Poisson process

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Entry type	Definition
Classification	msc 60G51
Synonym	homogeneous Poisson process
Defines	simple Poisson process
Defines	intensity

A counting process $\{X(t) \mid t \in \mathbb{R}^+ \cup \{0\}\}$ is called a *simple Poisson*, or simply a *Poisson process* with parameter λ, also known as the *intensity*, if

1. $X(0) = 0$,
2. $\{X(t)\}$ has stationary independent increments,
3. $P(X(t) = 1) = \lambda t + o(t)$,
4. $P(X(t) > 1) = o(t)$,

where $o(t)$ is the O notation.

Remarks.

- The intensity λ is assumed to be a constant in terms of t.
- Condition 3 above says that the *rate* in which the an event occurs once in time interval t, as t approaches 0, is λ. Condition 4 says that the event occurs more than once is very unlikely (the rate approaches zero as the time interval shrinks to zero).
- It can be shown that $X(t)$ has a Poisson distribution (hence the name of the stochastic process) with parameter λt:

$$P(X(t) = n) = e^{-\lambda t} \frac{(\lambda t)^n}{n!}.$$

- Therefore, $\mathrm{E}[X(t)] = \lambda t$.