A Note on Ultimately-Periodic Finite Interval Temporal Logic Model Checking

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Introduction

In its standard formulation, model checking (MC) is the problem of verifying if a formula is satisfied by a given model.

MC has been intensively studied for point-based temporal logics, such as LTL or CTL, but it is less known for interval-based temporal logics, such as HS (or its fragments).

Model Checking

$$M, (w) \Vdash \varphi$$

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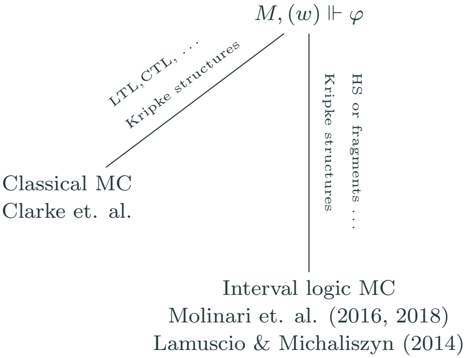
$M, (w) \Vdash \varphi$

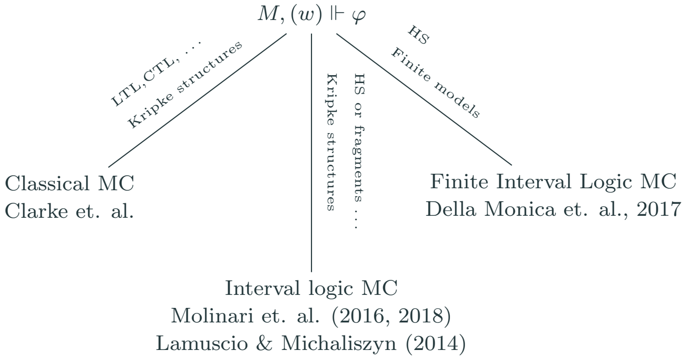
LTL,CTL, ...

Kripke structures

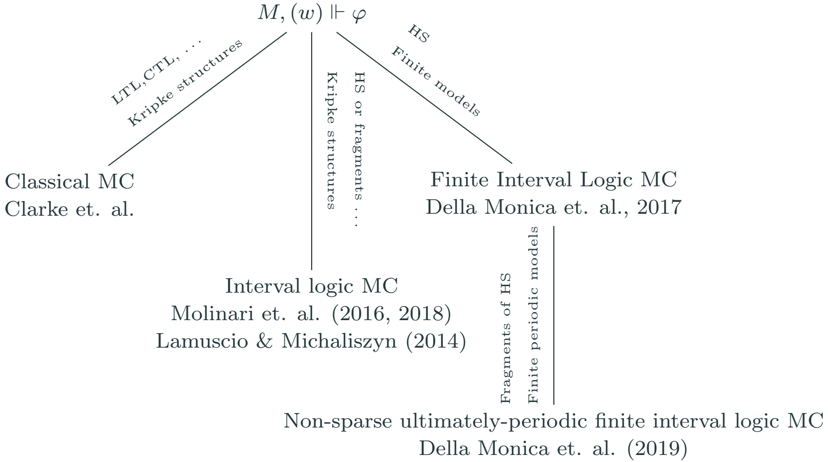
Classical MC
Clarke et. al.


$M, (w) \Vdash \varphi$
LTL,CTL, ...
Kripke structures
Kripke structures
HS or fragments ...
Classical MC
Clarke et. al.
Interval logic MC
Molinari et. al. (2016, 2018)
Lamuscio & Michaliszyn (2014)

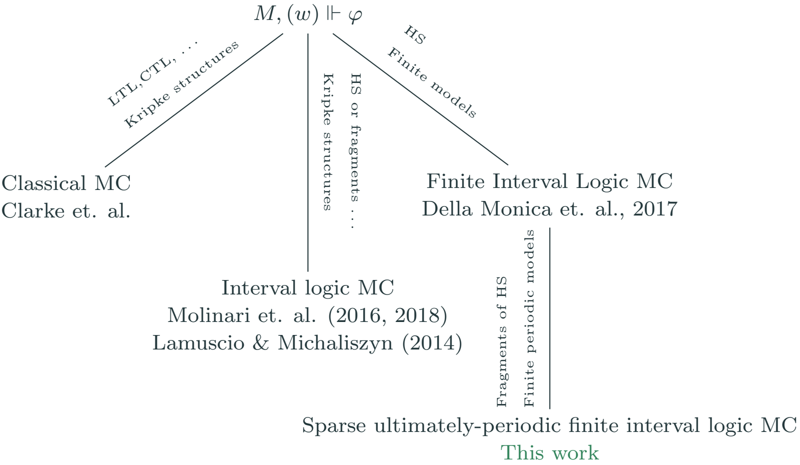
Model Checking



Model Checking



Model Checking



Interval-based MC differs from classical (point-based) model checking in two ways:

- formulas are written in interval temporal logic HS, and
- models are interval-based rather than point-based.

