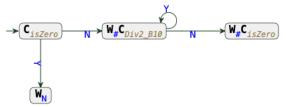
Turing Machines

#### Is Power of 2

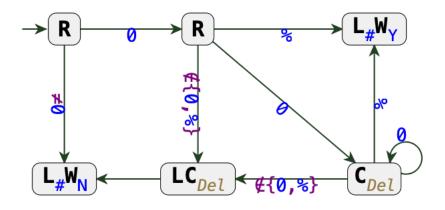
This machine starts in a configuration: #num% Where num is a sequence of digits that represents the number N. It ends in a configuration: RW Where R is Y if N is a power of 2, and R is N otherwise. We do not care what is left to the right of R



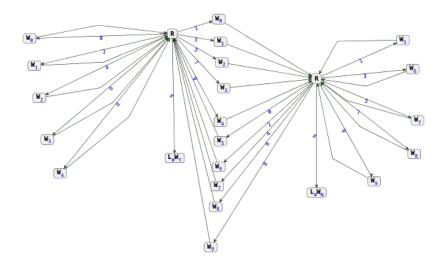
## **Auxiliary Machines**

- isZero: Verifies whether or not the machine is reading digits that represent the value 0. Additionally it removes leading 0's.
- Div\_B10: Divides by 2 in base 10. Additionally, writes Y if the number is even, N otherwise.
  - SL: Shiftleft: Shifts the string one position to the right of the head one position to the left, thus removing the firs symbol to the right. The machine must start reading a blanck. The machine ends in the same absolute position from which it startw]s.
  - Del: Delete this: Deletes the symbol being read. The machine ends in the same absolute position from which it starts. The machine cannot be reading the first cell of the tape when it starts.

## Auxiliary Machines: isZero



## Auxiliary Machines: isZero



**#**1024%

► #1024% ==> Y512%

- ► #1024% ==> Y512%
- **#**512%

- ► #1024% ==> Y512%
- ► #512% ==> Y256%

- ► #1024% ==> Y512%
- ► #512% ==> Y256%
- **256%**

- ► #1024% ==> Y512%
- ► #512% ==> Y256%
- ► #256% ==> Y128%

- ► #1024% ==> Y512%
- ► #512% ==> Y256%
- ► #256% ==> Y128%
- **#**128%

- ► #1024% ==> Y512%
- ► #512% ==> Y256%
- ► #256% ==> Y128%
- ► #128% ==> Y64%

- ► #1024% ==> Y512%
- ► #512% ==> Y256%
- ► #256% ==> Y128%
- ► #128% ==> Y64%
- **\(\begin{align\*}
  \pm 464\)\\\
  \end{align\*}**

- ► #1024% ==> Y512%
- ► #512% ==> Y256%
- ► #256% ==> Y128%
- ► #128% ==> Y64%
- ► #64% ==> Y32%

- ► #1024% ==> Y512%
- ► #512% ==> Y256%
- ► #256% ==> Y128%
- ► #128% ==> Y64%
- ► #64% ==> Y32%
- **#**32%

- ► #1024% ==> Y512%
- ► #512% ==> Y256%
- ► #256% ==> Y128%
- ► #128% ==> Y64%
- ► #64% ==> Y32%
- ► #32% ==> Y16%

- ► #1024% ==> Y512%
- ► #512% ==> Y256%
- ► #256% ==> Y128%
- ► #128% ==> Y64%
- ► #64% ==> Y32%
- ► #32% ==> Y16%
- **1** #16%

- ► #1024% ==> Y512%
- ► #512% ==> Y256%
- ► #256% ==> Y128%
- ► #128% ==> Y64%
- ► #64% ==> Y32%
- ► #32% ==> Y16%
- ► #16% ==> Y8%

- ► #1024% ==> Y512%
- ► #512% ==> Y256%
- ► #256% ==> Y128%
- ► #128% ==> Y64%
- ► #64% ==> Y32%
- ► #32% ==> Y16%
- ► #16% ==> Y8%
- **>** #8%

