convergence in probability

Canonical name	ConvergenceInProbability
Date of creation	2013-03-22 15:01:05
Last modified on	2013-03-22 15:01:05
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	7
Author	CWoo (3771)
Entry type	Definition
Classification	msc 60B10
Synonym	converge in probability
Synonym	converges in measure
Synonym	converge in measure
Synonym	convergence in measure
Defines	converges in probability

Let $\{X_i\}$ be a sequence of random variables defined on a probability space $(\Omega, \mathcal{F}, P)$ taking values in a separable metric space (Y, d), where d is the metric. Then we say the sequence X_i *converges in probability* or *converges in measure* to a random variable X if for every $\varepsilon > 0$,

$$\lim_{i \to \infty} P(d(X_i, X) \geq \varepsilon) = 0.$$

We denote convergence in probability of X_i to X by

$$X_i \xrightarrow{pr} X.$$

Equivalently, $X_i \xrightarrow{pr} X$ iff every subsequence of $\{X_i\}$ contains a subsequence which converges to X almost surely.

Remarks.

- Unlike ordinary convergence, $X_i \xrightarrow{pr} X$ and $X_i \xrightarrow{pr} Y$ only implies that $X = Y$ almost surely.
- The need for separability on Y is to ensure that the metric, $d(X_i, X)$, is a random variable, for all random variables X_i and X.
- Convergence almost surely implies convergence in probability but not conversely.

References

[1] R. M. Dudley, *Real Analysis and Probability*, Cambridge University Press (2002).

[2] W. Feller, *An Introduction to Probability Theory and Its Applications. Vol. 1*, Wiley, 3rd ed. (1968).