2.0

$$\int_0^1\int_0^1 \frac{2}{3}(x+2y)\,\partial x\,\partial y$$

$$\frac{2}{3}\int_0^1 \left[\frac{x^2}{2} + 2xy\right]_0^1 \rightarrow \frac{1}{2} + 2y$$

$$\frac{2}{3}\int_0^1 \frac{1}{2} + 2y \rightarrow \left[\frac{y}{2} + y^2\right]_0^1 = \frac{1}{2} + 1 = \frac{3}{2}$$

$$\frac{2}{3} \cdot \frac{3}{2} = 1 \checkmark$$

Si es una funcion de densidad Conjunta Valida

a) Caso Continuo

$\rightarrow$ Fuera de este límite la función es 0

$$g(x) = \int_{-\infty}^{\infty} f(x,y)\,dy \rightarrow \int_0^1 \frac{2}{3}(x+2y)\,dy \rightarrow \frac{2}{3}\int_0^1 x + \int_0^1 2y\,dy$$

$$\frac{2}{3}\left[xy + y^2\right]_0^1 \rightarrow \frac{2}{3}(x+1) = g(x)$$

$$h(x) = \int_{-\infty}^{\infty} f(x,y)\,dx \rightarrow \int_0^1 \frac{2}{3}(x+2y)\,dx \rightarrow \frac{2}{3}\int_0^1 x + 2y\,dy$$

$$\frac{2}{3}\left[\frac{x^2}{2} + 2yx\right]_0^1 \rightarrow \frac{2}{3}\left(\frac{1}{2} + 2y\right) = h(y)$$

$$\frac{2}{3}\left(\frac{1}{2} + 2y\right) \cdot \frac{2}{3}(x+1) \rightarrow \frac{4}{6}\left(\frac{1}{2} + 2y\right)(x+1)$$

$$x + 2y \stackrel{?}{=} \frac{2}{3}\left(\frac{1}{2}x + \frac{1}{2} + 2yx + 2y\right)$$ evaluamos en $(1,1)$

$$3 = \frac{2}{3}\left(\frac{1}{2} + \frac{1}{2} + 2 + 2\right) \rightarrow 3 \neq \frac{10}{3}$$

Por contraejemplo se ve que son dependientes

b) Caso Continuo

$$\mu_x = \int_{-\infty}^{\infty} x \cdot \left(\frac{2}{3}(x+1)\right) = \frac{2}{3}\int_0^1 x^2 + x \, dx \rightarrow \left[\frac{x^3}{3} + \frac{x^2}{2}\right]_0^1$$

$$\frac{2}{3}\left(\frac{1}{3} + \frac{1}{2}\right) = \frac{2}{3} \cdot \left(\frac{5}{6}\right) = \frac{10}{18}$$

c)

$$\mu_y = \int_{-\infty}^{\infty} y \cdot \frac{2}{3}\left(\frac{1}{2} + 2y\right) dy \rightarrow \frac{2}{3}\int_0^1 \frac{y}{2} + 2y^2 \, dy$$

$$\frac{2}{3}\left[\frac{y^2}{4} + \frac{2y^3}{3}\right]_0^1 \rightarrow \frac{2}{3}\left(\frac{1}{4} + \frac{2}{3}\right) \rightarrow \frac{2}{3}\left(\frac{3+8}{12}\right) = \frac{11}{18}$$

d)

$$\int_0^1\int_0^1 xy\,\frac{2}{3}(x+2y)\,dx\,dy \rightarrow x^2y + 2y^2x \rightarrow \frac{x^3}{3}y + x^2y^2$$

$$\frac{2}{3}\int_0^1\left(\frac{y}{3} + y^2\right) dy \rightarrow \frac{2}{3}\left[\frac{y^2}{6} + \frac{y^3}{3}\right]_0^1 = \frac{2}{3}\left(\frac{1}{6} + \frac{1}{3}\right) = \frac{1}{2} \cdot \frac{2}{3}$$

$$\sigma_{xy} = \frac{1}{3} - \frac{11}{18} \cdot \frac{10}{18} = -\frac{1}{162} = -0.00617$$

e Pa despues

$$\int_0^1\int_0^1 \frac{2}{3}(x+2y)\left(x-\frac{10}{18}\right)\cdot\left(y-\frac{11}{18}\right)dx\,dy$$

$$\left(x^2-\frac{10x}{18}+2yx-\frac{20y}{18}\right)\left(y-\frac{11}{18}\right)$$

$$\frac{2}{3}\int_0^1\int_0^1 x^2y-\frac{10}{18}xy+2xy^2-\frac{20}{18}y^2-\frac{11}{18}x^2+\frac{110x}{324}-\frac{22yx}{18}+\frac{220y}{324}$$

$$\left[\frac{x^3}{3}y-\frac{10x^2}{36}y+x^2y^2-\frac{20}{18}y^2x-\frac{11}{54}x^3+\frac{110}{648}x^2-\frac{11}{18}x^2y+\frac{220}{324}xy\right]_0^1$$

$$\int_0^1 \frac{y}{3}-\frac{10}{36}y+y^2-\frac{20}{18}y^2-\frac{11}{54}+\frac{110}{648}-\frac{11}{18}y+\frac{220}{324}y$$

$$\int_0^1\left(-\frac{y^2}{9}+\frac{10}{81}y-\frac{11}{324}\right)dy=$$

$$\left[-\frac{y^3}{27}+\frac{5y^2}{81}-\frac{11y}{324}\right]_0^1$$

$$-\frac{1}{27}+\frac{5}{81}-\frac{11}{324}=-0.00925\cdot\frac{2}{3}=$$

$$-0.006172$$

1) Media $= \frac{1}{N}\sum_{i=1}^{N} X_i$

¿$Var\left(\frac{1}{N}\sum_{i=1}^{N} X_i\right)$?

Caso base $N=2$

$Var\left(\frac{X_1+X_2}{2}\right) = \left(\frac{1}{2}\right)^2 Var(X_1+X_2)$

$E[(X_1+X_2-E[X_1+X_2])^2]$

$E[((X_1-E(X_1))^2+(X_2-E[X_2]))^2+2$

$(X_1-E[X_1])(X_2-E(X_2))]$

$Var(X_1)+Var(X_2)+2\,cov(X_1,X_2)$

$Var\left(\frac{X_1+X_2}{2}\right) = Var(X_1)+V(X_2)+2\,Cov(X_1,X_2)$

Caso N

$Var\left(\frac{1}{N}\sum_{i=1}^{N} X_i\right) = \left(\frac{1}{N}\right)^2 Var\left(\sum_{i=1}^{N} X_i\right)$

$E[(\sum_{i=1}^{N} X_i - E[\sum_{i=1}^{N} X_i])^2]$

$E[\sum_{i=1}^{N}(X_i - E(X_i))+2\left(\sum_{i=1}^{N}\sum_{j=1}^{N}(X_i-E[X_i])\cdot\right.$
$(X_j-E[X_j])$

$$\frac{1}{N^2}\left(\sum_{i=1}^{N} Var(X_i) + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N} Cov(X_i, X_j)\right)$$

$$Vor\left(\frac{1}{N}\cdot\sum_{i=1}^{N} X_i\right) = \frac{1}{N^2}\cdot\sum_{i=1}^{N} Var(X_i) + \frac{2}{N^2}$$

$$\sum_{i=1}^{N}\sum_{j=i+1}^{N} Cov(X_i, X_j)$$