Números Complexos

Exercício 1

a) $4+2i$; *b)* $\frac{11}{17}-\frac{10i}{17}$; *c)* $\frac{1}{10}+\frac{7i}{10}$;

d) 1; *e)* $1-2i$; *f)* $\frac{47}{61}-\frac{13i}{61}$.

Exercício 2

a) $-\frac{61}{17}-\frac{227i}{17}$; *b)* $\frac{3}{2}-\frac{i}{2}$.

Exercício 3

a) $\sqrt{3}+(88-14i)$, $88+\sqrt{3}$; *b)* $-\frac{4}{7}-\frac{6\sqrt{3}}{7}+i\left(\frac{2\sqrt{3}}{7}-\frac{12}{7}\right)$, $\frac{2\sqrt{3}}{7}-\frac{12}{7}$.

Exercício 5

a) $1-i$; *b)* 1.

Exercício 6

a) $-\frac{244}{13}-\frac{1194i}{13}$; *b)* $\frac{s}{4s^2+1}+i\left(\frac{2s^2}{4s^2+1}+\frac{1}{4s^2+1}-\frac{3}{4}\right)$.

Exercício 8

a) $|z|=2, \quad \arg(z)=0$; *b)* $|z|=2, \quad \arg(z)=\frac{\pi}{2}$; *c)* $|z|=3, \quad \arg(z)=\pi$;

d) $|z|=4, \quad \arg(z)=\frac{3\pi}{2}$; *e)* $|z|=2\sqrt{2} \quad \arg(z)=\frac{3\pi}{4}$; *f)* $|z|=\sqrt{2}, \quad \arg(z)=\frac{5\pi}{4}$;

g) $|z|=\sqrt{2}, \quad \arg(z)=\frac{7\pi}{4}$; *h)* $|z|=1, \quad \arg(z)=\frac{11\pi}{6}$; *i)* $|z|=\sqrt{2}, \quad \arg(z)=\frac{3\pi}{4}$;

j) $|z|=2, \quad \arg(z)=\frac{3\pi}{4}$.

Exercício 9

a) -1;

b) $-5I$;

c) $\frac{1}{\sqrt{2}} + i\sqrt{\frac{3}{2}}$;

d) $-\frac{3}{2} - \frac{3i\sqrt{3}}{2}$;

e) $\frac{3}{2} + \frac{3i\sqrt{3}}{2}$;

f) $-\sqrt{2}$;

g) $4i$;

h) $-\frac{\sqrt{3}}{2} + \frac{i}{2}$.

Exercício 10

a) $z = 4\operatorname{cis}(\frac{7\pi}{6})$;

$w = 2\operatorname{cis}(\frac{2\pi}{3})$;

$zw = 8\operatorname{cis}(\frac{11\pi}{6})$;

$\frac{z}{w} = 2\operatorname{cis}(\frac{\pi}{2})$;

b) $z = \sqrt{2}\operatorname{cis}(\frac{7\pi}{4})$;

$w = 2\sqrt{3}\operatorname{cis}(\frac{\pi}{3})$;

$zw = 2\sqrt{6}\operatorname{cis}(\frac{\pi}{12})$;

$\frac{z}{w} = \frac{1}{\sqrt{6}}\operatorname{cis}(\frac{17\pi}{12})$.

Exercício 11 $3\operatorname{cis}(\frac{3\pi}{2})$.

Exercício 12

a) $1^{10}\operatorname{cis}(10.\frac{2\pi}{3}) = \operatorname{cis}(\frac{20\pi}{3}) = \operatorname{cis}(\frac{2\pi}{3})$;

b) $1^{10}\operatorname{cis}(10.(-\frac{\pi}{6})) = \operatorname{cis}(\frac{\pi}{3})$;

c) $1^{20}\operatorname{cis}(20.(-\frac{\pi}{4})) = \operatorname{cis}(\pi)$;

d) $1^{30}cis(30.\frac{\pi}{6}) = \operatorname{cis}(\pi)$.

