Interest, Prices, and Risk

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Abstract

Monetary policy is often said to be responsible for setting or targeting the nominal risk-free interest rate of an economy even though it is operated in a risky environment through assets that may be risky. This paper studies how policy-asset risk can be transmitted to monetary policy under a neo-Wisksellian approach. We introduce risk in the policy asset of a partial equilibrium model and show that monetary policy power w.r.t prices and inflation is reduced, as well as a positive correlation between inflation and that risk emerges. Moreover, uncompensated risk premium induces an inflationary bias. These results constitute a novel argument in favor of a more hawkish stance in case of a fiscal or political crisis, since monetary policy can only be conditionally active in the presence of risk in its underlying asset. Overall, we find that monetary policy's activeness depends on whether the central bank sets and targets a risky or a risk-free rate.

1 Introduction

Real ex-ante interest rate spreads between emerging and advanced economies sovereign bonds have been mostly positive throughout the years, implying that there are consider-

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able premia on them.[1] If emerging bonds had the same objective characteristics – such as maturity, coupon structure, and currency denomination – as well as the same desirable features from the perspective of the same marginal investor, then they should represent perfect substitute cash-flows to bonds of advanced economies. This equivalence would lead them to be identically priced in markets with free mobility of capital, rational investors, and no other financial friction. However, that does not seem to be the case, what suggests that the *(real) natural interest rate*,[2] hereafter defined as the real interest rate consistent with flexible prices, should also differ significantly across emerging and developed economies.

A simple attempt to estimate the natural rate using univariate filters gives us a glimpse of such discrepancy in Figure 1. There seems to be comovement, it is true, with most notably a downward trend[3], but, at least from 2000 to 2020, the spread is likely to have remained sizable and positive.[4] The details of this estimation are available in Appendix A.

[1] Du and Schreger (2016) calculate for 10 emerging economies between 2005 and 2014 the yield spread of local currency bonds over synthetic local currency risk-free rates constructed with cross-currency swaps, assuming U.S. rates as risk-free. They find that spreads are positive and sizable. Ferreira and León-Ledesma (2007) investigate the real interest parity hypothesis empirically in a small unbalanced panel of 5 advanced economies and 5 developing ones, ranging from the early 1970s to the early 2000s, by conducting unit-root tests. The authors find that while interest rate differentials to the U.S. rate tend to revert to zero in the first group, it reverts to a positive wedge in the second. With an older sample, Mishkin (1984) had already found that real interest rates differed across developed economies after conducting econometric tests on their equality, as well as on an ex-ante version of the purchasing power parity, on the uncovered interest parity condition, and on the unbiasedness of forward rate forecasts of exchange rates.

[2] Some other names for the (real) natural interest rate are (real) neutral interest rate and (real) structural interest rate. This paper will use all of them as synonyms.

[3] Rachel and Smith (2015) find similar results subtracting professional 1-year ahead inflation forecasts, used as a proxy for 10-year inflation expectations, from 10-year nominal government bond yields in a larger sample comprising 20 advanced economies and 17 emerging ones. They also estimate that the global real interest rate would have fallen by 450 bps from 1980 to 2015, of which they can explain 400 bps as follows: lower output growth (-100 bps), demographic forces (-90 bps), higher inequality within countries (-45 bps), a preference shift towards higher saving by emerging economies after the Asian crisis (-25 bps), a decline in the relative price of capital goods (-50 bps), a preference shift away from public investment projects (-20 bps), and an increase in the spread between the risk-free rate and the return on capital (-70 bps). From a historical point of view, Schmelzing (2020) finds that real rates have been on a declining trend since the 14th century at the speed of 0.6-1.6 b.p. per annum using data for countries that compose a large share of advanced economies output in the period.

[4] Beyond visual inspection, a paired-sample t-test of the difference in means between emerging real neutral rates and their advanced counterparts results in the rejection at the 1% level of the null hypothesis that

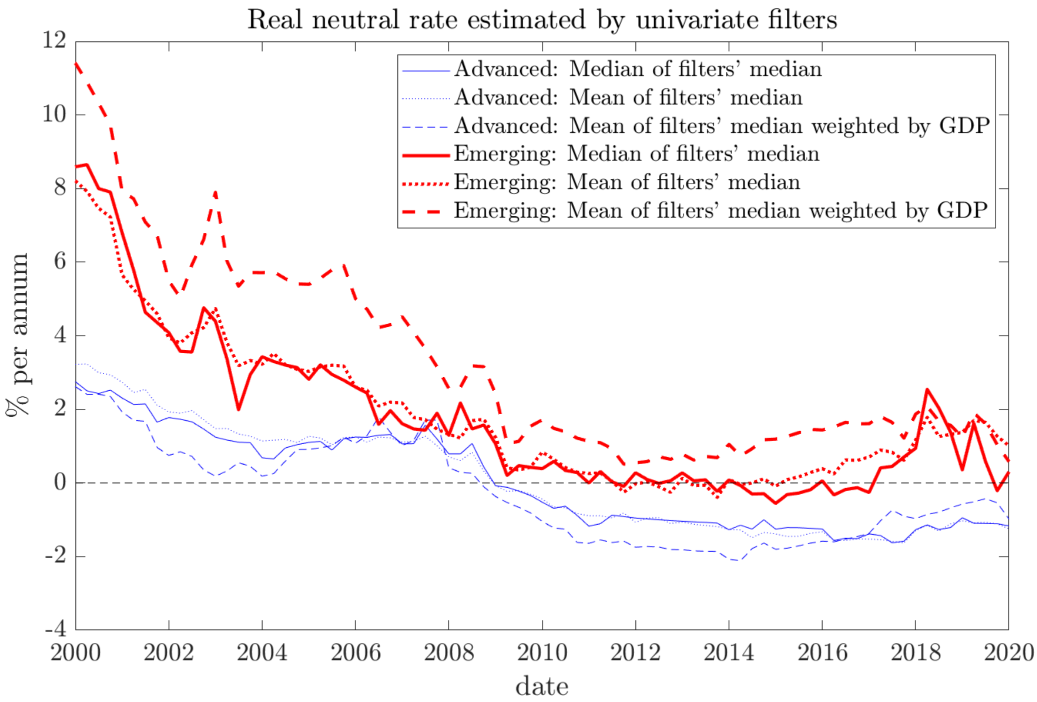


Figure 1: Real neutral rates estimated by univariate filters

Moreover, since the natural interest rate constitutes the theoretical reference for the intercept of several monetary policy rules adopted in the New-Keynesian literature (Woodford (2003); Galí (2015); etc.) and in actual central banking (Taylor and Williams (2010)), there are concerns on the adequacy of prescribing such rules to emerging economies without understanding the dynamics of the natural interest rate wedge. In the specific case of emerging-economy central banks that target short-term nominal interest rates in local currency, like those which abide by some inflation-targeting regime, that is a particularly relevant theme. For these institutions, setting or targeting nominal interest rates too low/high could ignite capital outflows/inflows with pertinent impacts on the level of domestic prices. This is so as debt sustainability, exchange rate stability, and foreign capital disposal are directly affected by domestic and foreign investors' desire for retaining local assets in their portfolio (Fraga, Goldfajn and Minella (2003)).

All these arguments, raised both in the literature and in actual monetary policymaking, constitute open-economy explanations for the importance of picking the right target for the

they are equal in favor of the alternative hypothesis that emerging rates are higher for all aggregations. Moreover, estimating ARIMA(0 to 4, 0 to 1, 0 to 4) models of the wedges and picking the one with the lowest BIC (ARIMA(2,0,1)) provides evidence at the 1% level in favor of such wedge being different from zero for the median of the filters' median. For the other aggregations, the evidence is weaker, though.

policy rate. Nonetheless, they frequently ignore that some of the very own assets employed by monetary policy in its monetary operations may be perceived as risky. The exact source of that risk may stem from the credibility of the issuer of the policy asset (i.e. outright or repo operations involving risky federal government bonds)[5], or still from the fact that liquid assets are easy candidates to confiscation in case a government wishes to collect extraordinary revenues or curb inflationary liquidity – as long as the government perceives it as a better choice than the alternatives on the table.[6] Moreover, in some countries part of government bonds is indexed to the policy rate, configuring an important link of the transmission channel of monetary policy, so that defaulting on them is to some extent defaulting on the policy rate. For the skeptical reader who considers the possibility of default in policy assets only a theoretical curiosity, it may be helpful to note that a financial asset with risk of retroactive tax is priced alike a defaultable bond.[7]

If the policy asset is risky, then, how is monetary policy affected? This is a question that we believe has been neither properly posed nor addressed in the literature, not even in the

[5]Domestic debt denominated in local currency, including short-term federal government bonds, have been defaulted on, *de facto* and *de jure*, throughout history, as shown by Reinhart and Rogoff (2009). While the specific case of default via unexpected inflation has received some attention from the literature – most notably, Miller, Paron and Wachter (2020) directly associates the downward trend observed in Figure 1 to the stabilization of inflation in many countries in the same period, which would have shrunk inflation premia – the possibility of default *de facto* in domestic currency is still mostly discredited, in denial of the empirical evidence. Focusing on local-currency defaults of large emerging economies, there is Brazil in 1990 (broad confiscation of financial assets), Russia in 1998-99 (suspension of payments of Treasuries), and Turkey in 1999 (retroactive withholding tax on financial assets). In a more extreme example, Ghana in 1979 implemented a monetary reform in which the exchange rate from old to new banknotes was worse than for bank deposits (currency wealth confiscation).

[6]In 1990, Brazil implemented the Collor Plan, a monetary reform intended to contain a hyperinflation process perceived as the result of excessive liquidity in the economy. Among its measures, it temporarily confiscated financial resources that amounted to up to 80% of the M4 (M1 plus all financial assets), or approximately 30% of the country's GDP at that time (Pastore, 1991).

[7]Default can be seen as a stochastic retroactive tax on wealth. In that sense, when adopted, it is like if the fiscal policy is going an extra mile to balance the budget. One actual example of that kind of confiscation would be the Turkish local-currency default of 1999. A Letter of Intent of the government of Turkey addressed to the IMF on December 9, 1999, in the context of its request for financial support, implausibly explained: "Finally, a withholding tax on government securities issued before December 1, 1999 has been introduced to reduce the windfall gain accruing to securities holders from the reduction in inflation and interest rates in 2000." Önal and Erçel (1999).

realm of a closed economy. Some further complications, however, exist for monetary policy. First of all, the natural interest rate is non-observable. Secondly, episodes of exacerbate risk in the policy asset shall have a severe impact on business activity in general. Once we take these hurdles into consideration, it is possible to conclude that in the presence of risk, investors will demand a premium for investing in risky assets, as well as will rebalance their portfolio to hedge against undesirable fluctuations of wealth.[8]

In this paper, the mechanism through which risk influences monetary policy is put under scrutiny. First, we expand a partial equilibrium closed-economy monetary model found in Woodford (2003, sec. 4.3 of ch. 1) to encompass risk in the asset used by monetary policy. We find that in that case the power of monetary policy w.r.t. inflation is reduced at the same time that the price level is higher than in an otherwise identical scenario. The same applies to inflation in case of inflation targeting. These results in conjunction generate a novel argument in favor of more hawkish monetary policy in case of, say, a fiscal or political crisis, which we believe is the main contribution to the literature of this study. Finally, we find that assuming that the policy asset is defaultable and that the central bank accommodates such risk to some extent is enough to generate virtually any positive correlation between inflation and default risk, a feature that is found in empirical data. If expanded this connection between default risk and inflation to an open economy, it could potentially offer an additional explanation for why some countries experience a positive correlation between currency risk and default risk. In that case, causality would flow from default risk to inflation risk to currency risk, and not from default risk to currency risk to inflation risk, as usually assumed in the literature through an uncovered interest rate parity equation.[9]

A simple way of giving the reader the intuition behind our results can be achieved starting from the basics of monetary policy conducted with interest rate rules in a linearized model. Given, at any period t, the real natural interest rate, r_t^n, and the time-varying intercept of the policy rule, $\bar{\imath}_t$, the nominal policy rate, i_t, and the inflation expectation for the

[8]For instance, an expected reaction of households to an increase in uncertainty about the future is to expand their (precautionary) savings, so that they can better smooth out consumption through time. De Paoli and Zabczyk (2013) incorporate this feature into a New-Keynesian model.

[9]Examples of New-Keynesian models which follow this approach are Justiniano and Preston (2010) and de Carvalho and Vilela (2015). In Blanchard (2004), the exchange-rate pass-through to inflation is essential to generate his fiscal dominance result. For a list of competing explanations for the positive correlation, check Lowenkron and Garcia (2005).

next period, $\pi^e_{t,t+1}$, are simultaneously determined by the Fisher equation and the central bank (CB) policy rule:[10]

$$\begin{aligned} i_t &= r^n_t + \pi^e_{t,t+1} \\ i_t &= \bar{\imath}_t + \phi\pi^e_{t,t+1} \end{aligned} \quad \Rightarrow \quad r^n_t - \bar{\imath}_t = \left(\phi - 1\right)\pi^e_{t,t+1} \tag{1}$$

Note that by the Taylor principle – required for the determination of inflation in a model like this – the coefficient $\left(\phi - 1\right)$ must be positive, what leads to a positive correlation between $r^n_t - \bar{\imath}_t$ and $\pi^e_{t,t+1}$. In this paper, we develop the idea that the policy asset is risky, so there is a wedge, Φ_t, between the rate of that asset and the risk-free rate.

$$\begin{aligned} i_t &= r^n_t + \Phi_t + \pi^e_{t,t+1} \\ i_t &= \bar{\imath}_t + \phi\pi^e_{t,t+1} \end{aligned} \quad \Rightarrow \quad \left(r^n_t + \Phi_t\right) - \bar{\imath}_t = \left(\phi - 1\right)\pi^e_{t,t+1} \tag{2}$$

Then, an important insight comes from the fact that inflation expectation is not only a function of $(r^n_t - \bar{\imath}_t)$, but also of the risk premium demanded to hold the policy asset. If the central bank does not compensate the policy-asset risk in its choice for $\bar{\imath}_t$, then it introduces a bias in inflation expectations. Here, we will focus on the case where $\Phi_t > 0$, which leads to an inflationary bias, but a negative wedge is also possible in case investors are willing to pay for a convenience yield.

As we develop, in this paper, a partial equilibrium monetary model, we suppose that the implicit fiscal policy is kept passive, using the terminology of Leeper (1991), at all times.[11] We show that, even though introducing default risk in the underlying asset of monetary policy reduces its activeness in directing inflation, the insight from Leeper (1991) for that macroeconomic stability only requires a mismatching active/passive combination of fiscal and monetary policy remains unchanged under very mild conditions for the default probability stochastic process.

The rest of this paper is structured as follows. Section 2 presents related literature. Section 3 exposes the theoretical background of neo-Wicksellian models. Section 4 introduces

[10] Without loss of generality, we implicitly assume that the inflation target is zero. Assuming otherwise or that the policy rule reacts to current inflation changes the dynamic of prices, but not our results.

[11] Monetary policy is not constrained by the fiscal one, while the latter automatically adjusts to any given trajectory for the price level, so that the real value of government liabilities equals the present value of all its present and future real surpluses. In Appendix C, we derive a simple adaptation of the canonical cashless flexible-price model of Woodford (2003, ch. 2, sec. 1) with a balanced-budget fiscal rule to show that it can underpin this paper's model.

policy asset risk. Section 5 derives the same model under an inflation target rule. Section 6 exposes some testable implications. Finally, Section 7 concludes. In Appendix A, we describe the calculation of univariate filters for the neutral real rate; Appendix B provides all proofs for this paper; Appendix C adapts a canonical cashless flexible-price model to show that it can underpin this paper's model; Appendix D analyzes the case of targeting a risky rate with safe assets; Appendix E simulates the model to investigate the power of monetary policy; and Appendix F gives more details on testable implications of the model.

2 Related literature

In the literature, this paper can be seen as an extension of Woodford (2003)'s interpretation of Wicksell (1898)'s work on the natural interest rate and its relation with monetary policy. The introduction of risk in the policy asset of a neo-Wicksellian model is our main contribution.

As of the consequences of taming inflation with defaultable assets, this paper is more closely related to Bi, Leeper and Leith (2018), which investigates for the case of a closed economy how the role of monetary policy in an inflation-targeting regime conducted with risk-free assets changes when default risk is not neglectful. They employ a New-Keynesian model with fixed intercept Taylor rules, and find that, on the one hand, if the central bank targets the risk-free rate, by presumably controlling a risk-free instrument, it can still manage to bring inflation to the target after a contractionary monetary shock while persistently depressing output. On the other hand, if the central bank targets the (default-) risky interest rate, by implicitly controlling a risk-free instrument whose rate passes through to the risky one, it will be as if it had lifted the inflation target above the publicly announced one, inflating away the government debt. Remarkably, the authors argue that monetary policy activeness increases at the same time that their model points to a positive link between inflation and default risk.[12] Commenting on that paper, Reis (2018) notes that the intercept of the risky Taylor rule actually lacks an adjustment for default risk, biasing its results. In Appendix D of our paper, we provide the adequate fix, and find that the effective inflation target now coincides with the announced one, weakening the relation between inflation

[12] We interpret that, in a Taylor-like rule, activeness is measured by the effective coefficient on its reaction terms, usually on inflation. The more active, the more elastic is the policy rate to deviations from the steady state, what should also reduce inflation volatility, implying higher monetary policy power w.r.t. inflation.

and default risk in that model. Moreover, activeness still increases, but it is only due to the fact that the policy asset remains risk-free, since policy-asset default risk would also reduce monetary policy power in that paper. Differently from Bi, Leeper and Leith (2018), this paper focuses on the role played by default risk embedded in the monetary policy asset. More strikingly, this paper models the policy asset as a defaultable bond and defines a policy rule that specifically targets the return of the bond in case the issuer repays the debt. We leave general equilibrium considerations for a companion paper.

As we show that neglecting significant policy-asset risk is not without consequences for a central bank's mandate, this paper contributes to the literature on the implications of mismeasurement and model misspecification for monetary policy. Orphanides and Williams (2002) seek to find policy rules that are robust to the uncertainty in the precise estimation of natural rates, both of interest and unemployment. The authors find that policies that underestimate such uncertainty end up generating higher costs, in terms of stabilizing inflation and unemployment, than policies that tend to overestimate it, tilting preference toward policy strategies that act as uncertainty is greater than baseline estimations, such as, for example, rules that react to first differences instead of deviations from natural levels. In Orphanides and Williams (2008), the same authors extend the analysis to when agents do not know the complete structure of the economy and instead must learn through forecasting models updated every period, finding additional evidence in favor of a "conservative" approach to the informational constraint. Although our paper does not elaborate on welfare considerations, it presents a new source of mismeasurement and misspecification that challenges the optimal control of monetary policy.

Despite we assume that implicit fiscal policy is passive, this paper still relates to the literature on the interaction between fiscal and monetary policy when debt is risky. Schabert and Van Wijnbergen (2014) find that, if the default probability process is allowed to depend on the debt level in a reduced form, a weak feedback mechanism from debt surprises to the primary surplus associated with active monetary policy jeopardizes macroeconomic stability, a situation described in Blanchard (2004). Despite that, the authors show analytically that active monetary policy can still contribute to stabilization if the slope of fiscal feedback intensity increases with the default probability, by increasing the passiveness of fiscal policy. Concerning their policy recommendation, our finding that default risk may endogenously reduce monetary policy activeness introduces a soothing channel that could counterbalance the need of draconian fiscal consolidation under the Blanchard effect.

Still on the same topic, Bonam and Lukkezen (2019) study how fiscal policy's cyclicality affects equilibrium stability and uniqueness. They show analytically in a closed-economy New-Keynesian model that, after growing budget deficits, debt-elastic interest rates increase in response to a now higher default-risk premium crowding out consumption, what, by its turn, decreases inflation requiring a reduction of interest rates. These effects lead to the shrinkage of the parameter space that supports macroeconomic stability. The latter can be avoided by adopting either a more aggressive fiscal consolidation or a more active monetary policy, or yet both. They conclude that procyclical fiscal policies are more prone to deliver macroeconomic stability. Here, again, our endogenous channel through which default risk reduces monetary policy activeness may be relevant, as their policy recommendations could be reinforced by it.

In Bonam and Lukkezen (2019)'s model, risk is contractionary, implying a negative correlation between default risk and inflation, while in Schabert and Van Wijnbergen (2014) it is expansionary, implying in correlation with the opposite sign. We show in Section 6 that, to the extent in which local-currency risk can be proxied by usual country-risk measures, empirical data go in favor of positive correlation for inflation-targeting emerging economies. In our model, such a positive correlation emerges naturally as the central bank reacts only to to the natural real rate and to inflation, ignoring that the policy asset is risky.

Finally, this paper may also be marginally correlated with the unpleasant monetarist arithmetic of Sargent and Wallace (1981); the tight money paradox of Loyo (1999), in which a bounded equilibrium is only obtained after either monetary or fiscal policy stops playing "chicken"; as well as with the fiscal theory of the price level (FTPL) of Sims (1994); Woodford (1994); Uribe (2006), who, like Blanchard (2004), analyze the interaction of monetary policy under an active fiscal policy regime in the definition of Leeper (1991). We leave for further research how the endogenous reduction in monetary policy activeness w.r.t. prices due to policy-asset risk affects their results. As far as we know, this paper is the first to explore that channel under the mix of active monetary policy and passive fiscal policy (not explicitly modeled), or under any other mix. In a companion paper, we endogenize the risk in the policy asset as a fiscal risk.

3 Theoretical background

We, now, start a "crash course" on the basics of Wicksell's monetary theory, describing Woodford (2003)'s mathematization of it, to afterward incorporate policy-asset risk to the neo-Wicksellian model.

3.1 Wicksell's monetary theory

In 1898, Knut Wicksell published his seminal book "Interest and Prices: A Study of the Causes Regulating the Value of Money", in which he introduced the concept of a natural interest rate. For him, in an economy based on pure credit, such that money as we know did not exist, and all exchanges and loans were conducted through real capital goods, there would be a unique interest rate with the particular property of clearing the market for these goods. In that economy, this interest rate, named by him as the natural interest rate, would be determined in an efficient market by the marginal productivity of real capital goods. In his own words:

> THERE is a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them. This is necessarily the same as the rate of interest which would be determined by supply and demand if no use were made of money and all lending were effected in the form of real capital goods. It comes to much the same thing to describe it as the current value of the natural rate of interest on capital (Wicksell, 1898, p. 102).

Going from a pure credit to a cash economy means that there are now two types of assets, money (or financial capital) and real capital goods. It is trivial to think that the existence of two assets may give birth to an exchange rate, i.e., n units of money for 1 unit of capital. This exchange rate is the price of capital, which means that the interest rate on loans with money (Wicksell called the "money rate") and the interest rate on loans with real capital goods can be compared.

For Wicksell, in an otherwise constant world, arbitrage would bring these interest rates to become equal, stabilizing the price of capital. However, in the presence of a money-rate setter with deep pockets or the ability to create them as required, say a contemporaneous central bank, such an equalization would fail to happen. Wicksell envisioned that whenever the money rate was higher than the natural rate, the same arbitrageurs of before would seek to exchange real capital goods for money, forcing the price of real capital to go down (deflation) indefinitely while the difference in the rates remained. Likewise, when the money rate was lower than the natural one, arbitrageurs would seek to convert money into real capital, forcing the price of real capital to go up (inflation) indefinitely until the money-rate setter changed its mind. Since the world is not constant, or more precisely, the factors that affect the natural rate or the money-rate setter's mind, prices in the actual economy fluctuate, alternating inflationary with deflationary periods — beware that Wicksell had in mind the price history available at his time, the nineteenth century.

3.2 Neo-Wicksellian monetary theory

A little more than a century after Wicksell's seminal book, Michael Woodford synthesized what he called the neo-Wicksellian monetary theory. Inspired by the fact that central banks around the world had largely abandoned monetary aggregate targets in favor of interest rate rules that depended on inflation, the so-called Taylor rules[13], as well as the prospect that electronic payment media were to become increasingly more popular, Woodford saw the similarity between this new reality and the work of Wicksell on the price level dynamics. In his own words:

> "(...) Wicksell's approach is a particularly congenial one for thinking about our present circumstances — a world of purely fiat currencies in which central banks adjust their operating targets for nominal interest rates in response to perceived risks of inflation, but pay little if any attention to the evolution of monetary aggregates — to say nothing of the one to which we may be headed, in which monetary frictions become negligible", (Woodford, 2003, p. 49).

Woodford (2003) illustrates in a monetary partial equilibrium model the way a central bank can determine the equilibrium price level without any consideration for money supply and demand. The model laid out in Chapter 1 Section 4.3 of the reference can be systematically exposed as follows. Take assumptions 1 to 5 as valid.

Assumption 1 (Flexible prices)**.** *at every period t prices are fully flexible.*

[13]Taylor (1993) estimates an interest rate rule for the Alan Greenspan's chairmanship at the Federal Reserve as a function of inflation and of a measure of output gap. Rules of this kind have been known as Taylor rules.

Assumption 2 (Exogenous real interest rate)**.** *the equilibrium real interest rate r_t is determined only by exogenous real factors affecting saving and investment; it is also the natural one because of Assumption 1, $r_t \equiv r_t^n$. Overall, it follows an exogenous process $\{r_j\}_{j=t}^{\infty}$.*

Assumption 3 (Rational agents)**.** *at every period t agents form forward-looking rational expectations about the future using all the information set available at period t.*

Assumption 4 (Fisher equation)**.** *at every period t the Fisher equation is valid.*

Assumption 5 (Central bank sets the nominal interest rate)**.** *at every period t the central bank successfully sets the nominal interest rate, i_t.*

The Fisher equation can be written up to a first-order approximation in its log-linear form as

$$p_t = \mathbb{E}_t p_{t+1} + r_t^n - i_t \tag{3}$$

where $\mathbb{E}_t$ is the mathematical expectation operator conditional on all information available at time t, p_t is the log of the price level at period t, r_t^n is the equilibrium real rate of interest (and the natural one) at period t, while i_t is the short-term nominal interest rate at period t. Since the central bank sets i_t, and r_t^n is exogenously determined, the common way of interpreting this equation as an *identity* can now be replaced by its understanding as an *equilibrium condition* between aggregate saving (supply of capital) and investment (demand for capital). In other words, at every instant t, the price level that clears the capital goods market (and equates saving and investment) is the one given by equation 3.

Now, assume the central bank sets i_t, the money rate in Wicksell's language, as arguably envisioned by the latter[14], such that p_t and i_t should move in the same direction and targeted inflation is zero. The central bank is able to do so by setting the nominal interest rate paid by a risk-free asset, a usual assumption whose importance will be made clearer later in this paper. For a derivation of the same model under an inflation target rule (or Taylor rule), check Section 5.

Assumption 6 (Wicksellian risk-free policy rule)**.** *there is a central bank that sets the nominal interest rate of a risk-free asset, i_t, through a Wicksellian rule of the type $i_t = \bar{\imath}_t + \phi\left(p_t - \overline{p}\right)$, where $\{\bar{\imath}_j\}_{j=t}^{\infty}$ is an exogenous process for a time-varying intercept determined independently*

[14]This is Woodford's interpretation of Wicksell's work, since the latter did not mathematically specify such a rule, even more its log-linear approximation.

of the evolution of prices which may or may not be correlated with $\left\{r_j^n\right\}_{j=t}^{\infty}$*;* $\overline{p}$ *is the log of the price level target; and* $\phi > 0$.

Without loss of generality, hereafter we set $\overline{P} = 1$, where $\overline{P}$ is the price level target. Inserting Assumption 6 into the Fisher relation (3) to eliminate i_t, we obtain the law of movement of the equilibrium price level:

$$p_t = \alpha \mathbb{E}_t p_{t+1} + \alpha \left(r_t^n - \bar{\imath}_t\right) \tag{4}$$

where $\alpha \equiv \frac{1}{1+\phi}$ is a coefficient that satisfies $0 < \alpha < 1$. If the process $\left\{r_t^n, \bar{\imath}_t\right\}$ is bounded then p_t is unique and bounded[15], obtained by iterating forward equation 4, which gives us

$$p_t = \sum_{j=0}^{\infty} \alpha^{j+1} \mathbb{E}_t \left(r_{t+j}^n - \bar{\imath}_{t+j}\right) \tag{5}$$

As a consequence, the equilibrium price level will fluctuate in a bounded interval around its long-run average value:

$$\overline{p} \equiv \frac{\overline{r^n} - \bar{\imath}}{\phi} \tag{6}$$

where $\overline{p}$, $\overline{r^n}$ and $\bar{\imath}$ are the long-run average of the log price level, of the natural interest rate and of the time-varying intercept, respectively.

The behavior of Woodford (2003)'s simple model is summarized in Figure 2 ($\phi = 0.1$) and Figure 3 ($\phi = 0.2$), in which all panels are plotted assuming an economy starting from the steady state at period $t = 0$, whose only shocks are the ones specified in each panel, and that $\bar{\imath}_t = \overline{r^n}$ at all periods unless otherwise stated. The panels can be synthesized as follows:

- There is no distinction in this model between the equilibrium price level and the price level since prices are flexible. If $\bar{\imath}_t = r_t^n \; \forall t$, then the price level never leaves its steady-state value (top-left panel and red line of bottom-right panel of Figure 2);

- An expected increase (decrease) in the equilibrium real rate at any time from $t = 1$ to $t = \infty$ will increase (decrease) equilibrium prices today (top-right panel of Figure 2). If the sequence of shocks is anticipated at period $t = 1$ (blue line), the increase (decrease) of the price level is larger than if shocks are individually seen as MIT shocks[16] (red line);

[15] A proof is available in Woodford (2003)'s Appendix A.4.

[16] An MIT shock is an unexpected shock, usually inflicted at the deterministic steady-state equilibrium, that once it happens it is not expected to happen again by the agents.

- An expected tightening (loosening) of the policy rule, represented in the model by a raise (reduction) of the rule's intercept, at any time from $t = 1$ to $t = \infty$ will decrease (increase) equilibrium prices today (bottom-left panel of Figure 2). Here rational expectations play a role again, since if the sequence of shocks is anticipated by the agents (blue line), the reduction (increase) of the price level is more intense than if they consider shocks to be individually MIT shocks (red line);

- r_t^n is a *sufficient statistic* for how real factors affect p_t in the model as any additional information about real variables does not expand the information set. By closely tracking the natural interest rate, monetary policy is able to stabilize prices (red line in bottom-right panel of Figure 2) when $\{r_t^n\}$ follows a stochastic process. Any intercept that differs from the natural interest rate will fail to stabilize at all periods the level of prices, where the specific combination of a stationary $\{r_t^n\}$ with an intercept fixed at the steady-state value of r_t^n results in the price level fluctuating around the target (blue line);

- The higher (lower) the reaction coefficient ϕ of the policy rule, the more reduced (augmented) is the intensity of deviations from the price level target. One can see this by comparing the panels of Figure 2, whose $\phi = 0.1$, with the respective panels of Figure 3, whose $\phi = 0.2$.

