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predictable process

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A predictable process is a real-valued stochastic process whose values are known, in a sense, just in advance of time. Predictable processes are also called *previsible*.

1 predictable processes in discrete time

Suppose we have a `http://planetmath.org/FiltrationOfSigmaAlgebras`filtration $(\mathcal{F}_n)_{n\in\mathbb{Z}_+}$ on a measurable space $(\Omega,\mathcal{F})$. Then a stochastic process X_n is predictable if X_n is $\mathcal{F}_{n-1}$-`http://planetmath.org/MeasurableFunctions`measurable for every $n\geq 1$ and X_0 is $\mathcal{F}_0$-measurable. So, the value of X_n is known at the previous time step. Compare with the definition of adapted processes for which X_n is $\mathcal{F}_n$-measurable.

2 predictable processes in continuous time

In continuous time, the definition of predictable processes is a little more subtle. Given a filtration $(\mathcal{F}_t)$ with time index t ranging over the non-negative real numbers, the class of predictable processes forms the smallest set of real valued stochastic processes containing all left-continuous $\mathcal{F}_t$-adapted processes and which is closed under taking limits of a sequence of processes.

Equivalently, a real-valued stochastic process

$$\begin{aligned} X\colon \mathbb{R}_+\times\Omega &\to \mathbb{R}\\ (t,\omega) &\mapsto X_t(\omega) \end{aligned}$$

is predictable if it is measurable with respect to the predictable sigma algebra $\wp$. This is defined as the smallest σ-algebra on $\mathbb{R}_+\times\Omega$ making all left-continuous and adapted processes measurable.

Alternatively, $\wp$ is generated by either of the following collections of subsets of $\mathbb{R}_+\times\Omega$

$$\begin{aligned} \wp &= \sigma\left(\{(t,\infty)\times A : t\geq 0, A\in\mathcal{F}_t\}\cup\{\{0\}\times A : A\in\mathcal{F}_0\}\right)\\ &= \sigma\left(\{(T,\infty) : T \text{ is a stopping time}\}\cup\{\{0\}\times A : A\in\mathcal{F}_0\}\right)\\ &= \sigma\left(\{[T,\infty) : T \text{ is a predictable stopping time}\}\right) \end{aligned}$$

Note that in these definitions, the sets (T,∞) and $[T,\infty)$ are stochastic intervals, and subsets of $\mathbb{R}_+\times\Omega$.

3 general predictable processes

The definition of predictable process given above can be extended to a filtration $(\mathcal{F}_t)$ with time index t lying in an arbitrary subset $\mathbb{T}$ of the extended real numbers. In this case, the predictable sets form a σ-algebra on $\mathbb{T}\times\Omega$. If $\mathbb{T}$ has a minimum element t_0 then let S be the collection of sets of the form $\{t_0\}\times A$ for $A\in\mathcal{F}_{t_0}$, otherwise let S be the empty set.Then, the predictable σ-algebra is defined by

$$\begin{aligned}\wp &= \sigma\left(\{(t,\infty]\times A : t\in\mathbb{T}, A\in\mathcal{F}_t\}\cup S\right)\\ &= \sigma\left(\{(T,\infty] : T\colon\Omega\to\mathbb{T} \text{ is a stopping time}\}\cup S\right).\end{aligned}$$

Here, $(t,\infty]$ and $(T,\infty]$ are understood to be intervals containing only times in the index set $\mathbb{T}$. If $\mathbb{T}$ is an interval of the real numbers then $\wp$ can be equivalently defined as the σ-algebra generated by the class of left-continuous and adapted processes with time index ranging over $\mathbb{T}$.

A stochastic process $X\colon\mathbb{T}\times\Omega\to\mathbb{R}$ is predictable if it is $\wp$-measurable. It can be verified that in the cases where $\mathbb{T}=\mathbb{Z}_+$ or $\mathbb{T}=\mathbb{R}_+$ then this definition agrees with the ones given above.