Prohorov inequality

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Theorem (Prohorov inequality, 1959):

Let $\{X_i\}_{i=1}^n$ be a collection of independent random variables satisfying the conditions:

a) $E[X_i^2] < \infty \; \forall i$, so that one can write $\sum_{i=1}^n E[X_i^2] = v^2$
b) $\Pr\{|X_i| \leq M\} = 1 \;\; \forall i$.

Then, for any $\varepsilon \geq 0$,

$$\begin{aligned}
\Pr\left\{\sum_{i=1}^n (X_i - E[X_i]) > \varepsilon\right\} &\leq \exp\left[-\frac{\varepsilon}{2M}\operatorname{arsinh}\left(\frac{\varepsilon M}{2v^2}\right)\right] \\
\Pr\left\{\left|\sum_{i=1}^n (X_i - E[X_i])\right| > \varepsilon\right\} &\leq 2\exp\left[-\frac{\varepsilon}{2M}\operatorname{arsinh}\left(\frac{\varepsilon M}{2v^2}\right)\right]
\end{aligned}$$

(See `http://planetmath.org/AreaFunctions`here for the meaning of $\operatorname{arsinh}(x)$)