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6.21

a) A expansão em multipolos do potencial pode ser expressa por

30.0

$$\Phi(\vec{x}) = \frac{q}{r} + \frac{\vec{p}\cdot\vec{x}}{r^3} + \dots$$

onde $r \equiv |\vec{x}|$. Como estamos trabalhando no CGS temos a eq. de Poisson:

$$\nabla^2 \Phi = -4\pi\rho$$

e temos as derivadas

$$\vec{\nabla}\left(\frac{1}{r}\right) = \hat{x}\,\frac{\partial}{\partial x}\,\frac{1}{x} = -\hat{x}\cdot\frac{1}{x^2} = -\frac{\vec{r}}{x^3} \quad (i)$$

$$\nabla^2\left(\frac{1}{r}\right) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\left(\frac{1}{r}\right)\right) = \frac{1}{r^2}\frac{\partial}{\partial r}(-1) = 0 \quad \text{em } r \neq 0.$$

$$\int_V \nabla^2\left(\frac{1}{r}\right) dV = \int_{S(V)} \vec{\nabla}\left(\frac{1}{r}\right)\cdot d\vec{s} \Longrightarrow \int_{S(V)} -\frac{\hat{r}}{r^2}\cdot\hat{r}\cdot R^2\, d\Omega$$

$$\int_0^{2\pi} d\theta \int_0^{\pi} d\varphi\, \frac{1}{R^2}\cdot R^2 \sin\theta = -4\pi.$$

portanto,

$$\int_V \nabla^2\left(\frac{1}{r}\right) dV = -4\pi \Longrightarrow \nabla^2\left(\frac{1}{r}\right) = -4\pi\,\delta(\vec{x}). \quad (ii)$$

Calculando agora o segundo termo da expansão.

$$\nabla^2\left(\frac{\vec{p}\cdot\vec{x}}{r^3}\right) = \vec{\nabla}\cdot\left[\vec{\nabla}\left(\frac{\vec{p}\cdot\vec{x}}{r^3}\right)\right] = \vec{\nabla}\cdot\left[(\vec{p}\cdot\vec{\nabla})\left(\frac{\vec{x}}{r^3}\right)\right]$$

$$= \vec{p} \cdot \vec{\nabla}\left[\vec{\nabla} \cdot \left(\frac{\vec{x}}{r^3}\right)\right]$$ substituindo pelo resultado (i)

$$= \vec{p} \cdot \vec{\nabla}\left[\vec{\nabla} \cdot \left(\vec{\nabla}\frac{1}{r}\right)\right] = -\vec{p} \cdot \vec{\nabla}\left[\nabla^2 \frac{1}{r}\right] = 4\pi(\vec{p} \cdot \vec{\nabla})\delta(\vec{x})$$

substituindo na eq. de Poisson temos

$$-4\pi\rho = 4\pi\,(\vec{p} \cdot \vec{\nabla})\,\delta(\vec{x})$$

$$\boxed{\rho(\vec{x}, t) = -(\vec{p} \cdot \vec{\nabla})\,\delta(\vec{x} - \vec{r}_0(t)).}$$

Para demonstrarmos a densidade de corrente vamos considerar a equação de conservação de carga que nos diz que

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla}\vec{J}$$

logo

$$\frac{\partial \rho}{\partial t} = -(\vec{p} \cdot \vec{\nabla})\left[\frac{\partial}{\partial t}\,\delta(\vec{x} - \vec{r}_0(t))\right] = -\vec{p} \cdot \vec{\nabla}\left[-\vec{v}\,\vec{\nabla}\,\delta(\vec{x} - \vec{r}_0(t))\right]$$

$$= \vec{\nabla} \cdot \left[\vec{v}\,(\vec{p} \cdot \vec{\nabla})\,\delta(\vec{x} - r_0(t))\right] = -\vec{\nabla}\vec{J}$$

$$\boxed{\therefore \quad \vec{J} = -\vec{v}\,(\vec{p} \cdot \vec{\nabla})\,\delta(\vec{x} - r_0(t))}$$

