**Supplemental Materials for “Testing Measurement Invariance in Longitudinal Data with Ordered-Categorical Measures”**

**[to be placed on website]**

**Supplemental Materials 1A**

**R Function for Calculating the Estimated Probabilities for Choosing Each Response Category for Each Indicator**

# Takes in output from the baseline/ loading invariance/ threshold invariance/ unique factor

# invariance model to evaluate the longitudinal measurement equivalence of a one-factor

# ordered-categorical CFA model with theta parameterization.

# Calculates the predicted probabilities of choosing each response category of each indicator

# at ONE measurement occasion.

# Assumes that unique factors within a measurement occasion are uncorrelated.

# The function can handle cases where some indicators have one less threshold than others;

# however, it currently cannot handle cases where the discrepancy in the number of response

# categories across different indicators is greater than one;

######################################################################################

ThresholdProbability = function(n.Item, Loadings, Common.Factor.Mean, Common.Factor.Variance, Unique.Factor.Variances, n.Threshold, Thresholds){

# Vector of loadings

LAMDA <- matrix(Loadings,n.Item,1)

# Common factor mean

KAPPA <- Common.Factor.Mean

# Common factor variance

PHI <- Common.Factor.Variance

# Covariance matrix for the unique factors of indicators that load on C1FAMO

THETA <- diag(Unique.Factor.Variances)

# Latent response variable means

MU <- LAMDA%\*%KAPPA

# Latent response variable covariance matrix.

SIGMA <- (LAMDA%\*%PHI%\*%t(LAMDA) + THETA)

# Latent response variable variances

Item.Variances <- diag(SIGMA)

Item.SDs <- sqrt(Item.Variances)

# Declare the matrices storing the p values

# p values associated with scoring under a certain threshold

p.LE.Threshold<-matrix(NA,nrow=n.Item, ncol=n.Threshold)

# p values associated with scoring within certain thresholds, i.e., choosing a category

p.within.Thresholds<-matrix(NA,nrow=n.Item, ncol=n.Threshold+1)

# Calculation of p values

for (i in 1:(n.Item)){

for (j in 1:(n.Threshold)){

p.LE.Threshold[i,j]<-pnorm(Thresholds[i,j], mean=MU[i], sd=Item.SDs[i])

}

}

for (i in 1:(n.Item)){

p.within.Thresholds[i,1]<-p.LE.Threshold[i,1]

if (is.na(p.LE.Threshold[i,n.Threshold])){

p.within.Thresholds[i,n.Threshold+1]<- NA

p.within.Thresholds[i,n.Threshold]<- (1-p.LE.Threshold[i,n.Threshold-1])

for (j in 2:(n.Threshold-1)){

p.within.Thresholds[i,j]<-(p.LE.Threshold[i,j]-p.LE.Threshold[i,j-1])

}

}

else {

p.within.Thresholds[i,n.Threshold+1]<- (1-p.LE.Threshold[i,n.Threshold])

for (j in 2:(n.Threshold)){

p.within.Thresholds[i,j]<-(p.LE.Threshold[i,j]-p.LE.Threshold[i,j-1])

}

}

}

return(p.within.Thresholds)

}

**Supplemental Materials 1B**

**R Code for Calculating the Discrepancies in the Estimated Probabilities between Two Models at One Measurement Occasion**

########################################################

# User input for familism obligations

########################################################

######################### Time 1 #######################

###########################

# Loading invariance model

###########################

# Number of items that load on the factor

n.Item <- 5

# Factor loadings at the measurement occasion under examination

Loadings <- c(0.89, 1.15, 1.000, 0.667, 0.697)

# Mean of the common factor at the measurement occasion under examination

Common.Factor.Mean <- 0.000

# Variance of the common factor at the measurement occasion under examination

Common.Factor.Variance <- 0.586

# Variances of the unique factors at the measurement occasion under examination

Unique.Factor.Variances <- c(1, 1, 1, 1, 1)

# Number of thresholds per item: assumed to be the same across items

# Can handle cases where some items have ONE less threshold than others:

# In such cases, enter the maximum number of thresholds

n.Threshold <- 3

# Matrix of thresholds for items that load on the common factor;

# each row for a different item, ordered from lowest to highest threshold.

Thresholds <- matrix(c(-2.452,-1.789,-0.618,

-2.735,-1.879,-0.24,

-2.381,-1.6,-0.396,

-1.948,-1.651,-0.661,

-2.058,-1.476,-0.381),

n.Item,n.Threshold,byrow=TRUE)

# The source file containing the function

source("D:/CurrentFolder/ThresholdProbability.txt")

# Calling the function

p.LoadingInv <- ThresholdProbability(n.Item, Loadings, Common.Factor.Mean, Common.Factor.Variance, Unique.Factor.Variances, n.Threshold, Thresholds)

###########################

# Threshold invariance model

###########################

# Number of items that load on the factor

n.Item <- 5

# Factor loadings at the measurement occasion under examination

Loadings <- c(0.901, 1.021, 1.000, 0.649, 0.675)

# Mean of the common factor at the measurement occasion under examination

Common.Factor.Mean <- 0.000

# Variance of the common factor at the measurement occasion under examination

Common.Factor.Variance <- 0.628

# Variances of the unique factors at the measurement occasion under examination

Unique.Factor.Variances <- c(1, 1, 1, 1, 1)

# Number of thresholds per item: assumed to be the same across items

# Can handle cases where some items have ONE less threshold than others:

# In such cases, enter the maximum number of thresholds

n.Threshold <- 3

# Matrix of thresholds for items that load on the common factor;

# each row for a different item, ordered from lowest to highest threshold.

Thresholds <- matrix(c(-2.668,-1.704,-0.537,

-2.757,-1.653,-0.478,

-2.439,-1.55,-0.463,

-2.114,-1.493,-0.654,

-2.262,-1.346,-0.274),

n.Item,n.Threshold,byrow=TRUE)

# Calling the function

p.ThresholdInv <- ThresholdProbability(n.Item, Loadings, Common.Factor.Mean, Common.Factor.Variance, Unique.Factor.Variances, n.Threshold, Thresholds)

##############################################

# Calculate differenceS in predicted p values

##############################################

p.Difference<-matrix(NA,nrow=n.Item,ncol=n.Threshold+1);

p.Difference<-p.ThresholdInv - p.LoadingInv

# Round to the third decimal place

p.Difference<-round(p.Difference, digits = 3)

**Supplemental Materials 2A**

**M*plus* Code for the Empirical Illustration: Baseline Model**

TITLE: Longitudinal invariance of Child Familism: Obligation;

Baseline model;

Ordered-categorical CFA;

Lagged unique factor covariances modeled;

DATA:

! Response categories 1 & 2 are collapsed for all indicators;

file = ChildFamilism.dat;

VARIABLE:

names are (omitted for brevity);

Missing are all (-999);

usevariables are (omitted for brevity);

categorical are

V1T1 V2T1 V3T1 V4T1 V5T1

V1T2 V2T2 V3T2 V4T2 V5T2

V1T3 V2T3 V3T3 V4T3 V5T3

V1T4 V2T4 V3T4 V4T4 V5T4;

ANALYSIS:

PARAMETERIZATION = THETA;

ITERATIONS=3000;

ESTIMATOR=WLSMV;

MODEL:

! Factor loadings;

C1FAMo by V1T1\*

V2T1\*

V3T1@1

V4T1\*

V5T1\*;

C2FAMo by V1T2\*

V2T2\*

V3T2@1

V4T2\*

V5T2\*;

C3FAMo by V1T3\*

V2T3\*

V3T3@1

V4T3\*

V5T3\*;

C4FAMo by V1T4\*

V2T4\*

V3T4@1

V4T4\*

V5T4\*;

! Thresholds;

[V1T1$1 V1T2$1 V1T3$1 V1T4$1];

[V1T1$2 V1T2$2 V1T3$2 V1T4$2] (1);

[V1T1$3 V1T2$3 V1T3$3 V1T4$3];

[V2T1$1 V2T2$1 V2T3$1 V2T4$1];

[V2T1$2 V2T2$2 V2T3$2 V2T4$2] (2);

[V2T1$3 V2T2$3 V2T3$3 V2T4$3];

[V3T1$1 V3T2$1 V3T3$1 V3T4$1];

[V3T1$2 V3T2$2 V3T3$2 V3T4$2] (3);

[V3T1$3 V3T2$3 V3T3$3 V3T4$3] (4);

[V4T1$1 V4T2$1 V4T3$1 V4T4$1];

[V4T1$2 V4T2$2 V4T3$2 V4T4$2];

[V4T1$3 V4T2$3 V4T3$3 V4T4$3] (5);

[V5T1$1 V5T2$1 V5T3$1 V5T4$1];

[V5T1$2 V5T2$2 V5T3$2 V5T4$2] (6);

[V5T1$3 V5T2$3 V5T3$3 V5T4$3];

! Common factor covariance matrix;

C1Famo C2Famo C3Famo C4Famo with

C1Famo C2Famo C3Famo C4Famo;

! Common factor means;

[C1FAMo@0 C2FAMo\* C3FAMo\* C4FAMo\*];

! Unique variances;

V1T1@1 V2T1@1 V3T1@1 V4T1@1 V5T1@1;

V1T2 V2T2 V3T2 V4T2 V5T2;

V1T3 V2T3 V3T3 V4T3 V5T3;

V1T4 V2T4 V3T4 V4T4 V5T4;

! Lagged unique factor covariances;

V1T1 with V1T2\* V1T3\* V1T4\*;

V1T2 with V1T3\* V1T4\*;

V1T3 with V1T4\*;

V2T1 with V2T2\* V2T3\* V2T4\*;

V2T2 with V2T3\* V2T4\*;

V2T3 with V2T4\*;

V3T1 with V3T2\* V3T3\* V3T4\*;

V3T2 with V3T3\* V3T4\*;

V3T3 with V3T4\*;

V4T1 with V4T2\* V4T3\* V4T4\*;

V4T2 with V4T3\* V4T4\*;

V4T3 with V4T4\*;

V5T1 with V5T2\* V5T3\* V5T4\*;

V5T2 with V5T3\* V5T4\*;

V5T3 with V5T4\*;

SAVEDATA: DIFFTEST IS obligation\_baseline.dat;

OUTPUT: sampstat residual mod(all 10);

**Supplemental Materials 2B**

**M*plus* Code for the Empirical Illustration: Loading Invariance Model**

TITLE: Longitudinal invariance of Child Familism: Obligation;

Loading invariance model;

Ordered-categorical CFA;

Lagged unique factor covariances modeled;

DATA:

! Response categories 1 & 2 are collapsed for all indicators;

file = ChildFamilism.dat;

VARIABLE:

names are (omitted for brevity);

Missing are all (-999) ;

usevariables are (omitted for brevity);

categorical are

V1T1 V2T1 V3T1 V4T1 V5T1

V1T2 V2T2 V3T2 V4T2 V5T2

V1T3 V2T3 V3T3 V4T3 V5T3

V1T4 V2T4 V3T4 V4T4 V5T4;

ANALYSIS:

PARAMETERIZATION = THETA;

ITERATIONS=3000;

ESTIMATOR=WLSMV;

DIFFTEST IS obligation\_baseline.dat;

MODEL:

! Factor loadings;

C1FAMo by V1T1\* (7)

V2T1\* (8)

V3T1@1

V4T1\* (9)

V5T1\* (10);

C2FAMo by V1T2\* (7)

V2T2\* (8)

V3T2@1

V4T2\* (9)

V5T2\* (10);

C3FAMo by V1T3\* (7)

V2T3\* (8)

V3T3@1

V4T3\* (9)

V5T3\* (10);

C4FAMo by V1T4\* (7)

V2T4\* (8)

V3T4@1

V4T4\* (9)

V5T4\* (10);

! Thresholds;

[V1T1$1 V1T2$1 V1T3$1 V1T4$1];

[V1T1$2 V1T2$2 V1T3$2 V1T4$2] (1);

