Kolmogorov's extension theorem

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For all $t_1, \cdots, t_k,\ k \in \mathbb{N}$, let $v_{t_1,\cdots,t_k}$ be probability measures on $\mathbb{R}^{nk}$ satisfying the following properties (consistency conditions):

1. $v_{t_{\sigma(1)},\cdots,t_{\sigma(k)}}(F_1 \times \cdots \times F_k) = v_{t_1,\cdots t_k}(F_{\sigma^{-1}(1)} \times \cdots F_{\sigma^{-1}(k)})$ for all permutations σ of $\{1, 2, \cdots, k\}$ and for all Borel sets F_i of $\mathbb{R}^n$

2. $v_{t_1,\cdots,t_k}(F_1 \times \cdots \times F_k) = v_{t_1,\cdots,t_k,t_{k+1},\cdots t_{k+m}}(F_1 \times \cdots \times F_k \times \mathbb{R}^n \times \cdots \times \mathbb{R}^n)$ for all $m \in \mathbb{N}$ and for all Borel sets F_i of $\mathbb{R}^n$

Then there exists a probability space $(\Omega, \mathcal{F}, P)$ and a stochastic process X_t on Ω, indexed by T, taking values in $\mathbb{R}^n$ such that

$$v_{t_1,\cdots,t_k}(F_1 \times \cdots \times F_k) = P(X_{t_1} \in F_1, \cdots, X_{t_k} \in F_k)$$

for all $t_i \in T, k \in \mathbb{R}^n$ and all Borel sets F_i of $\mathbb{R}^n$