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recurrence in a Markov chain

Canonical name	RecurrenceInAMarkovChain
Date of creation	2013-03-22 16:24:43
Last modified on	2013-03-22 16:24:43
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	5
Author	CWoo (3771)
Entry type	Definition
Classification	msc 60J10
Synonym	null recurrent
Synonym	positive recurrent
Synonym	strongly ergodic
Synonym	weakly ergodic
Defines	recurrent state
Defines	persistent state
Defines	transient state
Defines	null state
Defines	positive state
Defines	ergodic state

Let $\{X_n\}$ be a http://planetmath.org/StationaryProcessstationary Markov chain and I the state space. Given $i, j \in I$ and any non-negative integer n, define a number F_{ij}^n as follows:

$$F_{ij}^n := \begin{cases} 0 & \text{if } n = 0, \\ P(X_n = j \text{ and } X_m \neq j \text{ for } 0 < m < n \mid X_0 = i) & \text{otherwise.} \end{cases}$$

In other words, F_{ij}^n is the probability that the process *first* reaches state j at time n from state i at time 0.

From the definition of F_{ij}^n, we see that the probability of the process reaching state j *within and including* time n from state i at time 0 is given by

$$\sum_{m=0}^{n} F_{ij}^m.$$

As $n \to \infty$, we have the limiting probability of the process reaching j *eventually* from the initial state of i at 0, which we denote by F_{ij}:

$$F_{ij} := \sum_{m=0}^{\infty} F_{ij}^m.$$

Definitions. A state $i \in I$ is said to be *recurrent* or *persistent* if $F_{ii} = 1$, and *transient* otherwise.

Given a recurrent state i, we can further classify it according to "how soon" the state i returns after its initial appearance. Given F_{ii}^n, we can calculate the expected number of steps or transitions required to *return* to state i by time n. This expectation is given by

$$\sum_{m=0}^{n} mF_{ii}^m.$$

When $n \to \infty$, the above expression may or may not approach a limit. It is the expected number of transitions needed to return to state i *at all* from the beginning. We denote this figure by μ_i:

$$\mu_i := \sum_{m=0}^{\infty} mF_{ii}^m.$$

Definitions. A recurrent state $i \in I$ is said to be or *strongly ergodic* if $\mu_i < \infty$, otherwise it is called *null* or *weakly ergodic*. If a stronly ergodic state is in addition http://planetmath.org/PeriodicityOfAMarkovChainaperiodic, then it is said to be an *ergodic state*.