[V1T1$3 V1T2$3 V1T3$3 V1T4$3];

[V2T1$1 V2T2$1 V2T3$1 V2T4$1];

[V2T1$2 V2T2$2 V2T3$2 V2T4$2] (2);

[V2T1$3 V2T2$3 V2T3$3 V2T4$3];

[V3T1$1 V3T2$1 V3T3$1 V3T4$1];

[V3T1$2 V3T2$2 V3T3$2 V3T4$2] (3);

[V3T1$3 V3T2$3 V3T3$3 V3T4$3] (4);

[V4T1$1 V4T2$1 V4T3$1 V4T4$1];

[V4T1$2 V4T2$2 V4T3$2 V4T4$2];

[V4T1$3 V4T2$3 V4T3$3 V4T4$3] (5);

[V5T1$1 V5T2$1 V5T3$1 V5T4$1];

[V5T1$2 V5T2$2 V5T3$2 V5T4$2] (6);

[V5T1$3 V5T2$3 V5T3$3 V5T4$3];

! Common factor covariance matrix;

C1Famo C2Famo C3Famo C4Famo with

C1Famo C2Famo C3Famo C4Famo;

! Common factor means;

[C1FAMo@0 C2FAMo\* C3FAMo\* C4FAMo\*];

! Unique variances;

V1T1@1 V2T1@1 V3T1@1 V4T1@1 V5T1@1;

V1T2 V2T2 V3T2 V4T2 V5T2;

V1T3 V2T3 V3T3 V4T3 V5T3;

V1T4 V2T4 V3T4 V4T4 V5T4;

! Lagged unique factor covariances;

V1T1 with V1T2\* V1T3\* V1T4\*;

V1T2 with V1T3\* V1T4\*;

V1T3 with V1T4\*;

V2T1 with V2T2\* V2T3\* V2T4\*;

V2T2 with V2T3\* V2T4\*;

V2T3 with V2T4\*;

V3T1 with V3T2\* V3T3\* V3T4\*;

V3T2 with V3T3\* V3T4\*;

V3T3 with V3T4\*;

V4T1 with V4T2\* V4T3\* V4T4\*;

V4T2 with V4T3\* V4T4\*;

V4T3 with V4T4\*;

V5T1 with V5T2\* V5T3\* V5T4\*;

V5T2 with V5T3\* V5T4\*;

V5T3 with V5T4\*;

SAVEDATA: DIFFTEST IS obligation\_loading.dat;

OUTPUT: sampstat residual mod(all 10);

**Supplemental Materials 2C**

**M*plus* Code for the Empirical Illustration: Threshold Invariance Model**

TITLE: Longitudinal invariance of Child Familism: Obligation;

Threshold invariance model;

Ordered-categorical CFA;

Lagged unique factor covariances modeled;

DATA:

! Response categories 1 & 2 are collapsed for all indicators;

file = ChildFamilism.dat;

VARIABLE:

names are (omitted for brevity);

Missing are all (-999) ;

usevariables are (omitted for brevity);

categorical are

V1T1 V2T1 V3T1 V4T1 V5T1

V1T2 V2T2 V3T2 V4T2 V5T2

V1T3 V2T3 V3T3 V4T3 V5T3

V1T4 V2T4 V3T4 V4T4 V5T4;

ANALYSIS:

PARAMETERIZATION = THETA;

ITERATIONS=3000;

ESTIMATOR=WLSMV;

DIFFTEST IS obligation\_loading.dat;

MODEL:

! Factor loadings;

C1FAMo by V1T1\* (7)

V2T1\* (8)

V3T1@1

V4T1\* (9)

V5T1\* (10);

C2FAMo by V1T2\* (7)

V2T2\* (8)

V3T2@1

V4T2\* (9)

V5T2\* (10);

C3FAMo by V1T3\* (7)

V2T3\* (8)

V3T3@1

V4T3\* (9)

V5T3\* (10);

C4FAMo by V1T4\* (7)

V2T4\* (8)

V3T4@1

V4T4\* (9)

V5T4\* (10);

! Thresholds;

[V1T1$1 V1T2$1 V1T3$1 V1T4$1] (11);

[V1T1$2 V1T2$2 V1T3$2 V1T4$2] (1);

[V1T1$3 V1T2$3 V1T3$3 V1T4$3] (12);

[V2T1$1 V2T2$1 V2T3$1 V2T4$1] (13);

[V2T1$2 V2T2$2 V2T3$2 V2T4$2] (2);

[V2T1$3 V2T2$3 V2T3$3 V2T4$3] (14);

[V3T1$1 V3T2$1 V3T3$1 V3T4$1] (15);

[V3T1$2 V3T2$2 V3T3$2 V3T4$2] (3);

[V3T1$3 V3T2$3 V3T3$3 V3T4$3] (4);

[V4T1$1 V4T2$1 V4T3$1 V4T4$1] (16);

[V4T1$2 V4T2$2 V4T3$2 V4T4$2] (17);

[V4T1$3 V4T2$3 V4T3$3 V4T4$3] (5);

[V5T1$1 V5T2$1 V5T3$1 V5T4$1] (18);

[V5T1$2 V5T2$2 V5T3$2 V5T4$2] (6);

[V5T1$3 V5T2$3 V5T3$3 V5T4$3] (19);

! Common factor covariance matrix;

C1Famo C2Famo C3Famo C4Famo with

C1Famo C2Famo C3Famo C4Famo;

! Common factor means;

[C1FAMo@0 C2FAMo\* C3FAMo\* C4FAMo\*];

! Unique variances;

V1T1@1 V2T1@1 V3T1@1 V4T1@1 V5T1@1;

V1T2 V2T2 V3T2 V4T2 V5T2;

V1T3 V2T3 V3T3 V4T3 V5T3;

V1T4 V2T4 V3T4 V4T4 V5T4;

! Lagged unique factor covariances;

V1T1 with V1T2\* V1T3\* V1T4\*;

V1T2 with V1T3\* V1T4\*;

V1T3 with V1T4\*;

V2T1 with V2T2\* V2T3\* V2T4\*;

V2T2 with V2T3\* V2T4\*;

V2T3 with V2T4\*;

V3T1 with V3T2\* V3T3\* V3T4\*;

V3T2 with V3T3\* V3T4\*;

V3T3 with V3T4\*;

V4T1 with V4T2\* V4T3\* V4T4\*;

V4T2 with V4T3\* V4T4\*;

V4T3 with V4T4\*;

V5T1 with V5T2\* V5T3\* V5T4\*;

V5T2 with V5T3\* V5T4\*;

V5T3 with V5T4\*;

OUTPUT: sampstat residual mod(all 10);

**Supplemental Materials 2D**

**M*plus* Code for the Empirical Illustration: Unique Factor Invariance Model**

TITLE: Longitudinal invariance of Child Familism: Obligation;

Unique factor invariance model;

Ordered-categorical CFA;

Lagged unique factor covariances modeled;

DATA:

! Response categories 1 & 2 are collapsed for all indicators;

file = ChildFamilism.dat;

VARIABLE:

names are (omitted for brevity);

Missing are all (-999) ;

usevariables are (omitted for brevity);

categorical are

V1T1 V2T1 V3T1 V4T1 V5T1

V1T2 V2T2 V3T2 V4T2 V5T2

V1T3 V2T3 V3T3 V4T3 V5T3

V1T4 V2T4 V3T4 V4T4 V5T4;

ANALYSIS:

PARAMETERIZATION = THETA;

ITERATIONS=3000;

ESTIMATOR=WLSMV;

DIFFTEST IS obligation\_loading.dat;

MODEL:

! Factor loadings;

C1FAMo by V1T1\* (7)

V2T1\* (8)

V3T1@1

V4T1\* (9)

V5T1\* (10);

C2FAMo by V1T2\* (7)

V2T2\* (8)

V3T2@1

V4T2\* (9)

V5T2\* (10);

C3FAMo by V1T3\* (7)

V2T3\* (8)

V3T3@1

V4T3\* (9)

V5T3\* (10);

C4FAMo by V1T4\* (7)

V2T4\* (8)

V3T4@1

V4T4\* (9)

V5T4\* (10);

! Thresholds;

[V1T1$1 V1T2$1 V1T3$1 V1T4$1] (11);

[V1T1$2 V1T2$2 V1T3$2 V1T4$2] (1);

[V1T1$3 V1T2$3 V1T3$3 V1T4$3] (12);

[V2T1$1 V2T2$1 V2T3$1 V2T4$1] (13);

[V2T1$2 V2T2$2 V2T3$2 V2T4$2] (2);

[V2T1$3 V2T2$3 V2T3$3 V2T4$3] (14);

[V3T1$1 V3T2$1 V3T3$1 V3T4$1] (15);

[V3T1$2 V3T2$2 V3T3$2 V3T4$2] (3);

[V3T1$3 V3T2$3 V3T3$3 V3T4$3] (4);

[V4T1$1 V4T2$1 V4T3$1 V4T4$1] (16);

[V4T1$2 V4T2$2 V4T3$2 V4T4$2] (17);

[V4T1$3 V4T2$3 V4T3$3 V4T4$3] (5);

[V5T1$1 V5T2$1 V5T3$1 V5T4$1] (18);

[V5T1$2 V5T2$2 V5T3$2 V5T4$2] (6);

[V5T1$3 V5T2$3 V5T3$3 V5T4$3] (19);

! Common factor covariance matrix;

C1Famo C2Famo C3Famo C4Famo with

C1Famo C2Famo C3Famo C4Famo;

! Common factor means;

[C1FAMo@0 C2FAMo\* C3FAMo\* C4FAMo\*];

! Unique variances;

V1T1@1 V2T1@1 V3T1@1 V4T1@1 V5T1@1;

V1T2@1 V2T2@1 V3T2@1 V4T2@1 V5T2@1;

V1T3@1 V2T3@1 V3T3@1 V4T3@1 V5T3@1;

V1T4@1 V2T4@1 V3T4@1 V4T4@1 V5T4@1;

! Lagged unique factor covariances;

V1T1 with V1T2\* V1T3\* V1T4\*;

V1T2 with V1T3\* V1T4\*;

V1T3 with V1T4\*;

V2T1 with V2T2\* V2T3\* V2T4\*;

V2T2 with V2T3\* V2T4\*;

V2T3 with V2T4\*;

V3T1 with V3T2\* V3T3\* V3T4\*;

V3T2 with V3T3\* V3T4\*;

V3T3 with V3T4\*;

V4T1 with V4T2\* V4T3\* V4T4\*;

V4T2 with V4T3\* V4T4\*;

V4T3 with V4T4\*;

V5T1 with V5T2\* V5T3\* V5T4\*;

V5T2 with V5T3\* V5T4\*;

V5T3 with V5T4\*;

OUTPUT: sampstat residual mod(all 10);

**Supplemental Materials 3A**

**lavaan Code for the Empirical Illustration: Baseline Model**

library(lavaan)

# Begin recoding for missing data

ChildFam$V1T1[ChildFam$V1T1==-999] <- NA

ChildFam$V1T2[ChildFam$V1T2==-999] <- NA

ChildFam$V1T3[ChildFam$V1T3==-999] <- NA

ChildFam$V1T4[ChildFam$V1T4==-999] <- NA

ChildFam$V2T1[ChildFam$V2T1==-999] <- NA

ChildFam$V2T2[ChildFam$V2T2==-999] <- NA

ChildFam$V2T3[ChildFam$V2T3==-999] <- NA

ChildFam$V2T4[ChildFam$V2T4==-999] <- NA

ChildFam$V3T1[ChildFam$V3T1==-999] <- NA

ChildFam$V3T2[ChildFam$V3T2==-999] <- NA

ChildFam$V3T3[ChildFam$V3T3==-999] <- NA

ChildFam$V3T4[ChildFam$V3T4==-999] <- NA

ChildFam$V4T1[ChildFam$V4T1==-999] <- NA

ChildFam$V4T2[ChildFam$V4T2==-999] <- NA

ChildFam$V4T3[ChildFam$V4T3==-999] <- NA

ChildFam$V4T4[ChildFam$V4T4==-999] <- NA

ChildFam$V5T1[ChildFam$V5T1==-999] <- NA

ChildFam$V5T2[ChildFam$V5T2==-999] <- NA

ChildFam$V5T3[ChildFam$V5T3==-999] <- NA

ChildFam$V5T4[ChildFam$V5T4==-999] <- NA

# End recoding for missing data

Baseline.Model <- '

