Local Consensus Algorithm for Modeling Security Perception Propagation in Communities

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This study explores the use of one-dimensional and two-dimensional cellular automata (CA) to model and analyze the propagation of security and insecurity perception in communities. We hypothesize that the propagation of these perceptions can be modeled and analyzed through CA using local totalistic rules. Two possible states are considered: security perception (-1) and insecurity perception (1). Various rules are evaluated, and the results obtained are discussed in terms of consensus efficacy and classification efficacy. This analysis can serve as a foundation for future research in modeling the propagation of security and insecurity perceptions.

A. Consensus Dynamics:

In recent years, various technological and collective social phenomena have emerged because of the interaction dynamics between multiple elements. One such phenomenon is consensus, a macroscopic state in which all elements of the system exhibit the same microscopic state, which could explain the emergence of a dominant majority in a population of individuals. In this context, examples can be cited such as blockchain-based applications, opinion formation, physiological and ecological systems, gene networks, the voting problem, and transportation, among others.

Blockchain-based applications, for example, can benefit from consensus dynamics to ensure the integrity and security of transactions on the network [2]. As for opinion formation, reality-inspired voter models have proven useful in understanding how opinions can spread and reach a consensus in a population [6,7]. Physiological and ecological systems can also exhibit consensus dynamics, as seen in the synchronization of circadian rhythms in organisms [2]. Gene networks, on the other hand, can reach a state of consensus through gene regulation and protein interactions [2].

The voting problem has been studied in the context of majority rules and how they can influence consensus formation in high dimensions [3,4,5,8]. Finally, in transportation, consensus dynamics can be relevant in the context of route optimization and coordination of multiple agents, as seen in multi-swarm-based approaches to solving high-dimensional optimization problems [8].

Although several approaches exist for modeling these dynamics, most mechanisms are related to the consideration of local majority. In this sense, two opinions (or states) can be assumed for everyone, represented by an agent or a node in a network model or cellular automaton: +1 and -1 in a mathematical formulation. Each individual interacts locally with individuals in their neighborhood, assuming the most common opinion in this set, and in case of a tie, maintains their state unchanged. This procedure is performed by selecting every individual and repeating until the dynamics reach an asymptotic state. Two of the most interesting asymptotic states (fixed points) are those in which the entire population assumes the same state, i.e., the states +1* (all individuals have the state +1) and -1* (all individuals have the state -1), in our notation. We refer to any of these situations as consensus. In addition, other fixed points distinct from consensus can arise, termed in this work as spurious points, in which the two opinions appear once the asymptotic state is reached.

Building on this foundation, we propose the hypothesis that the propagation of security and insecurity perception in communities can be modeled and analyzed using cellular automata and totalistic local rules. To test this hypothesis, we present an approach based on one-dimensional and two-dimensional cellular automata to model and analyze the propagation of security and insecurity perception. In this research, we employed a range of update rules and local interaction mechanisms to delve into how distinct configurations and algorithms might affect the capacity to reach consensus and the emergence of fixed states. We executed the modes of iteration in three diverse manners: through synchronous or parallel updates where all cells are refreshed simultaneously, through sequential or asynchronous

updates where cells are updated one at a time, and through a random update process where a cell is chosen randomly for each update. Further, we scrutinized how the topological and dynamic properties of cellular automata shape the quality and speed of consensus formation, along with their relationship to classification. The objective was to construct a more comprehensive understanding of these intricate systems and to investigate methods to enhance their function for potential future applications.
This paper is structured as follows: after this brief introduction, we present some definitions, metrics, and introduce the algorithm. Then, we present our theoretical and simulation results. Finally, we provide a discussion and draw conclusions.

B. One-dimensional Cellular Automata:

This study aims to analyze the propagation of security and insecurity perception in communities using one-dimensional cellular automata by applying different local totalistic rules to each cell, considering their neighbors' states. The simulations were conducted using the synchronous mode of iteration, where the updates for all cells occur simultaneously in each generation:

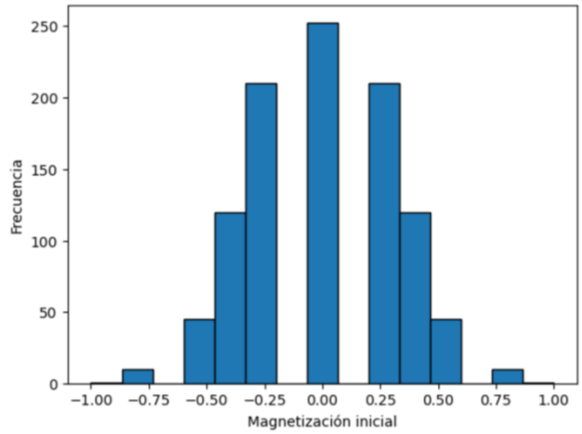
Definition of the cellular space and initial states: A one-dimensional cellular space (vector) was created, composed of a finite number of discrete cells, each with two possible states: -1 (representing security perception) and 1 (representing insecurity perception). Various random initial states were generated to represent different configurations of security and insecurity perceptions in the community.

Establishment of strict totalistic rules: A strict totalistic rule was implemented, where the sum of a cell's neighbors (within a certain distance r) determined the cell's next state. The next state would be 1 (insecurity perception) if the sum was greater than 0, -1 (security perception) if the sum was less than 0, and unchanged if the sum was 0.

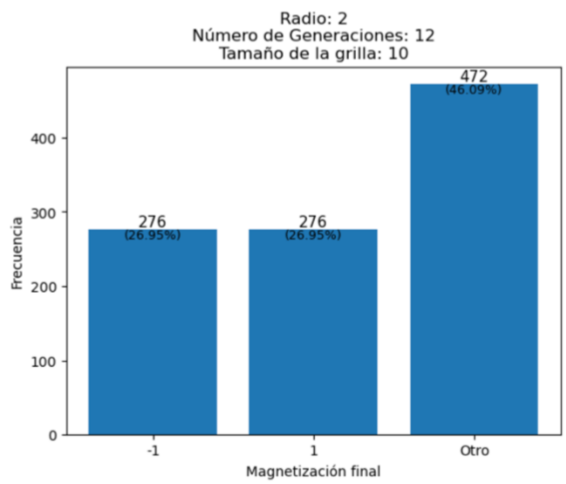
The following equation represents the condition previously described.

$$fv(x_u|u\epsilon A) = \begin{cases} -1, & \text{si } \sum_{u\epsilon A} x_u < 0, \\ x_u, & \text{si } \sum_{u\epsilon A} x_u = 0, \\ +1, & \text{si } \sum_{u\epsilon A} x_u > 0, \end{cases}$$

Combinatorial analysis and selection of initial states: A combinatorial analysis of 2 (binary) raised to the power of 10 (1024 combinations) evaluated the cellular automaton's behavior for each initial configuration using the strict totalistic rule. In this analysis, all possible combinations of cells in the one-dimensional space with security and insecurity perceptions (-1 and 1) were created to represent different community configurations.
In the following figure, a frequency table (histogram) is displayed, which illustrates the initial magnetization of each state based on its occurrence level. It can be observed that the distribution is normal, with a high concentration at level 0 and a lower concentration at levels -1 and 1. The magnetization is calculated as the average of all states. It is important to note that the magnetization is directly related to the number of 1s and -1s present in the system. If the quantities of 1s and -1s are balanced, the magnetization would tend to be zero.



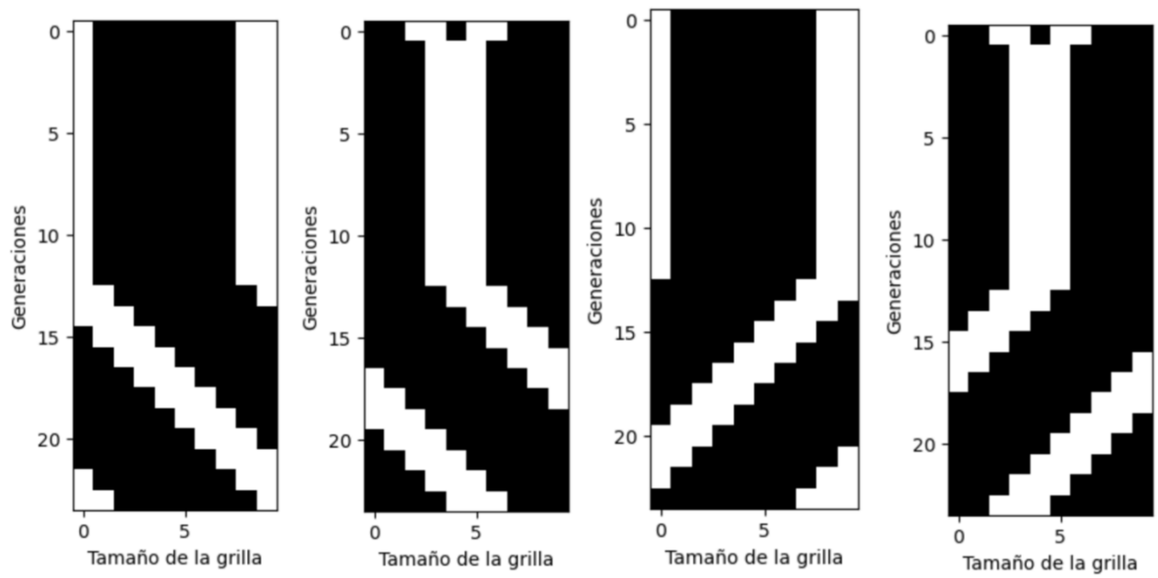
The strict totalistic rule was applied over the course of 12 generations, using a radius of 2, and the final magnetization was then measured. This allowed for the verification of the number of simulations that reached consensus. As observed in the table, 472 combinations reached consensus (276 with a final magnetization of 1 and 276 with a final magnetization of -1). The third column represents those that did not reach consensus, with their range of values falling between >-1 and <1.



The following summary table shows the magnetization levels divided into quarters (0.25), providing an overview of the concentration of magnetization. It can be observed that, when applying the strict totalistic rule, the magnetization levels close to -1 predominantly reach fixed points of -1, and those close to 1 predominantly reach fixed points of 1. Notably, within the range between >0 and <=0.5, there are 110 states that did not achieve consensus. This highlights the importance of exploring alternative update rules to address these challenging configurations to reach consensus more effectively.

