A Location-Allocation-Improvement Matheuristic for a Territorial Design Problem in Microfinance Institutions

Eduardo Salazar-Treviño
(AMPL Optimization Inc., Universitat Politècnica de Catalunya [soon])
Roger Z. Ríos-Mercado
(Graduate Program in Electrical Engineering, Universidad Autónoma de Nuevo León)
Diana L. Huerta-Muñoz
(Independent Researcher)

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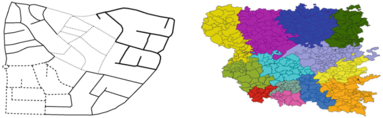
Agenda

- Classic problem, also called "Districting Problem" [4].
- Organizes a set B of *basic units* (BUs) and a set S of territorial centers into p larger territories or districts.
- Key constraints:
  - Spatial: compactness, contiguity.
  - Planning: activity balance (e.g., sales, workload).
- Objective: Achieve compact territories while balancing various metrics, which arise in political, commercial, and public service domains, [1].

Territory Design Problem Examples



(a) Districting plan for streets and postal codes

(b) Population graph for an electoral districting problem in Germany

Figure: TDPs and their applications

- Motivated by previous TDP application to a real microfinancial institution [2].
- Role of Microfinancial Institutions (MFIs):
  - Alleviate income inequality and poverty.
  - Provide credit and financial services to high-risk markets.
  - Typically avoided by commercial banks due to high risk and low profitability.
- Natural extension of my undergraduate thesis which was a GRASP

Special Requirements of Microfinancial Institutions

Problem Formulation

- Role of TDP: Identify potential branch locations and assign customers to their office.
- Importance of balance:
  - Centers distributed among different business lines of the host.
  - Activity metrics distributed uniformly (e.g., loan amounts, number of clients).
  - Manage loan risks due to volatility.
- Goal: Minimize distance between BUs and institution branches.

TDP Example for MFI

Instance Example

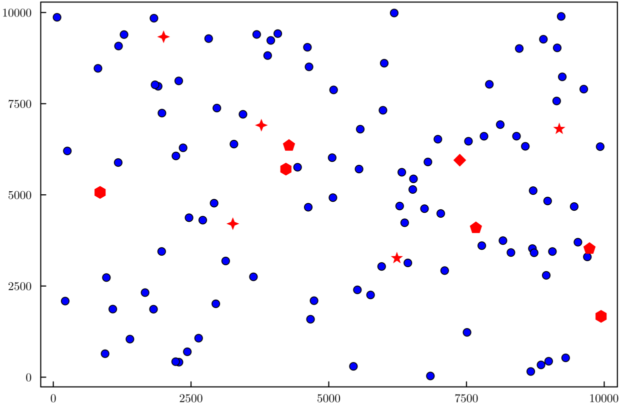


Figure: Instance with 100 clients and 12 centers

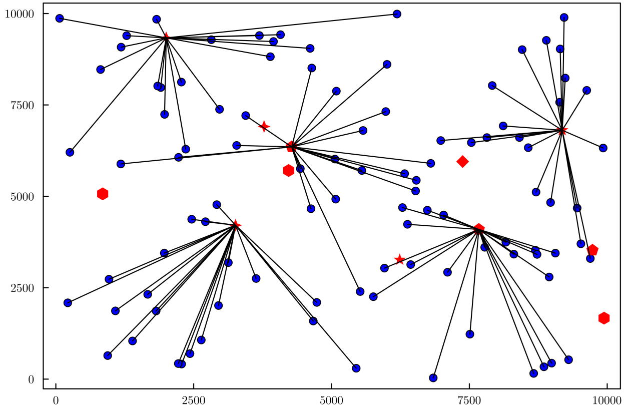


Figure: Optimal solution for the instance

Combinatorial Model

Sets and Parameters

- S: set of territorial centers, B: set of BUs
- d_{ij} : distance between center i and BU j
- $M = \{1, 2, ..., m\}$: number of activities to measure
- $K = \{1, 2, ..., k\}$: number of center types.
- $S_k \subseteq S$: set of center indices of type k, $\forall k \in K$
- μ_m^i: activity m target at center i, $\forall m \in M$
- v_m^j: activity m measure at BU j, $\forall m \in M$
- t_m: tolerance for activity m, $\forall m \in M$
- r_j : risk value at BU j
- β_i : risk threshold at center i
- L_k, U_k: bounds for centers in $T_i k$, $\forall k \in K$
- p: number of centers to be used

Combinatorial Model

Decision Variables

- $Y_i = 1$ if center i is used; $= 0$ otherwise.
- $X_{ij} = 1$ if BU j is assigned to center i; $= 0$ otherwise.