# Latent common factor definitions

C1FAMo =~ NA\*V1T1 + V2T1 + 1\*V3T1 + V4T1 + V5T1

C2FAMo =~ NA\*V1T2 + V2T2 + 1\*V3T2 + V4T2 + V5T2

C3FAMo =~ NA\*V1T3 + V2T3 + 1\*V3T3 + V4T3 + V5T3

C4FAMo =~ NA\*V1T4 + V2T4 + 1\*V3T4 + V4T4 + V5T4

# Latent common factor variances and covariances

C1FAMo ~~ C1FAMo + C2FAMo + C3FAMo + C4FAMo

C2FAMo ~~ C2FAMo + C3FAMo + C4FAMo

C3FAMo ~~ C3FAMo + C4FAMo

C4FAMo ~~ C4FAMo

# Latent common factor means

# Fix T1 common factor mean to zero

C1FAMo ~ 0\*1

# Freely estimate T2-T4 common factor means

C2FAMo ~ 1

C3FAMo ~ 1

C4FAMo ~ 1

# Thresholds

# t1, t2, and t3 after \* represent the first, second, and third

# thresholds of an indicator, respectively

# Threshold parameters that are freely estimated have labels VaTbtc,

# with a representing indicator number, b representing time,

# c representing ordering of the threshold (first, second, third)

# Threshold parameters that are constrained to be equal over time

# are given the labels Vatc, with a representing indicator number,

# c representing ordering of the threshold (first, second, third)

V1T1 | V1T1t1\*t1 + V1t2\*t2 + V1T1t3\*t3

V1T2 | V1T2t1\*t1 + V1t2\*t2 + V1T2t3\*t3

V1T3 | V1T3t1\*t1 + V1t2\*t2 + V1T3t3\*t3

V1T4 | V1T4t1\*t1 + V1t2\*t2 + V1T4t3\*t3

V2T1 | V2T1t1\*t1 + V2t2\*t2 + V2T1t3\*t3

V2T2 | V2T2t1\*t1 + V2t2\*t2 + V2T2t3\*t3

V2T3 | V2T3t1\*t1 + V2t2\*t2 + V2T3t3\*t3

V2T4 | V2T4t1\*t1 + V2t2\*t2 + V2T4t3\*t3

V3T1 | V3T1t1\*t1 + V3t2\*t2 + V3t3\*t3

V3T2 | V3T2t1\*t1 + V3t2\*t2 + V3t3\*t3

V3T3 | V3T3t1\*t1 + V3t2\*t2 + V3t3\*t3

V3T4 | V3T4t1\*t1 + V3t2\*t2 + V3t3\*t3

V4T1 | V4T1t1\*t1 + V4T1t2\*t2 + V4t3\*t3

V4T2 | V4T2t1\*t1 + V4T2t2\*t2 + V4t3\*t3

V4T3 | V4T3t1\*t1 + V4T3t2\*t2 + V4t3\*t3

V4T4 | V4T4t1\*t1 + V4T4t2\*t2 + V4t3\*t3

V5T1 | V5T1t1\*t1 + V5t2\*t2 + V5T1t3\*t3

V5T2 | V5T2t1\*t1 + V5t2\*t2 + V5T2t3\*t3

V5T3 | V5T3t1\*t1 + V5t2\*t2 + V5T3t3\*t3

V5T4 | V5T4t1\*t1 + V5t2\*t2 + V5T4t3\*t3

# Intercepts

# Fix all intercepts to zero

V1T1 + V2T1 + V3T1 + V4T1 + V5T1 ~ 0\*1

V1T2 + V2T2 + V3T2 + V4T2 + V5T2 ~ 0\*1

V1T3 + V2T3 + V3T3 + V4T3 + V5T3 ~ 0\*1

V1T4 + V2T4 + V3T4 + V4T4 + V5T4 ~ 0\*1

# Unique variances

# Fix unique variances at T1 to 1.00

V1T1 ~~ 1\*V1T1

V2T1 ~~ 1\*V2T1

V3T1 ~~ 1\*V3T1

V4T1 ~~ 1\*V4T1

V5T1 ~~ 1\*V5T1

# lavaan constrains all indicator unique variances to 1.00 by default

# Free this constraint at T2-T4

V1T2 ~~ NA\*V1T2

V2T2 ~~ NA\*V2T2

V3T2 ~~ NA\*V3T2

V4T2 ~~ NA\*V4T2

V5T2 ~~ NA\*V5T2

V1T3 ~~ NA\*V1T3

V2T3 ~~ NA\*V2T3

V3T3 ~~ NA\*V3T3

V4T3 ~~ NA\*V4T3

V5T3 ~~ NA\*V5T3

V1T4 ~~ NA\*V1T4

V2T4 ~~ NA\*V2T4

V3T4 ~~ NA\*V3T4

V4T4 ~~ NA\*V4T4

V5T4 ~~ NA\*V5T4

# Lagged unique factor covariances

V1T1 ~~ V1T2 + V1T3 + V1T4

V1T2 ~~ V1T3 + V1T4

V1T3 ~~ V1T4

V2T1 ~~ V2T2 + V2T3 + V1T4

V2T2 ~~ V2T3 + V1T4

V2T3 ~~ V1T4

V3T1 ~~ V3T2 + V3T3 + V3T4

V3T2 ~~ V3T3 + V3T4

V3T3 ~~ V3T4

V4T1 ~~ V4T2 + V4T3 + V4T4

V4T2 ~~ V4T3 + V4T4

V4T3 ~~ V4T4

V5T1 ~~ V5T2 + V5T3 + V5T4

V5T2 ~~ V5T3 + V5T4

V5T3 ~~ V5T4

'

Fit.Baseline.Model <- cfa(Baseline.Model, data = ChildFam,

ordered=c("V1T1", "V2T1", "V3T1", "V4T1", "V5T1",

"V1T2", "V2T2", "V3T2", "V4T2", "V5T2",

"V1T3", "V2T3", "V3T3", "V4T3", "V5T3",

"V1T4", "V2T4", "V3T4", "V4T4", "V5T4"),

parameterization = "theta", estimator = "wlsmv",

missing = "pairwise"

# Default is missing = "listwise"

)

# Print out parameter estimates and fit statistics

summary(Fit.Baseline.Model, fit.measures = TRUE)

**Supplemental Materials 3B**

**lavaan Code for the Empirical Illustration: Loading Invariance Model**

LoadingInv.Model <- '

# Latent common factor definitions

# Give factor loadings the same label over time to constrain them to be equal

# The first indicators of each measurement occasion are mentioned twice;

# the first time freeing the default constraint (1.00),

# the second time giving them the same label;

C1FAMo =~ NA\*V1T1 + V1Loading\*V1T1 + V2Loading\*V2T1 + 1\*V3T1 + V4Loading\*V4T1 + V5Loading\*V5T1

C2FAMo =~ NA\*V1T2 + V1Loading\*V1T2 + V2Loading\*V2T2 + 1\*V3T2 + V4Loading\*V4T2 + V5Loading\*V5T2

C3FAMo =~ NA\*V1T3 + V1Loading\*V1T3 + V2Loading\*V2T3 + 1\*V3T3 + V4Loading\*V4T3 + V5Loading\*V5T3

C4FAMo =~ NA\*V1T4 + V1Loading\*V1T4 + V2Loading\*V2T4 + 1\*V3T4 + V4Loading\*V4T4 + V5Loading\*V5T4

# Latent common factor variances and covariances

C1FAMo ~~ C1FAMo + C2FAMo + C3FAMo + C4FAMo

C2FAMo ~~ C2FAMo + C3FAMo + C4FAMo

C3FAMo ~~ C3FAMo + C4FAMo

C4FAMo ~~ C4FAMo

# Latent common factor means

# Fix T1 common factor mean to zero

C1FAMo ~ 0\*1

# Freely estimate T2-T4 common factor means

C2FAMo ~ 1

C3FAMo ~ 1

C4FAMo ~ 1

# Thresholds

# t1, t2, and t3 after \* represent the first, second, and third thresholds of an indicator,

# respectively

# Threshold parameters that are freely estimated have labels VaTbtc, with a representing indicator

# number, b representing time, and c representing ordering of the threshold (first, second,

# third)

# Threshold parameters that are constrained to be equal over time are given the labels Vatc, with

# a representing indicator number and c representing ordering of the threshold (first, second,

# third)

V1T1 | V1T1t1\*t1 + V1t2\*t2 + V1T1t3\*t3

V1T2 | V1T2t1\*t1 + V1t2\*t2 + V1T2t3\*t3

V1T3 | V1T3t1\*t1 + V1t2\*t2 + V1T3t3\*t3

V1T4 | V1T4t1\*t1 + V1t2\*t2 + V1T4t3\*t3

V2T1 | V2T1t1\*t1 + V2t2\*t2 + V2T1t3\*t3

V2T2 | V2T2t1\*t1 + V2t2\*t2 + V2T2t3\*t3

V2T3 | V2T3t1\*t1 + V2t2\*t2 + V2T3t3\*t3

V2T4 | V2T4t1\*t1 + V2t2\*t2 + V2T4t3\*t3

V3T1 | V3T1t1\*t1 + V3t2\*t2 + V3t3\*t3

V3T2 | V3T2t1\*t1 + V3t2\*t2 + V3t3\*t3

V3T3 | V3T3t1\*t1 + V3t2\*t2 + V3t3\*t3

V3T4 | V3T4t1\*t1 + V3t2\*t2 + V3t3\*t3

V4T1 | V4T1t1\*t1 + V4T1t2\*t2 + V4t3\*t3

V4T2 | V4T2t1\*t1 + V4T2t2\*t2 + V4t3\*t3

V4T3 | V4T3t1\*t1 + V4T3t2\*t2 + V4t3\*t3

V4T4 | V4T4t1\*t1 + V4T4t2\*t2 + V4t3\*t3

V5T1 | V5T1t1\*t1 + V5t2\*t2 + V5T1t3\*t3

V5T2 | V5T2t1\*t1 + V5t2\*t2 + V5T2t3\*t3

V5T3 | V5T3t1\*t1 + V5t2\*t2 + V5T3t3\*t3

V5T4 | V5T4t1\*t1 + V5t2\*t2 + V5T4t3\*t3

# Intercepts

# Fix all intercepts to zero

V1T1 + V2T1 + V3T1 + V4T1 + V5T1 ~ 0\*1

V1T2 + V2T2 + V3T2 + V4T2 + V5T2 ~ 0\*1

V1T3 + V2T3 + V3T3 + V4T3 + V5T3 ~ 0\*1

V1T4 + V2T4 + V3T4 + V4T4 + V5T4 ~ 0\*1

# Unique variances

# Fix unique variances at T1 to 1.00

V1T1 ~~ 1\*V1T1

V2T1 ~~ 1\*V2T1

V3T1 ~~ 1\*V3T1

V4T1 ~~ 1\*V4T1

V5T1 ~~ 1\*V5T1

# lavaan constrains all the indicator unique variances to be 1.00 by default.

# Free this constraint at T2-T4

V1T2 ~~ NA\*V1T2

V2T2 ~~ NA\*V2T2

V3T2 ~~ NA\*V3T2

V4T2 ~~ NA\*V4T2

V5T2 ~~ NA\*V5T2

V1T3 ~~ NA\*V1T3

V2T3 ~~ NA\*V2T3

V3T3 ~~ NA\*V3T3

V4T3 ~~ NA\*V4T3

V5T3 ~~ NA\*V5T3

V1T4 ~~ NA\*V1T4

V2T4 ~~ NA\*V2T4

V3T4 ~~ NA\*V3T4

V4T4 ~~ NA\*V4T4

V5T4 ~~ NA\*V5T4

# Lagged unique factor covariances

V1T1 ~~ V1T2 + V1T3 + V1T4

V1T2 ~~ V1T3 + V1T4

V1T3 ~~ V1T4

V2T1 ~~ V2T2 + V2T3 + V1T4

V2T2 ~~ V2T3 + V1T4

V2T3 ~~ V1T4

V3T1 ~~ V3T2 + V3T3 + V3T4

V3T2 ~~ V3T3 + V3T4

V3T3 ~~ V3T4

V4T1 ~~ V4T2 + V4T3 + V4T4

V4T2 ~~ V4T3 + V4T4

V4T3 ~~ V4T4

V5T1 ~~ V5T2 + V5T3 + V5T4

V5T2 ~~ V5T3 + V5T4

V5T3 ~~ V5T4

'

