Polinomio progresivo - Newton

Conocemos $n+1$ puntos $x_0, x_1, \dots x_n$ tal que $\boxed{x_i - x_{i-1} = h}$ y sus imágenes $f(x_0), f(x_1), \dots f(x_n)$.

Se propone el siguiente polinomio interpolante.

$$P(x) = a_0 + a_1(x - x_0) + a_2(x-x_0)(x-x_1)$$
$$+ a_n(x-x_0)(x-x_1)\dots(x-x_{n-1})$$

Coeficientes.

$$P(x_0) = f_0 = a_0 + a_1\cancel{(x_0 - x_0)} + \dots$$

$$f_0 = a_0$$

$$a_1(x_1 - x_0)$$

$$P(x_1) = f_1$$

$$f_1 = f_0 + a_1(x_1 - x_0) + a_2\cancel{(x_1 - x_0)(x_1 - x_1)}^{\,0}$$

$$f_1 - f_0 = a_1 h$$

$$a_1 = \frac{f_1 - f_0}{h} \quad \checkmark$$

$$P(x_2) = f_2$$

$$f_2 = f_0 + \left(\frac{f_1 - f_0}{h}\right)\overset{2h}{(x_2 - x_0)} + a_2\overset{2h}{(x_2 - x_0)}\overset{h}{(x_2 - x_1)}$$

$$f_2 = f_0 + (f_1 - f_0)2 + a_2 2h^2$$

$$\frac{f_2 - 2f_1 + f_0}{2h^2} = a_2$$

Término general: $a_i = \frac{1}{i!\,h^i}$

$\Delta f_i = f_{i+1} - f_i$ al orden n:

$$\Delta^{n+1} f_i = \Delta^n f_{i+1} - \Delta^n f_i$$

$$\Delta^2 f_i = \Delta^1 f_{i+1} - \Delta^1 f_i$$

$$= f_{i+2}^{0} - f_{i+1}^{0} - \{ f_{i+1}^{0} - f_i^{0} \}$$

$$= f_{i+2} - 2f_{i+1} + f_i$$

Conclusión:

$$a_i = \frac{\Delta^i f_0}{i!\, h^i}$$

Δ^0	Δ^1	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
39	-20	-20	24	0	0	0
19	-40	4	24	0	0	
-21	-36	28	24	0		
-57	-8	52	24			
-65	44	76				
-21	120					
99						

$$P(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1)$$
$$a_3(x-x_0)(x-x_1)(x-x_2)$$

$$a_0 = 39$$

$$a_1 = \frac{-20}{1!\,2^1} = -10$$

$$a_2 = \frac{-20}{2!\,2^2} = -5/2$$

$$a_3 = \frac{24}{3!\,2^3} = 1/2$$