Buffon's needle

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The plane is ruled by parallel lines 2 inches apart and a 1-inch long needle is dropped at random on the plane. What is the probability that it hits parallel lines?

Solution.

The first issue is to find some appropriate probability space $(\Omega, \mathcal{F}, P)$. For this,

- $h =$ distance from the center of the needle to the nearest line
- $\theta =$ the angle that the needle makes with the horizontal ranging from 0 to $\frac{\pi}{2}$.

These fully determine the position of the needle. Let us next take the

1. The probability space is $\Omega = [0,1] \times [0, \frac{\pi}{2})$
2. The probability of an event B is denoted by $P[B]$ is equal to $\frac{area\ \ of\ \ B}{\frac{\pi}{2}}$

Now we denote by A the event that the needle hits a horizontal line. It is easily seen that this happens when $\sin\theta \geq \frac{h}{1/2}$. Consequently $A = \{(\theta, h) \in \Omega : h \leq \frac{\sin\theta}{2}\}$ and then we get $P[A] = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \frac{1}{2}\sin\theta d\theta = \frac{1}{\pi}$□

In general case, when the length of needle is l and the distance of parallel lines is d provided that $l < d$, the probability we want is $\frac{2l}{\pi d}$. This is obvious just taking the l/d-point from one edge instead of the center of the needle.