- Métodos computacionales:
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- **Derivación**

  a) Incluir el código Notebook (.ipynb).

  b) Guardar la información en una carpeta llamada Semana3_Nombre1_Nombre2

  c) Comprimir en formato `zip` la carpeta para tenga el nombre final Semana3_Nombre1_Nombre2.zip

  d) **Hacer una sola entrega por grupo**.

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1 Derivation

1.1 Derivadas

1. **(20 Puntos)** Usando el método de diferencias finitas centrales estime la derivada de la función (usando $h = 0.05$):

$$f(x) = \frac{1}{\sqrt{1 + e^{-x^2}}}, \tag{1}$$

   a) En el punto $x = 0$
   b) En el intervalo $-10 \leq x \leq 10$
   c) Para el intervalo anterior, estimar el error en cada nodo y el error global de la aproximación.

2. **(20 Puntos)** Demuestre la formula alternativa para la estimación de la segunda derivada discreta:

$$\frac{d^2 f(x_j)}{dx^2} = \frac{f(x_{j+2}) - 2f(x_j) + f(x_{j-2})}{4h^2} \tag{2}$$

1.2 Derivatives, Method of images

1. **(30 Points)** A classical electrodynamics problem is the following: suppose a point charge q held a distance d above an infinite grounded conducting plane. The electric potential is given by:

$$V(x,y) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{\sqrt{(x-\vec{r_q}[0])^2+(y-d)^2}} - \frac{1}{\sqrt{(x-\vec{r_q}[0])^2+(y+d)^2}}\right] \tag{3}$$

For this numerical estimation the position of the charge is $\vec{r_q} = (0.51, 0.21)\ m$. Note that $d = \vec{r_q}[1]$.

a) Calculate the electric field in arbitrary units where $\frac{q}{4\pi\epsilon_0} = 1$.

$$\vec{E} = -\nabla V(x,y) \tag{4}$$

The derivation region is $R \in [0., 1.] \times [0., 1.]\ m^2$ and the step of discretization is $h = 0.05$.

b) Plot the vectorial field using: `ax.quiver(x[i],y[j],Ex[i,j],Ey[i,j])`
The result looks like this 1:

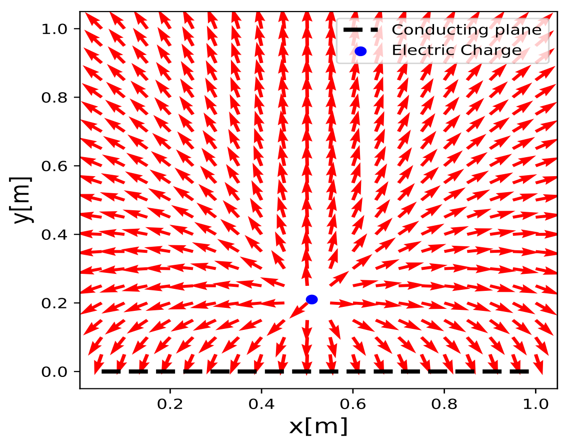

Figure 1: Electric field of an electric charge close to grounded conducting plane. Note that the Gauss' Law is satisfied because of the electric field is normal in the surface. Nice!

1.3 $D^4 f$ operator

1. **(30 Points)** Show (analytically) that the $D^4 f$ operator is given by:

$$D^4 f(x_j) \cong \frac{f(x_{j+2}) - 4f(x_{j+1}) + 6f(x_j) - 4f(x_{j-1}) + f(x_{j-2})}{h^4} \tag{5}$$

2. For this operator, what is the order ($\mathcal{O}(h^k)$) of the approximation?