Example of stochastic matrix of mapping

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Owner	rspuzio (6075)
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Author	rspuzio (1000683)
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In order to understand the notion of stochastic matrix associated to a mapping and its dual, we will work through a simple example. Let $X = \{a, b, c\}$ and let $Y = \{d, e\}$, and define the mapping $f\colon X \to Y$ as follows:

$$
\begin{aligned}
f(a) &= d \\
f(b) &= d \\
f(c) &= e
\end{aligned}
$$

Then $\mathcal{V}X$ is a 3-dimensional real vector space with basis

$$\delta_a = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \qquad \delta_b = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \qquad \delta_c = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

and $\mathcal{V}Y$ is a 3-dimensional real vector space with basis

$$\delta_c = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \delta_d = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

and

$$\mathcal{V}f = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

To form the dual, we first renormalize the rows to sum to unity, then transpose:

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{ren} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{*} \begin{pmatrix} \frac{1}{2} & 0 \\ \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}$$

Next, to illustrate inclusions, we shall examine the map $i\colon Y \hookrightarrow X$ defined as follows:

$$
\begin{aligned}
f(d) = &\qquad a \\
f(e) = &\qquad b
\end{aligned}
$$

Following the same procedures as above, for this map we find that

$$\mathcal{V}i = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

and

$$(\mathcal{V}i)^\natural = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$