symmetric random variable

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Defines	symmetric distribution function

Let $(\Omega, \mathcal{F}, P)$ be a probability space and X a real random variable defined on Ω. X is said to be *symmetric* if $-X$ has the same distribution function as X. A distribution function $F : \mathbb{R} \to [0, 1]$ is said to be *symmetric* if it is the distribution function of a symmetric random variable.

Remark. By definition, if a random variable X is symmetric, then $E[X]$ exists ($< \infty$). Furthermore, $E[X] = E[-X] = -E[X]$, so that $E[X] = 0$. Furthermore, let F be the distribution function of X. If F is continuous at $x \in \mathbb{R}$, then

$$F(-x) = P(X \leq -x) = P(-X \leq -x) = P(X \geq x) = 1-P(X \leq x) = 1-F(x),$$

so that $F(x) + F(-x) = 1$. This also shows that if X has a density function $f(x)$, then $f(x) = f(-x)$.

There are many examples of symmetric random variables, and the most common one being the normal random variables centered at 0. For any random variable X, then the difference ΔX of two independent random variables, identically distributed as X is symmetric.