moment

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Defines	central moment
Defines	skewness
Defines	kurtosis
Defines	platykurtic
Defines	leptokurtic

Moments

Given a random variable X, the k**th moment** of X is the value $E[X^k]$, if the expectation exists.

Note that the expected value is the first moment of a random variable, and the variance is the second moment minus the first moment squared.

The kth moment of X is usually obtained by using the moment generating function.

Central moments

Given a random variable X, the k**th central moment** of X is the value $E\big[(X - E[X])^k\big]$, if the expectation exists. It is denoted by μ_k.

Note that the $\mu_1 = 0$ and $\mu_2 = Var[X] = \sigma^2$. The third central moment divided by the standard deviation cubed is called the *skewness* τ:

$$\tau = \frac{\mu_3}{\sigma^3}$$

The skewness measures how "symmetrical", or rather, how "skewed", a distribution is with respect to its mode. A non-zero τ means there is some degree of skewness in the distribution. For example, $\tau > 0$ means that the distribution has a longer positive tail.

The fourth central moment divided by the fourth power of the standard deviation is called the *kurtosis* κ:

$$\kappa = \frac{\mu_4}{\sigma^4}$$

The kurtosis measures how "peaked" a distribution is compared to the standard normal distribution. The standard normal distribution has $\kappa = 3$. $\kappa < 3$ means that the distribution is "flatter" than then standard normal distribution, or *platykurtic*. On the other hand, a distribution with $\kappa > 3$ can be characterized as being more "peaked" than $N(0, 1)$, or *leptokurtic*.