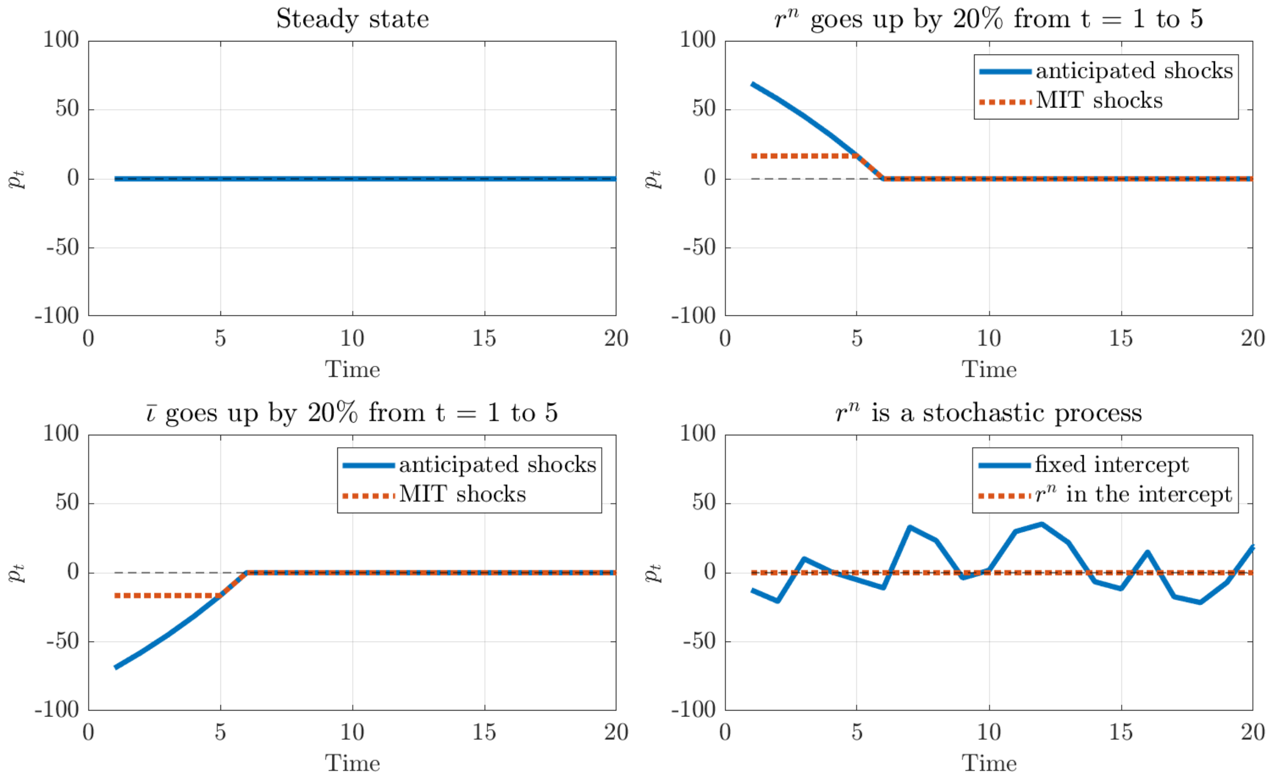


Figure 2: The price level under neo-Wicksellian monetary theory ($\phi = 0.1$)

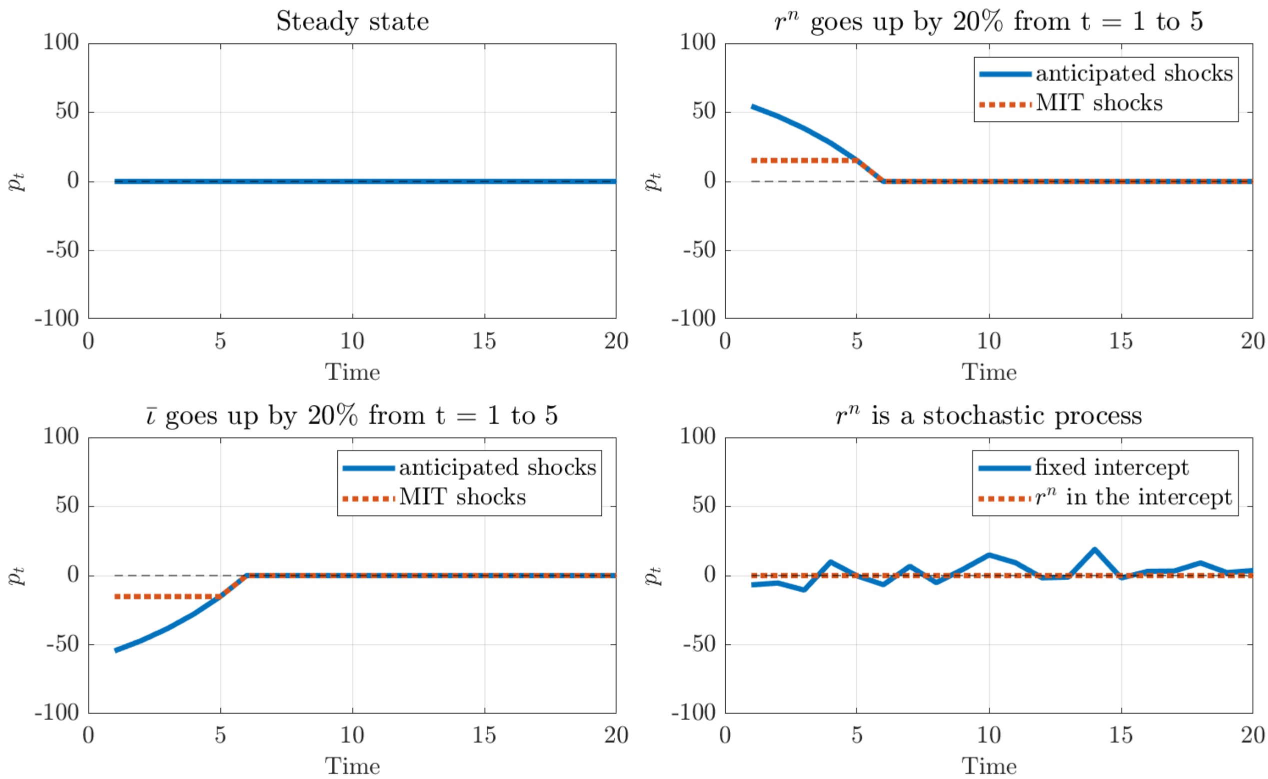


Figure 3: The price level under neo-Wicksellian monetary theory ($\phi = 0.2$)

4 Neo-Wicksellian monetary theory with risky policy assets

Proceeding with the partial equilibrium monetary model of Woodford (2003), we advance substituting the risk-free policy asset for a defaultable bond, so as to incorporate policy-asset risk into the model.

4.1 A defaultable bond

Duffie and Singleton (1999) model recursively the arbitrage-free value of a defaultable bond whose probability of default and recovery rate in case of default are given by processes independent of the value, V_t, of the defaultable claim itself. Under discrete time, they provide the following expression for the value of that defaultable bond[17]

$$V_t\left(1+i_t^{RF}\right)=\left(1-\mathbb{E}_t^Q\mathscr{D}_{t+1}\right)\mathbb{E}_t^Q\left(V_{t+1}\right)+\mathbb{E}_t^Q\mathscr{D}_{t+1}\omega_{t+1} \tag{7}$$

where i_t^{RF} is the risk-free net nominal interest rate at period t; $\mathbb{E}_t^Q\mathscr{D}_{t+1}$ is the conditional probability of default under a risk-neutral probability measure Q between periods t and $t+1$ given the information available at period t and that no default has happened until that same period; while ω_{t+1} is the recovery value in case of default measured in the same unit of account as V_t. Moreover, Duffie and Singleton (1999) prove that if we impose that the risk-neutral expected recovery rate $\mathbb{E}_t^Q\omega_{t+1}$ is given by $(1-L_t)\mathbb{E}_t^Q(V_{t+1})$, where L_t is an adapted process[18] bounded by 1, a case known as recovery of market value (RMV), we can price a default-risky nominal return i_{t+1}^R as

$$1=\left(1-\mathbb{E}_t^Q\mathscr{D}_{t+1}\right)\frac{1+i_{t+1}^R}{1+i_t^{RF}}+\mathbb{E}_t^Q\mathscr{D}_{t+1}\frac{1+i_{t+1}^R}{1+i_t^{RF}}\left(1-L_t\right) \tag{8}$$

where $i_{t+1}^R\approx i_t^{RF}+\mathbb{E}_t^Q\mathscr{D}_{t+1}L_t$. Furthermore, there is no restriction for $\mathbb{E}_t^Q\mathscr{D}_{t+1}$ or L_t to not depend on or to not be correlated with i_t^{RF}. Building the bridge between finance and macroeconomics, this fact means that both processes can depend on the state vector of an economic model.

[17] After adapting notation, rearranging terms, undoing an exponential approximation of the risk-free return, and imposing risk-neutral expectations on the recovery rate.

[18] A stochastic process X is adapted iif for every realization and every t, X_t is known at time t.

4.2 The policy asset as a defaultable bond

As it is often the case, the central bank of our model operates monetary policy through open market operations, by selling and buying federal government bonds, which we will assume are outright operations for simplicity.[19] These operations allow the central bank to target the liquidity and the interest rate of short-term financial markets[20], which, by their turn, will impact, through the *expectations hypothesis*, medium to long-term credit markets (not modeled here) on which the real economy relies on. This transmission mechanism is usually assumed to be risk-free at the beginning of the yield curve, since the central bank and federal government bonds are also usually assumed to enjoy a safe status. Nonetheless, governments have defaulted on domestic debt in the past, including domestic currency liabilities – as extensively listed by Reinhart and Rogoff (2009). If agents with rational expectations forecast a future scenario with probability greater than zero in which the asset used by the central bank to conduct monetary policy fails to reward i_t, then policy assets become risky.

Continuing with our model, to a world in which assumptions (1) to (5) hold, we add the no-arbitrage hypothesis for pricing the policy asset as a defaultable bond. From Cochrane (2009),

Assumption 7 (No-arbitrage hypothesis)**.** *Given a payoff space X and pricing function $p(x)$, every payoff x that is always nonnegative, $x \geq 0$ (almost surely), and positive, $x > 0$, with some positive probability, has positive price, $p(x) > 0$.*

We are, now, ready to depart from the risk-free assumption. Let us say policy assets are expected to remunerate at a deterministic gross rate $(1 + i_t)$ with *expected repayment probability* $(1 - \mathbb{E}_t \mathscr{D}_{t+1})$, and expected to remunerate at a non-deterministic gross rate $(1 + i_t)(1 - \mathbb{E}_t \delta_{t+1})$ with *expected default probability* $\mathbb{E}_t \mathscr{D}_{t+1}$. That way, the payoff is uncertain only in the case that the issuer of the policy asset defaults on the asset. This is closer to the actual situation faced by a real central bank that conducts monetary policy through open-market operations using government bonds. In that sense, the central bank can only set the nominal interest

[19] In the case of a central bank that conducts monetary policy paying an interest rate on reserves instead of outright transactions or repos, the default risk of the specific issuer is analogous. If not the federal government who can directly default on the debt, the central bank may hypothetically confiscate either part or the total of the reserves.

[20] As part of the new non-conventional monetary policy toolbox developed after the Great Recession, repos with longer maturities and even with foreign-currency assets have become more common.

paid by the policy asset in case of repayment by the issuer, having no control over its outcome in case of default, which will ultimately be determined by the fiscal authority. We define, next, the interest rate demanded in equilibrium for holding that asset.

Definition 4.1. *A net risky policy-asset interest rate, i_t^{Risky}, is a net nominal interest rate that in a market under no-arbitrage hypothesis satisfies the identity:*

$$\mathbb{E}_t^Q\left(1+i_{t+1}^{Risky}\right)=\left[\left(1-\mathbb{E}_t^Q\mathscr{D}_{t+1}\right)(1+i_t)+\mathbb{E}_t^Q\mathscr{D}_{t+1}(1+i_t)(1-\delta_{t+1})\right]=\left(1-\mathbb{E}_t^Q\mathscr{D}_{t+1}\delta_{t+1}\right)(1+i_t)$$

where i_t is the net risky policy-asset interest rate in case of non-default; $\mathscr{D}_{t+1}$ is the probability that the risky policy asset will default at maturity; and δ_{t+1} is the haircut in case of default.

Additionally, we have to restrain the policy default probability distribution to be independent of the current value of the bond. Note that we allow $\mathscr{D}_t$ and δ_t to be correlated, like in Duffie and Singleton (1999).

Assumption 8 (Policy default is exogenous to the price of the bond)**.** *The default probability, $\mathscr{D}_t$, and the haircut, δ_t, embedded in the risky policy-asset interest rate, i_t^{Risky}, follow distributions independent of the current value of the bond, while the latter is normalized to* 1.

As we work with a log-linearized model, we have no additional loss in focusing only in up to first-order effects of default. Therefore, we also assume that all agents are risk-neutral, so that risk-neutral probabilities coincide with objective ones, and risk-neutral returns are just mathematical expected returns.

Assumption 9 (Risk-neutral agents)**.** *at every period t agents are indifferent between choices with equal mathematical expected payoffs.*

4.3 Monetary policy with risk in the policy asset

We, now, redefine the Wicksellian policy rule so as to incorporate the fact that the policy asset is risky.

Assumption 10 (Wicksellian risky policy rule)**.** *there is a central bank that sets the nominal interest rate of a risky asset, i_t, through a Wicksellian rule of the type $i_t=\bar{\imath}_t+\phi\left(p_t-\overline{p}\right)$, where $\{\bar{\imath}_j\}_{j=t}^{\infty}$ is an exogenous process for a time-varying intercept determined independently of the evolution of prices which may or may not be correlated with $\left\{r_j^n\right\}_{j=t}^{\infty}$; $\overline{p}$ is the log of the price level target; and $\phi>0$.*

Taking assumptions 1 to 5 and 7 to 10 as valid, the expected net return of the policy asset in line with Definition 4.1 is

$$\mathbb{E}_t\, i_{t+1}^{\text{Risky}} = (1 - \mathbb{E}_t \mathscr{D}_{t+1}\delta_{t+1})\, i_t - \mathbb{E}_t \mathscr{D}_{t+1}\delta_{t+1} \tag{9}$$

Next, relying on Assumption 7 of no arbitrage opportunities, nominal returns must equalize in expectations

$$i_t^{RF} = \mathbb{E}_t\, i_{t+1}^{\text{Risky}} \tag{10}$$

where, as before, i_t^{RF} is the risk-free net nominal interest rate.

Substituting the policy rule of Assumption 10 into the policy asset expected return (9) and then through the no-arbitrage condition (10) into the Fisher equation (3), we obtain the *equilibrium price level in the risky policy asset economy*

$$\begin{aligned} p_t = \sum_{j=0}^{\infty} \Upsilon_{t,j+1} \mathbb{E}_t \Big(r_{t+j}^n - \big(1 - \mathbb{E}_t \mathscr{D}_{t+j+1}\delta_{t+j+1}\big)\, \bar{\imath}_{t+j} + \mathbb{E}_t \mathscr{D}_{t+j+1}\delta_{t+j+1} \Big) \\ \Upsilon_{t,j+1} \equiv \Pi_{k=1}^{j+1} \left(\frac{1}{1 + (1 - \mathbb{E}_t \mathscr{D}_{t+k}\delta_{t+k})\,\phi} \right) \quad \forall j \geq 0, \forall t \end{aligned} \tag{11}$$

For p_t to exist and be unique, the process $\{r_t^n, \mathscr{D}_t, \delta_t, \bar{\imath}_t\}$ must be bounded and more assumptions must be made about $\{\mathscr{D}_t\}$, $\{\delta_t\}$, and ϕ. Imposing further that $\phi > 0$, like in the canonical case, and that for all t there is at least one infinite sequence $k_n \subset [1, \infty)$ such that $0 \leq \mathbb{E}_t \mathscr{D}_{t+k}\delta_{t+k} < 1$ in that sequence is sufficient for assuring determinacy since we have already assumed that the object $\{\mathscr{D}_t\}$ represents a probability and $\{\delta_t\}$ a fraction, which implies that $0 \leq \{\mathscr{D}_t\delta_t\} \leq 1$ for all t. These conditions assure for all t that $0 < \frac{1}{1+(1-\mathbb{E}_t \mathscr{D}_{t+k}\delta_{t+k})\phi} < 1$ for at least one infinite sequence $k_n \subset [1, \infty)$. Overall, determinacy is guaranteed under rather mild restrictions. For one, when full (not partial) default is expected at the next period with certainty at all periods, the policy-asset market simply collapses. In that extreme case, $\mathbb{E}_t \mathscr{D}_{t+k}\delta_{t+k} = 1$ for all t and all k, such that we have an explosive process. For two, assuming that all sequences of $k_n \subset [1, \infty)$ such that $0 \leq \mathbb{E}_t \mathscr{D}_{t+k}\delta_{t+k} < 1$ are finite imposes that after a certain $k = \tilde{k}$ the risky policy asset becomes strictly a full confiscation *ad eternum* by the government, what is quite unrealistic. For three, such sufficient requirements for determinacy still allow for full default to be expected for certain at any amount of periods into the future. Finally, note that under the implicit assumption that ϕ is time-invariant these are not only sufficient but also necessary conditions. Allowing ϕ to vary in time would allow for equilibrium determinacy even with $\phi < 0$ at some periods.[21]

[21] Davig and Leeper (2007) study the case in which the value of ϕ switches according to different mone-

Next, we give a formal proof of determinacy conditions for the price level targeting case. A proof for when the central bank adopts inflation targeting is available in Proposition 5.1.

Proposition 4.2 (Price level determinacy under price-level targeting)**.** *A bounded process* $\{r_t^n, \mathscr{D}_t, \delta_t, \bar{\imath}_t\}$*, a monetary policy rule such as the one proposed in Assumption 10 with* $\phi > 0$*, the fact that* $\mathscr{D}_t$ *is a process that represents a probability and* δ_t *is a process that represents a fraction, and the condition that for all t there is at least one infinite sequence* $k_n \subset [1,\infty)$ *such that* $0 \leq \mathbb{E}_t \mathscr{D}_{t+k} \delta_{t+k} < 1$ *in that sequence are necessary and sufficient conditions so that the price level equilibrium exists and is unique.*

Proof. Available in Appendix B. □

Now, we prove which intercept for the Wicksellian rule is able to stabilize the price level at all times. The equivalent proof for a central bank that targets inflation is developed in Proposition 5.2.

Proposition 4.3 (Optimal intercept under price-level targeting)**.** *In an economy in which the marginal investor is risk-neutral, monetary policy can stabilize prices through operations with the risky asset if it can track the sequence of natural interest rates* $\left\{r_{t+j}^n\right\}_{j=0}^{\infty}$*, the sequence of one-period-ahead policy-asset default probabilities* $\{\mathscr{D}_{t+j+1}\}_{j=0}^{\infty}$*, the sequence of one-period-ahead expected haircuts* $\{\delta_{t+j+1}\}_{j=0}^{\infty}$*, and it credibly adopts at period t a Wickellian rule with the sequence of time-varying intercepts* $\{\bar{\imath}_{t+j}\}_{j=0}^{\infty} \equiv \left\{\frac{r_{t+j}^n + \mathbb{E}_t \mathscr{D}_{t+j+1}\delta_{t+j+1}}{1-\mathbb{E}_t \mathscr{D}_{t+j+1}\delta_{t+j+1}}\right\}_{j=0}^{\infty}$.

Proof. Available in Appendix B. □

From the equilibrium price level equation (11) and Figures 4 ($\phi = 0.1$) and 5 ($\phi = 0.2$), one can see that default expectation affects prices in four ways:

- First, it directly changes the level of prices: $\Upsilon_{j+1} > \alpha^{j+1} \quad \forall j \geq 0$, the higher the default probability, the higher the price level (top-left panel of Figure 4).
- Second, it reduces the power of monetary policy w.r.t. prices (middle-right panel of Figure 4):

tary regimes, and also finds that the Taylor principle does not have to be attended at every regime for a New-Keynesian model to be determined. Schabert (2010) finds that an interest rate policy rule makes prices indeterminate when the government can default on the debt, but that is only because he does not have an additional equation for default probability to close the model. Here, we assume it is exogenous.

$\frac{(1-\mathbb{E}_t \mathscr{D}_{t+j+1}\delta_{t+j+1})}{\Pi_{k=1}^{j+1}[1+(1-\mathbb{E}_t \mathscr{D}_{t+k}\delta_{t+k})\phi]}\bar{\iota}_{t+j} < \frac{1}{\Pi_{k=1}^{j+1}[1+\phi]}\bar{\iota}_{t+j} \quad \forall \mathbb{E}_t \mathscr{D}_{t+1}\delta_{t+1} > 0$. In Appendix E, we simulate the model for different correlations between r_t^n and $\mathbb{E}_t \mathscr{D}_{t+1}$, and conclude that the power reduction in monetary policy is robust even when considered the whole time-sequence of expected default probabilities. One way of seeing the inflationary bias that policy-asset risk introduces is to compare the multiplier of r_{t+j}^n, $\Upsilon_{t,j+1}$, with the multiplier of $\bar{\iota}_{t+j}$, $\Upsilon_{t,j+1}\left(1-\mathbb{E}_t \mathscr{D}_{t+j+1}\delta_{t+j+1}\right)$, and note that the first one is larger for any positive $\mathbb{E}_t \mathscr{D}_{t+j+1}\delta_{t+j+1}$.

- Third, the expected haircut term amplifies the reduction of power of monetary policy, as the higher $\mathbb{E}_t \mathscr{D}_{t+j+1}\delta_{t+j+1}$ the higher is the price level in equilibrium (bottom-left panel at Figure 4). Nevertheless, its effect is numerically dominated in reasonable calibrations, where $\left(1-\mathbb{E}_t \mathscr{D}_{t+j+1}\delta_{t+j+1}\right) \gg \mathbb{E}_t \mathscr{D}_{t+j+1}\delta_{t+j+1}$.

- Fourth, monetary policy is no longer able to stabilize prices at all periods by just tracking $\{r_t^n\}$; it has to track a function of the natural interest rate, the default probability, and the haircut (Proposition 4.3). Despite that, note that even though tracking $\{r_t^n\}$ does not fully stabilize prices, it does reduce the volatility of prices under the assumption of independence between $\left\{r_t^n\right\}$ and $\{\mathscr{D}_t\}$ (bottom-right panel of Figure 4).

- Finally, by comparing the panels of Figure 4) ($\phi = 0.08$) with their equivalents in Figure 5 ($\phi = 0.15$), one can see the consequence of increasing ϕ. Considering only the trajectory of the price level, the effects of conducting monetary policy with defaultable bonds can be consistently attenuated with a more hawkish stance.

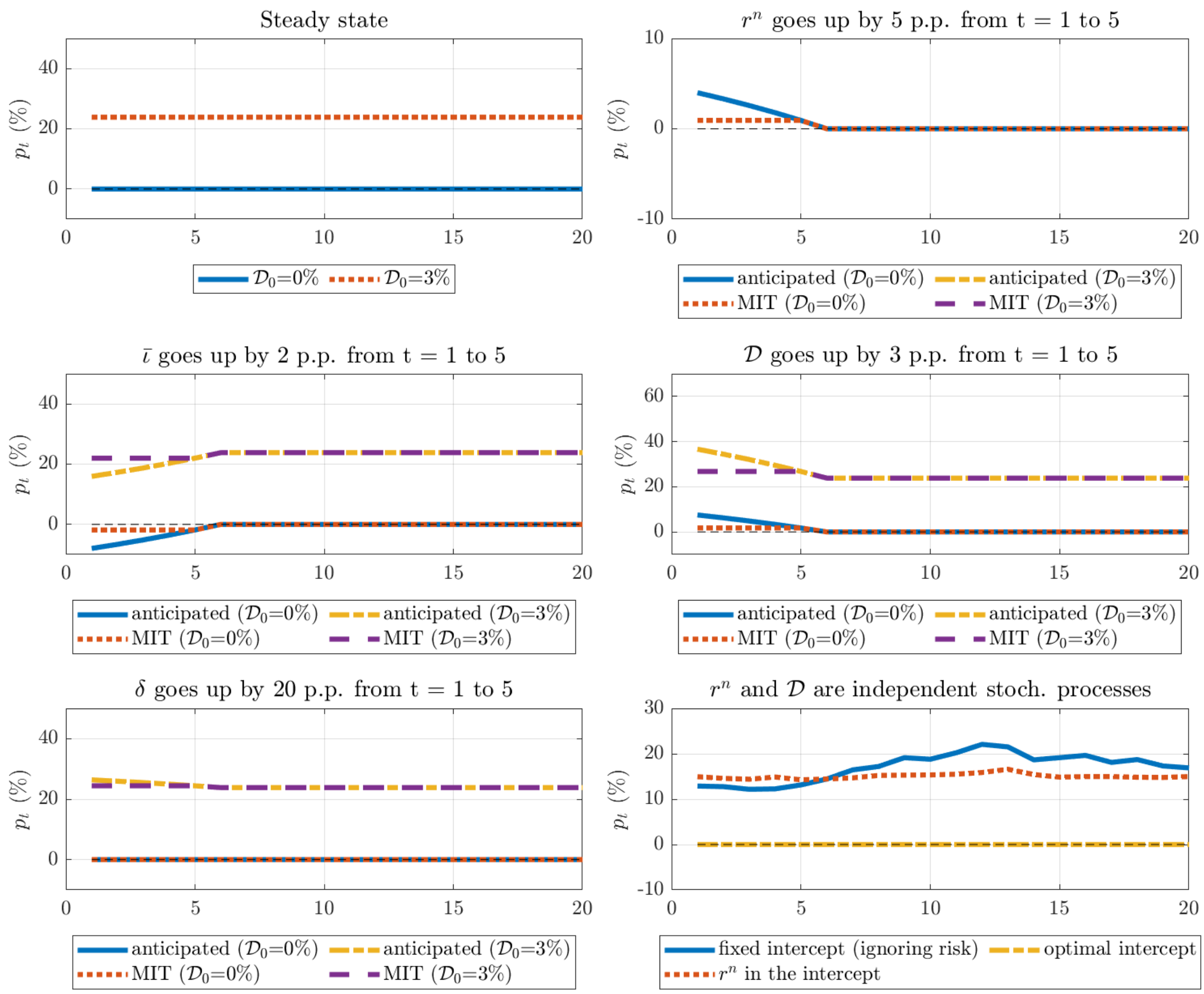


Figure 4: The price level under neo-Wicksellian monetary theory augmented with policy-asset risk ($\phi = 0.08$)

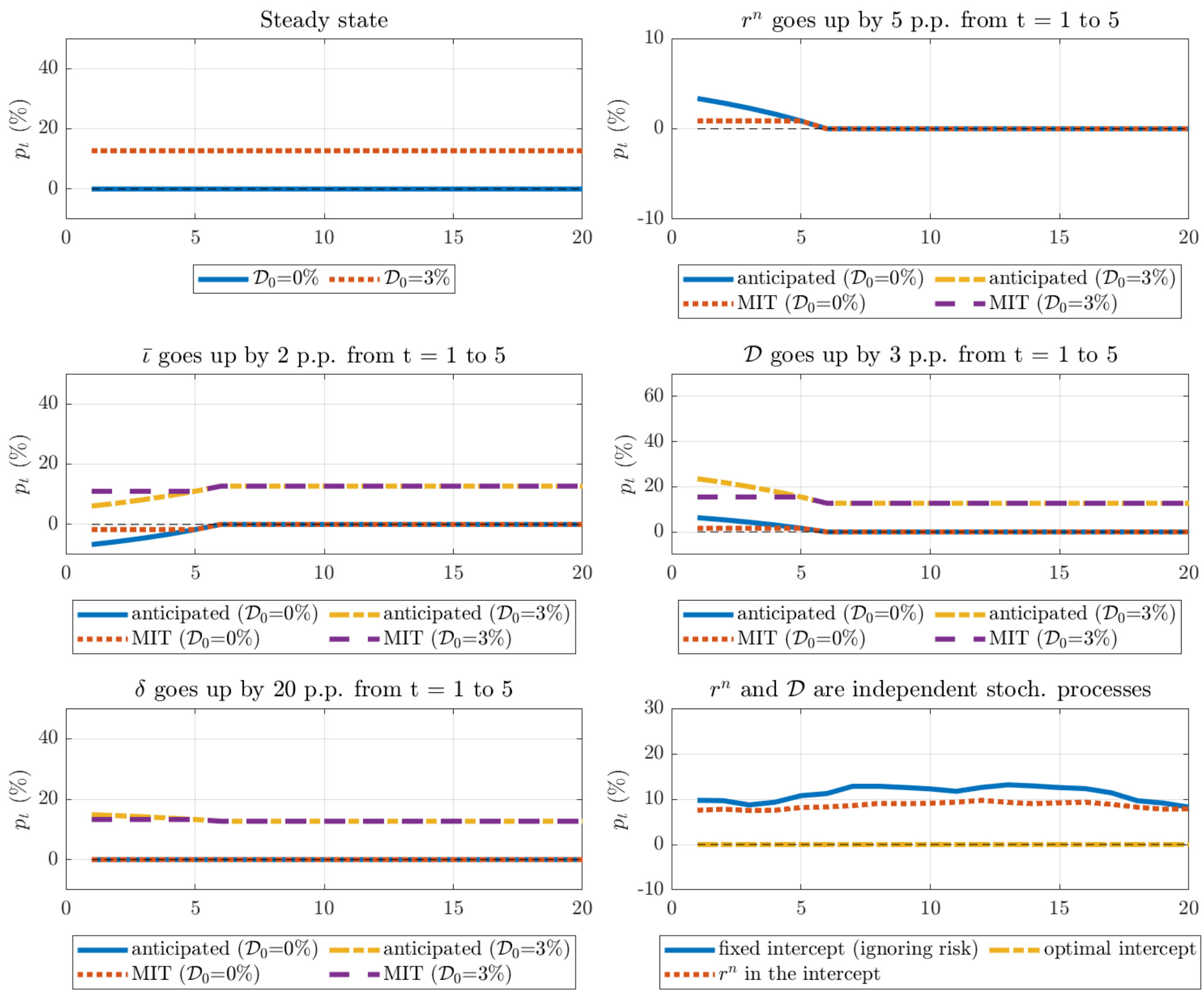


Figure 5: The price level under neo-Wicksellian monetary theory augmented with policy-asset risk ($\phi = 0.15$)

The *intuition* for why the risk level of the asset used by monetary policy matters is straightforward after we have built this model. Starting from the end of period t, let us say that a continuum of states can represent all possible states of nature that can happen at period $t+1$. Only in those future states where there is no default of the policy asset is that the central bank can affect the ex-post return obtained by an investor who holds the policy asset. In all other states, in which the policy asset defaults, the return obtained with the policy asset is entirely independent of the action of the central bank, and, therefore, monetary policy must be weaker compared to a scenario where the policy asset has no risk. The firepower of a traditional model central bank relies on its capacity of setting a nominal return of reference in all future states. After introducing risk in the policy asset, the latter is no longer true.

Rationalizing in terms of an active vs. passive monetary policy à la Leeper (1991), we have that when the policy asset carries risk of default, monetary policy is only active conditional to the repayment of the debt by the issuer, and, thus, default risk makes it "less active".

As far as we know, the reduction of the power of monetary policy w.r.t. prices and inflation in case of a surge in the perceived risk of the policy asset, such as what happens with risky debt during political or fiscal crises, is a new argument in favor of raising interest rates during such episodes in the literature. An obvious caveat, though, is that so far we have worked with only a partial equilibrium model. The interaction of interest rates with the trajectory of the debt is a fundamental mechanism that can revert this conclusion. In Appendix C, we derive a simple adaptation of the somewhat larger cashless flexible-price model of Woodford (2003, ch. 2, sec. 1) under the assumption of a balanced-budget fiscal rule and show how our partial equilibrium default-risky cashless model dialogues well with the microfoundations of canonical neo-Wicksellian models. When fiscal policy is passive à la Leeper (1991), as we implicitly assume here, this is a result that we believe may survive in more realistic set-ups.

4.4 The high-price-level bias

The high-price-level bias inherent to conducting monetary policy with risky assets partially disconnects the unconditional mean of the price level from the central bank's target. We show this, next, by recovering the recursive formulation of (11) and taking the unconditional mean of both sides of that equation. Up to first order, these are the results.

$$\begin{aligned} \mathbb{E}\,p = \Upsilon\left(\mathbb{E}\,r^n - (1-\mathbb{E}\mathscr{D}\delta)\,\mathbb{E}\bar{\imath} + \mathbb{E}\mathscr{D}\delta + \mathbb{E}\,p\right) \quad \text{where} \quad \Upsilon \equiv \left(\frac{1}{1+(1-\mathbb{E}\mathscr{D}\delta)\phi}\right) \\ \mathbb{E}\,p = \frac{\Upsilon}{1-\Upsilon}\left(\mathbb{E}\,r^n - (1-\mathbb{E}\mathscr{D}\delta)\,\mathbb{E}\bar{\imath} + \mathbb{E}\mathscr{D}\delta\right) \\ \mathbb{E}\,p = \frac{1}{(1-\mathbb{E}\mathscr{D}\delta)\,\phi}\left(\mathbb{E}\,r^n - (1-\mathbb{E}\mathscr{D}\delta)\,\mathbb{E}\bar{\imath} + \mathbb{E}\mathscr{D}\delta\right) \end{aligned} \tag{12}$$

To make explicit the bias, we assume that the intercept of the policy rule is the same of the canonical model, that is $\mathbb{E}\bar{\imath} = \mathbb{E}\,r^n$, and we obtain

$$\mathbb{E}\,p = \underbrace{\frac{\mathbb{E}\mathscr{D}\delta}{(1-\mathbb{E}\mathscr{D}\delta)\,\phi}\left(1+\mathbb{E}\,r^n\right)}_{\text{price-level bias}} \tag{13}$$

which will collapse to the price level target ($\overline{p} = 0$) only when $\mathbb{E}\mathscr{D}\delta = 0$, that is, only when the policy asset is non-defaultable, which brings us back to the canonical case. As, by definition, $r^n > -1$, the price-level bias is positive for any positive default probability lower than 1. Trivially, if the unconditional mean is full confiscation ($\mathbb{E}\mathscr{D}\delta = 1$), then prices are not determined. Finally, note that the bias reduces with the size of ϕ, suggesting that more active monetary policy can mitigate it. The bias also increases with the default probability and reduces with the recovery rate.

In Section 5, we show that, under inflation targeting, conducting monetary policy with risky assets results in a non-accelerating inflationary bias, which can be sizable if the central bank opts for being only slightly active. This last result could explain a certain preference of risky-economy central banks for higher inflation-reaction coefficients in their policy rules, as such a parametric range is more likely to successfully meet inflation targets even with less than optimal intercepts. Alternatively, these central banks could opt to raise the intercept itself. Both monetary policy strategies constitute what some may call "conservatism" of monetary policy, but in reality they may be required to deliver inflation on the target.