- ► #1024% ==> Y512%
- ► #512% ==> Y256%
- ► #256% ==> Y128%
- ► #128% ==> Y64%
- ► #64% ==> Y32%
- ► #32% ==> Y16%
- ► #16% ==> Y8%
- ▶ #8% ==> Y4%

- ► #1024% ==> Y512%
- ► #512% ==> Y256%
- ► #256% ==> Y128%
- ► #128% ==> Y64%
- ► #64% ==> Y32%
- ► #32% ==> Y16%
- ► #16% ==> Y8%
- ► #8% ==> Y4%
- **4**4%

- ► #1024% ==> Y512%
- ► #512% ==> Y256%
- ► #256% ==> Y128%
- ► #128% ==> Y64%
- ► #64% ==> Y32%
- ► #32% ==> Y16%
- ► #16% ==> Y8%
- ► #8% ==> Y4%
- ► #4% ==> Y2%

- ► #1024% ==> Y512%
- ► #512% ==> Y256%
- ► #256% ==> Y128%
- ► #128% ==> Y64%
- ► #64% ==> Y32%
- ► #32% ==> Y16%
- ► #16% ==> Y8%
- ► #8% ==> Y4%
- ► #4% ==> Y2%
- **2**%

- ► #1024% ==> Y512%
- ► #512% ==> Y256%
- ► #256% ==> Y128%
- ► #128% ==> Y64%
- ► #64% ==> Y32%
- ► #32% ==> Y16%
- ► #16% ==> Y8%
- ► #8% ==> Y4%
- ▶ #4% ==> Y2%
- ► #2% ==> Y1%

- ► #1024% ==> Y512%
- ► #512% ==> Y256%
- ► #256% ==> Y128%
- ► #128% ==> Y64%
- ► #64% ==> Y32%
- ► #32% ==> Y16%
- ► #16% ==> Y8%
- ► #8% ==> Y4%
- ▶ #4% ==> Y2%
- ► #2% ==> Y1%
- ► #1%

- ► #1024% ==> Y512%
- ► #512% ==> Y256%
- ► #256% ==> Y128%
- ► #128% ==> Y64%
- ► #64% ==> Y32%
- ► #32% ==> Y16%
- ► #16% ==> Y8%
- ► #8% ==> Y4%
- ▶ #4% ==> Y2%
- ► #2% ==> Y1%
- ► #1% ==> N0%

- ► #1024% ==> Y512%
- ► #512% ==> Y256%
- ► #256% ==> Y128%
- ► #128% ==> Y64%
- ► #64% ==> Y32%
- ► #32% ==> Y16%
- ► #16% ==> Y8%
- ► #8% ==> Y4%
- ► #4% ==> Y2%
- ▶ #2% ==> Y1%
- ► #1% ==> N0%
- ► N0%



- ► #1024% ==> Y512%
- ► #512% ==> Y256%
- ► #256% ==> Y128%
- ► #128% ==> Y64%
- ► #64% ==> Y32%
- ► #32% ==> Y16%
- ► #16% ==> Y8%
- ▶ #8% ==> Y4%
- ► #4% ==> Y2%
- ► #2% ==> Y1%
- ► #1% ==> N0%
- ► N0% ==> Y0%

- ► #1024% ==> Y512%
- ► #512% ==> Y256%
- ► #256% ==> Y128%
- ► #128% ==> Y64%
- ► #64% ==> Y32%
- ► #32% ==> Y16%
- ► #16% ==> Y8%
- ▶ #8% ==> Y4%
- ► #4% ==> Y2%
- ► #2% ==> Y1%
- ► #1% ==> N0%
- ► N0% ==> Y0%
- ▶ Since the value to the right of N was 0 then it is a power of 2!

