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conditional independence

Canonical name	ConditionalIndependence
Date of creation	2013-03-22 16:25:09
Last modified on	2013-03-22 16:25:09
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	4
Author	CWoo (3771)
Entry type	Definition
Classification	msc 60A05
Defines	conditionally independent

Let $(\Omega, \mathcal{F}, P)$ be a probability space.

Conditional Independence Given an Event

Given an event $C \in \mathcal{F}$:

1. Two events A and B in $\mathcal{F}$ are said to be *conditionally independent given* C if we have the following equality of conditional probabilities:

$$P(A \cap B|C) = P(A|C)P(B|C).$$

2. Two sub sigma algebras $\mathcal{F}_1, \mathcal{F}_2$ of $\mathcal{F}$ are *conditionally independent given* C if any two events $A \in \mathcal{F}_1$ and $B \in \mathcal{F}_2$ are conditionally independent given C.

3. Two real random variables $X, Y : \Omega \to \mathbb{R}$ are *conditionally independent given event* C if $\mathcal{F}_X$ and $\mathcal{F}_Y$, the sub `http://planetmath.org/MathcalFMeasurableFunctio` algebras generated by X and Y are conditionally independent given C.

Conditional Independence Given a Sigma Algebra

Given a sub sigma algebra $\mathcal{G}$ of $\mathcal{F}$:

1. Two events A and B in $\mathcal{F}$ are said to be *conditionally independent given* $\mathcal{G}$ if we have the following equality of `http://planetmath.org/ProbabilityConditioningOn` probabilities (as random variables):

$$P(A \cap B|\mathcal{G}) = P(A|\mathcal{G})P(B|\mathcal{G}).$$

2. Two sub sigma algebras $\mathcal{F}_1, \mathcal{F}_2$ of $\mathcal{F}$ are *conditionally independent given* $\mathcal{G}$ if any two events $A \in \mathcal{F}_1$ and $B \in \mathcal{F}_2$ are conditionally independent given $\mathcal{G}$.

3. Two real random variables $X, Y : \Omega \to \mathbb{R}$ are *conditionally independent given event* $\mathcal{G}$ if $\mathcal{F}_X$ and $\mathcal{F}_Y$, the sub sigma algebras generated by X and Y are conditionally independent given $\mathcal{G}$.

4. Finally, we can define *conditional idependence given a random variable*, say $Z : \Omega \to \mathbb{R}$ in each of the above three items by setting $\mathcal{G} = \mathcal{F}_Z$.