	Magnetización Final		
Mag. inicial	-1	1	Otro
[-1 , -0.5]	56 (100%)	-	-
] - 0.5 , 0[	220 (67%)	-	110 (33%)
0	-	-	252 (100%)
] 0 , 0.5]	-	220 (67%)	110 (33%)
] 0.5 , 1]	-	56 (100%)	-
[-1 , 1]	276 (27%)	276 (27%)	472 (46%)

Application of new update rules: new update rules were applied to the 110 initial states that did not reach consensus to investigate their impact on the cellular automaton's behavior.
In the next figure, an application of a modification to the update rule is shown, which consists of hiding neighbors (left or right). To demonstrate this, two different states were selected, and this modification was applied starting from generation 12 for an additional 12 generations, reaching generation 24. The figure illustrates that negating neighbors is not effective for achieving consensus, only causing a displacement effect to the left or right of the automaton while maintaining the magnetization.

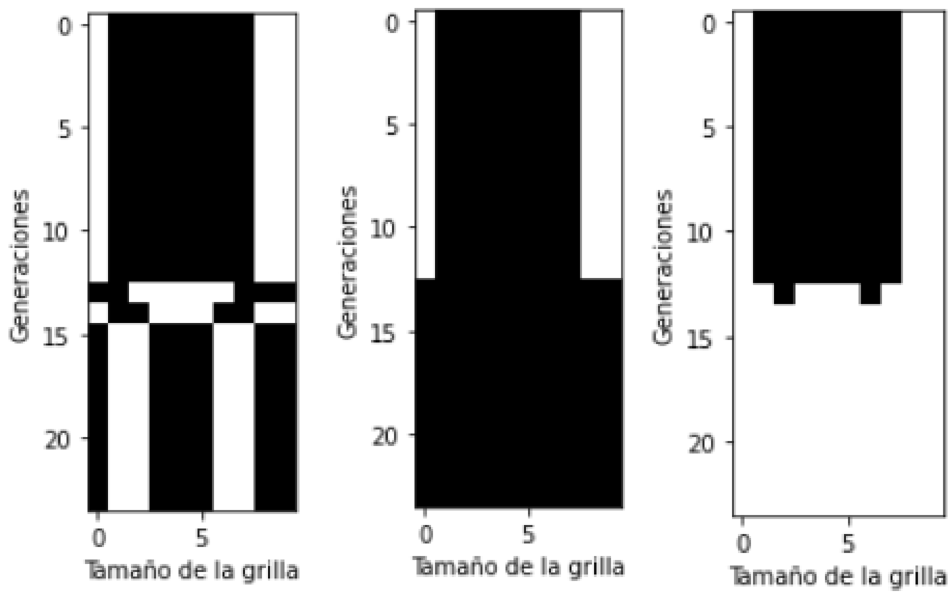


Considering that the previously mentioned modification was not effective in achieving consensus, the following figure demonstrates the effects of three additional rules (rule 0, rule 024, and rule 2) on the magnetization and the automaton's ability to reach fixed points. These rules are defined as follows:

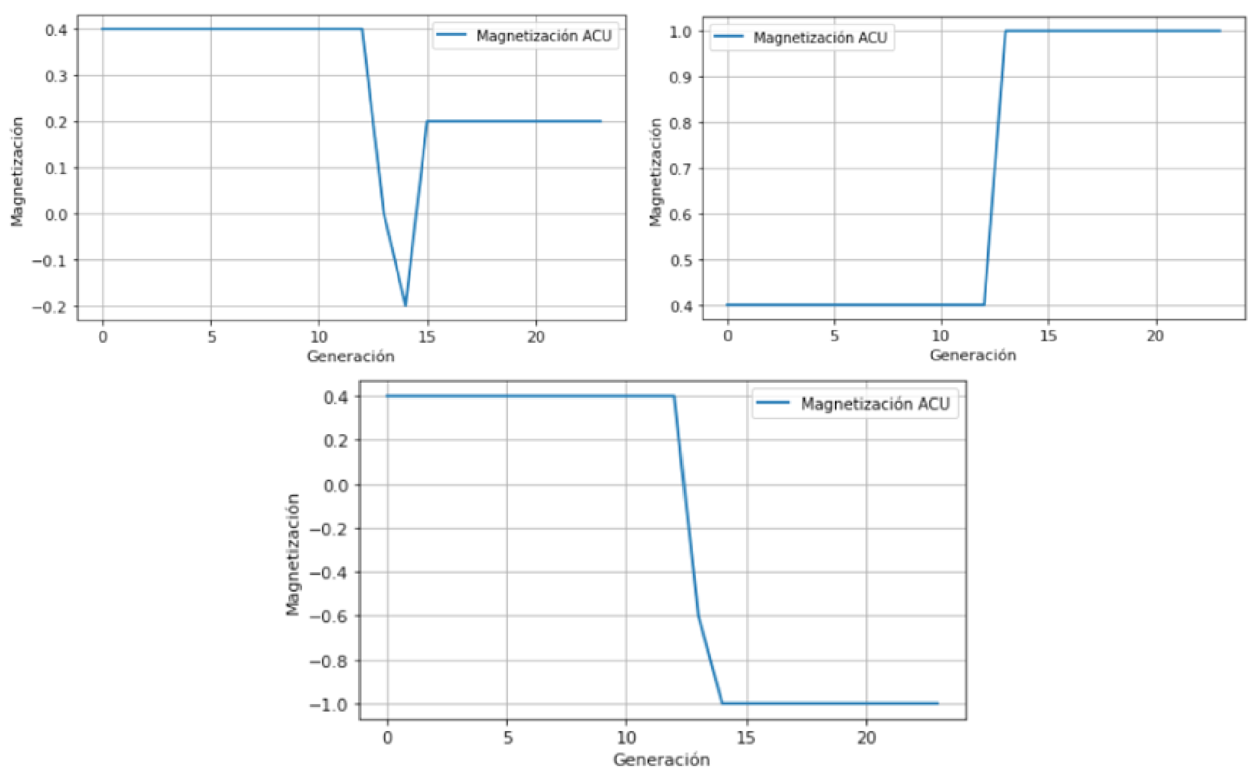
Rule 0 (Left): The next state depends on the sum of the neighboring cells. If the sum is 0, the next state becomes 1; otherwise, it becomes -1.

Rules 024 (Center) and 2 (Right): These rules employ the same logic, with the next state depending on the sum of the neighbors.

The figure illustrates the outcomes of applying these rules. Rule 0 fails to reach a consensus as it is unable to guide the system toward fixed points. On the other hand, Rule 024 successfully achieves consensus, acting similarly to an OR function in logical operations. Lastly, Rule 2 does manage to reach a consensus; however, it disregards the initial magnetization level, impacting its classification efficacy. This aspect will be further discussed in the two-dimensional cellular automaton section of this paper.



Evaluation of metrics: The analysis employed magnetization between each generation as a crucial metric to evaluate the performance of the update rules. The following figure offers a more comprehensive understanding of this metric by illustrating the effects of rule 0 (Left), rule 024 (Center), and rule 2 (Right) on the change in magnetization between generations. This visualization enhances the explanations provided in the previous sections, providing a deeper understanding of the rules' effectiveness in reaching consensus.



This methodology offers a comprehensive approach to understanding the propagation of security and insecurity perception in communities using one-dimensional cellular automata. It meticulously outlines the steps involved, from defining the cellular space to analyzing the results, ensuring that all aspects of the problem are considered and discussed. Furthermore, the study will proceed to examine the problem from the perspective of two-dimensional cellular automata, enriching the analysis and providing a more robust understanding of the dynamics at play.

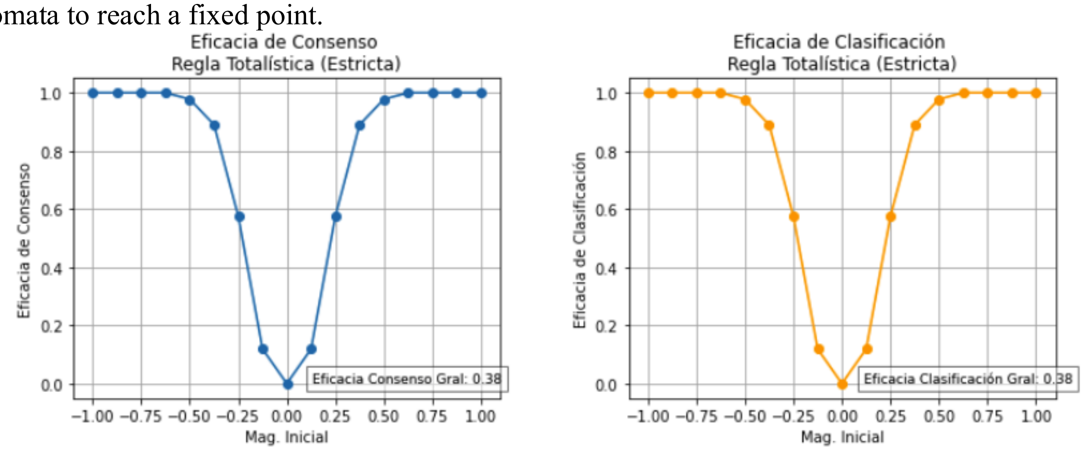
C. Two-dimensional Cellular Automata:

To extend the analysis of the propagation of security perception in communities using two-dimensional cellular automata, different local totalistic rules were applied to each cell, considering their neighbors' states in two dimensions. The simulations were conducted using three different modes of iteration: asynchronous, synchronous, and random. These modes allowed for exploring the effects of different update strategies and interaction patterns on the dynamics of the cellular automata and their ability to reach consensus.

Definition of the cellular space and initial states: A two-dimensional cellular space (matrix) was created, composed of a finite number of discrete cells, each with two possible states: -1 (representing security perception) and 1 (representing insecurity perception). All possible initial state combinations were generated in a 4x4 grid, representing 65,536 configurations of security and insecurity perceptions in the community.

Establishment of strict totalistic rules: A strict totalistic rule was implemented, where the sum of a cell's neighbors (Von Neumann) determined the cell's next state in the matrix. The next state would be 1 if the sum was greater than 0, -1 if the sum was less than 0, and unchanged if the sum was 0.

