memoryless random variable

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A non-negative-valued random variable X is *memoryless* if $P(X > s+t \mid X > s) = P(X > t)$ for $s, t \geq 0$.

In words, given that a certain event did not occur during time period s *in the past*, the chance that an event will occur after an additional time period t *in the future* is the same as the chance that the event would occur after a time period t from the beginning, regardless of how long or how short the time period s is; the memory is *erased*.

From the definition, we see that

$$P(X > t) = P(X > s+t \mid X > s) = \frac{P(X > s+t \text{ and } X > s)}{P(X > s)} = \frac{P(X > s+t)}{P(X > s)},$$

so $P(X > s+t) = P(X > s)P(X > t)$ iff X is memoryless.

An example of a discrete memoryless random variable is the geometric random variable, since $P(X > s+t) = (1-p)^{s+t} = (1-p)^s(1-p)^t = P(X > s)P(X > t)$, where p is the probability of X=success. The exponential random variable is an example of a continuous memoryless random variable, which can be proved similarly with $1-p$ replaced by $e^{-\lambda}$. In fact, the exponential random variable is the only continuous random variable having the memoryless property.