Borel space

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Defines	rigid Borel space
Defines	Borel subset space

Definition 0.1. A *Borel space* $(X; \mathcal{B}(X))$ is defined as a set X, together with a Borel `http://planetmath.org/SigmaAlgebra`σ-algebra $\mathcal{B}(X)$ of subsets of X, called Borel sets. The Borel algebra on X is the smallest σ-algebra containing all open sets (or, equivalently, all closed sets if the topology on closed sets is selected).

Remark 0.1. Borel sets were named after the French mathematician Emile Borel.

Remark 0.2. A subspace of a Borel space $(X; \mathcal{B}(X))$ is a subset $S \subset X$ endowed with the relative Borel structure, that is the σ-algebra of all subsets of S of the form $S \bigcap E$, where E is a Borel subset of X.

Definition 0.2. A *rigid Borel space* $(X_r; \mathcal{B}(X_r))$ is defined as a Borel space whose only automorphism $f : X_r \to X_r$ (that is, with f being a bijection, and also with $f(A) = f^{-1}(A)$ for any $A \in \mathcal{B}(X_r)$) is the identity function $1_{(X_r; \mathcal{B}(X_r))}$ (ref.[**?**]).

Remark 0.3. R. M. Shortt and J. Van Mill provided the first construction of a rigid Borel space on a 'set of large cardinality'.

References

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