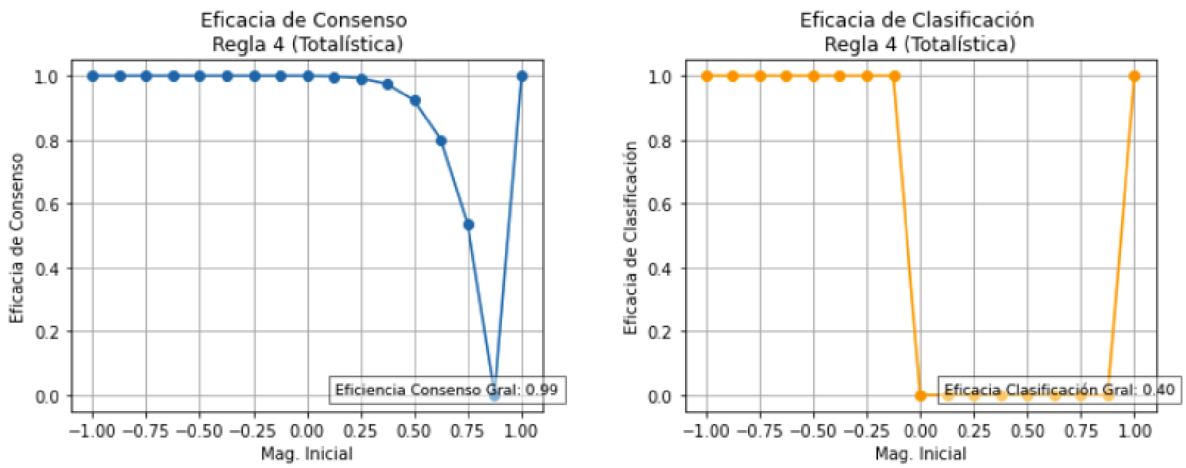
Simulations and metrics analysis: Simulations were initially conducted in asynchronous mode of iteration with the different initial state combinations in a 4x4 grid for a total of 50 generations, applying the strict totalistic rule. Two main metrics were measured: consensus efficacy and classification efficacy.
Consensus efficacy measures the ability of the cellular automaton to reach a fixed point, or consensus, where the system stabilizes, and no further changes occur. This metric evaluates the proportion of initial states that successfully achieve this fixed point and is used to determine the overall effectiveness of the applied rule in driving the system towards a stable state.
Classification efficacy, on the other hand, focuses on the proportion of initial states that not only reach a fixed point but also preserve their initial magnetization. In other words, this metric assesses how well the cellular automaton retains its original properties while converging to a fixed point. This is important for understanding the system's behavior in terms of its responsiveness to the initial conditions and the rules applied.
Additionally, a third metric, the average time to reach a fixed point, was introduced to provide insights into the speed at which consensus is achieved (transient level). This metric calculates the average number of generations required for the automata to reach a fixed point.



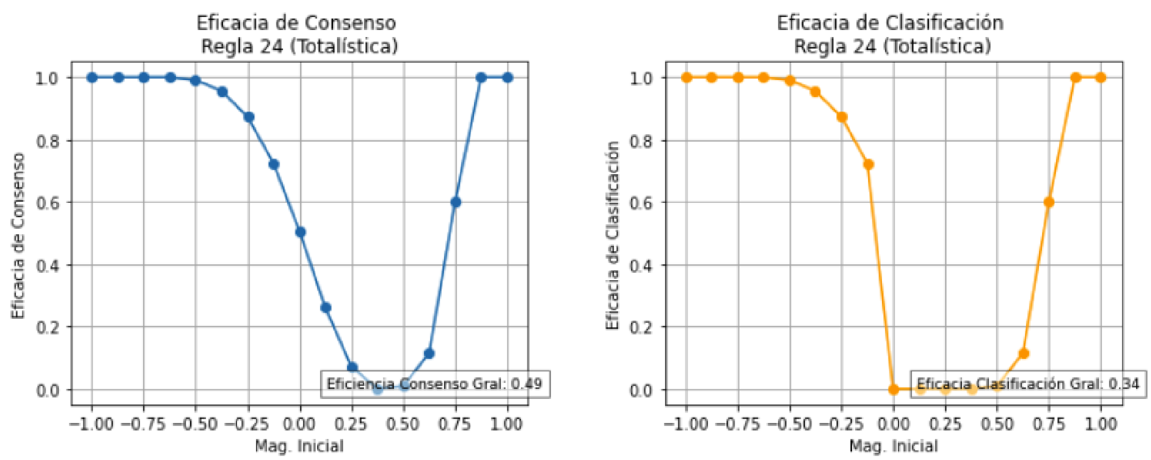
In both cases, a summary value is provided to showcase the comprehensive analysis of the two main metrics. These values offer an overarching view of the system's performance under the applied rules, presenting the outcomes for consensus efficacy and classification efficacy across different initial states and rule combinations.

Application of new rules: The same combinatorial operation was performed for eight additional rules: rule 4, rule 24, rule 04, rule 024, rule -24, rule -224, rule -204, and rule -2024. The creation of these rules follows the same logic as the one-dimensional cellular automata rules, where the next state depends on whether the sum of the neighbors matches the rule number. In cases where the sum of the neighbors is equal to the rule number, the next state is 1, and in all other cases, the next state is -1. The results obtained by applying these update rules were analyzed and compared to the strict totalistic rule results, providing further insights into their impact on consensus efficacy and classification efficacy.

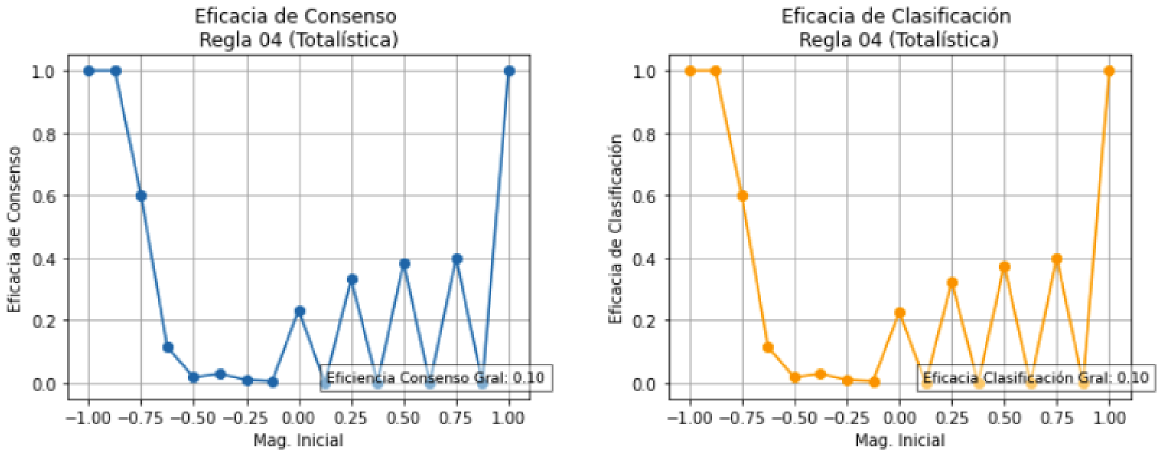
Rule 4:



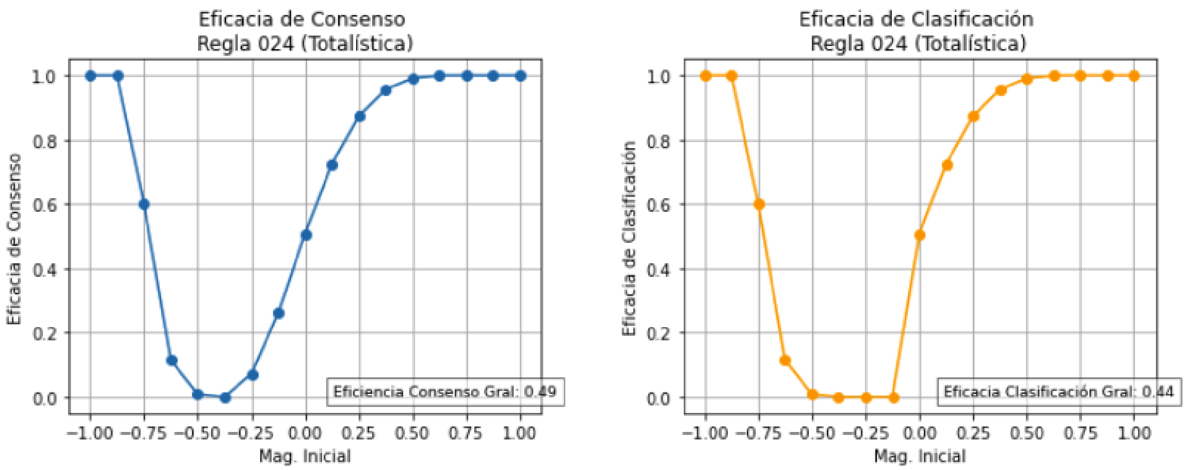
Rule 24:



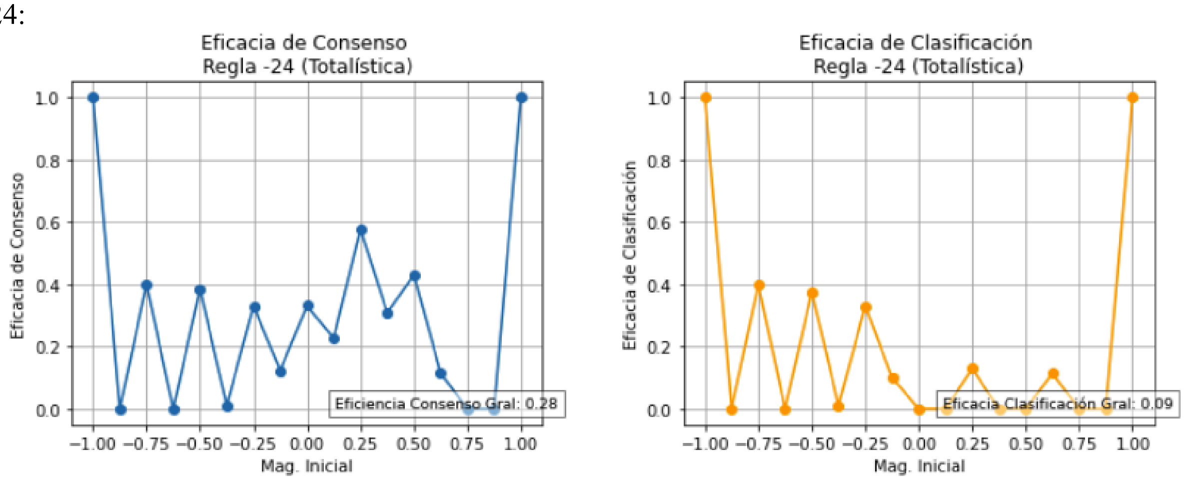
Rule 04:



