wavelet representation of Brownian motion

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First we define the function

$$H(t) = \begin{cases} 1 & \text{for } 0 \leq t < \frac{1}{2} \\ -1 & \text{for } \frac{1}{2} \leq t \leq 1 \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

and the sequence of functions

$$H_n(t) = 2^{j/2} H(2^j t - k) \tag{2}$$

for $n = 2^j + k$ where $j > 0$ and $0 \leq k \leq 2^j$. We also set $H_0(t) = 1$.

Wavelet Representation of Brownian Motion. *If* $\{Z_n : 0 \leq n < \infty\}$ *is a sequence of independent Gaussian random variables with mean* 0 *and variance* 1*, then the series defined by*

$$X_t = \sum_{n=0}^{\infty} \left(Z_n \int_0^t H_n(s)\, ds \right) \tag{3}$$

converges uniformly on $[0, 1]$ *with probability one. Moreover, the process* $\{X_t\}$ *defined by the limit is a* Brownian motion *for* $0 \leq t \leq 1$.