stochastic process

Canonical name	StochasticProcess
Date of creation	2013-03-22 14:39:10
Last modified on	2013-03-22 14:39:10
Owner	gel (22282)
Last modified by	gel (22282)
Numerical id	14
Author	gel (22282)
Entry type	Definition
Classification	msc 60G05
Classification	msc 60G60
Synonym	random process
Related topic	DistributionsOfAStochasticProcess
Defines	discrete-time process
Defines	continuous-time process
Defines	state
Defines	time series
Defines	state space
Defines	random function
Defines	jointly measurable

Let $(\Omega, \mathcal{F}, \mathbf{P})$ be a probability space. A *stochastic process* is a collection

$$\{X_t \mid t \in T\}$$

of random variables X_t defined on $(\Omega, \mathcal{F}, \mathbf{P})$, where T is a set, called the index set of the process $\{X_t \mid t \in T\}$. T is usually (but not always) a subset of $\mathbb{R}$. X is sometimes known as a *random function*.

Given any t, the possible values of X_t are called the *states* of the process at t. The set of all states (for all t) of a stochastic process is called its *state space*.

If T is discrete, then the stochastic process is a *discrete-time process*. If T is an interval of $\mathbb{R}$, then $\{X_t \mid t \in T\}$ is a *continuous-time process*. If T can be linearly ordered, then t is also known as the *time*.

A stochastic process X with state space S can be thought of in either of following three ways.

- As a collection of random variables, X_t, for each t in the index set T.

- As a function in two variables $t \in T$ and $\omega \in \Omega$,

  $$X\colon T \times \Omega \to S,\ (t, \omega) \mapsto X_t(\omega).$$

  The process is said to be measurable, or, jointly measurable if it is $\mathcal{B}(T) \otimes \mathcal{F}/\mathcal{B}(S)$-measurable. Here, $\mathcal{B}(T)$ and $\mathcal{B}(S)$ are the Borel σ-algebras on T and S respectively.

- In terms of the sample paths. Each $\omega \in \Omega$ maps to a function

  $$T \to S,\ t \mapsto X_t(\omega).$$

  Many common examples of stochastic processes have sample paths which are either continuous or cadlag.

Examples. The following list is some of the most common and important stochastic processes:

1. a random walk, as well as its limiting case, a Brownian motion, or a Wiener process

2. Poisson process

3. Markov process; a Markov chain is a Markov process whose state space is discrete

4. renewal process

Remarks.

- Sometimes, a stochastic process is also called a *random process*, although a stochastic process is generally linked to any "time" dependent process. In a random process, the index set may not be linearly ordered, as in the case of a random field, where the index set may be, for example, the unit sphere $S^2 \subseteq \mathbb{R}^3$.

- In statistics, a stochastic process is often known as a *time series*, where the index set is a finite (or at most countable) ordered sequence of real numbers.