Levy martingale characterization

Canonical name	LevyMartingaleCharacterization
Date of creation	2013-03-22 15:12:48
Last modified on	2013-03-22 15:12:48
Owner	skubeedooo (5401)
Last modified by	skubeedooo (5401)
Numerical id	5
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Entry type	Theorem
Classification	msc 60J65

Theorem (Levy's martingale characterisation)**.** *Let* $W(t), t \geq 0$*, be a stochastic process and let* $\mathcal{F}_t = \sigma(W_s, s \leq t)$ *be the filtration generated by it. Then* $W(t)$ *is a Wiener process if and only if the following conditions hold:*

1. $W(0) = 0$ *almost surely;*
2. *The sample paths* $t \mapsto W(t)$ *are continuous almost surely;*
3. $W(t)$ *is a martingale with respect to the filtration* $\mathcal{F}_t$*;*
4. $|W(t)|^2 - t$ *is a martingale with respect to* $\mathcal{F}_t$*.*