Oficina_2 - Cálculos

(2) Funsão de transferência do sistema

$m\ddot{x} + c\dot{x} + Kx = f(t)$ Domínio do tempo

Considerando condisões iniciais nulas $\frac{d^n}{dx} x(t) = s^n X(s)$

$ms^2X(s) + csX(s) + KX(s) = F(s)$

$X(s)[ms^2 + cs + K] = F(s)$

$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + cs + K}$ Domínio de Laplace

(3). Funsão de transferência do sistema

$m\ddot{x} + c\dot{x} + Kx = c\dot{y} + Ky$ Domínio do tempo

Condisões iniciais nulas

$ms^2X(s) + csX(s) + KX(s) = csY(s) + KY(s)$

$X(s)[ms^2 + cs + K] = Y(s)[cs + K]$

$\frac{X(s)}{Y(s)} = \frac{cs + K}{ms^2 + cs + K} = G(s)$ Domínio de laplace

. Resposta temporal $x(t)$, $m = 1$, $c = 2$, $K = 4$

$$X(s) = \underbrace{\frac{2s + 4}{(s^2 + 2s + 4)}}_{G(s)} \times \underbrace{\frac{2}{s^2 + 1}}_{Y(s)} = \frac{A}{\left(s - \frac{1}{2} + j\sqrt{3}\right)} + \frac{B}{\left(s - \frac{1}{2} - j\sqrt{3}\right)} + \frac{C}{(s + j)} + \frac{D}{(s - j)}$$

$s = \frac{-2 \pm \sqrt{4 - 4 \times 1 \times 4}}{2}$

$s = -\frac{1}{2} \pm j\sqrt{3}$

④ $F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)\, dt$

$f(t) = \begin{cases} 0, & t < 0 \\ 2t, & 0 \le t \le 0,5 \\ 1, & t > 0,5 \end{cases}$

$f(t) = mt + b$

$m = \frac{f(0,5) - f(0)}{0,5 - 0} = 2$

$F(s) = \underbrace{\int_0^{0,5} e^{-st} \cdot 2t\, dt}_{F_1(s)} + \underbrace{\int_{0,5}^{\infty} e^{-st}\, dt}_{F_2(s)}$

$\int u\,dv = uv - \int v\,du$

$-st = u \quad du = -s\,dt$

$\frac{du}{-s} = dt$

$F_1(s) = 2 \int_0^{0,5} t e^{-st}\, dt \xrightarrow{\text{Integração por partes}} F_1(s) = 2 \cdot \left[-\frac{t e^{-st}}{s} \Big|_0^{0,5} + \frac{1}{s} \int_0^{0,5} e^{-st}\, dt \right]$

$u = t \quad du = dt$

$v = -\frac{e^{-st}}{s} \quad dv = e^{-st}$

$F_1(s) = 2 \left[-\frac{0,5 e^{-0,5s}}{s} - \frac{1}{s^2} e^{-st} \Big|_0^{0,5} \right]$

$F_1(s) = 2 \left[-\frac{0,5 e^{-0,5s}}{s} - \frac{1}{s^2} \left(e^{-0,5s} - 1 \right) \right]$

$F_2(s) = \int_{0,5}^{\infty} e^{-st}\, dt = -\frac{1}{s} e^{-st} \Big|_{0,5}^{\infty} = -\frac{1}{s} \left(0 - e^{-0,5s} \right) = \frac{e^{-0,5s}}{s}$

$F(s) = \frac{-e^{-0,5s}}{s} + \frac{2}{s^2} \left(1 - e^{-0,5s} \right) + \frac{e^{-0,5s}}{s} = \frac{2 \left(1 - e^{-0,5s} \right)}{s^2}$

⑤ Intervalo de 0 - 1 $f(t) = mt + b$

$$m = \frac{f(1) - f(0)}{t_1 - t_0} = \frac{2 - 0}{1 - 0} = 2 \quad , \quad f(t) = 2t$$

Intervalo de 1 - 2

$$m = \frac{f(2) - f(1)}{t_2 - t_1} = \frac{0 - 2}{2 - 1} = -2 \quad , \quad f(t) = -2t + b = -2t + 4$$

$$0 = -2 \cdot 2 + b$$

$$b = 4$$

$$f(t) = \begin{cases} 0, & t < 0 \\ 2t, & 0 \leq t \leq 1 \\ -2t + 4 & 1 \leq t \leq 2 \\ 0, & t > 2 \end{cases}$$

$$u = -st$$

$$-\frac{du}{s} = dt$$

$$F(s) = \underbrace{\int_0^1 2t\, e^{-st} dt}_{F_1(s)} + \underbrace{\int_1^2 -2t\, e^{-st} dt}_{F_2(s)} + \underbrace{4\int_1^2 e^{-st} dt}_{F_3(s)}$$

