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Borel measure

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Definition 1 - Let X be a topological space and \mathcal{B} be its <http://planetmath.org/BorelSigma> σ -algebra. A **Borel measure** on X is a measure on the measurable space (X, \mathcal{B}) .

In the literature one can find other different definitions of Borel measure, like the following:

Definition 2 - Let X be a topological space and \mathcal{B} be its Borel σ -algebra. A **Borel measure** on X is a measure μ on the measurable space (X, \mathcal{B}) such that $\mu(K) < \infty$ for all compact subsets $K \subset X$. (ref.[?]).

Definition 3 - Let X be a topological space and \mathcal{B} be the σ -algebra generated by all compact sets of X . A **Borel measure** on X is a measure μ on the measurable space (X, \mathcal{B}) such that $\mu(K) < \infty$ for all compact subsets $K \subset X$.

Definition 4 - The <http://planetmath.org/RestrictionOfAFunctionrestriction> of the Lebesgue measure to the Borel σ -algebra of \mathbb{R}^n is also sometimes called “the” Borel measure of \mathbb{R}^n .

Remark - Definitions 2 and 3 are technically different. For example, when constructing a Haar measure on a locally compact group one considers the σ -algebra generated by all compact subsets, instead of all closed (or open) sets.

References

- [1] M.R. Buneci. 2006., <http://www.utgjiu.ro/math/mbuneci/preprint/p0024.pdf> Groupoid C^* -Algebras., *Surveys in Mathematics and its Applications*, Volume 1: 71–98.
- [2] A. Connes.1979. Sur la théorie noncommutative de l’ integration, *Lecture Notes in Math.*, Springer-Verlag, Berlin, **725**: 19-14.