Objective Function:

$$\min \sum_{i \in S, j \in B} d_{ij} X_{ij} \tag{1}$$

Subject to

$$\sum_{i \in S} X_{ij} = 1, \quad \forall j \in B \tag{2}$$

Unique assignment of a BU j to a center i

$$X_{ij} \leq Y_i, \quad \forall i \in S, \quad j \in B \tag{3}$$

BUs can only be assigned to centers that are open

$$\mu_m^i (1 - t^m) Y_i \leq \sum_{j \in B} v_m^j X_{ij} \leq \mu_m^i (1 + t^m) Y_i, \quad \forall i \in S, \forall m \in M \tag{4}$$

Activity measures for each territory must be within a tolerance range

Mathematical Programming Model

Formulation

$$L_k \leq \sum_{i \in S_k} Y_i \leq U_k, \quad \forall k \in K \tag{5}$$

Selected center types must respect the lower and upper bounds for each type k.

$$\sum_{i \in S} Y_i = p \tag{6}$$

The number of centers to open must equal p

$$\sum_{j \in B} r_j X_{ij} \leq \beta_i, \quad \forall i \in S \tag{7}$$

The risk measure of each territory must not exceed the risk threshold

Location-Allocation-Improvement Models

Previous works [3] introduce the concept of location-allocation and improvement heuristics successfully for TDPs with p-median type objectives.
The core of the algorithm consists of solving the assignment phase as a transportation problem, fixing the located centers and relaxing the integrality of X, obtaining fractional values for **some** BUs, obtaining *splits* that must be resolved.
The activity measure tolerances (t_m) are further adjusted, attempting to form perfectly balanced centers.

Pseudocode: LAI Matheuristic

General Algorithm

Algorithm Location-Allocation-Improvement Matheuristic

Require: Sets S, B, parameters d_{ij}, v_m^j, μ_m^i, t_m, r_j, β_i, p
Ensure: Solution (Y^*, X^*) and objective value z^*

$z^* \leftarrow \infty$, $Y^* \leftarrow \emptyset$, $X^* \leftarrow \emptyset$
for each location method $\ell \in \{1, \dots, 5\}$ **do**
    $Y \leftarrow \textsc{SolveLocation}(S, B, p, \ell)$
    $(X, t'_m) \leftarrow \textsc{SolveAssignment}(Y, S, B, t_m)$
    $X \leftarrow \textsc{ResolveSplits}(X, Y)$
    $(X, Y) \leftarrow \textsc{RepairFeasibility}(X, Y)$
    $(Y, X, z) \leftarrow \textsc{LocalSearch}(Y, X)$
    if $z < z^*$ **then**
        $z^* \leftarrow z$, $Y^* \leftarrow Y$, $X^* \leftarrow X$
    end if
end for
return (Y^*, X^*, z^*)

Pseudocode: Location Phase

Center Selection Methods

Algorithm SolveLocation(S, B, p, ℓ)

Require: Sets S, B, instance parameters, method ℓ
Ensure: Location solution Y

```
Y ← {Y_i = 0 | ∀i ∈ S}
if ℓ = 1 then
    Method 1: p-dispersion heuristic with types
else if ℓ = 2 then
    Method 2: Colored p-dispersion model
else if ℓ = 3 then
    Method 3: Colored p-median model
else if ℓ = 4 then
    Method 4: Colored p-median model with Benders decomposition
    Decomposition comes from another work with José Emmanuel Gómez-Rocha and José Fernando Camacho-Vallejo.
    At ÓPTIMA we gave a talk on it.
else if ℓ = 5 then
    Method 5: Original model with relaxed X
end if
return Y
```

Variables:

- $Y_i \in \{0,1\}$: 1 if center i is selected
- $u \geq 0$: minimum distance between selected centers (maximize)
- $w_k \in \mathbb{Z}_+$: number of type k centers selected