Exercício 13 $8\operatorname{cis}(\frac{13\pi}{12})$.

Exercício 14

a) $12\operatorname{cis}(\frac{7\pi}{6})$;

b) $\frac{1}{2}\operatorname{cis}(\frac{\pi}{3})$.

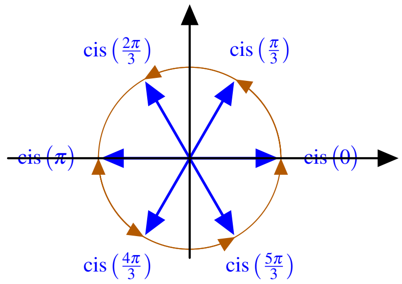
Exercício 15

a)
$$\begin{aligned}
\bar{z} = z^3 &\iff \rho\operatorname{cis}(-\theta) = \rho^3\operatorname{cis}(3\theta)\\
&\iff \rho = \rho^3 \ \wedge\ -\theta = 3\theta + 2k\pi,\, k \in \mathbb{Z}\\
&\iff \rho(1-\rho^2) = 0 \ \wedge\ -4\theta = 2k\pi\\
&\iff (\rho = 0 \ \vee\ \rho = \pm 1) \ \wedge\ \theta = \tfrac{k\pi}{2}\\
&\iff (\rho = 0 \ \vee\ \rho = 1) \ \wedge\ \theta = \tfrac{k\pi}{2};\\
&\iff z = 0 \ \vee\ z = 1\operatorname{cis}\left(\tfrac{k\pi}{2}\right),\, k = 0, 1, 2, 3.
\end{aligned}$$

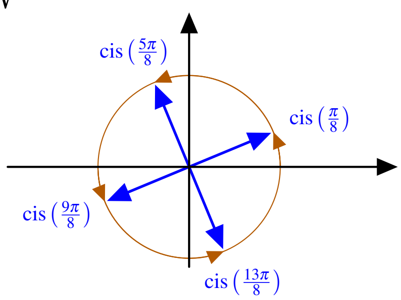
b) $z^2 = |z|^2 \iff \rho^2 \operatorname{cis}(2\theta) = \rho^2 \operatorname{cis}(0)$
$\iff \rho^2 = \rho^2 \ \wedge \ 2\theta = 2k\pi$
$\iff \theta = k\pi\,,\ k \in \mathbb{Z}$

Exercício 16

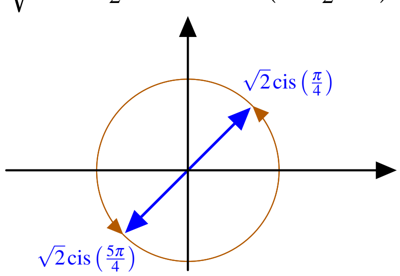
a) $\sqrt[6]{1\operatorname{cis}(0)} = 1\operatorname{cis}\left(\frac{0+2k\pi}{6}\right) = \operatorname{cis}\left(\frac{k\pi}{3}\right)\,,\ k = 0,\,1,\,2,\,3,\,4,\,5\,.$



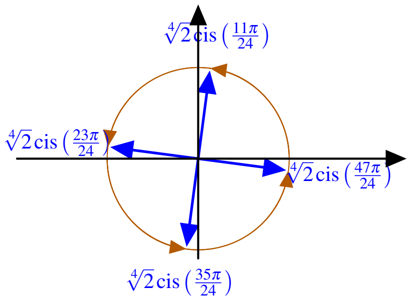
b) $\sqrt[4]{1\operatorname{cis}(\frac{\pi}{2})} = 1\operatorname{cis}\left(\frac{\pi/2+2k\pi}{4}\right) = \operatorname{cis}\left(\frac{(1+4k)\pi}{8}\right)\,,\ k = 0,\,1,\,2,\,3\,.$



