# optional process

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| Defines | optional |

Suppose we are given a `http://planetmath.org/FiltrationOfSigmaAlgebras`filtration $(\mathcal{F})_{t\in\mathbb{T}}$ on a measurable space $(\Omega,\mathcal{F})$. A stochastic process is said to be adapted if $X_t$ is $\mathcal{F}_t$-measurable for every time $t$ in the index set $\mathbb{T}$. For an arbitrary, uncountable, index set $\mathbb{T}\subseteq\mathbb{R}$, this property is too restrictive to be useful. Instead, we can impose measurability conditions on $X$ considered as a map from $\mathbb{T}\times\Omega$ to $\mathbb{R}$. For instance, we could require $X$ to be progressively measurable, but that is still too weak a condition for many purposes. A stronger condition is for $X$ to be *optional*. The index set $\mathbb{T}$ is assumed to be a closed subset of $\mathbb{R}$ in the following definition.

The class of optional processes forms the smallest set containing all adapted and right-continuous processes, and which is closed under taking limits of sequences of processes.

The $\sigma$-algebra, $\mathcal{O}$, on $\mathbb{T}\times\Omega$ generated by the right-continuous and adapted processes is called the *optional* $\sigma$-algebra. Then, a process is optional if and only if it is $\mathcal{O}$-measurable.

Alternatively, the optional $\sigma$-algebra may be defined as

$$\mathcal{O}=\sigma\left(\{[T,\infty):T\text{ is a stopping time}\}\right).$$

Here, $[T,\infty)$ is a stochastic interval, consisting of the pairs $(t,\omega)\in\mathbb{T}\times\Omega$ such that $T(\omega)\leq t$. In continuous-time, the equivalence of these two definitions for $\mathcal{O}$ does require mild conditions on the filtration — it is enough for $\mathcal{F}_t$ to be universally complete.

In the discrete-time case where the index set $\mathbb{T}$ countable, then the definitions above imply that a process $X_t$ is optional if and only if it is adapted.