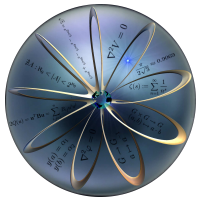

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Borel-Cantelli lemma

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Let $A_1, A_2, \ldots$ be random events in a probability space.

1. If $\sum_{n=1}^{\infty} P(A_n) < \infty$, then $P(A_n \,\text{i. o.}) = 0$;
2. If $A_1, A_2, \ldots$ are independent, and $\sum_{n=1}^{\infty} P(A_n) = \infty$, then $P(A_n \,\text{i. o.}) = 1$

where $A = [A_n \,\text{i. o.}]$ represents the event "A_n happens for infinitely many values of n." Formally, $A = \limsup A_n$, which is a limit superior of sets.