Objective Function:

$$\max \quad u \tag{8}$$

Constraints:

$$u \leq d_{ij} + D_{max}(2 - Y_i - Y_j), \quad \forall i < j \in S \tag{9}$$

$$\sum_{i \in S} Y_i = p \tag{10}$$

$$w_k = \sum_{i \in S_k} Y_i, \quad \forall k \in K \tag{11}$$

$$L_k \leq w_k \leq U_k, \quad \forall k \in K \tag{12}$$

$$Y_i \in \{0,1\}, \quad u \geq 0, \quad w_k \in \mathbb{Z}_+ \tag{13}$$

Interpretation: Dispersed centers reduce cannibalization and improve territorial coverage

Colored P-Median Model

Method 3: Mathematical Formulation

Variables:

- $Y_i \in \{0, 1\}$: 1 if center i is selected
- $X_{ij} \in \{0, 1\}$: 1 if BU j is assigned to center i

Objective Function:

$$\min \sum_{i \in S, j \in B} d_{ij} X_{ij} \tag{14}$$

Constraints:

$$\sum_{i \in S} X_{ij} = 1, \quad \forall j \in B \tag{15}$$

$$X_{ij} \leq Y_i, \quad \forall i \in S, j \in B \tag{16}$$

$$\sum_{i \in S} Y_i = p \tag{17}$$

$$L_k \leq \sum_{i \in S_k} Y_i \leq U_k, \quad \forall k \in K \tag{18}$$

$$X_{ij}, Y_i \in \{0, 1\} \tag{19}$$

Advantage: Directly considers the final objective (minimize total distance)

Benders decomposition

Consider the following problem:

$$\min \sum_{j \in J} \theta_j(y) \tag{20}$$

$$s.t. \quad \sum_{i \in I} y_i = p, \tag{21}$$

$$y_i \in \{0,1\}, \qquad i \in I \tag{22}$$

Where $y = (y_i, i \in I)$, and $\sum_{i \in I} \theta_i(y)$ is defined as the following problem given a feasible solution y_i:

$$\sum_{j \in J} \theta_j(y) = \min \sum_{i \in I} \sum_{j \in J} d_j c_{ij} x_{ij} \tag{23}$$

$$s.t. \quad \sum_{i \in I} x_{ij} = 1, \qquad j \in J \tag{24}$$

$$x_{ij} \leq y_i, \qquad i \in I, j \in J \tag{25}$$

$$x_{ij} \in [0,1], \qquad i \in I, j \in J \tag{26}$$

Benders decomposition

Let λ_j be the dual variable associated with constraint (24) and π_{ij} be the dual variable associated with constraint (25). Thus the subproblem in the Benders decomposition can be formulated as follows:

$$
\begin{aligned}
& \sum_{j \in J} \theta_j(y) = \max \sum_{j \in J} \left(\lambda_j - \sum_{i \in I} \bar{y}_i \pi_{ij} \right) && && (27) \\
s.t.\ & \lambda_j - \pi_{ij} \leq d_j c_{ij}, && i \in I, j \in J && (28) \\
& \lambda_j \geq 0, && j \in J && (29) \\
& \pi_{ij} \geq 0, && i \in I, j \in J && (30)
\end{aligned}
$$

Where $\bar{y}_i$ is a feasible solution. This transportation problem in its dual form can be solved independently for each customer.

Thus, the master problem can be expressed as follows.

$$\min \sum_{j \in J} \theta_j(y) \tag{31}$$

$$s.t. \sum_{i \in I} y_i = p, \tag{32}$$

$$\theta_j(y) \geq \bar{\lambda}_j - \sum_{i \in I} y_i \bar{\pi}_{ij}, \qquad j \in J \tag{33}$$

$$y_i \in \{0, 1\}, \qquad i \in I \tag{34}$$

Where constraint (33) are the optimality cuts, with $\bar{\lambda}_j$ and $\bar{\pi}_{ij}$ being the extreme points previously obtained in the subproblems.