Fit.LoadingInv.Model <- cfa(LoadingInv.Model, data = ChildFam,

ordered=c("V1T1", "V2T1", "V3T1", "V4T1", "V5T1",

"V1T2", "V2T2", "V3T2", "V4T2", "V5T2",

"V1T3", "V2T3", "V3T3", "V4T3", "V5T3",

"V1T4", "V2T4", "V3T4", "V4T4", "V5T4"),

parameterization = "theta", estimator = "wlsmv",

missing = "pairwise"

# Default is missing = "listwise"

)

# Print out parameter estimates and fit statistics

summary(Fit.LoadingInv.Model, fit.measures = TRUE)

# Request nested model test

anova(Fit.Baseline.Model, Fit.LoadingInv.Model)

# Request modification indices

# The modindices() function returns a data frame

LoadingInv.Model.MI <- modindices(Fit.LoadingInv.Model)

# Select those rows with modification indices greater than 10

LoadingInv.Model.MI[LoadingInv.Model.MI$mi >= 10,]

**Supplemental Materials 3C**

**lavaan Code for the Empirical Illustration: Threshold Invariance Model**

ThresholdInv.Model <- '

# Latent common factor definitions

# Give factor loadings the same label over time to constrain them to be equal

# The first indicators of each measurement occasion are mentioned twice;

# the first time freeing the default constraint (1.00),

# the second time giving them the same label;

C1FAMo =~ NA\*V1T1 + V1Loading\*V1T1 + V2Loading\*V2T1 + 1\*V3T1 + V4Loading\*V4T1 + V5Loading\*V5T1

C2FAMo =~ NA\*V1T2 + V1Loading\*V1T2 + V2Loading\*V2T2 + 1\*V3T2 + V4Loading\*V4T2 + V5Loading\*V5T2

C3FAMo =~ NA\*V1T3 + V1Loading\*V1T3 + V2Loading\*V2T3 + 1\*V3T3 + V4Loading\*V4T3 + V5Loading\*V5T3

C4FAMo =~ NA\*V1T4 + V1Loading\*V1T4 + V2Loading\*V2T4 + 1\*V3T4 + V4Loading\*V4T4 + V5Loading\*V5T4

# Latent common factor variances and covariances

C1FAMo ~~ C1FAMo + C2FAMo + C3FAMo + C4FAMo

C2FAMo ~~ C2FAMo + C3FAMo + C4FAMo

C3FAMo ~~ C3FAMo + C4FAMo

C4FAMo ~~ C4FAMo

# Latent common factor means

# Fix T1 common factor mean to zero

C1FAMo ~ 0\*1

# Freely estimate T2-T4 common factor means

C2FAMo ~ 1

C3FAMo ~ 1

C4FAMo ~ 1

# Thresholds

# t1, t2, and t3 after \* represent the first, second, and third thresholds, respectively

# Threshold parameters that are constrained to be equal over time are given the labels Vatc, with

# a representing indicator number and c representing ordering of the threshold (first, second,

# third)

V1T1 | V1t1\*t1 + V1t2\*t2 + V1t3\*t3

V1T2 | V1t1\*t1 + V1t2\*t2 + V1t3\*t3

V1T3 | V1t1\*t1 + V1t2\*t2 + V1t3\*t3

V1T4 | V1t1\*t1 + V1t2\*t2 + V1t3\*t3

V2T1 | V2t1\*t1 + V2t2\*t2 + V2t3\*t3

V2T2 | V2t1\*t1 + V2t2\*t2 + V2t3\*t3

V2T3 | V2t1\*t1 + V2t2\*t2 + V2t3\*t3

V2T4 | V2t1\*t1 + V2t2\*t2 + V2t3\*t3

V3T1 | V3t1\*t1 + V3t2\*t2 + V3t3\*t3

V3T2 | V3t1\*t1 + V3t2\*t2 + V3t3\*t3

V3T3 | V3t1\*t1 + V3t2\*t2 + V3t3\*t3

V3T4 | V3t1\*t1 + V3t2\*t2 + V3t3\*t3

V4T1 | V4t1\*t1 + V4t2\*t2 + V4t3\*t3

V4T2 | V4t1\*t1 + V4t2\*t2 + V4t3\*t3

V4T3 | V4t1\*t1 + V4t2\*t2 + V4t3\*t3

V4T4 | V4t1\*t1 + V4t2\*t2 + V4t3\*t3

V5T1 | V5t1\*t1 + V5t2\*t2 + V5t3\*t3

V5T2 | V5t1\*t1 + V5t2\*t2 + V5t3\*t3

V5T3 | V5t1\*t1 + V5t2\*t2 + V5t3\*t3

V5T4 | V5t1\*t1 + V5t2\*t2 + V5t3\*t3

# Intercepts

# Fix all intercepts to zero

V1T1 + V2T1 + V3T1 + V4T1 + V5T1 ~ 0\*1

V1T2 + V2T2 + V3T2 + V4T2 + V5T2 ~ 0\*1

V1T3 + V2T3 + V3T3 + V4T3 + V5T3 ~ 0\*1

V1T4 + V2T4 + V3T4 + V4T4 + V5T4 ~ 0\*1

# Unique variances

# Fix unique variances at T1 to 1.00

V1T1 ~~ 1\*V1T1

V2T1 ~~ 1\*V2T1

V3T1 ~~ 1\*V3T1

V4T1 ~~ 1\*V4T1

V5T1 ~~ 1\*V5T1

# lavaan constrains all the indicator unique variances to be 1.00 by default.

# Free this constraint at T2-T4

V1T2 ~~ NA\*V1T2

V2T2 ~~ NA\*V2T2

V3T2 ~~ NA\*V3T2

V4T2 ~~ NA\*V4T2

V5T2 ~~ NA\*V5T2

V1T3 ~~ NA\*V1T3

V2T3 ~~ NA\*V2T3

V3T3 ~~ NA\*V3T3

V4T3 ~~ NA\*V4T3

V5T3 ~~ NA\*V5T3

V1T4 ~~ NA\*V1T4

V2T4 ~~ NA\*V2T4

V3T4 ~~ NA\*V3T4

V4T4 ~~ NA\*V4T4

V5T4 ~~ NA\*V5T4

# Lagged unique factor covariances

V1T1 ~~ V1T2 + V1T3 + V1T4

V1T2 ~~ V1T3 + V1T4

V1T3 ~~ V1T4

V2T1 ~~ V2T2 + V2T3 + V1T4

V2T2 ~~ V2T3 + V1T4

V2T3 ~~ V1T4

V3T1 ~~ V3T2 + V3T3 + V3T4

V3T2 ~~ V3T3 + V3T4

V3T3 ~~ V3T4

V4T1 ~~ V4T2 + V4T3 + V4T4

V4T2 ~~ V4T3 + V4T4

V4T3 ~~ V4T4

V5T1 ~~ V5T2 + V5T3 + V5T4

V5T2 ~~ V5T3 + V5T4

V5T3 ~~ V5T4

'

Fit.ThresholdInv.Model <- cfa(ThresholdInv.Model, data = ChildFam,

ordered=c("V1T1", "V2T1", "V3T1", "V4T1", "V5T1",

"V1T2", "V2T2", "V3T2", "V4T2", "V5T2",

"V1T3", "V2T3", "V3T3", "V4T3", "V5T3",

"V1T4", "V2T4", "V3T4", "V4T4", "V5T4"),

parameterization = "theta", estimator = "wlsmv",

missing = "pairwise"

# Default is missing = "listwise"

)

# Print out parameter estimates and fit statistics

summary(Fit.ThresholdInv.Model, fit.measures = TRUE)

# Request nested model test

anova(Fit.LoadingInv.Model, Fit.ThresholdInv.Model)

# Request modification indices

# The modindices() function returns a data frame

ThresholdInv.Model.MI <- modindices(Fit.ThresholdInv.Model)

# Select those rows with modification indices greater than 10

ThresholdInv.Model.MI[ThresholdInv.Model.MI$mi >= 10,]

**Supplemental Materials 3D**

**lavaan Code for the Empirical Illustration: Unique Factor Invariance Model**

UniquenessInv.Model <- '

# Latent common factor definitions

# Give factor loadings the same label over time to constrain them to be equal

# The first indicators of each measurement occasion are mentioned twice;

# the first time freeing the default constraint (1.00),

# the second time giving them the same label;

C1FAMo =~ NA\*V1T1 + V1Loading\*V1T1 + V2Loading\*V2T1 + 1\*V3T1 + V4Loading\*V4T1 + V5Loading\*V5T1

C2FAMo =~ NA\*V1T2 + V1Loading\*V1T2 + V2Loading\*V2T2 + 1\*V3T2 + V4Loading\*V4T2 + V5Loading\*V5T2

C3FAMo =~ NA\*V1T3 + V1Loading\*V1T3 + V2Loading\*V2T3 + 1\*V3T3 + V4Loading\*V4T3 + V5Loading\*V5T3

C4FAMo =~ NA\*V1T4 + V1Loading\*V1T4 + V2Loading\*V2T4 + 1\*V3T4 + V4Loading\*V4T4 + V5Loading\*V5T4