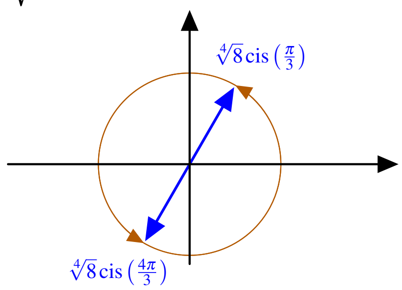
c) $\sqrt{2\operatorname{cis}(\frac{\pi}{2})} = \sqrt{2}\operatorname{cis}\left(\frac{\pi/2+2k\pi}{2}\right) = \sqrt{2}\operatorname{cis}\left(\frac{(1+4k)\pi}{4}\right)\,,\ k = 0,\,1\,.$



d) $\sqrt[4]{2\operatorname{cis}(\frac{11\pi}{6})} = \sqrt[4]{2}\operatorname{cis}\left(\frac{11\pi/6+2k\pi}{4}\right) = \sqrt[4]{2}\operatorname{cis}\left(\frac{(11+12k)\pi}{24}\right)\,,\ k = 0,\,1,\,2,\,3\,.$



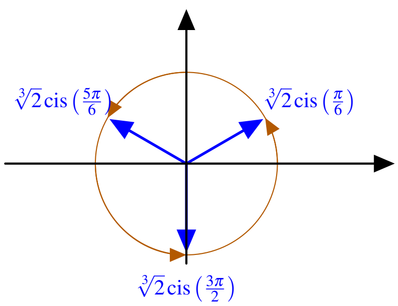
e) $\sqrt{\sqrt{8}\operatorname{cis}(\frac{2\pi}{3})} = \sqrt[4]{8}\operatorname{cis}\left(\frac{2\pi/3+2k\pi}{2}\right) = \sqrt[4]{8}\operatorname{cis}\left(\frac{(2+6k)\pi}{6}\right)\,,\ k = 0,\, 1\,.$



Exercício 17

a) $\sqrt[3]{2\operatorname{cis}(\frac{\pi}{2})} = \sqrt[3]{2}\operatorname{cis}\left(\frac{\pi/2+2k\pi}{3}\right) = \sqrt[3]{2}\operatorname{cis}\left(\frac{(1+4k)\pi}{6}\right)\,,\ k = 0,\, 1,\, 2\,.$

$= \sqrt[3]{2}\operatorname{cis}\left(\frac{\pi}{6}\right) \vee \sqrt[3]{2}\operatorname{cis}\left(\frac{5\pi}{6}\right) \vee \sqrt[3]{2}\operatorname{cis}\left(\frac{3\pi}{2}\right)\,.$

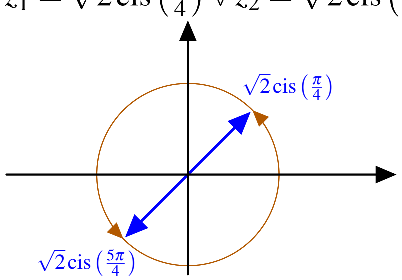


b) $\sqrt[6]{2\operatorname{cis}(\frac{\pi}{2})} = \sqrt[6]{2}\operatorname{cis}\left(\frac{\pi/2+2k\pi}{6}\right) = \sqrt[6]{2}\operatorname{cis}\left(\frac{(1+4k)\pi}{12}\right)\,,\ k = 0,\, 1,\, 2,\, 3,\, 4,\, 5\,.$

$\left(\sqrt[6]{2\operatorname{cis}(\frac{\pi}{2})}\right)^3 = (\sqrt[6]{2})^3\operatorname{cis}\left(3\frac{(1+4k)\pi}{12}\right)\,,\ k = 0,\, 1,\, 2,\, 3,\, 4,\, 5\,.$

