The Larson et al. (2016) shoreline change model: sensitivity analysis

Eduardo Gomez

The Larson et al. (2016) model was developed to simulate cross-shore transport and the resulting profile response at **decadal** time scales. The Larson et al. (2016) model explicitly accounts for morphological changes in diverse beach profile features (see Fig. 1 and Table 1). The Larson et al. (2016) model employs equations of both sediment transport and sediment volume conservation based on physics and empirical observations. The modules of the model describes: 1) cliff/dune erosion and overwash, 2) bar-berm material exchange, and 3) wind-blown sand transport.

Parameter	Description	Units
	Sediment transport	
q_{wL}	Constant wind-blown transport on the shoreward side	$m^3s^{-1}m^{-1}$
q_L	Overwash transport	$m^3s^{-1}m^{-1}$
q_D	Cross-shore transport rate	$m^3s^{-1}m^{-1}$
q_s	Seaward transport resulting from erosion of the dune (backwash)	$m^3s^{-1}m^{-1}$
q_{ws}	Constant wind-blown transport towards the dune	$m^3s^{-1}m^{-1}$
q_B	Berm-bar transport	$m^3s^{-1}m^{-1}$
	Morphological locations	
y_L	Location of landward end of the dune/barrier	m
y_s	Location of seaward end of the dune/barrier (dune foot)	m
y_B	Location of berm crest (represents the shoreline position)	m
y_G	Location of still-water shoreline	m
D_C	Depth of closure	m
	Morphological features	
S	Dune height	m
D_B	Berm crest height	m
V_B	Bar volume	m^3m^{-1}

Table 1: List of symbols on Figure 1 .

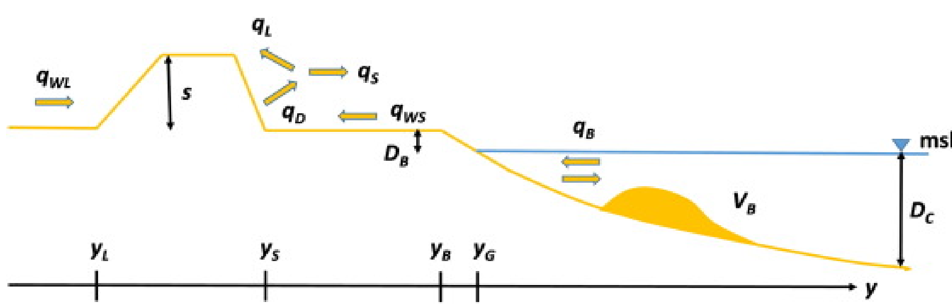

Figure 1: Sketch to calculate cross-shore transport and beach profile evolution (Larson et al., 2016).

Sensitivity analysis

The sensitivity of a model (S) can be measured by the relative change in a state variable of interest (y) with respect to changing a certain parameter of interest (x) from a baseline value. A simple index of sensitivity is given by:

$$S_{y,x} = \frac{(\Delta y/y)}{(\Delta x/x)} \tag{1}$$

Where y and x are, respectively, the baseline values of the state variable and the parameter of interest, while Δy and Δx represent the change obtained by modifying the parameter x. When $S_{x,y} < 1$ there is no sensitivity by changing parameter x. On the other hand, when $S_{x,y} > 1$ the sensitivity can have a linear (same value of S for different magnitudes of x change) or a non-linear response.

To understand the role of the parameters in the Larson et al. (2016) model, a sensitivity analysis was made following eq.(1). The state variables (y, q) and related parameters (x) that were studied are specified on table 2 , while table 3 contains detailed information of the parameters of interest.

Transport rates of interest (q)		Related parameters (x)
q_D	cross-shore transport rate	C_s
q_L	overwash transport	A
q_B	berm-bar transport	λ_0, m, C_B
State variables of interest (y)		**Related transport rates (q)**
y_G	still-water shoreline	q_D
y_L	landward dune end	q_L
y_s	seaward dune end	q_B

Table 2: State variables and transport rates evaluated.