b) Temos o momento de dipolo

$$\vec{m} = \frac{1}{2c}\int_V d^3r' \, \vec{r}\,' \times \vec{J}(\vec{r}\,')$$

do resultado do item (a) podemos considerar

$$\vec{m} = \frac{1}{2c}\int_V d^3r' \, \vec{r}\,' \times -\vec{v}\,(\vec{p}\cdot\vec{\nabla})\,\delta(\vec{x}\,'-\vec{x}_0)$$

temos o dipolo em

$$\rho(\vec{x},t) = q\,\delta^{(3)}(\vec{x}-\vec{r}_0(t)-\vec{\pi}) - q\,\delta^{(3)}(\vec{x}-\vec{r}_0(t))$$

logo

$$\vec{m} = \frac{1}{2c}\int_V d^3r' \, \vec{r}\,' \times \left[-q\,\vec{v}\,\delta^{(3)}(\vec{r}-\vec{r}_0(t))\right] + \frac{1}{2c}\int d^3r' \, \vec{r}\,' \times \left[q\vec{v}\,\delta^{(3)}(\vec{r}\,'-\vec{r}_0(t)-\vec{\pi})\right]$$

$$= \frac{1}{2c}\left[-\vec{r}_0(t)\times\vec{v}q\right] + \frac{1}{2c}\left(\vec{r}_0(t)+\vec{\pi}\right)\times\vec{v}q$$

$$= \frac{1}{2c}q\vec{\pi}\times\vec{v} = \frac{1}{2c}\left[\vec{p}\times\vec{v}\right]$$

O próximo resultado é um pouco mais chato e não entendi muito bem a lógica, mas seguimos um procedimento e é possível demonstrar o que queremos:

No quadrupolo temos:

$$Q_{ij} = \int d^3r\,(3x_ix_j - r^2\delta_{ij})\,\rho(\vec{r},t)$$

temos então

$$Q_{ij} = \int d^3r \left(3x_i x_j - \vec{r}^2 \delta_{ij}\right)\left(-\vec{p}\cdot\vec{\nabla}\right) \delta(\vec{r}-\vec{r}_0)$$

$$= \left(3 X_{0i} X_{0j} - \vec{r}_0^{\,2} \delta_{ij}\right)\cdot\left(-\vec{p}\cdot\vec{\nabla}\right)$$

Calculando os derivados temos

$$Q_{ij} = -\vec{p}\cdot\left[3\left(\delta_{jk} X_{0i} + \delta_{ik} X_{0j}\right) - 2\vec{r}_0 \delta_{ij}\right]$$

$$\boxed{Q_{ij} = -3\left(p_j X_{0i} + p_i X_{0j}\right) + 2\left(\vec{p}\cdot\vec{r}_0\right)\delta_{ij}}$$

c) Temos o campo elétrico calculado por

$$\vec{E} = -\vec{\nabla}\Phi = -\hat{e}_k \, \partial_k \left(\sum_{ij} \frac{Q_{ij}}{2} \frac{x_i x_j}{r^5} \right) = -\frac{\hat{e}_k}{2} \sum_{ij} Q_{ij} \, \partial_k \frac{(x_i x_j)}{r^5}$$

Temos

$$\partial_k \left(\frac{x_i x_j}{r^5} \right) = \frac{\delta_{ik} x_j + \delta_{jk} x_i}{r^5} - \frac{5 \, x_i x_j x_k}{r^7}$$

e

$$Q_{ij} = 3(x_{0i} p_j + x_{0j} p_i) - 2\vec{r_0} \cdot \vec{p} \, \delta_{ij}$$

Agora basta substituir tudo em multiplas manipulações algébricas.

$$\vec{E} = -\frac{\hat{e}_k}{2} \sum_{ij} \left[3(x_{0i} p_j + x_{0j} p_i) - 2\vec{r_0} \cdot \vec{p} \, \delta_{ij} \right] \cdot \left(\frac{\delta_{ik} x_j + \delta_{jk} x_i}{r^5} - 5 \frac{x_i x_j x_k}{r^7} \right)$$

$$= -\frac{\hat{e}_k}{2} \sum_{ij} \left(\frac{3(x_{0i} p_j + x_{0j} p_i)(\delta_{ik} x_j + \delta_{jk} x_i)}{r^5} - \frac{15(x_{0i} p_j + x_{0j} p_i)(x_i x_j x_k)}{r^7} \right.$$

