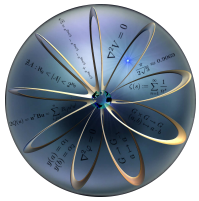

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gale

Canonical name	Gale
Date of creation	2013-03-22 16:43:37
Last modified on	2013-03-22 16:43:37
Owner	skubeedooo (5401)
Last modified by	skubeedooo (5401)
Numerical id	5
Author	skubeedooo (5401)
Entry type	Definition
Classification	msc 60G46
Classification	msc 60G44
Classification	msc 60G42
Defines	supergale
Defines	gale
Defines	supermartingale
Defines	succeed
Defines	succeed strongly
Defines	success set
Defines	strong success set

Let ν be a probability measure on Cantor space $\mathbf{C}$, and let $s \in [0, \infty)$.

1. A *ν-s-supergale* is a function $d : \{0,1\}^* \to [0, \infty)$ that satisfies the condition
$$d(w)\nu(w)^s \geq d(w0)\nu(w0)^s + d(w1)\nu(w1)^s \tag{1}$$
for all $w \in \{0,1\}^*$, the set of all finite strings of 0's and 1's (including e, the empty string).

2. A *ν-s-gale* is a *ν-s-supergale* that satisfies the condition with equality for all $w \in \{0,1\}^*$.

3. A *ν-supermartingale* is a ν-1-supergale.

4. A *ν-martingale* is a ν-1-gale.

5. An *s-supergale* is a μ-s-supergale, where μ is the uniform probability measure.

6. An *s-gale* is a μ-s-gale.

7. A *supermartingale* is a 1-supergale.

8. A *martingale* is a 1-gale.

Put in another way, a martingale is a function $d : \{0,1\}^* \to [0, \infty)$ such that, for all $w \in \{0,1\}^*$, $d(w) = (d(w0) + d(w1))/2$.

Let d be a *ν-s-supergale*, where ν is a probability measure on $\mathbf{C}$ and $s \in [0, \infty)$. We say that d *succeeds* on a sequence $S \in \mathbf{C}$ if

$$\limsup_{n \to \infty} d(S[0..n-1]) = \infty.$$

The *success set* of d is $S^\infty[d] = \{S \in \mathbf{C} \big| d \text{ succeeds on } S\}$. d succeeds on a language $A \subseteq \{0,1\}^*$ if d succeeds on the characteristic sequence χ_A of A. We say that d *succeeds strongly* on a sequence $S \in \mathbf{C}$ if

$$\liminf_{n \to \infty} d(S[0..n-1]) = \infty.$$

The *strong success set* of d is $S^\infty_{\mathrm{str}}[d] = \{S \in \mathbf{C} \big| d \text{ succeeds strongly on } S\}$.

Intuitively, a supergale d is a betting strategy that bets on the next bit of a sequence when the previous bits are known. s is the parameter that tunes

the fairness of the betting. The smaller s is, the less fair the betting is. If d succeeds on a sequence, then the bonus we can get from applying d as the betting strategy on the sequence is unbounded. If d succeeds strongly on a sequence, then the bonus goes to infinity.