Given a set of open centers (fixed by Y from the location phase), solve:

$$\min \sum_{i\in S, j\in B} d_{ij}X_{ij}, X_{ij} \in [0,1], \quad \forall i \in S, j \in B \tag{35}$$

s.t.

$$\sum_{i\in S} X_{ij} = 1, \quad \forall j \in B \qquad \text{(complete assignment)}$$

$$\sum_{j\in B} r_j X_{ij} \leq \beta_i Y_i, \quad \forall i \in S \qquad \text{(risk)}$$

$$\sum_{j\in B} v_m^j X_{ij} \geq (1-\varepsilon) Y_i \mu_m^i, \quad \forall i \in S, m \in M \qquad \text{(lower balance)}$$

$$\sum_{j\in B} v_m^j X_{ij} \leq (1+\varepsilon) Y_i \mu_m^i, \quad \forall i \in S, m \in M \qquad \text{(upper balance)}$$

where $\varepsilon = 0.01$ is the adjusted tolerance.

Pseudocode: Split Resolution

Solution Integralization

Algorithm $\textsc{ResolveSplits}(X, Y)$

Require: Fractional assignment $X \in [0,1]^{S \times B}$, location Y
Ensure: Integer assignment $\hat{X} \in \{0,1\}^{S \times B}$

Initialize $\hat{X}_{ij} \leftarrow 0, \quad \forall i \in S, j \in B$

for each BU $j \in B$ **do**
    $i^* \leftarrow \arg\max_{i \in S} X_{ij}$
    $\hat{X}_{i^*,j} \leftarrow 1$
end for

return $\hat{X}$

Strategy: Greedy heuristic that assigns each BU j to the center i^* that had the largest fraction X_{ij} in the continuous solution of the transportation problem.
Note: This solution may violate balance and risk constraints, therefore it requires the feasibility repair phase.

Repair Heuristic Characteristics

Strategies and Decisions

Violation Prioritization:

1. **Risk first:** Risk violations are repaired before activity violations
2. **Magnitude:** Within each type, the largest violations are addressed first

Reassignment Strategy:

- **Risk:** Reassign high-risk BUs from the violated center to centers with capacity
- **Low activity:** Import BUs from other centers that increase the deficient activity
- **High activity:** Export high-activity BUs to other centers

Selection Criteria:

- Minimize the increase in the objective function (total distance)
- Ensure that no new violations are created in the process
- Validate all constraints (activity, risk) before reassigning

Local Search: Solution Improvement

Cyclic Search

With a feasible solution, we implement the following moves cyclically, exploring each neighborhood until finding the local optimum, moving to the next move until all 3 are at local optima:

Move 1: BU Reassignment (i, j)

Reassign BU j from center i to another center $\tilde{i}$, where $\tilde{i} \neq i$, if and only if $X_{ij} = 1$.

Move 2: BU Exchange $(i, j, \tilde{i}, \tilde{j})$

Exchange assignments of BUs j and $\tilde{j}$ such that j is assigned to $\tilde{i}$ and $\tilde{j}$ to i, if and only if $X_{ij} = 1 \wedge X_{\tilde{i}\tilde{j}} = 1$.

Move 3: Center Exchange $(i, \tilde{i})$

Close center i and open the previously unused center $\tilde{i}$. Assign BUs to $\tilde{i}$ until meeting lower bounds, and reassign BUs from i to the best-fitting centers, if and only if $Y_i = 1 \wedge Y_{\tilde{i}} = 0$.

Development Environment:

- *Language*: Julia 1.11.2. *Platform*: Ryzen 9 7940HS, 96 GB RAM
- *Solver*: Gurobi 12.0.1

Experiment Instances:

Size	$\lvert B \rvert$	$\lvert S \rvert$	p	Number of instances	Time limit (s)
1	1000	200	15,20,25	25	1800
2	2000	400	30,40,50	25	3600
3	3000	600	40,60,80	25	7200

Random coordinates in [5, 10000]. Types, risk, and activities with random ranges informed by the original work

