

Pre-class Work - Lesson “Red-black trees—part II” CS110

Create a Google document to record all your answers. Be ready to paste a link to your Google document into a class poll (make sure the document's settings are such that one can assess its content from the link).

Question 1. (Exercise 13.2-1, Cormen et al.)

[Estimate: 3 minutes] Write pseudocode for RIGHT-ROTATE.

Right Rotate (T, x):

$y = x.\text{left}$

$x.\text{left} = y.\text{right}$ # make y right subtree the subtree of x 's left subtree

If $y.\text{right} \neq T.\text{nil}$:

$y.\text{right}.p = x$

$y.p = x.p$

if $x.p == T.\text{nil}$:

$T.\text{root} = y$

elif: $x == x.p.\text{right}$:

$x.p.\text{right} = y$

else:

$x.p.\text{left} = y$

$y.\text{right} = x$ # perform switch

$x.p = y$

Question 2. (Exercise 13.2-2, Cormen et al.)

[Estimate: 3 minutes] Argue that in every n -node binary search tree, there are exactly $(n-1)$ possible rotations.

Given that a rotation is essentially a switch/rotation of a subtree, we know that it is possible to make a rotation from any node to any other node (even if it is not convenient to do so). If we can make a rotation from any node to any other node, we can make $n-1$ rotations (-1 accounts for the current node, since we can't rotate it on itself).

Question 3. (Exercise 13.2-3, Cormen et al.)

[Estimate: 4 minutes] How do the depths of nodes in a BST change when a rotation is performed? Explain your answer.

The depth of the root node of a subtree is reduced by 1, as well as all its children and other nodes beneath it. On the other hand, the node that is rotated onto (put as a new child of the subtree's root) gains 1 extra depth. The reason for this is easy to understand when visualized. E.g.

Before Rotation

3

\

2

\

1

Post Rotation

2

/ \

3

1

2 loses 1 depth, and so does 1. 3 gains 1 depth.

Question 4. (Exercise 13.3-2, Cormen et al.)

[Estimate: 5 minutes] Write down or illustrate the red-black trees that result after successively inserting the keys 41; 38; 31; 12; 19; 8 into an initially empty red-black tree.

