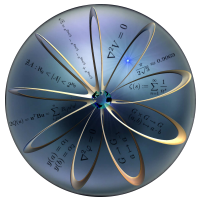

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Bonferroni inequalities

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Let $E(1)$, $E(2), \ldots, E(n)$ be events in a sample space. Define

$$S_1 := \sum_{i=1}^{n} \Pr(E(i))$$

$$S_2 := \sum_{i<j} \Pr(E(i) \cap E(j)),$$

and for $2 < k \leq n$,

$$S_k := \sum \Pr(E(i_1) \cap \cdots \cap E(i_k))$$

where the summation is taken over all ordered k-tuples of distinct integers.

Theorem

For odd k, $1 \leq k \leq n$,

$$\Pr(E(1) \cup \cdots \cup E(n)) \leq \sum_{j=1}^{k} (-1)^{j+1} S_j,$$

and for even k, $2 \leq k \leq n$,

$$\Pr(E(1) \cup \cdots \cup E(n)) \geq \sum_{j=1}^{k} (-1)^{j+1} S_j,$$

Remark When $k = 1$, the Bonferroni inequality is also known as the union bound. When $k = n$, we have an equality, also known as the inclusion-exclusion principle.