So, how large is the price-level bias? Under the near-optimal intercept rule $\left(\mathbb{E}\bar{\imath} = \mathbb{E}\, r^n\right)$, Tables 1 and 2 calculate that variable in percentage points for some combinations of $\mathbb{E}\mathscr{D}\delta$ and ϕ, arbitrarily assuming $\mathbb{E}\, r^n = 4\%$, for $\delta = 0.05$ and $\delta = 0.6$, respectively. For policy rules with low values of ϕ, the bias is not neglectful at all, while for rules with relatively high values of ϕ it is mostly attenuated. We leave further considerations for when we analyze the inflation-targeting case.

Table 1: Price-level bias (p.p.) with $\delta = 0.05$

	$\phi = 0.2$	$\phi = 0.5$	$\phi = 1.0$	$\phi = 1.5$	$\phi = 2.0$
$\mathscr{D} = 0.0\%$	0.0	0.0	0.0	0.0	0.0
$\mathscr{D} = 2.5\%$	0.7	0.3	0.1	0.1	0.1
$\mathscr{D} = 5.0\%$	1.3	0.5	0.3	0.2	0.1
$\mathscr{D} = 7.5\%$	2.0	0.8	0.4	0.3	0.2
$\mathscr{D} = 10.0\%$	2.6	1.1	0.5	0.3	0.3

Table 2: Price-level bias (p.p.) with $\delta = 0.60$

	$\phi = 0.2$	$\phi = 0.5$	$\phi = 1.0$	$\phi = 1.5$	$\phi = 2.0$
$\mathcal{D} = 0.0\%$	0.0	0.0	0.0	0.0	0.0
$\mathcal{D} = 2.5\%$	7.9	3.2	1.6	1.1	0.8
$\mathcal{D} = 5.0\%$	16.1	6.4	3.2	2.1	1.6
$\mathcal{D} = 7.5\%$	24.5	9.8	4.9	3.3	2.5
$\mathcal{D} = 10.0\%$	33.2	13.3	6.6	4.4	3.3

5 Model derivation under an inflation target rule

The same model we derived in Section 4 under Assumption 6 of a price-level target (or Wicksellian) rule can easily be derived under an inflation-target (or Taylor) rule. We start from Assumptions 1 to 5, and expose the Fisher equation in its log-linear form:

$$\mathbb{E}_t \pi_{t+1} = i_t - r_t^n \tag{14}$$

where, exactly as before, $\mathbb{E}_t$ is the mathematical expectation operator conditional on all information available at time t, π_t is the log of the gross inflation at period t, r_t^n is the equilibrium real rate of interest (and the natural one) at period t, while i_t is the short-term nominal interest rate at period t. We can interpret this equation as an *equilibrium condition* instead of as an *identity* for the same reasons exposed for the price-level target derivation. In other words, at every instant t, the inflation that clears the goods market (and equates saving and investment) is the one given by equation 14.

5.1 Neo-Wicksellian model with inflation targeting

Now, assume the central bank sets i_t for a risk-free asset such that π_t and i_t should move in the same direction and targeted inflation is zero.

Assumption 11 (Taylor risk-free policy rule)**.** *there is a central bank that sets the nominal interest rate of a risk-free asset, i_t, through a Taylor rule of the type $i_t = \bar{\imath}_t + \phi^\pi \left(\pi_t - \overline{\pi}\right)$, where $\left\{\bar{\imath}_j\right\}_{j=t}^{\infty}$ is an exogenous process for a time-varying intercept determined independently of the evolution of prices which may or may not be correlated with $\left\{r_j^n\right\}_{j=t}^{\infty}$; $\overline{\pi}$ is the log of the gross inflation target; and $\phi^\pi > 1$.*

Without loss of generality, hereafter, we set $\overline{\pi} = 0$. Inserting Assumption 11 into the Fisher relation (14) to eliminate i_t, we obtain the law of movement of the equilibrium inflation:

$$\pi_t = \alpha^{\pi}\mathbb{E}_t\pi_{t+1} + \alpha^{\pi}\left(r_t^n - \bar{\imath}_t\right) \tag{15}$$

where $\alpha^{\pi} \equiv \frac{1}{\phi^{\pi}}$ is a coefficient that satisfies $0 < \alpha^{\pi} < 1$. If the process $\{r_t^n, \bar{\imath}_t\}$ is bounded then π_t is unique and bounded[22], obtained by iterating forward equation 15, which gives us

$$\pi_t = \sum_{j=0}^{\infty} \alpha^{\pi\, j+1}\mathbb{E}_t\left(r_{t+j}^n - \bar{\imath}_{t+j}\right) \tag{16}$$

As a consequence, the equilibrium inflation will fluctuate in a bounded interval around its long-run average value:

$$\overline{\pi} \equiv \frac{\overline{r^n} - \bar{\imath}}{\phi^{\pi} - 1} \tag{17}$$

where $\overline{\pi}$, $\overline{r^n}$, and $\bar{\imath}$ are the long-run average of the net inflation, of the natural interest rate and of the time-varying intercept, respectively.

Note in Figure 6 ($\phi^{\pi} = 1.1$) and Figure 7 ($\phi^{\pi} = 1.2$) that inflation exhibits the same behavior as the price level did under the price level targeting rule in Figure 2 ($\phi = 0.1$) and Figure 3 ($\phi = 0.2$).

[22] Proof follows the same procedure as the one available in Woodford (2003)'s Appendix A.4. for the price level target case.

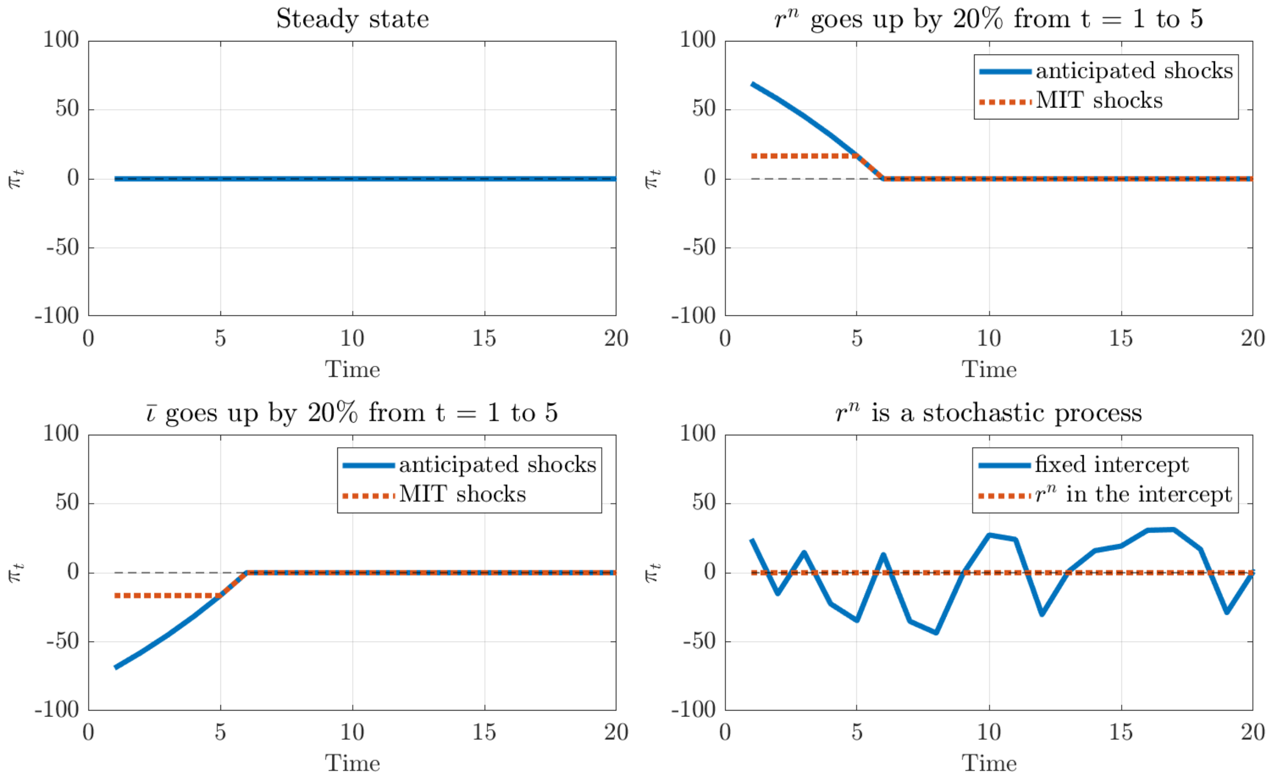


Figure 6: Inflation under neo-Wicksellian monetary theory ($\phi^\pi = 1.1$)

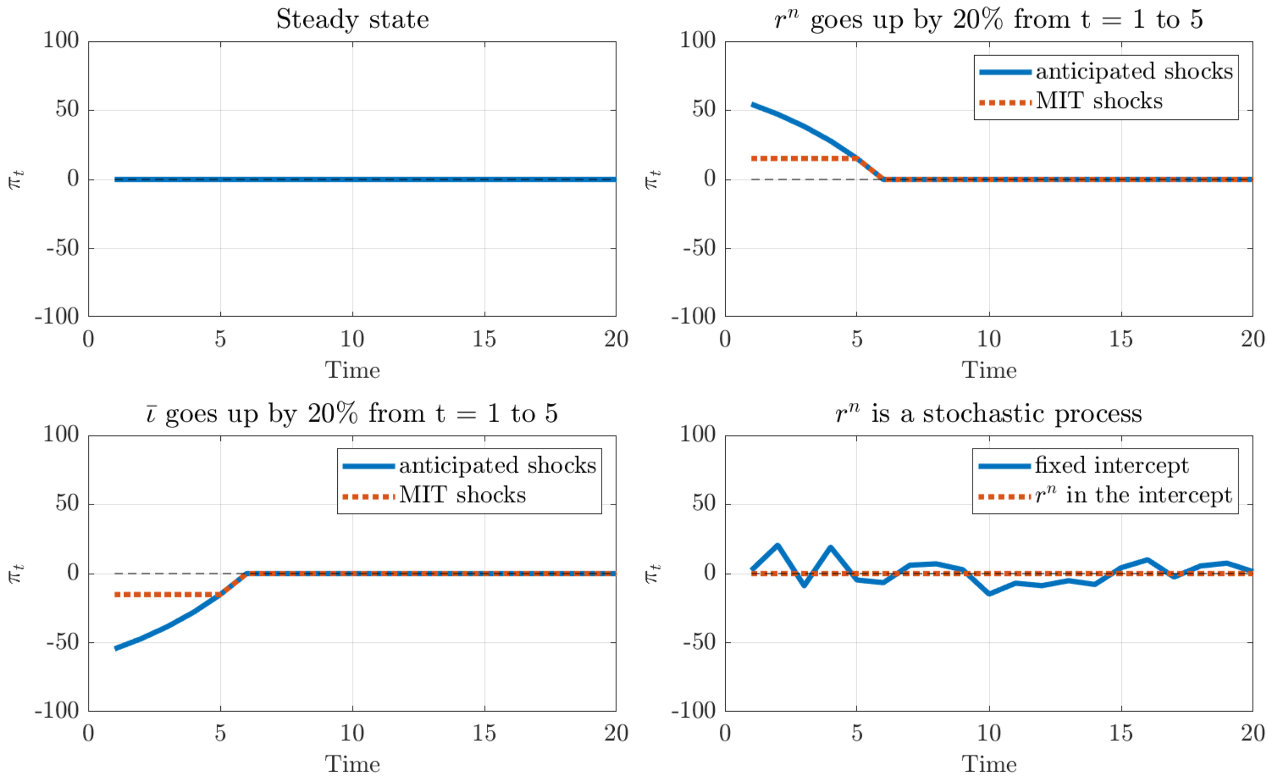


Figure 7: Inflation under neo-Wicksellian monetary theory ($\phi^\pi = 1.2$)

5.2 Monetary policy with risk in the policy asset under inflation targeting

We derive now the partial equilibrium monetary model with risk in the policy asset under an inflation target rule, by building on previous assumptions 1 to 5, as well as 7 (no arbitrage), 8 (policy default is exogenous to the price of the bond), and 9 (agents are risk-neutral). We re-define the Taylor rule of Assumption 11 to reflect the risk in the policy asset with Assumption 12.

Assumption 12 (Taylor risky policy rule)**.** *there is a central bank that sets the nominal interest rate of a risky asset, i_t, through a Taylor rule of the type $i_t = \bar{\imath}_t + \phi^\pi (\pi_t - \overline{\pi})$, where $\{\bar{\imath}_j\}_{j=t}^{\infty}$ is an exogenous process for a time-varying intercept determined independently of the evolution of prices which may or may not be correlated with $\left\{r_j^n\right\}_{j=t}^{\infty}$; $\overline{\pi}$ is the log of the gross inflation target; and $\phi^\pi > \frac{1}{1-\mathbb{E}_t\mathscr{D}_{t+1}\delta_{t+1}}$ for all t.*

Again, we normalize the net inflation target to $\overline{\pi} = 0$. Having set the independence of the policy asset default probability, then:[23]

$$\mathbb{E}_t\, i_{t+1}^{\text{Risky}} = (1 - \mathbb{E}_t \mathscr{D}_{t+1}\delta_{t+1})\, i_t - \mathbb{E}_t \mathscr{D}_{t+1}\delta_{t+1} \tag{9}$$

Next, relying on Assumption 7 of no arbitrage opportunities, nominal returns must equalize in expectations

$$i_t^{RF} = \mathbb{E}_t\, i_{t+1}^{\text{Risky}} \tag{10}$$

where, as before, i_t^{RF} is the risk-free net nominal interest rate.

Substituting the policy rule of Assumption 12 into the policy asset expected return (9) and then through the no-arbitrage condition (10) into the Fisher equation (14), we obtain the *equilibrium inflation in the risky-policy-asset economy*

$$\begin{gathered}\pi_t = \sum_{j=0}^{\infty} \Upsilon_{j+1}^{\pi} \mathbb{E}_t \left(r_{t+j}^n - \left(1 - \mathbb{E}_t \mathscr{D}_{t+j+1}\delta_{t+j+1}\right) \bar{\imath}_{t+j} + \mathbb{E}_t \mathscr{D}_{t+j+1}\delta_{t+j+1} \right) \\ \Upsilon_{j+1}^{\pi} \equiv \Pi_{k=1}^{j+1} \left(\frac{1}{(1 - \mathbb{E}_t \mathscr{D}_{t+k}\delta_{t+k})\, \phi^\pi} \right) \quad \forall j \geq 0\end{gathered} \tag{18}$$

As we did in Proposition 4.2, now we prove necessary and sufficient conditions for inflation to be determined.

Proposition 5.1 (Inflation determinacy under inflation targeting)**.** *A bounded process $\{r_t^n, \mathscr{D}_t, \delta_t, \bar{\imath}_t\}$, a monetary policy rule such as the one proposed in Assumption 12 with $\phi^\pi > \frac{1}{1-\mathbb{E}_t\mathscr{D}_{t+1}\delta_{t+1}}$*

[23] Equations 9 and 10 are reproduced here for the help of the reader.

for all t, the fact that $\mathscr{D}_t$ is a process that represents a probability and δ_t is a process that represents a fraction, and the condition that for all t there is at least one infinite sequence $k_n \subset [1,\infty)$ such that $0 \leq \mathbb{E}_t \mathscr{D}_{t+k} \delta_{t+k} < 1$ in that sequence are necessary and sufficient conditions so that equilibrium inflation exists and it is unique.

Proof. Available in Appendix B. □

Now, we show what intercept for the Taylor Rule is able to stabilize inflation at all periods, like we did in Proposition 4.3 for the price level when the latter is targeted.

Proposition 5.2 (Optimal intercept under inflation targeting)**.** *In an economy in which the marginal investor is risk-neutral, monetary policy is able to stabilize inflation through operations with the risky asset if it can track the sequence of natural interest rates $\left\{r^n_{t+j}\right\}_{j=0}^{\infty}$, the sequence of one-period-ahead policy-default probabilities $\{\mathscr{D}_{t+j+1}\}_{j=0}^{\infty}$, the sequence of one-period-ahead expected haircuts $\{\delta_{t+j+1}\}_{j=0}^{\infty}$, and it credibly adopts at period t a Taylor rule with the sequence of time-varying intercepts $\{\bar{\imath}_{t+j}\}_{j=0}^{\infty} \equiv \left\{\frac{r^n_{t+j}+\mathbb{E}_t \mathscr{D}_{t+j+1}\delta_{t+j+1}}{1-\mathbb{E}_t \mathscr{D}_{t+j+1}\delta_{t+j+1}}\right\}_{j=0}^{\infty}$.*

Proof. Available in Appendix B. □

Figure 8 ($\phi^{\pi} = 1.08$) and Figure 9 ($\phi^{\pi} = 1.15$) show the dynamics of the model under the specific shock sequences described in each panel of the figures.

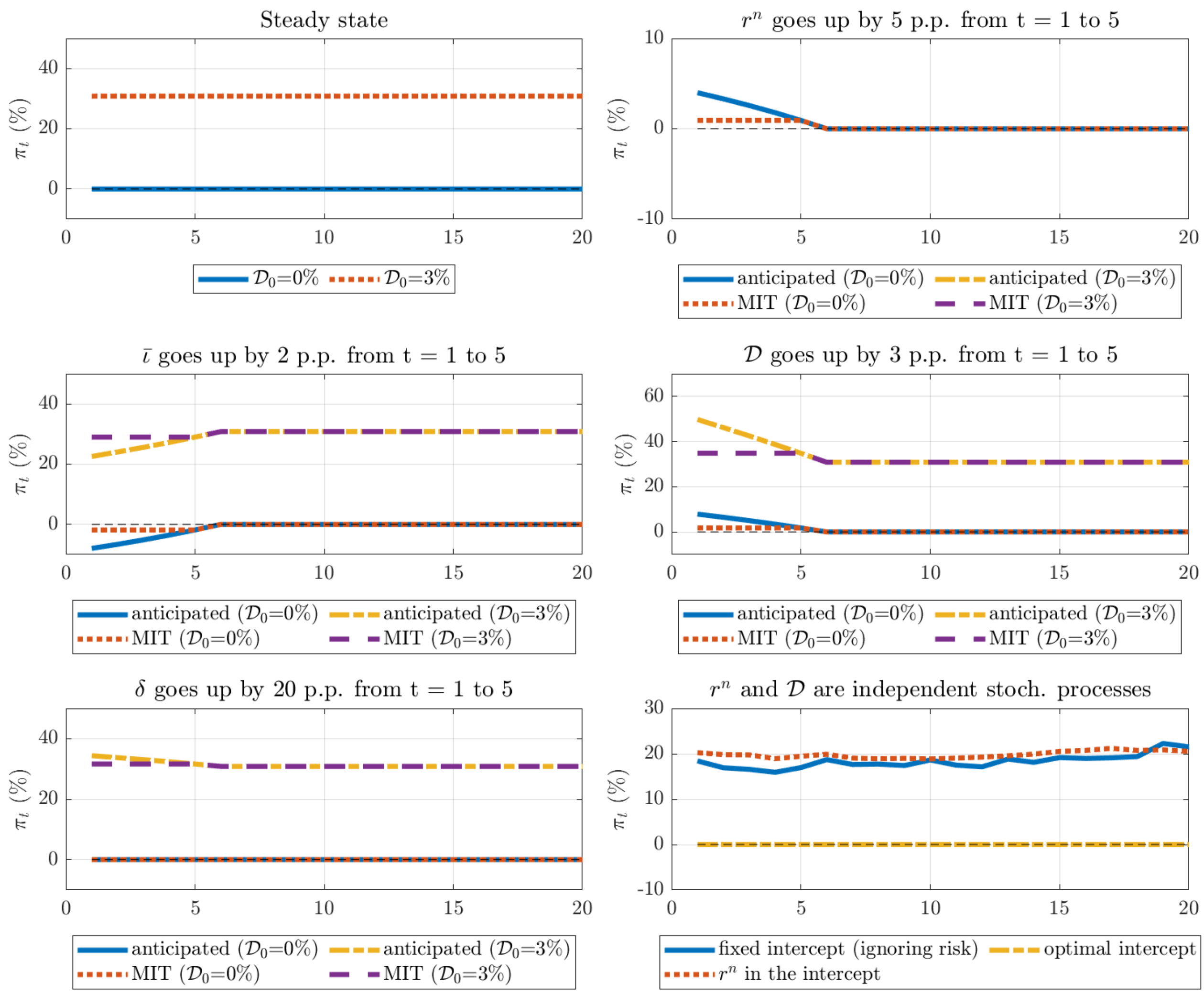


Figure 8: Inflation under neo-Wicksellian monetary theory augmented with policy-asset default (ϕ^{π} = 1.08)

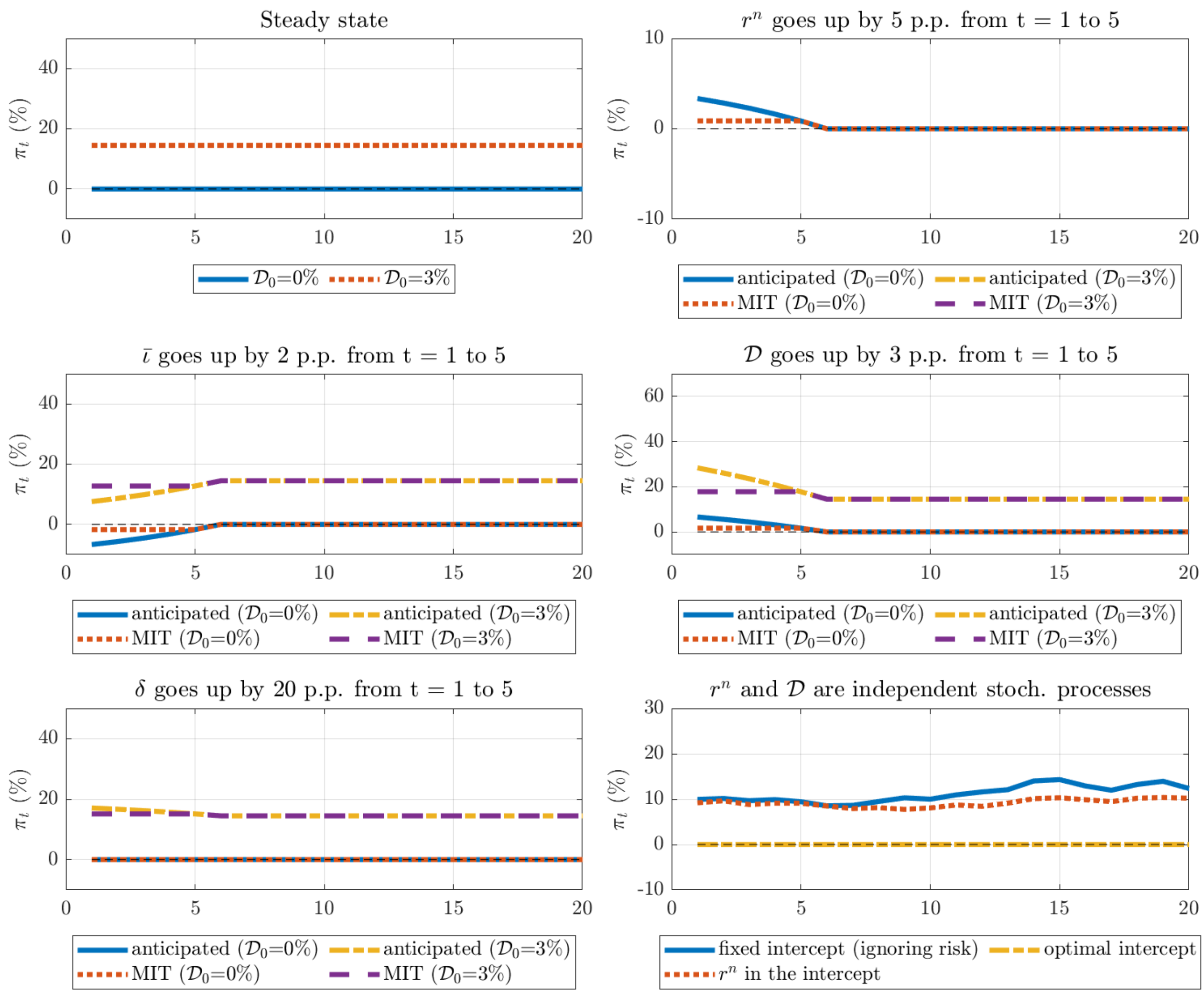


Figure 9: Inflation under neo-Wicksellian monetary theory augmented with policy-asset default ($\phi^\pi = 1.15$)

5.3 The inflation bias

Conducting monetary policy with risky assets under inflation targeting partially disconnects the unconditional mean of inflation from the central bank's target. We show this, next, by recovering the recursive formulation of (18) and taking the unconditional mean of both sides of that equation. Up to first order, these are the results.

$$\begin{aligned}
\mathbb{E}\pi = \Upsilon^\pi \left(\mathbb{E}r^n - (1-\mathbb{E}\mathscr{D}\delta)\,\mathbb{E}\bar{\imath} + \mathbb{E}\mathscr{D}\delta + \mathbb{E}\pi\right) \quad \text{where} \quad \Upsilon^\pi \equiv \left(\frac{1}{(1-\mathbb{E}\mathscr{D}\delta)\,\phi^\pi}\right) \\
\mathbb{E}\pi = \frac{\Upsilon^\pi}{1-\Upsilon^\pi}\left(\mathbb{E}r^n - (1-\mathbb{E}\mathscr{D}\delta)\,\mathbb{E}\bar{\imath} + \mathbb{E}\mathscr{D}\delta\right) \qquad (19) \\
\mathbb{E}\pi = \frac{1}{(1-\mathbb{E}\mathscr{D}\delta)\,\phi^\pi - 1}\left(\mathbb{E}r^n - (1-\mathbb{E}\mathscr{D}\delta)\,\mathbb{E}\bar{\imath} + \mathbb{E}\mathscr{D}\delta\right)
\end{aligned}$$

To make explicit the inflation bias, we assume that the intercept of the policy rule is the same of the canonical model, that is $\mathbb{E}\bar{\imath} = \mathbb{E}\, r^n$, and we obtain

$$\mathbb{E}\pi = \underbrace{\frac{\mathbb{E}\mathscr{D}\delta}{(1-\mathbb{E}\mathscr{D}\delta)\,\phi^{\pi}-1}\left(1+\mathbb{E}\, r^n\right)}_{\text{inflation bias}} \tag{20}$$

which will collapse to the inflation target ($\overline{\pi} = 0$) only when $\mathbb{E}\mathscr{D}\delta = 0$, that is, only when the policy asset is non-defaultable, showing that the canonical case is nested here. As, by definition, $r^n > -1$, the inflation bias is positive for any positive default probability lower than 1 as long as $\phi^{\pi} > \frac{1}{1-\mathbb{E}\mathscr{D}\delta}$. In case monetary policy is less active to the point that $\phi^{\pi} < \frac{1}{1-\mathbb{E}\mathscr{D}\delta}$, the inflation bias is negative, while when the expression is evaluated with equality the bias is indeterminate, just like inflation itself. If full confiscation is all to be expected ($\mathbb{E}\mathscr{D}\delta = 1$), inflation is not determined as it requires $\phi^{\pi} > +\infty$. Finally, note that the bias reduces with the size of ϕ^{π}, suggesting that more active monetary policy can mitigate it. The bias also increases with the default probability and reduces with the recovery rate. It is important to note that as long as inflation is determined the bias tilts inflation upward, but it does not make it accelerate.

So, how large is the inflation bias? Under the near-optimal intercept rule $\left(\mathbb{E}\bar{\imath} = \mathbb{E}\, r^n\right)$, Tables 3 and 4 calculate that variable in percentage points for some combinations of $\mathbb{E}\mathscr{D}\delta$ and ϕ^{π}, arbitrarily assuming $\mathbb{E}\, r^n = 4\%$, for $\delta = 0.05$ and $\delta = 0.6$, respectively. For policy rules with low values of ϕ^{π}, the bias is not neglectful at all, while for rules with relatively high values of ϕ^{π} it is mostly attenuated. Our interpretation of these results is that central banks of risky economies, when taking the temperature of the room in the monetary market, may end up heuristically picking high values of ϕ^{π} as these values are more likely to bring inflation to the target more often even when the calibration of the policy rule's intercept is not quite accurate.

Table 3: Inflation bias (p.p.) with $\delta = 0.05$

	$\phi^{\pi} = 1.2$	$\phi^{\pi} = 1.5$	$\phi^{\pi} = 2.0$	$\phi^{\pi} = 2.5$	$\phi^{\pi} = 3.0$
$\mathscr{D} = 0.0\%$	0.0	0.0	0.0	0.0	0.0
$\mathscr{D} = 2.5\%$	0.7	0.3	0.1	0.1	0.1
$\mathscr{D} = 5.0\%$	1.3	0.5	0.3	0.2	0.1
$\mathscr{D} = 7.5\%$	2.0	0.8	0.4	0.3	0.2
$\mathscr{D} = 10.0\%$	2.7	1.1	0.5	0.3	0.3

Table 4: Inflation bias (p.p.) with $\delta = 0.60$

	$\phi^\pi = 1.2$	$\phi^\pi = 1.5$	$\phi^\pi = 2.0$	$\phi^\pi = 2.5$	$\phi^\pi = 3.0$
$\mathscr{D} = 0.0\%$	0.0	0.0	0.0	0.0	0.0
$\mathscr{D} = 2.5\%$	8.6	3.3	1.6	1.1	0.8
$\mathscr{D} = 5.0\%$	19.0	6.9	3.3	2.2	1.6
$\mathscr{D} = 7.5\%$	32.0	10.8	5.1	3.4	2.5
$\mathscr{D} = 10.0\%$	48.8	15.2	7.1	4.6	3.4

6 Testable implications of the model

Our expansion of the neo-Wicksellian model with the inclusion of risk in the underlying asset of monetary policy allows for some testable implications: actual and expected prices and inflation should be positively correlated with policy-asset risk. In case a good measure of $\mathbb{E}_t \delta_{t+1}$ were constructed, it should also be positively correlated with them. Lastly, inflation volatility should reduce with larger values of ϕ (or ϕ^π), like in the canonical model, but risky economies should have a harder time to achieve that result.

In Section 1, we showed how the combination of the Fisher equation with an interest rate rule already revealed the predicted correlation between inflation and policy-asset risk. A possible counterfactual set-up would consist in a risk-free policy asset rate, i_t, but a central bank which still reacts to the risk wedge. Suppose an "unnecessarily reactive" central bank.

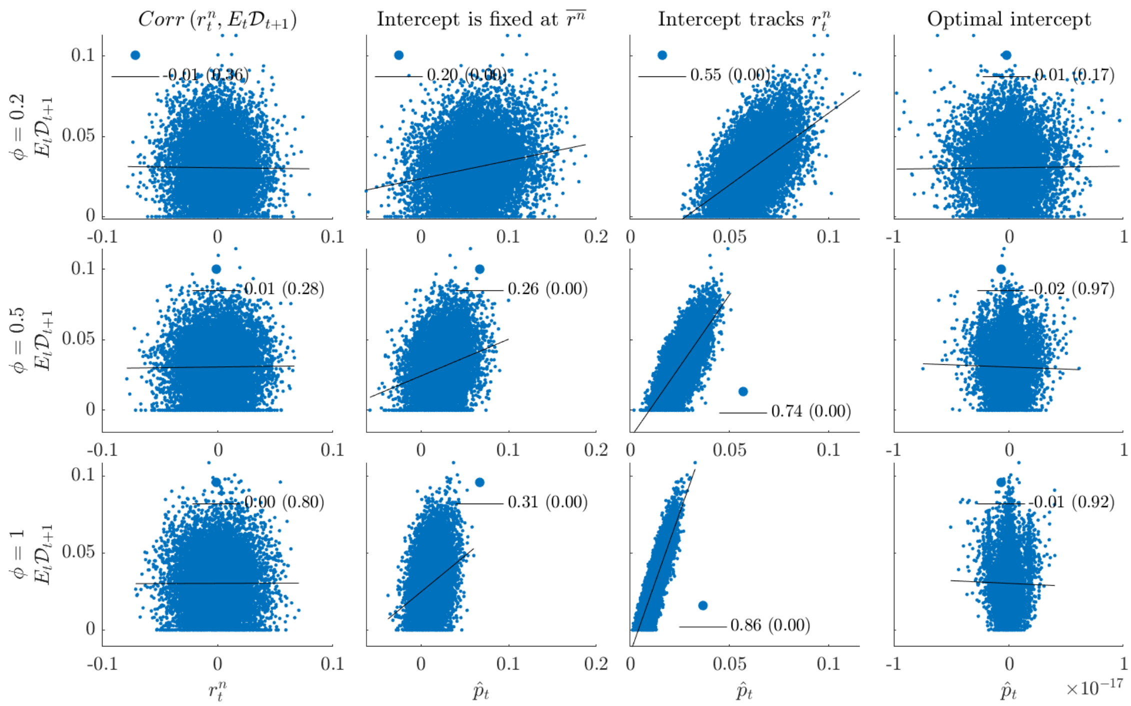
$$\begin{aligned} i_t &= r_t^n + \pi_{t,t+1}^e \\ i_t &= \bar{\imath}_t + \Phi_t + \phi \pi_{t,t+1}^e \end{aligned} \quad \Rightarrow \quad r_t^n - (\bar{\imath}_t + \Phi_t) = \left(\phi - 1\right) \pi_{t,t+1}^e \tag{21}$$

As long as $r_t^n - (\bar{\imath}_t + \Phi_t) < 0$, the correlation between policy-asset risk and inflation expectation is negative, at odds with what is commonly found in empirical data for emerging economies. Although unusual this set-up, we must remember that r_t^n is not observable by the central bank, and that for very low levels of risk premium or even negative ones (convenience yields), this scenario could actually come up. In this section, we test the empirical implications of our model and show that, while developed economies tend to present correlation indistinguishable from zero, negative correlations are not impossible. Most importantly, if emerging central banks "unnecessarily" reacted to risk perceptions, than the correlation with inflation should be negative. We show, next, that this is not the case.

