joint cumulative distribution function

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Let $X_1, X_2, ..., X_n$ be n random variables all defined on the same probability space. The **joint cumulative distribution function** of $X_1, X_2, ..., X_n$, denoted by $F_{X_1,X_2,...,X_n}(x_1, x_2, ..., x_n)$, is the following function:

$$F_{X_1,X_2,...,X_n} : R^n \to R$$
$$F_{X_1,X_2,...,X_n}(x_1, x_2, ..., x_n) = P[X_1 \leq x_1, X_2 \leq x_2, ..., X_n \leq x_n]$$

As in the unidimensional case, this function satisfies:

1. $\lim_{(x_1,...,x_n)\to(-\infty,...,-\infty)} F_{X_1,X_2,...,X_n}(x_1, ..., x_n) = 0$ and $\lim_{(x_1,...,x_n)\to(\infty,...,\infty)} F_{X_1,X_2,...,X_n}(x_1, ..., x_n$ 1
2. $F_{X_1,X_2,...,X_n}(x_1, ..., x_n)$ is a monotone, nondecreasing function.
3. $F_{X_1,X_2,...,X_n}(x_1, ..., x_n)$ is continuous from the right in each variable.

The way to evaluate $F_{X_1,X_2,...,X_n}(x_1, ..., x_n)$ is the following:

$$F_{X_1,X_2,...,X_n}(x_1, ..., x_n) = \int_{-\infty}^{x_1} \int_{-\infty}^{x_2} \cdots \int_{-\infty}^{x_n} f_{X_1,X_2,...,X_n}(u_1, ..., u_n) du_1 du_2 \cdots du_n$$

(if F is continuous) or

$$F_{X_1,X_2,...,X_n}(x_1, ..., x_n) = \sum_{i_1 \leq x_1,...,i_n \leq x_n} f_{X_1,X_2,...,X_n}(i_1, ..., i_n)$$

(if F is discrete),

where $f_{X_1,X_2,...,X_n}$ is the *joint density function* of $X_1, ..., X_n$.