Notes on the Yates et al. (2009) shoreline change model

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The Yates et al. (2009) model belongs to the equilibrium-type shoreline change models. These types of models assume that a beach exposed to steady wave conditions will evolve toward a unique equilibrium beach profile. Once the equilibrium is reached, no further change occurs!

In the Yates et al. (2009) model, the instantaneous shoreline position Y change rate is assumed proportional to both the wave energy E and the wave energy disequilibrium ΔE.

$$\frac{dY}{dt} = CE^{\frac{1}{2}}\Delta E \tag{1}$$

where Y is the shoreline position, t is time, C is the erosion/accretion coefficient for accretion (C^+ for $\Delta E < 0$) and erosion (C^- for $\Delta E < 0$), and ΔE is the energy disequilibrium expressed by:

$$\Delta E(Y) = E - E_{eq}(Y) \tag{2}$$

where E_{eq} is the equilibrium wave energy, which is defined as a linear function of the shoreline position Y:

$$E_{eq}(Y) = aY + b \tag{3}$$

where a and b are the slope and y intercept. For a given shoreline position Y there is an equilibrium wave energy E_{eq} that causes no change.

Modification of the Yates model

Vitousek et al. (2020) reformulated the Yates et al. (2009) model (although retaining *exactly* the same model dynamics) to improve the physicial interpretation of the model parameters a, b and C by introducing the following modified parameters :

- The equilibrium adjustment time scale $\Delta T \equiv \frac{1}{Cab^{\frac{1}{2}}}$
- The equilibrium shoreline excursion parameter $\Delta Y \equiv \frac{b}{-a}$
- The background (e.g. mean) wave height $\overline{H_s} \equiv \sqrt{b} = \sqrt{\overline{E}}$

The modified Yates et al. (2009) model becomes:

$$\frac{dY}{dt} = \frac{1}{\tau}(Y_{eq} - Y) \tag{4}$$

where Y_{eq} is obtained rearrenging equation (3):

$$Y_{eq} = \frac{E-b}{a} \Rightarrow \frac{b}{b}\frac{E-b}{a} \Rightarrow \frac{b}{a}\frac{E-b}{b} \Rightarrow -\Big(-\frac{b}{a}\Big)\frac{E-b}{b} \Rightarrow -\Delta Y\frac{H_s^2 - \overline{H_s^2}}{\overline{H_s^2}} \tag{5}$$

and

$$\tau = \frac{1}{CaE^{\frac{1}{2}}} \Rightarrow \Delta T \Big(\frac{H_s}{\overline{H_s}}\Big)^{-1} \tag{6}$$

where τ is the equilibrium time scale. Note: the instantaneous wave energy E is equivalent to the wave height H_s^2, following Long and Plant (2012).

Equilibrium experiment [1]

Vitousek et al. (2020) make an interesting remark on the equilibrium nature of the shoreline dynamics expressed in equation (4). What would happen if the shoreline could instantaneously respond to come into equilibrium with the wave energy? In other words, what would happen if the instantaneous shoreline position Y would be given by :

$$Y = Y_{eq} \tag{7}$$

Using equation (5) in equation (7) we obtain:

$$Y = -\Delta Y \frac{H_s^2 - \overline{H_s^2}}{\overline{H_s^2}} \tag{8}$$

We can then use equation (8) to predict how the shoreline would instanteneously respond to equilibrium and compare this with the observed behaviour. First, we need data to determine H_s and Y_{obs}, using the wave hindcast and the shoreline data at Tairua Beach, New Zealand for the period *1999-2016* (Figure 1), obtained at *Coast and Ocean Collective* (2020).

```
% Load wave hindcast data
load('Wave_hindcast_corrected.mat');

t=hindcast.time;
Hs=hindcast.Hs; %instantaneous wave height

% Load shoreline observations
load('Shorecast_complete.mat')

%Observed shoreline at Tairua Beach
tobs=Shore.time;
Yobs=Shore.average-nanmean(Shore.average);
```

Calculating $\overline{H_s}$ with the average of H_s, and setting the equilibrium shoreline excursion parameter $\Delta Y \approx 10$ as cited in Vitousek et al. (2020), we are ready to calculate the shoreline position Y under the assumptions in equation (8):

```
% Calculate wave height average
Hsb=nanmean(Hs);
% Specify characteristic cross-shore length
DY=10;

%Equation 8
Ypred=(Hs.^2-Hsb.^2)/Hsb^2*-DY;
```

[1] This experiment follows directly the equilibrium remarks of Vitousek et al. (2020). The original MATLAB® scripts were obtained from the *Coast and Ocean Collective* data access site `https://coastalhub.science/data`, (Vitousek et al., 2020).

