Notes on the Bruun Rule for shoreline change. Part II

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Numerical experiments: varying sea level rise.

Model for shoreline response with Vitousek et al. (2020) rearrangement of the Yates et al. (2009) model and the Bruun (1962) Rule:

$$\underbrace{\frac{\partial Y}{\partial t}}_{\substack{\text{shoreline}\\ \text{change}}} = \underbrace{\frac{1}{\tau}(Y_{eq} - Y)}_{\substack{\text{cross-shore}\\ \text{transport}}} - \overbrace{\frac{c}{tan\beta}\frac{\partial S}{\partial t}}^{\substack{\text{shoreline migration}\\ \text{due to sea-level rise}}} \tag{1}$$

```
%% Set model step and parameters (Yates)
dt=nanmean(diff(t));  % model time step
Hs_bar=nanmean(Hs(:));            % mean wave height
DT=28;   % model time scale
DY=10;   % model shoreline excursion parameter
Nsteps=length(t);
```

Bruun Rule discretization

$$-\overbrace{\frac{c}{tan\beta}\frac{\partial S}{\partial t}}^{\substack{\text{shoreline migration}\\ \text{due to sea-level rise}}} \tag{2}$$

```
%% RUN FORWARD YATES + BRUUN MODEL
Sref = 0.003 / 365;
Y_b=NaN(Nsteps,1); Y_b(1,:)=Yobs(1);

for n=1:Nsteps-1

    % Y at equilibrium
    Yeq_b=-DY*(Hs(n,:).^2-Hs_bar^2)./Hs_bar^2;
    tau=DT*(Hs_bar./Hs(n,:));

    % Yates (+ Bruun)
    Y_b(n+1,:)=Y_b(n,:)+dt./tau.*(Yeq_b-Y_b(n,:))-dt./tanb.*(Sref);
end
```

What would happen if instead of Tairua we look at Miami beach $tan\beta < 0.001$ (Athanasiou et al., 2019) and we take S= 12 mm per year (Church et al., 2013) in RCP 8.5 (year 2100) ? See Figure 1.

As we can see, this code for the Bruun Rule only accounts for a constant divergence of the shoreline as time goes by. We can introduce a correction on the sea level rise differential to change this.

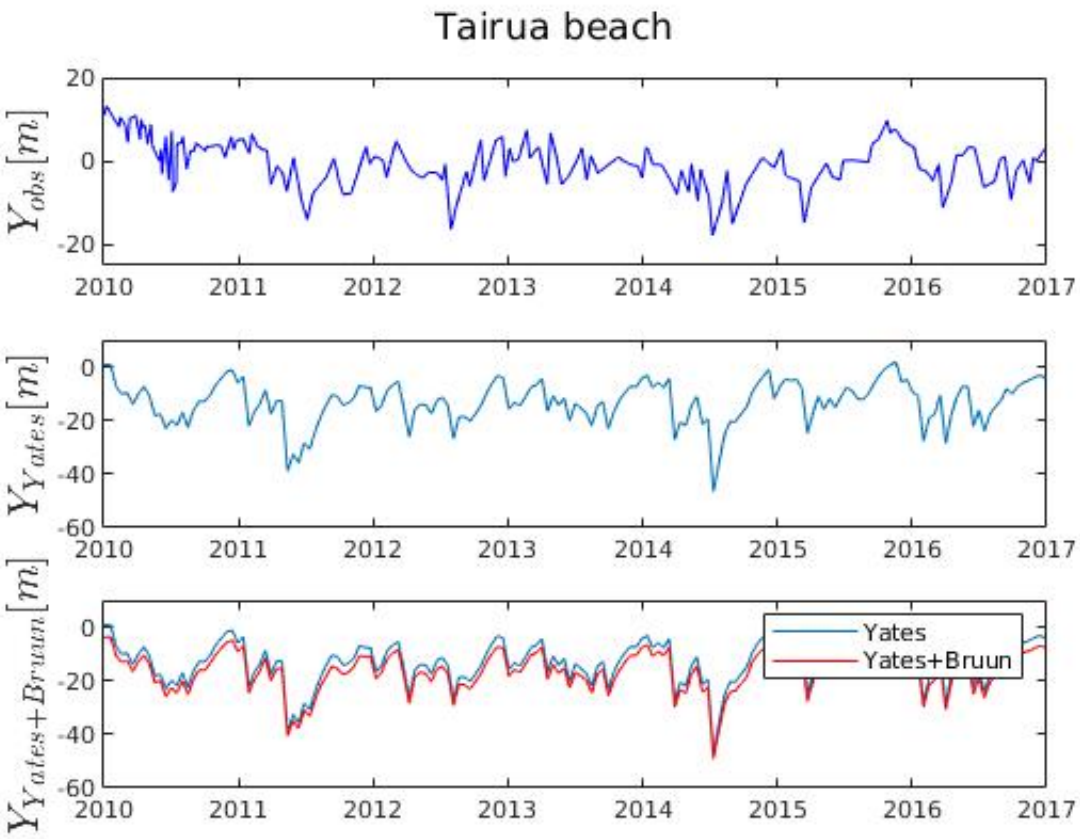

Figure 1: $tan\beta = 0.0003$, $S = 12$ millimeters per year, RCP 8.5 IPCC, world average.

