Bernstein inequalities

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1) Let $\{X_i\}_{i=1}^n$ be a collection of independent random variables satisfying the conditions:
a) $E[X_i^2] < \infty \ \forall i$, so that one can write $\sum_{i=1}^n E[X_i^2] = v^2$
b) $\exists c \in \mathbb{R} : \sum_{i=1}^n E[|X_i|^k] \leq \frac{1}{2} k! v^2 c^{k-2}$ for all integers $k \geq 3$

Then, for any $\varepsilon \geq 0$,

$$\Pr\left\{\sum_{i=1}^n (X_i - E[X_i]) > \varepsilon\right\} \leq \exp\left[-\frac{v^2}{c^2}\left(1 + \frac{c\varepsilon}{v^2} - \sqrt{1 + 2\frac{c\varepsilon}{v^2}}\right)\right] \leq \exp\left(-\frac{\varepsilon^2}{2\left(v^2 + c\varepsilon\right)}\right)$$

$$\Pr\left\{\left|\sum_{i=1}^n (X_i - E[X_i])\right| > \varepsilon\right\} \leq 2\exp\left[-\frac{v^2}{c^2}\left(1 + \frac{c\varepsilon}{v^2} - \sqrt{1 + 2\frac{c\varepsilon}{v^2}}\right)\right] \leq 2\exp\left(-\frac{\varepsilon^2}{2\left(v^2 + c\varepsilon\right)}\right)$$

2) Let $\{X_i\}_{i=1}^n$ be a collection of independent, `http://planetmath.org/AlmostSurelyBoundedR` surely absolutely bounded random variables, that is $\Pr\{|X_i| \leq M\} = 1 \ \ \forall i$. Then, for any $\varepsilon \geq 0$,

$$\Pr\left\{\sum_{i=1}^n (X_i - E[X_i]) > \varepsilon\right\} \leq \exp\left[-\frac{9v^2}{M^2}\left(1 + \frac{M\varepsilon}{3v^2} - \sqrt{1 + 2\frac{M\varepsilon}{3v^2}}\right)\right] \leq \exp\left(-\frac{\varepsilon^2}{2\left(v^2 + \frac{M}{3}\varepsilon\right)}\right)$$

$$\Pr\left\{\left|\sum_{i=1}^n (X_i - E[X_i])\right| > \varepsilon\right\} \leq 2\exp\left[-\frac{9v^2}{M^2}\left(1 + \frac{M\varepsilon}{3v^2} - \sqrt{1 + 2\frac{M\varepsilon}{3v^2}}\right)\right] \leq 2\exp\left(-\frac{\varepsilon^2}{2\left(v^2 + \frac{M}{3}\varepsilon\right)}\right)$$