stopped process

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Synonym	optional stopping
Defines	pre-stopped process
Defines	prestopped process

A stochastic process $(X_t)_{t\in\mathbb{T}}$ defined on a measurable space $(\Omega,\mathcal{F})$ can be stopped at a random time $\tau\colon\Omega\to\mathbb{T}\cup\{\infty\}$. The resulting stopped process is denoted by X^τ,

$$X^\tau_t\equiv X_{\min(t,\tau)}.$$

The random time τ used is typically a stopping time.

If the process X_t has http://planetmath.org/CadlagProcessleft limits for every $t\in\mathbb{T}$, then it can alternatively be stopped just before the time τ, resulting in the pre-stopped process

$$X^{\tau-}\equiv\begin{cases}X_t, & \text{if } t<\tau,\\ X_{\tau-}, & \text{if } t\geq\tau.\end{cases}$$

Stopping is often used to enforce boundedness or integrability constraints on a process. For example, if B is a Brownian motion and τ is the first time at which $|B_\tau|$ hits some given positive value, then the stopped process B^τ will be a continuous and bounded martingale. It can be shown that many properties of stochastic processes, such as the martingale property, are stable under stopping at any stopping time τ. On the other hand, a pre-stopped martingale need not be a martingale.

For continuous processes, stopping and pre-stopping are equivalent procedures. If τ is the first time at which $|X_\tau|\geq K$, for any given real number K, then the pre-stopped process $X^{\tau-}$ will be uniformly bounded. However, for some noncontinuous processes it is not possible to find a stopping time $\tau>0$ making X^τ into a uniformly bounded process. For example, this is the case for any http://planetmath.org/LevyProcessLevy process with unbounded jump distribution.

Stopping is used to generalize properties of stochastic processes to obtain the related localized property. See, for example, local martingales.