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independent sigma algebras

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Let $(\Omega, \mathcal{B}, P)$ be a probability space. Let $\mathcal{B}_1$ and $\mathcal{B}_2$ be two sub sigma algebras of $\mathcal{B}$. Then $\mathcal{B}_1$ and $\mathcal{B}_2$ are said to be if for any pair of events $B_1 \in \mathcal{B}_1$ and $B_2 \in \mathcal{B}_2$:

$$P(B_1 \cap B_2) = P(B_1)P(B_2).$$

More generally, a finite set of sub-σ-algebras $\mathcal{B}_1, \ldots, \mathcal{B}_n$ is *independent* if for any set of events $B_i \in \mathcal{B}_i$, $i = 1, \ldots, n$:

$$P(B_1 \cap \cdots \cap B_n) = P(B_1) \cdots P(B_n).$$

An arbitrary set $\mathcal{S}$ of sub-σ-algebras is *mutually independent* if any finite subset of $\mathcal{S}$ is independent.

The above definitions are generalizations of the notions of `http://planetmath.org/Independen` for events and for random variables:

1. Events $B_1, \ldots, B_n$ (in Ω) are *mutually independent* if the sigma algebras $\sigma(B_i) := \{\varnothing, B_i, \Omega - B_i, \Omega\}$ are mutually independent.
2. Random variables $X_1, \ldots, X_n$ defined on Ω are *mutually independent* if the `http://planetmath.org/MathcalFMeasurableFunction`sigma algebras $\mathcal{B}_{X_i}$ generated by the X_i's are mutually independent.

In general, mutual independence among events B_i, random variables X_j, and sigma algebras $\mathcal{B}_k$ means the mutual independence among $\sigma(B_i)$, $\mathcal{B}_{X_j}$, and $\mathcal{B}_k$.

Remark. Even when random variables $X_1, \ldots, X_n$ are defined on different probability spaces $(\Omega_i, \mathcal{B}_i, P_i)$, we may form the `http://planetmath.org/InfiniteProductMea` of these spaces $(\Omega, \mathcal{B}, P)$ so that X_i (by abuse of notation) are now defined on Ω and their independence can be discussed.