Paul Lévy continuity theorem

Canonical name	PaulLevyContinuityTheorem
Date of creation	2013-03-22 13:14:31
Last modified on	2013-03-22 13:14:31
Owner	Koro (127)
Last modified by	Koro (127)
Numerical id	7
Author	Koro (127)
Entry type	Theorem
Classification	msc 60E10

Let $F_1, F_2, \ldots$ be distribution functions with characteristic functions $\varphi_1, \varphi_2, \ldots$, respectively. If φ_n converges pointwise to a limit φ, and if $\varphi(t)$ is continuous at $t = 0$, then there exists a distribution function F such that $F_n \to F$ `http://planetmath.org/ConvergenceInDistribution`weakly, and the characteristic function associated to F is φ.

Remark. The reciprocal of this theorem is a corollary to the Helly-Bray theorem; hence $F_n \to F$ weakly if and only if $\varphi_n \to \varphi$ pointwise; but this theorem says something stronger than the sufficiency of that : it says that the limit of a sequence of characteristic functions is a characteristic function whenever it is continuous at 0.