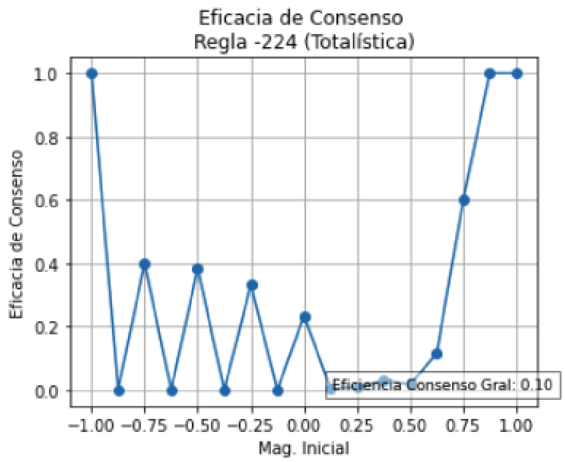
Rule 024:



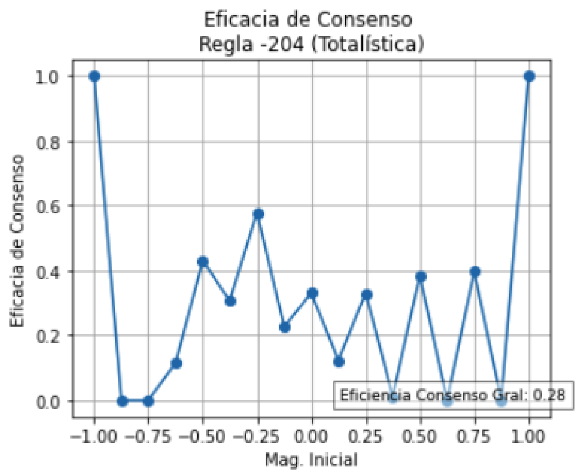
Rule -24:



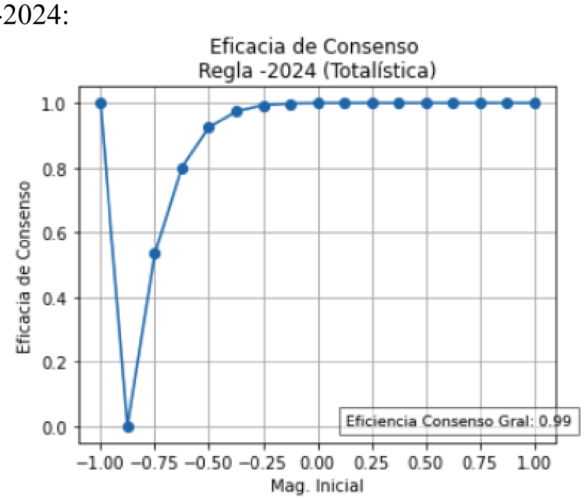
Rule -224:



Rule -204:

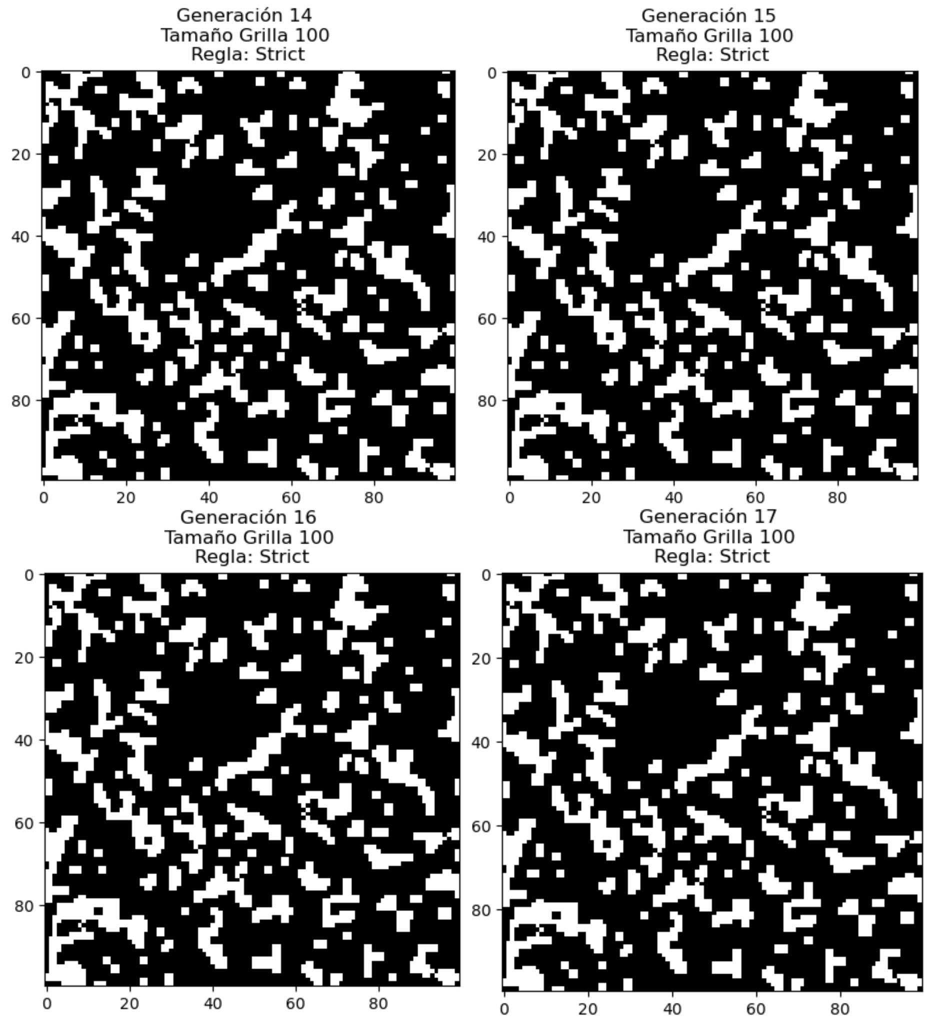


Rule -2024:

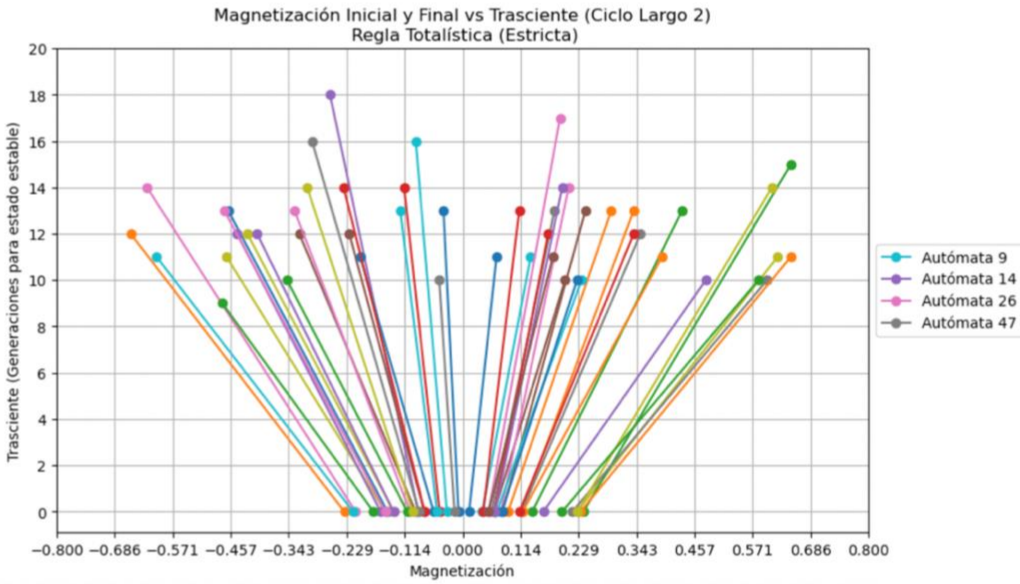


Upon examining the graphs, it becomes evident that rules 24 and 024 outperform the others in creating favorable conditions for consensus attainment. Not only do they facilitate consensus, but they also effectively preserve the initial magnetization levels, thus demonstrating a higher level of classification efficacy.

Analysis of specific cases: In the analysis, a sample of 50 automata that did not reach consensus under the strict rule and exhibited an initial magnetization within the range of -0.25 to +0.25 were selected. These automata were examined in terms of cycles, which represent the repetition of magnetization levels and the distribution of 1's and -1's on the grid. The figure below depicts a cellular automaton with a specific initial configuration, displaying a cycle length of 2. This means that generations 14 and 16 are identical, and generations 15 and 17 are also identical. Importantly, this state is stable and will persist if the strict rule continues to be applied.

