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conditional probability

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Let $(\Omega, \mathfrak{B}, \mu)$ be a probability space, and let $X, Y \in \mathfrak{B}$ be events. The *conditional probability* of X given Y is defined as

$$\mu(X|Y) = \frac{\mu(X \cap Y)}{\mu(Y)} \tag{1}$$

provided $\mu(Y) > 0$. (If $\mu(Y) = 0$, then $\mu(X|Y)$ is not defined.)

If $\mu(X) > 0$ and $\mu(Y) > 0$, then

$$\mu(X|Y)\mu(Y) = \mu(X \cap Y) = \mu(Y|X)\mu(X), \tag{2}$$

and so also

$$\mu(X|Y) = \frac{\mu(Y|X)\mu(X)}{\mu(Y)}, \tag{3}$$

which is Bayes' Theorem.

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