$= \sqrt{2}\operatorname{cis}\left(\frac{(1+4k)\pi}{4}\right)\,,$

$z_1 = \sqrt{2}\operatorname{cis}\left(\frac{\pi}{4}\right) \vee z_2 = \sqrt{2}\operatorname{cis}\left(\frac{5\pi}{4}\right) \vee z_3 = \sqrt{2}\operatorname{cis}\left(\frac{9\pi}{4}\right) = z_1\sqrt{2}\operatorname{cis}\left(\frac{\pi}{4}\right)\,.$



Exercício 18

a) $z = -1 \vee z = 0 \vee z = 1 \vee z = -\frac{1}{2} - \frac{i\sqrt{3}}{2} \vee z = \frac{1}{2} - \frac{i\sqrt{3}}{2} \vee z = -\frac{1}{2} + \frac{i\sqrt{3}}{2} \vee z = \frac{1}{2} + \frac{i\sqrt{3}}{2}$

b) $z = \sqrt[8]{2}\operatorname{cis}\left(\frac{7\pi}{8}\right) \vee z = \sqrt[8]{2}\operatorname{cis}\left(\frac{15\pi}{8}\right)$

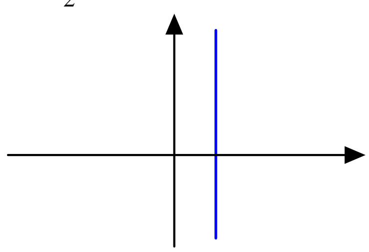
c) $z=\sqrt{8}\operatorname{cis}\left(\frac{7\pi}{4}+\frac{2k\pi}{3}\right), k=0,1,2. \Longleftrightarrow$[1] $z=2+2i \vee z=-1+\sqrt{3}+i\left(1+\sqrt{3}\right) \vee z=-1-\sqrt{3}+i\left(1-\sqrt{3}\right)$

d) $z_1=-\sqrt{2}+\sqrt{2}i \vee z_2=\sqrt{2}-\sqrt{2}i$

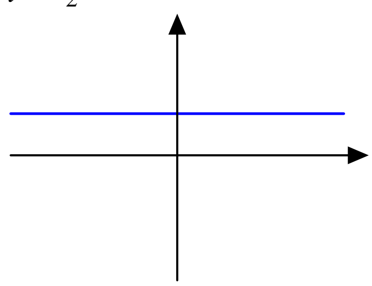
e) $z_1=\frac{3-\sqrt{17}}{4}(1+i) \vee z_2=\frac{3+\sqrt{17}}{4}(1+i)$

