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eigenvalues of stochastic matrix

Canonical name	EigenvaluesOfStochasticMatrix
Date of creation	2013-03-22 16:18:02
Last modified on	2013-03-22 16:18:02
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Numerical id	7
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Entry type	Theorem
Classification	msc 60G99
Classification	msc 15A51

Theorem: The spectrum of a stochastic matrix is contained in the unit disc in the complex plane.

Proof. Let A be a stochastic matrix and let m be an eigenvalue of A , with v eigenvector; then, for any self-consistent matrix norm $\|\cdot\|$, we have:

$$|m| \|v\| = \|mv\| = \|Av\| \leq \|A\| \|v\| ,$$

that is, since v is nonzero,

$$|m| \leq \|A\| .$$

Now, for a (doubly) stochastic matrix,

$$\|A\|_1 = \max_j \left(\sum_i |a_{ij}| \right) = 1$$

whence the conclusion. □