tight and relatively compact measures

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Definition 1. Let $\mathcal{M} = \{\mu_i, i \in I\}$ be a family of finite measures on the Borel subsets of a metric space Ω. We say that $\mathcal{M}$ is tight iff for each $\epsilon > 0$ there is a compact set K such that $\mu_i(\Omega - K) < \epsilon$ for all i. We say that $\mathcal{M}$ is relatively compact iff each sequence in $\mathcal{M}$ has a subsequence converging weakly to a finite measure on $\mathcal{B}(\Omega)$.

If $\{F_i, i \in I\}$ is a family of distribution functions, relative compactness or tightness of $\{F_i\}$ refers to relative compactness or tightness of the corresponding measures.

Theorem. *Let $\{F_i, i \in I\}$ be a family of distribution functions with $F_i(\infty) - F_i(-\infty) < M < \infty$ for all i. The family is tight iff it is relatively compact.*

Proof. Coming soon...(needs other theorems before) $\square$