



A stochastic process  $\{X(t) \mid t \in \mathbb{R}^+ \cup \{0\}\}$  is called a *counting process* if, for each outcome  $\omega$  in the sample space  $\Omega$ ,

1.  $X(t) \in \mathbb{Z}^+ \cup \{0\}$  for all  $t$ ,
2.  $X(t)(\omega)$  is piecewise constant,
3.  $X(t)(\omega)$  is non-decreasing,
4.  $X(t)(\omega)$  is right continuous (continuous from the right), and
5. for any  $t$ , there is an  $s \in \mathbb{R}$  such that  $t < s$  and  $X(t)(\omega) + 1 = X(s)(\omega)$ .

**Remark.** For any  $t$ , the random variable  $X(t)$  is usually called the number of occurrences of some event by time  $t$ . Then, for  $s < t$ ,  $X(t) - X(s)$  is the number of occurrences in the half-open interval  $(s, t]$ .