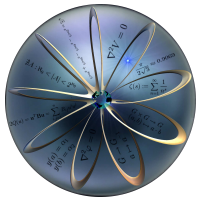

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## median of a distribution

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|---|---|
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Given a probability distribution (density) function $f_X(x)$ on $\Omega$ over a random variable $X$, with the associated probability measure $P$, a *median* $m$ of $f_X$ is a real number such that

1. $P(X \leq m) \geq \frac{1}{2}$,
2. $P(X \geq m) \geq \frac{1}{2}$.

The median is also known as the $50^{\text{th}}$-percentile or the second quartile.
**Examples:**

- An example from a discrete distribution. Let $\Omega = \mathbb{R}$. Suppose the random variable $X$ has the following distribution: $P(X = 0) = 0.99$ and $P(X = 1000) = 0.01$. Then we can easily see the median is 0.
- Another example from a discrete distribution. Again, let $\Omega = \mathbb{R}$. Suppose the random variable $X$ has distribution $P(X = 0) = 0.5$ and $P(X = 1000) = 0.5$. Then we see that the median is not unique. In fact, all real values in the interval $[0, 1000]$ are medians.
- In practice, however, the median may be calculated as follows: if there are $N$ numeric data points, then by ordering the data values (either non-decreasingly or non-increasingly),
  1. the $(\frac{N+1}{2})$-th data point is the median if $N$ is odd, and
  2. the midpoint of the $(N-1)$th and the $(N+1)$th data points is the median if $N$ is even.
- The median of a normal distribution (with mean $\mu$ and variance $\sigma^2$) is $\mu$. In fact, for a normal distribution, mean = median = mode.
- The median of a uniform distribution in the interval $[a, b]$ is $(a + b)/2$.
- The median of a Cauchy distribution with location parameter t and scale parameter s is the location parameter.
- The median of an exponential distribution with location parameter $\mu$ and scale parameter $\beta$ is the scale parameter times the natural log of 2, $\beta \ln 2$.
- The median of a Weibull distribution with shape parameter $\gamma$, location parameter $\mu$, and scale parameter $\alpha$ is $\alpha(\ln 2)^{1/\gamma} + \mu$.