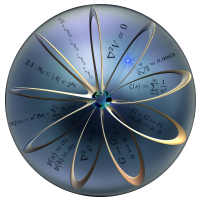

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almost everywhere

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Let $(X, \mathfrak{B}, \mu)$ be a measure space. A condition holds *almost everywhere* on X if it holds "with probability 1," i.e. if it holds everywhere except for a subset of X with measure 0. For example, let f and g be nonnegative functions on X. Suppose we want a sufficient condition on functions $f(x)$ and $g(x)$ such that the relation

$$\int_X f d\mu(x) \leq \int_X g d\mu(x) \tag{1}$$

holds. Certainly $f(x) \leq g(x)$ for all $x \in X$ is a sufficient condition, but in fact it's enough to have $f(x) \leq g(x)$ almost surely on X. In fact, we can loosen the above non-negativity condition to only require that f and g are almost surely nonnegative as well.

If $X = [0, 1]$, then g might be less than f on the Cantor set, an uncountable set with measure 0, and still satisfy the condition. We say that $f \leq g$ almost everywhere (often abbreviated *a.e.*).

Note that this is the of the "almost surely" from probabilistic measure .