predictable stopping time

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A predictable, or *previsible* stopping time is a random time which is possible to predict just before the event. Letting $(\mathcal{F}_t)_{t\in\mathbb{R}_+}$ be a http://planetmath.org/FiltrationOfS on a measurable space $(\Omega, \mathcal{F})$, then, a stopping time τ is predictable if there exists an increasing sequence of stopping times τ_n satisfying the following.

- $\tau_n < \tau$ whenever $\tau > 0$.
- $\tau_n \to \tau$ as $n \to \infty$.

The sequence τ_n is said to announce or foretell τ.

For example, if X is a continuous adapted process with $X_0 = 0$, such as Brownian motion, then the first time τ at which it hits a given level $K \neq 0$ is a predictable stopping time. In this case, if τ_n is the first time at which X hits the level $K(1 - 1/n)$, then the sequence τ_n announces τ.

On the other hand, if X is a Poisson process then the first time τ at which it is nonzero is not predictable. To show this, suppose that $\tau_n < \tau$ are stopping times. The fact that $X_t - \lambda t$ is a martingale means that Doob's optional sampling theorem can be applied, giving $\mathbb{E}[X_{\tau_n} - \lambda\tau_n] = 0$. Then, $X_t = 0$ for $t < \tau$ gives $\mathbb{E}[\tau_n] = 0$. So, $\tau_n = 0$ with probability one, and the sequence τ_n cannot announce τ.

In discrete time, where the filtration $(\mathcal{F}_t)$ has time t running over the index set $\mathbb{Z}_+$, then a stopping time is said to be predictable if $\{\tau \leq t\}$ is $\mathcal{F}_{t-1}$-measurable for every time $t = 1, 2, \ldots$.

This can be generalized to an arbitrary index set $\mathbb{T}$, where a stopping time $\tau\colon \Omega \to \mathbb{T} \cup \{\infty\}$ is predictable if there exists an increasing sequence of stopping times $\tau_n \leq \tau$ such that $\tau_n < \tau$ whenever τ is not equal to a minimal element of $\mathbb{T}$, and $\bigcap_n (\tau_n, \tau)$ contains no elements of $\mathbb{T}$.