

# Algoritmi e Strutture Dati

## Lezione 3

3 ottobre 2022

# MOLTIPLICAZIONE mediante SOMME ITERATIVE

$$a * b = \underbrace{a + a + \dots + a}_{b \text{ volte}}$$

ALGORITMO moltiplicazione (intero a, intero b)  $\rightarrow$  intero

```
1 prod  $\leftarrow$  0
2 WHILE b > 0 DO
3   |   prod  $\leftarrow$  prod + a
4   |   b  $\leftarrow$  b - 1
5 RETURN prod
```

se b = 0 linee 1, 2, 5  $\rightarrow 3$

se b > 0 linee 1, 5 1 volta  $\rightarrow 2$

linee 3, 4 b volte  $\rightarrow 2b$

linea 2 b + 1 volte  $\rightarrow b + 1$   

---

 $3b + 3$

$$T(a, b) = 3b + 3$$

MULTIPLICANDO (a)

\*2↓

19  
38  
76  
152  
304  
608  
1216

MULTIPLICARE (b)

✓2↓

114  
57  
28  
14  
7  
3  
1  
1

38

304

608

1216

2166

$$a \cdot b = 2a \cdot \frac{b}{2}$$

divisione reale

$$a \cdot b = \begin{cases} 2a \cdot \left\lfloor \frac{b}{2} \right\rfloor \\ 2a \cdot \left\lfloor \frac{b}{2} \right\rfloor + a \end{cases}$$

se b pari:

se b dispari,  $b \neq 1$

se  $b = 1$

$$\text{b dispari: } \left\lfloor \frac{b}{2} \right\rfloor = \frac{b-1}{2}$$

$$2a \cdot \frac{b-1}{2} = ab - a$$

# MOLTIPLICAZIONE

"ALLA RUSSA"

a	b	prod
14	21	0
28	10	14
56	5	"
112	2	70
224	1	"
448	0	294

ALGORITHM  $\text{multiplication}(\text{intero } a, \text{intero } b) \rightarrow \text{intero}$

prod  $\leftarrow 0$

WHILE  $b > 0$  DO

IF  $b$  dispari THEN

prod  $\leftarrow$  prod +  $a$

$b \leftarrow b / 2$

$a \leftarrow a * 2$

RETURN prod

$$a_i \cdot b_i + \text{prod}_i = a \cdot b$$

ALGORITHM multiplication (info a, info b)  $\rightarrow$  info

```

prod ← 0
WHILE b > 0 DO
    IF b dispari THEN
        prod ← prod + a
    b ← b / 2
    a ← a * 2
RETURN prod
    
```

ind. su i

$$\begin{array}{l} i=0 \\ \hline a_0 = a \quad b_0 = b \\ \text{prod}_0 = 0 \end{array}$$

$$a_i \cdot b_i + \text{prod}_i = a \cdot b + 0 = a \cdot b$$

RETURN prod

$i-1 \rightarrow i$

• Se  $b_{i-1}$  è pari

$$\begin{aligned} b_i &= \frac{b_{i-1}}{2} & a_i &= a_{i-1} * 2 \\ \text{prod}_i &= \text{prod}_{i-1} \end{aligned}$$

$$\begin{aligned} a_i \cdot b_i + \text{prod}_i &= a_{i-1} \cdot \frac{b_{i-1}}{2} + \text{prod}_{i-1} = \\ &= a_{i-1} \cdot b_{i-1} + \text{prod}_{i-1} = a \cdot b \end{aligned}$$

$\leftarrow$  in ind.

• Se  $b_{i-1}$  è dispari

$$\begin{aligned} b_i &= \left\lfloor \frac{b_{i-1}}{2} \right\rfloor = \frac{b_{i-1}-1}{2} \\ a_i &= 2 \cdot a_{i-1} & \text{prod}_i &= \text{prod}_{i-1} + a_{i-1} \end{aligned}$$

$$\begin{aligned} a_i \cdot b_i + \text{prod}_i &= 2a_{i-1} \cdot \frac{b_{i-1}-1}{2} + \text{prod}_{i-1} + a_{i-1} = \\ &= a_{i-1} \cdot b_{i-1} - a_{i-1} + \text{prod}_{i-1} + a_{i-1} = a \cdot b \end{aligned}$$

quando  $b_i = 0$

$$\text{prod}_i = a \cdot b$$

ALGORITHM multiplication (intero  $a$ , intero  $b$ )  $\rightarrow$  intero

```

1 prod  $\leftarrow$  0
2 WHILE  $b > 0$  DO
3   IF  $b$  dispari THEN
4     prod  $\leftarrow$  prod +  $a$ 
5    $b \leftarrow b/2$ 
6    $a \leftarrow a \times 2$ 
7 RETURN prod
    
```

