multivariate distribution function

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Defines	multivariate cumulative distribution function
Defines	joint distribution function
Defines	margin

A function $F : \mathbb{R}^n \to [0,1]$ is said to be a *multivariate distribution function* if

1. F is non-decreasing in each of its arguments; i.e., for any $1 \leq i \leq n$, the function $G_i : \mathbb{R} \to [0,1]$ given by $G_i(x) := F(a_1, \ldots, a_{i-1}, x, a_{i+1}, \ldots, a_n)$ is non-decreasing for any set of $a_j \in \mathbb{R}$ such that $j \neq i$.

2. $G_i(-\infty) = 0$, where G_i is defined as above; i.e., the limit of G_i as $x \to -\infty$ is 0

3. $F(\infty, \ldots, \infty) = 1$; i.e. the limit of F as each of its arguments approaches infinity, is 1.

Generally, right-continuty of F in each of its arguments is added as one of the conditions, but it is not assumed here.

If, in the second condition above, we set $a_j = \infty$ for $j \neq i$, then $G_i(x)$ is called a (one-dimensional) *margin* of F. Similarly, one defines an m-dimensional ($m < n$) *margin* of F by setting $n - m$ of the arguments in F to ∞. For each $m < n$, there are $\binom{n}{m}$ m-dimensional margins of F. Each m-dimensional margin of a multivariate distribution function is itself a multivariate distribution function. A one-dimensional margin is a distribution function.

Multivariate distribution functions are typically found in probability theory, and especially in statistics. An example of a commonly used multivariate distribution function is the multivariate Gaussian distribution function. In $\mathbb{R}^2$, the standard bivariate Gaussian distribution function (with zero mean vector, and the identity matrix as its covariance matrix) is given by

$$F(x,y) = \frac{1}{2\pi} \int_{-\infty}^{x} \int_{-\infty}^{y} \exp\big(-\frac{s^2+t^2}{2}\big) ds dt$$

B. Schweizer and A. Sklar have generalized the above definition to include a wider class of functions. The generalization has to do with the weakening of the coordinate-wise non-decreasing condition (first condition above). The attempt here is to study a class of functions that can be used as models for distributions of distances between points in a "probabilistic metric space".

References

[1] B. Schweizer and A. Sklar, *Probabilistic Metric Spaces*, Dover Publications, (2005).