6.1 Model simulation

We start with the model under price-level targeting. To show these correlations in the model, we assume that both the real natural interest rate and the expected default probability at the next period follow independent normal AR(1) processes from which we simulate 10,000 vectors containing 100 periods each. Figure 10 shows the dispersion of these variables for three different values of ϕ (0.2, 0.5, and 1.0) under three different monetary policy rules: intercept is fixed at the steady-state value[24], intercept tracks the real natural rate, and intercept tracks r_t^n adjusting by the risk underlying the policy asset. Note that for the first policy rule, both variables are somewhat positively correlated. When the central bank tracks r_t^n, this increases the positive correlation between $\mathbb{E}_t\mathscr{D}_{t+1}$ and P_t. Finally, when the central bank tracks r_t^n adjusting by the risk in the policy asset, the correlation disappears. As we transition through the rules in that same order (from the second to the fourth column in the picture) price dispersion is increasingly reduced until virtually zero, showing the benefits of updating the policy rule intercept according to the evolution of r_t^n and $\mathbb{E}_t\mathscr{D}_{t+1}$. Moreover, concerning the activeness of monetary policy w.r.t. prices, the higher the ϕ the higher is the correlation. All these results are robust to the sign of the correlation between r_t^n and $\mathbb{E}_t\mathscr{D}_{t+1}$ as can be seen in Appendix F.

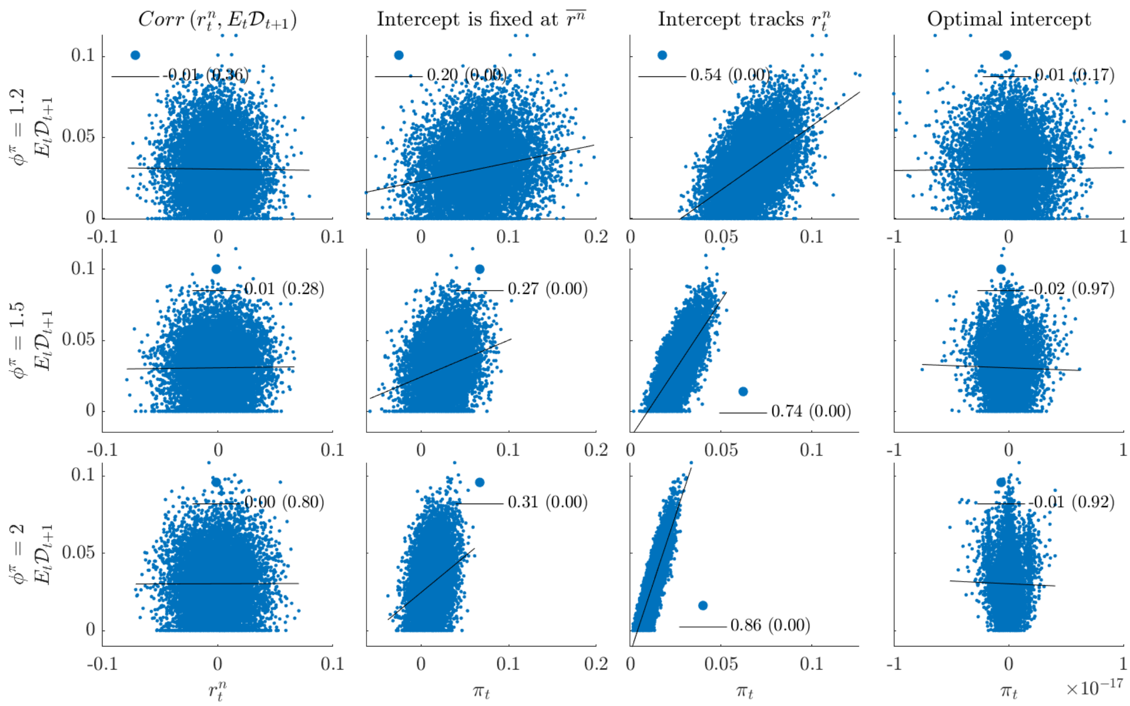
[24]We impose $\overline{\mathscr{D}} = 0$.



Note: p-values between parentheses.

Figure 10: Correlation between the default probability and the price level under price level targeting: $Corr\left(r_t^n, \mathbb{E}_t\mathscr{D}_{t+1}\right) = 0$

Since price level targeting is more of a theoretical device than a practical one, we repeat the same simulation under an inflation targeting rule, picking $\phi^{\pi} \in \{1.2, 1.5, 2.0\}$, and plot the correlations replacing the price level by inflation in Figure 11. The correlations we had seen for the price level targeting case between the price level and the expected default probability are now found between the latter and inflation. Robustness checks to the sign of the correlation between r_t^n and $\mathbb{E}_t\mathscr{D}_{t+1}$ are also available in Appendix F.



Note: p-values between parentheses.

Figure 11: Correlation between the default probability and inflation under inflation targeting: $Corr\left(r_t^n, \mathbb{E}_t \mathscr{D}_{t+1}\right) = 0$

6.2 Empirical validation

The theoretical predictions of our model are actually empirically found by Galli (2020) in a set of 10 large emerging economies[25] in the period 2004Q1-2018Q4, although for some countries data start later. In his analysis, besides actual inflation data, he uses CDS data as an indicator of default risk, and fixed-for-fixed Cross-Currency Swaps (XCSs) for representing the expected depreciation of the local currency against the U.S. dollar, which, by its turn, he interprets as a proxy for expected inflation. He concludes that, first, countries with high default risk also exhibit high inflation and high expected inflation levels; second, within each country at a quarterly frequency, he finds positive pairwise correlations between default risk and actual inflation, default risk and inflation risk, as well as default risk and exchange rate depreciation. What is away from Galli (2020)'s scope, but is important in this paper, is that of

[25] Brazil (1999), Colombia (1999), Indonesia (2005), Mexico (2001), Malaysia, Poland (1998), Russia (2014), Thailand (2000), Turkey (2006), and South Africa (2000), where the year in parentheses is the one in which each country adopted the inflation-targeting regime.

the 10 countries in the sample 9 of them have adopted the inflation-targeting regime during at least part of the interval analyzed.[26] In that line, Arellano et al. (2018) also find positive pairwise correlations between nominal interest rates, inflation, and default risk analyzing a sample of quarterly data from 2004-2017 comprehending the 10 inflation-targeting emerging countries that compose the JP-Morgan Emerging Market Bond Index (EMBI), which, by its turn, is used as the measure of default risk.[27]

As our model extends the very basic framework of neo-Wicksellian models, so important for the inflation-targeting rationale, the aforementioned empirical facts can be reconciled with the inflation-targeting regime without the need of adding to the model money and a seignorage channel through which defaulting on the debt and later inflating the economy would play complementary roles like in Galli (2020). Moreover, also different from what is done in Arellano et al. (2018), where these positive correlations are obtained by merging the small-open-economy New-Keynesian model of Gali and Monacelli (2005) with the strategic sovereign default RBC model of Arellano (2008), we get the same sign for the correlations still in the realm of a closed-economy flexible-price model. To conclude, embedding monetary policy's underlying asset with risk is sufficient for generating correlations with the same sign as the data.

We conduct, now, our own empirical exercise on the subject. Collecting a CDS dataset of emerging economies from Bloomberg[28] and merging it with the same data we used for plotting Figure 1 – our estimation of the natural interest rate of emerging and advanced economies using univariate filters – we can calculate for each country in the sample the contemporaneous four-quarter moving correlation between quarterly-mean nominal policy rates, Q/(Q-4) CPI inflation, and a measure of risk. For emerging economies, we use 5-year CDS in USD (usually the most liquid), while for advanced economies we opt for the 1-year nominal interest rate spread w.r.t. to 1-year nominal U.S. Treasuries, as CDS contracts are not liquid for these economies.[29] We show the correlations from pooling these observa-

[26]Malaysia, who adopted an "inflation-anchoring" regime in the same period, is the only exception. Under that regime, the central bank does not target a specific price index, adopting a more discretionary attitude toward price stability.

[27]The EMBI spread is measured as the difference in yields between foreign-currency emerging-economy government bonds and a comparable U.S. government bond. Arellano et al. (2018)'s sample comprises Brazil, Chile, Colombia, Indonesia, Korea, Mexico, Peru, Philippines, Poland, and South Africa.

[28]Data source description is available in Appendix F.

[29]Even though CDS data reflect the insurance premium on debt issued in USD, they are widely used by

tions in Figures 12 (emerging economies) and 13 (advanced economies), as well as country-specific in Figures 14 to 18. Our sample ranges from 2000Q1 to 2019Q4, and includes 12 of the 20 largest emerging economies, in addition to 7 of the 20 largest developed economies, where the missing countries were due to lack of data.[30] We opt to remove outliers identified as observations more than three scaled median absolute deviations (MAD) from the median as such extreme values can have a large effect on correlations.[31] In Appendix F, we reproduce this exercise using Du and Schreger (2016)'s measure of 5-year local-currency credit spread, which arguably controls for exchange-rate expectations and exchange-rate risk, and confirm the robustness of the difference in correlation pattern between emerging and advanced economies. From a country-specific perspective, the sign and the significance of correlations may vary from country to country depending on the risk measure chosen, but the pattern that default risk in emerging economies tends to exhibit positive correlation with inflation while in advanced ones that correlations is mostly non-significant (or negative) remains for any of these risk measures.

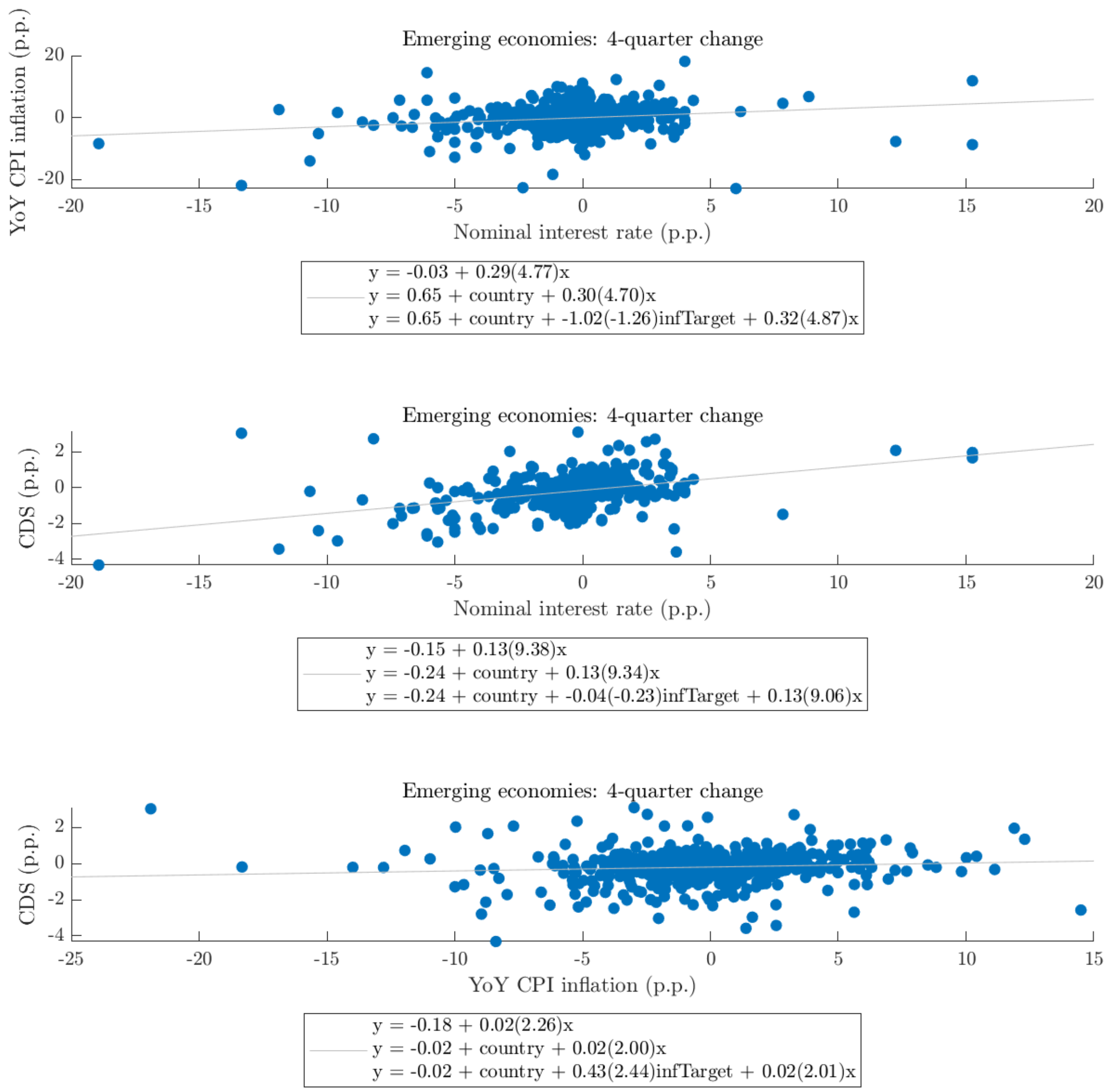
We find that the regression coefficient between inflation and default risk is significantly positive at the 5% level for the aggregate of emerging economies.[32] Individually in the same group, it is significant at least at the 10% level for seven of them (Brazil, Russian Federation, Poland, South Africa, the Philippines, Colombia, and Egypt); it is indistinguishable from zero for China, Mexico, Saudi Arabia, Turkey, and Malaysia; while it is significantly negative for none. Furthermore, for the aggregate of advanced economies no linear relation is found between inflation and default risk, reinforcing that for that group default risk may be perceived as too small to influence prices. At the individual country level, Sweden presents positive regression coefficient significant at the 10% level; Hong Kong presents negative one significant at the 5% level; while the coefficient is indistinguishable from zero for the remaining countries in that group. Besides, aggregate results are robust to country-fixed effects, whereas adopting inflation targeting is positively correlated with significant reduction of the level of default risk for emerging economies.

financial market practitioners as a general measure of country risk.

[30]We adopted a small arbitrary minimum threshold: 10 available observations.

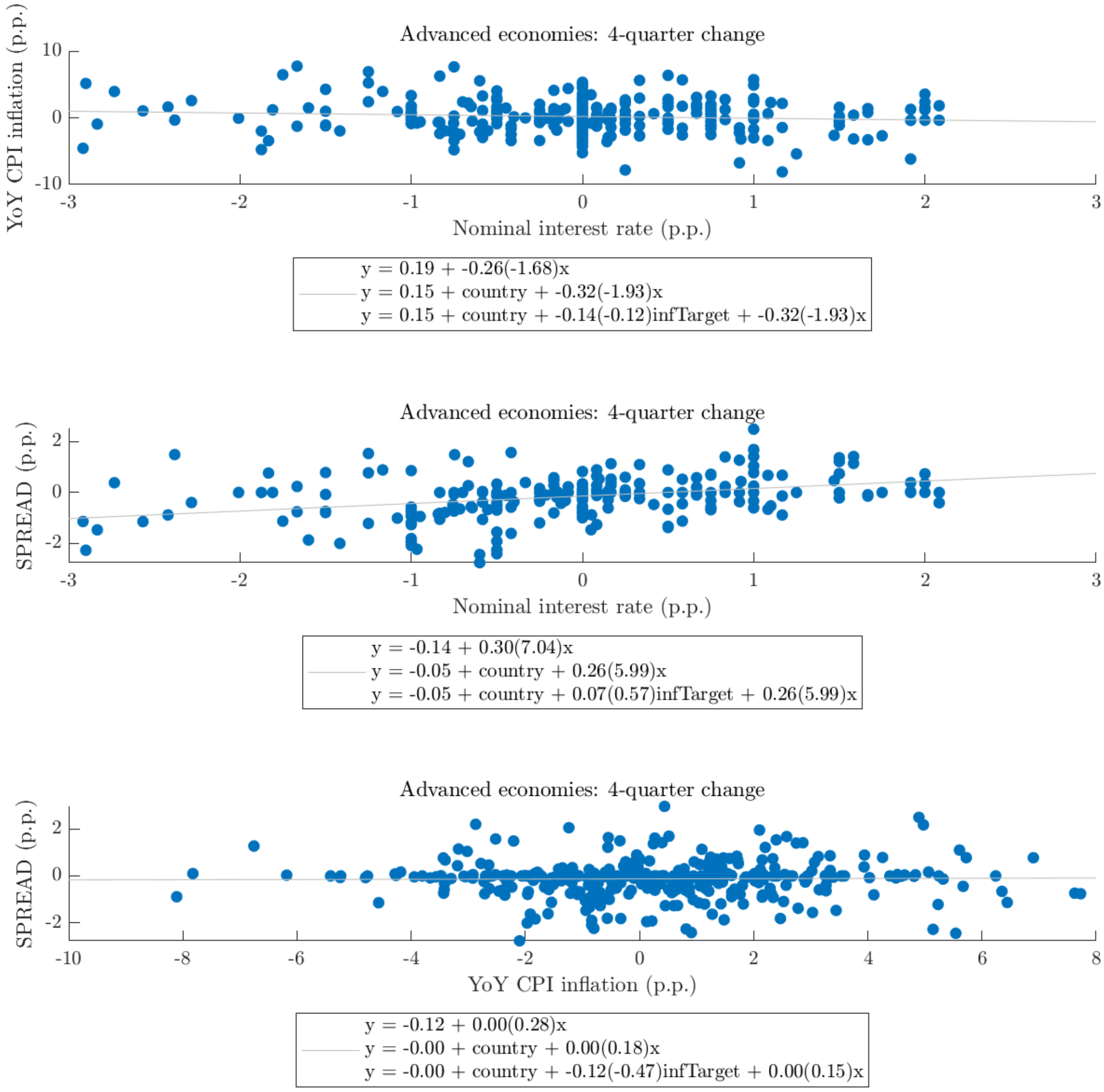
[31]The scaled MAD formula is given by c*median(abs(A-median(A))), where A is the vector of observations, and c=-1/(sqrt(2)*erfcinv(3/2)).

[32]Replacing CDS spreads by 1-year nominal interest rate spreads increases significance up to the 1% level.



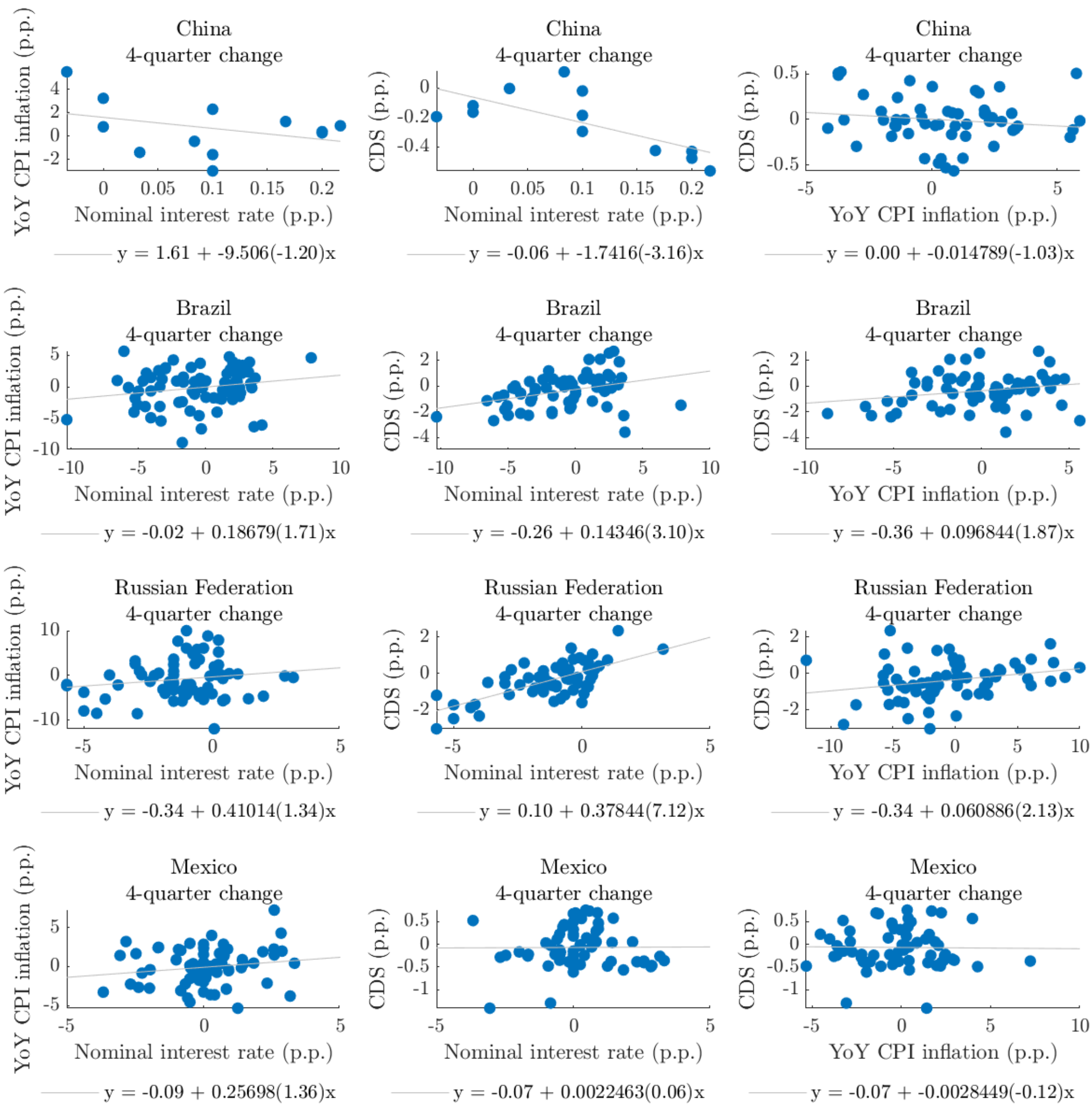
Note: between parentheses are t-statistics; "country" is a country-fixed effect; "infTarget" is a dummy that equals 1 when inflation targeting is adopted at the observation.

Figure 12: Scatter plot of pooled emerging economies: nominal interest rate, inflation, and default risk



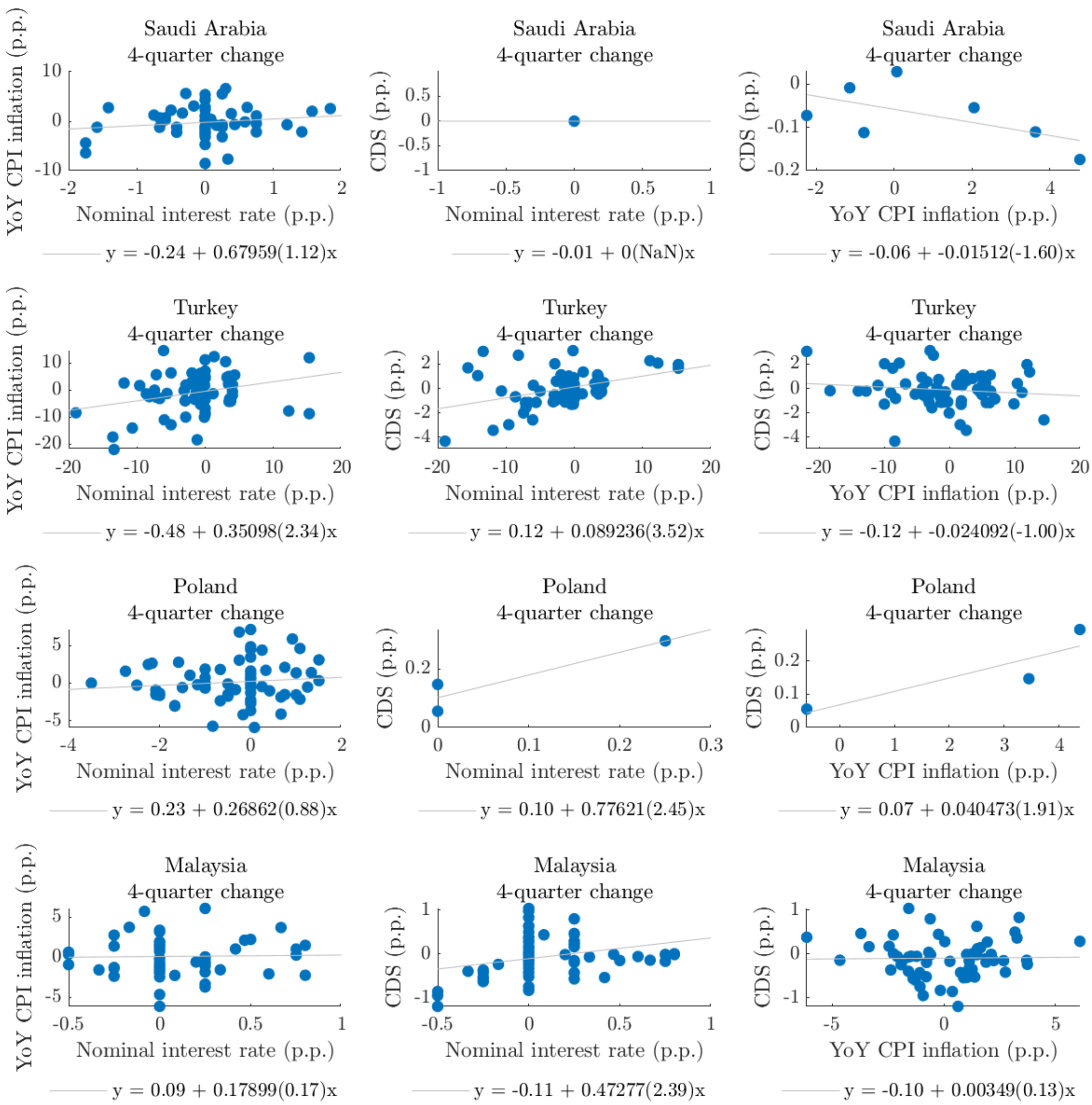
Note: between parentheses are t-statistics; "country" is a country-fixed effect; "infTarget" is a dummy that equals 1 when inflation targeting is adopted at the observation.

Figure 13: Scatter plot of pooled advanced economies: nominal interest rate, inflation, and default risk



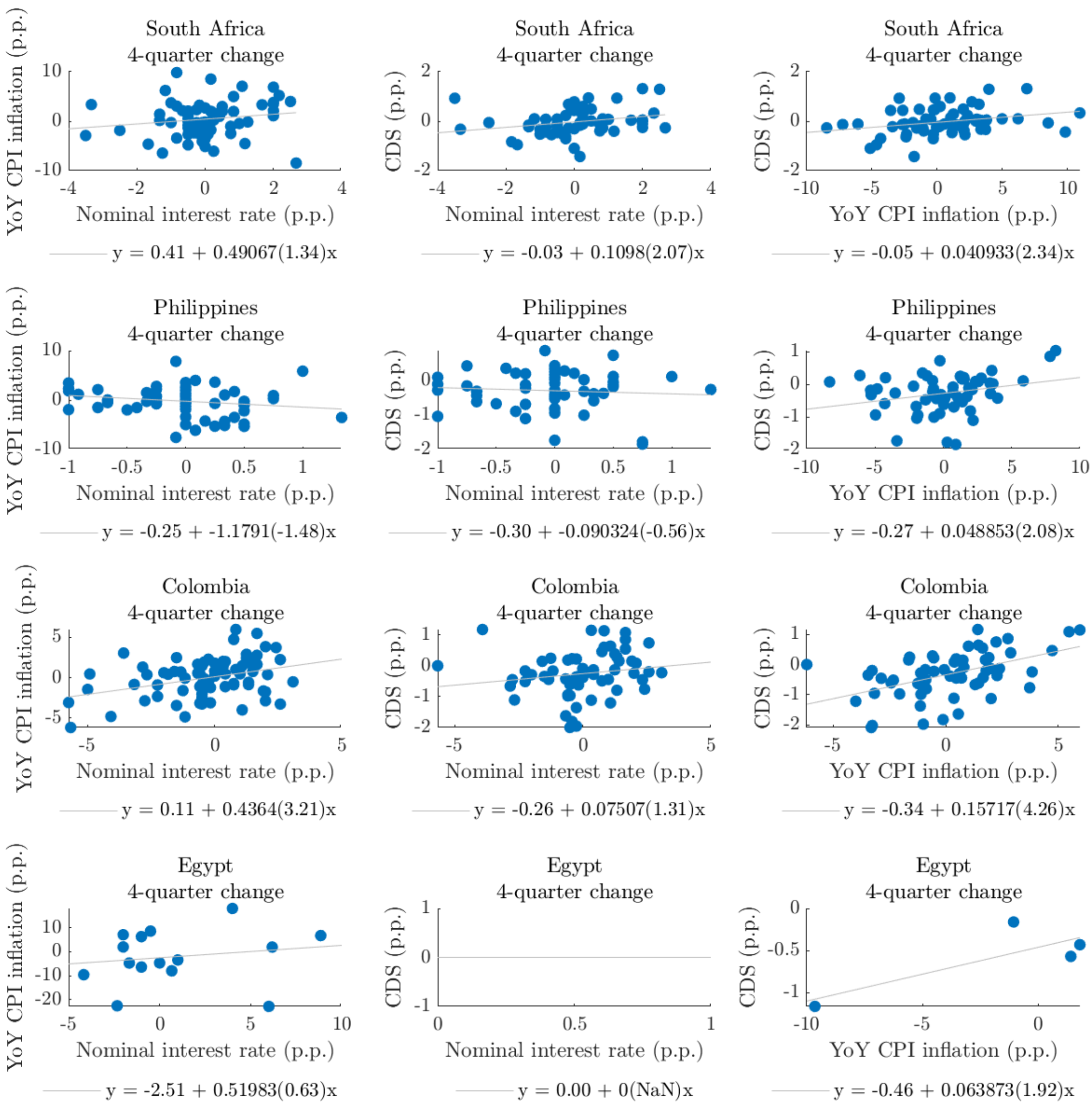
Note: t-statistics between parentheses; NaN means that there are not enough observations.

