probability conditioning on a sigma algebra

Canonical name	ProbabilityConditioningOnASigmaAlgebra
Date of creation	2013-03-22 16:25:05
Last modified on	2013-03-22 16:25:05
Owner	CWoo (3771)
Last modified by	CWoo (3771)
Numerical id	7
Author	CWoo (3771)
Entry type	Definition
Classification	msc 60A99
Classification	msc 60A10

Let $(\Omega, \mathfrak{B}, \mu)$ be a probability space and $B \in \mathfrak{B}$ an event. Let $\mathfrak{D}$ be a sub sigma algebra of $\mathfrak{B}$. The *of B given $\mathfrak{D}$* is defined to be the conditional expectation of the random variable 1_B defined on Ω, given $\mathfrak{D}$. We denote this conditional probability by $\mu(B|\mathfrak{D}) := E(1_B|\mathfrak{D})$. 1_B is also known as the indicator function.

Similarly, we can define a conditional probability given a random variable. Let X be a random variable defined on Ω. The *conditional probability of B given X* is defined to be $\mu(B|\mathfrak{B}_X)$, where $\mathfrak{B}_X$ is the sub sigma algebra of $\mathfrak{B}$, `http://planetmath.org/MathcalFMeasurableFunction`generated by X. The conditional probability of B given X is simply written $\mu(B|X)$.

Remark. Both $\mu(B|\mathfrak{D})$ and $\mu(B|X)$ are random variables, the former is $\mathfrak{D}$-measurable, and the latter is $\mathfrak{B}_X$-measurable.