stable random variable

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Defines	stable distribution function
Defines	strictly stable random variable
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A real random variable X defined on a probability space $(\Omega, \mathcal{F}, P)$ is said to be *stable* if

1. X is not trivial; that is, the range of the distribution function of X strictly includes $\{0, 1\}$, and

2. given any positive integer n and $X_1, \ldots, X_n$ random variables, iid as X:
$$S_n := X_1 + \cdots + X_n \stackrel{t}{=} X.$$
In other words, there are real constants a, b such that S_n and $aX + b$ have the same distribution functions; S_n and X are of the same type.

Furthermore, X is *strictly stable* if X is stable and the b given above can always be take as 0. In other words, X is strictly stable if S_n and X belong to the same scale family.

A distribution function is said to be *stable* (*strictly stable*) if it is the distribution function of a stable (strictly stable) random variable.

Remarks.

- If X is stable, then $aX + b$ is stable for any $a, b \in \mathbb{R}$.

- If X and Y are independent, stable, and of the same type, then $X + Y$ is stable.

- X is stable iff for any independent X_1, X_2, identically distributed as X, and any $a, b \in \mathbb{R}$, there exist $c, d \in \mathbb{R}$ such that $aX_1 + bX_2$ and $cX + d$ are identically distributed.

- A stable distribution function is `http://planetmath.org/AbsolutelyContinuousFunction2` continuous and infinitely divisible.

Some common stable distribution functions are the normal distributions and Cauchy distributions.