# Latent common factor variances and covariances

C1FAMo ~~ C1FAMo + C2FAMo + C3FAMo + C4FAMo

C2FAMo ~~ C2FAMo + C3FAMo + C4FAMo

C3FAMo ~~ C3FAMo + C4FAMo

C4FAMo ~~ C4FAMo

# Latent common factor means

# Fix T1 common factor mean to zero

C1FAMo ~ 0\*1

# Freely estimate T2-T4 common factor means

C2FAMo ~ 1

C3FAMo ~ 1

C4FAMo ~ 1

# Thresholds

# t1, t2, and t3 after \* represent the first, second, and third thresholds, respectively

# Threshold parameters that are constrained to be equal over time are given the labels Vatc, with

# a representing indicator number and c representing ordering of the threshold (first, second,

# third)

V1T1 | V1t1\*t1 + V1t2\*t2 + V1t3\*t3

V1T2 | V1t1\*t1 + V1t2\*t2 + V1t3\*t3

V1T3 | V1t1\*t1 + V1t2\*t2 + V1t3\*t3

V1T4 | V1t1\*t1 + V1t2\*t2 + V1t3\*t3

V2T1 | V2t1\*t1 + V2t2\*t2 + V2t3\*t3

V2T2 | V2t1\*t1 + V2t2\*t2 + V2t3\*t3

V2T3 | V2t1\*t1 + V2t2\*t2 + V2t3\*t3

V2T4 | V2t1\*t1 + V2t2\*t2 + V2t3\*t3

V3T1 | V3t1\*t1 + V3t2\*t2 + V3t3\*t3

V3T2 | V3t1\*t1 + V3t2\*t2 + V3t3\*t3

V3T3 | V3t1\*t1 + V3t2\*t2 + V3t3\*t3

V3T4 | V3t1\*t1 + V3t2\*t2 + V3t3\*t3

V4T1 | V4t1\*t1 + V4t2\*t2 + V4t3\*t3

V4T2 | V4t1\*t1 + V4t2\*t2 + V4t3\*t3

V4T3 | V4t1\*t1 + V4t2\*t2 + V4t3\*t3

V4T4 | V4t1\*t1 + V4t2\*t2 + V4t3\*t3

V5T1 | V5t1\*t1 + V5t2\*t2 + V5t3\*t3

V5T2 | V5t1\*t1 + V5t2\*t2 + V5t3\*t3

V5T3 | V5t1\*t1 + V5t2\*t2 + V5t3\*t3

V5T4 | V5t1\*t1 + V5t2\*t2 + V5t3\*t3

# Intercepts

# Fix all intercepts to zero

V1T1 + V2T1 + V3T1 + V4T1 + V5T1 ~ 0\*1

V1T2 + V2T2 + V3T2 + V4T2 + V5T2 ~ 0\*1

V1T3 + V2T3 + V3T3 + V4T3 + V5T3 ~ 0\*1

V1T4 + V2T4 + V3T4 + V4T4 + V5T4 ~ 0\*1

# Unique variances

# Constrain unique variances to be equal across time

# Since unique variances are fixed to 1.00 at T1,

# essentially this model constrains all unique variance at all occasions to 1.00

V1T1 ~~ 1\*V1T1

V2T1 ~~ 1\*V2T1

V3T1 ~~ 1\*V3T1

V4T1 ~~ 1\*V4T1

V5T1 ~~ 1\*V5T1

V1T2 ~~ 1\*V1T2

V2T2 ~~ 1\*V2T2

V3T2 ~~ 1\*V3T2

V4T2 ~~ 1\*V4T2

V5T2 ~~ 1\*V5T2

V1T3 ~~ 1\*V1T3

V2T3 ~~ 1\*V2T3

V3T3 ~~ 1\*V3T3

V4T3 ~~ 1\*V4T3

V5T3 ~~ 1\*V5T3

V1T4 ~~ 1\*V1T4

V2T4 ~~ 1\*V2T4

V3T4 ~~ 1\*V3T4

V4T4 ~~ 1\*V4T4

V5T4 ~~ 1\*V5T4

# Lagged unique factor covariances

V1T1 ~~ V1T2 + V1T3 + V1T4

V1T2 ~~ V1T3 + V1T4

V1T3 ~~ V1T4

V2T1 ~~ V2T2 + V2T3 + V1T4

V2T2 ~~ V2T3 + V1T4

V2T3 ~~ V1T4

V3T1 ~~ V3T2 + V3T3 + V3T4

V3T2 ~~ V3T3 + V3T4

V3T3 ~~ V3T4

V4T1 ~~ V4T2 + V4T3 + V4T4

V4T2 ~~ V4T3 + V4T4

V4T3 ~~ V4T4

V5T1 ~~ V5T2 + V5T3 + V5T4

V5T2 ~~ V5T3 + V5T4

V5T3 ~~ V5T4

'

Fit.UniquenessInv.Model <- cfa(UniquenessInv.Model, data = ChildFam,

ordered=c("V1T1", "V2T1", "V3T1", "V4T1", "V5T1",

"V1T2", "V2T2", "V3T2", "V4T2", "V5T2",

"V1T3", "V2T3", "V3T3", "V4T3", "V5T3",

"V1T4", "V2T4", "V3T4", "V4T4", "V5T4"),

parameterization = "theta", estimator = "wlsmv",

missing = "pairwise"

# Default is missing = "listwise"

)

# Print out parameter estimates and fit statistics

summary(Fit.UniquenessInv.Model, fit.measures = TRUE)

# Request nested model test

anova(Fit.ThresholdInv.Model, Fit.UniquenessInv.Model)

# Request modification indices

# The modindices() function returns a data frame

UniquenessInv.Model.MI <- modindices(Fit.UniquenessInv.Model)

# Select those rows with modification indices greater than 10

UniquenessInv.Model.MI[UniquenessInv.Model.MI$mi >= 10,]

**Supplemental Materials 4A**

**OpenMx Code for the Empirical Illustration: Baseline Model**

# Download the beta version of OpenMx

source('http://openmx.psyc.virginia.edu/getOpenMxBeta.R')

# Load library

library(OpenMx)

# Convert items to categorical variables

ChildFam$V1T1 <- mxFactor(ChildFam$V1T1, levels = c(0,1,2,3))

ChildFam$V1T2 <- mxFactor(ChildFam$V1T2, levels = c(0,1,2,3))

ChildFam$V1T3 <- mxFactor(ChildFam$V1T3, levels = c(0,1,2,3))

ChildFam$V1T4 <- mxFactor(ChildFam$V1T4, levels = c(0,1,2,3))

ChildFam$V2T1 <- mxFactor(ChildFam$V2T1, levels = c(0,1,2,3))

ChildFam$V2T2 <- mxFactor(ChildFam$V2T2, levels = c(0,1,2,3))

ChildFam$V2T3 <- mxFactor(ChildFam$V2T3, levels = c(0,1,2,3))

ChildFam$V2T4 <- mxFactor(ChildFam$V2T4, levels = c(0,1,2,3))

ChildFam$V3T1 <- mxFactor(ChildFam$V3T1, levels = c(0,1,2,3))

ChildFam$V3T2 <- mxFactor(ChildFam$V3T2, levels = c(0,1,2,3))

ChildFam$V3T3 <- mxFactor(ChildFam$V3T3, levels = c(0,1,2,3))

ChildFam$V3T4 <- mxFactor(ChildFam$V3T4, levels = c(0,1,2,3))

ChildFam$V4T1 <- mxFactor(ChildFam$V4T1, levels = c(0,1,2,3))

ChildFam$V4T2 <- mxFactor(ChildFam$V4T2, levels = c(0,1,2,3))

ChildFam$V4T3 <- mxFactor(ChildFam$V4T3, levels = c(0,1,2,3))

ChildFam$V4T4 <- mxFactor(ChildFam$V4T4, levels = c(0,1,2,3))

ChildFam$V5T1 <- mxFactor(ChildFam$V5T1, levels = c(0,1,2,3))

ChildFam$V5T2 <- mxFactor(ChildFam$V5T2, levels = c(0,1,2,3))

ChildFam$V5T3 <- mxFactor(ChildFam$V5T3, levels = c(0,1,2,3))

ChildFam$V5T4 <- mxFactor(ChildFam$V5T4, levels = c(0,1,2,3))