Aproveitando a Resposta da questão anterior

$$F_1(s) = \frac{2(1 - e^{-0,5s})}{s^2}$$

$$F_2(s) = -\frac{2(1 - e^{-0,5s})}{s^2}$$

$$F_3(s) = -\frac{4}{s} e^{-st}\Big|_1^2 = -\frac{4}{s}(e^{-2t} - e^{-t})$$

$$F(s) = -\frac{4}{s}(e^{-2t} - e^{-t})$$

(b) Temos uma função quadrática

$F(t) = at^2 + bt + c$

$c \rightarrow$ Termo constante : interseção da curva com o eixo y

$c = 0$

$a \rightarrow$ coeficiente quadrático

$a < 0$ concavidade para baixo

$b \rightarrow$ coeficiente linear

Pontos no gráfico

1. Origem : $(t_1, f_1) = (0,0)$ $\rightarrow a(0)^2 + b(0) + c = 0 \rightarrow c = 0$
2. Vértice : $(t_v, f_v) = (0{,}5, 1)$ $\rightarrow a(0{,}5)^2 + b(0{,}5) + c = 0 \rightarrow 0{,}25a + 0{,}5b = 0$
3. Ponto : $(t_2, f_2) = (1, 0)$ $\rightarrow a(1)^2 + b(1) + c = 0 \rightarrow a + b = 1$

Sistema de equações

$$\begin{cases} a + b = 0 \\ 0{,}25a + 0{,}5b = 1 \end{cases}$$

$x - 0{,}25$

$0{,}25b = 1 \rightarrow b = 4$

$a + 4 = 0$

$a = -4$

Equação da parábola $f(t) = -4t^2 + 4t$

$$\int u\,dv = uv\Big| - \int v\,du$$

$$F(s) = -\int_0^1 4t^2 e^{-st}\,dt + \underbrace{\int_0^1 4t\,e^{-st}\,dt}_{F_2(s)}$$

$F_1(s)$

$$F_1(s) = -4\int_0^1 t^2 e^{-st}\,dt = 4\left[-\frac{t^2 e^{-st}}{s}\Big|_0^1 - \frac{2}{s}\int_0^1 t\,e^{-st}\,dt\right] = 4\left[-\frac{t^2 e^{-st}}{s}\Big|_0^1 + \frac{2te^{-st}}{s^2}\Big|_0^1 - \int_0^1 \frac{e^{-st}}{s}\,dt\right]$$

$u = t^2$ $\quad dv = e^{-st}$

$du = 2t\,dt$ $\quad v = -\frac{e^{-st}}{s}$

$u = t$ $\quad dv = e^{-st}dt$

$du = dt$ $\quad v = -\frac{e^{-st}}{s}$

$$F_1(s) = 4\left[-\frac{t^2 e^{-st}}{s}\Big|_0^1 + \frac{2te^{-st}}{s^2}\Big|_0^1 - \frac{1}{s^2}e^{-st}\Big|_0^1\right] = 4\left[-\frac{t^2e^{-t}}{s} + \frac{2e^{-t}}{s^2} - \frac{(e^{-t}-1)}{s^2}\right]$$

$$F_2(s) = 4\left[-\frac{te^{-st}}{s}\Big|_0^1 + \frac{1}{s}\int_0^1 e^{-st}\right] = 4\left[-\frac{e^{-t}}{s} - \frac{1}{s^2}(e^{-t}-1)\right]$$

$$F(s) = \frac{4t^2e^{-t}}{s} + \frac{2e^{-t}}{s^2} - \frac{4(e^{-t}-1)}{s^2} - \frac{4e^{-t}}{s} - \frac{4(e^{-t}-1)}{s^2} = \frac{4}{s^2} - \frac{8}{s^3}$$

① $[0, t_1]$ Reta Crescente $f(t) = mt + b$

$b = 0$

$m = \frac{F_0 - 0}{t_1 - 0} = \frac{F_0}{t_1}$ $f(t) = \frac{F_0}{t_1} t$

$[t_1, t_2]$ - F_0 Constante

$[t_2, t_3]$ Reta Decrescente $f(t) = mt + b$

Pontos (t_2, F_0) $F_0 = mt_2 + b$ $= mt_2 - mt_3 = m(t_2 - t_3)$

$(t_3, 0)$ $0 = mt_3 + b$

$b = -mt_3$

$m = \frac{F_0}{t_2 - t_3}$, $b = -\left(\frac{F_0}{t_2 - t_3}\right) t_3 = \frac{-F_0 t_3}{(t_2 - t_3)}$

$f(t) = mt + b$

$f(t) = \left(\frac{F_0}{t_2 - t_3}\right) t - \frac{F_0 t_3}{(t_2 - t_3)} = \frac{F_0}{(t_2 - t_3)} (t - t_3) = \frac{F_0 (t_3 - t)}{(t_3 - t_2)}$