0$
ii) em $\hat{\theta}$ é nulo pois $\partial A_r / \partial \varphi = 0$ e $A_\varphi = 0$
iii) em $\hat{\varphi}$ é:

$$\vec{B}_{DE} = -ik \left[\frac{1}{r} \frac{\partial (r A_\theta)}{\partial r} - \frac{1}{r} \frac{\partial (A_r)}{\partial \theta} \right] \hat{\varphi}$$

$$= -ik \left[\frac{1}{r} \frac{\partial}{\partial r} \left(\cancel{r} \frac{\exp(ikr)}{\cancel{r}} p_c (-\sin\theta) \right) - \frac{1}{r} \frac{\partial}{\partial \theta} \left(\frac{\exp(ikr)}{r} p_c \cos\theta \right) \right] \hat{\varphi}$$

$$= -ik \left[\frac{-1}{r} (ik) \exp(ikr) p_c \sin\theta + \frac{1}{r^2} \exp(ikr) p_c \sin\theta \right]$$

(4)

$$= \left[-\frac{k^2}{r} \exp(ikr)\, p_c \sin\theta - \frac{ik}{r^2} \exp(ikr)\, p_c \sin\theta \right] \hat{\varphi}$$

$$= -p_c \exp(ikr) \sin\theta \left(\frac{k^2}{r} - \frac{ik}{r^2} \right) \hat{\varphi} = \vec{B}_{DE}$$

Calculando o fluxo:

$$\oint da\, \hat{r} \cdot \vec{B}_{DE} = 0 \qquad \boxed{\hat{r} \cdot \hat{\varphi} = 0}$$

Fluxo do campo de Dipolo é nulo

Considerando apenas os termos de radiação:

$$\vec{\nabla}\phi^{rad} = Lq\left(\frac{\cos\theta}{r}\right)\left(\frac{w}{c}\right)^2 \cdot \cos\left[w\left(t-\frac{r}{c}\right)\right]\hat{r}$$

$$\frac{1}{c}\frac{\partial \vec{A}}{\partial t} = \left(-\hat{r}\cos\theta + \hat{\theta}\sin\theta\right)\frac{1}{c^2 r}Lqw\frac{\partial}{\partial t}\left(\sin\left[w\left(t-\frac{r}{c}\right)\right]\right)$$

$$= \left(-\hat{r}\cos\theta + \hat{\theta}\sin\theta\right)\frac{Lq}{c^2 r}w^2\cos\left[w\left(t-\frac{r}{c}\right)\right] \quad \propto r^{-1}$$

Finalmente:

$$\vec{E}_{rad} = -\vec{\nabla}\phi^{rad} - \frac{\partial \vec{A}}{\partial t}$$

$$= -Lq\left(\frac{\cos\theta}{r}\right)\left(\frac{w}{c}\right)^2\cos\left[w\left(t-\frac{r}{c}\right)\right]\hat{r} +$$

$$+ Lq\left(\frac{\cos\theta}{r}\right)\frac{w^2}{c^2}\cos\left[w\left(t-\frac{r}{c}\right)\right]\hat{r}$$

$$- Lq\left(\frac{\sin\theta}{r}\right)\frac{w^2}{c^2}\cos\left[w\left(t-\frac{r}{c}\right)\right]\hat{\theta}$$

$$\boxed{\begin{aligned}\vec{E}_{rad} &= -Lq\left(\frac{\sin\theta}{r}\right)\left(\frac{w^2}{c^2}\right)\cos\left[w\left(t-\frac{r}{c}\right)\right]\hat{\theta} \\ &\text{e} \\ \vec{B}_{rad} &= -Lq\left(\frac{\sin\theta}{r}\right)\left(\frac{w}{c}\right)^2\cos\left[w\left(t-\frac{r}{c}\right)\right]\hat{\varphi}\end{aligned}}$$

b) Vamos calcular o vetor de Poynting:

$$\vec{S}_{rad} = \frac{c}{4\pi}\vec{E}_{rad}\times\vec{B}_{rad} = \frac{c}{4\pi}\left[Lq\frac{\sin\theta}{r}\left(\frac{w}{c}\right)^2\cos\left[w\left(t-\frac{r}{c}\right)\right]\right]^2(\hat{\theta}\times\hat{\varphi})$$

$$= \frac{\left(Lq\sin\theta\, w^2\cos\left[w\left(t-\frac{r}{c}\right)\right]\right)^2}{4\pi r^2 c^3}\hat{r}$$

6

Agora basta calcular a potência por meio da integração no ângulo sólido:

$$P = \int_{4\pi} d\Omega \; r^2 \; \hat{r} \cdot \vec{S}_{rad} = \frac{(Lqw^2)^2}{4\pi c^3} \int_{4\pi} d\Omega \; \sin^2\theta \cos^2\left(w\left(t - \frac{r}{c}\right)\right)$$

$$= \frac{(Lqw^2)^2}{4\pi c^3} \cos^2\left[w\left(t - \frac{r}{c}\right)\right] \int_{4\pi} d\Omega \; \sin^2\theta$$

Calculando a integral:

$$\int_{4\pi} d\Omega \; \sin^2\theta = \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\theta \; \sin^2\theta = 2\pi \int_0^{\pi} d\theta \; \sin^3\theta$$

$$u = -\cos\theta \qquad du = \sin\theta \, d\theta \qquad \sin^2\theta = 1 - \cos^2\theta$$

$$\sin^2\theta = 1 - u^2$$

$$\sin^2\theta \cdot \sin\theta \, d\theta = \sin^2\theta \, du \Rightarrow (1-u^2)\,du$$

$$2\pi \int_{-1}^{1} (1-u^2)\,du = 2\pi\left(u - \frac{u^3}{3}\right)\Bigg|_{-1}^{1} = 2\pi\left(1 - \frac{1}{3} + 1 - \frac{1}{3}\right) = 2\pi\left(2 - \frac{2}{3}\right)$$

$$= 2\pi\left(\frac{6-2}{3}\right) = \frac{8\pi}{3}$$

$$P = \frac{(Lqw^2)^2}{4\pi c^3} \cos^2\left[w\left(t - \frac{r}{c}\right)\right] \cdot \frac{8\pi}{3} =$$

$$\boxed{P = \frac{2(Lqw^2)^2}{3c^3} \cos^2\left[w\left(t - \frac{r}{c}\right)\right]}$$