# progressively measurable process

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A stochastic process $(X_t)_{t\in\mathbb{Z}_+}$ is said to be adapted to a `http://planetmath.org/FiltrationOf` $(\mathcal{F}_t)$ on the measurable space $(\Omega,\mathcal{F})$ if $X_t$ is an $\mathcal{F}_t$-measurable random variable for each $t=0,1,\ldots$. However, for continuous-time processes, where the time $t$ ranges over an arbitrary index set $\mathbb{T}\subseteq\mathbb{R}$, the property of being adapted is too weak to be helpful in many situations. Instead, considering the process as a map

$$X\colon \mathbb{T}\times\Omega\to\mathbb{R},\ (t,\omega)\mapsto X_t(\omega)$$

it is useful to consider the measurability of $X$.

The process $X$ is *progressive* or *progressively measurable* if, for every $s\in\mathbb{T}$, the stopped process $X^s_t\equiv X_{\min(s,t)}$ is $\mathcal{B}(\mathbb{T})\otimes\mathcal{F}_s$-measurable. In particular, every progressively measurable process will be adapted and jointly measurable. In discrete time, when $\mathbb{T}$ is countable, the converse holds and every adapted process is progressive.

A set $S\subseteq\mathbb{T}\times\Omega$ is said to be progressive if its characteristic function $1_S$ is progressive. Equivalently,

$$S\cap((-\infty,s]\times\Omega)\in\mathcal{B}(\mathbb{T})\otimes\mathcal{F}_s$$

for every $s\in\mathbb{T}$. The progressively measurable sets form a $\sigma$-algebra, and a stochastic process is progressive if and only if it is measurable with respect to this $\sigma$-algebra.