Métodos Computacionales 1 - Tarea Semana 3

1.3 D^4f operator

1.

Según la definición de la segunda derivada central de las Notas de Clase:

$$f^2(x_j) = \frac{f(x_{j+1}) - 2f(x_j) + f(x_{j-1})}{h^2} \qquad (1.15)$$

Del mismo modo:

$$f^4(x_j) = \frac{f^2(x_{j+1}) - 2f^2(x_j) + f^2(x_{j-1})}{h^2} \qquad (2)$$

De nuevo usando segunda derivada:

$$f^2(x_{j+1}) = \frac{f(x_{j+2}) - 2f(x_{j+1}) + f(x_j)}{h^2} \qquad (3)$$

$$f^2(x_j) = \frac{f(x_{j+1}) - 2f(x_j) + f(x_{j-1})}{h^2} \qquad (4)$$

$$f^2(x_{j-1}) = \frac{f(x_j) - 2f(x_{j-1}) + f(x_{j-2})}{h^2} \qquad (5)$$

Reemplazando (3) (4) y (5) en (2):

$$f^4(x_j) = \frac{\frac{f(x_{j+2}) - 2f(x_{j+1}) + f(x_j)}{h^2} - 2\frac{f(x_{j+1}) - 2f(x_j) + f(x_{j-1})}{h^2} + \frac{f(x_j) - 2f(x_{j-1}) + f(x_{j-2})}{h^2}}{h^2}$$

$$f^4(x_j) = \frac{f(x_{j+2}) - 2f(x_{j+1}) + f(x_j) - 2f(x_{j+1}) + 4f(x_j) - 2f(x_{j-1}) + f(x_j) - 2f(x_{j-1}) + f(x_{j-2})}{h^4}$$

$$f^4(x_j) = \frac{f(x_{j+2}) - 4f(x_{j+1}) + 6f(x_j) - 4f(x_{j-1}) + f(x_{j-2})}{h^4}$$

2.

El orden de la aproximación es $O(h^2)$