Notes on the Bruun Rule for shoreline change

Eduardo Gomez

Bruun (1962) suggested that, in response to sea level rise, the beach profile would retain its equilibrium concave shape while wave action transports sediment from the upper part of the profile to the lower part. In this model, the shoreline retreat $\partial y/\partial t$ is related to sea level rise $\partial z_{sea}/\partial t$ by the ratio of the horizontal and vertical dimensions of the active profile (Figure 1), L_s and H_s respectively:

$$\frac{\partial y}{\partial t} = -\frac{\partial z_{sea}}{\partial t}\frac{L_s}{H_s} \Longrightarrow -\frac{\partial z_{sea}}{\partial t}\frac{1}{tan\beta} \tag{1}$$

where the average nearshore slope S_s is $tan\beta$.

$$\Longrightarrow \frac{\partial y}{\partial t} = -\frac{\partial z_{sea}}{\partial t}\frac{1}{S_s} \tag{2}$$

The Bruun rule (eq. 2) predicts that shoreline retreat $\partial y/\partial t$ parallels the average nearshore slope S_s of the equilibrium profile, independent of the inland slope.

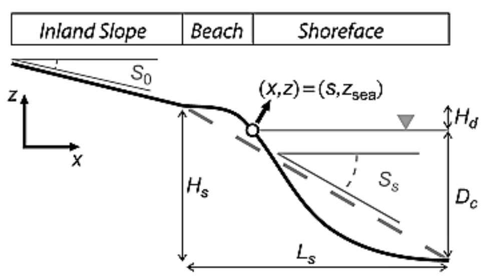

Figure 1: Conceptual model of a coastal system consisting of a terrestrial inland slope S_0, a nearshore beach, and a shoreface complex. The shoreline at $x = s$ (white circle) is the intersection of the nearshore profile with sea level z = z_{sea}. The Bruun model assumes an equilibrium nearshore profile with relief H_s and length L_s, extending from the berm (dune) top H_d landward to the closure depth D_c seaward. The nearshore slope S_s controls shoreline retreat in the Bruun model Wolinsky (2009).

In its classic formulation, the Bruun Rule requires the following assumptions:

1. The shoreface, beach, and substrate must have a homogeneous composition.
2. The shoreface and beach must maintain a fixed equilibrium profile.
3. This profile must be closed with respect to external sources and sinks of sediment.

Much of the controversy surrounding the Bruun Rule concerns the validity of the closed equilibrium profile concept (assumptions 2 and 3). Despite its initial breakthrough in identifying the entire shoreface as responsive to sea level rise, the subsequent advances have shown that the Bruun Rule is an inadequate descriptor of shoreface response to sea level rise (Cooper and Pilkey, 2004).

Modifications of the Bruun Rule

Passive inundation (Wolinsky, 2009)

Passive inundation occurs when the land surface is static during transgression, in these cases the shoreline retreat follows the slope of the inland topography, (e.g. during storm surge flooding). This passive inundation is simple to model, but shoreline retreat is usually accompanied by erosion and deposition which drive morphologic changes that impact future retreat.

The passive inundation model predicts much more pronounced shoreline retreat on gentle coasts versus steep coasts. In contrast, for a given nearshore profile the Bruun model predicts identical shoreline trajectories. This is due to differences between the models; while the passive inundation follows the inland slope S_0, the Bruun Rule shoreline retreat follows the average nearshore slope S_s (Figure 2).

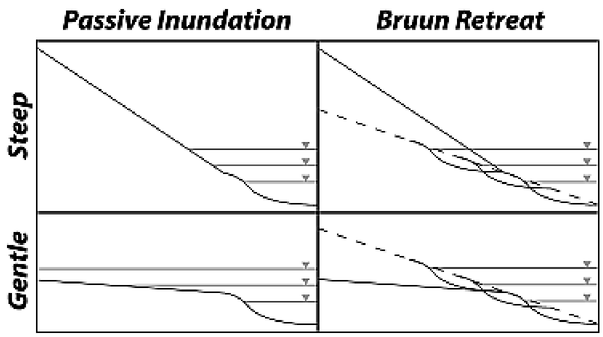

Figure 2: Shoreline transgression due to sea level rise. In passive inundation transgression follows the inland slope S_0 while in Bruun retreat trangression follows the average slope of the nearshore equilibrium profile S_s.

The Bruun rule fails to predict long-term shoreline retreat! To solve this, Wolinsky (2009) derive generalizations of the Bruun rule for steep and gentle coasts, which consists basically on extending the equilibrium profile landward:

$$\frac{\partial y}{\partial t} = -\frac{\partial z_{sea}}{\partial t}\frac{1}{\overline{S}} \tag{3}$$

where $\overline{S}$ is the average extended profile slope, in contrast with eq. 2, where the nearshore slope S_s is used. Wolinsky (2009) named eq. 3 the *essential* Bruun Rule.