Figure 14: Scatter plot of emerging economies: nominal interest rate, inflation, and default risk (Part 1)



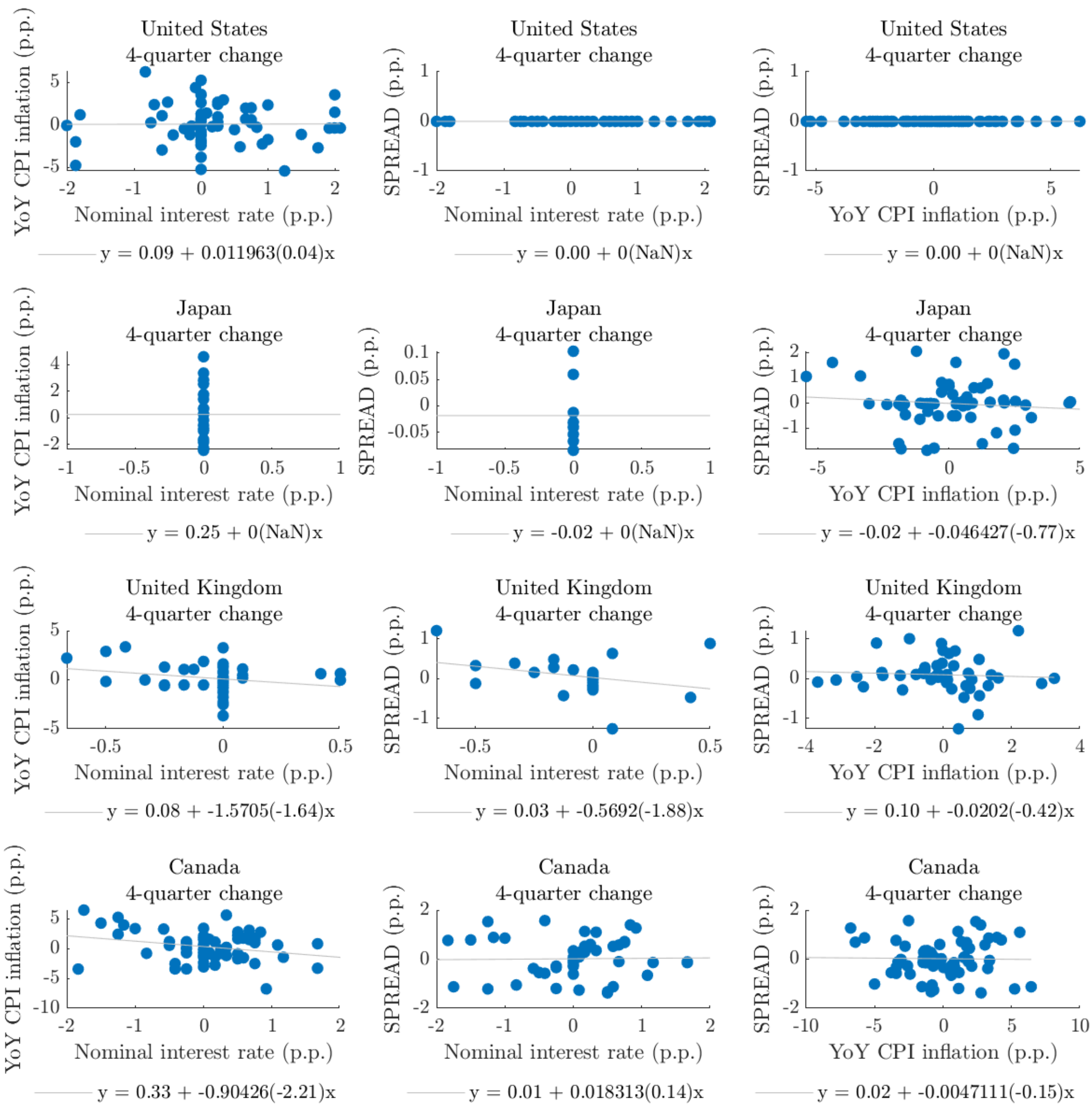
Note: t-statistics between parentheses; NaN means that there are not enough observations.

Figure 15: Scatter plot of emerging economies: nominal interest rate, inflation, and default risk (Part 2)



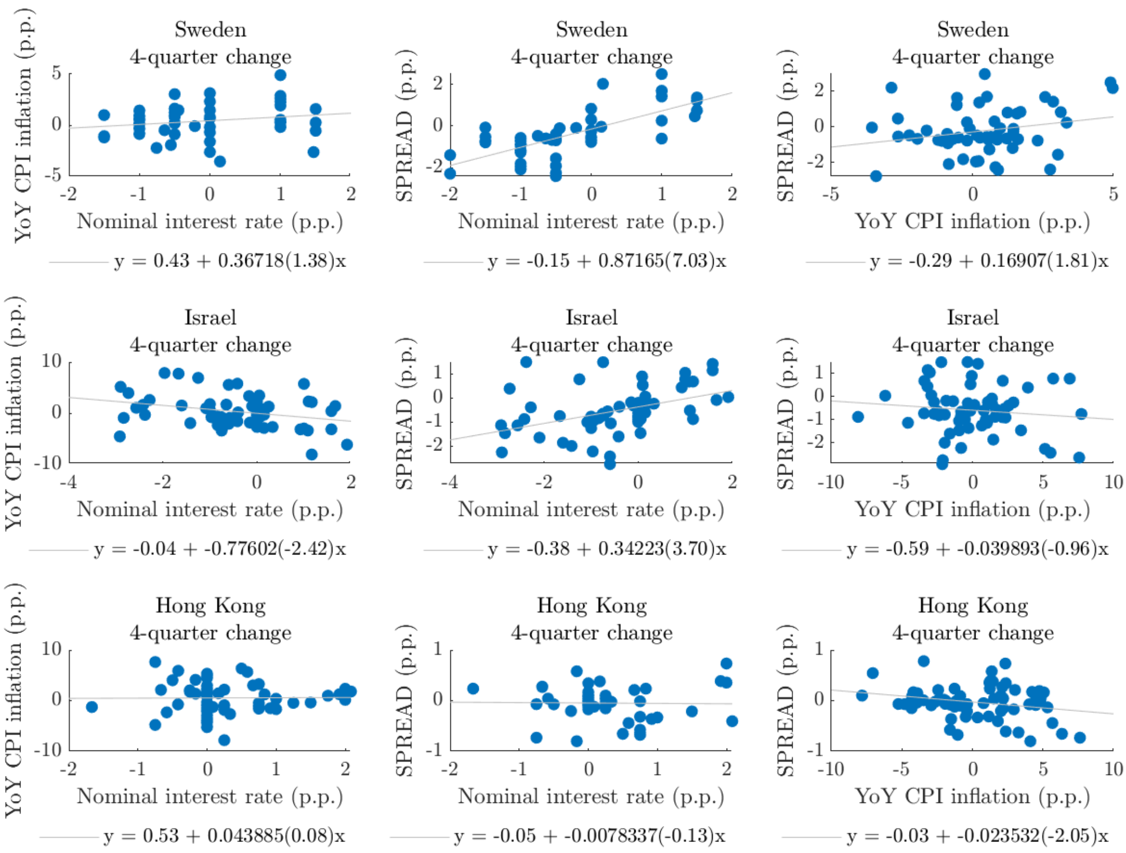
Note: t-statistics between parentheses; NaN means that there are not enough observations.

Figure 16: Scatter plot of emerging economies: nominal interest rate, inflation, and default risk (Part 3)



Note: t-statistics between parentheses; NaN means that there are not enough observations.

Figure 17: Scatter plot of advanced economies: nominal interest rate, inflation, and default risk (Part 1)

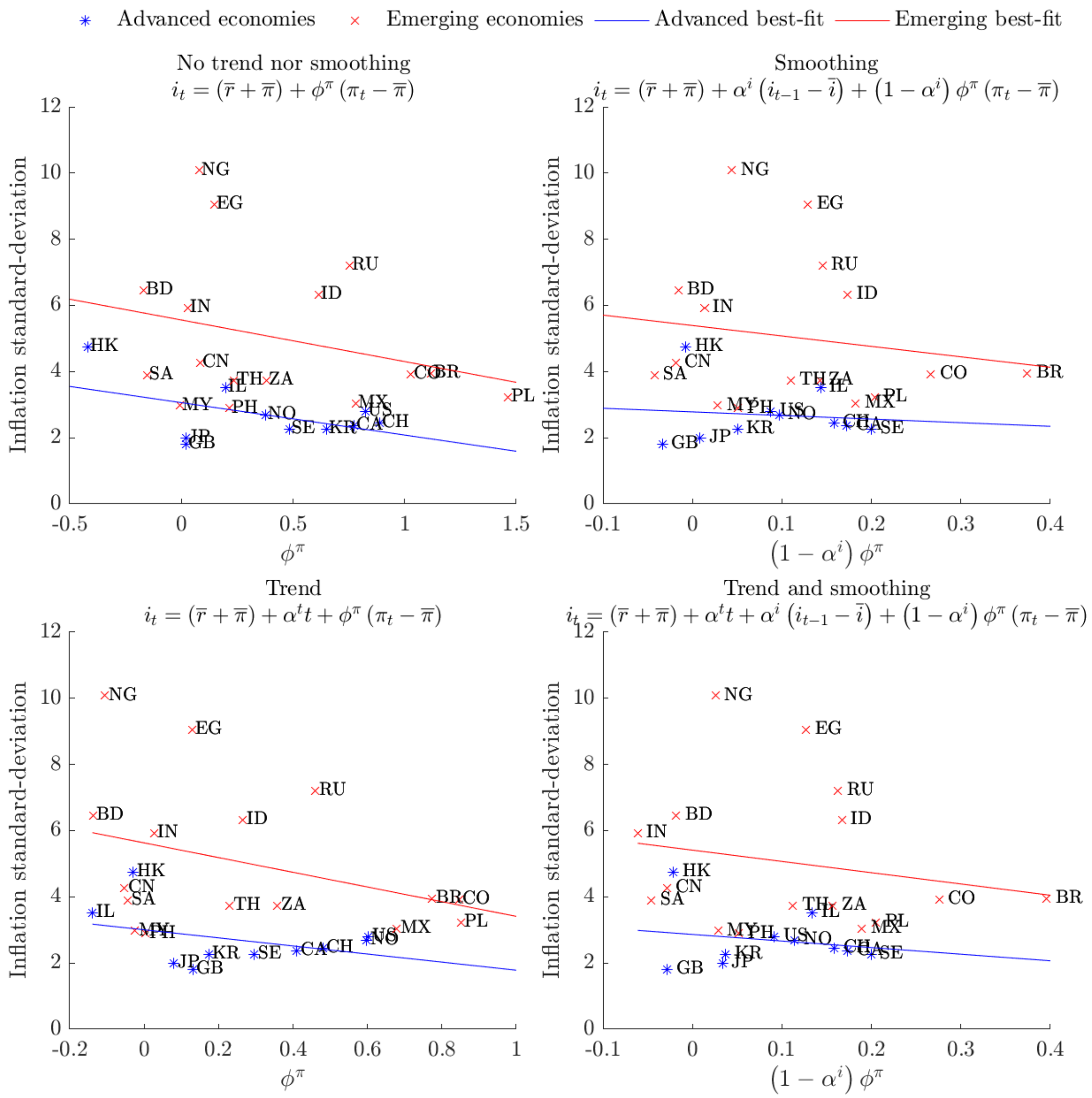


Note: t-statistics between parentheses; NaN means that there are not enough observations.

Figure 18: Scatter plot of advanced economies: nominal interest rate, inflation, and default risk (Part 2)

The last testable implication of the model that we explore is the one that relates inflation volatility with the inflation response coefficient, ϕ^{π}, in the interest rate rule adopted by the central bank. Our model predicts that increasing ϕ^{π} reduces that volatility, but when the policy asset is risky, it should occur with less intensity, because monetary policy power is attenuated by the default probability. To verify that prediction, we estimate four different Taylor rules using OLS for an unbalanced panel of 27 countries, not necessarily inflationtargeters.[33] Even though the Taylor rule's endogeneity implies that regressors are correlated with the error term, resulting in an asymptotic bias, Carvalho, Nechio and Tristao (2019) argue that such a bias should be small since monetary policy shocks explain only a small fraction of the variance of the regressors usually employed in this type of rule. Quarterly data is obtained from the IMF-IFS and it ranges from 2000Q1 to 2019Q4. For π_t, we use CPI YoY inflation, while for i_t, we use the quarterly mean of the annual nominal rate on 1-year Treasury Bills. We plot our results in Figure 19. Each panel is a scatter plot of estimated coefficient ϕ^{π} against the standard-deviation of CPI inflation under a different specification for the Taylor rule: no trend nor smoothing (top-left panel), smoothing (top-right panel), trend (bottom-left paenl), and trend and smoothing (bottom-right panel). The reason for including a linear trend is to allow for the possibility that real natural interest rates have been declining during the sample period, as we have seen in Figure 1. The graphs confirm our model's prediction. Inflation volatility does reduce with the size of ϕ^{π}, what can be verified by the declining best-fit lines estimated, whereas emerging (risky) economies enjoy higher inflation volatility than advanced (risk-free) ones for the same ϕ^{π} values, what can be verified by the positive shift between the respective best-fit lines. It does not go without notice, though, that estimated ϕ^{π} values in the graphs without smoothing are mostly lower than 1, in apparent contradiction with the Taylor principle. To a great extent, this is explained by the fact that central banks tend to smooth interest rates out across the business cycle. The ϕ^{π} values in the graphs with smoothing (not displayed) are much more likely to attend the Taylor principle. Additionally, our Taylor rules still miss some of the elements to which central banks respond, i.e. output gaps and time-varying inflation targets. It is also possible that our selected empirical series do not strongly positively correlate with the exact indicators tracked by each country's central bank. Above all, this empirical exercise sought to demonstrate broad stylized facts, more than to obtain precise country-specific estimations.

[33]We start from a sample with the 20 largest advanced and the 20 largest emerging economies, classified like in Appendix A. Countries without data available for the sample period are excluded. The remaining economies are the following. Advanced economies: Canada (CA), Hong Kong (HK), Israel (IL), Japan (JP), Norway (NO), South Korea (KR), Sweden (SE), United Kingdom (GB), United States (US). Emerging economies: Argentina (AR), Bangladesh (BD), Brazil (BR), Colombia (CO), Chile (CL), China (CN), Egypt (EG), India (IN), Indonesia (ID), Malaysia (MY), Mexico (MX), Nigeria (NG), Phillipines (PH), Poland (PL), Russia (RU), Saudi Arabia (SA), South Africa (ZA), Thailand (TH). Moreover, we exclude Turkey (TR) from the sample because it is clearly an outlier.



Note: estimation is done with OLS (ordinary least squares). Data sample ranges from 2000Q1 to 2019Q4. We use the CPI YoY inflation (π_t) and the quarterly mean of the annual nominal rate on Treasury Bills (i_t), both collected from the IMF-IFS. The Treasury Bills we use are local-currency 1-year-maturity federal government bonds, and the panel is unbalanced. $(\bar{r} + \bar{\pi})$ is the intercept of each regression; α^i is the smoothing coefficient; α^t is the time-trend coefficient; t is a linear time trend; ϕ^π is the inflation-reaction coefficient; and $\bar{x}$ is the sample mean of any variable x. Advanced economies: Canada (CA), Hong Kong (HK), Israel (IL), Japan (JP), Norway (NO), South Korea (KR), Sweden (SE), United Kingdom (GB), United States (US). Emerging economies: Argentina (AR), Bangladesh (BD), Brazil (BR), Colombia (CO), Chile (CL), China (CN), Egypt (EG), India (IN), Indonesia (ID), Malaysia (MY), Mexico (MX), Nigeria (NG), Phillipines (PH), Poland (PL), Russia (RU), Saudi Arabia (SA), South Africa (ZA), Thailand (TH).

Figure 19: Empirically estimated Taylor rules

7 Conclusion

The assumption that monetary policy is conducted through risk-free instruments may be unrealistic for an emerging economy whose central bank targets inflation. This paper showed using a cashless closed-economy model with flexible prices that when the policy instrument is modeled as risky the power of monetary policy w.r.t the price level and inflation is reduced compared to a counterfactual without that risk. Moreover, uncompensated policy-asset risk distinguishes the unconditional mean of the price level, or inflation, from its target, resulting in a bias that induces a more aggressive reaction from the central bank. That reaction ultimately consists of either adopting a Taylor rule more elastic to inflation, or with a higher estimation for the unobserved real natural rate in the intercept, or both. This phenomenon perceived by some pundits in risky economies as "monetary policy conservatism" is actually necessary to bringing inflation to the target in the presence of risk in the policy asset.

By modeling the policy asset as a defaultable bond instead of a risk-free asset, monetary policy is not only less active, but it also becomes conditionally active to the issuer repaying its debt. Ignoring that fact leads to non-optimal monetary policy inducing positive correlation between default risk and inflation, what we showed is consistent with empirical data. That correlation, frequently motivated in the literature as an exchange rate transmission, emerges here without the need of opening the economy. In the context of active monetary policy combined with passive fiscal policy, our results represent a novel argument in favor of a more hawkish stance for monetary policy in the event of a fiscal or political crisis. Under the light of our results, central banks in the disposal of some independence should consider switching to remunerated deposits to mitigate exposure to government default risk.

The fact that the underlying asset of monetary policy can carry significant risk raises challenges to emerging economy central banks that have so far been either neglected or treated as of little relevance by the literature. This paper is a first attempt to tackle such challenges. However, many questions remain open. For instance, what other mechanisms allow policy-asset risk to interacting with the natural interest rate? How do nominal frictions can affect our results? Does the reduction of power we find for monetary policy remains relevant in a quantitative general equilibrium model? These and other questions we leave for future research.

A Estimation of the real neutral rate using univariate filters

Estimation is conducted with quarterly country data from the IMF-IFS as follows. All countries are sorted in descending order by forecasted nominal Q42020 GDP measured in USD as of 27/01/2020. Countries are classified using the IMF-WEO definition of emerging and advanced economies. The largest 20 advanced and the largest 20 emerging economies are selected from the dataset. The real ex-ante interest rate for each country is calculated according to a linearized Fisher equation, that is, subtracting a series of nominal interest rate by a series of inflation expectation, both with the same time horizon. For the nominal interest rate, we chose the annualized rate on national Treasury Bills (local-currency 1-year-maturity federal government bonds). For the inflation expectation, we estimated AR(p) models for each country's quarterly CPI inflation series with p ranging from 0 to 4 lags. Then, for each country, we selected the model with the lowest BIC information criterion. Since not all selected countries had enough observations of the aforementioned time series available, some of the initially selected countries had to be removed from the sample. Having defined each country's inflation forecast model, we built a series of 1-year-ahead inflation forecast for each country. A summary of the data sample is available in Table 5.

After calculating all country-specific ex-ante real rate series, we applied three statistical filters on them: HP (λ= 1600), Baxter-King (min=6, max=32, order=12), and Christiano-Fitzgerald (min=6, max=32).[34] We calculated the median of these filters and their range as shown in Figures 20 (Advanced Economies) and 21 (Emerging Economies). Finally, we obtained with the country-specific medians three statistics for each country group: the group median, the arithmetic group mean, and the country-weighted group mean using the aforementioned GDP values (Figure 1).

[34] The original reference of each filter is, in order, Hodrick and Prescott (1997), Baxter and King (1999), and Christiano and Fitzgerald (2003).

	Name	Status	NR Start	NR End	CPI Start	CPI End
US	United States of America	Advanced	1950Q1	2020Q1	1955Q2	2020Q1
GB	United Kingdom	Advanced	1964Q1	2016Q3	1955Q2	2020Q1
IT	Italy	Advanced	1977Q1	2020Q1	1955Q2	2020Q1
CA	Canada	Advanced	1950Q1	2017Q2	1949Q2	2020Q1
ES	Spain	Advanced	1987Q3	2020Q1	1954Q2	2020Q1
SE	Sweden	Advanced	1960Q1	2017Q2	1955Q2	2020Q1
BE	Belgium	Advanced	1957Q1	2017Q4	1955Q2	2020Q1
IL	Israel	Advanced	1995Q1	2020Q1	1952Q2	2020Q1
HK	Hong Kong	Advanced	1992Q4	2018Q4	1981Q1	2020Q1
BR	Brazil	Emerging	1995Q1	2020Q1	1980Q1	2020Q1
MX	Mexico	Emerging	1978Q1	2020Q1	1957Q2	2020Q1
SA	Saudi Arabia	Emerging	2009Q2	2018Q1	1980Q2	2020Q1
PL	Poland	Emerging	1992Q1	2017Q1	1988Q2	2020Q1
TH	Thailand	Emerging	2001Q1	2020Q1	1965Q2	2020Q1
ZA	South Africa	Emerging	1957Q1	2020Q1	1957Q2	2020Q1
PH	Philippines	Emerging	1976Q1	2019Q4	1957Q2	2020Q1
BD	Bangladesh	Emerging	2006Q3	2020Q1	1993Q4	2020Q1
EG	Egypt	Emerging	1997Q1	2020Q1	1957Q2	2019Q4

Table 5: Filters: Summary of the data sample

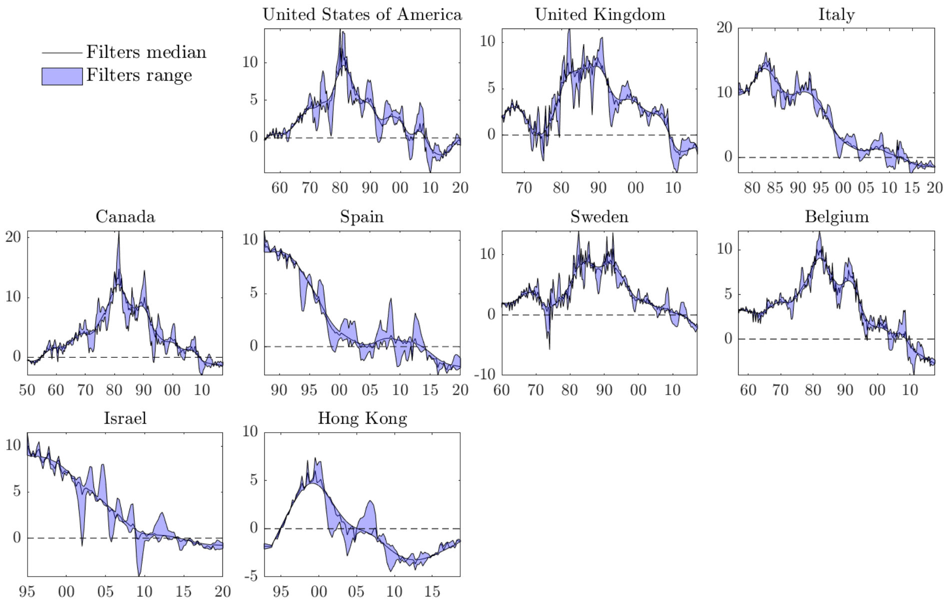


Figure 20: Advanced Economies: Real neutral rates estimated by univariate filters

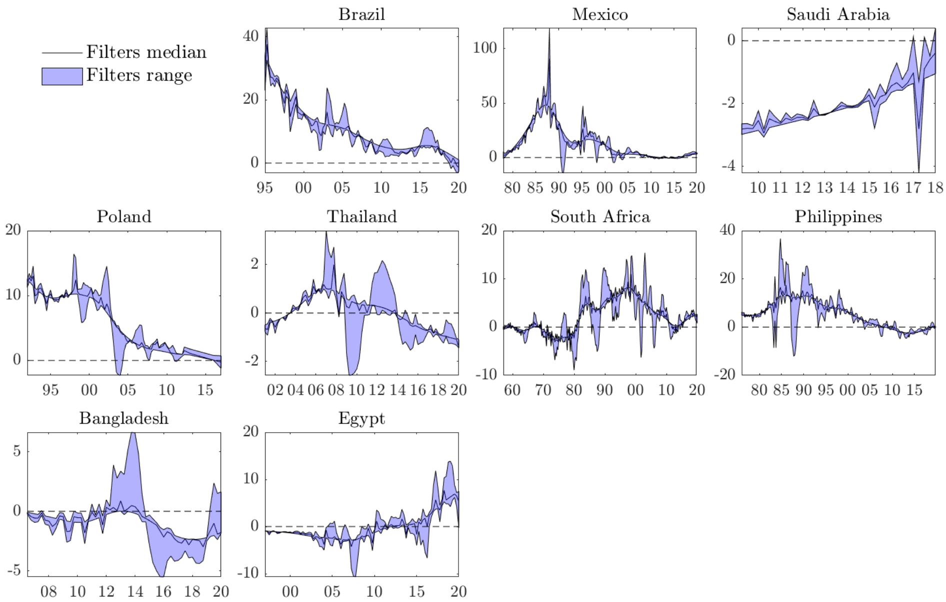


Figure 21: Emerging Economies: Real neutral rates estimated by univariate filters

B Propositions and proofs

Here, we provide the proofs for some of the propositions in this paper. We keep their original numbering.

Proposition 4.2 (Price level determinacy under price-level targeting)**.** *A bounded process* $\left\{r_t^n, \mathscr{D}_t, \delta_t, \bar{\iota}_t\right\}$*, a monetary policy rule such as the one proposed in Assumption 10 with* $\phi > 0$*, the fact that* $\mathscr{D}_t$ *is a process that represents a probability and* δ_t *is a process that represents a fraction, and the condition that for all t there is at least one infinite sequence* $k_n \subset [1, \infty)$ *such that* $0 \leq \mathbb{E}_t \mathscr{D}_{t+k}\delta_{t+k} < 1$ *in that sequence are necessary and sufficient conditions so that the price level equilibrium exists and is unique.*

Proof. We start by proving that conditions are sufficient. $0 < \frac{1}{1+(1-\mathbb{E}_t \mathscr{D}_{t+k}\delta_{t+k})\phi} < 1$ for all t and at least one infinite sequence $k_n \subset [1, \infty)$. This leads to $\Upsilon_{t,j+1} \leq 1$ and $\Upsilon_{t,j+1} \leq \Upsilon_{t,j}$ for all t and all j. Moreover, $\lim_{j\to\infty} \Upsilon_{t,j+1} = 0$ for all t. Since $\{r_t^n, \mathscr{D}_t, \delta_t, \bar{\iota}_t\}$ is a bounded process, $\Gamma_{t+j} = \mathbb{E}_t\left(r_{t+j}^n - \left(1 - \mathbb{E}_t \mathscr{D}_{t+j+1}\delta_{t+j+1}\right)\bar{\iota}_{t+j} + \mathbb{E}_t \mathscr{D}_{t+j+1}\delta_{t+j+1}\right)$ is also a bounded process. The sum of Equation 11 can be seen as the product of two bounded processes, Υ_{t+j+1} and Γ_{t+j}, which therefore will also be a bounded process. Let now a number $M > 0$ be such that $|\Gamma_n| \leq M$. Let $\{\Upsilon_n\}$ be such that $\Upsilon_n \to 0$. Given $\epsilon > 0$, for the positive number $\frac{\epsilon}{M}$, there exists some N such that $|\Upsilon_n| < \frac{\epsilon}{M}$ for all $n \geq N$, so $|\Upsilon_n \Gamma_n| \leq M|\Upsilon_n| < M.\frac{\epsilon}{M} = \epsilon$ for all such an n, what shows that $\Upsilon_n \Gamma_n \to 0$. This assures that the price level equilibrium exists and is unique, and therefore it is determined.

Now, we prove the proposed conditions are also necessary by contradiction. If $\phi = 0$ then $\Upsilon_{t,j+1} = 1$ for all t and all j. Since $\Gamma_n \to \mathbb{E}(\Gamma_n) = \overline{L}$ where $\overline{L}$ is a finite number, the convergence of the sum will only happen in case $\mathbb{E}(\Gamma_n) = 0$, as if the latter expectation is positive or negative then p_t diverges to ∞ or $-\infty$, respectively, constituting an unstable equilibrium. If $\phi < 0$, then $\Upsilon_{t,j+1} > 1$ in at least one infinite sequence $j_n \subset [1, \infty)$, and then $\Upsilon_n \to \infty$, while $\Gamma_n \to \mathbb{E}(\Gamma_n) = \overline{L}$. Their product will diverge. Thus, $\phi > 0$ is a necessary condition. If there is no infinite sequence in which $k_n \subset [1, \infty)$ such that $0 \leq \mathbb{E}_t \mathscr{D}_{t+k}\delta_{t+k} < 1$ in that sequence then there may be only finite sequences of that kind, none of them converging to zero, and there is still an infinite sequence in which $k_n \subset [1, \infty)$ such that $0 \leq \mathbb{E}_t \mathscr{D}_{t+k}\delta_{t+k} = 1$. The sum diverges. This assures that the proposed conditions are not only sufficient but also necessary. □

Proposition 4.3 (Optimal intercept under price-level targeting)**.** *In an economy in which the marginal investor is risk-neutral, monetary policy can stabilize prices through operations with the risky asset if it can track the sequence of natural interest rates* $\left\{r^n_{t+j}\right\}_{j=0}^{\infty}$*, the sequence of one-period-ahead policy-asset default probabilities* $\{\mathscr{D}_{t+j+1}\}_{j=0}^{\infty}$*, the sequence of one-period-ahead expected haircuts* $\{\delta_{t+j+1}\}_{j=0}^{\infty}$*, and it credibly adopts at period t a Wickellian rule with the sequence of time-varying intercepts* $\{\bar{\iota}_{t+j}\}_{j=0}^{\infty} \equiv \left\{\frac{r^n_{t+j}+\mathbb{E}_t\mathscr{D}_{t+j+1}\delta_{t+j+1}}{1-\mathbb{E}_t\mathscr{D}_{t+j+1}\delta_{t+j+1}}\right\}_{j=0}^{\infty}$.

Proof. Substituting $\bar{\iota}_{t+j} = \frac{r^n_{t+j}+\mathbb{E}_t\mathscr{D}_{t+j+1}\delta_{t+j+1}}{1-\mathbb{E}_t\mathscr{D}_{t+j+1}\delta_{t+j+1}}$ into the equilibrium price level for the risky economy (11), all terms of the respective summation collapse to zero. □

Proposition 5.1 (Inflation determinacy under inflation targeting)**.** *A bounded process* $\{r^n_t, \mathscr{D}_t, \delta_t, \bar{\iota}_t\}$*, a monetary policy rule such as the one proposed in Assumption 12 with* $\phi^\pi > \frac{1}{1-\mathbb{E}_t\mathscr{D}_{t+1}\delta_{t+1}}$ *for all t, the fact that* $\mathscr{D}_t$ *is a process that represents a probability and* δ_t *is a process that represents a fraction, and the condition that for all t there is at least one infinite sequence* $k_n \subset [1,\infty)$ *such that* $0 \le \mathbb{E}_t\mathscr{D}_{t+k}\delta_{t+k} < 1$ *in that sequence are necessary and sufficient conditions so that equilibrium inflation exists and it is unique.*

Proof. We start by proving that conditions are sufficient. $0 < \frac{1}{(1-\mathbb{E}_t\mathscr{D}_{t+k}\delta_{t+k})\phi^\pi} < 1$ for all t and at least one infinite sequence $k_n \subset [1,\infty)$. This leads to $\Upsilon^\pi_{t,j+1} \le 1$ and $\Upsilon^\pi_{t,j+1} \le \Upsilon^\pi_{t,j}$ for all t and all j. Moreover, $\lim_{j\to\infty}\Upsilon^\pi_{t,j+1} = 0$ for all t. Since $\{r^n_t, \mathscr{D}_t, \delta_t, \bar{\iota}_t\}$ is a bounded process, $\Gamma_{t+j} = \mathbb{E}_t\left(r^n_{t+j} - \left(1-\mathbb{E}_t\mathscr{D}_{t+j+1}\delta_{t+j+1}\right)\bar{\iota}_{t+j} + \mathbb{E}_t\mathscr{D}_{t+j+1}\delta_{t+j+1}\right)$ is also a bounded process. The sum of Equation 18 can be seen as the product of two bounded processes, Υ^π_{t+j+1} and Γ_{t+j}, which therefore will also be a bounded process. Let now a number $M > 0$ be such that $|\Gamma_n| \le M$. Let $\{\Upsilon^\pi_n\}$ be such that $\Upsilon^\pi_n \to 0$. Given $\epsilon > 0$, for the positive number $\frac{\epsilon}{M}$, there exists some N such that $|\Upsilon^\pi_n| < \frac{\epsilon}{M}$ for all $n \ge N$, so $|\Upsilon^\pi_n\Gamma_n| \le M|\Upsilon^\pi_n| < M.\frac{\epsilon}{M} = \epsilon$ for all such an n, this shows that $\Upsilon^\pi_n\Gamma_n \to 0$. This assures that equilibrium inflation exists and is unique, and therefore it is determined.

Now, we prove the proposed conditions are also necessary by contradiction. If $\phi^\pi = \frac{1}{1-\mathbb{E}_t\mathscr{D}_{t+1}\delta_{t+1}}$ for all t then $\Upsilon^\pi_{t,j+1} = 1$ for all t and all j. Since $\Gamma_n \to \mathbb{E}(\Gamma_n) = \overline{L}$ where $\overline{L}$ is a finite number, the convergence of the sum will only happen in case $\mathbb{E}(\Gamma_n) = 0$, as if the latter expectation is positive or negative then π_t (and p_t) diverge to ∞ or $-\infty$, respectively, constituting an unstable equilibrium. If $\phi^\pi < \frac{1}{1-\mathbb{E}_t\mathscr{D}_{t+1}\delta_{t+1}}$ for all t, then $\Upsilon^\pi_{t,j+1} > 1$ in at least one infinite sequence $j_n \subset [1,\infty)$, and then $\Upsilon^\pi_n \to \infty$, while $\Gamma_n \to \mathbb{E}(\Gamma_n) = \overline{L}$. Their product will diverge. Thus, $\phi^\pi > \frac{1}{1-\mathbb{E}_t\mathscr{D}_{t+1}\delta_{t+1}}$ for all t is a necessary condition. If there is no infinite sequence in which $k_n \subset [1,\infty)$ such that $0 \leq \mathbb{E}_t \mathscr{D}_{t+k} \delta_{t+k} < 1$ in that sequence then there may be only finite sequences of that kind, none of them converging to zero, and there is still an infinite sequence in which $k_n \subset [1,\infty)$ such that $0 \leq \mathbb{E}_t \mathscr{D}_{t+k} \delta_{t+k} = 1$. The sum diverges. This assures that the proposed conditions are not only sufficient but also necessary. □

Proposition 5.2 (Optimal intercept under inflation targeting)**.** *In an economy in which the marginal investor is risk-neutral, monetary policy is able to stabilize inflation through operations with the risky asset if it can track the sequence of natural interest rates* $\left\{r^n_{t+j}\right\}_{j=0}^{\infty}$*, the sequence of one-period-ahead policy-default probabilities* $\{\mathscr{D}_{t+j+1}\}_{j=0}^{\infty}$*, the sequence of one-period-ahead expected haircuts* $\{\delta_{t+j+1}\}_{j=0}^{\infty}$*, and it credibly adopts at period t a Taylor rule with the sequence of time-varying intercepts* $\{\bar{\imath}_{t+j}\}_{j=0}^{\infty} \equiv \left\{\frac{r^n_{t+j}+\mathbb{E}_t \mathscr{D}_{t+j+1}\delta_{t+j+1}}{1-\mathbb{E}_t \mathscr{D}_{t+j+1}\delta_{t+j+1}}\right\}_{j=0}^{\infty}$.

