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Markov chain

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**Definition** We begin with a probability space  $(\Omega, \mathcal{F}, \mathbb{P})$ . Let  $I$  be a countable set,  $(X_n : n \in \mathbb{Z})$  be a collection of random variables taking values in  $I$ ,  $\mathbf{T} = (t_{ij} : i, j \in I)$  be a stochastic matrix, and  $\lambda$  be a distribution. We call  $(X_n)_{n \geq 0}$  a *Markov chain* with initial distribution  $\lambda$  and *transition matrix*  $\mathbf{T}$  if:

1.  $X_0$  has distribution  $\lambda$ .
2. For  $n \geq 0$ ,  $\mathbb{P}(X_{n+1} = i_{n+1} | X_0 = i_0, \dots, X_n = i_n) = t_{i_n i_{n+1}}$ .

That is, the next value of the chain depends only on the current value, not any previous values. This is often summed up in the pithy phrase, “Markov chains have no memory.”

As a special case of (2) we have that  $\mathbb{P}(X_{n+1} = i | X_n = j) = t_{ij}$  whenever  $i, j \in I$ . The values  $t_{ij}$  are therefore called *transition probabilities* for the Markov chain.

**Discussion** Markov chains are arguably the simplest examples of random processes. They come in discrete and continuous versions; the discrete version is presented above.