1) (a) 2.0
b) 1.5
2) 3.0
3) 1.5
8.0

Segundo Exame de Eletromagnetismo — Página 1

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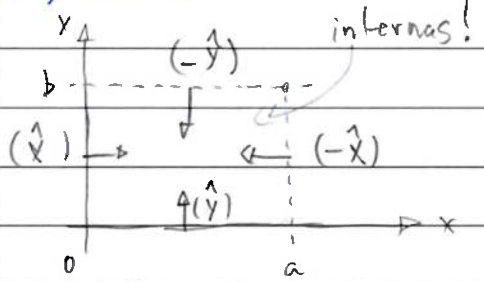
Segunda Questão:



Para calcular as densidades de carga nessa guia de onda devemos considerar a condição de contorno dessa superfície onde:

$$\sigma = \hat{n} \cdot \vec{D} \quad ; \quad \vec{D} = \varepsilon_0 \vec{E}$$

e $\vec{E} = \mathrm{Re}(\vec{\mathcal{E}})$. De forma análoga para a densidade de corrente j:

$$\vec{j} = \hat{n} \times \vec{H} = \hat{n} \times \frac{\vec{B}}{\mu_0} = \hat{n} \times \frac{\mathrm{Re}(\vec{\mathcal{B}})}{\mu_0}$$

Vamos então calcular a parte real dos campos:

$$\mathcal{E}_x = \frac{-i k_z}{k_z^2 - \frac{w^2}{c^2}} \frac{\pi}{a} \mathcal{E}_0 \left[\cos(k_z z - wt) + i \sin(k_z z - wt)\right] \cos\left(\frac{\pi}{a} x\right) \sin\left(\frac{\pi}{b} y\right)$$

$$= \frac{k_z}{k_z^2 - \frac{w^2}{c^2}} \frac{\pi}{a} \mathcal{E}_0 \left[i \cos(k_z z - wt) + \sin(k_z z - wt)\right] \cos\left(\frac{\pi}{a} x\right) \sin\left(\frac{\pi}{b} y\right)$$

$$\mathrm{Re}(\mathcal{E}_x) = \frac{k_z}{k_z^2 - \frac{w^2}{c^2}} \frac{\pi}{a} \mathcal{E}_0 \sin(k_z z - wt) \cos\left(\frac{\pi}{a} x\right) \sin\left(\frac{\pi}{b} y\right)$$

Analogamente para $\mathcal{E}_y$

$$\mathrm{Re}(\mathcal{E}_y) = \frac{k_z}{k_z^2 - \frac{w^2}{c^2}} \frac{\pi}{b} \mathcal{E}_0 \sin(k_z z - wt) \sin\left(\frac{\pi}{a} x\right) \cos\left(\frac{\pi}{b} y\right)$$

Considerando então a densidade de carga em TM_{11}:

$$\sigma(x=0) = (\hat{x} \cdot \hat{x}) \varepsilon_0 \frac{k_z}{k_z^2 - \frac{w^2}{c^2}} \frac{\pi}{a} \sin(k_z \cdot z - wt) \sin\left(\frac{\pi}{b} y\right)$$

②

$$\sigma(x=a) = \left[(-\hat{x})\cdot\hat{x}\right] \varepsilon_0 \frac{K_z}{K_z^2 - \frac{w^2}{c^2}} \frac{\pi}{a} \sin(K_z z - wt)\,(-1)\,\sin\left(\frac{\pi}{b}y\right)$$

$$= \varepsilon_0 \frac{K_z}{K_z^2 - \frac{w^2}{c^2}} \frac{\pi}{a} \sin(K_z z - wt) \sin\left(\frac{\pi}{b}y\right)$$

$$\therefore \quad \sigma(x=0) = \sigma(x=a)$$

Em y.

$$\sigma(y=0) = (\hat{x}\cdot\hat{x})\,\varepsilon_0 \frac{K_z}{K_z^2 - \frac{w^2}{c^2}} \frac{\pi}{b} \sin(K_z\cdot z - wt) \sin\left(\frac{\pi}{a}x\right)$$

$$= \varepsilon_0 \frac{K_z}{K_z^2 - \frac{w^2}{c^2}} \frac{\pi}{b} \sin(K_z\cdot z - wt) \sin\left(\frac{\pi}{a}x\right)$$

$$\sigma(y=b) = (-\hat{x}\cdot\hat{x})\,\varepsilon_0 \frac{K_z}{K_z^2 - \frac{w^2}{c^2}} \frac{\pi}{b} \sin(K_z z - wt) \sin\left(\frac{\pi}{a}x\right)(-1)$$

$$= \varepsilon_0 \frac{K_z}{K_z^2 - \frac{w^2}{c^2}} \frac{\pi}{b} \sin(K_z z - wt) \sin\left(\frac{\pi}{a}x\right)$$

$$\therefore \quad \sigma(y=0) = \sigma(y=b)$$

Vamos agora calcular a densidade de corrente $\vec{J}$, começando calculando a parte real dos campos.