**48%** 

► #48% ==> Y24%

- ► #48% ==> Y24%
- **424%**

- ► #48% ==> Y24%
- ► #24% ==> Y12%

- ▶ #48% ==> Y24%
- ▶ #24% ==> Y12%
- **12%**

- ► #48% ==> Y24%
- ▶ #24% ==> Y12%
- ► #12% ==> Y6%

- ► #48% ==> Y24%
- ▶ #24% ==> Y12%
- ► #12% ==> Y6%
- **\>** #6%

- ► #48% ==> Y24%
- ▶ #24% ==> Y12%
- ▶ #12% ==> Y6%
- ► #6% ==> Y3%

- ► #48% ==> Y24%
- ▶ #24% ==> Y12%
- ► #12% ==> Y6%
- ► #6% ==> Y3%
- **>** #3%

- ► #48% ==> Y24%
- ► #24% ==> Y12%
- ► #12% ==> Y6%
- ► #6% ==> Y3%
- ► #3% ==> N1%

- ► #48% ==> Y24%
- ▶ #24% ==> Y12%
- ► #12% ==> Y6%
- ► #6% ==> Y3%
- ► #3% ==> N1%
- ► N1%

- ► #48% ==> Y24%
- ► #24% ==> Y12%
- ► #12% ==> Y6%
- ► #6% ==> Y3%
- ► #3% ==> N1%
- ► N1% ==> N1%

- ► #48% ==> Y24%
- ► #24% ==> Y12%
- ► #12% ==> Y6%
- ► #6% ==> Y3%
- ► #3% ==> N1%
- ► N1% ==> N1%
- Since the value to the right of N was not 0 then it is not a power of 2!

**>** #50%

- ► #50% ==> Y25%
- **25**%

- ► #50% ==> Y25%
- ► #12% ==> N12%

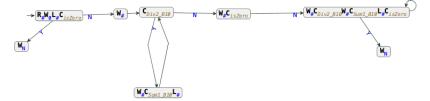
- ▶ #50% ==> Y25%
- ► #12% ==> N12%
- ► N12%

- ► #50% ==> Y25%
- ► #12% ==> N12%
- ► N12% ==> N12%

- ► #50% ==> Y25%
- ► #12% ==> N12%
- ► N12% ==> N12%
- ➤ Since the value to the right of N was not 0 then it is not a power of 2!

### Log<sub>2</sub>

This machine starts in a configuration: #num% Where num is a sequence of digits that represents the number N. It ends in a configuration:  $R_0\%log$  Where R is Y if N is a power of 2, and R is N otherwise. Additionally, log is a sequence of digits that represents:  $\lfloor log_2(N) \rfloor$ 



**#**1024%0

► #1024%0 ==> Y512%1

- ► #1024%0 ==> Y512%1
- **\$\pm\$** #512%1

- ► #1024%0 ==> Y512%1
- ► #512%1 ==> Y256%2

- ► #1024%0 ==> Y512%1
- ► #512%1 ==> Y256%2
- **2**56%2

- ► #1024%0 ==> Y512%1
- ► #512%1 ==> Y256%2
- ► #256%2 ==> Y128%3

- ► #1024%0 ==> Y512%1
- ► #512%1 ==> Y256%2
- ► #256%2 ==> Y128%3
- **#**128%3

- ► #1024%0 ==> Y512%1
- ► #512%1 ==> Y256%2
- ► #256%2 ==> Y128%3
- ► #128%3 ==> Y64%4

- ► #1024%0 ==> Y512%1
- ► #512%1 ==> Y256%2
- ► #256%2 ==> Y128%3
- ► #128%3 ==> Y64%4
- **#**64%4

- ► #1024%0 ==> Y512%1
- ► #512%1 ==> Y256%2
- ► #256%2 ==> Y128%3
- ► #128%3 ==> Y64%4
- ► #64%4 ==> Y32%5

- ► #1024%0 ==> Y512%1
- ► #512%1 ==> Y256%2
- ► #256%2 ==> Y128%3
- ► #128%3 ==> Y64%4
- ► #64%4 ==> Y32%5
- **#**32%5

- ► #1024%0 ==> Y512%1
- ► #512%1 ==> Y256%2
- ► #256%2 ==> Y128%3
- ► #128%3 ==> Y64%4
- ► #64%4 ==> Y32%5
- ► #32%5 ==> Y16%6

