density function

Canonical name	DensityFunction
Date of creation	2013-03-22 13:02:49
Last modified on	2013-03-22 13:02:49
Owner	drini (3)
Last modified by	drini (3)
Numerical id	12
Author	drini (3)
Entry type	Definition
Classification	msc 60E05
Synonym	probability function
Synonym	density
Synonym	probabilities function
Related topic	DistributionFunction
Related topic	CumulativeDistributionFunction
Related topic	RandomVariable
Related topic	Distribution
Related topic	GeometricDistribution2

Let X be a discrete random variable with sample space $\{x_1, x_2, \ldots\}$. Let p_k be the probability of X taking the value x_k.

The function

$$f(x) = \begin{cases} p_k & \text{if } x = x_k \\ 0 & \text{otherwise} \end{cases}$$

is called the *probability function* or *density function*.

It must hold:

$$\sum_{j=1}^{\infty} f(x_j) = 1$$

If the density function for a random variable is known, we can calculate the probability of X being on certain interval:

$$P[a < X \leq b] = \sum_{a<x_j\leq b} f(x_j) = \sum_{a<x_j\leq b} p_j.$$

The definition can be extended to continuous random variables in a direct way: The probability of x being on a given interval is calculated with an integral instead of using a summation:

$$P[a < X \leq b] = \int_a^b f(x)dx.$$

For a more formal approach using measure theory, look at probability distribution function entry.