measure

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Defines	measure space
Defines	probability space
Defines	probability measure
Defines	countably additive
Defines	finitely additive
Defines	σ-additive
Defines	positive measure

Let $(E, \mathcal{B}(E))$ be a measurable space. A *measure* on $(E, \mathcal{B}(E))$ is a function $\mu\colon \mathcal{B}(E) \to \mathbb{R} \cup \{\infty\}$ with values in the extended real numbers such that:

1. $\mu(A) \geq 0$ for $A \in \mathcal{B}(E)$, with equality if $A = \emptyset$
2. $\mu(\bigcup_{i=0}^{\infty} A_i) = \sum_{i=0}^{\infty} \mu(A_i)$ for any sequence of pairwise disjoint sets $A_i \in \mathcal{B}(E)$.

Occasionally, the term *positive measure* is used to distinguish measures as defined here from more general notions of measure which are not necessarily restricted to the non-negative extended reals.

The second property above is called countable additivity, or σ-additivity. A *finitely additive measure* μ has the same definition except that $\mathcal{B}(E)$ is only required to be an algebra and the second property above is only required to hold for finite unions. Note the slight abuse of terminology: a finitely additive measure is not necessarily a measure.

The triple $(E, \mathcal{B}(E), \mu)$ is called a *measure space*. If $\mu(E) = 1$, then it is called a *probability space*, and the measure μ is called a *probability measure*.

Lebesgue measure on $\mathbb{R}^n$ is one important example of a measure.