8.8 Error normalized LMS algorithms

A new class of LMS algorithms based on error normalization has been reported by the authors in IEEE conferences in 2004 and 2005. These algorithms are:

1. Error Normalized Step-Size (ENSS) LMS Algorithm

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\mu}{1+\mu \left\| \mathbf{e}_{\mathrm{L}}(n) \right\|^2} \mathbf{x}(n)\, e(n) \tag{8.8.1}$$

2. Robust Variable Step-Size (RVSS) LMS Algorithm

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\mu \left\| \mathbf{e}_{\mathrm{L}}(n) \right\|^2}{\alpha \left\| \mathbf{e}(n) \right\|^2 + (1-\alpha) \left\| \mathbf{x}(n) \right\|^2} \mathbf{x}(n) e(n) \tag{8.8.2}$$

3. Error-Data Normalized Step-Size (EDNSS) LMS Algorithm

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\mu}{\alpha \left\| \mathbf{e}_{\mathrm{L}}(n) \right\|^2 + (1-\alpha) \left\| \mathbf{x}(n) \right\|^2} \mathbf{x}(n) e(n) \tag{8.8.3}$$

where

$$\left\| \mathbf{e}_{\mathrm{L}}(n) \right\|^2 = \sum_{i=0}^{\mathrm{L}-1} \left| e(n-i) \right|^2 \tag{8.8.4}$$

and

$$\left\| \mathbf{e}(n) \right\|^2 = \sum_{i=0}^{n-1} \left| e(n-i) \right|^2 \tag{8.8.5}$$

Comments

- The parameters α, L, and μ in all of these algorithms are appropriately chosen to achieve the best trade-off between rate of convergence and low final MSE. L could be constant or variable (L = n, for example), depending on whether the underlying environment is stationary or nonstationary.
- The variable step-sizes in all of these algorithms should vary between two predetermined hard limits. The lower value guarantees the capability of the algorithm to respond to an abrupt change that could happen at a very large value of iteration number n, while the maximum value maintains stability of the algorithm.