# Baseline model

FAMo.Baseline.omx <- mxModel("Baseline",

type = "RAM",

mxData(observed=ChildFam, type="raw"),

manifestVars = c('V1T1','V1T2','V1T3','V1T4',

'V2T1','V2T2','V2T3','V2T4',

'V3T1','V3T2','V3T3','V3T4',

'V4T1','V4T2','V4T3','V4T4',

'V5T1','V5T2','V5T3','V5T4'),

latentVars = c('C1FAMo','C2FAMo','C3FAMo','C4FAMo'),

# Unique variances at T1: Fixed to 1

mxPath(from=c('V1T1','V2T1','V3T1','V4T1','V5T1'),

arrows=2, free=FALSE, values=1),

# Unique variances at other waves: Freely estimated

mxPath(from=c('V1T2','V1T3','V1T4',

'V2T2','V2T3','V2T4',

'V3T2','V3T3','V3T4',

'V4T2','V4T3','V4T4',

'V5T2','V5T3','V5T4'),

arrows=2, free=TRUE, values=c(1.23,1.18,2.01,

0.70,0.59,0.46,

0.62,0.58,0.64,

0.41,0.50,0.49,

0.96,1.33,1.09)),

# Lagged unique factor covariances

mxPath(from=c('V1T1','V1T1','V1T1','V1T2','V1T2','V1T3'),

to = c('V1T2','V1T3','V1T4','V1T3','V1T4','V1T4'),

arrows=2, free=TRUE,

values=c(0.26,0.23,0.22,0.37,0.38,0.60)),

mxPath(from=c('V2T1','V2T1','V2T1','V2T2','V2T2','V2T3'),

to = c('V2T2','V2T3','V2T4','V2T3','V2T4','V2T4'),

arrows=2, free=TRUE,

values=c(0.02,0.15,0.05,0.02,0.08,0.10)),

mxPath(from=c('V3T1','V3T1','V3T1','V3T2','V3T2','V3T3'),

to = c('V3T2','V3T3','V3T4','V3T3','V3T4','V3T4'),

arrows=2, free=TRUE,

values=c(0.12,0.13,0.08,0.16,0.16,0.20)),

mxPath(from=c('V4T1','V4T1','V4T1','V4T2','V4T2','V4T3'),

to = c('V4T2','V4T3','V4T4','V4T3','V4T4','V4T4'),

arrows=2, free=TRUE,

values=c(0.17,0.10,0.04,0.06,0.07,0.17)),

mxPath(from=c('V5T1','V5T1','V5T1','V5T2','V5T2','V5T3'),

to = c('V5T2','V5T3','V5T4','V5T3','V5T4','V5T4'),

arrows=2, free=TRUE,

values=c(0.15,0.12,0.12,0.39,0.26,0.56)),

# Latent common factor covariances

mxPath(from=c('C1FAMo','C2FAMo','C3FAMo','C4FAMo'),

connect='unique.pairs', arrows=2,

free=TRUE, values=c(0.59,0.22,0.14,0.18,

0.52,0.30,0.26,

0.58,0.43,

0.63),

labels=c('psi\_11','psi\_21','psi\_31','psi\_41',

'psi\_22','psi\_32','psi\_42',

'psi\_33','psi\_43',

'psi\_44')),

# Factor loadings

mxPath(from='C1FAMo', to=c('V1T1','V2T1','V3T1','V4T1','V5T1'),

arrows=1,free=c(TRUE,TRUE,FALSE,TRUE,TRUE),

values=c(0.79,1.51,1.000,0.78,0.80),labels=

c('loading1T1','loading2T1',NA,'loading4T1','loading5T1')),

mxPath(from='C2FAMo', to=c('V1T2','V2T2','V3T2','V4T2','V5T2'),

arrows=1,free=c(TRUE,TRUE,FALSE,TRUE,TRUE),

values=c(0.93,1.23,1.000,0.62,0.65),labels=

c('loading1T2','loading2T2',NA,'loading4T2','loading5T2')),

mxPath(from='C3FAMo', to=c('V1T3','V2T3','V3T3','V4T3','V5T3'),

arrows=1,free=c(TRUE,TRUE,FALSE,TRUE,TRUE),

values=c(0.95,1.14,1.000,0.66,0.69),labels=

c('loading1T3','loading2T3',NA,'loading4T3','loading5T3')),

mxPath(from='C4FAMo', to=c('V1T4','V2T4','V3T4','V4T4','V5T4'),

arrows=1,free=c(TRUE,TRUE,FALSE,TRUE,TRUE),

values=c(0.90,1.19,1.000,0.62,0.73),labels=

c('loading1T4','loading2T4',NA,'loading4T4','loading5T4')),

# Latent common factor means

mxPath(from='one',to=c('C2FAMo','C3FAMo','C4FAMo'),

arrows=1, free=TRUE,

values=c(-.40,-.59,-.65),

labels=c('alpha\_2','alpha\_3','alpha\_4')),

# Thresholds

mxThreshold(vars=c('V1T1','V1T2','V1T3','V1T4'),nThresh=3,

free=TRUE,values=c(-2.45,-1.79,-0.62,

-2.75,-1.79,-0.57,

-2.82,-1.79,-0.38,

-3.48,-1.79,-0.31),

labels=c('tau\_V1T1\_t1','tau\_V1\_t2','tau\_V1T1\_t3',

'tau\_V1T2\_t1','tau\_V1\_t2','tau\_V1T2\_t3',

'tau\_V1T3\_t1','tau\_V1\_t2','tau\_V1T3\_t3',

'tau\_V1T4\_t1','tau\_V1\_t2','tau\_V1T4\_t3')),

mxThreshold(vars=c('V2T1','V2T2','V2T3','V2T4'),nThresh=3,

free=TRUE,values=c(-2.74,-1.88,-0.24,

-3.20,-1.88,-0.40,

-3.59,-1.88,-0.58,

-3.32,-1.88,-0.55),

labels=c('tau\_V2T1\_t1','tau\_V2\_t2','tau\_V2T1\_t3',

'tau\_V2T2\_t1','tau\_V2\_t2','tau\_V2T2\_t3',

'tau\_V2T3\_t1','tau\_V2\_t2','tau\_V2T3\_t3',

'tau\_V2T4\_t1','tau\_V2\_t2','tau\_V2T4\_t3')),

mxThreshold(vars=c('V3T1','V3T2','V3T3','V3T4'),nThresh=3,

free=TRUE,values=c(-2.38,-1.60,-0.40,

-2.49,-1.60,-0.40,

-2.72,-1.60,-0.40,

-2.68,-1.60,-0.40),

labels=c('tau\_V3T1\_t1','tau\_V3\_t2','tau\_V3\_t3',

'tau\_V3T2\_t1','tau\_V3\_t2','tau\_V3\_t3',

'tau\_V3T3\_t1','tau\_V3\_t2','tau\_V3\_t3',

'tau\_V3T4\_t1','tau\_V3\_t2','tau\_V3\_t3')),

mxThreshold(vars=c('V4T1','V4T2','V4T3','V4T4'),nThresh=3,

free=TRUE,values=c(-1.95,-1.65,-0.66,

-2.23,-1.49,-0.66,

-2.57,-1.59,-0.66,

-2.82,-1.79,-0.66),

labels=c('tau\_V4T1\_t1','tau\_V4T1\_t2','tau\_V4\_t3',

'tau\_V4T2\_t1','tau\_V4T2\_t2','tau\_V4\_t3',

'tau\_V4T3\_t1','tau\_V4T3\_t2','tau\_V4\_t3',

'tau\_V4T4\_t1','tau\_V4T4\_t2','tau\_V4\_t3')),

mxThreshold(vars=c('V5T1','V5T2','V5T3','V5T4'),nThresh=3,

free=TRUE,values=c(-2.06,-1.48,-0.38,

-2.60,-1.48,-0.25,

-2.89,-1.48,-0.07,

-2.68,-1.48,-0.19),

labels=c('tau\_V5T1\_t1','tau\_V5\_t2','tau\_V5T1\_t3',

'tau\_V5T2\_t1','tau\_V5\_t2','tau\_V5T2\_t3',

'tau\_V5T3\_t1','tau\_V5\_t2','tau\_V5T3\_t3',

'tau\_V5T4\_t1','tau\_V5\_t2','tau\_V5T4\_t3'))

) # Close model

FAMo.Baseline.out <- mxRun(FAMo.Baseline.omx)

summary(FAMo.Baseline.out)

**Supplemental Materials 4B**

**OpenMx Code for the Empirical Illustration: Loading Invariance Model**

# Loading invariance model

FAMo.loadingInv.omx <- mxModel("LoadingInv",

type = "RAM",

mxData(observed=ChildFam, type="raw"),

manifestVars = c('V1T1','V1T2','V1T3','V1T4',

'V2T1','V2T2','V2T3','V2T4',

'V3T1','V3T2','V3T3','V3T4',

'V4T1','V4T2','V4T3','V4T4',

'V5T1','V5T2','V5T3','V5T4'),

latentVars = c('C1FAMo','C2FAMo','C3FAMo','C4FAMo'),

# Unique variances at T1: Fixed to 1

mxPath(from=c('V1T1','V2T1','V3T1','V4T1','V5T1'),

arrows=2, free=FALSE, values=1),

# Unique variances at other waves: Freely estimated

mxPath(from=c('V1T2','V1T3','V1T4',

'V2T2','V2T3','V2T4',

'V3T2','V3T3','V3T4',

'V4T2','V4T3','V4T4',

'V5T2','V5T3','V5T4'),

arrows=2, free=TRUE, values=c(1.23,1.18,2.01,

0.70,0.59,0.46,

0.62,0.58,0.64,

0.41,0.50,0.49,

0.96,1.33,1.09)),

# Lagged unique factor covariances

mxPath(from=c('V1T1','V1T1','V1T1','V1T2','V1T2','V1T3'),

to = c('V1T2','V1T3','V1T4','V1T3','V1T4','V1T4'),

arrows=2, free=TRUE, values=c(0.26,0.23,0.22,0.37,0.38,0.60)),

mxPath(from=c('V2T1','V2T1','V2T1','V2T2','V2T2','V2T3'),

to = c('V2T2','V2T3','V2T4','V2T3','V2T4','V2T4'),

arrows=2, free=TRUE, values=c(0.02,0.15,0.05,0.02,0.08,0.10)),

mxPath(from=c('V3T1','V3T1','V3T1','V3T2','V3T2','V3T3'),

to = c('V3T2','V3T3','V3T4','V3T3','V3T4','V3T4'),

arrows=2, free=TRUE, values=c(0.12,0.13,0.08,0.16,0.16,0.20)),

mxPath(from=c('V4T1','V4T1','V4T1','V4T2','V4T2','V4T3'),

to = c('V4T2','V4T3','V4T4','V4T3','V4T4','V4T4'),

arrows=2, free=TRUE, values=c(0.17,0.10,0.04,0.06,0.07,0.17)),

mxPath(from=c('V5T1','V5T1','V5T1','V5T2','V5T2','V5T3'),

to = c('V5T2','V5T3','V5T4','V5T3','V5T4','V5T4'),

arrows=2, free=TRUE, values=c(0.15,0.12,0.12,0.39,0.26,0.56)),

# Latent common factor covariances

mxPath(from=c('C1FAMo','C2FAMo','C3FAMo','C4FAMo'),

connect='unique.pairs', arrows=2,

free=TRUE, values=c(0.59,0.22,0.14,0.18,

0.52,0.30,0.26,

0.58,0.43,

0.63),

labels=c('psi\_11','psi\_21','psi\_31','psi\_41',

'psi\_22','psi\_32','psi\_42',

'psi\_33','psi\_43',

'psi\_44')),

# Factor loadings

mxPath(from='C1FAMo', to=c('V1T1','V2T1','V3T1','V4T1','V5T1'),

arrows=1,free=c(TRUE,TRUE,FALSE,TRUE,TRUE),

values=c(0.89,1.15,1.000,0.67,0.70),

labels=c('loading1','loading2',NA,'loading4','loading5')),

mxPath(from='C2FAMo', to=c('V1T2','V2T2','V3T2','V4T2','V5T2'),

arrows=1,free=c(TRUE,TRUE,FALSE,TRUE,TRUE),

values=c(0.89,1.15,1.000,0.67,0.70),

labels=c('loading1','loading2',NA,'loading4','loading5')),

mxPath(from='C3FAMo', to=c('V1T3','V2T3','V3T3','V4T3','V5T3'),

arrows=1,free=c(TRUE,TRUE,FALSE,TRUE,TRUE),

values=c(0.89,1.15,1.000,0.67,0.70),

labels=c('loading1','loading2',NA,'loading4','loading5')),

mxPath(from='C4FAMo', to=c('V1T4','V2T4','V3T4','V4T4','V5T4'),

arrows=1,free=c(TRUE,TRUE,FALSE,TRUE,TRUE),

values=c(0.89,1.15,1.000,0.67,0.70),

labels=c('loading1','loading2',NA,'loading4','loading5')),

# Latent common factor means

mxPath(from='one',to=c('C2FAMo','C3FAMo','C4FAMo'),

arrows=1, free=TRUE,

values=c(-.40,-.59,-.65),

labels=c('alpha\_2','alpha\_3','alpha\_4')),

# Thresholds

mxThreshold(vars=c('V1T1','V1T2','V1T3','V1T4'),nThresh=3,

free=TRUE,values=c(-2.45,-1.79,-0.62,

-2.75,-1.79,-0.57,

-2.82,-1.79,-0.38,

-3.48,-1.79,-0.31),

labels=c('tau\_V1T1\_t1','tau\_V1\_t2','tau\_V1T1\_t3',

'tau\_V1T2\_t1','tau\_V1\_t2','tau\_V1T2\_t3',

'tau\_V1T3\_t1','tau\_V1\_t2','tau\_V1T3\_t3',

'tau\_V1T4\_t1','tau\_V1\_t2','tau\_V1T4\_t3')),

mxThreshold(vars=c('V2T1','V2T2','V2T3','V2T4'),nThresh=3,

free=TRUE,values=c(-2.74,-1.88,-0.24,

-3.20,-1.88,-0.40,

-3.59,-1.88,-0.58,

-3.32,-1.88,-0.55),

labels=c('tau\_V2T1\_t1','tau\_V2\_t2','tau\_V2T1\_t3',

'tau\_V2T2\_t1','tau\_V2\_t2','tau\_V2T2\_t3',

'tau\_V2T3\_t1','tau\_V2\_t2','tau\_V2T3\_t3',

'tau\_V2T4\_t1','tau\_V2\_t2','tau\_V2T4\_t3')),

mxThreshold(vars=c('V3T1','V3T2','V3T3','V3T4'),nThresh=3,

free=TRUE,values=c(-2.38,-1.60,-0.40,

-2.49,-1.60,-0.40,

-2.72,-1.60,-0.40,

-2.68,-1.60,-0.40),

labels=c('tau\_V3T1\_t1','tau\_V3\_t2','tau\_V3\_t3',

'tau\_V3T2\_t1','tau\_V3\_t2','tau\_V3\_t3',

'tau\_V3T3\_t1','tau\_V3\_t2','tau\_V3\_t3',

'tau\_V3T4\_t1','tau\_V3\_t2','tau\_V3\_t3')),

mxThreshold(vars=c('V4T1','V4T2','V4T3','V4T4'),nThresh=3,

free=TRUE,values=c(-1.95,-1.65,-0.66,

-2.23,-1.49,-0.66,

-2.57,-1.59,-0.66,

-2.82,-1.79,-0.66),

labels=c('tau\_V4T1\_t1','tau\_V4T1\_t2','tau\_V4\_t3',

'tau\_V4T2\_t1','tau\_V4T2\_t2','tau\_V4\_t3',

'tau\_V4T3\_t1','tau\_V4T3\_t2','tau\_V4\_t3',

'tau\_V4T4\_t1','tau\_V4T4\_t2','tau\_V4\_t3')),

mxThreshold(vars=c('V5T1','V5T2','V5T3','V5T4'),nThresh=3,

free=TRUE,values=c(-2.06,-1.48,-0.38,

-2.60,-1.48,-0.25,

-2.89,-1.48,-0.07,

-2.68,-1.48,-0.19),

labels=c('tau\_V5T1\_t1','tau\_V5\_t2','tau\_V5T1\_t3',

'tau\_V5T2\_t1','tau\_V5\_t2','tau\_V5T2\_t3',

'tau\_V5T3\_t1','tau\_V5\_t2','tau\_V5T3\_t3',

'tau\_V5T4\_t1','tau\_V5\_t2','tau\_V5T4\_t3'))

) # Close model

FAMo.loadingInv.out <- mxRun(FAMo.loadingInv.omx)

summary(FAMo.loadingInv.out)