$u$  iterazioni

Linea 1, 2  $\rightarrow$  1 volta 2

Linea 2  $\rightarrow$   $u+1$  volte  $u+1$

Linea 3, 5, 6  $\rightarrow$   $u$  volte  $3u$

Linea 4  $\rightarrow$   $\leq u$  volte  $\leq u$

$\leq 5u+3$

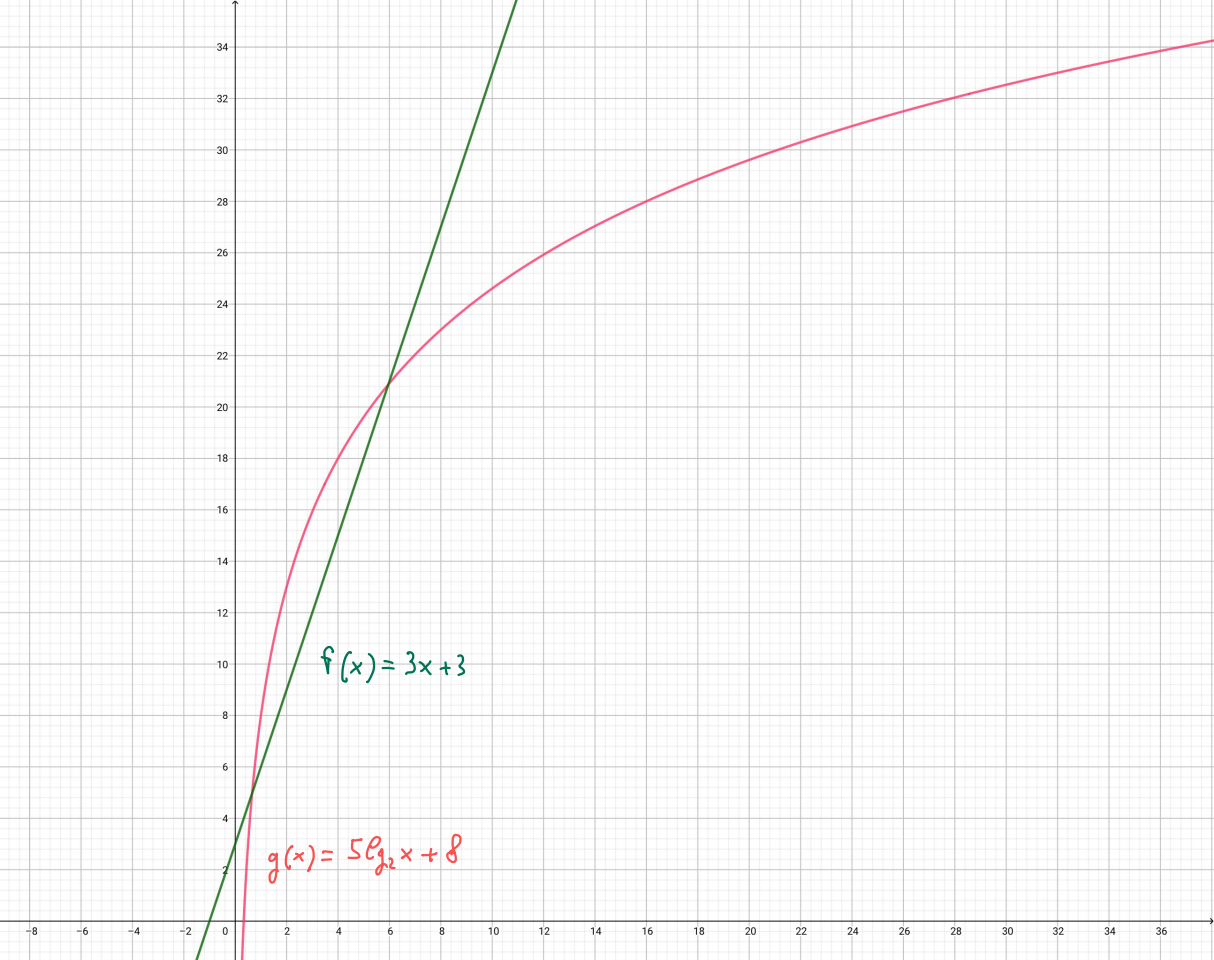
$b$	$u$
0	0
1	1
2	2
3	2
4	3
5	3
$\vdots$	$\vdots$

