a)
Temos o potencial vetorial dado por uma expansão:

$$\vec{A}_c(\vec{r}) \approx \frac{\exp(ikr)}{rc}\int_V d^3r' \, \vec{J}_c(\vec{r}') - ik\frac{\exp(ikr)}{rc}\int_V d^3r' (\hat{r}\cdot\vec{r}')\vec{J}_c(\vec{r}')$$

e vamos olhar apenas para o ~~primeiro~~ segundo termo da expressão

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$$\int_V d^3r' \, (\hat{r}\cdot\vec{r}')\vec{J}_c(\vec{r}')$$

$$(\hat{r}\cdot\vec{r}')\vec{J}_c = \frac{1}{2}(\hat{r}\cdot\vec{r}')\vec{J}_c + \frac{1}{2}(\hat{r}\cdot\vec{r}')\vec{J}_c + \frac{1}{2}\vec{r}'(\hat{r}\cdot\vec{J}_c) - \frac{1}{2}\vec{r}'(\hat{r}\cdot\vec{J}_c)$$

logo

$$= \underbrace{\frac{1}{2}(\hat{r}\cdot\vec{r}')\vec{J}_c - \frac{1}{2}\vec{r}'(\hat{r}\cdot\vec{J}_c)}_{\mathrm{I}} + \underbrace{\frac{1}{2}(\hat{r}\cdot\vec{r}')\vec{J}_c + \frac{1}{2}\vec{r}'(\hat{r}\cdot\vec{J}_c)}_{\mathrm{II}}$$

(I) $\vec{A}\times\vec{B}\times\vec{C} = (A\cdot C)B - (A\cdot B)C \Rightarrow \hat{r}\times\vec{J}_c\times\vec{r}' = -\hat{r}\times\vec{J}_c\times\vec{r}'$

Temos então

$$-\int_V d^3r' \, \hat{r}\times\vec{J}_c\times\vec{r}' \Rightarrow \frac{ik\exp(ikr)}{2rc}\hat{r}\times\int_V d^3r' \, \vec{J}_c\times\vec{r}'$$

~~[illegible]~~ que nos dá:

$$\frac{ik\exp(ikr)}{r}\hat{r}\times\int_V d^3r' \frac{1}{2c}\vec{J}_c\times\vec{r}' = \boxed{\frac{ik\exp(ikr)}{r}\hat{r}\times\vec{m}_c}$$

A_{DM}

b) Agora basta olhar para o termo (II) da expressão anterior. Reescrevendo:

$$\frac{1}{2}\int\left[(\hat{r}\cdot\vec{r}\,')\vec{J}_c + \vec{r}\,'(\hat{r}\cdot\vec{J}_c)\right]d^3r'$$

Assumindo os cálculos das notas de aula do prof. philippe Courtelli do IFSC temos a expressão ~~da~~ Eq 8.53

$$\frac{1}{2}\int d^3r'\left[(\hat{r}\cdot\vec{r}\,')\vec{J}_c + \vec{r}\,'(\hat{r}\cdot\vec{J}_c)\right] = -\frac{i\omega}{2}\int \vec{r}\,'(\hat{r}\cdot\vec{r}\,')\rho(\vec{r}\,')d^3r'$$

Reescrevendo nas convenções de Einstein:

$$= -\frac{i\omega}{2}\int_V d^3r' \; x_i'\hat{x}_j x_j'\rho(\vec{r}\,')$$

Trabalhando um pouco com a Díade de Dipolo

$$\overleftrightarrow{Q} = \hat{x}_m Q_{mn}\hat{x}_n \Rightarrow Q_{mn} = \int_V d^3r'\left[3x_m'x_n' - \delta_{mn}(r')^2\right]\rho_c(\vec{r}\,')$$

$$\frac{Q_{ij}}{3} + \int_V \frac{\delta_{ij}(\vec{r}\,')^2}{3}\rho(\vec{r}\,') = \int_V d^3r' \; x_i'x_j'\rho_c(\vec{r}\,')$$

substituindo

$$\hat{x}_j\frac{Q_{ij}}{3} + \int \hat{x}_j\cdot\frac{\delta_{ij}(\vec{r}\,')^2}{3}\rho(\vec{r}) = \hat{x}_j\int d^3r' \; x_i'x_j'\rho_c(\vec{r}\,')$$

Finalmente

$$-ik\frac{\exp(ikr)}{rc}\left(\frac{-i\omega}{2}\right)\cdot\frac{1}{3}\left[\hat{r}\cdot\overleftrightarrow{Q} + \hat{r}\int_V d^3r'(r')^2\rho(\vec{r}\,')\right]; \quad k=\frac{\omega}{c}$$

$$\boxed{= -\frac{k^2\exp(ikr)}{6r}\left[\hat{r}\cdot\overleftrightarrow{Q} + \hat{r}\int_V d^3r'(r')^2\rho(\vec{r}\,')\right]} \rightarrow A_{QE}$$