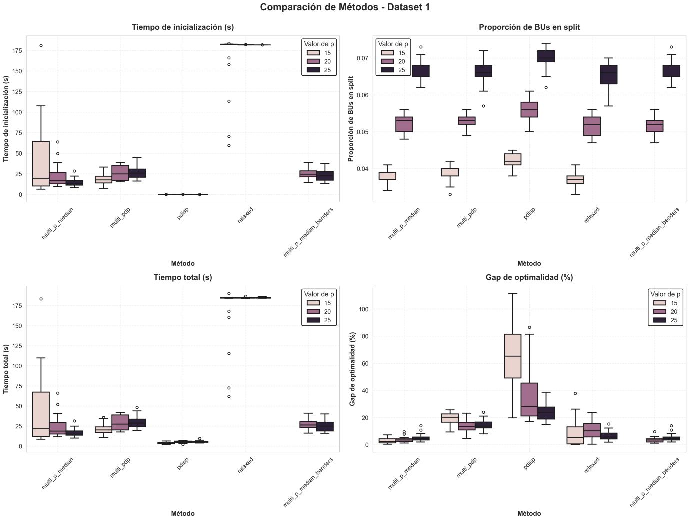
Experiment Instances:

Size	$\|B\|$	$\|S\|$	p	Instances	Average Gap
1	1000	200	15	25	0.25%
			20	25	1.01%
			25	25	2.01%
2	2000	400	30	25	2.67%
			40	25	3.74%
			50	25	6.11%
3	3000	600	40	25	13.97%
			60	25	14.89%
			80	25	20.80%

- 5 small instances (p=15) were solved to optimality
- 3 large instances did not find a feasible solution within the time limit

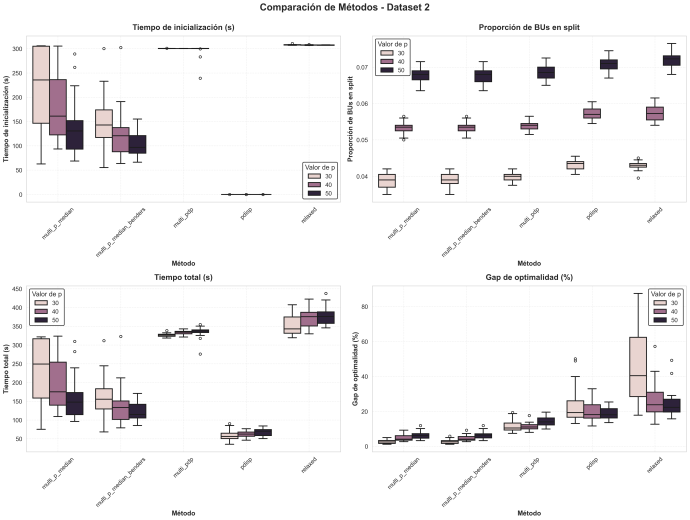
Comparison of Construction Methods

Size 1 - All instances



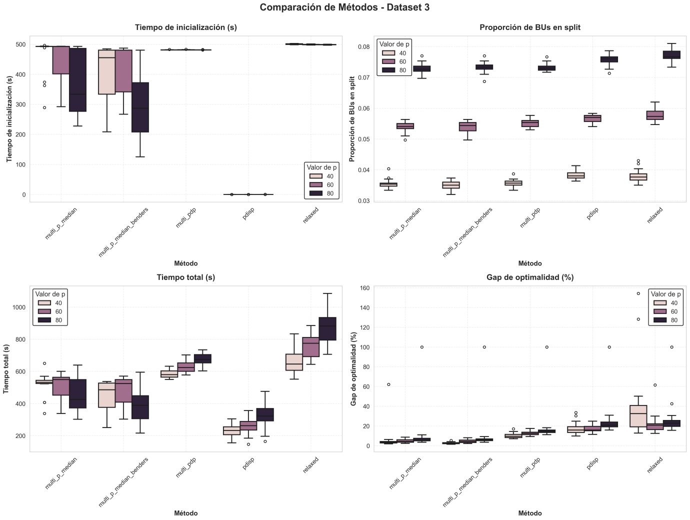
Comparison of Construction Methods

Size 2 - All instances



Comparison of Construction Methods

Size 3 - All instances



Construction Components:

Colored p-median model was chosen for small instances, and the colored p-median Benders model for medium and large instances.

As a note, the proportion of BUs in split is well-behaved, or predictably, for no size or p does it rise above 10%

Local Search Experiment Results

Adding moves does have an impact on total time, but also on the optimality gap. All moves is the Pareto optimal combination in gap, so we decided to take it, even though it represents longer execution time.