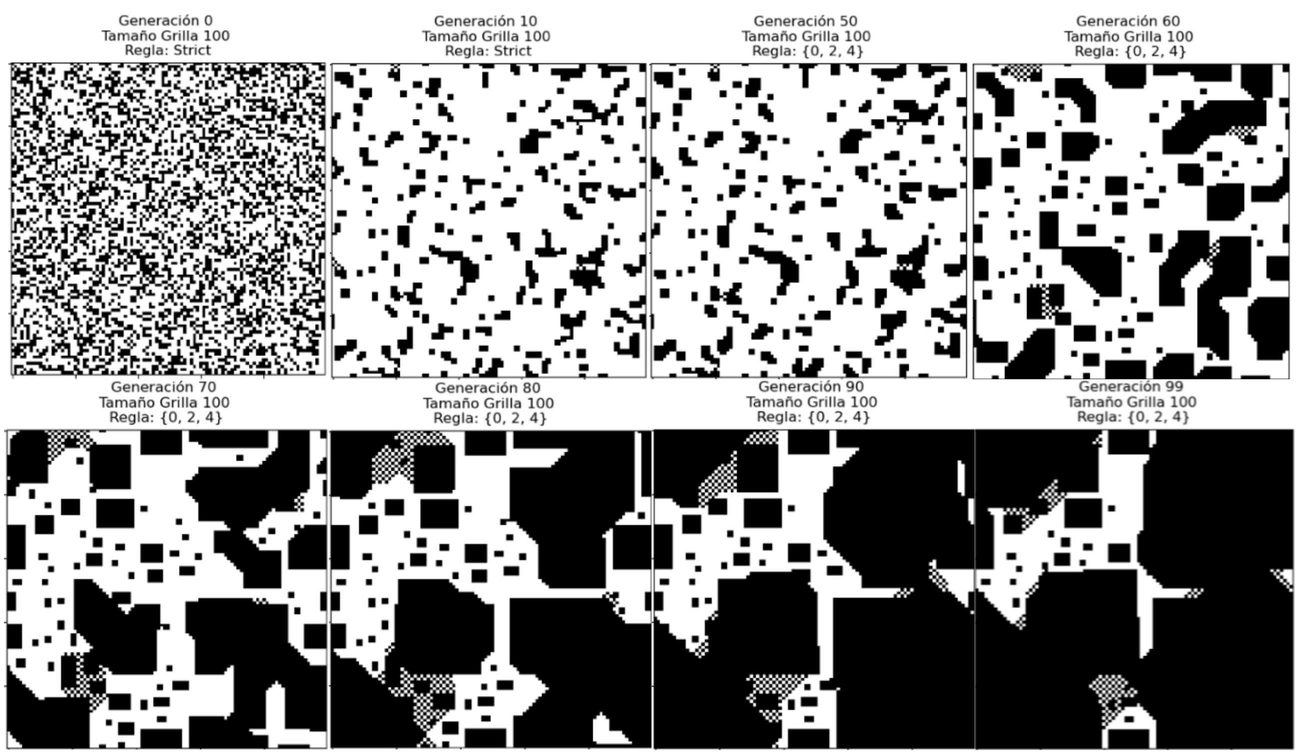


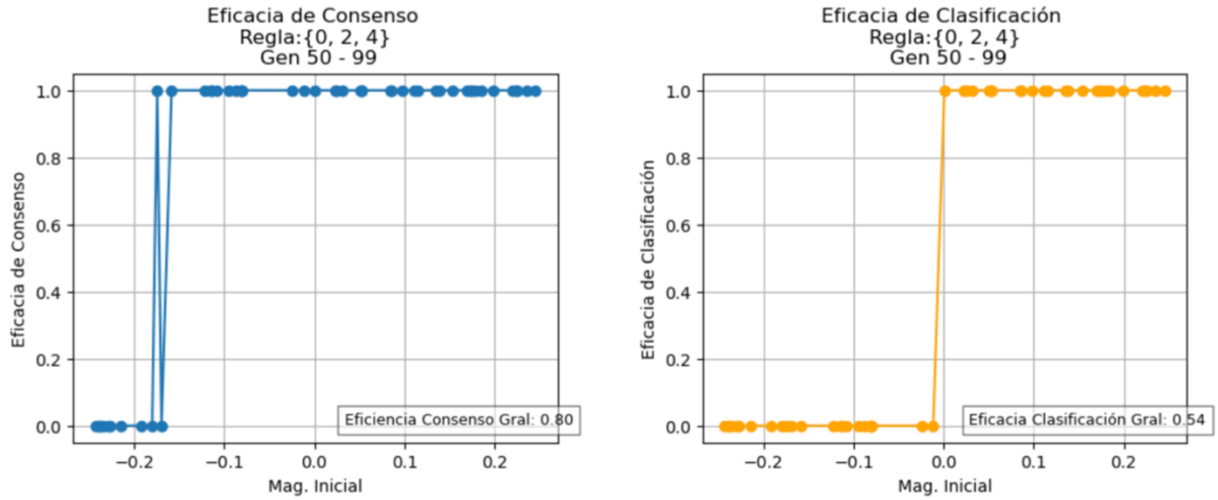
Upon observing the previous findings, simulations were conducted on the entire sample of 50 automata, with all reaching a cycle 2 state. In the subsequent figure, each automaton is displayed with its initial and final magnetization levels, as well as the transient level, which represents the number of generations required to achieve the cycle. Interestingly, the magnetization levels consistently followed their respective direction, indicating that an automaton with an initial magnetization of 0.15 would result in a final magnetization greater than 0.15, while an automaton with an initial magnetization of -0.15 would result in a final magnetization lower than -0.15.
The legend shows the only four automata whose transient levels were higher than 15 generations.



Combined simulations: To enhance the potential for reaching consensus, combined simulations were performed, where the strict totalistic rule was initially applied for 50 generations. Subsequently, rules 024 and 24 were applied for the next 50 or 100 generations. This approach aimed to examine the influence of combined rules on the system's dynamics, specifically in terms of consensus efficacy and classification efficacy metrics.

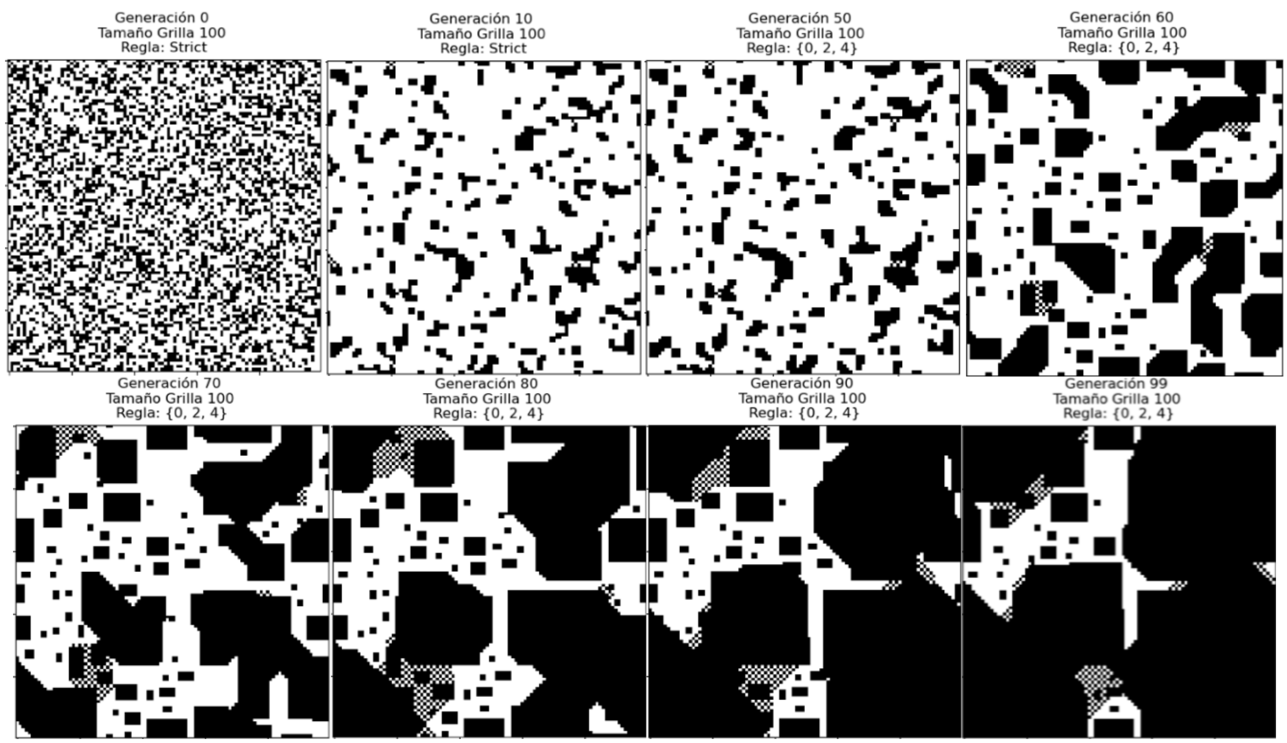
Combined:
Rule Strict : Gen [0 - 49]
Rule 024 : Gen [50 - 99]

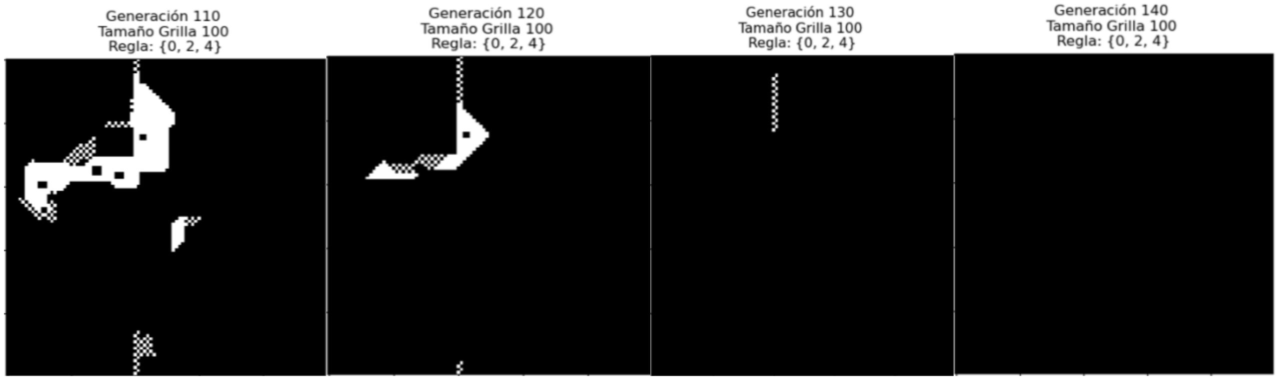


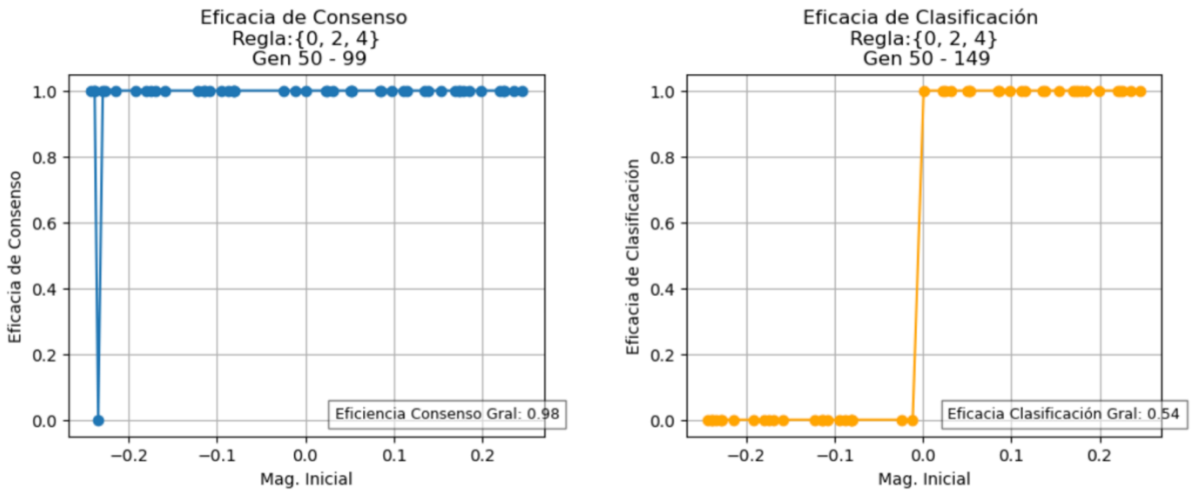


The following figure illustrates the consensus efficacy and classification efficacy graphs for the sample of 50 automata, applying a combination of rules: the strict totalistic rule from generation 0 to 49, and rule 024 from generation 50 to 99. As shown in the figure, consensus is not achieved for this initial configuration. The metrics reveal that the application of rule 024 enables most cases to reach consensus (0.80); however, the classification efficacy (0.54) indicates that only automata with an initial magnetization level above 0 are accurately classified in relation to their initial concentration of 1s or -1s.