Proof. Substituting $\bar{\imath}_{t+j} = \frac{r^n_{t+j}+\mathbb{E}_t \mathscr{D}_{t+j+1}\delta_{t+j+1}}{1-\mathbb{E}_t \mathscr{D}_{t+j+1}\delta_{t+j+1}}$ into the equilibrium inflation for the risky economy (18), all terms of the respective summation collapse to zero. □

C A default-risky cashless economy

We derive a simple adaptation of the canonical cashless flexible-price model of Woodford (2003, ch. 2, sec. 1) to show that it can underpin the partial equilibrium default-risky cashless model developed in this paper. We keep the same notation whenever possible.

There is a cashless economy with a goods market, a financial market, infinitely-lived households, a government, and a central bank. Time is discrete. Both markets are completely frictionless, that is, they are perfectly competitive, prices adjust continuously to clear them, and state-contingent securities of any kind can be traded. The goods market exchanges a single good whose endowment at every period is exogenous. All prices are quoted in terms of a single monetary unit of account defined in terms of a claim to a certain quantity of a one-period liability issued exclusively by the central bank, who can also issue other one-period liabilities that promise to pay additional units of that same liability at the next period – this allows the central bank to control both the nominal interest yield and the quantity of these liabilities. The government issues every period one-period nominal bonds and imposes lump-sum nominal taxes, which can be negative, to the households. Government is tricky, though. There are random times in which it fails to repay its bonds. Worse than that, it also controls the central bank, and when it fails to repay its bonds the central bank fails to repay its liabilities. As usual, arbitrage relations describe the conditions under which any agent is willing to hold any liability at any time. As there are no impediments to arbitrage, these conditions are always satisfied in equilibrium, so there is a well-defined exchange rate at any time between units of government bonds, goods, and central bank's liabilities.

A representative household summarizes a continuum of identical households who maximizes

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(C_t; \xi_t) \right\} \tag{22}$$

where $\mathbb{E}_0$ is the expectation conditional upon the state of the economy at period 0; β is a subjective discount factor, $u(.)$ is a one-period utility function, C_t represents consumption, and ξ_t is an exogenous stochastic disturbance to $u(.)$. We restrict $u(.)$ to be concave and strictly increasing in C. Under complete markets, a household's flow budget constraint is expressed by

$$M_t + B_t \le W_t + P_t Y_t - T_t - P_t C_t \tag{23}$$

where M_t is end-of-period balances of the central bank's asset; B_t is the end-of-period nominal value of the portfolio of assets not issued by the central bank; W_t is beginning-of-period financial wealth; Y_t is the exogenous endowment of the good; P_t is its price; and T_t is the net nominal tax collection by the government. Financial wealth evolves according to

$$W_{t+1} = \left(1 + i_{t+1}^{\text{Risky}}\right) M_t + A_{t+1} \tag{24}$$

such that i_{t+1}^{Risky} is the net nominal interest rate actually paid on the central bank's asset held at the end of period t; and A_{t+1} is the value of the portfolio of assets not issued by the central bank held at the end of period t after the realization of the state of the economy at period $t+1$. It is helpful to revisit the derived relation (9) for $\mathbb{E}_t\, i_{t+1}^{\text{Risky}}$ and the arbitrage condition (10). Note that the timing of i_{t+1}^{Risky} is defined as that as it is only known with certainty at period $t+1$.

The assumption of no arbitrage opportunities implies the existence of a unique stochastic discount factor, $Q_{t,t+1}$, which prices at the end of period t any asset maturing at period $t+1$, say B_t,

$$B_t = \mathbb{E}_t \left[Q_{t,t+1} A_{t+1}\right] \tag{25}$$

whose net interest rate, i_t^A can be written as

$$1 = \mathbb{E}_t \left[Q_{t,t+1} \left(1 + i_t^A\right)\right] \tag{26}$$

like any other return, such as

$$1 = \mathbb{E}_t \left[Q_{t,t+1} \left(1 + i_{t+1}^{\text{Risky}}\right)\right] \tag{27}$$

The conditional expectation of the stochastic discount factor is also the inverse of the gross rate at the end of period t, $1 + i_t^{RF}$, of an asset that pays with certainty 1 unit of the monetary unit of account at period $t+1$

$$\frac{1}{1 + i_t^{RF}} = \mathbb{E}_t \left[Q_{t,t+1}\right] \tag{28}$$

Equations (23), (24), and (25) completely describe the resources constraint. Using (24) and (25) to eliminate B_t in (23), we can rewrite the flow budget constraint as

$$P_t C_t + M_t \left(1 - \mathbb{E}_t \left[Q_{t,t+1} \left(1 + i_{t+1}^{\text{Risky}}\right)\right]\right) + \mathbb{E}_t \left[Q_{t,t+1} W_{t+1}\right] \leq W_t + \left[P_t Y_t - T_t\right] \tag{29}$$

The household can choose any non-negative value for C_t and M_t as long as they are jointly consistent with (29) given her expected state-contingent wealth at the beginning of next period, W_{t+1}. Note that if we imposed the assumption of risk-neutrality to the households or, still, if we assumed that the risky component of i_{t+1}^{Risky} is completely exogenous, then we could rewrite (29) as

$$P_t C_t + \Delta_t M_t + \mathbb{E}_t \left[Q_{t,t+1} W_{t+1} \right] \leq W_t + \left[P_t Y_t - T_t \right] \tag{30}$$

where $\Delta_t \equiv \frac{i_t^{RF} - \mathbb{E}_t\, i_{t+1}^{\text{Risky}}}{1+i_t^{RF}}$, which represents the opportunity cost of holding wealth in risky-monetary form if the risk-free asset exists.

We proceed further by imposing two restrictions. First, a limit on borrowing to avoid Ponzi schemes at all times (and all states of nature),

$$\sum_{T=t}^{\infty} \mathbb{E}_t \left[Q_{t,T} \left(P_T Y_T - T_T \right) \right] < \infty \tag{31}$$

where $Q_{t,T} \equiv \prod_{s=t+1}^{T} Q_{s-1,s}$. Second, we formalize the no arbitrage assumption. If the risk-free asset does exits, then at all times it must be true that

$$\mathbb{E}_t \left[Q_{t,t+1} \left(1 + i_t^{RF} \right) \right] \geq \mathbb{E}_t \left[Q_{t,t+1} \left(1 + i_t^{A} \right) \right] \geq \mathbb{E}_t \left[Q_{t,t+1} \left(1 + i_{t+1}^{\text{Risky}} \right) \right] \tag{32}$$

so the household cannot borrow without limits at the risk-free rate to finance unlimited consumption and still pay back its loan at the next period. A proof that the infinite sequence of flow budget constraints, (29) or (30), are equivalent to a single intertemporal budget constraint is obtainable by following the same steps of the one available in Woodford (2003, ch. 2, Appendix, Proof of Proposition 2.1). The optimizing problem of the representative household consists in choosing $C_t \geq 0$ and $M_t \geq 0$ for all periods $t \geq 0$ satisfying the sequence of (29), or (30), given its initial wealth, W_0, in addition to both goods and assets expected prices to maximize (22) subject to (31) and (32) at all periods. Finally, we impose the cashless limit, which in our economy requires that at all periods either $M_t = 0$ or (32) holds with equality (in case the risk-free asset exists). Given the optimal path for the policy variables (C_t and M_t), the sequence of budget constraints delivers a path for W_t, which, by its turn, results in a path for A_t, with (24), and B_t, with (25). The first-order conditions of this optimization are

$$\frac{u_c \left(C_t; \xi_t \right)}{u_c \left(C_{t+1}; \xi_{t+1} \right)} = \frac{\beta}{Q_{t,t+1}} \frac{P_t}{P_{t+1}} \tag{33}$$

which must hold for all periods and states of nature, the sequence of flow budget constraints, (29) or (30), evaluated with equality, and the transversality condition

$$\lim_{T \to \infty} \mathbb{E}_t \left[Q_{t,T} W_T \right] = 0 \tag{34}$$

From (28), we have that the nominal risk-free rate will be given by

$$1 + i_t^{RF} = \beta^{-1} \left\{ \mathbb{E}_t \left[\frac{u_c(C_{t+1};\xi_{t+1})}{u_c(C_t;\xi_t)} \frac{P_t}{P_{t+1}} \right] \right\}^{-1} \tag{35}$$

We can establish, now, all the conditions for a rational expectations equilibrium in the model. In addition to the conditions previously stated for the representative household, markets must clear at all periods. Therefore, for all periods it must be that

$$C_t = Y_t \tag{36}$$

$$M_t = M_t^S \tag{37}$$

$$A_{t+1} = A_{t+1}^S \tag{38}$$

where M_t^S is the supply of the central bank's asset; and A_{t+1}^S is the aggregate value at the beginning of period $t+1$ of all assets not issued by the central bank at the end of period t. Had we not already limited bonds maturity to 1 period, A_{t+1}^S could be defined more broadly as

$$A_{t+1}^S \equiv \sum_{j=1}^{\infty} \mathbb{E}_{t+1} \left[Q_{t+1} B_{t,t+j}^S \right] \tag{39}$$

where $B_{t,t+j}^S$ refers to the supply of nominal coupons to be paid at period $t+j$ from bonds outstanding at the end of period t.

Moving further, if $M_t^S > 0$, then market-clearing will force (32) to be true with equality. We proceed by substituting (36) into (35) so to obtain the equilibrium condition for the nominal interest rate

$$1 + i_t^{RF} = \beta^{-1} \left\{ \mathbb{E}_t \left[\frac{u_c(Y_{t+1};\xi_{t+1})}{u_c(Y_t;\xi_t)} \frac{P_t}{P_{t+1}} \right] \right\}^{-1} \tag{40}$$

In case of a real risk-free asset, the equilibrium condition reduces to

$$1 + r_t^{RF} = \beta^{-1} \left\{ \mathbb{E}_t \left[\frac{u_c(Y_{t+1};\xi_{t+1})}{u_c(Y_t;\xi_t)} \right] \right\}^{-1} \tag{41}$$

Finally, the transversality condition (34) in equilibrium can be written in terms of the end-of-period value of total liabilities, $D_t \equiv M_t^S + B_t^S$, as

$$\lim_{T\to\infty} \beta^T \mathbb{E}_t \left[u_c(Y_T;\xi_T) \frac{D_T}{P_T} \right] = 0 \tag{42}$$

and the Ponzi scheme restriction can be described as

$$\beta^T \sum_{T=t}^{\infty} \mathbb{E}_t \left[u_c(Y_T;\xi_T)\, Y_T \right] < \infty \tag{43}$$

We proceed further by characterizing policy in this economy. Although not necessary, we assume risk-neutrality as in Assumption 9 to ease the algebra. The monetary policy in this economy is conducted with a (linearized) Wicksellian risky policy rule as proposed by Assumption 10. The policy-asset default probability, $\{\mathscr{D}_t\}$, is exogenous as in Assumption 8, and we also assume exogeneity for the haircut $\{\delta_t\}$ but imposing that $0 \leq \delta_t \leq 1 \, \forall \, t$, so the return in case of default must be lower or equal to the policy rate in case of repayment at all times. The no-arbitrage Assumption 7 further implies the relation (9) between the conditional expected value of the risky rate, $\mathbb{E}_t \, i_{t+1}^{\text{Risky}}$, and the realized value in each possible future state of nature (default or not default). The supply of central bank's assets, M_t^S, does not enter in the equilibrium conditions, for we simplify assuming it to also follow an exogenous path. Finally, we assume in this paper that fiscal policy is always kept passive. One simple way of characterizing such a policy is assuming a balanced-budget rule so that $\Delta D_t = 0$ at all periods. This sets an exogenous path for $\{D_t\}$. We are ready to define the rational-expectations equilibrium of our model.

Definition C.1. *A rational-expectations equilibrium of the default-risky cashless economy is a set of processes* $\left\{P_t, i_t^{Risky}\right\}$ *that satisfy the no-arbitrage condition (32) with equality, the Euler equation of the nominal risk-free asset (40), the transversality condition (42), and the monetary policy rule (Assumption 10) at all periods* $t \geq 0$*, given the exogenous processes* $\{Y_t, \xi_t\}$*, which satisfy the no-Ponzi scheme condition (43),* $\{0 \leq \mathscr{D}_t \leq 1\}$*,* $\{0 \leq \delta_t \leq 1\}$*, and* $\left\{M_t^S, D_t\right\}$*.*

D Monetary policy targets the risky rate using safe assets

In Bi, Leeper and Leith (2018), the authors portray the interest rate rule as targeting either a default-risk-free rate or a default-risky rate, which slightly adapted to our notation result in rules (44) and (45), respectively, where π^{Target} is the fixed net inflation target, φ is the response coefficient to inflation deviation, i_t^{RF} is the nominal net risk-free rate, and i_{t+1}^{R} is the nominal net default-risky rate. They do not assume, though, that the central bank uses a risky asset to conduct monetary policy, instead they claim that through open market operations, with a presumably safe policy asset, the central bank can affect the risky asset interest rate.

$$\frac{1}{1+i_t^{RF}} = \frac{1}{1+\bar{\imath}_t} + \varphi\left(\frac{1}{1+\pi_t} - \frac{1}{1+\pi^{\text{Target}}}\right) \tag{44}$$

$$\frac{1}{1+i_{t+1}^{R}} = \frac{1}{1+\bar{\imath}_t} + \varphi\left(\frac{1}{1+\pi_t} - \frac{1}{1+\pi^{\text{Target}}}\right) \tag{45}$$

In their exposition of the interest rate mechanism, they shut down the nominal rigidity of their cashless New-Keynesian model, assuming then flexible prices, and replace the production function by a constant endowment specification. In this simplified economy, the Euler equation generates the following Fisher equations for each policy rule, respectively

$$\frac{1}{1+i_t^{RF}} = \beta\mathbb{E}_t\left[\frac{1}{1+\pi_{t+1}}\right] \tag{46}$$

$$\frac{1}{1+i_{t+1}^{R}} = \beta\mathbb{E}_t\left[\frac{1-\delta_{t+1}}{1+\pi_{t+1}}\right] \tag{47}$$

where β is the subjective discount factor, and δ_{t+1} is the stochastic fraction at which the government defaults at period $t+1$. In case of full repayment, $\delta_{t+1}=0$.

Next, they derive a law of movement for inflation when each rule is adopted by the monetary authority, respectively equations (48) and (49).

$$\left(\frac{1}{1+\pi_t} - \frac{1}{1+\pi^{\text{Target}}}\right) = \frac{\beta}{\varphi}\mathbb{E}_t\left[\frac{1}{1+\pi_{t+1}} - \frac{1}{1+\pi^{\text{Target}}}\right] \tag{48}$$

$$\left(\frac{1}{1+\pi_t} - \frac{1}{1+\pi^{\text{Target}}}\right) = \frac{\beta}{\varphi}\mathbb{E}_t\left[\frac{1-\delta_{t+1}}{1+\pi_{t+1}} - \frac{1}{1+\pi^{\text{Target}}}\right] \tag{49}$$

As Reis (2018) points out, in this derivation, they make an implicit assumption that will bias their results. They assume that the intercept of both policy rules is fixed as well as it is the same and given by $\bar{\imath}_t = \frac{1+\pi^{\text{Target}}}{\beta} - 1$. This intercept only makes sense, though, for when the central bank targets a risk-free rate. When the targeted rate is risky, the intercept should adjust to the risk premium embedded in the risky bond: $\bar{\imath}_t = \frac{1+\pi^{\text{Target}}}{(1-\delta_{t+1})\beta} - 1$. Afterward, the authors iterate forward the laws of movement of inflation, and obtain for the risk-free and the risky rate targets, respectively, equations (50) and (51).

$$\frac{1}{1+\pi_t} = \frac{1}{1+\pi^{\text{Target}}} \tag{50}$$

$$\frac{1}{1+\pi_t} = \frac{1}{1+\pi^{\text{Target}}}\left(1-\frac{\beta}{\varphi}\right)\left[1+\sum_{j=1}^{\infty}\left(\frac{\beta}{\varphi}\right)^j \Pi_{k=1}^{j}(1-\delta_{t+k})\right] \tag{51}$$

To make the analysis more tractable, the authors impose to equation 51 a constant default rate for all periods ($\delta_t = \delta$), and obtain a biased expression for inflation (52).

$$(1+\pi_t) = \left(1+\pi^{\text{Target}}\right)\left(\frac{1-\frac{\beta(1-\delta)}{\varphi}}{1-\frac{\beta}{\varphi}}\right) \tag{52}$$

Finally, Bi, Leeper and Leith (2018) rewrite the risky interest rate policy rule in terms of the risk-free rate.

$$\frac{1}{1+i_t^{RF}} = \frac{1}{1+\bar{\imath}_t} + \frac{\varphi}{1-\delta}\left[\frac{1}{1+\pi_t} - \left(\frac{1}{1+\pi^{\text{Target}}} - \frac{\delta}{\varphi(1+\bar{\imath}_t)}\right)\right] \tag{53}$$

From equations (50) and (53), we can summarize the main results of Bi, Leeper and Leith (2018) concerning monetary policy. First, if the central bank targets the risk-free rate using a risk-free instrument, the presence of government default risk does not prevent the central bank from hitting the inflation target. Second, default raises the effective gross inflation target from $1+\pi^{\text{Target}}$ to $\frac{1+\pi^{\text{Target}}}{1-\delta\frac{\beta}{\varphi}}$, so that the announced inflation target is unattainable ($1-\delta\frac{\beta}{\varphi} < 1$). One can find that result by substituting the intercept used by the authors into the right-hand side of equation 53, and comparing the announced inflation target with the effective one. Third, the authors say that default makes monetary policy "more active" since the effective inflation feedback coefficient becomes higher: $\frac{\varphi}{1-\delta} > \varphi$.

Now, we correct the intercept of equation (51), as suggested by Reis (2018), for the case in which the central bank targets the risky interest rate.

$$\left(\frac{1}{1+\pi_t} - \frac{1}{1+\pi^{\text{Target}}}\right) = \frac{\beta}{\varphi}\mathbb{E}_t(1-\delta_{t+1})\left[\frac{1}{1+\pi_{t+1}} - \frac{1}{1+\pi^{\text{Target}}}\right] \tag{54}$$

Iterating forward equation (54), and imposing a constant default rate ($\delta_t = \delta$), we get

$$\frac{1}{1+\pi_t} = \frac{1}{1+\pi^{\text{Target}}} \tag{55}$$

just like when the central bank targets the risk-free rate (50), what invalidates Bi, Leeper and Leith (2018)'s second result. Thus, after adjusting the intercept in Bi, Leeper and Leith

(2018), whether the central bank targets a risk-free or a risky rate, the announced inflation target and the effective one will be the same. Rewriting the policy rule that targets the risky interest rate in terms of the risk-free rate it becomes now

$$\begin{aligned}\frac{1}{1+i_t^{RF}} &= \frac{1}{(1-\delta)(1+\bar{\imath}_t)} + \frac{\varphi}{1-\delta}\left[\frac{1}{1+\pi_t} - \frac{1}{1+\pi^{\text{Target}}}\right] \\ &= \frac{\beta}{1+\pi^{\text{Target}}} + \frac{\varphi}{1-\delta}\left[\frac{1}{1+\pi_t} - \frac{1}{1+\pi^{\text{Target}}}\right] \\ &\text{(after a log-approximation of interest rates)} \\ i_t^{RF} &\approx r^n + \pi^{\text{Target}} + \frac{\varphi}{1-\delta}\left[\pi_t - \pi^{\text{Target}}\right]\end{aligned} \tag{56}$$

Equation (56) provides us the risk-free rule that would be equivalent to targeting the risky rate. As one can see from that equation, the effective intercept is now the gross nominal risk-free rate, $\left(\frac{1+\pi^{\text{Target}}}{\beta} = \left(1+\pi^{\text{Target}}\right)(1+r^n)\right)$. Moreover, Bi, Leeper and Leith (2018)'s third result, that default makes monetary policy "more active", remains even after we have fixed the intercept, in apparent divergence with our finding that it makes it "less active". However, the difference lies in the fact that our policy asset is risky while theirs is still risk-free although specified in terms of a risky rate. A proper comparison between the inflation-reaction coefficients would be $\frac{\varphi}{1-\delta}$ against $\phi(1-\delta)$. Under our specification, default weakens the effective response of inflation to changes in the policy rate.

Naturally, fixing the intercept will also change the dynamics of Bi, Leeper and Leith (2018)'s model, for we actually have a different rule now than the one proposed in that paper, more specifically, the positive link between default and inflation seems, at least, partially compromised.

E Monetary policy power

In this appendix, we simulate monetary policy power under a price level targeting regime for 5 different correlation values (-1.0, -0.5, 0.0, 0.5, 1.0) between r_t^n and $\mathbb{E}_t \mathscr{D}_{t+1}$ in Figures 22, 23, 24, 25, and 26. We randomly draw 10,000 vectors of r_t^n and $\mathbb{E}_t \mathscr{D}_{t+1}$ with 40 periods each from a multivariate normal distribution. We calibrate $\mathbb{E}\, r_t^n = 0.04$, $\text{Std}(r^n) = 0.005$, and set the intercept of the policy rule to be equal to $\mathbb{E}\, r_t^n$ at all periods. Additionally, we calibrate $\mathbb{E}\mathscr{D}_{t+1} = 0.10$ and $\text{Std}(\mathscr{D}_{t+1}) = 0.01$, while we suppose no recovery rate ($\delta_t = 1\, \forall\, t$). We replace negative numbers drawn with their absolute values.

In the graphs, we plot only 100 vectors and 40 periods to avoid too much cluttering. In the first column of each figure, we plot the scatter plot of the vectors drawn. In the second column, we plot the difference of the multiplier $\Upsilon_{t,j+1}$, which multiplies the natural interest rate, in the default-risky scenario against in the risk-free one for each term period. In the third column, we compare the same scenarios, but from the perspective of the coefficient that multiplies the policy rule intercept. Finally, our results show that monetary policy power reduces no matter the correlation between r_t^n and $\mathbb{E}_t \mathscr{D}_{t+1}$, what is easily observed in the graphs at the last column of each figure, as the sum of all terms from $t = 1$ to $t = 40$ aggregates to the total change of the price level at $t = 0$, which is largely positive for all values of ϕ simulated (0.1, 0.5, 1.0). Note that increasing ϕ reduces the total change of the price level due to the presence of default in the policy asset, while it makes more unlikely that individual period contributions are negative.

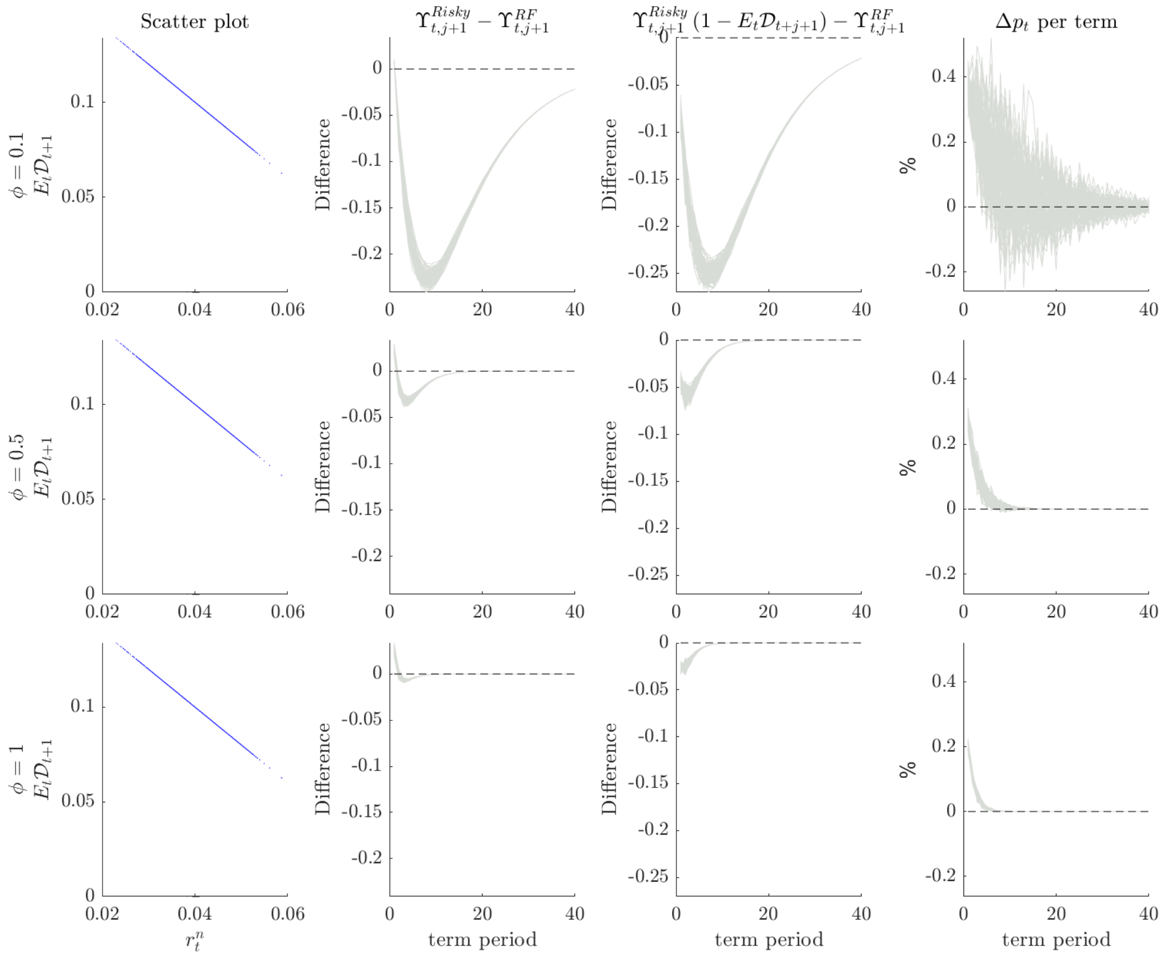


Figure 22: Monetary policy power under price level targeting: $Corr\left(r_t^n, \mathbb{E}_t \mathscr{D}_{t+1}\right)$ = -1

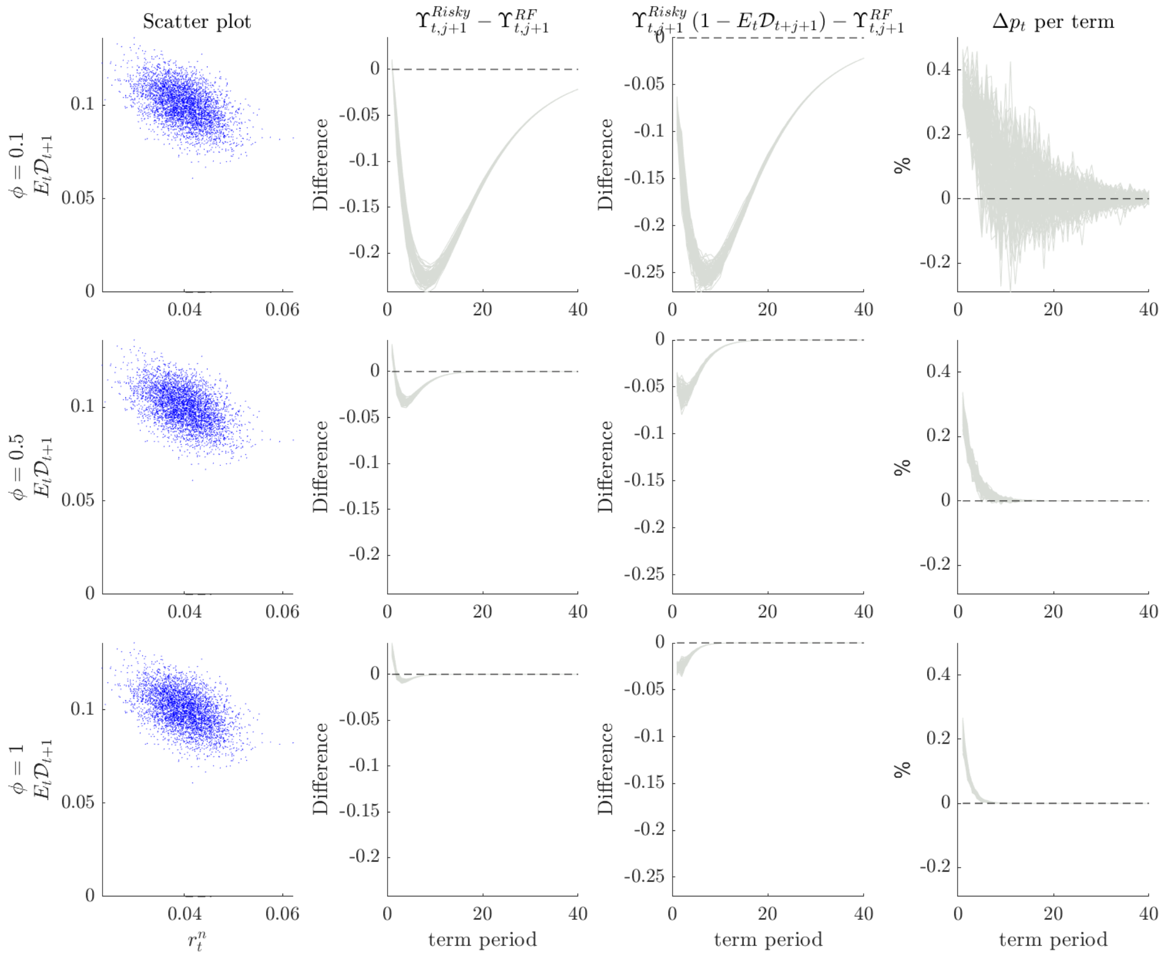


Figure 23: Monetary policy power under price level targeting: $Corr\left(r^n_t, \mathbb{E}_t\mathcal{D}_{t+1}\right)$ = -0.5

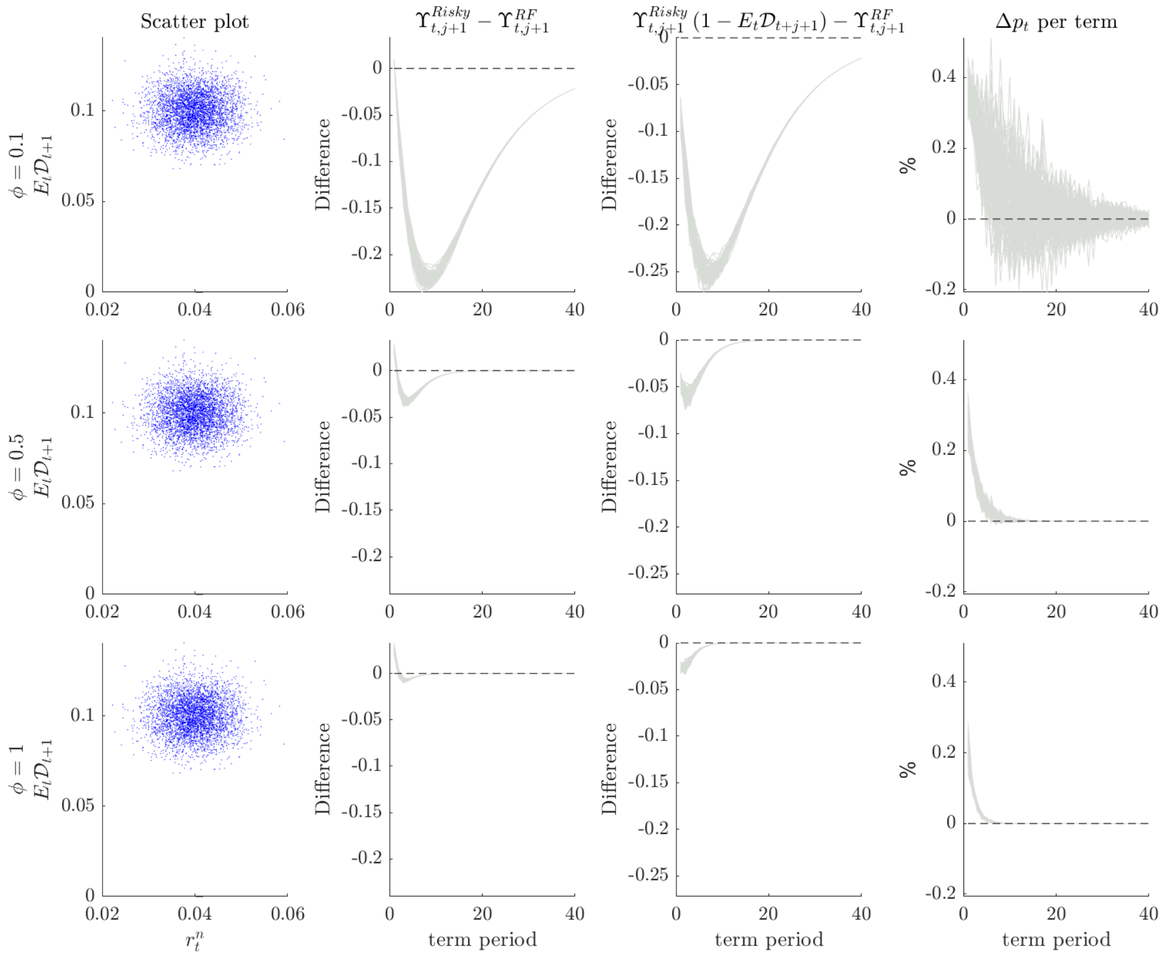


Figure 24: Monetary policy power under price level targeting: $Corr\left(r^n_t, \mathbb{E}_t\mathscr{D}_{t+1}\right) = 0$