Exercício 19

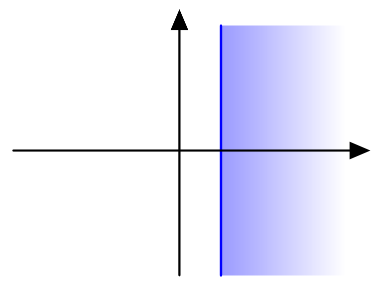
a) $x=\frac{1}{2}$



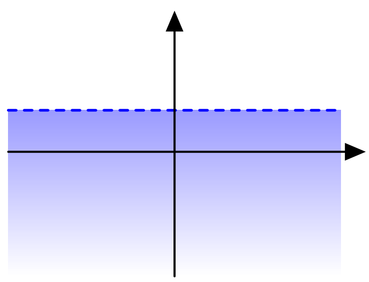
b) $y=\frac{1}{2}$



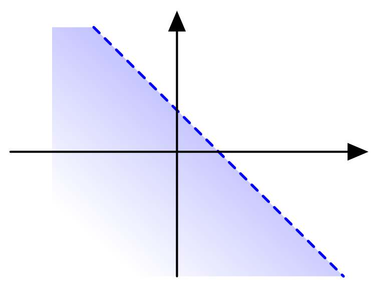
c) $x \geq a$



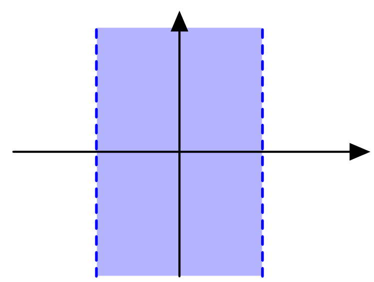
d) $y<b$



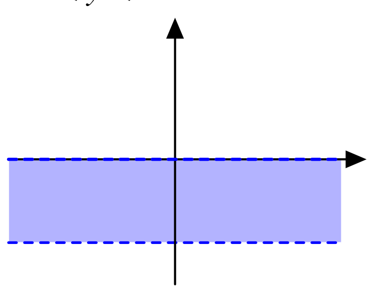
e) $y<1-x$



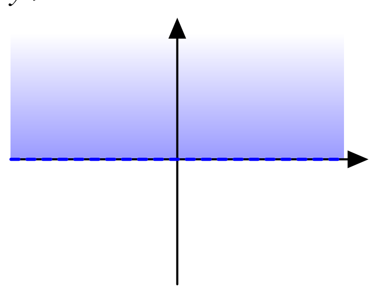
f) $-1<x<1$



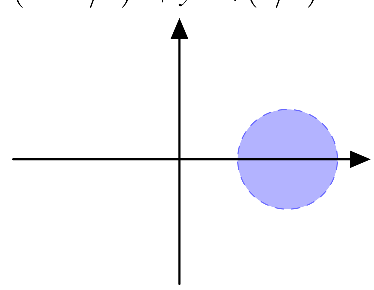
g) $-1<y<0$



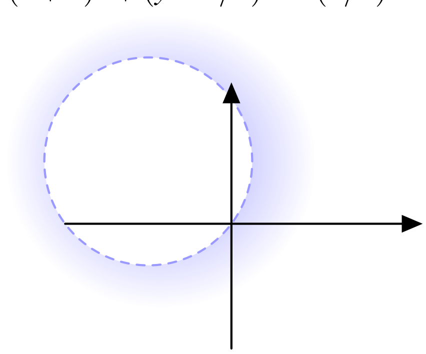
h) $y>0$



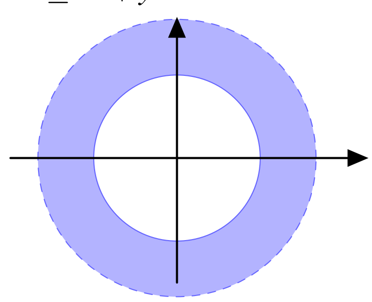
i) $(x-4/3)^2+y^2<(2/3)^2$



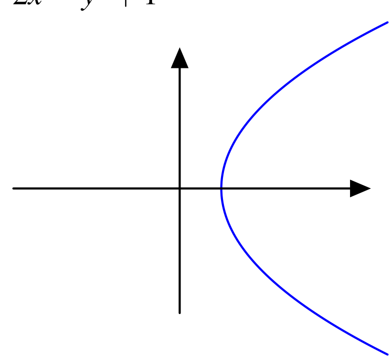
j) $(x+2)^2+(y-3/2)^2>(5/2)^2$



k) $3^2 \leq x^2+y^2<5^2$

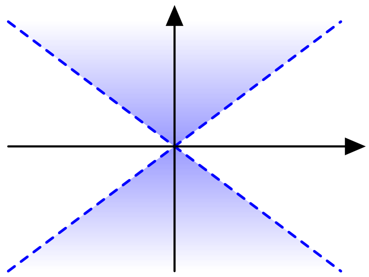


l) $2x=y^2+1$

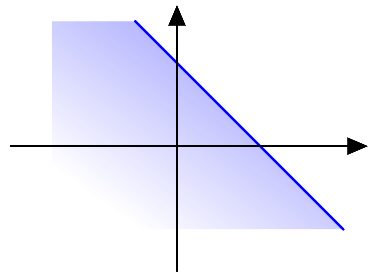


[1] Considerando as fórmulas $\operatorname{sen}(\alpha+\beta)=\operatorname{sen}(\alpha)\cos(\beta)+\operatorname{sen}(\beta)\cos(\alpha), \quad \cos(\alpha+\beta)=\cos(\alpha)\cos(\beta)-\operatorname{sen}(\alpha)\operatorname{sen}(\beta)$