**Supplemental Materials 4C**

**OpenMx Code for the Empirical Illustration: Threshold Invariance Model**

# Threshold invariance model

FAMo.thresholdInv.omx <- mxModel("ThresholdInv",

type = "RAM",

mxData(observed=ChildFam, type="raw"),

manifestVars = c('V1T1','V1T2','V1T3','V1T4',

'V2T1','V2T2','V2T3','V2T4',

'V3T1','V3T2','V3T3','V3T4',

'V4T1','V4T2','V4T3','V4T4',

'V5T1','V5T2','V5T3','V5T4'),

latentVars = c('C1FAMo','C2FAMo','C3FAMo','C4FAMo'),

# Unique variances at T1: Fixed to 1

mxPath(from=c('V1T1','V2T1','V3T1','V4T1','V5T1'),

arrows=2, free=FALSE, values=1),

# Unique variances at other waves: Freely estimated

mxPath(from=c('V1T2','V1T3','V1T4',

'V2T2','V2T3','V2T4',

'V3T2','V3T3','V3T4',

'V4T2','V4T3','V4T4',

'V5T2','V5T3','V5T4'),

arrows=2, free=TRUE, values=c(1.04,0.93,1.20,

0.49,0.40,0.33,

0.53,0.42,0.47,

0.37,0.36,0.28,

0.70,0.80,0.73)),

# Lagged unique factor covariances

mxPath(from=c('V1T1','V1T1','V1T1','V1T2','V1T2','V1T3'),

to = c('V1T2','V1T3','V1T4','V1T3','V1T4','V1T4'),

arrows=2, free=TRUE, values=c(0.23,0.21,0.17,0.30,0.26,0.41)),

mxPath(from=c('V2T1','V2T1','V2T1','V2T2','V2T2','V2T3'),

to = c('V2T2','V2T3','V2T4','V2T3','V2T4','V2T4'),

arrows=2, free=TRUE, values=c(0.03,0.12,0.06,0.02,0.06,0.09)),

mxPath(from=c('V3T1','V3T1','V3T1','V3T2','V3T2','V3T3'),

to = c('V3T2','V3T3','V3T4','V3T3','V3T4','V3T4'),

arrows=2, free=TRUE, values=c(0.11,0.11,0.07,0.13,0.13,0.15)),

mxPath(from=c('V4T1','V4T1','V4T1','V4T2','V4T2','V4T3'),

to = c('V4T2','V4T3','V4T4','V4T3','V4T4','V4T4'),

arrows=2, free=TRUE, values=c(0.16,0.09,0.03,0.06,0.05,0.11)),

mxPath(from=c('V5T1','V5T1','V5T1','V5T2','V5T2','V5T3'),

to = c('V5T2','V5T3','V5T4','V5T3','V5T4','V5T4'),

arrows=2, free=TRUE, values=c(0.13,0.09,0.10,0.26,0.18,0.35)),

# Latent common factor covariances

mxPath(from=c('C1FAMo','C2FAMo','C3FAMo','C4FAMo'),

connect='unique.pairs', arrows=2,

free=TRUE, values=c(0.63,0.21,0.12,0.16,

0.45,0.24,0.21,

0.44,0.32,

0.45),

labels=c('psi\_11','psi\_21','psi\_31','psi\_41',

'psi\_22','psi\_32','psi\_42',

'psi\_33','psi\_43',

'psi\_44')),

# Factor loadings

mxPath(from='C1FAMo', to=c('V1T1','V2T1','V3T1','V4T1','V5T1'),

arrows=1,free=c(TRUE,TRUE,FALSE,TRUE,TRUE),

values=c(0.90,1.02,1.000,0.65,0.68),

labels=c('loading1','loading2',NA,'loading4','loading5')),

mxPath(from='C2FAMo', to=c('V1T2','V2T2','V3T2','V4T2','V5T2'),

arrows=1,free=c(TRUE,TRUE,FALSE,TRUE,TRUE),

values=c(0.90,1.02,1.000,0.65,0.68),

labels=c('loading1','loading2',NA,'loading4','loading5')),

mxPath(from='C3FAMo', to=c('V1T3','V2T3','V3T3','V4T3','V5T3'),

arrows=1,free=c(TRUE,TRUE,FALSE,TRUE,TRUE),

values=c(0.90,1.02,1.000,0.65,0.68),

labels=c('loading1','loading2',NA,'loading4','loading5')),

mxPath(from='C4FAMo', to=c('V1T4','V2T4','V3T4','V4T4','V5T4'),

arrows=1,free=c(TRUE,TRUE,FALSE,TRUE,TRUE),

values=c(0.90,1.02,1.000,0.65,0.68),

labels=c('loading1','loading2',NA,'loading4','loading5')),

# Latent common factor means

mxPath(from='one',to=c('C2FAMo','C3FAMo','C4FAMo'),

arrows=1, free=TRUE,

values=c(-.46,-.65,-.70),

labels=c('alpha\_2','alpha\_3','alpha\_4')),

# Thresholds

mxThreshold(vars=c('V1T1','V1T2','V1T3','V1T4'),nThresh=3,

free=TRUE,values=c(-2.67,-1.70,-0.54),

labels=c('tau\_V1\_t1','tau\_V1\_t2','tau\_V1\_t3')),

mxThreshold(vars=c('V2T1','V2T2','V2T3','V2T4'),nThresh=3,

free=TRUE,values=c(-2.77,-1.65,-0.48),

labels=c('tau\_V2\_t1','tau\_V2\_t2','tau\_V2\_t3')),

mxThreshold(vars=c('V3T1','V3T2','V3T3','V3T4'),nThresh=3,

free=TRUE,values=c(-2.44,-1.55,-0.46),

labels=c('tau\_V3\_t1','tau\_V3\_t2','tau\_V3\_t3')),

mxThreshold(vars=c('V4T1','V4T2','V4T3','V4T4'),nThresh=3,

free=TRUE,values=c(-2.11,-1.49,-0.60),

labels=c('tau\_V4\_t1','tau\_V4\_t2','tau\_V4\_t3')),

mxThreshold(vars=c('V5T1','V5T2','V5T3','V5T4'),nThresh=3,

free=TRUE,values=c(-2.26,-1.35,-0.27),

labels=c('tau\_V5\_t1','tau\_V5\_t2','tau\_V5\_t3'))

) # Close model

FAMo.thresholdInv.out <- mxRun(FAMo.thresholdInv.omx)

summary(FAMo.thresholdInv.out)

**Supplemental Materials 4D**

**OpenMx Code for the Empirical Illustration: Unique Factor Invariance Model**

# Unique factor invariance model

FAMo.UniqueFactorInv.omx <- mxModel("UniqueFactorInv",

type = "RAM",

mxData(observed=ChildFam, type="raw"),

manifestVars = c('V1T1','V1T2','V1T3','V1T4',

'V2T1','V2T2','V2T3','V2T4',

'V3T1','V3T2','V3T3','V3T4',

'V4T1','V4T2','V4T3','V4T4',

'V5T1','V5T2','V5T3','V5T4'),

latentVars = c('C1FAMo','C2FAMo','C3FAMo','C4FAMo'),

# Unique variances at T1: Fixed to 1

mxPath(from=c('V1T1','V2T1','V3T1','V4T1','V5T1'),

arrows=2, free=FALSE, values=1),

# Unique variances at other waves: Fixed to 1

mxPath(from=c('V1T2','V1T3','V1T4',

'V2T2','V2T3','V2T4',

'V3T2','V3T3','V3T4',

'V4T2','V4T3','V4T4',

'V5T2','V5T3','V5T4'),

arrows=2, free=FALSE, values=1),

# Lagged unique factor covariances

mxPath(from=c('V1T1','V1T1','V1T1','V1T2','V1T2','V1T3'),

to = c('V1T2','V1T3','V1T4','V1T3','V1T4','V1T4'),

arrows=2, free=TRUE,

values=c(0.23,0.21,0.17,0.30,0.26,0.41)),

mxPath(from=c('V2T1','V2T1','V2T1','V2T2','V2T2','V2T3'),

to = c('V2T2','V2T3','V2T4','V2T3','V2T4','V2T4'),

arrows=2, free=TRUE,

values=c(0.03,0.12,0.06,0.02,0.06,0.09)),

mxPath(from=c('V3T1','V3T1','V3T1','V3T2','V3T2','V3T3'),

to = c('V3T2','V3T3','V3T4','V3T3','V3T4','V3T4'),

arrows=2, free=TRUE,

values=c(0.11,0.11,0.07,0.13,0.13,0.15)),

mxPath(from=c('V4T1','V4T1','V4T1','V4T2','V4T2','V4T3'),

to = c('V4T2','V4T3','V4T4','V4T3','V4T4','V4T4'),

arrows=2, free=TRUE,

values=c(0.16,0.09,0.03,0.06,0.05,0.11)),

mxPath(from=c('V5T1','V5T1','V5T1','V5T2','V5T2','V5T3'),

to = c('V5T2','V5T3','V5T4','V5T3','V5T4','V5T4'),

arrows=2, free=TRUE,

values=c(0.13,0.09,0.10,0.26,0.18,0.35)),

# Latent common factor covariances

mxPath(from=c('C1FAMo','C2FAMo','C3FAMo','C4FAMo'),

connect='unique.pairs', arrows=2,

free=TRUE, values=c(0.63,0.21,0.12,0.16,

0.45,0.24,0.21,

0.44,0.32,

0.45),

labels=c('psi\_11','psi\_21','psi\_31','psi\_41',

'psi\_22','psi\_32','psi\_42',

'psi\_33','psi\_43',

'psi\_44')),

# Factor loadings

mxPath(from='C1FAMo', to=c('V1T1','V2T1','V3T1','V4T1','V5T1'),

arrows=1,free=c(TRUE,TRUE,FALSE,TRUE,TRUE),

values=c(0.90,1.02,1.000,0.65,0.68),

labels=c('loading1','loading2',NA,'loading4','loading5')),

mxPath(from='C2FAMo', to=c('V1T2','V2T2','V3T2','V4T2','V5T2'),

arrows=1,free=c(TRUE,TRUE,FALSE,TRUE,TRUE),

values=c(0.90,1.02,1.000,0.65,0.68),

labels=c('loading1','loading2',NA,'loading4','loading5')),

mxPath(from='C3FAMo', to=c('V1T3','V2T3','V3T3','V4T3','V5T3'),

arrows=1,free=c(TRUE,TRUE,FALSE,TRUE,TRUE),

values=c(0.90,1.02,1.000,0.65,0.68),

labels=c('loading1','loading2',NA,'loading4','loading5')),

mxPath(from='C4FAMo', to=c('V1T4','V2T4','V3T4','V4T4','V5T4'),

arrows=1,free=c(TRUE,TRUE,FALSE,TRUE,TRUE),

values=c(0.90,1.02,1.000,0.65,0.68),

labels=c('loading1','loading2',NA,'loading4','loading5')),

# Latent common factor means

mxPath(from='one',to=c('C2FAMo','C3FAMo','C4FAMo'),

arrows=1, free=TRUE,

values=c(-.46,-.65,-.70),

labels=c('alpha\_2','alpha\_3','alpha\_4')),

# Thresholds

mxThreshold(vars=c('V1T1','V1T2','V1T3','V1T4'),nThresh=3,

free=TRUE,values=c(-2.67,-1.70,-0.54),

labels=c('tau\_V1\_t1','tau\_V1\_t2','tau\_V1\_t3')),

mxThreshold(vars=c('V2T1','V2T2','V2T3','V2T4'),nThresh=3,

free=TRUE,values=c(-2.77,-1.65,-0.48),

labels=c('tau\_V2\_t1','tau\_V2\_t2','tau\_V2\_t3')),

mxThreshold(vars=c('V3T1','V3T2','V3T3','V3T4'),nThresh=3,

free=TRUE,values=c(-2.44,-1.55,-0.46),

labels=c('tau\_V3\_t1','tau\_V3\_t2','tau\_V3\_t3')),

mxThreshold(vars=c('V4T1','V4T2','V4T3','V4T4'),nThresh=3,

free=TRUE,values=c(-2.11,-1.49,-0.60),

labels=c('tau\_V4\_t1','tau\_V4\_t2','tau\_V4\_t3')),

mxThreshold(vars=c('V5T1','V5T2','V5T3','V5T4'),nThresh=3,

free=TRUE,values=c(-2.26,-1.35,-0.27),

labels=c('tau\_V5\_t1','tau\_V5\_t2','tau\_V5\_t3'))

) # Close model

FAMo.UniqueFactorInv.out <- mxRun(FAMo.UniqueFactorInv.omx)

summary(FAMo.UniqueFactorInv.out)