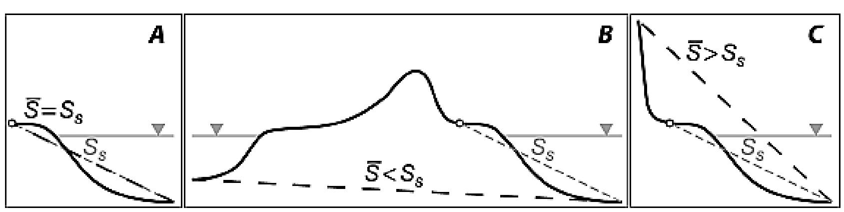

Figure 3: Applications of the essential Bruun rule to different control volumes. (a) Application to a shoreface and beach gives the classic Bruun rule. (b) Application to a beach and shoreface backed by a barrier island gives a barrier Bruun rule. (c) Application to a beach and shoreface backed by a cliff gives a cliff Bruun rule.

When applying the essential Bruun Rule (Figure 3) the implied transgression slope (black dashed line) equals the average profile slope $\overline{S}$ but only in the classic Bruun rule is this equal to the nearshore slope S_s (gray dotted line). The generalized cliff ($\overline{S} > S_s$) and barrier ($\overline{S} < S_s$) Bruun rules give physically reasonable predictions of long-term shoreline retreat on steep and gentle coasts. However this requires a sufficiently tall cliff (or long barrier) so that $\overline{S} = S_0$ while providing no explanation of how this may happen. Wolinsky (2009) derived simplified morphokinematic models to account for this intermediate stages of shoreline transgression and thus gain insight on evolving nearshore geometry.

Landward transport (Rosati et al., 2013)

One of the problematic assumptions of the classic Bruun Rule is that the beach profile must be closed with respect to external sources and sinks of sediment. The Bruun Rule predicts that in response to sea level rise, sand will be transported and deposited seaward. However, there are several processes that evince landward sediment transport due to sea level rise (Rosati et al., 2013):

- Overwash and aeolian transport.
- Barrier island formation due to excess of sand in profile.
- Lack of offshore sand deposits predicted by the Bruun Rule.
- Profile deepening with time.

Rosati et al. (2013) propose a modified form of the Bruun Rule to include profile response due to landward transport associated with aeolian and/or overwash processes:

$$\frac{\partial y}{\partial t} = -\frac{\partial z_{sea}}{\partial t} \frac{L_s + V_D / \frac{\partial z_{sea}}{\partial t}}{H_s} \tag{4}$$

It is noted that this relationship includes an additive term to the original Bruun Rule, thus landward transport increases the shoreline retreat. Application of Eq.(4) only requires quantification of the volume per unit length of the landward deposition V_D.

In other words, for profiles in which both landward and seaward transports occur,the implication of the modified Bruun Rule is that beach recession $\partial y/\partial t$ will be greater than with the original Bruun Rule, as the modification represents an additive term.

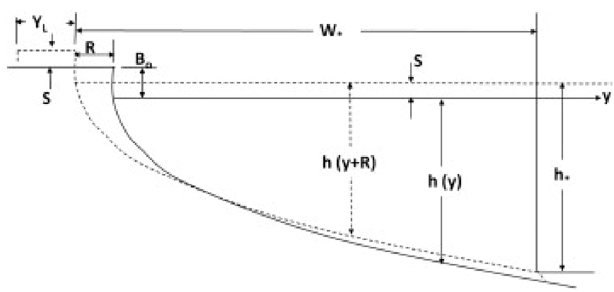

Figure 4: Landward transport Bruun Rule

Numerical experiment

Hybrid model for shoreline response CoSMoS-COAST (Vitousek et al., 2017):

$$\underbrace{\frac{\partial Y}{\partial t}}_{\substack{\text{shoreline}\\ \text{change}}} = \overbrace{-\frac{1}{d}\frac{\partial Q}{\partial X}}^{\substack{\text{longshore}\\ \text{transport}}} + \underbrace{CE^{\frac{1}{2}}\Delta E}_{\substack{\text{cross-shore}\\ \text{transport}}} - \overbrace{\frac{c}{tan\beta}\frac{\partial S}{\partial t}}^{\substack{\text{shoreline migration}\\ \text{due to sea-level rise}}} + \underbrace{v_{lt}}_{\substack{\text{long-term shoreline trend;}\\ \text{unresolved processes}}} \tag{5}$$

```
%% Ensemble Wave conditions %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
load('Wave_hindcast_corrected.mat');
t=hindcast.time;
Hs=hindcast.Hs;
Tp=hindcast.tm02;
Dir=hindcast.Dir;
%% Load shoreline observations
load('Shorecast_complete.mat')
tobs=Shore.time;
Yobs=Shore.average-nanmean(Shore.average);
```

