associativity of stochastic integration

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The chain rule for expressing the derivative of a variable z with respect to x in terms of a third variable y is

$$\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx}.$$

Equivalently, if $dy = \alpha\,dx$ and $dz = \beta\,dy$ then $dz = \beta\alpha\,dx$. The following theorem shows that the stochastic integral satisfies a generalization of this.

Theorem. *Let X be a semimartingale and α be an X-integrable process. Setting $Y = \int \alpha\,dX$ then Y is a semimartingale. Furthermore, a predictable process β is Y-integrable if and only if $\beta\alpha$ is X-integrable, in which case*

$$\int \beta\,dY = \int \beta\alpha\,dX. \tag{1}$$

Note that expressed in alternative notation, (**??**) becomes

$$\beta \cdot (\alpha \cdot X) = (\beta\alpha) \cdot X$$

or, in differential notional,

$$\beta(\alpha\,dX) = (\beta\alpha)\,dX.$$

That is, stochastic integration is associative.