$$\left. - \frac{2(\vec{r_0} \cdot \vec{p}) \delta_{ij} (\delta_{ik} x_j + \delta_{jk} x_i)}{r^5} + \frac{10 (\vec{r_0} \cdot \vec{p}) \delta_{ij} x_i x_j x_k}{r^7} \right)$$

$$= -\frac{\hat{e}_k}{2} \left[\sum_{ij}^{r} \frac{3}{r^5} \left(x_{0i} p_j \delta_{ik} x_j + x_{0i} p_j \delta_{jk} x_i + x_{0j} p_i \delta_{ik} x_j + x_{0j} p_i \delta_{jk} x_i \right) \right.$$

$$- \sum_{ij} \frac{15}{r^7} \left(x_{0i} p_j x_i x_j x_k + x_{0j} p_i x_i x_j x_k \right) - \sum_{ij} \frac{2}{r^5} (\vec{r} \cdot \vec{p}) (\delta_{ij} \delta_{ik} x_j + \delta_{ij} \delta_{jk} x_j)$$

$$\left. + \sum_{ij} \frac{10}{r^7} (\delta_{ij} x_i x_j x_k)(\vec{r} \cdot \vec{p}) \right]$$

$$= -\frac{\hat{e}_k}{2}\Big[\frac{3}{r^5}\left(X_{0k} P_j X_j + X_{0i} P_k X_i + X_{0j} P_k X_j + X_{0k} P_i X_i\right)$$

$$- \frac{15}{r^7}\left(X_{0i} X_i X_j P_j X_k + X_{0j} X_j X_i P_i X_k\right) - \frac{2}{r^5}(\vec{r}_0 \cdot \vec{p})(X_k + X_k)$$

$$+ \frac{10}{r^7}(\vec{r} \cdot \vec{p})\, X_i X_i X_k\Big]$$

$$= -\frac{\hat{e}_k}{2}\Bigg(\frac{3\vec{r}_0(\vec{p}\cdot\vec{r}) + 3\vec{p}(\vec{r}_0\cdot\vec{r}) + 3\vec{p}(\vec{r}_0\cdot\vec{r}) + 3\vec{r}_0(\vec{p}\cdot\vec{r})}{r^5}$$

$$- \frac{15\left(\vec{r}(\vec{r}\cdot\vec{r}_0)(\vec{r}\cdot\vec{p}) + \vec{r}(\vec{r}\cdot\vec{r}_0)(\vec{r}\cdot\vec{p})\right)}{r^7} - \frac{4}{r^5}(\vec{r}\cdot\vec{p})\vec{r}$$

$$+ \frac{10}{r^7}(\vec{r}_0\cdot\vec{p})\, X_i X_j X_k\Bigg)$$

$$= -\frac{1}{2}\Bigg[\frac{6\vec{r}_0(\vec{p}\cdot\vec{r}) + 6\vec{p}(\vec{r}_0\cdot\vec{r})}{r^5} - \frac{4}{r^5}(\vec{r}_0\cdot\vec{p})\vec{r} - \frac{30\vec{r}(r_0\cdot\vec{p})|\vec{r}|^2}{r^7}$$

$$+ \frac{10}{r^7}(\vec{r}_0\cdot\vec{p})|\vec{r}|^2\vec{r}\Bigg]$$

$$= \frac{15\hat{r}(\hat{r}\cdot\vec{r}_0)(\hat{r}\cdot\vec{p}) - 3 r_0(\vec{p}\cdot\hat{r}) - 3\vec{p}(\vec{r}_0\cdot\hat{r}) - 3\hat{r}(\vec{p}\cdot\vec{r}_0)}{r^4}$$

9.15

a) $\vec{r}_1 = a(\hat{x} + i\hat{y})e^{-iwt}$ $\quad \vec{r}_2 = -a(\hat{x} + i\hat{y})e^{-iwt}$

Temos as velocidades

$$\vec{v}_1 = \frac{d\vec{r}_1}{dt} = -aw(-i\hat{x} + \hat{y})e^{iwt} \quad \vec{v}_2 = -aw(i\hat{x} - \hat{y})e^{-iwt}$$