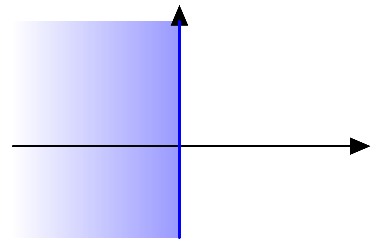
m) $y^2 > x^2$



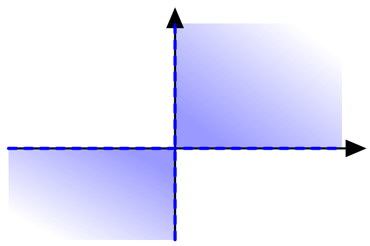
n) $y \leq 2 - x$



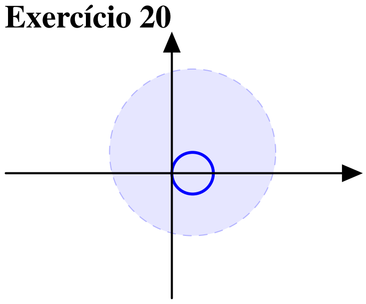
o) $x \leq 0$



p) $2xy > 0$



Exercício 20



Exercício 21

$5\sqrt{\frac{2}{13}}$

Exercício 22

$x = 0$

Exercício 23

a) $A' = \{z \in \mathbb{C} : Im(z) = 0\}$

b) $B' = \{z \in \mathbb{C} : |z| < 1\}$

Exercício 24

a) $A' = \{z \in \mathbb{C} : x^2 + (y+1)^2 < 1\}$

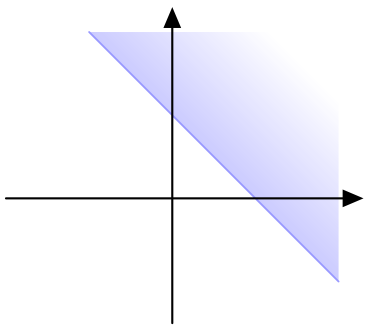
b) $A' = \{z \in \mathbb{C} : 3\left(x - \frac{2}{3}\right)^2 + 3\left(y - \frac{1}{3}\right)^2 \leq \frac{2}{3}\}$

c) $A' = \{z \in \mathbb{C} : x^2 - 2x + y^2 - 1 = 0\}$

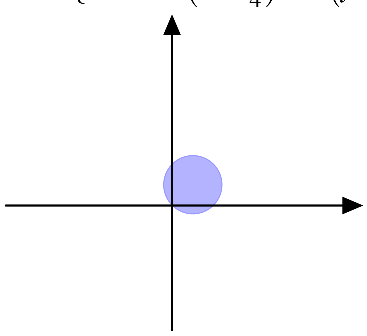
d) $A' = \{z \in \mathbb{C} : x^2 + y^2 > 1 \wedge x < 0\}$

Exercício 25

a) $A = \{z \in \mathbb{C} : x + y \geq 2\}$, $B = \{z \in \mathbb{C} :: \left(x + \frac{4}{3}\right)^2 + y^2 \geq \frac{4}{9}\}$, $A \cap B = \{z \in \mathbb{C} : x + y \geq 2\}$

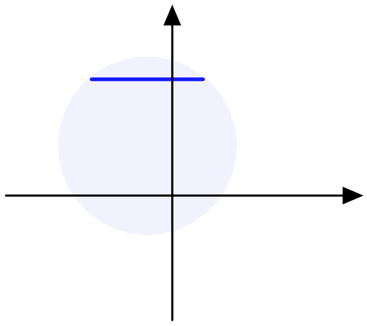


b) $A' = \{z \in \mathbb{C} : \left(x - \frac{1}{4}\right)^2 + \left(y - \frac{1}{4}\right)^2 \leq \frac{1}{8}\}$