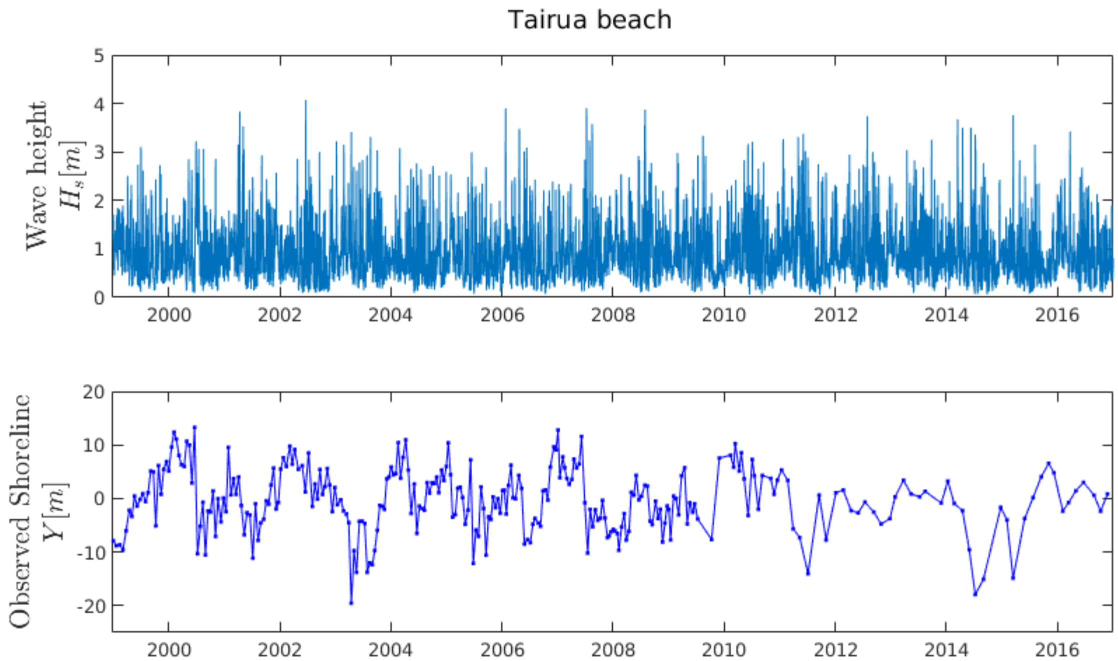

Figure 1: Data retreived from (*Coast and Ocean Collective*, 2020).

Plotting the prediction and the observed shorelines, we obtain Figure 2.

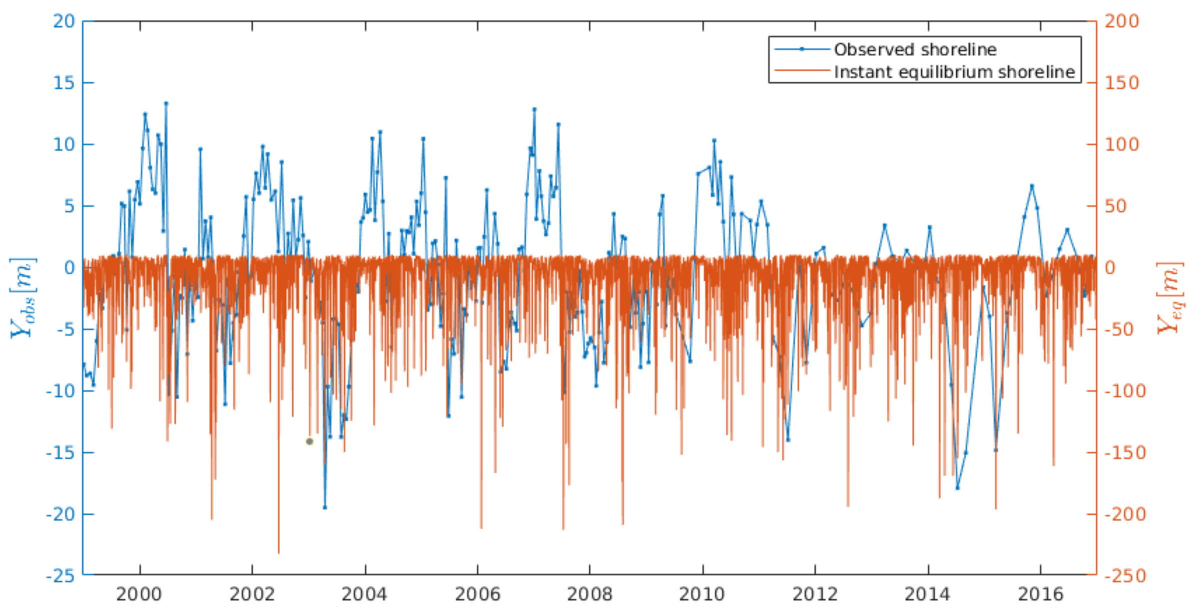

Figure 2: Observed and model behaviour of shoreline position Y.

In reality, the observed relative wave-energy anomaly is $max\left(\frac{H_s^2(t)-\overline{H_s^2}}{\overline{H_s^2}}\right) \approx 10$, yet the observed maximally eroded shoreline position is $\approx 20[m]$, which is obviously far less than the $-10\Delta Y \approx -100m$ eroded position that would be predicted by equation 8 under the assumption of instantaneous adjustment to equilibrium (see Figure 2).

What's happening? The **time scale** for equilibrium adjustment plays an exceedingly important role in the dynamics of shoreline change Vitousek et al. (2020). Although these models are called *equilibrium shoreline models*, truly reaching an *equilibrium state* associated with large wave conditions may never occur; instead, the beach perpetually exists in a transient state determined by the recent memory of large wave events that have forced the system out of its baseline state Vitousek et al. (2020).

References

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