Boole inequality, proof of

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Let $\{B_1, B_2, \cdots\}$ be a sequence defined by:

$$B_i = A_i \setminus \bigcup_{k=1}^{i-1} A_k$$

Clearly $B_i \in \mathcal{F}, \forall i \in \mathbb{N}$, since $\mathcal{F}$ is σ-algebra, they are a disjoint family and :

$$\bigcup_{n=1}^{i} A_n = \bigcup_{n=1}^{i} B_n, \forall i \in \mathbb{N}$$

and since P is a measure over $\mathcal{F}$ it follows that :

$$P(\bigcup_{n=1}^{i} B_n) = \sum_{n=1}^{i} P(B_n), \forall i \in \mathbb{N}$$

Clearly $B_i \subset A_i$, then $P(B_i) \leq P(A_i)$ because measures are `http://planetmath.org/node/446` then it follows that :

$$P(\bigcup_{n=1}^{i} B_n) \leq \sum_{n=1}^{i} P(A_n), \forall i \in \mathbb{N}$$

finally taking $n \to \infty$:

$$P(\bigcup_{n=1}^{\infty} A_n) = P(\bigcup_{n=1}^{\infty} B_n) \leq \sum_{n=1}^{\infty} P(A_n)$$

the latter is valid because the measure continuity , and is the proof of the theorem