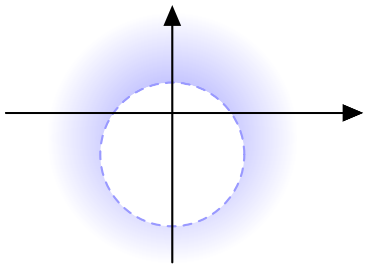


Exercício 26

a) $A = \{z \in \mathbb{C} : (x+1)^2 + (y-2)^2 < 9\}$, $B = \{z \in \mathbb{C} : y = 2\}$, $A \cap B = \{z \in \mathbb{C} : y = 2 \wedge -4 < x < 2\}$

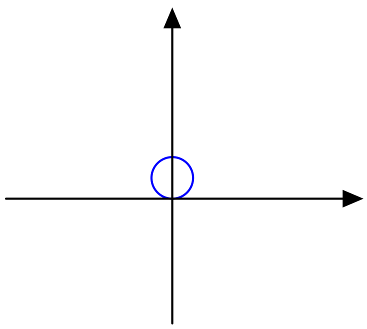


b) $A' = \{z \in \mathbb{C} : (y+\frac{2}{5})^2 + x^2 > \frac{9}{25}\}$

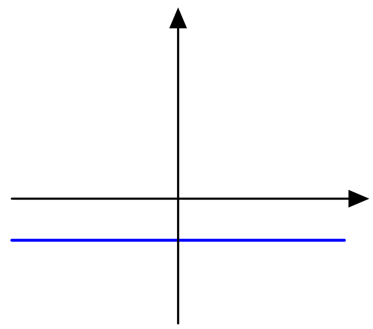


Exercício 27

a) $A = \{z \in \mathbb{C} : x^2 + (y+1/2)^2 = 1/4\}$

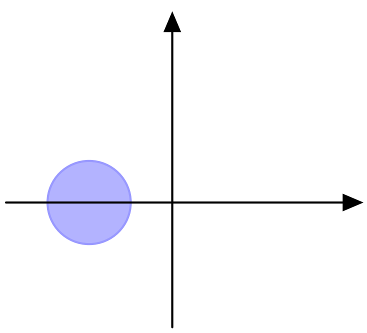


b) $A' = \{z \in \mathbb{C} : y = -1\}$

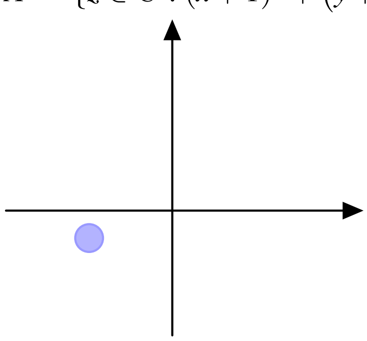


Exercício 28

a) $A = \{z \in \mathbb{C} : (x+4)^2 + y^2 \leq 4 \wedge \frac{1}{x^2+y^2} \leq 3\} = \{z \in \mathbb{C} : (x+4)^2 + y^2 \leq 4\}$

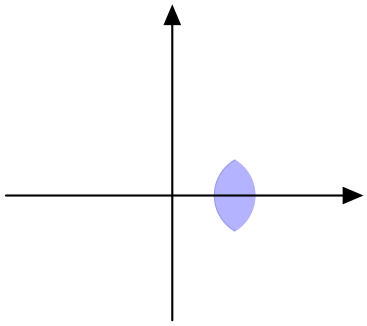


b) $A' = \{z \in \mathbb{C} : (x+1)^2 + \left(y+\frac{1}{3}\right)^2 \leq \frac{1}{36} \wedge (x+1)^2 + y^2 \leq 2\} = \{z \in \mathbb{C} : (x+1)^2 + \left(y+\frac{1}{3}\right)^2 \leq \frac{1}{36}\}$