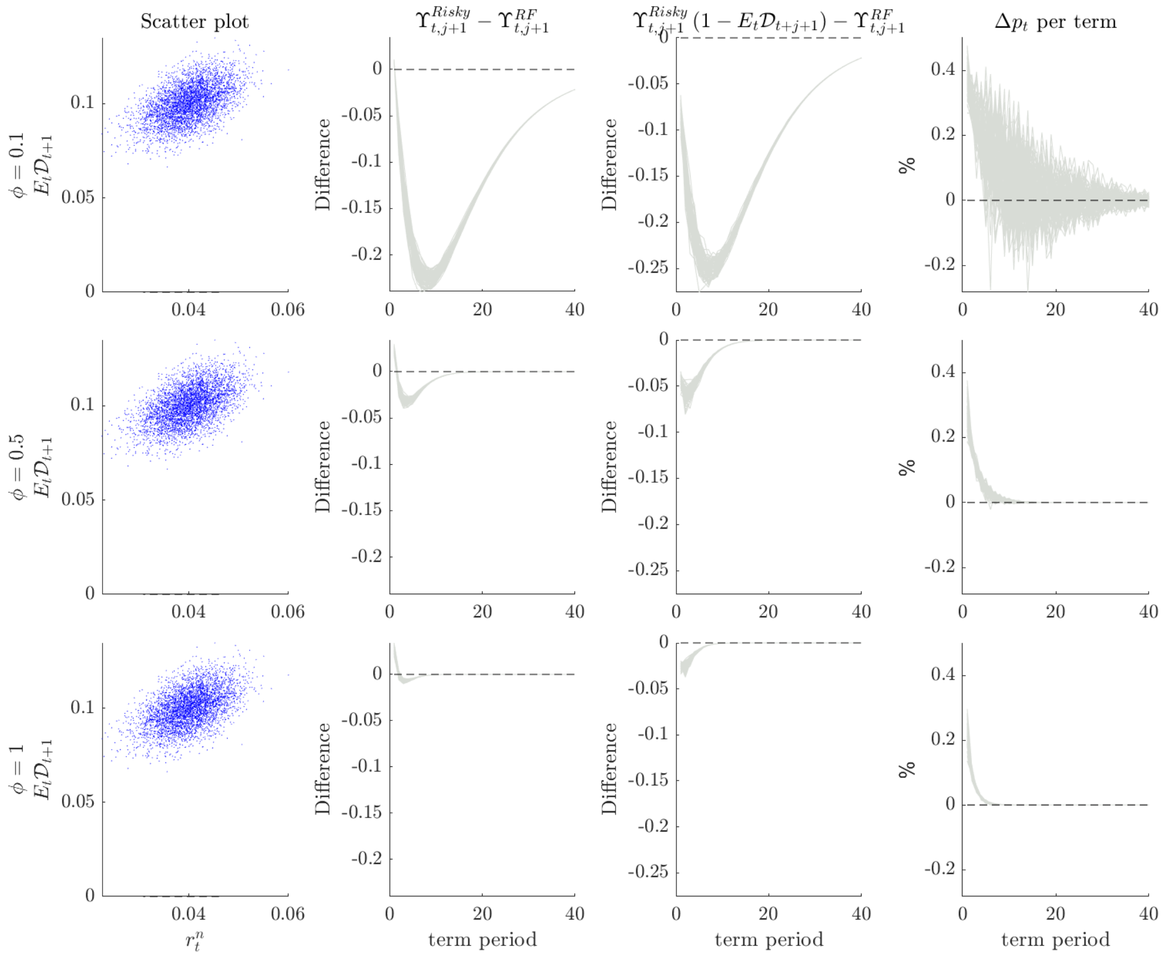


Figure 25: Monetary policy power under price level targeting: $Corr\left(r^n_t, \mathbb{E}_t \mathscr{D}_{t+1}\right) = 0.5$

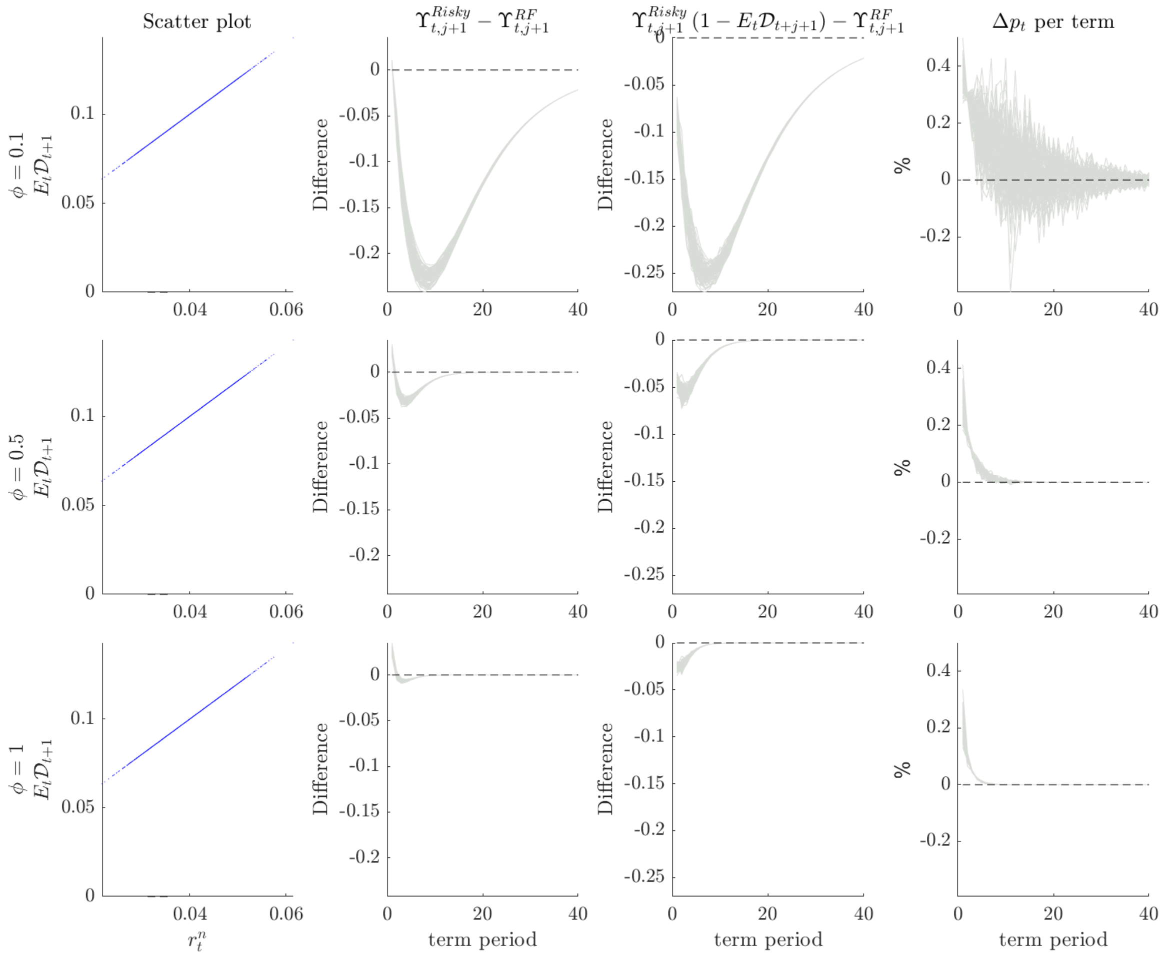


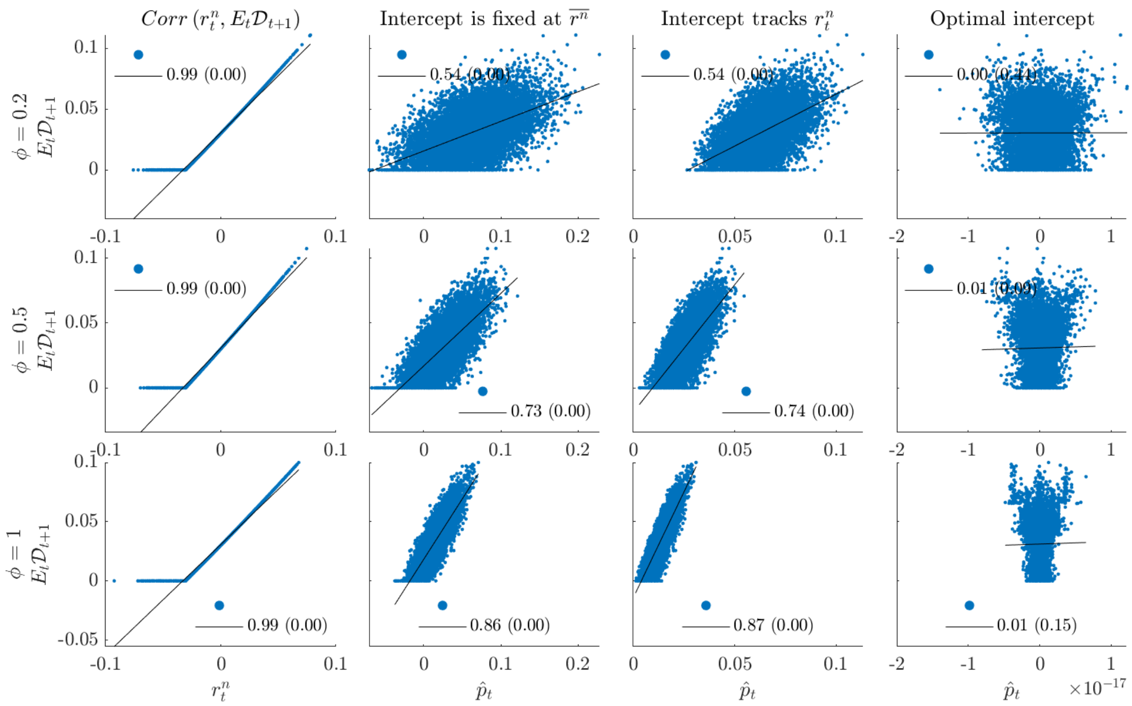
Figure 26: Monetary policy power under price level targeting: $Corr\left(r^n_t, \mathbb{E}_t\mathscr{D}_{t+1}\right) = 1$

F Testable implications of the model: correlations

In this section, we plot the correlation between the price level (or inflation) and the policy-asset risk for the cases when each r_t^n and $\mathscr{D}_{t+1}$ follows a normal AR(1) simulated process which is positively or negatively pairwise correlated, in complement to the uncorrelated case exposed in Figure 10.

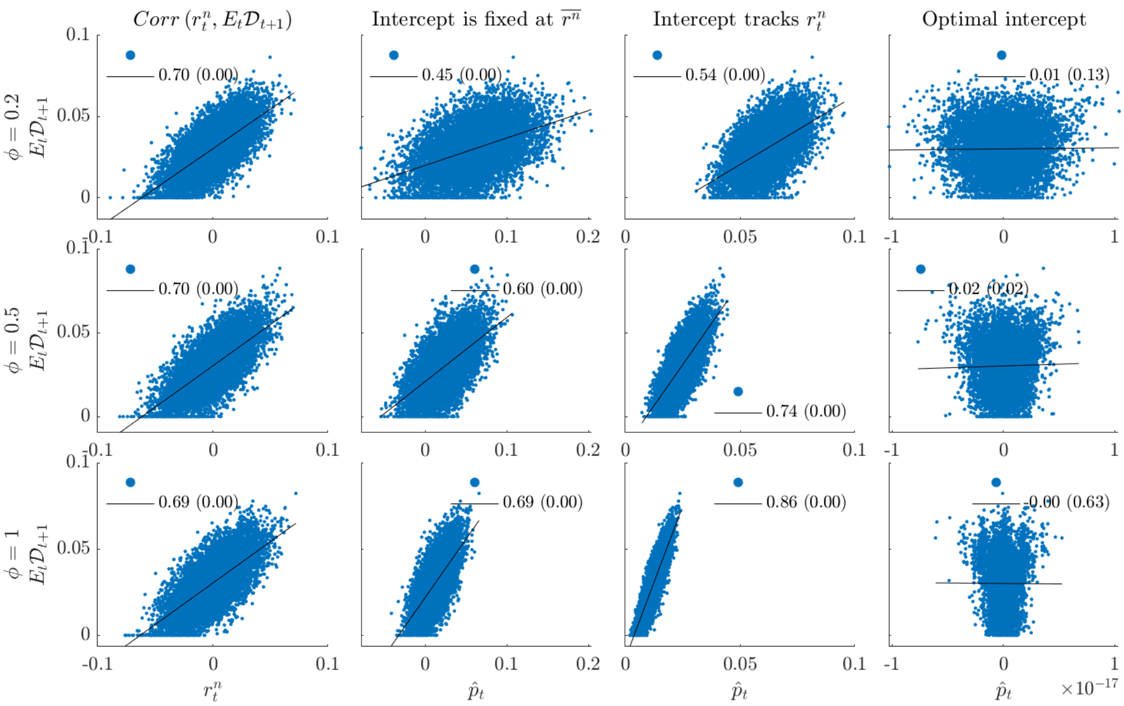
F.1 Price level targeting

Under price level targeting, we pick correlations to be 1 (Figure 27), 0.5 (Figure 28), -0.5 (Figure 29), and -1 (Figure 30). As one can see, the fact that tracking r_t^n induces positive correlation between these two variables is unchanged by the correlation of their underlying processes. The same is true for the fact that raising ϕ (the hawkishness of monetary policy) also raises that very same correlation.



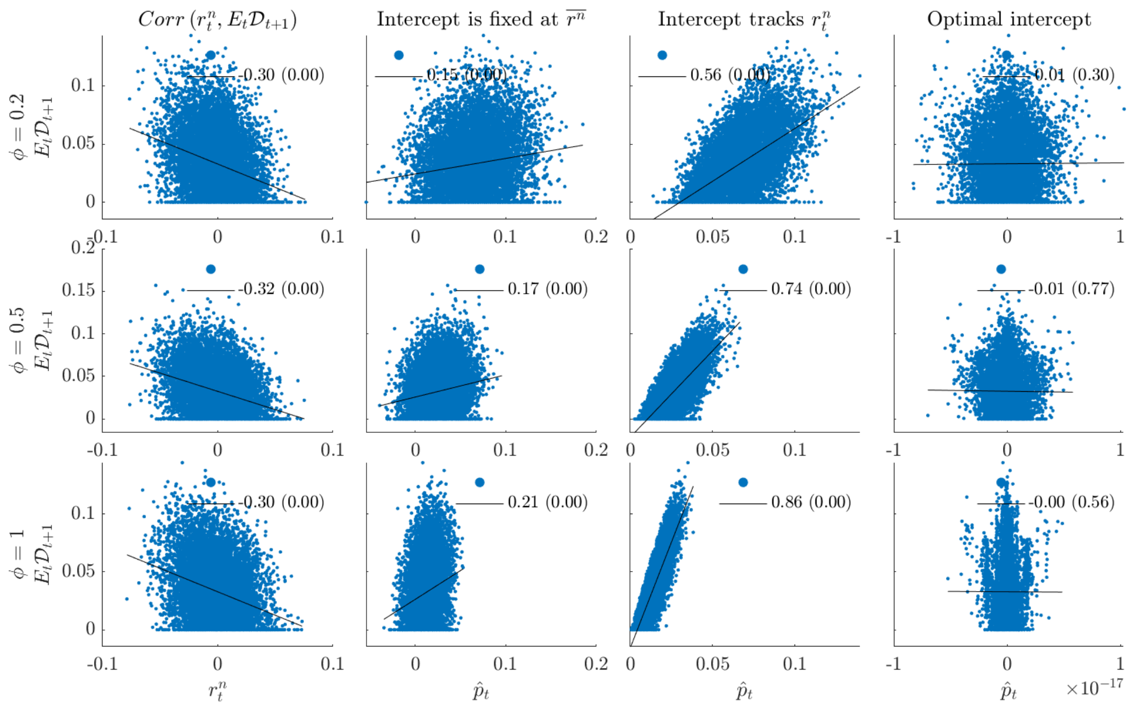
Note: p-values between parentheses.

Figure 27: Correlation between the default probability and the price level under price level targeting: $Corr\left(r_t^n, \mathbb{E}_t\mathscr{D}_{t+1}\right) = 1$



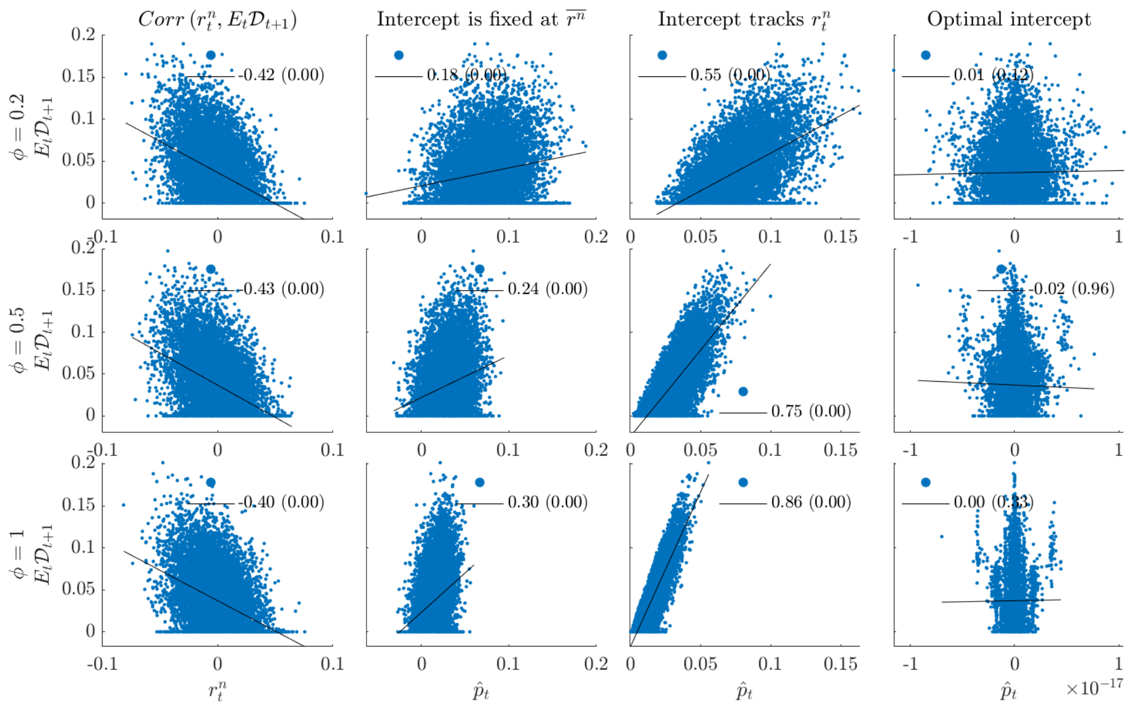
Note: p-values between parentheses.

Figure 28: Correlation between the default probability and the price level under price level targeting: $Corr\left(r_t^n, \mathbb{E}_t \mathscr{D}_{t+1}\right) = 0.5$



Note: p-values between parentheses.

Figure 29: Correlation between the default probability and the price level under price level targeting: $Corr\left(r_t^n, \mathbb{E}_t\mathscr{D}_{t+1}\right)$ = -0.5

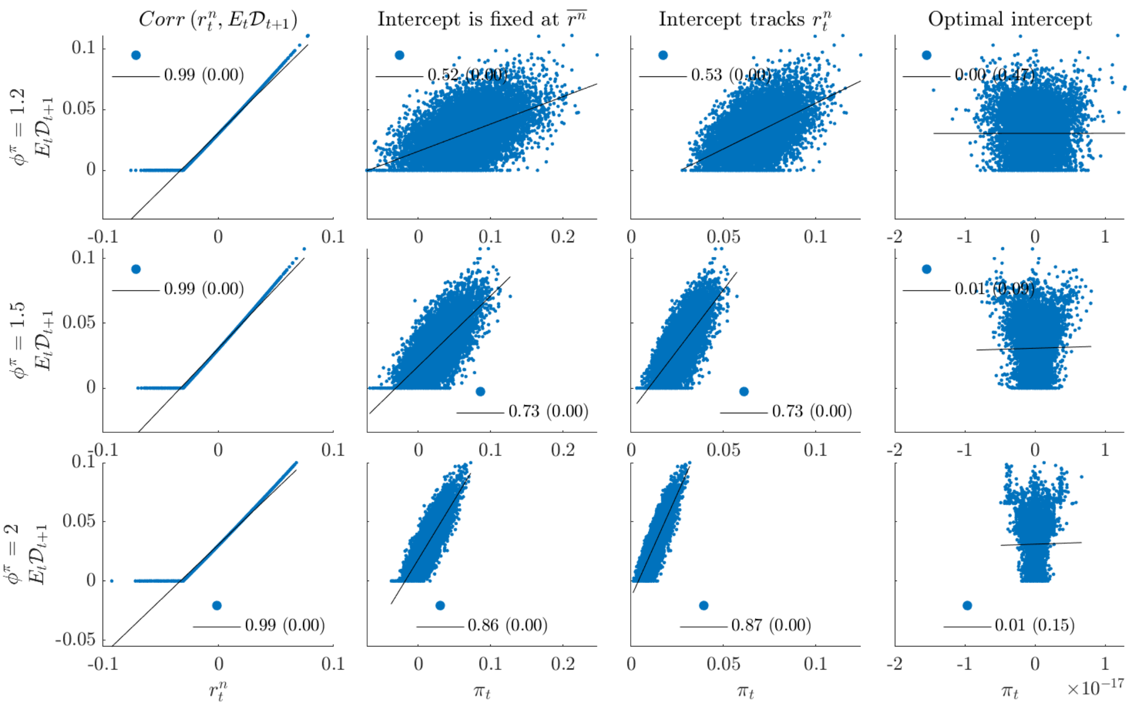


Note: p-values between parentheses.

Figure 30: Correlation between the default probability and the price level under price level targeting: $Corr\left(r_t^n, \mathbb{E}_t \mathscr{D}_{t+1}\right) = -1$

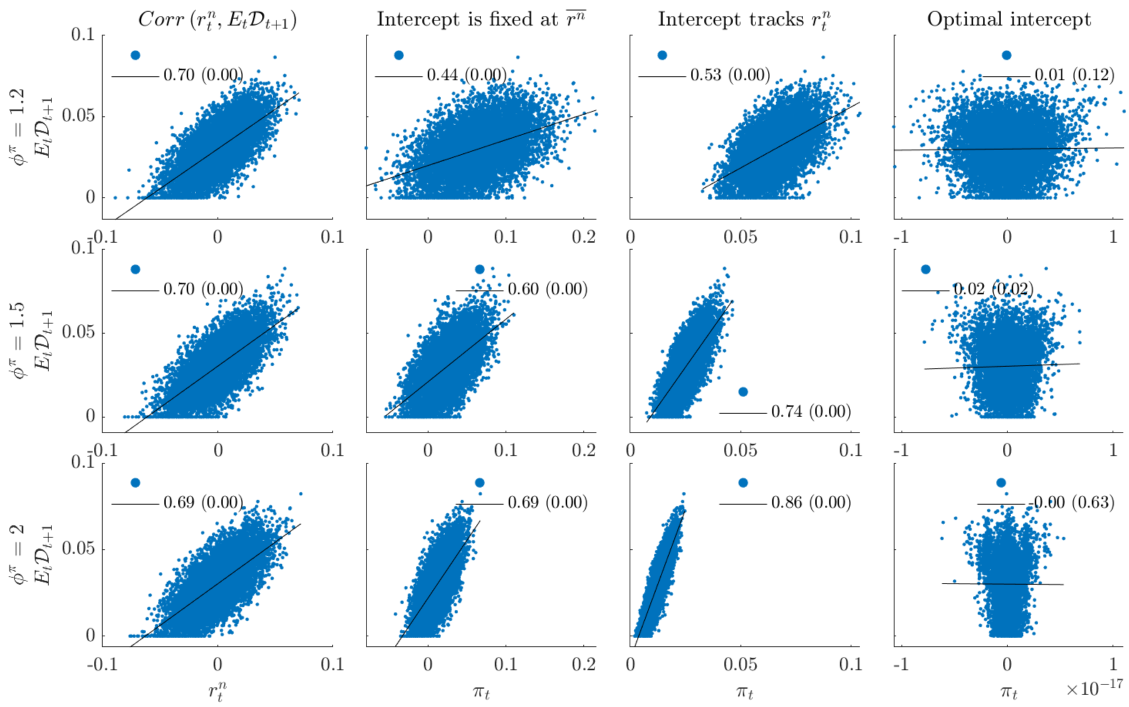
F.2 Inflation targeting

Under inflation targeting, we pick correlations to be 1 (Figure 31), 0.5 (Figure 32), -0.5 (Figure 33), and -1 (Figure 34). As one can see, the fact that tracking r_t^n induces positive correlation between these two variables is unchanged by the correlation of their underlying processes. The same is true for the fact that raising ϕ^π (the hawkishness of monetary policy) also raises that very same correlation.



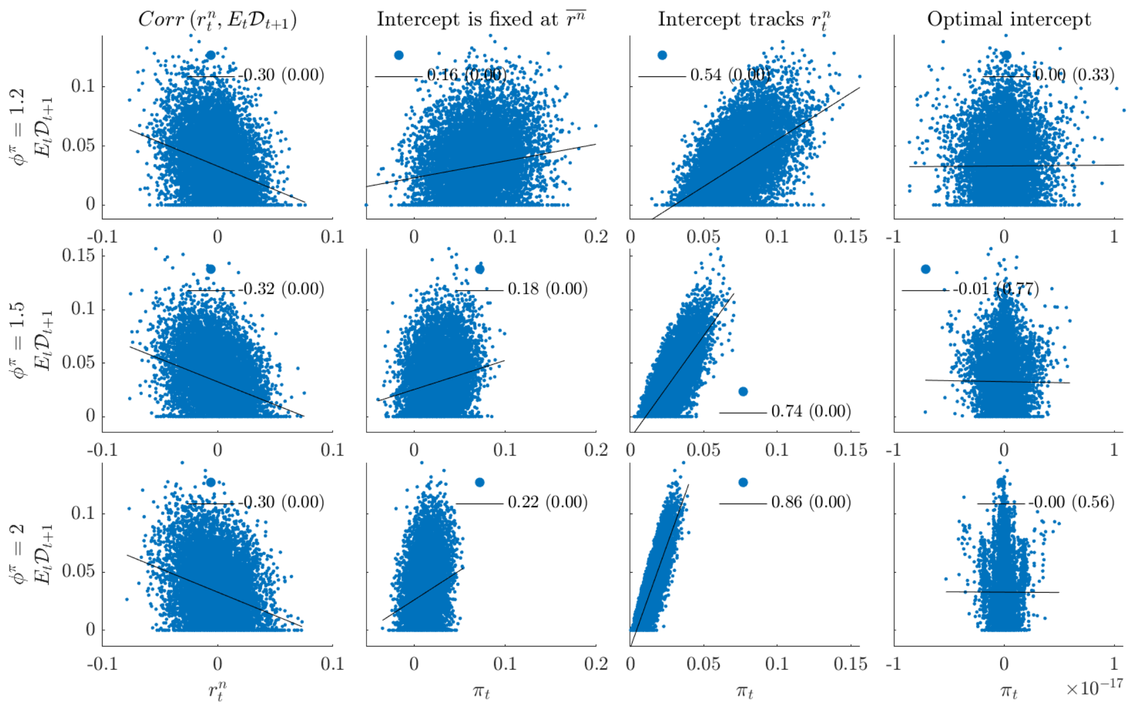
Note: p-values between parentheses.

Figure 31: Correlation between the default probability and inflation under inflation targeting: $Corr\left(r_t^n, \mathbb{E}_t\mathscr{D}_{t+1}\right) = 1$



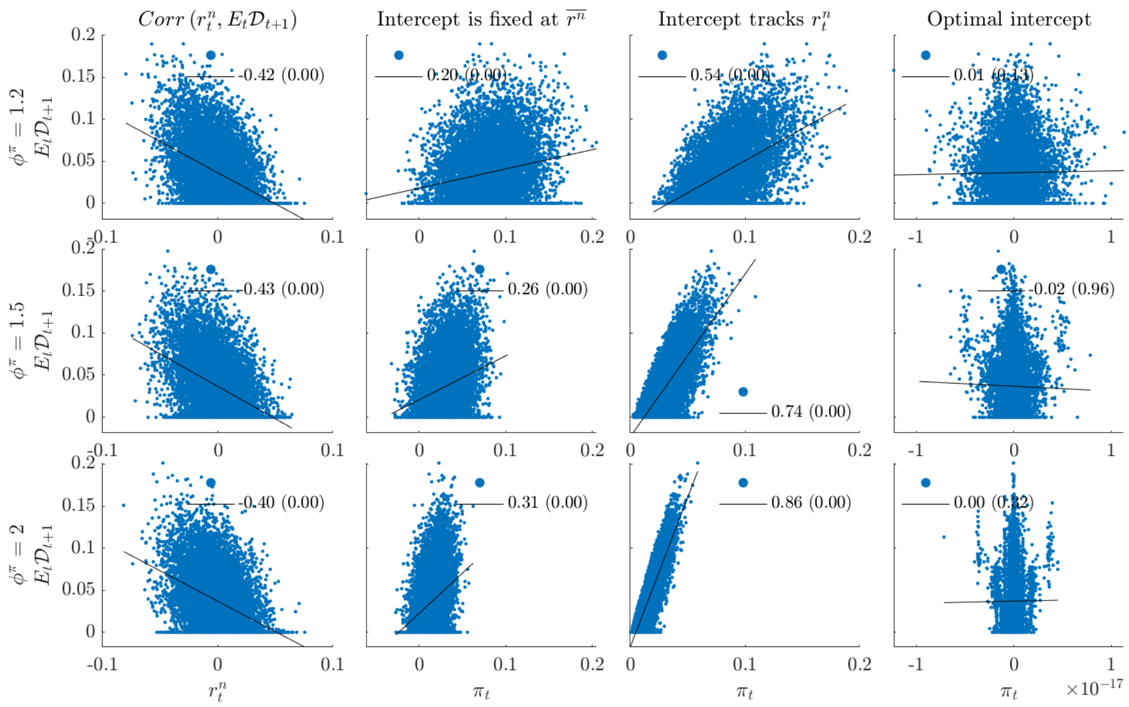
Note: p-values between parentheses.

Figure 32: Correlation between the default probability and inflation under inflation targeting: $Corr\left(r_t^n, \mathbb{E}_t \mathscr{D}_{t+1}\right) = 0.5$



Note: p-values between parentheses.

