random walk

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Defines	simple random walk
Defines	symmetric simple random walk

Definition. Let $(\Omega, \mathcal{F}, \mathbf{P})$ be a probability space and $\{X_i\}$ a discrete-time stochastic process defined on $(\Omega, \mathcal{F}, \mathbf{P})$, such that the X_i are iid real-valued random variables, and $i \in \mathbb{N}$, the set of natural numbers. The *random walk* defined on X_i is the sequence of partial sums, or partial series

$$S_n: = \sum_{i=1}^{n} X_i.$$

If $X_i \in \{-1, 1\}$, then the random walk defined on X_i is called a *simple random walk*. A *symmetric simple random walk* is a simple random walk such that $\mathbf{P}(X_i = 1) = 1/2$.

The above defines random walks in one-dimension. One can easily generalize to define higher dimensional random walks, by requiring the X_i to be vector-valued (in $\mathbb{R}^n$), instead of $\mathbb{R}$.

Remarks.

1. Intuitively, a random walk can be viewed as movement in space where the length and the direction of each step are random.

2. It can be shown that, the limiting case of a random walk is a Brownian motion (with some conditions imposed on the X_i so as to satisfy part of the defining conditions of a Brownian motion). By limiting case we mean, loosely speaking, that the lengths of the steps are very small, approaching 0, while the total lengths of the walk remains a constant (so that the number of steps is very large, approaching ∞).

3. If the random variables X_i defining the random walk w_i are integrable with zero mean $\mathrm{E}[X_i] = 0$, S_i is a martingale.