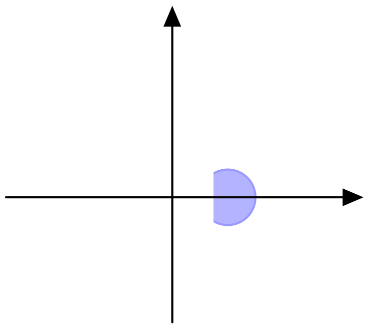


Exercício 29

a) $A = \{z \in \mathbb{C} : (x-1)^2 + y^2 \leq 1 \wedge (x-2)^2 + y^2 \leq 1\}$

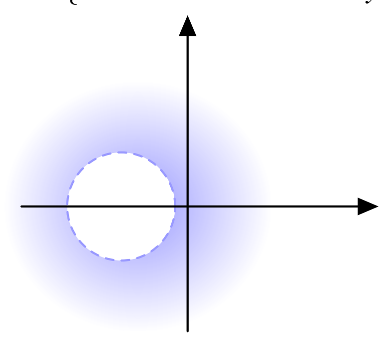


b) $A' = \{z \in \mathbb{C} : 3\left(x - \frac{2}{3}\right)^2 + 3y^2 \leq \frac{1}{3} \wedge x > \frac{1}{2}\}$

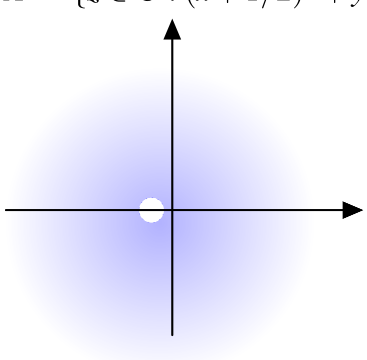


Exercício 30

a) $A = \{z \in \mathbb{C} : 9 + 10x + x^2 + y^2 > 0 \wedge (x+3)^2 + y^2 > 0\} = \{z \in \mathbb{C} : (x+5)^2 + y^2 > 16\}$

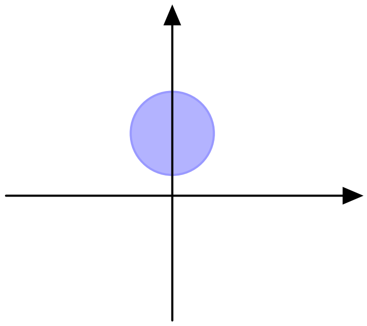


b) $A' = \{z \in \mathbb{C} : (x+1/2)^2 + y^2 > \frac{1}{16}\}$

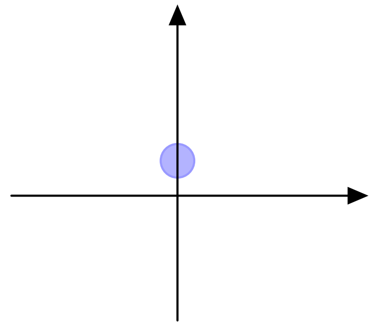


Exercício 31

- $C = \{z \in \mathbb{C} : x^2 + (y-3)^2 \leq 4\}$

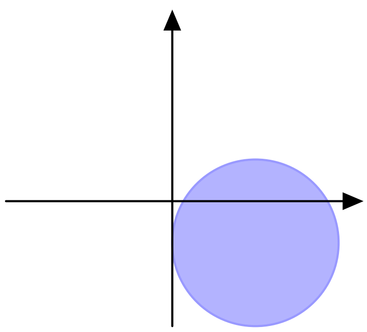


- $C' = \{z \in \mathbb{C} : x^2 + \left(y - \frac{3}{5}\right)^2 \leq \frac{4}{25}\}$



Exercício 32

- $A = \{z \in \mathbb{C} : (x-2)^2 + (y+1)^2 \leq 4\}$



- $A' = \{z \in \mathbb{C} : 2x + 2y \geq 3\}$