gamma random variable

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Synonym	gamma distribution
Defines	Erlang random variable

A **gamma random variable** with parameters $\alpha > 0$ and $\lambda > 0$ is one whose probability density function is given by

$$f_X(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}$$

for $x > 0$, and is denoted by $X \sim Gamma(\alpha, \lambda)$.

Notes:

1. Gamma random variables are widely used in many applications. Taking $\alpha = 1$ reduces the form to that of an exponential random variable. If $\alpha = \frac{n}{2}$ and $\lambda = \frac{1}{2}$, this is a chi-squared random variable.

2. The function $\Gamma : [0, \infty] \to R$ is the gamma function, defined as $\Gamma(t) = \int_0^\infty x^{t-1} e^{-x} dx$.

3. The expected value of a gamma random variable is given by $E[X] = \frac{\alpha}{\lambda}$, and the variance by $Var[X] = \frac{\alpha}{\lambda^2}$

4. The moment generating function of a gamma random variable is given by $M_X(t) = (\frac{\lambda}{\lambda - t})^\alpha$.

If the first parameter is a positive integer, the variate is usually called Erlang random variate. The sum of n exponentially distributed variables with parameter λ is a gamma (Erlang) variate with parameters n, λ.