O momento de dipolo magnético pode ser calculado na expressão encontrada do item (b) do problema 6.21. Logo

$$\vec{m} = \frac{1}{2c}\vec{p} \times \vec{v} = \frac{1}{2c}\left[(\vec{p}_1 \times \vec{v}_1) + (\vec{p}_2 \times \vec{v}_2)\right]$$

$$= \frac{1}{2c}\left\{\left[p\hat{z} \times (-i\hat{x} + \hat{y})(-aw\, e^{-iwt})\right] + \left[-p\hat{z} \times (i\hat{x} - \hat{y})(-aw\, e^{-iwt})\right]\right\}$$

$$= -\frac{apw}{c}(i\hat{x} - \hat{y})e^{-iwt}$$

No quadrupolo elétrico

$$Q_{ij} = 3(X_{0i}P_j + X_{0j}P_i) - 2\vec{r}_0 \cdot \vec{p}\,\delta_{ij}$$

Temos $\vec{r}_0 \cdot \vec{p} = \vec{r}_1 \cdot \vec{p}_1 = \vec{r}_2 \cdot \vec{p}_2 = 0$ uma vez temos $\vec{r}$ no plano xy e $\vec{p}$ na direção $\hat{z}$.

logo

$$Q_{ij} = 3(X_{0i}P_j + X_{0j}P_i)$$

$$\vec{P}_1 = (0,0,p), \quad \vec{P}_2 = (0,0,-p)$$

$$Q_{ij} \neq 0 \iff ij = xz,\ yz,\ zx \text{ e } zy$$

$$Q_{xz} = 3(x_{0x}p_z + x_{0z}p_x)_1 + 3(x_{0x}p_z + x_{0z}p_x)_2$$

$$= 3(ap) + 3(-a)(-p) = 6ap$$

Analogamente, $Q_{zx} = Q_{xz}$

$$Q_{zx} = 6ap$$

$$Q_{zy} = 6i\,ap$$

$$Q_{yz} = 6iap$$

Que nos dá o tensor

$$\overleftrightarrow{Q} = 6ap\, e^{-i\omega t}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & i \\ 1 & i & 0 \end{pmatrix}$$

b) Partindo do campo de radiação

$$\vec{B}^{rad}_{DM} = -\frac{k^2}{r} e^{ikr}\, \hat{r} \times (\hat{r} \times \vec{m}) = \frac{-k^2}{r} e^{ikr} \left(\frac{-ap\omega}{c}\right) e^{-i\omega t}$$
$$(\hat{r} \times [\hat{r} \times (i\hat{x} - \hat{y})])$$

Calculando o produto vetorial

$$\hat{r} \times [\hat{r} \times (i\hat{x} - \hat{y})] = \hat{r} \times [\sin\theta\cos\varphi\, \hat{x} \times (i\hat{x} - \hat{y})$$
$$+ \sin\theta\sin\varphi\, \hat{y} \times (i\hat{x} - \hat{y}) + \cos\hat{z} \times (i\hat{x} - \hat{y})$$

$$= \hat{r} \times [-\sin\theta\cos\varphi\, \hat{z} - i\sin\theta\sin\varphi\, \hat{z} + i\cos\theta\, \hat{y} + \cos\theta\, \hat{x}]$$

$$= [\sin\theta\cos\varphi\, \hat{x} + \sin\theta\sin\varphi\, \hat{y} + \cos\theta\, \hat{z}] \times [-\sin\theta\cos\varphi\, \hat{z} - i\sin\theta\sin\varphi\, \hat{z}$$
$$+ i\cos\theta\, \hat{y} + \cos\theta\, \hat{x}].$$

$$= \sin^2\theta\cos^2\theta\, \hat{y} + i\cos\varphi\sin^2\theta\sin\varphi\, \hat{y} + i\cos\theta\sin\theta\cos\varphi\, \hat{z}$$
$$- \sin^2\theta\sin\varphi\cos\varphi\, \hat{x} - i\sin^2\theta\sin^2\varphi\, \hat{x} - \sin\theta\cos\theta\sin\varphi\, \hat{z}$$
$$- i\cos^2\theta\, \hat{x} + \cos^2\theta\, \hat{y}$$