**Supplemental Materials 5A**

**M*plus* Code for the Empirical Illustration: More Constrained Null Model**

TITLE: Longitudinal invariance of Child Familism: Obligation;

NULL model with COMPLETE THRESHOLD INVARIANCE;

Ordered-categorical CFA;

DATA:

! Response categories 1 & 2 are collapsed for all indicators;

file = ChildFamilism.dat;

VARIABLE:

names are (omitted for brevity);

Missing are all (-999) ;

usevariables are (omitted for brevity);

categorical are

V1T1 V2T1 V3T1 V4T1 V5T1

V1T2 V2T2 V3T2 V4T2 V5T2

V1T3 V2T3 V3T3 V4T3 V5T3

V1T4 V2T4 V3T4 V4T4 V5T4;

ANALYSIS:

PARAMETERIZATION = THETA;

ITERATIONS=3000;

ESTIMATOR=WLSMV;

MODEL:

! Factor loadings;

! Constrain all factor loadings to 0;

C1FAMo by V1T1@0

V2T1@0

V3T1@0

V4T1@0

V5T1@0;

C2FAMo by V1T2@0

V2T2@0

V3T2@0

V4T2@0

V5T2@0;

C3FAMo by V1T3@0

V2T3@0

V3T3@0

V4T3@0

V5T3@0;

C4FAMo by V1T4@0

V2T4@0

V3T4@0

V4T4@0

V5T4@0;

! Thresholds;

! Constrain all thresholds to be invariant across time;

[V1T1$1 V1T2$1 V1T3$1 V1T4$1] (7);

[V1T1$2 V1T2$2 V1T3$2 V1T4$2] (1);

[V1T1$3 V1T2$3 V1T3$3 V1T4$3] (8);

[V2T1$1 V2T2$1 V2T3$1 V2T4$1] (9);

[V2T1$2 V2T2$2 V2T3$2 V2T4$2] (2);

[V2T1$3 V2T2$3 V2T3$3 V2T4$3] (10);

[V3T1$1 V3T2$1 V3T3$1 V3T4$1] (11);

[V3T1$2 V3T2$2 V3T3$2 V3T4$2] (3);

[V3T1$3 V3T2$3 V3T3$3 V3T4$3] (4);

[V4T1$1 V4T2$1 V4T3$1 V4T4$1] (12);

[V4T1$2 V4T2$2 V4T3$2 V4T4$2] (13);

[V4T1$3 V4T2$3 V4T3$3 V4T4$3] (5);

[V5T1$1 V5T2$1 V5T3$1 V5T4$1] (14);

[V5T1$2 V5T2$2 V5T3$2 V5T4$2] (6);

[V5T1$3 V5T2$3 V5T3$3 V5T4$3] (15);

! Common factor covariance matrix;

C1Famo@1; C2Famo@1; C3Famo@1; C4Famo@1;

C1Famo with C2Famo@0 C3Famo@0 C4Famo@0;

C2Famo with C3Famo@0 C4Famo@0;

C3Famo with C4Famo@0;

! Common factor means;

[C1FAMo@0 C2FAMo@0 C3FAMo@0 C4FAMo@0];

! Unique variances;

V1T1@1 V2T1@1 V3T1@1 V4T1@1 V5T1@1;

V1T2@1 V2T2@1 V3T2@1 V4T2@1 V5T2@1;

V1T3@1 V2T3@1 V3T3@1 V4T3@1 V5T3@1;

V1T4@1 V2T4@1 V3T4@1 V4T4@1 V5T4@1;

! NO lagged unique factor covariance;

V1T1 with V1T2@0 V1T3@0 V1T4@0;

V1T2 with V1T3@0 V1T4@0;

V1T3 with V1T4@0;

V2T1 with V2T2@0 V2T3@0 V2T4@0;

V2T2 with V2T3@0 V2T4@0;

V2T3 with V2T4@0;

V3T1 with V3T2@0 V3T3@0 V3T4@0;

V3T2 with V3T3@0 V3T4@0;

V3T3 with V3T4@0;

V4T1 with V4T2@0 V4T3@0 V4T4@0;

V4T2 with V4T3@0 V4T4@0;

V4T3 with V4T4@0;

V5T1 with V5T2@0 V5T3@0 V5T4@0;

V5T2 with V5T3@0 V5T4@0;

V5T3 with V5T4@0;

**Supplemental Materials 5B**

**lavaan Code for the Empirical Illustration: More Constrained Null Model**

Null.Model <- '

# latent variable definitions

# Constrain all factor loadings to 0;

C1FAMo =~ 0\*V1T1 + 0\*V2T1 + 0\*V3T1 + 0\*V4T1 + 0\*V5T1

C2FAMo =~ 0\*V1T2 + 0\*V2T2 + 0\*V3T2 + 0\*V4T2 + 0\*V5T2

C3FAMo =~ 0\*V1T3 + 0\*V2T3 + 0\*V3T3 + 0\*V4T3 + 0\*V5T3

C4FAMo =~ 0\*V1T4 + 0\*V2T4 + 0\*V3T4 + 0\*V4T4 + 0\*V5T4

# Latent common factor variances and covariances

C1FAMo ~~ 1\*C1FAMo + 0\*C2FAMo + 0\*C3FAMo + 0\*C4FAMo

C2FAMo ~~ 1\*C2FAMo + 0\*C3FAMo + 0\*C4FAMo

C3FAMo ~~ 1\*C3FAMo + 0\*C4FAMo

C4FAMo ~~ 1\*C4FAMo

# Latent common factor means

# Fix common factor means to zero

C1FAMo ~ 0\*1

C2FAMo ~ 0\*1

C3FAMo ~ 0\*1

C4FAMo ~ 0\*1

# Thresholds

# Constrain all thresholds to be invariant across time

V1T1 | V1t1\*t1 + V1t2\*t2 + V1t3\*t3

V1T2 | V1t1\*t1 + V1t2\*t2 + V1t3\*t3

V1T3 | V1t1\*t1 + V1t2\*t2 + V1t3\*t3

V1T4 | V1t1\*t1 + V1t2\*t2 + V1t3\*t3

V2T1 | V2t1\*t1 + V2t2\*t2 + V2t3\*t3

V2T2 | V2t1\*t1 + V2t2\*t2 + V2t3\*t3

V2T3 | V2t1\*t1 + V2t2\*t2 + V2t3\*t3

V2T4 | V2t1\*t1 + V2t2\*t2 + V2t3\*t3

V3T1 | V3t1\*t1 + V3t2\*t2 + V3t3\*t3

V3T2 | V3t1\*t1 + V3t2\*t2 + V3t3\*t3

V3T3 | V3t1\*t1 + V3t2\*t2 + V3t3\*t3

V3T4 | V3t1\*t1 + V3t2\*t2 + V3t3\*t3

V4T1 | V4t1\*t1 + V4t2\*t2 + V4t3\*t3

V4T2 | V4t1\*t1 + V4t2\*t2 + V4t3\*t3

V4T3 | V4t1\*t1 + V4t2\*t2 + V4t3\*t3

V4T4 | V4t1\*t1 + V4t2\*t2 + V4t3\*t3

V5T1 | V5t1\*t1 + V5t2\*t2 + V5t3\*t3

V5T2 | V5t1\*t1 + V5t2\*t2 + V5t3\*t3

V5T3 | V5t1\*t1 + V5t2\*t2 + V5t3\*t3

V5T4 | V5t1\*t1 + V5t2\*t2 + V5t3\*t3

# Intercepts

# Fix all intercepts to zero

V1T1 + V2T1 + V3T1 + V4T1 + V5T1 ~ 0\*1

V1T2 + V2T2 + V3T2 + V4T2 + V5T2 ~ 0\*1

V1T3 + V2T3 + V3T3 + V4T3 + V5T3 ~ 0\*1

V1T4 + V2T4 + V3T4 + V4T4 + V5T4 ~ 0\*1

# Unique variances

# Constrain all unique variance at all occasions to 1.00

V1T1 ~~ 1\*V1T1

V2T1 ~~ 1\*V2T1

V3T1 ~~ 1\*V3T1

V4T1 ~~ 1\*V4T1

V5T1 ~~ 1\*V5T1

V1T2 ~~ 1\*V1T2

V2T2 ~~ 1\*V2T2

V3T2 ~~ 1\*V3T2

V4T2 ~~ 1\*V4T2

V5T2 ~~ 1\*V5T2

V1T3 ~~ 1\*V1T3

V2T3 ~~ 1\*V2T3

V3T3 ~~ 1\*V3T3

V4T3 ~~ 1\*V4T3

V5T3 ~~ 1\*V5T3

V1T4 ~~ 1\*V1T4

V2T4 ~~ 1\*V2T4

V3T4 ~~ 1\*V3T4

V4T4 ~~ 1\*V4T4

V5T4 ~~ 1\*V5T4

# No lagged unique factor covariances

V1T1 ~~ 0\*V1T2 + 0\*V1T3 + 0\*V1T4

V1T2 ~~ 0\*V1T3 + 0\*V1T4

V1T3 ~~ 0\*V1T4

V2T1 ~~ 0\*V2T2 + 0\*V2T3 + 0\*V1T4

V2T2 ~~ 0\*V2T3 + 0\*V1T4

V2T3 ~~ 0\*V1T4

V3T1 ~~ 0\*V3T2 + 0\*V3T3 + 0\*V3T4

V3T2 ~~ 0\*V3T3 + 0\*V3T4

V3T3 ~~ 0\*V3T4

V4T1 ~~ 0\*V4T2 + 0\*V4T3 + 0\*V4T4

V4T2 ~~ 0\*V4T3 + 0\*V4T4

V4T3 ~~ 0\*V4T4

V5T1 ~~ 0\*V5T2 + 0\*V5T3 + 0\*V5T4

V5T2 ~~ 0\*V5T3 + 0\*V5T4

V5T3 ~~ 0\*V5T4

'

Fit.Null.Model <- cfa(Null.Model, data = ChildFam,

ordered=c("V1T1", "V2T1", "V3T1", "V4T1", "V5T1",

"V1T2", "V2T2", "V3T2", "V4T2", "V5T2",

"V1T3", "V2T3", "V3T3", "V4T3", "V5T3",

"V1T4", "V2T4", "V3T4", "V4T4", "V5T4"),

parameterization = "theta", estimator = "wlsmv",

missing = "pairwise"

# Default is missing = "listwise"

)

summary(Fit.Null.Model, fit.measures = TRUE)

**Supplemental Materials 5C**

**SAS Macro for Calculating the Corrected CFI: Adapted from Wu, West, & Taylor (2009)**

\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*

/\*\* This macro is to calculate the CFI using the chi-square test statistic \*\*/

/\*\* from the hypothesized and baseline models \*\*/

/\*\* One needs to run the hypothesized and more constrained null models \*\*/

/\*\* to obtain the values for the following macro variables: \*\*/

/\*\* NCP\_NULL: noncentrality parameter for the more constrained null model\*\*/

/\*\* NCP\_H1: noncentrality parameter for the hypothesized model\*\*/

/\*\* chisq\_H1: chi-square test statistic for the hypothesized model \*\*/

/\*\* df\_H1: degrees of freedom for the hypothesized model \*\*/

/\*\* chisq\_NULL: chi-square test statistic for the more constrained null model \*\*/

/\*\* df\_NULL: degrees of freedom for the more constrained null model \*\*/

\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*;

**%macro** CFI\_corrected(chisq\_H1,df\_H1,chisq\_NULL,df\_NULL);

data outfit;

NCP\_NULL = (&chisq\_NULL-&df\_NULL);

NCP\_H1 = (&chisq\_H1-&df\_H1);

if NCP\_H1 > **0** and NCP\_NULL > NCP\_H1 then CFI = **1** - NCP\_H1/NCP\_NULL;

else if NCP\_H1>**0** and NCP\_NULL < NCP\_H1 then CFI = **0**;

else if NCP\_H1<**0** then CFI = **1**;

keep CFI;

run;

proc print data = outfit;

var CFI;

run;

**%mend** CFI\_corrected;