Combined:
Rule Strict : Gen [0 - 49]
Rule 024 : Gen [50 - 149]



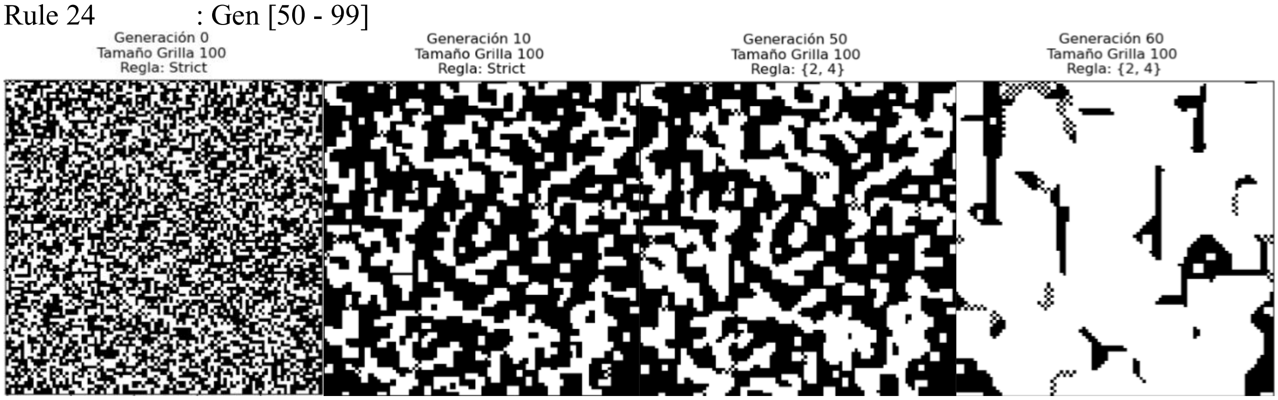


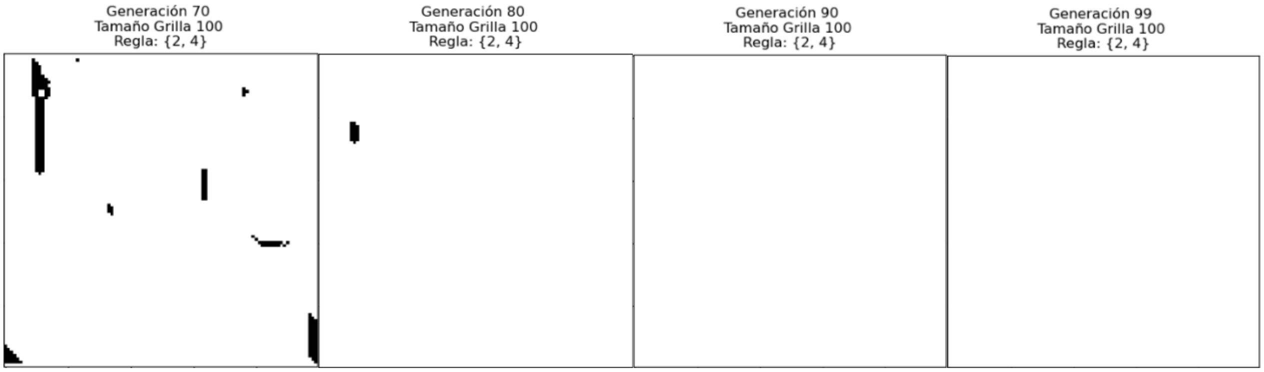


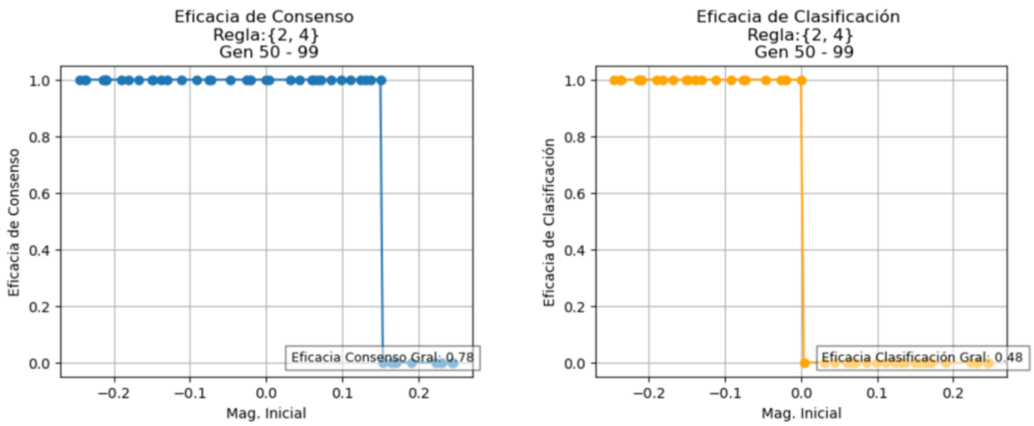
The previous figures demonstrate that increasing the number of generations allows more initial states to reach consensus, raising the consensus efficacy to 0.78. However, the classification efficacy remains almost unchanged, as the rule is unable to select or distinguish magnetizations based on the levels of 1s and -1s.

Combined:

Rule Strict : Gen [0 - 49]
Rule 24 : Gen [50 - 99]

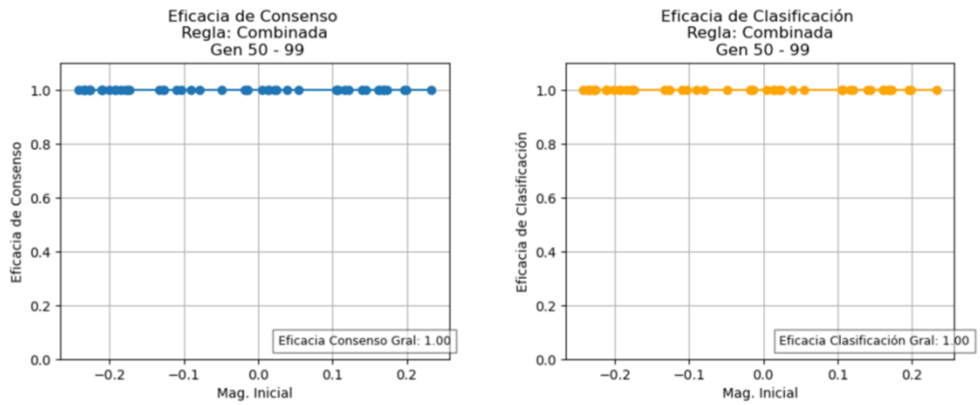






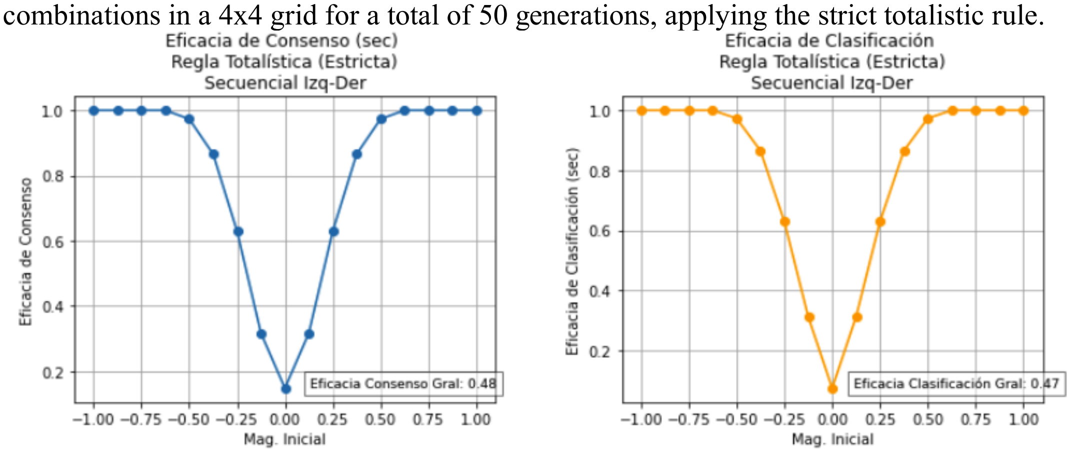
The most recent figures display the implementation of a combined update rule approach, applying the strict totalistic rule for the first 50 generations, followed by rule 24 for the subsequent 50 generations. As observed, this approach primarily achieves consensus towards the fixed point of -1. It exhibits a high consensus efficacy, while the classification efficacy is inversely proportional to that of rule 024. In this instance, the combined rule strategy allows for a favorable correlation between initial magnetizations of <0 and fixed points at -1.