- ► #1024%0 ==> Y512%1
- ► #512%1 ==> Y256%2
- ► #256%2 ==> Y128%3
- ► #128%3 ==> Y64%4
- ► #64%4 ==> Y32%5
- ► #32%5 ==> Y16%6
- **1**416%6

- ► #1024%0 ==> Y512%1
- ► #512%1 ==> Y256%2
- ► #256%2 ==> Y128%3
- ► #128%3 ==> Y64%4
- ► #64%4 ==> Y32%5
- ► #32%5 ==> Y16%6
- ► #16%6 ==> Y8%7

- ► #1024%0 ==> Y512%1
- ► #512%1 ==> Y256%2
- ► #256%2 ==> Y128%3
- ► #128%3 ==> Y64%4
- ► #64%4 ==> Y32%5
- ► #32%5 ==> Y16%6
- ► #16%6 ==> Y8%7
- **▶** #8%7

- ► #1024%0 ==> Y512%1
- ► #512%1 ==> Y256%2
- ► #256%2 ==> Y128%3
- ► #128%3 ==> Y64%4
- ► #64%4 ==> Y32%5
- ► #32%5 ==> Y16%6
- ► #16%6 ==> Y8%7
- ► #8%7 ==> Y4%8

- ► #1024%0 ==> Y512%1
- ► #512%1 ==> Y256%2
- ► #256%2 ==> Y128%3
- ► #128%3 ==> Y64%4
- ► #64%4 ==> Y32%5
- ► #32%5 ==> Y16%6
- ► #16%6 ==> Y8%7
- ► #8%7 ==> Y4%8
- **44**%8

- ► #1024%0 ==> Y512%1
- ► #512%1 ==> Y256%2
- ► #256%2 ==> Y128%3
- ► #128%3 ==> Y64%4
- ► #64%4 ==> Y32%5
- ► #32%5 ==> Y16%6
- ► #16%6 ==> Y8%7
- ► #8%7 ==> Y4%8
- ► #4%8 ==> Y2%9

- ► #1024%0 ==> Y512%1
- ► #512%1 ==> Y256%2
- ► #256%2 ==> Y128%3
- ► #128%3 ==> Y64%4
- ► #64%4 ==> Y32%5
- ► #32%5 ==> Y16%6
- ► #16%6 ==> Y8%7
- ► #8%7 ==> Y4%8
- ► #4%8 ==> Y2%9
- **42%9**

- ► #1024%0 ==> Y512%1
- ► #512%1 ==> Y256%2
- ► #256%2 ==> Y128%3
- ► #128%3 ==> Y64%4
- ► #64%4 ==> Y32%5
- ► #32%5 ==> Y16%6
- ► #16%6 ==> Y8%7
- ► #8%7 ==> Y4%8
- ► #4%8 ==> Y2%9
- ► #2%9 ==> Y1%10

- ► #1024%0 ==> Y512%1
- ▶ #512%1 ==> Y256%2
- ► #256%2 ==> Y128%3
- ► #128%3 ==> Y64%4
- ► #64%4 ==> Y32%5
- ► #32%5 ==> Y16%6
- ► #16%6 ==> Y8%7
- ► #8%7 ==> Y4%8
- ► #4%8 ==> Y2%9
- ► #2%9 ==> Y1%10
- **41%10**

