independent

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In a probability space, we say that the random events $A_1, \ldots, A_n$ are *independent* if

$$P(A_{i_1} \cap A_{i_2} \cap \cdots \cap A_{i_k}) = P(A_{i_1}) \ldots P(A_{i_k})$$

for all $i_1, \ldots, i_k$ such that $1 \leq i_1 < i_2 < \cdots < i_k \leq n$.

An arbitrary family of random events is independent if every finite subfamily is independent.

The random variables $X_1, \ldots, X_n$ are independent if, given any Borel sets $B_1, \ldots, B_n$, the random events $[X_1 \in B_1], \ldots, [X_n \in B_n]$ are independent. This is equivalent to saying that

$$F_{X_1,\ldots,X_n} = F_{X_1} \ldots F_{X_n}$$

where $F_{X_1}, \ldots, F_{X_n}$ are the distribution functions of $X_1, \ldots, X_n$, respectively, and $F_{X_1,\ldots,X_n}$ is the joint distribution function. When the density functions $f_{X_1}, \ldots, f_{X_n}$ and $f_{X_1,\ldots,X_n}$ exist, an equivalent condition for independence is that

$$f_{X_1,\ldots,X_n} = f_{X_1} \ldots f_{X_n}.$$

An arbitrary family of random variables is independent if every finite subfamily is independent.