Upon analyzing the performance of each update rule from generation 50 onwards, we observe that the classification efficacy metric allows the application of rule 024 to reach consensus with a high correlation when considering a positive magnetization of >=0. Conversely, implementing rule 24 facilitates reaching consensus with a high correlation when considering a negative magnetization.
The subsequent figure illustrates the impact of applying both rules concurrently: rule 024 for magnetization in generation 50 >=0, and rule 24 for magnetization in generation 50 <0. As displayed, implementing these rules allows for consensus achievement while also respecting the initial magnetization, ultimately resulting in a high classification efficacy metric.



Despite the high classification efficacy metric resulting from the application of these rules, which indicates their effectiveness in classifying the system's state, it's important to note that this is not the standard procedure as it utilizes global information about the system, rather than local information. This approach may not fully capture the inherent complexity and local interactions within the system. Therefore, alternative iteration modes will be explored to reach consensus.

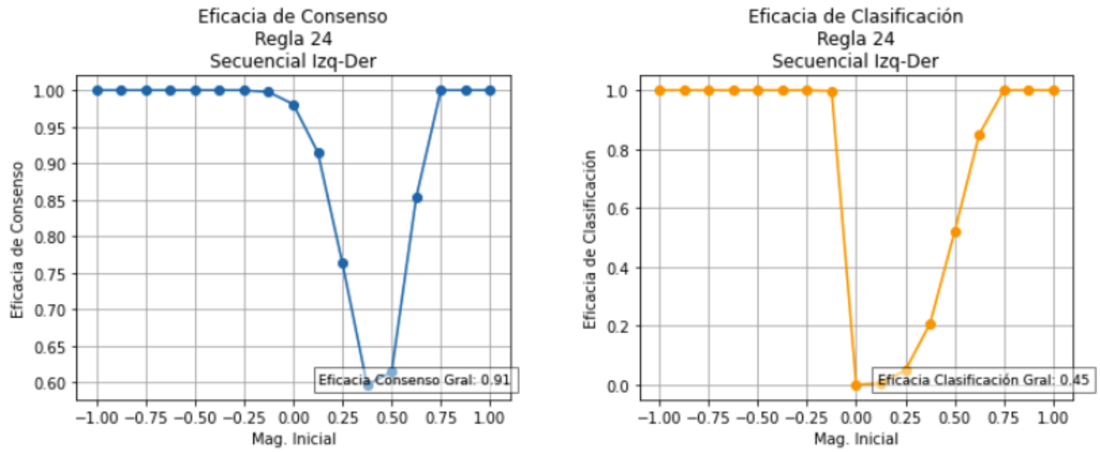
Synchronous Iteration Mode: Simulations were conducted in synchronous mode of iteration with the different initial state combinations in a 4x4 grid for a total of 50 generations, applying the strict totalistic rule.



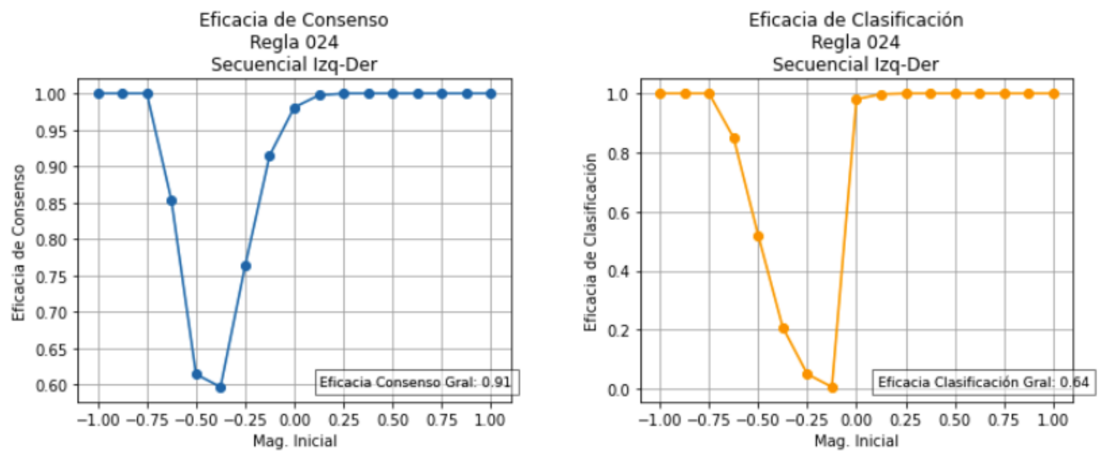
It's noteworthy to mention that the synchronous mode of iteration enhances the efficacy in both aspects. Regardless of whether the sequence starts from the left or the right, the same results are obtained in terms of magnetization. This indicates the robustness of the synchronous iteration mode, as it leads to consistent outcomes irrespective of the direction of sequence initiation.

Synchronous Application of New Rules: The same combinatorial operation was performed using the two most effective rules: rule 24 and rule 024. The results obtained from applying these update rules in a synchronous manner were analyzed and compared to the outcomes from the strict totalistic rule.

Rule 24:



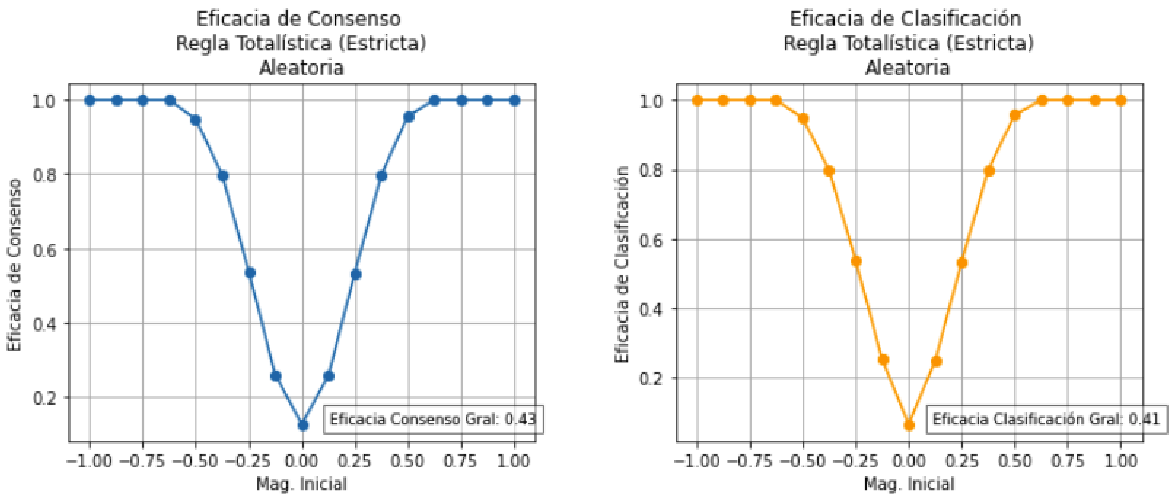
Rule 024:



Upon scrutinizing the graphical representations, it's clear that rules 24 and 024 excel in fostering conditions conducive to consensus attainment. These rules not only streamline the path to consensus but also adeptly maintain the initial magnetization levels, thereby exhibiting superior classification efficacy. Moreover, it's worth highlighting that the system's performance under synchronous iteration substantially bolsters the efficacy, leading to more robust and consistent outcomes.

Despite the evident benefits of synchronous iteration, it's crucial to venture into other iteration methods to ensure a holistic understanding of the system's dynamics. As such, an exploration into an iteration method premised on random assignment is on the horizon. This method introduces an element of unpredictability into the system's evolution by randomly selecting elements for update in each generation. The insights gleaned from this method will be juxtaposed with those from synchronous and asynchronous iterations, thereby painting a comprehensive picture of the potential strategies for achieving consensus in the system.

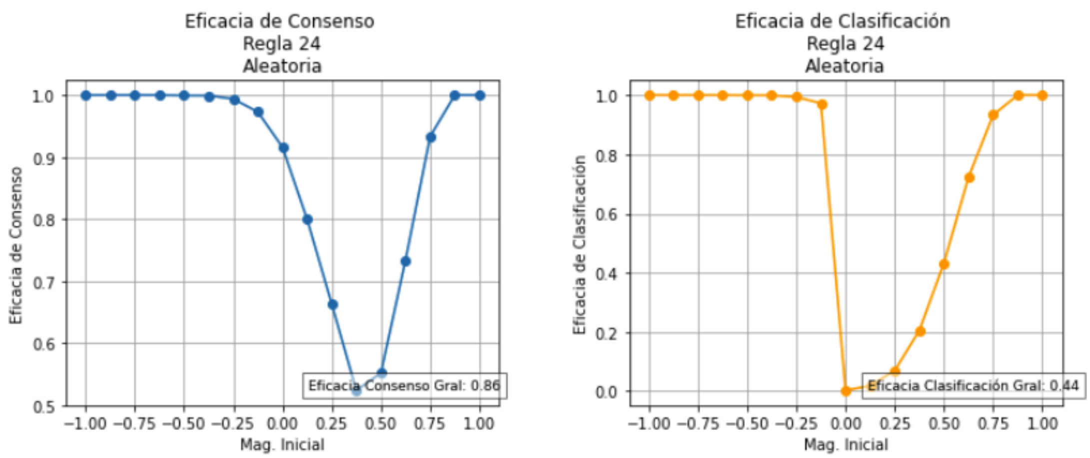
Random Iteration Mode: Simulations were also conducted in a random mode of iteration with the different initial state combinations in a 4x4 grid for a total of 50 generations, applying the strict totalistic rule.



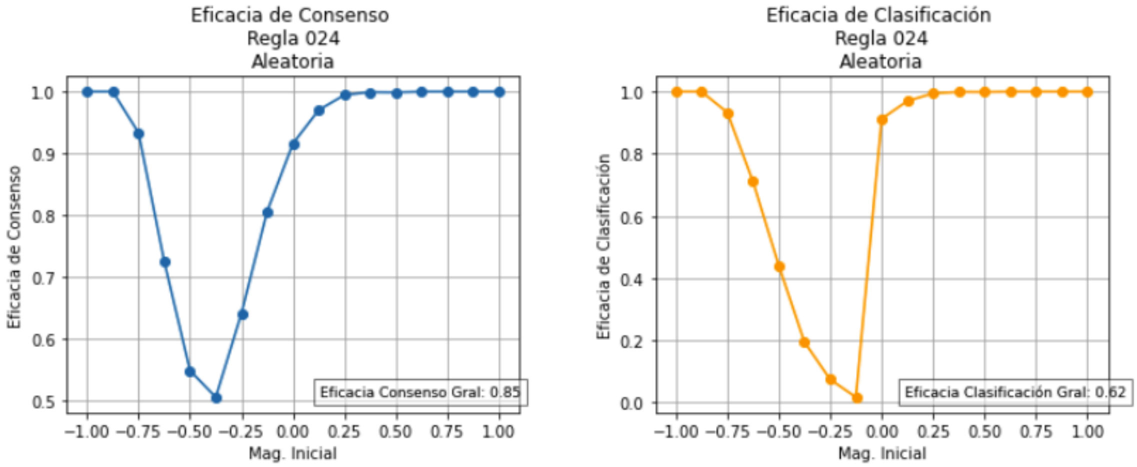
It's important to note that the random mode of iteration, while introducing an element of unpredictability, does not significantly improve the efficacy compared to the asynchronous mode. Regardless of the sequence's starting point, the results in terms of magnetization are similar to those obtained under asynchronous iteration. This suggests that the random iteration mode, while providing a different approach to system evolution, does not necessarily lead to better outcomes.

