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infinitely divisible random variable

Canonical name	InfinitelyDivisibleRandomVariable
Date of creation	2013-03-22 16:25:58
Last modified on	2013-03-22 16:25:58
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	8
Author	CWoo (3771)
Entry type	Definition
Classification	msc 60E07
Defines	$n$ -decomposable
Defines	$n$ -divisible
Defines	infinitely divisible distribution
Defines	infinitely divisible
Defines	decomposable random variable

Let  $n$  be a positive integer. A real random variable  $X$  defined on a probability space  $(\Omega, \mathcal{F}, P)$  is said to be

1. *n-decomposable* if there exist  $n$  independent random variables  $X_1, \dots, X_n$  such that  $X$  is identically distributed as the sum  $X_1 + \dots + X_n$ . A 2-decomposable random variable is also called a *decomposable random variable*;
2. *n-divisible* if  $X$  is  $n$ -decomposable and the  $X_i$ 's can be chosen so that they are identically distributed;
3. *infinitely divisible* if  $X$  is  $n$ -divisible for every positive integer  $n$ . In other words,  $X$  can be written as the sum of  $n$  iid random variables for any  $n$ .

A distribution function is said to be *infinitely divisible* if it is the distribution function of an infinitely divisible random variable.

**Remark.** Any stable random variable is infinitely divisible.

Some examples of infinitely divisible distribution functions, besides those that are stable, are the gamma distributions, negative binomial distributions, and compound Poisson distributions.