Heuristic Results - Small Instances

Size 1: $|B| = 1000$, $|S| = 200$ (Method: Colored p-median)

p	Inst.	Total time (s)			Split BUs prop.(%)			Post-LS gap (%)		
		Min	Avg	Max	Min	Avg	Max	Min	Avg	Max
15	25	8.56	40.66	183.31	3.40	3.80	4.10	0.35	2.80	7.24
20	25	11.61	24.17	65.98	4.80	5.22	5.60	1.16	3.75	9.51
25	25	9.90	16.76	31.28	6.20	6.60	7.30	1.96	5.10	13.93
Overall average		**10.02**	**27.20**	**93.52**	**4.80**	**5.21**	**5.67**	**1.16**	**3.89**	**10.23**

Heuristic Results - Medium Instances

Size 2: $|B| = 2000$, $|S| = 400$ (Method: Colored p-median Benders)

p	Inst.	Total time (s)			Split BUs prop. (%)			Post-LS gap (%)		
		Min	Avg	Max	Min	Avg	Max	Min	Avg	Max
30	25	68.20	161.45	311.68	3.50	3.91	4.20	1.16	2.63	5.86
40	25	79.06	139.33	322.77	5.05	5.33	5.65	2.70	4.69	9.24
50	25	85.50	123.21	170.85	6.35	6.78	7.15	3.28	6.40	11.86
Overall average		**77.59**	**141.33**	**268.43**	**4.97**	**5.34**	**5.67**	**2.38**	**4.57**	**8.99**

Heuristic Results - Large Instances

Size 3: $|B| = 3000$, $|S| = 600$ (Method: Colored p-median Benders)

p	Inst.	Total time (s)			Split BUs prop. (%)			Post-LS gap (%)		
		Min	Avg	Max	Min	Avg	Max	Min	Avg	Max
40	25	250.46	436.47	536.99	3.20	3.50	3.73	1.42	2.71	5.21
60	25	302.68	483.13	570.39	4.97	5.38	5.63	2.21	4.53	8.05
80	24	215.82	385.47	595.32	6.87	7.33	7.70	3.50	6.01	9.18
Overall average		**256.32**	**435.69**	**567.57**	**5.01**	**5.37**	**5.69**	**2.38**	**4.39**	**7.48**

Warmstart Results: Small Instances (B=1000, S=200)

	Avg Gap (%)		**Max Gap (%)**		**WS $\geq$ Plain**	
P	**Plain**	**WS**	**Plain**	**WS**	**Obj**	**Gap**
15	0.25	0.22	2.22	1.14	**21/25**	10/25
20	1.01	1.11	2.69	3.61	12/25	10/25
25	2.01	1.72	3.87	3.66	**15/25**	**14/25**
Total	1.09	1.02	—	—	**48/75**	**34/75**

WS $\geq$ Plain $=$ instances where warmstart wins or ties with plain Gurobi.

Warmstart Results: Medium Instances (B=2000, S=400)

	Avg Gap (%)		**Max Gap (%)**		**WS $\geq$ Plain**	
P	**Plain**	**WS**	**Plain**	**WS**	**Obj**	**Gap**
30	2.67	1.46	6.37	4.70	**18/25**	**17/25**
40	3.74	3.43	8.74	6.94	11/25	11/25
50	6.11	3.74	30.76	6.09	**18/25**	**18/25**
Total	4.17	2.88	—	—	**47/75**	**46/75**

WS $\geq$ Plain = instances where warmstart wins or ties with plain Gurobi.

Conclusions and Future Work

Conclusions:

- ✓ LAI Matheuristic outperforms Gurobi on medium and large instances
- ✓ Efficient times: 10-567s vs limits of 1800-7200s
- **In progress:** Is the matheuristic used as a warm-start beneficial?

Future Work:

- → Incorporate spatial contiguity constraints that are exponential
- → Introduce risk as a stochastic measure, not deterministic
- → Study further the posed problems of the colored location variants

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Thank You

Reach out via

- github.com/eduardosalaz
- eduardo@ampl.com

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Eduardo Salazar-Treviño
(AMPL Optimization Inc., Universitat Politècnica de Catalunya [soon])
Roger Z. Ríos-Mercado
(Graduate Program in Electrical Engineering, Universidad Autónoma de Nuevo León)
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