$$= \cos^2\theta\,(\hat{y} - x_i) - \sin^2\theta \sin\varphi\, e^{i\varphi}\,\hat{x} + \hat{y}\sin^2\theta\cos\varphi\, e^{i\varphi} + \hat{z}\sin\theta\cos\theta\,(i\cos\varphi - \sin\varphi)$$

logo

$$\vec{B}^{rad}_{DM} = \frac{k^2 a p w}{rc} e^{ikr} e^{-iwt}\left[\cos^2\theta\,(\hat{y} - \hat{x}_i) - \sin^2\theta\sin\varphi\, e^{i\varphi}\hat{x} + \sin^2\theta\cos\varphi\, e^{i\varphi}\,\hat{y} + \hat{z}\sin\theta\cos\theta\,(i\cos\varphi - \sin\varphi)\right]$$

Em quadrupolo:

$$B^{rad}_{QE} = \frac{-ik^3}{6r} e^{ikr}\left(\hat{r} \times \overleftrightarrow{Q}\cdot\hat{r}\right)$$

$$= \frac{-ik^3}{6r} e^{ikr}\left[\hat{r} \times \underbrace{6ape^{-iwt}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & i \\ 1 & i & 0 \end{pmatrix}\begin{pmatrix} \sin\theta\cos\theta \\ \sin\varphi\sin\theta \\ \cos\theta \end{pmatrix}}_{\mathrm{I}}\right]$$

$$\mathrm{I} \Rightarrow 6ape^{-iwt}\left(\cos\theta\,\hat{x} + i\cos\theta\,\hat{y} + \sin\theta\cos\varphi\,\hat{z} + i\sin\theta\sin\varphi\,\hat{z}\right)$$

$$\hat{r} \times (\overleftrightarrow{Q}\cdot\vec{r}) = 6ape^{-iwt}\left[\hat{z}\sin\theta\cos\theta\,(i\cos\varphi - \sin\varphi) - \hat{y}\sin^2\theta\cos\varphi\, e^{i\varphi} + \hat{y}\sin^2\theta\sin\varphi\, e^{i\varphi} + \cos^2\theta\,(\hat{y} - i\hat{x})\right]$$

Juntando os resultados temos os campos de radiação

$$\vec{B}^{rad} \simeq \vec{B}^{rad}_{DE} + \vec{B}^{rad}_{DM}$$

$$= 2apk^3\cos\theta\,\frac{e^{ikr}}{r} e^{-iwt}\left[(\hat{x} + i\hat{y})\cos\theta - e^{i\varphi}\sin\theta\,\hat{z}\right]$$

e)

$$\frac{dP}{d\Omega} = r^2\hat{r}\cdot\langle\vec{S}_{rad}\rangle$$

Temos

$$\langle\vec{S}_{rad}\rangle = \frac{c}{8\pi}|\vec{B}_{rad}|^2\hat{r} = \frac{c}{8\pi}4a^2p^2k^6(\cos^2\theta + \cos^4\theta)\frac{\hat{r}}{r^2}$$

$$= \frac{a^2p^2k^6c}{2\pi}(\cos^2\theta + \cos^4\theta)\frac{\hat{r}}{r}.$$

Integrando no ângulo sólido:

$$dP = \frac{ca^2 p^2 k^6}{2\pi} (\cos^2\theta + \cos^4\theta)\, d\Omega$$

$$P = \int dP = \int d\Omega\, \frac{ca^2 p^2 k^6}{2\pi} (\cos^2\theta + \cos^4\theta)$$

$$= \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta\, \sin\theta\, \frac{ca^2 p^2 k^6}{2\pi} (\cos^2\theta + \cos^4\theta)$$

$$= ca^2 p^2 k^6 \int_0^{\pi} d\theta\, \sin\theta (\cos^2\theta + \cos^4\theta)$$

Integrando (utilizei o wolfram nesse calculo) temos:

$$\boxed{P = \frac{16}{15} ca^2 p^2 k^6}$$

9.16

a) No caso de uma antena temos a densidade de corrente descrita por

$$\vec{J}_c(\vec{r}) = \hat{z}\, I\, \delta(x)\,\delta(y)\,\sin\left(\frac{kd}{2} - k|z|\right)$$

logo

$$\vec{J}_c(\vec{r}) = \hat{z}\, I\, \delta(x)\,\delta(y)\,\sin\left(k|z|\right)$$

