



mean square convergence of the sample mean
of a stationary process

Canonical name	MeanSquareConvergenceOfTheSampleMeanOfAStationaryProcess
Date of creation	2013-03-22 15:20:52
Last modified on	2013-03-22 15:20:52
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Last modified by	georgiosl (7242)
Numerical id	5
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Entry type	Theorem
Classification	msc 60G10

If $\{X_t, t \in T\}$ is a stationary process with mean μ and autocovariance function $\gamma(\cdot)$, then as $n \rightarrow \infty$ we have the following:

- $\text{var}[\bar{X}_n] = E[(\bar{X}_n - \mu)^2] \rightarrow 0$ if $\gamma(n) \rightarrow 0$
- $nE[(\bar{X}_n - \mu)^2] \rightarrow \sum_{h=-\infty}^{\infty} \gamma(h)$ if $\sum_{h=-\infty}^{\infty} |\gamma(h)| < \infty$ where

$$\bar{X}_n = \frac{1}{n} \sum_{k=1}^n X_k$$

is the sample mean which is a natural unbiased estimator of the mean μ of the stationary process $\{X_t\}$.

References

- [1] Peter J. Brockwell G., Richard A. Davis , *Time Series : Theory and Methods*.