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measurability of stochastic processes

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For a continuous-time stochastic process adapted to a given <http://planetmath.org/Filtration>  $(\mathcal{F}_t)_{t \in \mathbb{R}_+}$  on a measurable space  $(\Omega, \mathcal{F})$ , there are various conditions which can be placed either on its sample paths or on its measurability when considered as a function from  $\mathbb{R}_+ \times \Omega$  to  $\mathbb{R}$ . The following theorem lists the dependencies between these properties.

**Theorem.** *Let  $(X_t)_{t \in \mathbb{R}_+}$  be a real valued stochastic process. Then,  $X$  is optional if it is adapted and right-continuous, it is predictable if it is adapted and left-continuous. Furthermore, each of the following properties implies the next.*

1.  *$X$  is predictable.*
2.  *$X$  is optional.*
3.  *$X$  is progressive.*
4.  *$X$  is adapted and jointly measurable.*