stopping time

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Let $(\mathcal{F}_t)_{t\in\mathbb{T}}$ be a http://planetmath.org/FiltrationOfSigmaAlgebrasfiltration on a set Ω. A random variable τ taking values in $\mathbb{T}\cup\{\infty\}$ is a *stopping time* for the filtration $(\mathcal{F}_t)$ if the event $\{\tau \leq t\} \in \mathcal{F}_t$ for every $t \in \mathbb{T}$.

Remarks

- The set $\mathbb{T}$ is the index set for the time variable t, and the σ-algebra $\mathcal{F}_t$ is the collection of all events which are observable up to and including time t. Then, the condition that τ is a stopping time means that the outcome of the event $\{\tau \leq t\}$ is known at time t.

- In discrete time situations, where $\mathbb{T} = \{0, 1, 2, \ldots\}$, the condition that $\{\tau \leq t\} \in \mathcal{F}_t$ is equivalent to requiring that $\{\tau = t\} \in \mathcal{F}_t$. This is not true for continuous time cases where $\mathbb{T}$ is an interval of the real numbers and hence uncountable, due to the fact that σ-algebras are not in general closed under taking uncountable unions of events.

- A random time τ is a stopping time for a stochastic process (X_t) if it is a stopping time for the natural filtration of X. That is, $\{\tau \leq t\} \in \sigma(X_s : s \leq t)$.

- The first time that an adapted process X_t hits a given value or set of values is a stopping time. The inclusion of ∞ into the range of τ is to cover the case where X_t never hits the given values.

- Stopping time is often used in gambling, when a gambler stops the betting process when he reaches a certain goal. The time it takes to reach this goal is generally not a deterministic one. Rather, it is a random variable depending on the current result of the bet, as well as the combined information from all previous bets.

Examples. A gambler has \$1,000 and plays the slot machine at \$1 per play.

1. The gambler stops playing when his capital is depleted. The number $\tau = n_1$ of plays that it takes the gambler to stop is a stopping time.

2. The gambler stops playing when his capital reaches \$2,000. The number $\tau = n_2$ of plays that it takes the gambler to stop is a stopping time.

3. The gambler stops playing when his capital either reaches \$2,000, or is depleted, which ever comes first. The number $\tau = \min(n_1, n_2)$ of plays that it takes the gambler to stop is a stopping time.