Exercício 25, alínea a)

$$Re(z - iz) \geq 2 \Leftrightarrow Re(x + yi - xi + y) \geq 2 \Leftrightarrow Re[(x + y) + (y - x)i] \geq 2 \Leftrightarrow x + y \geq 2$$

$$\left|\frac{z}{z+1}\right| \leq 2 \Leftrightarrow |z| \leq 2|z+1| \Leftrightarrow x^2 + y^2 \leq 4((x+1)^2 + y^2)$$

$$\Leftrightarrow x^2 + y^2 \leq 4x^2 + 8x + 4 + 4y^2$$

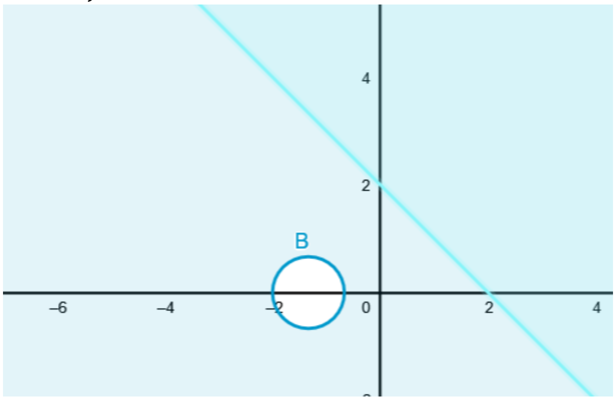
$$\Leftrightarrow 0 \leq 3x^2 + 8x + 4 + 3y^2$$

$$\Leftrightarrow x^2 + \frac{8}{3}x + \frac{16}{9} + y^2 \geq -\frac{4}{3} + \frac{16}{9}$$

$$\Leftrightarrow \left(x + \frac{4}{3}\right)^2 + y^2 \geq \frac{4}{9}$$

$$A \cap B = \left\{z = x + yi \in \mathbb{C}:\ x + y \geq 2\ \wedge\ \left(x + \tfrac{4}{3}\right)^2 + y^2 \geq \tfrac{4}{9}\right\}$$

$$= \{z = x + yi \in \mathbb{C}:\ x + y \geq 2\ \} = A$$



Exercício 25, alínea b)

$$w = \frac{1}{\overline{z}} \Leftrightarrow \overline{z} = \frac{1}{w} \Leftrightarrow z = \frac{1}{\overline{w}}$$

$$A = \{z = x + yi \in \mathbb{C}:\quad x + y \geq 2\ \} = \{z \in \mathbb{C}:\quad Re(z) + Im(z) \geq 2\ \}$$

$$A' = \left\{w \in \mathbb{C}:\quad Re\left(\frac{1}{\overline{w}}\right) + Im\left(\frac{1}{\overline{w}}\right) \geq 2\ \right\}$$

$$\frac{1}{\overline{w}} = \frac{w}{\overline{w} \times w} = \frac{w}{|w|^2} = \frac{x + yi}{x^2 + y^2} = \underbrace{\frac{x}{x^2 + y^2}}_{=Re(w)} + \underbrace{\frac{y}{x^2 + y^2}}_{=Im(w)} i$$

$$Re\left(\frac{1}{\overline{w}}\right) + Im\left(\frac{1}{\overline{w}}\right) \geq 2 \Leftrightarrow \frac{x}{x^2 + y^2} + \frac{y}{x^2 + y^2} \geq 2$$

$$\Leftrightarrow x + y \geq 2(x^2 + y^2)$$

$$\Leftrightarrow 2x^2 + 2y^2 - x - y \leq 0$$

$$\Leftrightarrow x^2 + y^2 - \frac{1}{2}x - \frac{1}{2}y \leq 0$$

$$\Leftrightarrow x^2 - \frac{1}{2}x + \frac{1}{16} + y^2 - \frac{1}{2}y + \frac{1}{16} \leq \frac{2}{16}$$

$$\Leftrightarrow \left(x - \frac{1}{4}\right)^2 + \left(y - \frac{1}{4}\right)^2 \leq \frac{1}{8}$$