Finite Interval Logic Model Checking

Finite interval-based MC is based on the same interval-based language, but for classical, linear orders.

In this case, models are completely represented and the MC process is immediate.

It becomes less trivial for sparse models: the representation of a (fully represented) model can be logarithmic in its size (Della Monica et. al., 2017).

Logic	Models	Complexity
HS	Sparse and non-sparse finite models	PTIME (Della Monica et. al., 2017)
MRPNL	Non-sparse ultimately-periodic finite models	PTIME (Della Monica et. al., 2019)
MRPNL	Sparse ultimately-periodic finite models	This work

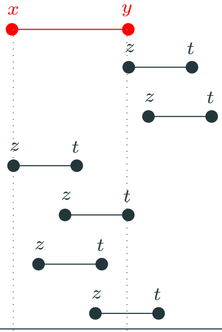
HS and Metric Right Propositional Neighborhood Logic

An interval over a linearly ordered set $\mathbb{D} = \langle D, < \rangle$ is an ordered pair $[x, y]$, where $x, y \in D$ and $x < y$. Let $\mathbb{I}(\mathbb{D})$ be the set of all intervals over $\mathbb{D}$.

Excluding the equality, there are 12 binary ordering relations between 2 intervals on a linear order, often called Allen's interval relations, which give rise to corresponding unary modalities over frames where intervals are primitive entities.

HS: The Modal Logic of Allen's Interval Relations

HS modality	Definition w.r.t. interval structure			Example
$\langle A \rangle$ (after)	$[x, y] R_A [z, t]$	$\Leftrightarrow$	$y = z$	
$\langle L \rangle$ (later)	$[x, y] R_L [z, t]$	$\Leftrightarrow$	$y < z$	
$\langle B \rangle$ (begins)	$[x, y] R_B [z, t]$	$\Leftrightarrow$	$x = z \wedge t < y$	
$\langle E \rangle$ (ends)	$[x, y] R_E [z, t]$	$\Leftrightarrow$	$y = t \wedge x < z$	
$\langle D \rangle$ (during)	$[x, y] R_D [z, t]$	$\Leftrightarrow$	$x < z \wedge t < y$	
$\langle O \rangle$ (overlaps)	$[x, y] R_O [z, t]$	$\Leftrightarrow$	$x < z < y < t$	



For each modality $\langle X \rangle$, its transpose corresponds to the inverse $R_{\overline{X}}$ of R_X, i.e., $R_{\overline{X}} = (R_X)^{-1}$.

With each subset of Allen's relations $\{R_{X_1}, \ldots, R_{X_k}\}$, we associate the HS fragment $\mathsf{X_1 X_2 \ldots X_k}$, whose formulas are defined by the grammar:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle X_1 \rangle \varphi \mid \ldots \mid \langle X_k \rangle \varphi,$$

where $p \in \mathcal{AP}$ (atomic proposition).

An interval model is a pair $M = \langle \mathbb{I}(\mathbb{D}), V \rangle$, where $\mathbb{D}$ is a linearly ordered set (i.e., the domain of M) and $V : \mathcal{AP} \to 2^{\mathbb{I}(\mathbb{D})}$ is a valuation function that assigns to each proposition $p \in \mathcal{AP}$ the set of interval $V(p)$ on which p holds.

For an interval model M, let N denote the size, or cardinality, of $\mathbb{D}$.

MPNL: Metric Propositional Neighborhood Logic

Among the many fragments of HS, Propositional Neighborhood Logic (PNL) has only two modalities $\langle A \rangle$ and $\langle \overline{A} \rangle$ whose satisfiability problem, unlike that of HS and of most of its fragments, is decidable.

Without losing its computational properties, the syntax of PNL can be extended, for each natural number k, with a pre-interpreted modal constant $\mathsf{len}_{<\mathsf{k}}$ (length constraint). The resulting logic is called Metric Propositional Neighborhood Logic (MPNL) whose formulas are generated by the grammar:

$$\varphi ::= \mathsf{len}_{<\mathsf{k}} \mid p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle A \rangle\varphi \mid \langle \overline{A} \rangle\varphi,$$

where $p \in \mathcal{AP}$ and $k \in \mathbb{N}$.