- ► #1024%0 ==> Y512%1
- ► #512%1 ==> Y256%2
- ► #256%2 ==> Y128%3
- ► #128%3 ==> Y64%4
- ► #64%4 ==> Y32%5
- ► #32%5 ==> Y16%6
- ► #16%6 ==> Y8%7
- ► #8%7 ==> Y4%8
- ► #4%8 ==> Y2%9
- ► #2%9 ==> Y1%10
- ► #1%10 ==> N0%10

- ► #1024%0 ==> Y512%1
- ► #512%1 ==> Y256%2
- ► #256%2 ==> Y128%3
- ► #128%3 ==> Y64%4
- ► #64%4 ==> Y32%5
- ► #32%5 ==> Y16%6
- ► #16%6 ==> Y8%7
- ► #8%7 ==> Y4%8
- ► #4%8 ==> Y2%9
- ► #2%9 ==> Y1%10
- ► #1%10 ==> N0%10
- ► N0%10

- ► #1024%0 ==> Y512%1
- ▶ #512%1 ==> Y256%2
- ► #256%2 ==> Y128%3
- ► #128%3 ==> Y64%4
- ► #64%4 ==> Y32%5
- ► #32%5 ==> Y16%6
- ► #16%6 ==> Y8%7
- ► #8%7 ==> Y4%8
- ► #4%8 ==> Y2%9
- ► #2%9 ==> Y1%10
- ► #1%10 ==> N0%10
- ► N0%10 ==> Y0%10

- ► #1024%0 ==> Y512%1
- ► #512%1 ==> Y256%2
- ► #256%2 ==> Y128%3
- ► #128%3 ==> Y64%4
- ► #64%4 ==> Y32%5
- ► #32%5 ==> Y16%6
- ► #16%6 ==> Y8%7
- ► #8%7 ==> Y4%8
- ► #4%8 ==> Y2%9
- ► #2%9 ==> Y1%10
- ► #1%10 ==> N0%10
- ► N0%10 ==> Y0%10
- $\log_2(1024) = 10!!!!$



**#**80%0

► #80%0 ==> Y40%1

- ► #80%0 ==> Y40%1
- **4**40%1

- ► #80%0 ==> Y40%1
- ► #40%1 ==> Y20%2

- ► #80%0 ==> Y40%1
- ► #40%1 ==> Y20%2
- **20%2**

- ► #80%0 ==> Y40%1
- ► #40%1 ==> Y20%2
- ► #20%2 ==> Y10%3

- ► #80%0 ==> Y40%1
- ► #40%1 ==> Y20%2
- ► #20%2 ==> Y10%3
- **10%3**

- ► #80%0 ==> Y40%1
- ► #40%1 ==> Y20%2
- ► #20%2 ==> Y10%3
- ► #10%3 ==> Y5%4

- ► #80%0 ==> Y40%1
- ► #40%1 ==> Y20%2
- ► #20%2 ==> Y10%3
- ► #10%3 ==> Y5%4
- **\*** #5%4

- ► N2%5

- ► N1%6

- ► #80%0 ==> Y40%1
- ► #40%1 ==> Y20%2
- ► #20%2 ==> Y10%3
- ► #10%3 ==> Y5%4
- ► #5%4 ==> N2%5
- ► N2%5 ==> N1%6
- ► N1%6 ==> N0%6
- ► N1%6

- ► #80%0 ==> Y40%1
- ► #40%1 ==> Y20%2
- ► #20%2 ==> Y10%3
- ► #10%3 ==> Y5%4
- ► #5%4 ==> N2%5
- ► N2%5 ==> N1%6
- ► N1%6 ==> N0%6
- ► N0%6 ==> N0%6

- ► #80%0 ==> Y40%1
- ► #40%1 ==> Y20%2
- ► #20%2 ==> Y10%3
- ► #10%3 ==> Y5%4
- ► #5%4 ==> N2%5
- ► N2%5 ==> N1%6
- ► N1%6 ==> N0%6
- ► N0%6 ==> N0%6
- ►  $log_2(80) = 6.321928$ ,  $\lfloor log_2(80) \rfloor = 6$ !!!!