Random Application of New Rules: The same combinatorial operation was performed using the two most effective rules: rule 24 and rule 024, but this time in a random manner. The results obtained from applying these update rules in a random fashion were analyzed and compared to the outcomes from both the strict totalistic rule and the synchronous application of the rules.

Rule 24:



Rule 024:



Upon examining the graphical representations, it's clear that while rules 24 and 024 still foster conditions conducive to consensus attainment under random iteration, they do not outperform their performance under synchronous iteration. These rules maintain the initial magnetization levels effectively, exhibiting good classification efficacy. However, the system's performance under random iteration does not significantly enhance the efficacy compared to asynchronous iteration.

In conclusion, while the exploration of different iteration methods provides valuable insights into the system's dynamics, the synchronous mode of iteration still stands out as the most effective method for achieving consensus and maintaining high classification efficacy. This highlights the importance of synchronous updates in systems where consensus attainment and state classification are crucial. The effectiveness of the synchronous mode of iteration in enhancing both consensus and classification efficacy can be further observed in the following summary table of metrics obtained from the simulations:

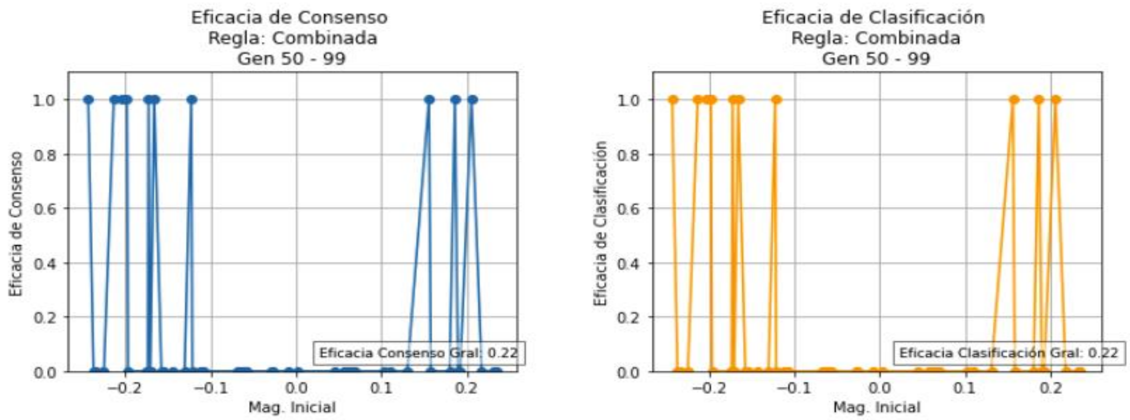
	Asynchronous Iteration		Synchronous Iteration		Random Iteration	
Rule	Consensus Efficacy	Classification Efficacy	Consensus Efficacy	Classification Efficacy	Consensus Efficacy	Classification Efficacy
Strict	0.38	0.38	0.48	0.47	0.43	0.41
24	0.49	0.34	0.91	0.45	0.86	0.44
024	0.49	0.44	0.91	0.64	0.85	0.62

This table provides a comprehensive overview of the performance of the different iteration methods and rule applications in terms of consensus attainment and classification efficacy. It clearly illustrates the superior performance of the synchronous mode of iteration, further emphasizing its importance in systems dynamics.

Combined simulations: To optimize the potential for reaching consensus, combined simulations were performed with a specific focus on identifying the iteration modes that facilitate consensus attainment. Initially, the strict totalistic rule was applied for 50 generations. Subsequently, rules 024 and 24 were applied for the next 50 or 100 generations.

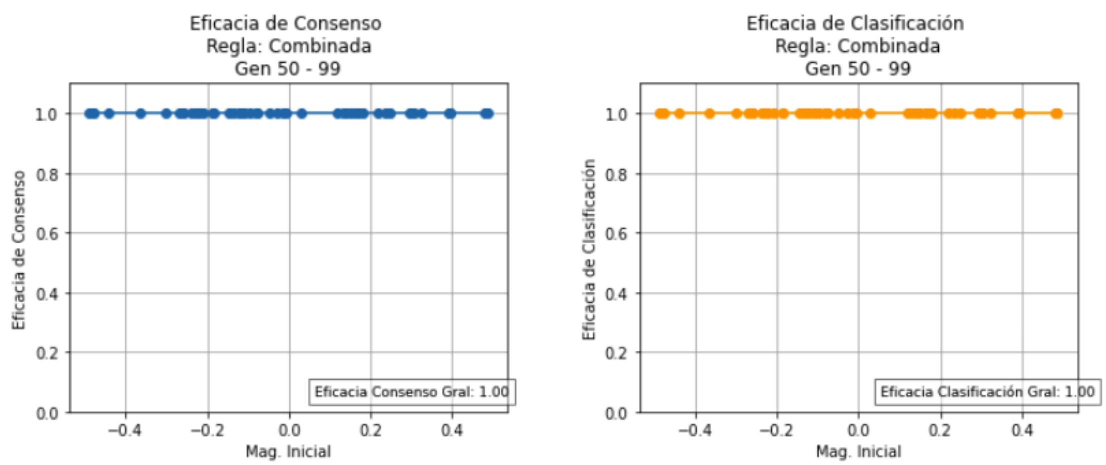
These combined simulations were conducted under various iteration modes, including synchronous and random. The results from each mode were meticulously analyzed and compared to discern the impact of the iteration mode on consensus attainment.

This comprehensive approach allowed for a more nuanced understanding of the system's dynamics and the influence of different iteration modes and rule applications on consensus achievement. The findings from these combined simulations will be instrumental in identifying the most effective strategies for consensus attainment in similar systems.



The previous figure illustrates the application of a combination of strict, 24, and 024 rules in a random assignment over 100 iterations, all with equal probability of occurrence. As can be observed, consensus is not achieved in this scenario. This underscores the complexity of consensus attainment in systems with random rule application and iteration, and further highlights the superior performance of the synchronous mode of iteration in facilitating consensus. Despite the randomness introducing an element of unpredictability into the system's evolution, it does not necessarily lead to better outcomes in terms of consensus attainment. This finding reinforces the importance of careful rule selection and iteration mode in the design and analysis of cellular automata systems.

In order to achieve consensus, a final combination was used that involved alternating cycles of each rule. First, the strict rule was applied, followed by rule 24, then the strict rule again, and finally rule 024. The length of the cycle is defined by the transient, that is, the number of generations it takes for the cellular automaton to reach consensus. For this example, an initial magnetization level ranging from -0.5 to +0.5 was used, demonstrating that all attractors reach a consensus. This can be observed in the following image:



It's noteworthy that once the attractors reach consensus, the application of a new rule 024/24 has no effect, maintaining an asymptotic state. This finding underscores the stability of the consensus state and the robustness of this rule combination in maintaining consensus, even when new rules are applied. This strategy of alternating rule application based on the transient provides a promising approach for achieving and maintaining consensus in cellular automata systems.

Considering these results, the implementation of two-dimensional cellular automata for modeling the propagation of security and insecurity perceptions in communities has demonstrated significant potential. By applying strict and combined update rules, identifying cycles, and employing key metrics to assess consensus and classification efficacy,

it is possible to better understand the dynamics underlying these complex systems. The combination of strict totalistic rules with other specific rules, such as rules 024 and 24, has shown promise in terms of achieving consensus and respecting initial magnetization levels. This comprehensive approach to analyzing cellular automata behavior can inform future research on consensus building and decision-making processes in various contexts, providing valuable insights and fostering a deeper understanding of the interactions among various components within a community.
To promote collaboration in research, all the code used for conducting the one-dimensional and two-dimensional cellular automata simulations in this study is made available for online access. You can find the code in a public GitHub repository at the following link: https://github.com/educarrascov/DISC_Complex

D. Conclusions:

In this study, we delved into the dynamics of security and insecurity perception within communities using one-dimensional and two-dimensional cellular automata. Our motivation stemmed from the understanding that while people often follow majority opinions about security perception, their assessment of which opinion constitutes the majority can vary.
We applied various update rules, including strict and combined rules, to investigate the propagation of security perception. Our findings revealed that a global consensus on security perception could be reached in densely connected networks, while multiple steady states of perception coexistence might be observed in sparser networks. Upon closer examination, we found that interconnected low-degree nodes and hub nodes played crucial roles in shaping security perceptions and building consensus on this issue.
A significant aspect of our study was the exploration of different modes of iteration - asynchronous, synchronous, and random. We found that the synchronous mode of iteration significantly enhanced the efficacy of consensus attainment and state classification, outperforming the other modes. This was particularly evident when combining rules 024 and 24 under synchronous iteration, which facilitated strong consensus formation while maintaining the initial magnetization levels.
However, it's important to note that while the use of cellular automata often prompts a desire to utilize global information, this should be avoided. The strength of cellular automata lies in their ability to model local interactions, and incorporating global information could potentially distort the inherent dynamics of the system.
By analyzing cellular automata behavior and implementing different rules, cycle recognition, and key metrics, we gained valuable insights into the consensus building in complex social systems. Although we might expect global consensus on security perception to be more achievable as social systems become increasingly interconnected, differing opinions on this matter continue to coexist due to factors such as community structures, temporal evolution, individual differences in influence evaluation, and resistance to majority effects.
Looking ahead, it is essential for future research to investigate the impacts of these factors in both synthetic and real-life networks to further deepen our understanding of security perception formation and consensus building in social systems.

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