Yates model, Vitousek et al. (2020)

$$\frac{dY}{dt} = \frac{1}{\tau}(Y_{eq} - Y) = \underbrace{CE^{\frac{1}{2}}\Delta E}_{\substack{\text{cross-shore} \\ \text{transport}}} \tag{6}$$

```
%% Set model step and parameters (Yates)
dt=nanmean(diff(t));  % model time step
Hs_bar=nanmean(Hs(:));            % mean wave height
DT=28;    % model time scale
DY=10;    % model shoreline excursion parameter
Nsteps=length(t);
```

Bruun Rule, Wolinsky (2009),Vitousek et al. (2017)

$$\overbrace{-\frac{c}{tan\beta}\frac{\partial S}{\partial t}}^{\substack{\text{shoreline migration} \\ \text{due to sea-level rise}}} \tag{7}$$

```
%% Set Bruun parameters
tanb = 0.13; %slope at Tairua (Blossier et al . (2017))
%tanb = 0.0003; %slope at Miami or similar Anthanasiou (2019) <0.001

%%%%%Sea level rise rate
%3.0    0.4 millimetres  per year for the period 1993  2017
%(Nerem et al. 2018)
%S =   0.003 / 365;
Sref = 0.003 / 365;

%IPCC RCP8.5 by 2100
%12 mm per year
S2100= 0.012 / 365;

%Miami
%9 mm per year after 2006 (Wdowinsky 2016)
```

$$\underbrace{\frac{\partial Y}{\partial t}}_{\substack{\text{shoreline}\\ \text{change}}} = +\underbrace{CE^{\frac{1}{2}}\Delta E}_{\substack{\text{cross-shore}\\ \text{transport}}} - \overbrace{\frac{c}{tan\beta}\frac{\partial S}{\partial t}}^{\substack{\text{shoreline migration}\\ \text{due to sea-level rise}}} \tag{8}$$

```
%% RUN FORWARD YATES + BRUUN MODEL

Y_b=NaN(Nsteps,1);
Y_b(1,:)=Yobs(1);

for n=1:Nsteps-1

    % Y at equilibrium
    Yeq_b=-DY*(Hs(n,:).^2-Hs_bar^2)./Hs_bar^2;
    tau=DT*(Hs_bar./Hs(n,:));

    % Yates (+ Bruun)
    Y_b(n+1,:)=Y_b(n,:)+dt./tau.*(Yeq_b-Y_b(n,:))-dt./tanb.*(Sref);
end
```

