Notes on the Larson et al. (2016) and Wolinsky and Murray (2009) shoreline change models

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Problems to solve on the long time-scale

The Bruun Rule suggests that shoreline (solely) recedes in response to sea level rise. However, in the long time-scale (decades) sea level rise may cause a **seaward** flux of sediment on beaches where cliff/dune erosion represents a source of sediment. In other words, the shoreline can advance in response to sea level rise.

How to account for these type of processes? Proposals

- Extend our coastal system to the dune/cliff. Account for dune/cliff morphodynamics (as in Larson et al. (2016)).
- Obtain a net mass balance equation for the extended coastal system (as in Wolinsky and Murray (2009)).
- Use geometry to relate time-varying (dune and foreshore) slopes to shoreline position.

Larson et al. (2016)

The Larson et al. (2016) model was developed to simulate cross-shore transport and the resulting profile response at **decadal** time scales. In contrast with other models described before (e.g. Vitousek et al. (2017), Yates et al. (2009)), the Larson et al. (2016) model explicitly accounts for morphological changes in diverse beach profile features (see Fig. 1 and Table 1). Since the Yates et al. (2009) model neglects geological feedbacks on shoreline evolution, the simulation of morphological features dynamics may be crucial to overcome current limitations on long-time scale shoreline change predictions.

The Larson et al. (2016) model employs equations of both sediment transport and sediment volume conservation based on physics and empirical observations. The modules of the model describes: 1) cliff/dune erosion and overwash, 2) bar-berm material exchange, and 3) wind-blown sand transport. The application of the Larson et al. (2016) model to diverse study sites is described in a companion paper (Palalane et al., 2016). **Note:** Although not explicitly addressed, the Larson et al. (2016) model includes an approach to simulate the effects of sea level rise.

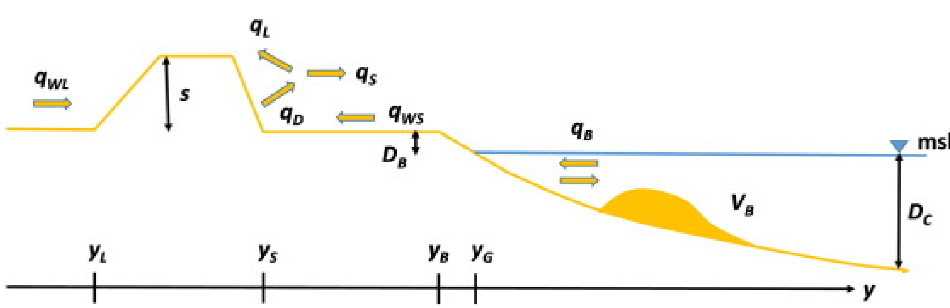

Figure 1: Sketch to calculate cross-shore transport and beach profile evolution (Larson et al., 2016).

Parameter	Description	Units
	Sediment transport	
q_{wL}	Constant wind-blown transport on the shoreward side	$m^3 s^{-1} m^{-1}$
q_L	Overwash transport	$m^3 s^{-1} m^{-1}$
q_D	Cross-shore transport rate	$m^3 s^{-1} m^{-1}$
q_s	Seaward transport resulting from erosion of the dune (backwash)	$m^3 s^{-1} m^{-1}$
q_{ws}	Constant wind-blown transport towards the dune	$m^3 s^{-1} m^{-1}$
q_B	Berm-bar transport	$m^3 s^{-1} m^{-1}$
	Morphological locations	
y_L	Location of landward end of the dune/barrier	m
y_s	Location of seaward end of the dune/barrier (dune foot)	m
y_B	Location of berm crest (represents the shoreline position)	m
y_G	Location of still-water shoreline	m
D_C	Depth of closure	m
	Morphological features	
S	Dune height	m
D_B	Berm crest height	m
V_B	Bar volume	$m^3 m^{-1}$

Table 1: List of symbols on Figure 1 .

Transport equations

- Dune erosion and overwash

$$q_D = 4C_s \frac{(R - Z_D)^s}{T} \tag{1}$$

The cross-shore transport rate q_D is related to the runup height R, the distance from the mean water level to the dune foot Z_D, the wave period T and an empirical coefficient C_s.

The overwash q_L and backwash q_s transports are derived from the cross-shore transport rate q_D.

- Bar-berm material exchange

$$\frac{dV_B}{dt} = \lambda(V_{BE} - V_B) \tag{2}$$

The change in the bar volume V_B is taken to be proportional to the deviation from its equilibrium value V_{BE}.

- Wind-blown sand

In all the study sites where Larson et al. (2016) model was applied (Palalane et al., 2016), constant values of wind-blown transport towards the dune on the seaward (q_{WS}) and landward (q_{WL}) sides were addopted.

Sediment conservation equations for morphological features

- Bar evolution

$$\frac{dV_B}{dt} = q_B \tag{3}$$

- Berm evolution

$$\frac{dy_B}{dt} = \frac{1}{D_B + D_C}\Big(- q_{ws} - q_B + q_s - \frac{dQ_L}{dx}\Big) \tag{4}$$

where Q_L is the longshore sand transport.

- Dune evolution

$$\frac{dy_s}{dt} = \frac{-q_D + q_{WS}}{s} \tag{5}$$

$$\frac{dy_L}{dt} = \frac{-q_L - q_{WL}}{s} \tag{6}$$

¿where is y_G?

If the beach profile has a fixed shape from the berm crest to the depth of closure, the following relationship holds:

$$y_G = y_B + \frac{D_B}{tan\beta_f} = y_B + L_B \tag{7}$$

where L_B is the horizontal distance from the berm crest to the shoreline (the foreshore length), and $tan\beta_f$ is the foreshore slope.

Wolinsky and Murray (2009): Shoreline Exner equation

Wolinsky (2009) adapted the Exner equation — which originally stated mass conservation in a river— to the shoreline system. The shoreline Exner equation is:

$$\underbrace{c_0\ H_s\frac{ds}{dt}}_{\substack{\text{shoreline}\\ \text{migration}}} = \underbrace{q_{x,s} - q_{x,u}}_{\substack{\text{cross-shore}\\ \text{flux divergence}}} + \overbrace{\partial_y Q_y}^{\substack{\text{alongshore}\\ \text{flux divergence}}} - \underbrace{\overline{c}LR'}_{\substack{\text{relative sea level rise}\\ \text{accomodation}}} \tag{8}$$

In a companion paper Wolinsky and Murray (2009) developed shoreline change models for the long-term (centuries) evolution of beaches with barrier islands and cliffs. The approach for model development in Wolinsky and Murray (2009) is systematic:

- Apply Eq. 8 to each module of the coastal system.
- Combine each module mass balance to obtain a net mass balance equation for the coastal system.
- If appropiate, use geometry to relate time-varying shoreline position to morphological features (e.g. cliff relief).

References

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