Portanto temos na zona de radiação

$$\vec{A}_c^{rad}(\vec{r}) = \hat{z}\,\frac{I\exp(ikr)}{rc}\int_V d^3r'\, \vec{J}_c(\vec{r}')\exp\left(-ik\hat{r}\cdot\vec{r}'\right)$$

$$= \hat{z}\,\frac{I\exp(ikr)}{rc}\int_V d^3r'\,\delta(x')\,\delta(y')\sin\left(k|z'|\right)\exp\left(-ik\hat{r}\cdot\vec{r}'\right)$$

$$= \hat{z}\,\frac{I\exp(ikr)}{rc}\int dz'\,\sin\left(k|z'|\right)\exp\left(-ikz'\cos\theta\right)$$

Resolvendo a integral entre o comprimento da barra:

$$\vec{A}_c^{rad}(\vec{r}) = \hat{z}\,\frac{I\exp(ikr)}{rc}\int_{-\frac{d}{2}}^{\frac{d}{2}} dz'\,\sin\left(k|z'|\right)\exp\left(-ikz'\cos\theta\right)$$

$$= \hat{z}\,\frac{I\exp(ikr)}{rc}\left[\int_{-\frac{d}{2}}^{0} dz'\,\sin(-kz')\exp\left(-ikz'\cos\theta\right) + \int_{0}^{\frac{d}{2}} dz'\,\sin(kz')\exp\left(-ikz'\cos\theta\right)\right]$$

$$= \hat{z}\,\frac{I\exp(ikr)}{rc}\left[-\int_{0}^{\frac{d}{2}} dz'\,\sin(kz)\exp\left(ikz'\cos\theta\right) + \int_{0}^{\frac{d}{2}} dz'\,\sin(kz')\exp\left(-ikz'\cos\theta\right)\right]$$

lembrando que

$$\sin(z) = \frac{e^{iz} - e^{-iz}}{2i} \Rightarrow -2i\sin(z) = e^{-iz} - e^{iz}$$

temos

$$\hat{z}\frac{I\exp(ikr)}{rc}\int_0^{d/2} dz' \sin(kz')(-2i)\sin(kz'\cos\theta) = \circledast$$

e também da propriedade que nos diz que

$$\sin(a)\sin(b) = -\frac{1}{2}[\cos(a+b) - \cos(a-b)]$$

$$\circledast\ \hat{z}\frac{I\exp(ikr)i}{rc}\int_0^{\frac{d}{2}} dz'\left[\cos(kz' + kz'\cos\theta) - \cos(kz' - kz'\cos\theta)\right]$$

$$\int_0^{\frac{d}{2}} dz' \cos(kz' + kz'\cos\theta) \Rightarrow \frac{1}{k(1+\cos\theta)}\int \cos u\, du = \frac{\sin u}{k(1+\cos\theta)}$$

$$= \frac{\sin(kz' + kz'\cos\theta)}{k(1+\cos\theta)}\Bigg|_0^{\frac{d}{2}} = \frac{\sin\left(\frac{kd}{2} + \frac{kd}{2}\cos\theta\right)}{k + k\cos\theta} = \frac{-\sin(k\cos\theta)}{k(1-\cos\theta)}$$

Analogamente

$$\int_0^{\frac{d}{2}} dz' \cos(kz' - kz'\cos\theta) \Rightarrow \frac{1}{k - k\cos\theta}\int \cos u\, du = \frac{\sin u}{k - k\cos\theta}$$

$$= \frac{\sin(kz' - kz'\cos\theta)}{k - k\cos\theta}\Bigg|_0^{\frac{d}{2}} = \frac{\sin\left(\frac{kd}{2} - \frac{kd}{2}\cos\theta\right)}{k - k\cos\theta} = \frac{\sin(k\cos\theta)}{k(1-\cos\theta)}$$

logo temos

$$\vec{A}_c(\vec{r}) = \frac{2i\hat{z}I\exp(ikr)}{krc}\frac{\sin(k\cos\theta)}{\sin^2\theta}$$

