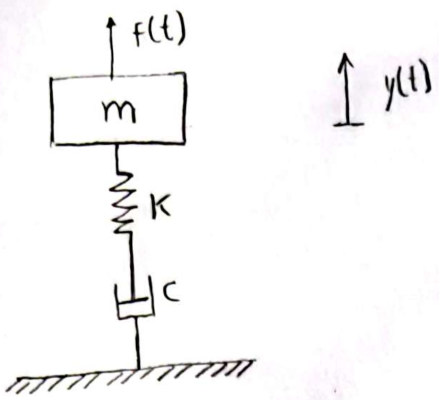
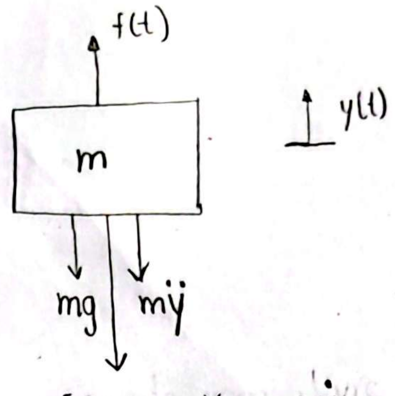
Tutorial 3 - Modelagem de Sistemas

1) massa - mola - amortecedor





$f_m = f_d = Ky = c\dot{y}$

- Sistema massa-mola-amortecedor

Lei de Hooke $F_m = -K\cdot x$

Força do amortecedor $F_d = -c\cdot\dot{x}$
(Damping)

- Como estão em série, as forças são iguais

$F_m = K(x-y)$ $\quad K(x-y) = c\dot{y} \rightarrow y = x - \frac{c}{K}\dot{y}$

$F_d = c\dot{y}$

- Diferenciando y

$$\frac{dy}{dt} = \frac{dx}{dt} - \frac{c}{K}\frac{d^2y}{dt^2} \rightarrow \dot{y} = \dot{x}(t) - \frac{c}{K}\ddot{y}$$

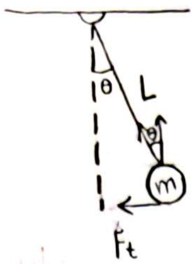
- A força resultante é $m\ddot{x} = f(t) - F_m$

$$m\ddot{x} = f(t) - c\left[\dot{x} - \frac{c}{K}\ddot{y}\right] \rightarrow m\ddot{x} + c\dot{x} - \frac{c^2}{K}\ddot{y} = f(t)$$

$$\frac{mc}{K}\dddot{x} + m\ddot{x} + c\dot{x} + Kx = f(t) + \frac{c}{K}\dot{f}$$

O sistema possui 2 graus de liberdade

② Pêndulo simples



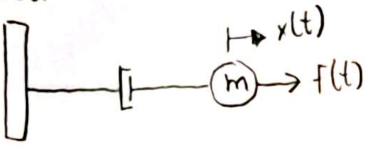
$$\sum F_t = m \cdot \ddot{x}$$

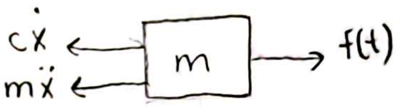
$$-mg\sin(\theta(t)) = m(L\ddot{\theta}(t))$$

$$L\ddot{\theta}(t) + g\sin(\theta(t)) = 0$$

$$\ddot{\theta}(t) + \frac{g}{L}\sin(\theta(t)) = 0$$

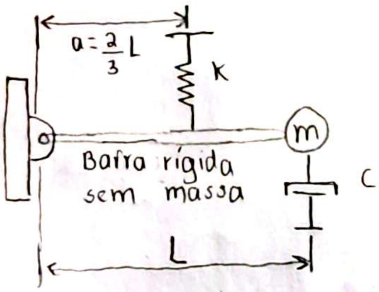
Massa - Amortecedor

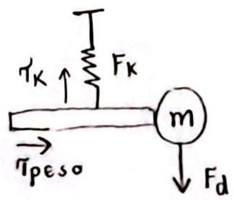




$$m\ddot{x}(t) + c\dot{x}(t) = f(t)$$

Sistema de suspensão





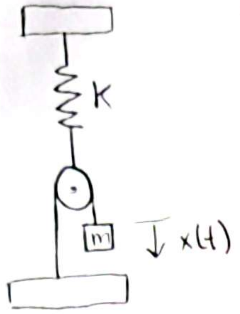
$$J \cdot \ddot{\theta}(t) = \sum \tau$$

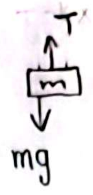
$$\tau_{peso} = -mgL\theta(t) \qquad \tau_K = -Kx(t) = -Ka\theta(t) = -K\left(\frac{4}{9}L^2\theta(t)\right)$$

$$\tau_c = -c\dot{x}_c(t) = -cL^2\dot{\theta}(t)$$

$$mL^2\ddot{\theta}(t) + cL^2\dot{\theta}(t) + \left[mgL + \frac{4}{9}KL^2\right]\theta(t) = 0$$

Sistema com polia





$$\sum F = m\ddot{x}$$

$$F_m = -K\,x(t) \qquad -K\,x(t) = 2T \qquad T = \frac{-K\,x(t)}{2}$$

$$mg + T = m\ddot{x}$$

$$mg - \frac{K\,x(t)}{2} = m\ddot{x} \quad \rightarrow \quad m\ddot{x} + \frac{K\,x(t)}{2} - mg = 0$$

3) Linearização para pequenas oscilações

$$\sin(\theta(t)) = \theta(t)$$

$$\ddot{\theta}(t) + \frac{g}{L}\,\theta(t) = 0 \quad \rightarrow \text{EDO Linear}$$