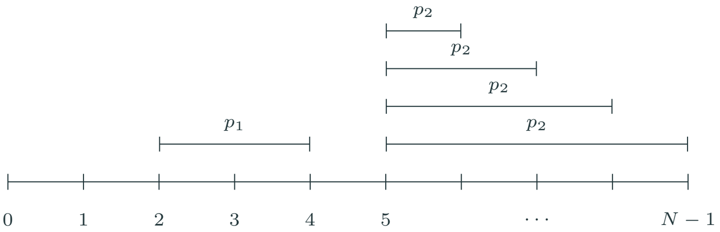
The future fragment of MPNL, called Metric Right Propositional Neighborhood Logic (MRPNL), consists of the formulas generated by the grammar:

$$\varphi ::= \mathsf{len}_{<\mathsf{k}} \mid p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle A \rangle \varphi,$$

where $p \in \mathcal{AP}$ and $k \in \mathbb{N}$.

The truth of a MRPNL formula φ on an interval $[x, y]$ of an interval model M is defined by structural induction on formulas:

$$\begin{array}{lll}
M, [x,y] \Vdash \mathsf{len}_{<\mathsf{k}} & \text{iff} & y - x < k, \\
M, [x,y] \Vdash p & \text{iff} & [x,y] \in V(p), \text{ for } p \in \mathcal{AP}, \\
M, [x,y] \Vdash \neg\psi & \text{iff} & M, [x,y] \nVdash \psi, \\
M, [x,y] \Vdash \psi \wedge \gamma & \text{iff} & M, [x,y] \Vdash \psi \text{ and } M, [x,y] \Vdash \gamma, \\
M, [x,y] \Vdash \langle A \rangle \psi & \text{iff} & \exists z \text{ s.t. } y < z \text{ and } M, [y,z] \Vdash \psi.
\end{array}$$



In the above model:

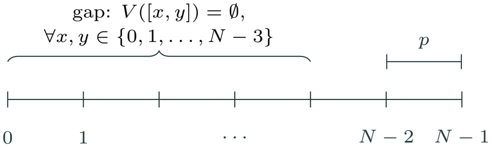
- $M, [x, 2] \Vdash \langle A \rangle (p_1 \wedge \mathsf{len}_{=2})$ for all $x \in \{0, 1\}$,
- $M, [x, 5] \Vdash [A] p_2$ for all $x \in \{0, 1, 2, 3, 4\}$,
- $M, [x, y] \nVdash \langle A \rangle (p_1 \wedge \mathsf{len}_{>2})$ for all $x, y \in \{0, 1, \ldots, N-1\}$ and $x < y$.

Finite and Ultimately-Periodic Interval Logic Model Checking

Definition 1 (FIMC, Della Monica et. al., 2017).

Given a pair (M, φ), where M is a finite interval model and φ is an HS formula, the finite interval logic model checking problem (FIMC) consists in deciding whether $M, [0,1] \Vdash \varphi$.

It is easy to devise a simple FIMC procedure whose running time is polynomial in the cardinality N of the model M (and in the size of the formula).



Representation:	**Space:**
N	$O(\log N)$
$p : [N-2, N-1]$	$O(\log N)$

Checking $\langle A\rangle\langle A\rangle p$ requires labeling with $\langle A\rangle p$ all intervals $[x, N-2]$, with $1 \leq x < N-2$, whose number is linear in N, and thus exponential in the size of the representation of M.

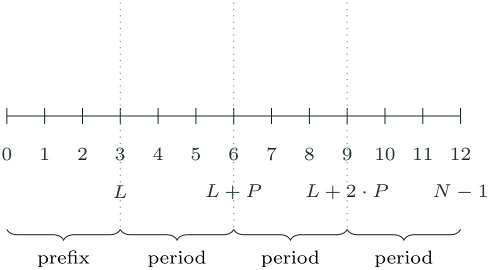
Every instance (M, φ) of FIMC, with M ranging over a sparse class of models, can be turned (in polynomial time) into an equivalent one, (M', φ), where M' ranges over a non-sparse class of models and such that:

$$M, [0,1] \Vdash \varphi \quad \text{iff} \quad M', [0,1] \Vdash \varphi.$$

Theorem 2 (Della Monica et. al., 2017).

FIMC is in PTIME.

UP-FIMC: Ultimately-Periodic FIMC



Definition 3 (UP-FIMC, Della Monica et. al., 2019).

Given a pair (M, φ), where M is a finite interval model and φ is a formula of MRPNL, the ultimately-periodic interval model checking problem (UP-FIMC) is the problem of enumerating the natural numbers P such that M can be extended, by repeating an infinite number of times its suffix of length P, into an infinite ultimately-periodic model $\mathcal{M}$ that satisfies φ, i.e., $\mathcal{M}, [0, 1] \Vdash \varphi$.