Partindo do campo de indução magnética

$$\vec{B}_c = \vec{\nabla} \times \vec{A}_c$$

$$\vec{\nabla} \times \frac{2i\exp(ikr)}{krc} \frac{\sin(k\cos\theta)}{\sin^2\theta}\hat{z}$$

$$-\hat{z} \times \vec{\nabla}\, \frac{2i\exp(ikr)}{krc} \frac{\sin(k\cos\theta)}{\sin^2\theta}$$

$$= -\hat{z} \times \hat{r} \frac{\partial}{\partial r} \frac{2i\exp(ikr)}{krc} \cdot \frac{\sin(k\cos\theta)}{\sin^2\theta}$$

$$= -\hat{z} \times \hat{r}\; ik\; \frac{i^{-1}\, 2I\exp(ikr)\sin(k\cos\theta)}{krc \quad \sin^2\theta} = (*)$$

$$\hat{z} \times \hat{r} = \hat{z} \times \hat{x}\sin\theta\cos\varphi + \hat{z} \times \hat{y}\sin\theta\sin\varphi + \hat{z} \times \hat{z}\cos\theta$$

$$= \hat{y}\sin\theta\cos\varphi - \hat{x}\sin\theta\sin\varphi$$

$$= \sin\theta\,\hat{\varphi}$$

logo

$$(*) = \hat{\varphi}\, \frac{2I\exp(ikr)}{rc} \frac{\sin(k\cos\theta)}{\sin\theta} = \vec{B}_c(\vec{r}).$$

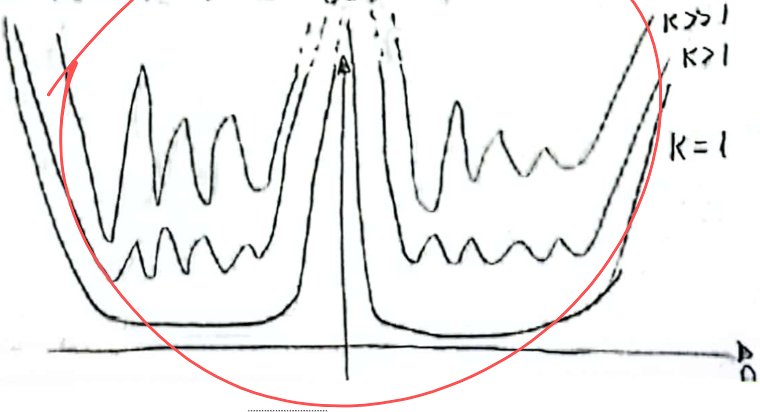
Finalmente calculamos

$$\frac{dP}{d\Omega} = r^2 \frac{c}{8\pi} |B_c(\vec{r})|^2 = r^2 \frac{c}{8\pi} \cdot \frac{4I^2\exp^2(ikr)}{r^2c^2} \frac{\sin^2(k\cos\theta)}{\sin^2\theta}$$

$$|\exp(ikr)|^2 = |\cos kr + i\sin kr|^2 = \cos^2 kr + \sin^2 kr = 1$$

por isso

$$\boxed{\frac{dP}{d\Omega} = \frac{I^2\sin^2(k\cos\theta)}{2\pi c\,\sin^2\theta}}$$



b) Temos que integrar no ângulo sólido para a potência irradiado.

$$P = \int d\Omega \frac{dP}{d\Omega} = \frac{I^2}{2\pi c} 2\pi \int_0^{\pi} \sin\theta \, d\theta \, \frac{\sin^2(K\cos\theta)}{\sin^2\theta}$$

Integrando temos

$$P = \frac{I^2}{c} \int_0^{\pi} \frac{\sin^2(K\cos\theta)}{\sin\theta} d\theta$$

A resistência de radiação é por definição $P = \frac{1}{2} R_{rad} I^2$, logo

$$R_{rad} = \frac{2}{c} \int_0^{\pi} \frac{\sin^2(K\cos\theta)}{\sin\theta} d\theta$$

Calculando um resultado numérico com $K = \pi$ temos

$$R_{rad} = \frac{2}{c} \cdot \frac{\pi}{2} = \frac{\pi}{c}$$

$\approx$