

proof of Chebyshev's inequality

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The proof of Chebyshev's inequality follows from the application of Markov's inequality.

Define $Y = (X - \mu)^2$. Then $Y \ge 0$ is a random variable, and

$$\mathbb{E}[Y] = \operatorname{Var}[X] = \sigma^2.$$

Applying Markov's inequality to Y, we see that

$$\mathbb{P}\left\{|X - \mu| \ge t\right\} = \mathbb{P}\left\{Y \ge t^2\right\} \le \frac{1}{t^2} \mathbb{E}[Y] = \frac{\sigma^2}{t^2}.$$