6. Discussion

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This paper developed techniques for analyzing the internal structure of distributed measurements. We introduced entanglement, which quantifies the extent to which a measurement is indecomposable. Entanglement can be shown to quantify context-dependence. Moreover, positive entanglement is necessary for a system to generate more information than the sum of its subsystems. Along the way, we constructed the quale, which geometrically represents the compositional structure of a distributed measurement. The information-theoretic approach developed here is dual, in a precise sense, to the algorithmic perspective on computation. Studying duals $\mathfrak{m}^\natural$ instead of mechanisms $\mathfrak{m}$ shifts the focus from *what* the algorithm does to *how* it does it: instead of analyzing rules we analyze functional dependencies.

The intuition driving the paper is that the structure presheaf $\mathcal{F}$ is an information-theoretic analogue of a tangent space. A particle moving in a manifold X defines a vector field – a section of the tangent space to X, which is a sheaf. The tangent vector at a point depends on the particle's location at "nearby time-points": it is computed by taking the limit of difference in positions at t and $t + h$ as $h \to 0$. Similarly, a system performing a measurement generates a quale, a section of the structure presheaf consisting of "nearby counterfactuals". The quale is computed by applying Bayes' rule to determine which inputs could have led to the output.[1] How far this analogy can be developed remains to be seen.

Entanglement can be loosely considered as an information-theoretic analogue of curvature: the extent to which interactions within a system "warp" sections of $\mathcal{F}$ away from a product structure. A related approach to geometrically analyzing the complexity of interactions was proposed in [?]. In fact, this project began as an attempt to reformulate [?] in terms of sheaf cohomology using ideas from [?]. We failed at the first step since the structure presheaf is not a sheaf. However, the failure was instructive since it is precisely the *obstruction* to forming a sheaf that is of interest since it is the obstruction (entanglement) that quantifies indecomposability and context-dependence, and only systems whose measurements are entangled are able to generate more information than the sum of their subsystems.

[1] A counterfactual input is "nearby" to an output if it causes (leads to) that output.

References

[1] N Ay, E Olbrich, N Bertschinger & J Jost (2006): *A unifying framework for complexity measures of finite systems*. In: *Proceedings of ECCS06, European Complex Systems Society*, Oxford, UK, pp. ECCS06–174.

[2] David Balduzzi & Giulio Tononi (2009): *Qualia: the geometry of integrated information*. *PLoS Comput Biol* 5(8), p. e1000462, doi:`10.1371/journal.pcbi.1000462`.