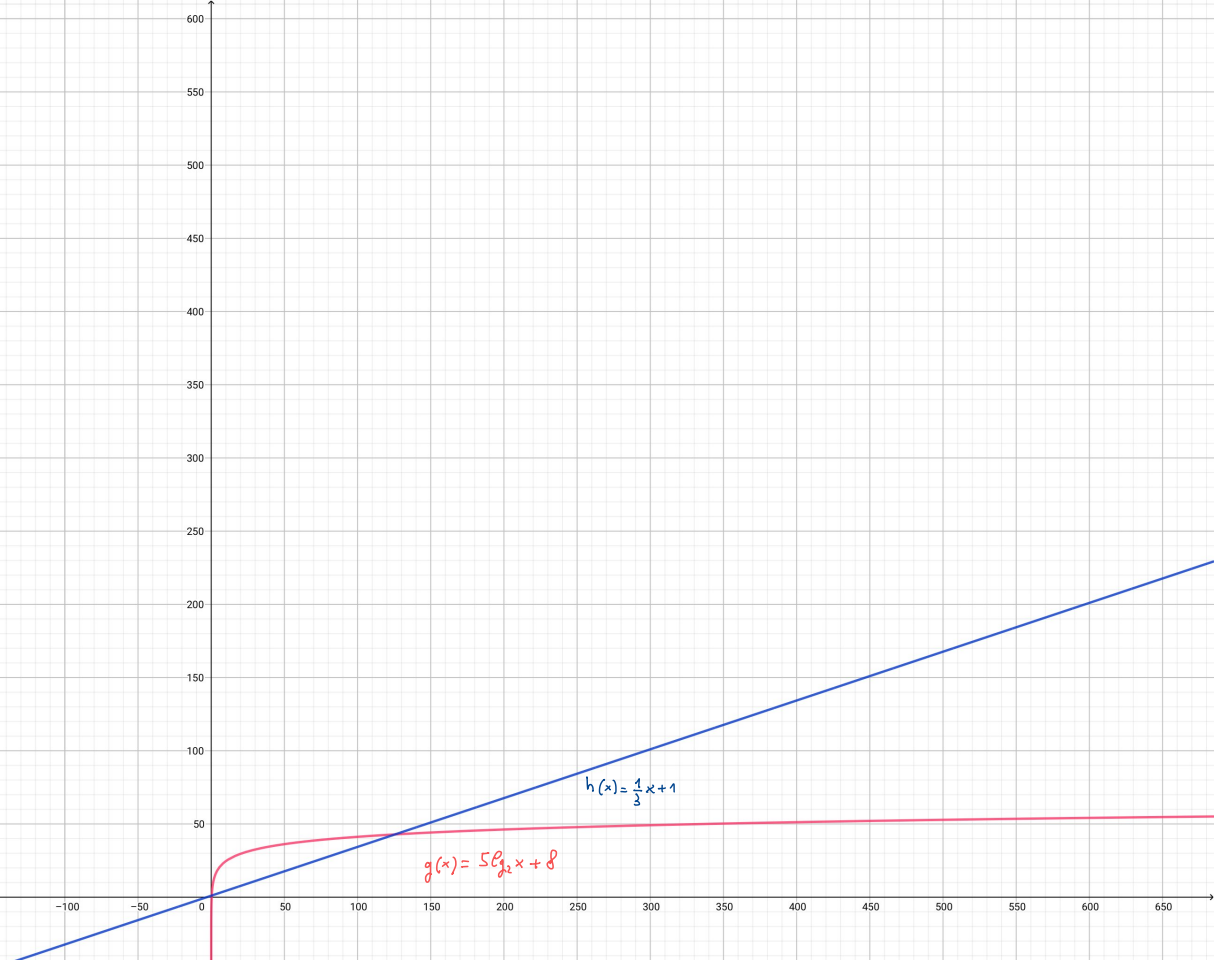
$$u = \lfloor \lg_2 b \rfloor + 1$$

$$T(a, b) = 5(\lfloor \lg_2 b \rfloor + 1) + 3$$

$$= 5 \lfloor \lg_2 b \rfloor + 8$$

$$T(a, b) = 3b + 3 \quad \text{per tutte iterazioni}$$







$$x, y \geq 0 \quad \text{intero} \quad x^y = \underbrace{x \cdot x \cdots x}_y \text{ volte}$$

ALGORITMO potenza (intero  $x$ , intero  $y$ )  $\rightarrow$  intero

1 power  $\leftarrow 1$

2 WHILE  $y > 0$  DO

3     power  $\leftarrow$  power  $\times x$

4      $y \leftarrow y - 1$

5 RETURN power

tempo  $T(x, y) = 3y + 3$

spazio 2 parametri + power = 3

$$x^{\frac{y}{2}} = x^{\frac{y}{2} \cdot 2} = \left(x^{\frac{y}{2}}\right)^2$$

$$x^y = \begin{cases} \left(x^{\frac{y}{2}}\right)^2 & \text{se } y \text{ pari, } y \neq 0 \\ \left(x^{\frac{y-1}{2}}\right)^2 \cdot x & \text{se } y \text{ dispari} \\ 1 & \text{se } y = 0 \end{cases}$$

