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outer measure

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**Definition** [?, ?, ?] Let  $X$  be a set, and let  $\mathcal{P}(X)$  be the power set of  $X$ . An *outer measure* on  $X$  is a function  $\mu^* : \mathcal{P}(X) \rightarrow [0, \infty]$  satisfying the properties

1.  $\mu^*(\emptyset) = 0$ .
2. If  $A \subset B$  are subsets in  $X$ , then  $\mu^*(A) \leq \mu^*(B)$ .
3. If  $\{A_i\}$  is a countable collection of subsets of  $X$ , then

$$\mu^*\left(\bigcup_i A_i\right) \leq \sum_i \mu^*(A_i).$$

Here, we can make two remarks. First, from (1) and (2), it follows that  $\mu^*$  is a positive function on  $\mathcal{P}(X)$ . Second, property (3) also holds for any finite collection of subsets since we can always append an infinite sequence of empty sets to such a collection.

## References

- [1] A. Mukherjea, K. Pothoven, *Real and Functional analysis*, Plenum press, 1978.
- [2] A. Friedman, *Foundations of Modern Analysis*, Dover publications, 1982.
- [3] G.B. Folland, *Real Analysis: Modern Techniques and Their Applications*, 2nd ed, John Wiley & Sons, Inc., 1999.