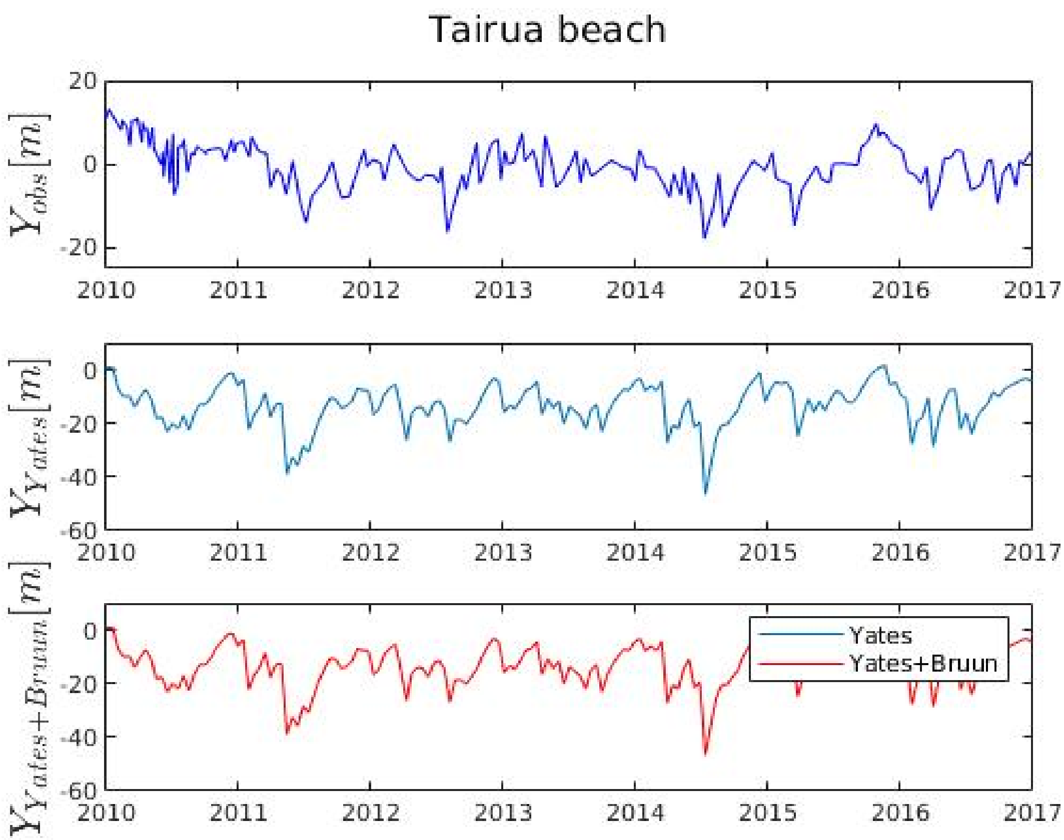

Figure 5: $tan\beta = 0.13$, $S = 3$ millimeters per year (world average 1993-2017).

What would happen if instead of Tairua we look at Miami beach $tan\beta < 0.001$ (Athanasiou et al., 2019) and we take S= 12 mm per year (Church et al., 2013) in RCP 8.5 (year 2100) ?

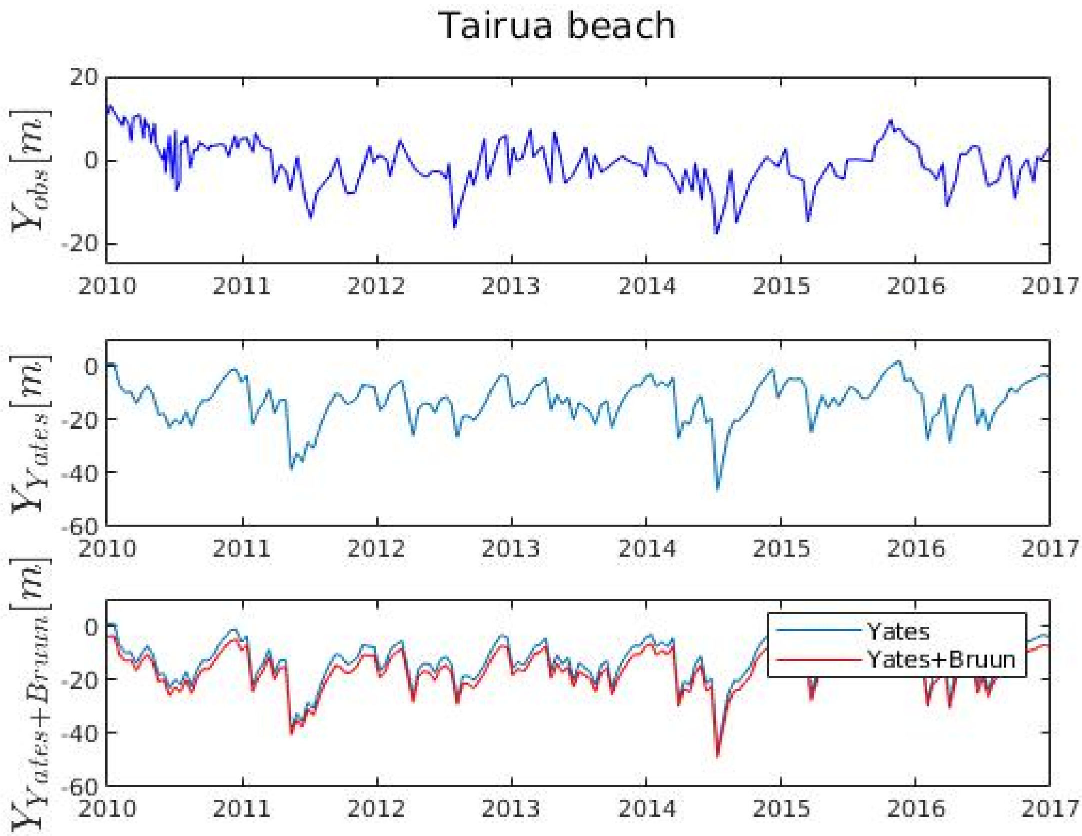

Figure 6: $tan\beta = 0.0003$, $S = 12$ millimeters per year, RCP 8.5 IPCC, world average.

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