Code correction

```
for n=1:Nsteps-1

    % Y at equilibrium
    Yeq=-DY*(Hs(n,:).^2-Hs_bar^2)./Hs_bar^2;
    tau=DT*(Hs_bar./Hs(n,:));

    % Yates (+ Bruun) wrong formulation
    % Y_b(n+1,:)=Y_b(n,:)+dt./tau.*(Yeq_b-Y_b(n,:))-(S2100/tanb);

    % Yates (+ Bruun), sea level rise change
    Y_b(n+1,:)=Y_b(n,:)+dt./tau.*(Yeq-Y_b(n,:))-(dsl(n)./tanb);
end
```

```
islope= slope_2060;  %SLR slope at the decade 2050-2060 (m/day)
fslope= slope_2100;  %SLR slope at the decade 2090-2100  (m/day)
dsl= linspace(islope,fslope,length(t)-1)';
```

Differences between sea level rise rates: past and future.

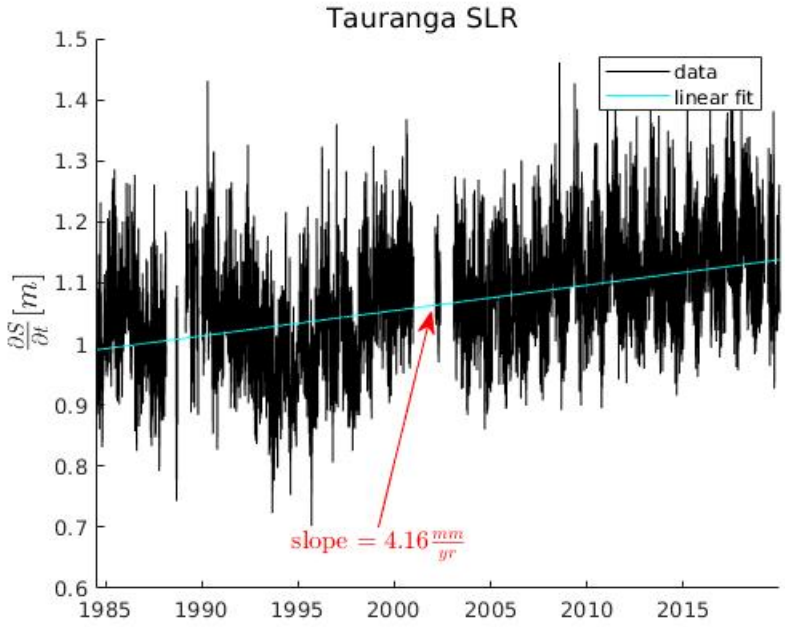

Figure 2: Tauranga sea level rise. Retrieved from: Caldwell et al. (2015).

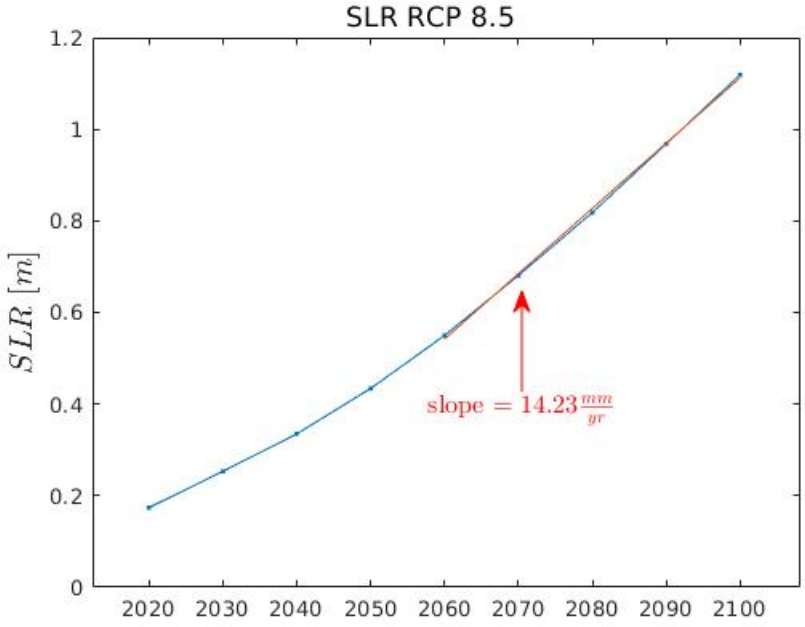

Figure 3: RCP 8.5 SLR scenarios. Retrieved from: *Coast and Ocean Collective* (2020)