$$B_x = \frac{iw}{K_z^2 c^2 - w^2} \frac{\pi}{b} \varepsilon_0 \left[\cos(K_z z - wt) + i\sin(K_z\cdot z - wt)\right] \sin\left(\frac{\pi}{a}x\right)\cos\left(\frac{\pi}{b}y\right)$$

$$\frac{w}{K_z^2 c^2 - w^2} \frac{\pi}{b} \varepsilon_0 \left[i\cos(K_z z - wt) - \sin(K_z\cdot z - wt)\right] \sin\left(\frac{\pi}{a}x\right)\cos\left(\frac{\pi}{b}y\right)$$

$$Re(B_x) = \frac{-w}{K_z^2 c^2 - w^2} \frac{\pi}{b} \varepsilon_0 \sin(K_z\cdot z - wt) \sin\left(\frac{\pi}{a}x\right)\cos\left(\frac{\pi}{b}y\right)$$

Analogamente

$$Re(B_y) = \frac{w}{K_z^2 c^2 - w^2} \frac{\pi}{a} \varepsilon_0 \sin(K_z\cdot z - wt) \cos\left(\frac{\pi}{a}x\right)\sin\left(\frac{\pi}{b}y\right)$$

Calculando a densidade de corrente por

$$\vec{j} = \hat{n} \times \vec{H}\Big|_S = \hat{n} \times \frac{1}{\mu_0} Re(\vec{B})\Big|_S ,$$

temos

$$\vec{j}(x=0) = \hat{x} \times \left[\hat{x}\frac{1}{\mu_0}(Bx) + \hat{y}\frac{1}{\mu_0} Re(By) \right]\Big|_{x=0}$$

$$= \hat{x} \times \hat{y} \frac{1}{\mu_0} Re(By)\Big|_{x=0}$$

$$= \frac{\hat{z}}{\mu_0} \frac{w}{k_z^2 - w^2} \frac{\pi}{a} e_0 \sin(k_z \cdot z - wt) \sin\left(\frac{\pi}{b} y\right)$$

$$\vec{j}(x=a) = \frac{(-\hat{x} \times \hat{y})}{\mu_0} Re(Bx)\Big|_{x=a} = \frac{-\hat{z}}{\mu_0} Re(By)\Big|_{x=a}$$

$$= \frac{-\hat{z}}{\mu_0} \frac{w}{k_z^2 - w^2} \frac{\pi}{a} e_0 \sin(k_z \cdot z - wt)(-1) \sin\left(\frac{\pi}{b} y\right)$$

$$= \frac{\hat{z}}{\mu_0} \frac{w}{k_z^2 - w^2} \frac{\pi}{a} e_0 \sin(k_z \cdot z - wt) \sin\left(\frac{\pi}{b} y\right)$$

$$\boxed{\therefore \vec{j}(x=0) = \vec{j}(x=a)}$$

Analogamente,

$$\vec{j}(y=0) = (\hat{y} \times \hat{x}) \frac{1}{\mu_0} Re(Bx)\Big|_{y=0}$$

$$= \frac{-\hat{z}}{\mu_0} \frac{-w}{k_z^2 - w^2} \frac{\pi}{b} e_0 \sin(k_z \cdot z - wt) \sin\left(\frac{\pi}{a} x\right)$$

$$\vec{j}(y=a) = (-\hat{y} \times \hat{x}) \frac{1}{\mu_0} Re(Bx)\Big|_{y=b}$$

$$= \frac{\hat{z}}{\mu_0} \frac{-w}{k_z^2 - w^2} \frac{\pi}{b} e_0 \sin(k_z z - wt)(-1) \sin\left(\frac{\pi}{b} y\right)$$

$$\boxed{\therefore \vec{j}(y=0) = \vec{j}(y=a)}$$

4

Primeira Questão

a) Podemos olhar para nosso problema da seguinte escrevendo os campos na sua forma complexa, na qual

$\vec{E}(x,y,z,t) = Re(\vec{\epsilon})$, logo

$$E_x = E_{0x} \cos\left(\frac{n_x \pi}{a} x\right) \sin\left(\frac{n_y \pi}{b} y\right) \sin\left(\frac{n_z \pi}{h} z\right) \exp(-iwt)$$

$$= \rho_0 \left(\frac{a}{\pi}\right) \cos\left(\frac{\pi}{a} x\right) \sin\left(\frac{\pi}{b} y\right) \sin\left(\frac{\pi}{h} z\right) \exp(-iwt)$$

$$\therefore E_{0x} = \rho_0 \left(\frac{a}{\pi}\right)$$

Analogamente, $E_{0y} = \rho_0 \left(\frac{b}{\pi}\right)$ e $E_{0z} = -2\rho_0 \left(\frac{h}{\pi}\right)$