ALGORITMO potenza (intero  $x$ , intero  $y$ )  $\rightarrow$  intero

IF  $y = 0$  THEN

    RETURN 1

ELSE

    power  $\leftarrow$  potenza ( $x$ ,  $y/2$ ) // divisione intera

    power  $\leftarrow$  power \* power

    IF  $y$  è dispari THEN

        power  $\leftarrow$  power \*  $x$

    RETURN power

$$x^y = \begin{cases} (x^{\frac{y}{2}})^2 & \text{se } y \text{ pari, } y \neq 0 \\ (x^{\frac{y-1}{2}})^2 \cdot x & \text{se } y \text{ dispari} \\ 1 & \text{se } y = 0 \end{cases}$$

ALGORITHM potenza (intero  $x$ , intero  $y$ )  $\rightarrow$  intero

1 IF  $y = 0$  THEN

2 | RETURN 1

ELSE

3 power  $\leftarrow$  potenza ( $x$ ,  $y/2$ ) // divisione intera

4 power  $\leftarrow$  power \* power

5 IF  $y$  è dispari THEN

6 power  $\leftarrow$  power \*  $x$

7 RETURN power

$T(x, y)$

- se  $y = 0$  linea 1? 2

- se  $y > 0$  linea 1, 3, 4, 5, 2  
1 volta 5

linea 6  
 $\leq 1$  volta  $\leq 1$

linee eseguite  
per calcolare  
potenza ( $x$ ,  $y/2$ )

$T(x, \lfloor \frac{y}{2} \rfloor)$

$$T(x, y) \leq \begin{cases} 2 & \text{se } y = 0 \\ T(x, \lfloor \frac{y}{2} \rfloor) + 6 & \text{altrimenti} \end{cases}$$

$$\leq 6 + T(x, \lfloor \frac{y}{2} \rfloor)$$

$\rightarrow$  Equazione di ricorrenza

$$T(x, 1) = T(x, 0) + 6 = 2 + 6 = 8$$

$$T(x, y) = \begin{cases} 8 \\ \underline{T(x, \frac{y}{2})} + 6 \end{cases}$$

se  $y=0$   
 se  $y=1$   
 altrimenti:

$$T(x, y) = \underline{T(x, \frac{y}{2})} + 6 = \underline{T(x, \frac{y}{2^2})} + 6 + 6$$

$$= \underline{T(x, \frac{y}{2^3})} + 6 + 6 + 6 = \dots = T(x, \frac{y}{2^k}) + \underbrace{6 + 6 + \dots + 6}_k$$

$$= T(x, \frac{y}{2^k}) + 6k$$

$$\frac{y}{2^k} = 1 \quad \text{per} \quad 2^k = y$$

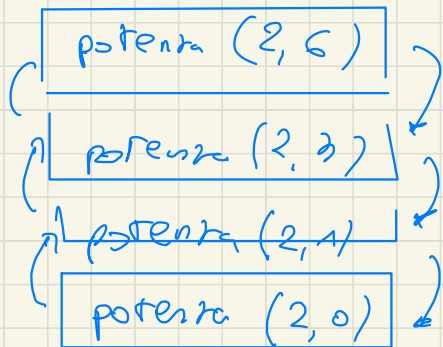
$$k = \lg_2 y$$

$$\begin{aligned} & \xrightarrow{k = \lg_2 y} \underline{T(x, 1)} + 6 \lg_2 y \\ & = 8 + 6 \lg_2 y \end{aligned}$$

$$T(x, y) \leq 6 \lg_2 y + 8$$

□ EIPD

Specie?



ALGORITHM potenza (intero x, intero y) → intero

1 IF  $y = 0$  THEN

2 | RETURN 1

ELSE

3 | power ← potenza (x,  $y/2$ ) // divisione intero

4 | power ← power \* power

5 | IF  $y$  è dispari THEN

6 | power ← power \* x

7 | RETURN power

$$H(x, y) = \begin{cases} 1 \\ 1 + H(x, \lfloor \frac{y}{2} \rfloor) \end{cases}$$

se  $y = 0$

altrimenti

$$H(x, 1) = 2$$

$$H = \begin{cases} 1 & \text{se } y = 0 \\ 2 & \text{se } y = 1 \\ 1 + H(x, \lfloor \frac{y}{2} \rfloor) & \text{altrimenti} \end{cases}$$