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Now we can see the divergence of shoreline change as time goes by:

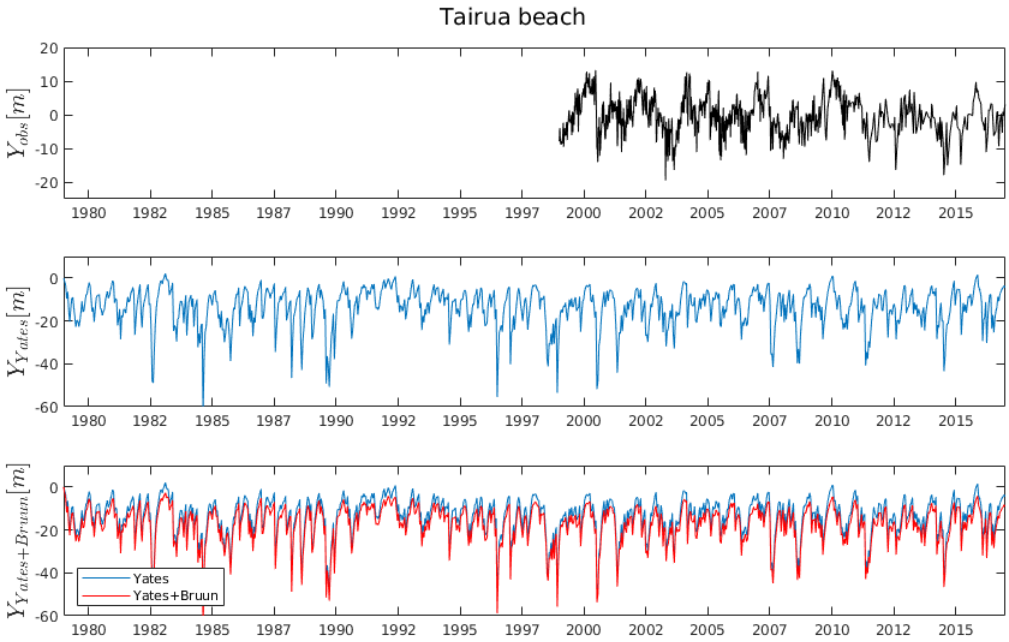

Figure 4: $tan\beta = 0.0003$, from SLR 2060 to SLR 2100, RCP 8.5 IPCC .

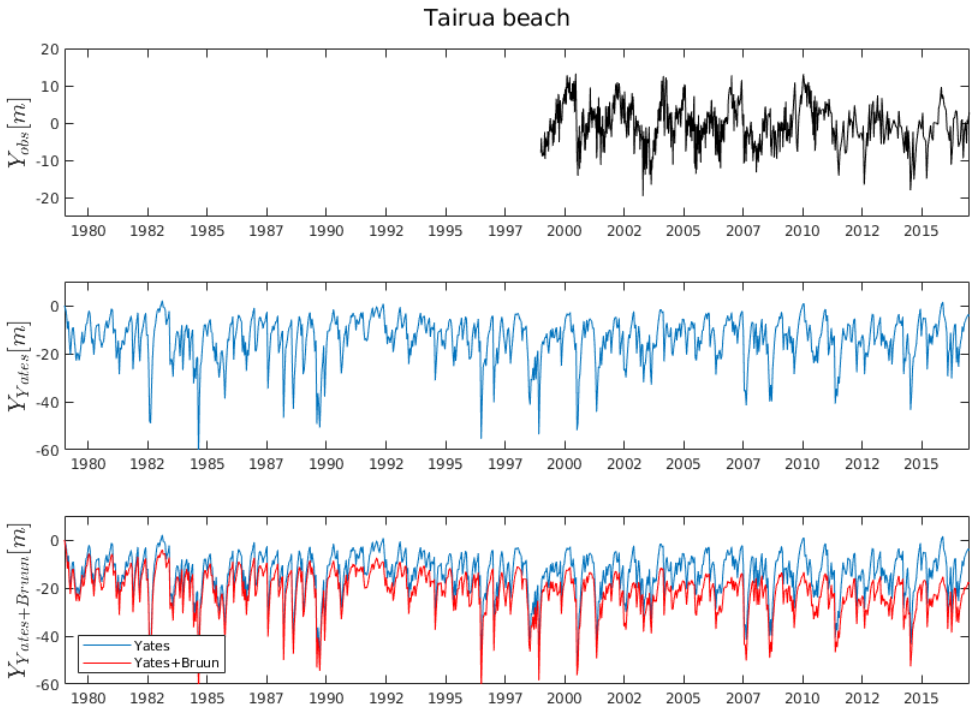

Figure 5: $tan\beta = 0.0003$, from SLR 2060 to SLR 2100*3, RCP 8.5 IPCC .