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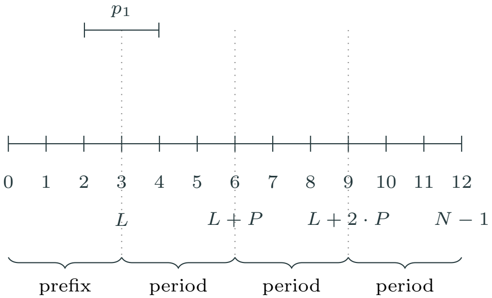

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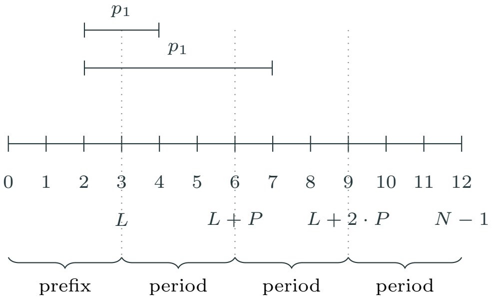

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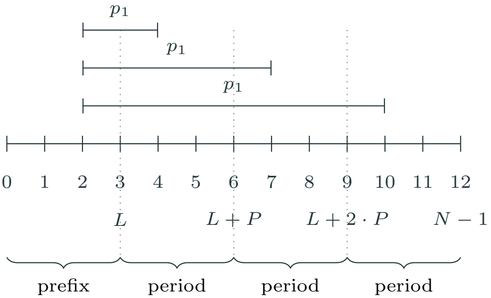

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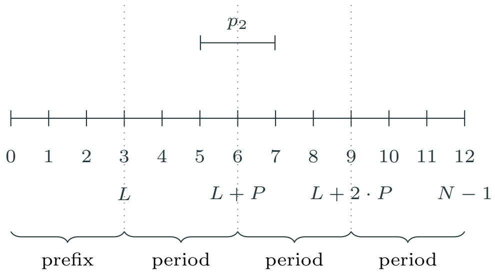

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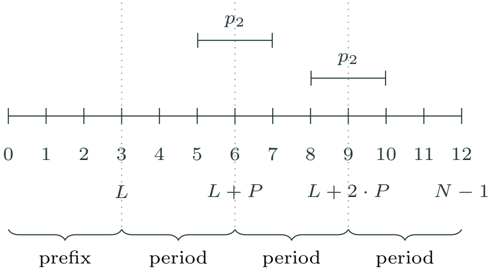

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Theorem 4 (Della Monica et. al., 2019).

UP-FIMC over non-sparse classes of models is in PTIME.

We generalize the above theorem to sparse classes of models by solving the following issues.

Issue

We need to be careful to only shrink gaps contained in either the prefix or the period (i.e., avoiding shrinking gaps spanning both prefix and period).

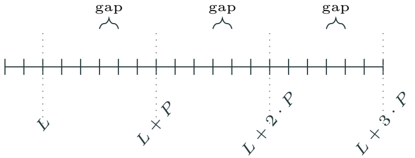
Solution: If a gap of size at least $2B$ exists, then we are sure that a gap of size at least B exists that is contained either in the prefix or in the period.

Issue

When shrinking a gap in the period, we obtain an infinite ultimately-periodic model whose period has also been shrunk.

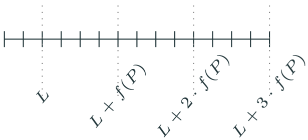
Solution:

Sparse instance (M, φ)



PTIME $\Longrightarrow$

Non-sparse instance (M', φ)
Invertible function $f : \mathbb{N} \to \mathbb{N}$



$\Downarrow$ PTIME

Run algorithm by Della Monica et. al. (2019) to produce a set of periods $\mathcal{P} = \{P_1, \ldots, P_s\}$

PTIME $\Longleftarrow$

For each $P \in \mathcal{P}$, apply $f^{-1}(P)$ to obtain a new set of periods $\mathcal{P}' = \{P'_1, \ldots, P'_s\}$ representing a solution to the original problem

UP-FIMC: Ultimately-Periodic FIMC

Theorem 5.

UP-FIMC is in PTIME.

Conclusions

Conclusions

We have presented:

- a generalization to the notion of FIMC by considering only a fragment of HS (MRPNL) for incomplete, infinite models;
- solved the sparsity problem of UP-FIMC;
- defined the UP-FIMC as enumerating one, in opposition to a decision one, allowing to rate a model, instead of just checking it.

We plan to:

- extend the sparse result to (full) MPNL
- generalize the MC problem from checking a single model to a set of models;
- apply the method to early and on-line classification of (multivariate) time series.