Kolmogorov's inequality

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Let $X_1, \ldots, X_n$ be independent random variables in a probability space, such that $\mathrm{E}[X_k] = 0$ and $\mathrm{Var}[X_k] < \infty$ for $k = 1, \ldots, n$. Then, for each $\lambda > 0$,

$$P\left(\max_{1 \leq k \leq n} |S_k| \geq \lambda\right) \leq \frac{1}{\lambda^2} \mathrm{Var}[S_n] = \frac{1}{\lambda^2} \sum_{k=1}^{n} \mathrm{Var}[X_k],$$

where $S_k = X_1 + \cdots + X_k$.