Parameters tested	Ref.value	Ocurrence	References
C_s	0.001	cross-shore transport equation (eq. 2)	developed (Larson et al., 2004)
A	3	q_L and q_s transport relationship (eq. 4)	field obs. (Larson et al., 2009)
C_B	0.022	Bar volume at equilibrium (eq. 7)	
λ_0	0.15	Bar volume time change (eq. 8)	LWT (Larson and Kraus, 1989)
m	-0.5	Bar volume time change (eq. 8)	

Table 3: Parameters tested.

Click on url to see sensitivity analysis on google spreadsheet G+:
https://docs.google.com/spreadsheets/d/1Ab0HnxCPEOuDnLChf-mHFLcn5nFKEREG-2UXvfo-wRQ/edit?usp=sharing

Equations review (hunting 🐞)

Transport equations

- **Dune erosion and overwash model**

The cross-shore transport rate q_D is related to the runup height R, the distance from the mean water level to the dune foot Z_D, the wave period T and an empirical coefficient C_s:

$$\cancel{q_D = 4C_s\frac{(R-Z_D)^s}{T}} \Rightarrow q_D = 4C_s\frac{(R-Z_D)s}{T} \tag{2}$$

where C_s expresses how much of the dune is removed by the wave impact. The cross-shore transport rate (q_D) is further decompossed to the overwash (q_L) and backwash (q_s) transports, where α is the relationship between q_L and q_s:

$$\alpha = q_L/q_s \Rightarrow q_L = \frac{\alpha q_D}{(1+\alpha)} \Rightarrow q_s = \frac{q_D}{(1+\alpha)} \tag{3}$$

Note : fow overwash (q_L) to occur $R - Z_D > s$, otherwise $\alpha = 0$, implying $q_D = q_s$. Larson et al. (2009) empirically approximated α by the following relationship:

$$\alpha = \frac{1}{A}\Big(\frac{R-Z_D}{s} - 1\Big) \tag{4}$$

- **Bar-berm exchange model**

The change in the bar volume V_B is taken to be proportional to the deviation from its equilibrium value V_{BE}:

$$\frac{dV_B}{dt} = \lambda(V_{BE} - V_B) = q_B \tag{5}$$

If V_{BE} and λ are constants, we obtain the solution:

$$V_B = V_{BE} - (V_{BE} - V_{B0})e^{-\lambda t} \tag{6}$$

where V_{B0} is the bar volume at $t = 0$. The bar volume at equilibrium V_{BE} is given by:

$$\frac{V_{BE}}{L_0^2} = C_B\Big(\frac{H_0}{wT}\Big)^{\frac{4}{3}}\frac{H_0}{L_0} \tag{7}$$

where w is the sediment fall speed and C_B a dimensionless coefficient. The parameter λ is the rate coefficient that depends on the wave and sediment properties:

$$\lambda = \lambda_0\Big(\frac{H_0}{wT}\Big)^m \tag{8}$$

where λ_0 and m are coefficients with values to be calibrated against data.

Sediment conservation equations for morphological features

- Bar evolution

$$\frac{dV_B}{dt} = q_B \tag{9}$$

- Berm evolution

$$\frac{dy_B}{dt} = \frac{1}{D_B + D_C}\Big(-q_{ws} - q_B + q_s - \frac{dQ_L}{dx}\Big) \tag{10}$$

where Q_L is the longshore sand transport.

- Dune evolution

$$\frac{dy_s}{dt} = \frac{-q_D + q_{WS}}{s} \tag{11}$$

$$\frac{dy_L}{dt} = \frac{-q_L - q_{WL}}{s} \tag{12}$$

¿where is y_G?

If the beach profile has a fixed shape from the berm crest to the depth of closure, the following relationship holds:

$$y_G = y_B + \frac{D_B}{tan\beta_f} = y_B + L_B \tag{13}$$

where L_B is the horizontal distance from the berm crest to the shoreline (the foreshore length), and $tan\beta_f$ is the foreshore slope.

References

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