exchangeable random variables

Canonical name	ExchangeableRandomVariables
Date of creation	2013-03-22 16:25:53
Last modified on	2013-03-22 16:25:53
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	5
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Entry type	Definition
Classification	msc 60G09
Synonym	exchangeable stochastic process
Defines	exchangeable
Defines	exchangeable process

A finite set of random variables $\{X_1, \dots, X_n\}$ defined on a common probablility space $(\Omega, \mathcal{F}, P)$ is said to be *exchangeable* if

$$P\big((X_1 \in B_1) \cap \cdots \cap (X_n \in B_n)\big) = P\big((X_{\sigma(1)} \in B_1) \cap \cdots \cap (X_{\sigma(n)} \in B_n)\big)$$

for every set of Borel sets $\{B_1, \dots, B_n\}$, and every permutation $\sigma \in S_n$. In other words, $X_1, \dots, X_n$ are exchangeable if their joint probability distribution function is the same regardless of their order.

A stochastic process $\{X_i\}$ is said to be *exchangeable* if every finite subset of $\{X_i\}$ is exchangeable.

Remarks

- If $S = \{X_1, \dots, X_n\}$ is exchangeable, then every subset of S is exchangeable (by picking suitable B_i and σ). In particular, all X_i are identically distributed, for

  $$P(X_i \in B) = P\big((X_i \in B) \cap (X_j \in \mathbb{R})\big) = P\big((X_j \in B) \cap (X_i \in \mathbb{R})\big) = P(X_j \in B).$$

- If $S = \{X_1, \dots, X_n\}$ is iid, then S is exchangeable, since the joint distribution of X_i is the product of the distributions of X_i:

  $$P\big((X_1 \in B_1) \cap \cdots \cap (X_n \in B_n)\big) = P(X_{\sigma(1)} \in B_1)\big) \cdots P(X_{\sigma(n)} \in B_n)\big).$$