IPG Politécnico da Guarda — Escola Superior de Tecnologia e Gestão	**ENUNCIADO DE AVALIAÇÃO**	MODELO PED.018.01

Curso	**Eng. Informática**					*Ano letivo*	2019/2020
Unidade curricular	**Álgebra e Geometria Analítica**						
Ano curricular	1º	*Semestre*	1º	*Data*	30/10/2019	*Duração*	1h 15m

1º Teste

(Cotação)

1. Considere os complexos $z = -\sqrt{3} + i$ e $w = 2\sqrt{3} + 2i$.

(2.0) **a)** Calcule os complexos $(z - w)$ e $(2z + \overline{w})$.

(3.0) **b)** Escreva z e w na forma trignométrica.

(3.0) **c)** Mostre que $\dfrac{w^2}{z^3} = -z$.

2. Determine os complexos que satisfazem as condições:

(4.0) **a)** $\overline{z} = 2z^3$.

(4.0) **b)** $|z + 1 + i| < |z - 2|$.

(4.0) **3.** Determine o transformado do conjunto,

$$S = \{z \in \mathbb{C} : |z - 1| < 2\},$$

por meio da transformação $w = \dfrac{1}{z - 3}$.

Formulário

$z=a+bi\,,\qquad w=c+di$	$a+bi=c+di \iff a=c \wedge b=d$
$z+w=(a+c)+(b+d)i$	$z\,w=(ac-bd)+(ad+bc)i$
$\bar{z}=a-bi$	$z^{-1}=\frac{1}{z\,\bar{z}}\,\bar{z}$
$\lvert z\rvert=\sqrt{a^2+b^2}$	$\arg(z)=\operatorname{arctg}\left(\frac{b}{a}\right)\in[0,2\pi]$
$z=\rho\operatorname{cis}(\theta)\,,\qquad w=\mu\operatorname{cis}(\alpha)$	$\rho\operatorname{cis}(\theta)=\mu\operatorname{cis}(\alpha) \iff \rho=\mu \wedge \alpha=\theta+2k\pi$
$\Big(\rho\operatorname{cis}(\theta)\Big).\Big(\mu\operatorname{cis}(\alpha)\Big)=\rho\mu\operatorname{cis}(\theta+\alpha)$	$\frac{\rho\operatorname{cis}(\theta)}{\mu\operatorname{cis}(\alpha)}=\frac{\rho}{\mu}\operatorname{cis}(\theta-\alpha)$
$\Big(\rho\operatorname{cis}(\theta)\Big)^n=\rho^n\operatorname{cis}(n\,\theta)$	$\sqrt[n]{\rho\operatorname{cis}(\theta)}=\sqrt[n]{\rho}\operatorname{cis}\left(\frac{\theta+2k\pi}{n}\right),\ k=0,..,n-1$

x	$\operatorname{sen}x$	$\cos x$	$\operatorname{tg}x$	$\operatorname{cot}x$
0	0	1	0	$(+\infty)$
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{2}$	1	0	$(+\infty)$	0

Resolução

1.

(2.0) **a)** $(z-w) = -\sqrt{3}+i-2\sqrt{3}-2i = -3\sqrt{3}-i$

$(2z+\overline{w}) = -2\sqrt{3}+2i+(2\sqrt{3}-2i) = -2\sqrt{3}+2i+2\sqrt{3}-2i = 0\,.$

(3.0) **b)** $\rho = \sqrt{1+3} = 2\,,\ \theta = \text{arctg}(-\frac{1}{\sqrt{3}}) = \frac{5\pi}{6}\,,\ z = 2\,\text{cis}(\frac{5\pi}{6})$

$\rho = \sqrt{12+4} = 4\,,\ \theta = \text{arctg}(\frac{1}{\sqrt{3}}) = \frac{\pi}{6}\,,\ w = 4\,\text{cis}(\frac{\pi}{6})\,.$

(3.0) **c)** $\dfrac{w^2}{z^3} = \dfrac{\left(4\,\text{cis}(\frac{\pi}{6})\right)^2}{\left(2\,\text{cis}(\frac{5\pi}{6})\right)^3} = \dfrac{4^2\,\text{cis}(\frac{2\pi}{6})}{2^3\,\text{cis}(\frac{15\pi}{6})} = \dfrac{16}{8}\text{cis}(-\frac{13\pi}{6}) = 2\,\text{cis}(-\frac{\pi}{6}) = 2(\frac{\sqrt{3}}{2}-\frac{1}{2}i) = -z\,.$

2.

(4.0) **a)** $\bar{z} = 2z^3 \iff \rho = 2\rho^3 \,\wedge\, -\theta = 3\theta+2k\pi \iff \rho(1-2\rho^2) = 0 \,\wedge\, -4\theta = 2k\pi \iff$

$(\rho = 0 \vee 1-2\rho^2 = 0) \,\wedge\, \theta = \frac{k\pi}{2} \iff \Big((\rho = 0 \vee \rho = \pm\frac{\sqrt{2}}{2}) \wedge \theta = \frac{k\pi}{2}\Big) \iff$

$z = 0\,,\ z = \frac{\sqrt{2}}{2}\text{cis}(0)\,,\ z = \frac{\sqrt{2}}{2}\text{cis}\left(\frac{\pi}{2}\right),\ z = \frac{\sqrt{2}}{2}\text{cis}(\pi)\,,\ z = \frac{\sqrt{2}}{2}\text{cis}\left(\frac{3\pi}{2}\right) \iff$

$z = 0\,,\ z = \frac{\sqrt{2}}{2}\,,\ z = \frac{\sqrt{2}}{2}i\,,\ z = -\frac{\sqrt{2}}{2}\,,\ z = -\frac{\sqrt{2}}{2}i$

(4.0) **b)** $|z+(1+i)|^2 = |z-2|^2 \iff |(x+1)+(y+1)i|^2 = |(x-2)+(y)i|^2 \iff$

$(x+1)^2+(y+1)^2 = (x-2)^2+y^2 \iff -2+6x+2y = 0 \iff y = 1-3x$

(4.0) **3.** $w = \dfrac{1}{z-3} \iff wz-3w = 1 \iff z = \dfrac{1+3w}{w}$

$\left|\dfrac{1+3w}{w}-1\right|^2 < 4 \iff |1+3w-w|^2 < 4|w|^2 \iff |1+2w|^2 < 4|w|^2 \iff$

$(1+2x)^2+(2y)^2 < 4(x^2+y^2) \iff 1+4x < 0 \iff x < -\frac{1}{4} \quad \longrightarrow \quad S' = \left\{w \in \mathbb{C} : \mathit{Re}(w) < -\frac{1}{4}\right\}$