Let's check Bruun Rule's sensibility to beach slope:

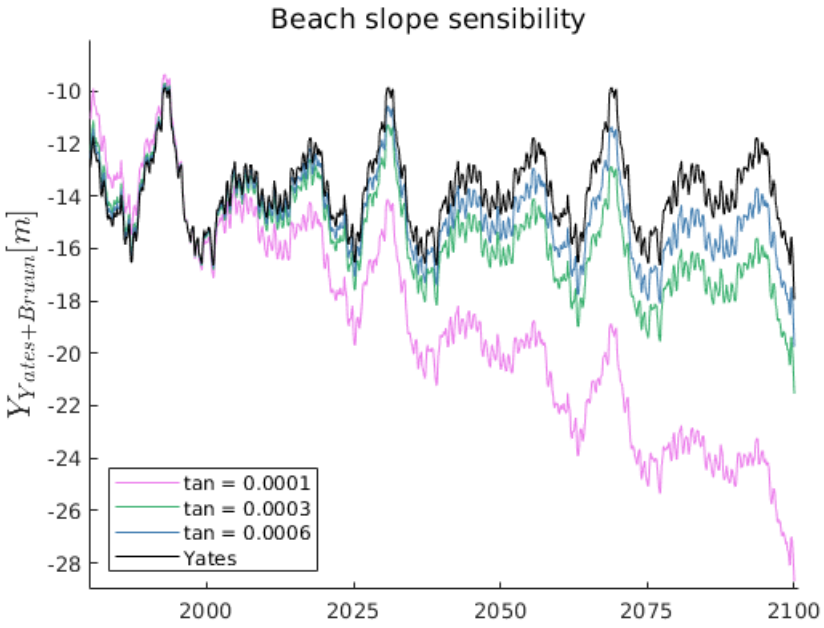

Figure 6: Bruun Rule and Yates model (moving averages), from SLR 1980 to SLR 2100, RCP 8.5 IPCC.

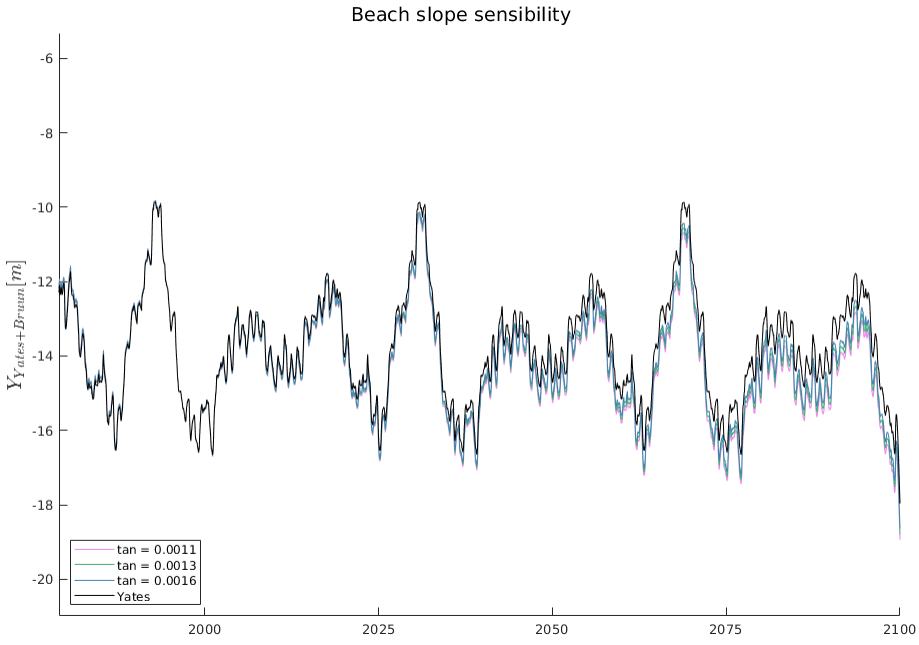

Figure 7: Bruun Rule and Yates model (moving averages), from SLR 1980 to SLR 2100, RCP 8.5 IPCC.

References

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Bruun, P. (1962). Sea-level rise as a cause of shore erosion. *Journal of the Waterways and Harbors division*, 88(1):117–132.

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