$$\rho_0 \left(\frac{a}{\pi}\right) \frac{n_x}{a} + \rho_0 \left(\frac{b}{\pi}\right) \frac{n_y}{b} - 2\rho_0 \left(\frac{h}{\pi}\right) \frac{n_z}{h} = 0$$

$$\Rightarrow n_x + n_y = 2n_z \iff n_x = n_y = n_z$$

logo podemos calcular as frequencias uma vez que

$$\frac{w^2}{c^2} = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{h}\right)^2 \Rightarrow w = c\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{h}\right)^2}$$

$$\boxed{w = c\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{h}\right)^2}}$$

b) Podemos calcular a média temporal da densidade de energia e muliplicarmos pelo volume total da cavidade

$$\langle u \rangle = \frac{\varepsilon}{16\pi} \vec{E} \cdot \vec{E}^* + \frac{1}{16\pi\mu} \vec{B} \cdot \vec{B}^* = \frac{1}{T}\int_0^T u \, dt'$$

entretanto,

$$\vec{B} \cdot \vec{B}^* = \mu\varepsilon \, (\hat{K} \times \vec{E}_0) \cdot (\hat{K} \times \vec{E}_0^*)$$
$$= \mu\varepsilon \, \hat{k} \cdot \left[\vec{E}_0 \times (\hat{k} \times \vec{E}_0^*)\right]$$
$$= \mu\varepsilon \cdot \left[\hat{k}\, \vec{E}_0 \cdot \vec{E}_0^* - \vec{E}_0^* \hat{k} \cdot \vec{E}_0\right]$$

$$\vec{B} \cdot \vec{B}^* = \mu\varepsilon \, \vec{E}_0 \cdot \vec{E}_0^*$$

logo, $$\langle u \rangle = \frac{\varepsilon}{16\pi} \vec{E} \cdot \vec{E}^* + \frac{1}{16\pi\mu} \mu\varepsilon \, \vec{E} \cdot \vec{E}^*$$

$$\langle u \rangle = \frac{\varepsilon}{8\pi} \vec{E} \cdot \vec{E}^*$$

Calculando o produto do campo elétrico complexo (utilizado no item (a))

$$\vec{E}^* = \left[\vec{f}(x,y,z) \exp(-iwt)\right]^* = \vec{f}(x,y,z) \exp(iwt)$$

$$\vec{E} \cdot \vec{E}^* = \left(\vec{f}(x,y,z) \cdot \vec{f}(x,y,z)\right) \exp(-iwt + iwt)$$
$$= \vec{f}(x,y,z) \cdot \vec{f}(x,y,z)$$
$$= \left(\rho_0 \frac{a}{\pi}\right)^2 \cos^2\left(\frac{\pi}{a}x\right) \sin^2\left(\frac{\pi}{b}y\right) \sin^2\left(\frac{\pi}{h}z\right)$$
$$+ \left(\rho_0 \frac{b}{\pi}\right)^2 \sin^2\left(\frac{\pi}{a}x\right) \cos^2\left(\frac{\pi}{b}y\right) \sin^2\left(\frac{\pi}{h}z\right) + \left(2\rho_0 \frac{h}{\pi}\right)^2 \sin^2\left(\frac{\pi}{a}x\right) \sin^2\left(\frac{\pi}{b}y\right) \cos\left(\frac{\pi}{h}z\right)$$

?

E o resto?

Terceira Questão

Temos de S para S'

$x' = \gamma(x - \beta c t)$ $\qquad x = \gamma(x' + \beta c t)$

$y' = y$

$z' = z$

$t' = \gamma(t - \beta x/c)$

$$\cancel{\boxed{\phi'(x,y,z,t) = \frac{q}{\sqrt{(\gamma x' + \vec{v} t)^2 + y'^2 + z'^2}}}}$$

Uma vez que

$$\vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$

$$-\vec{\nabla}\phi = q\frac{(x+y+z)}{(x^2+y^2+z^2)^{3/2}} = \vec{E}$$

$$\vec{E}' = q\frac{[(\gamma x + \vec{v}t) + y' + z']}{[(\gamma x + \vec{v}t)^2 + y'^2 + z'^2]^{3/2}} = -\vec{\nabla}\phi' - \frac{\partial \vec{A}'}{\partial t}$$

$$-\vec{\nabla}\phi = \frac{q}{\partial}\frac{\partial(\gamma x' + \vec{v}t)^2 + \partial y' + \partial z'}{[(\gamma x + \vec{v}t)^2 + y'^2 + z'^2]^{3/2}}$$

$$\therefore \frac{\partial \vec{A}'}{\partial t} = 0 \implies \cancel{\boxed{\vec{A}' = 0}}$$