proof of mean square convergence of the sample mean of a stationary process

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$$n\,\mathrm{var}(\bar{X}_n) = \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\mathrm{cov}(X_i, X_j) = \sum_{|h|<n}(1-\frac{|h|}{n})\gamma(h) \leq \sum_{|h|<n}|\gamma(h)|$$

If $\gamma(n) \to 0$ as $n \to \infty$ then $\lim_{n\to\infty}\frac{1}{n}\sum_{|h|<n}|\gamma(h)| = 2\lim_{n\to\infty}|\gamma(n)| = 0$, whence $\mathrm{var}[\bar{X}_n] \to 0$.
If $\sum_{h=-\infty}^{\infty}|\gamma(h)| < \infty$ then the dominated Convergence theorem gives

$$\lim_{n\to\infty}\sum_{|h|<n}(1-\frac{|h|}{n})\gamma(h) = \sum_{h=-\infty}^{\infty}\gamma(h)$$

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