Figure 33: Correlation between the default probability and inflation under inflation targeting: $Corr\left(r_t^n, \mathbb{E}_t \mathscr{D}_{t+1}\right)$ = -0.5

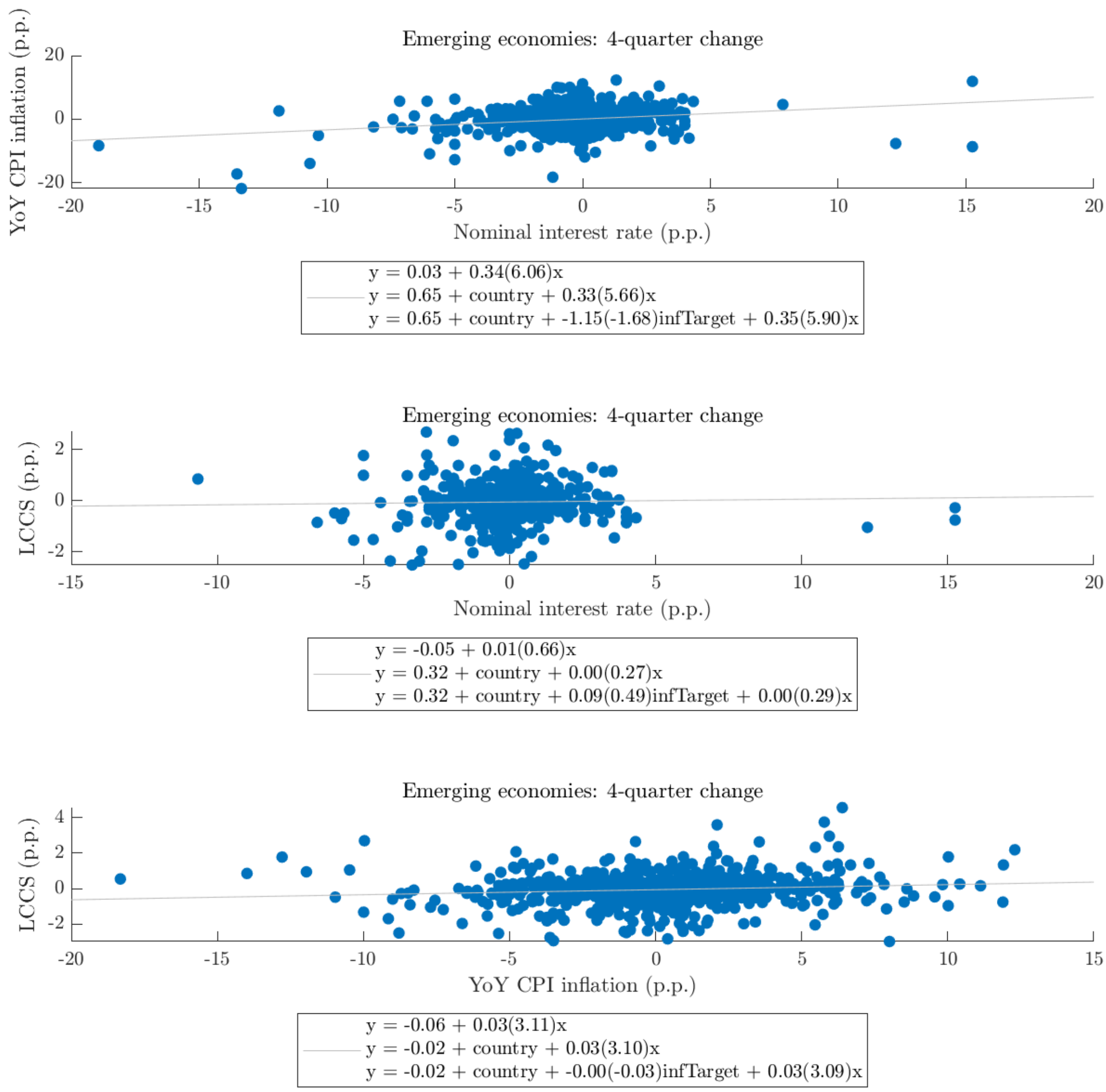


Note: p-values between parentheses.

Figure 34: Correlation between the default probability and inflation under inflation targeting: $Corr\left(r_t^n, \mathbb{E}_t \mathcal{D}_{t+1}\right)$ = -1

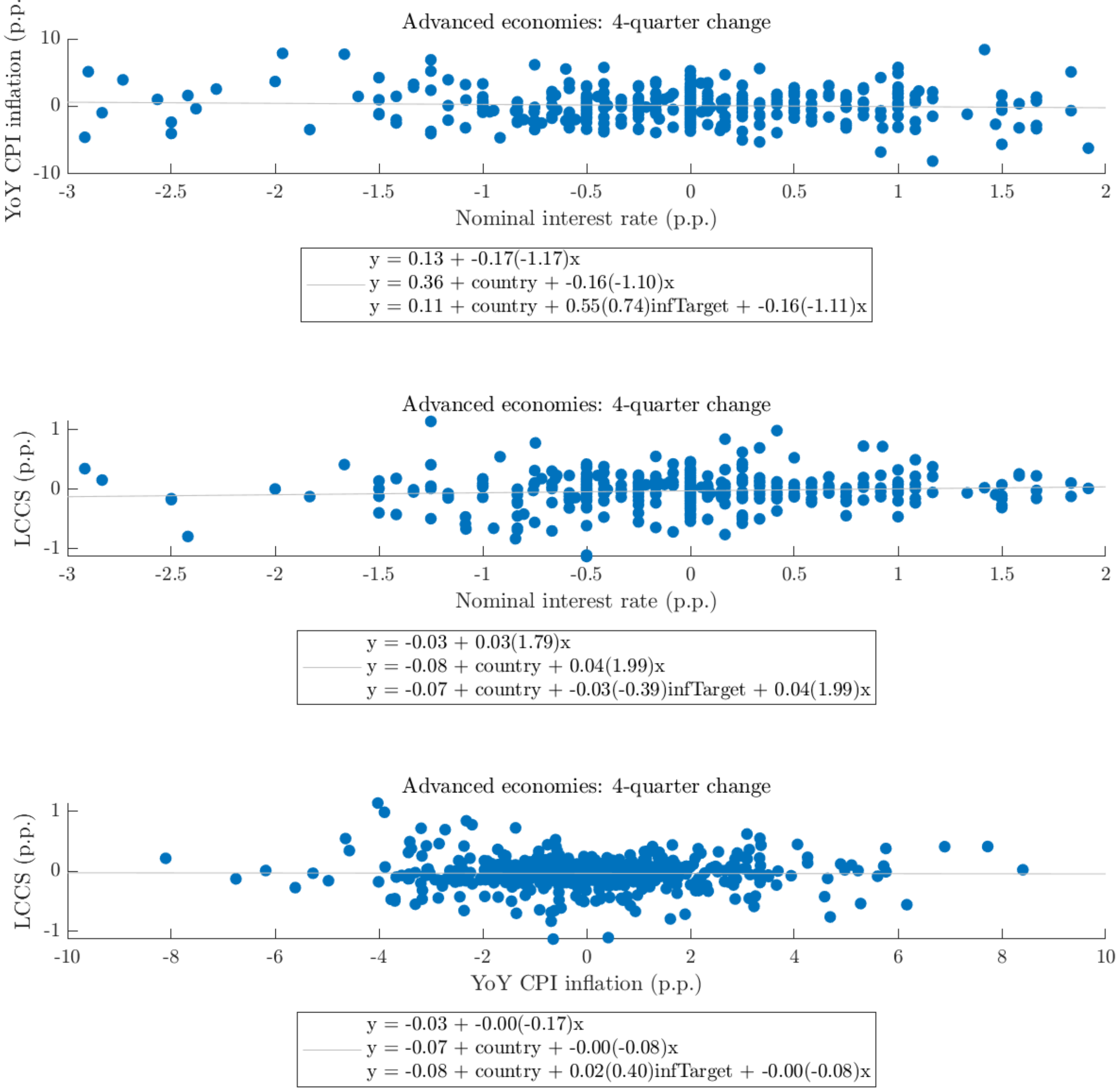
F.3 Alternative measures of risk

Here, we reproduce aggregate and country-specific correlations using Du and Schreger (2016)'s 5-year local-currency credit spread (LCCS) as a measure of risk.



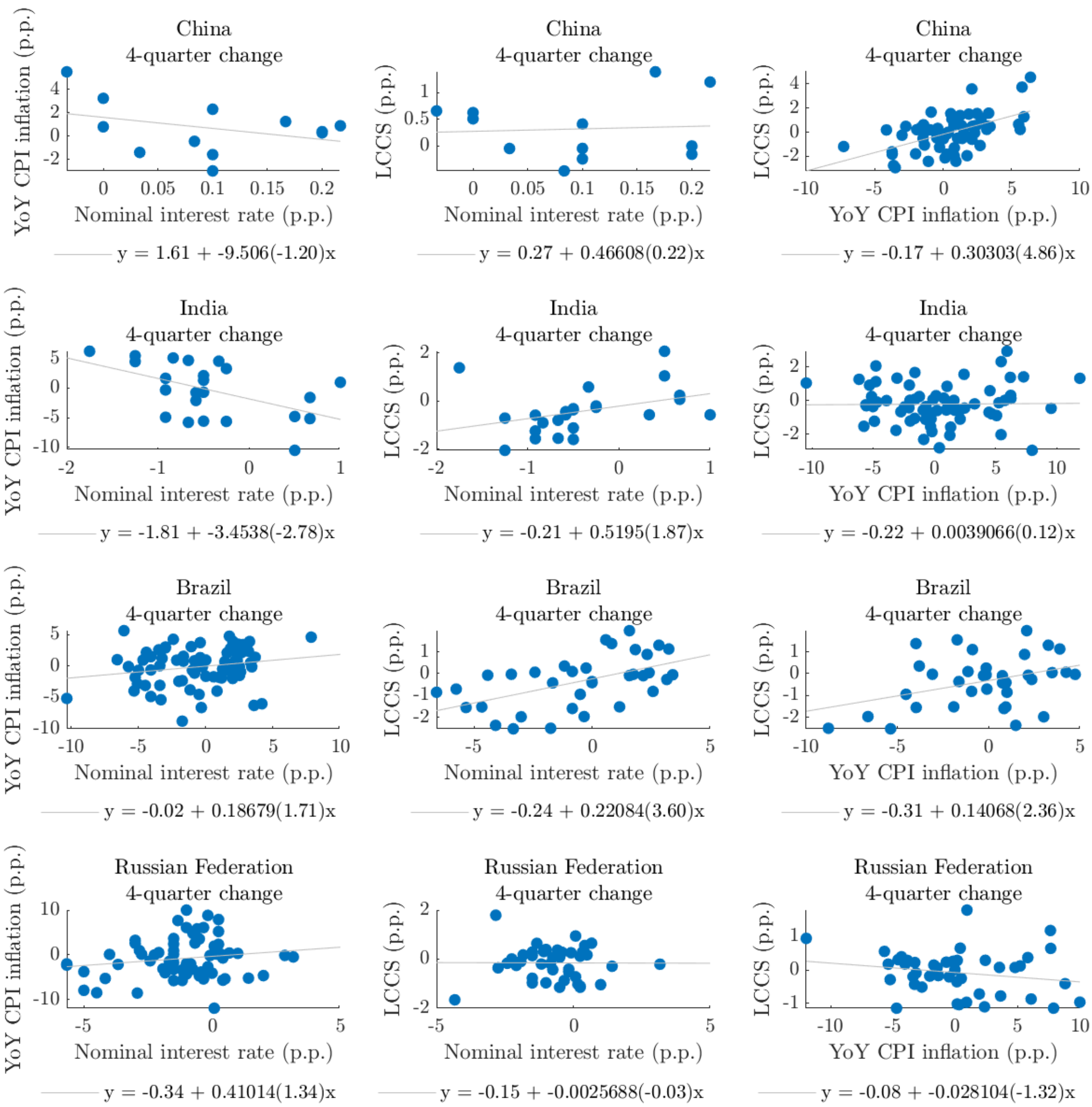
Note: between parentheses are t-statistics; "country" is a country-fixed effect; "infTarget" is a dummy that equals 1 when inflation targeting is adopted at the observation.

Figure 35: Scatter plot of pooled emerging economies: nominal interest rate, inflation, and LCCS



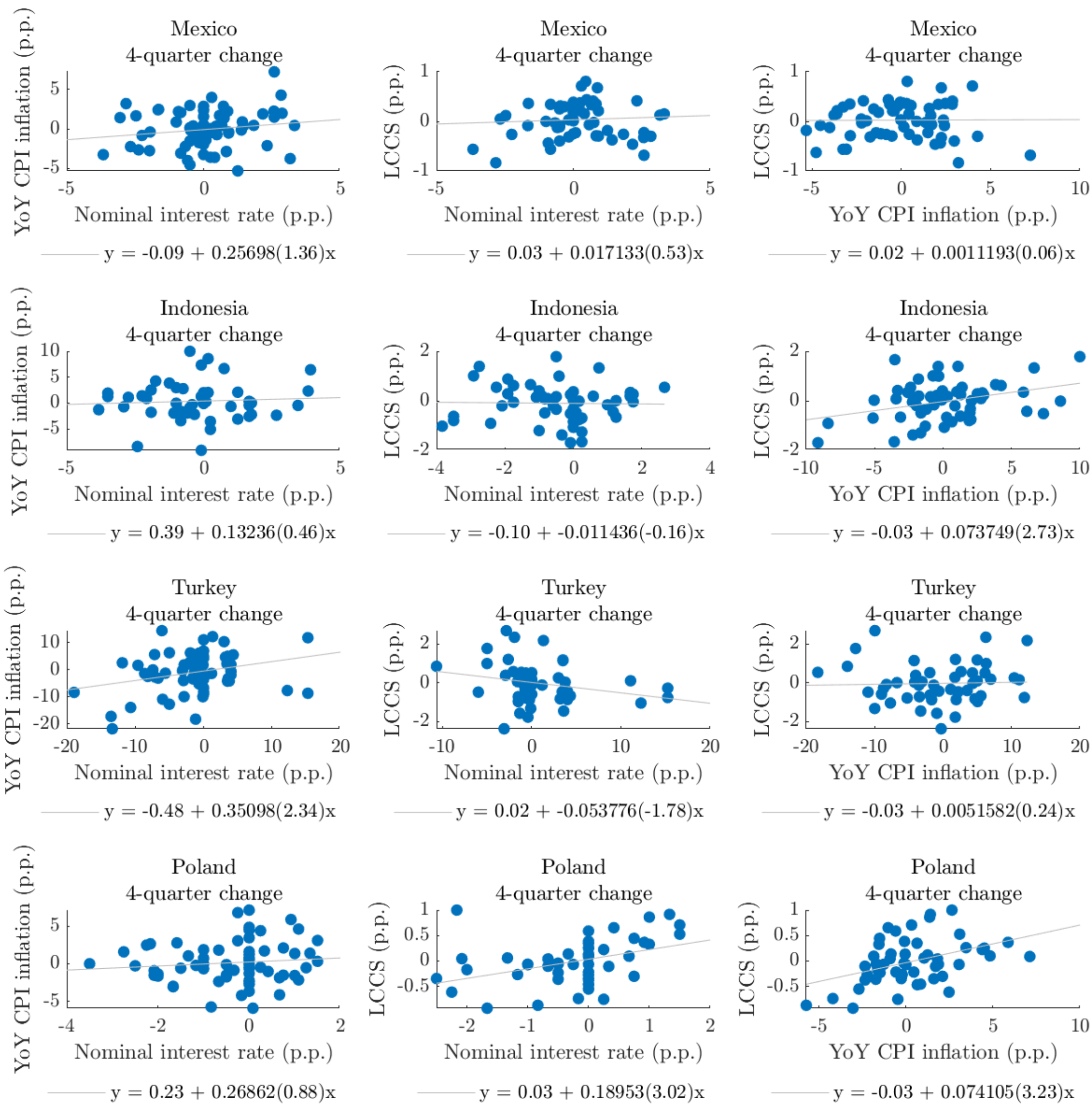
Note: between parentheses are t-statistics; "country" is a country-fixed effect; "infTarget" is a dummy that equals 1 when inflation targeting is adopted at the observation.

Figure 36: Scatter plot of pooled advanced economies: nominal interest rate, inflation, and LCCS



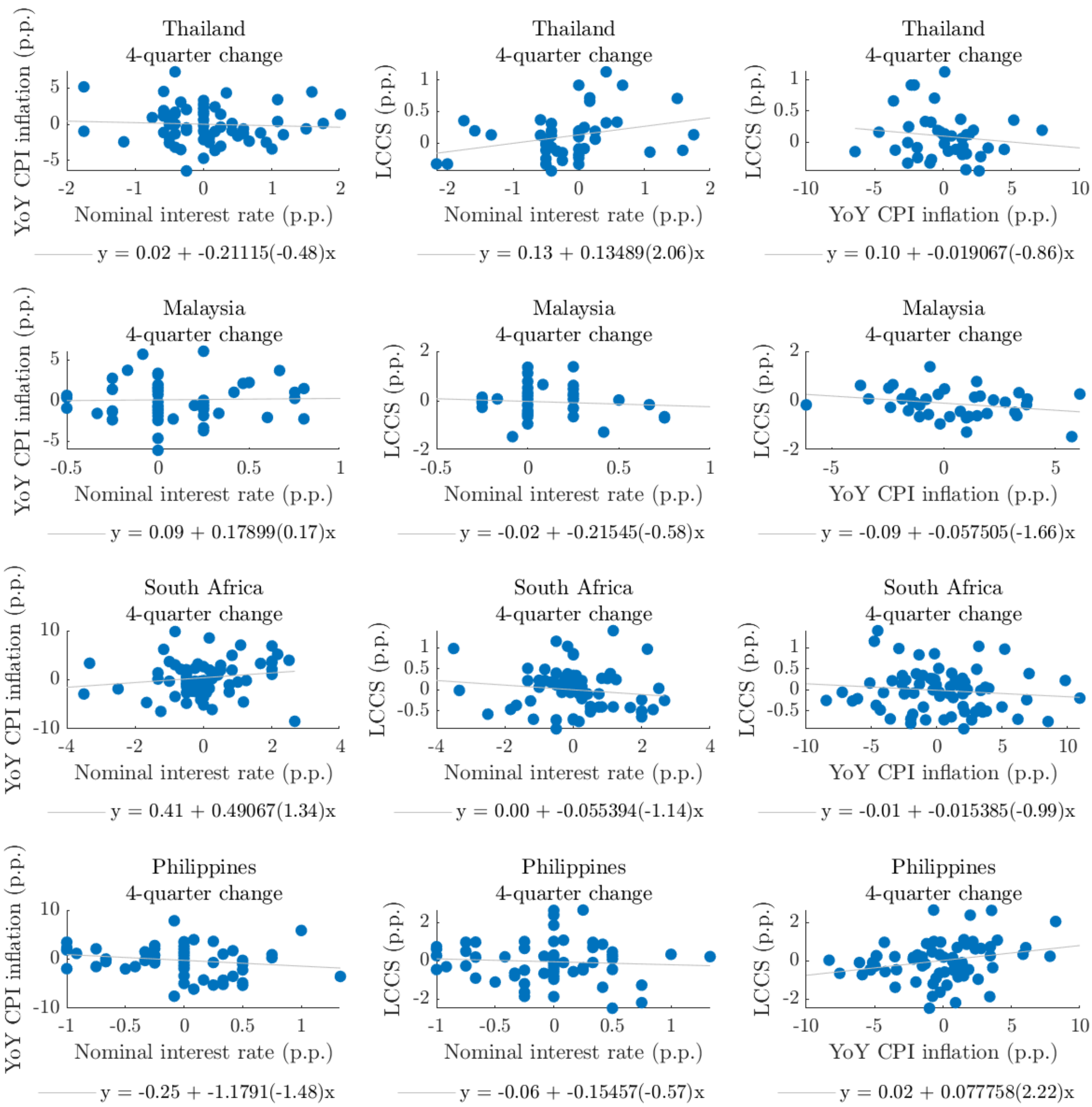
Note: t-statistics between parentheses; NaN means that there are not enough observations.

Figure 37: Scatter plot of emerging economies: nominal interest rate, inflation, and LCCS (Part 1)



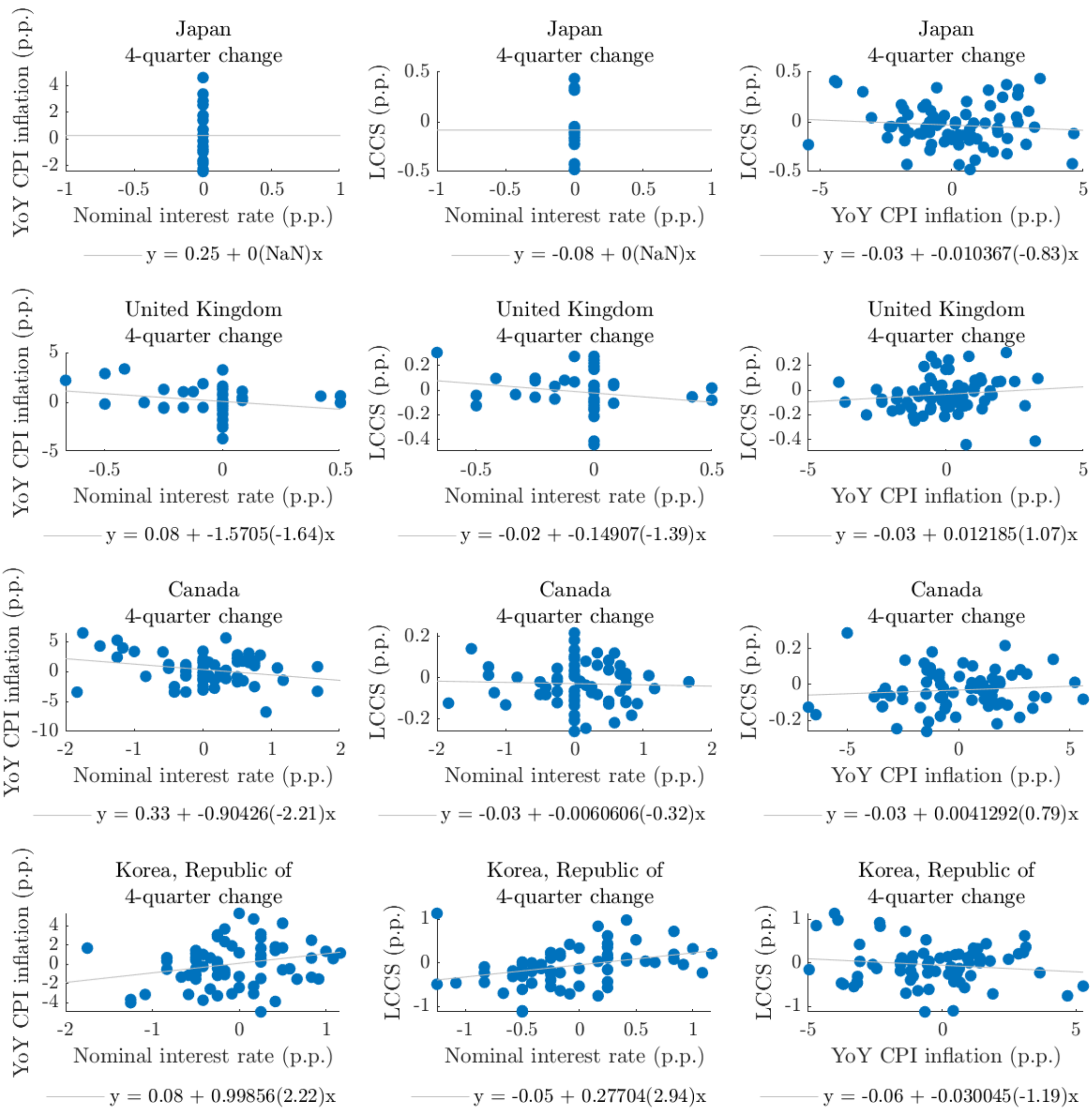
Note: t-statistics between parentheses; NaN means that there are not enough observations.

Figure 38: Scatter plot of emerging economies: nominal interest rate, inflation, and LCCS (Part 2)



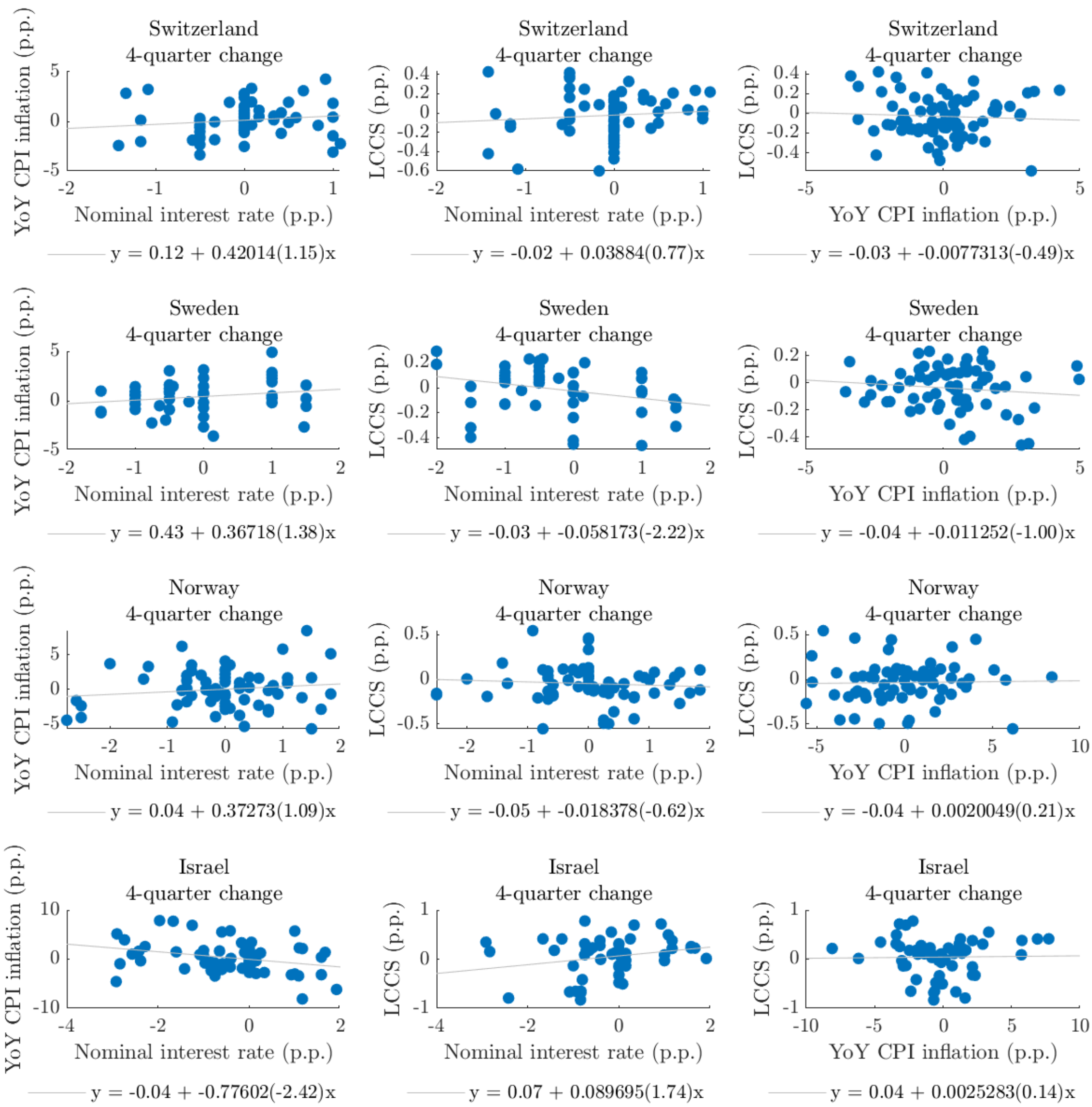
Note: t-statistics between parentheses; NaN means that there are not enough observations.

Figure 39: Scatter plot of emerging economies: nominal interest rate, inflation, and LCCS (Part 3)



Note: t-statistics between parentheses; NaN means that there are not enough observations.

Figure 40: Scatter plot of advanced economies: nominal interest rate, inflation, and LCCS (Part 1)



Note: t-statistics between parentheses; NaN means that there are not enough observations.

Figure 41: Scatter plot of advanced economies: nominal interest rate, inflation, and LCCS (Part 2)

F.4 CDS data source

The Bloomberg tickers of the CDS time series are listed in Tables 6, 7, and 8. Only emerging-economy CDS contracts were used in this paper as liquidity on advanced-economy contracts is very low.

Table 6: CDS data source: part 1

Country	Ticker Description	Bloomberg Ticker
United States	US CDS EUR SR 5Y D14	CT786896 CMAI Curncy
Brazil	BRAZIL CDS USD SR 5Y D14	CBRZ1U5 CBIN Curncy
Colombia	COLOM CDS USD SR 5Y D14	CCOL1U5 CBIN Curncy
Argentina	ARGENT CDS USD SR 5Y D14	CT350188 CBIN Curncy
Mexico	MEX CDS USD SR 5Y D14	CMEX1U5 CBIN Curncy
Chile	CHILE CDS USD SR 5Y D14	CCHIL1U5 CBIN Curncy
Peru	PERU CDS USD SR 5Y D14	CPERU1U5 CBIN Curncy
Venezuela	VENZ CDS USD SR 5Y D14	CVENZ1U5 CBIN Curncy
Canada	CANPAC CDS USD SR 5Y D14	CT718785 CMAI Curncy
Panama	PANAMA CDS USD SR 5Y D14	CPAN1U5 CBIN Curncy
Uruguay	URUGUA CDS USD SR 5Y D14	CX352614 CMAI Curncy
Costa Rica	COSTAR CDS USD SR 5Y D14	CT409282 CMAI Curncy
Guatemala	GUATEM CDS USD SR 5Y D14	CX352582 CMAI Curncy
El Salvador	ELSALV CDS USD SR 5Y D14	CX352550 CMAI Curncy
United Kingdom	UK CDS USD SR 5Y D14	CUKT1U5 CMAI Curncy
France	FRANCE CDS USD SR 5Y D14	CFRTR1U5 CMAI Curncy
Germany	GERMAN CDS USD SR 5Y D14	CDBR1U5 CMAI Curncy
Italy	ITALY CDS USD SR 5Y D14	CITLY1U5 CMAI Curncy
Spain	SPAIN CDS USD SR 5Y D14	CSPA1U5 CMAI Curncy
Portugal	PORTUG CDS USD SR 5Y D14	CPGB1U5 CMAI Curncy
Sweden	SWED CDS USD SR 5Y D14	CT777839 CMAI Curncy
Netherlands	NETHER CDS USD SR 5Y D14	CT425574 CMAI Curncy
Switzerland	SWISS CDS USD SR 5Y D14	CX991622 CMAI Curncy
Greece	GREECE CDS USD SR 5Y D14	CGGB1U5 CMAI Curncy
Turkey	TURKEY CDS USD SR 5Y D14	CTURK1U5 CBIN Curncy
South Africa	REPSOU CDS USD SR 5Y D14	CSOAF1U5 CBIN Curncy
Russia	RUSSIA CDS USD SR 5Y D14	CRUSS1U5 CBIN Curncy
Saudi Arabia	KSA CDS USD SR 5Y D14	CT965307 CBIN Curncy
Qatar	QATAR CDS USD SR 5Y D14	CQTA1U5 CBIN Curncy
Lebanon	LEBAN CDS USD SR 5Y D14	CT358551 CMAI Curncy
Hungary	HUNGARY CDS USD SR 5Y D14	CHUN1U5 CMAI Curncy
Poland	POLAND CDS USD SR 5Y D14	CPOLD1U5 CMAI Curncy

Table 7: CDS data source: part 2

Country	Ticker Description	Bloomberg Ticker
Egypt	EGYPT CDS USD SR 5Y D14	CEGY1U5 CMAI Curncy
Dubai	DUBAI CDS USD SR 5Y D14	CT421069 CMAI Curncy
Ireland	IRELND CDS USD SR 5Y D14	CT777651 CMAI Curncy
Bahrain	BHRAIN CDS USD SR 5Y D14	CT393413 CMAI Curncy
Ukraine	UKRAIN CDS USD SR 5Y D14	CUKR1U5 CMAI Curncy
Abu Dhabi	ABUDHAB CDS USD SR 5Y D14	CX855707 CMAI Curncy
Romania	ROMANI CDS USD SR 5Y D14	CROA1U5 CMAI Curncy
Oman	OMAN CDS USD SR 5Y D14	CT991547 CMAI Curncy
Belgium	BELG CDS USD SR 5Y D14	CBELG1U5 CMAI Curncy
Austria	AUST CDS USD SR 5Y D14	CAUT1U5 CMAI Curncy
Israel	ISRAEL CDS USD SR 5Y D14	CISR1U5 CMAI Curncy
Czech	CZECH CDS USD SR 5Y D14	CT349923 CMAI Curncy
Croatia	CROATB CDS USD SR 5Y D14	CCROA1U5 CMAI Curncy
Bulgaria	BGARIA CDS USD SR 5Y D14	CBULG1U5 CMAI Curncy
Kazakhstan	KAZAKS CDS USD SR 5Y D14	CKAZ1U5 CMAI Curncy
Nigeria	NIGERIA CDS USD SR 5Y D14	CT393726 CMAI Curncy
Norway	NORWAY CDS USD SR 5Y D14	CT777775 CMAI Curncy
Denmark	DENK CDS USD SR 5Y D14	CDEN1U5 CMAI Curncy
Finland	FINL CDS USD SR 5Y D14	CFIN1U5 CMAI Curncy
Kuwait	KUWAIT ST CDS USD SR 5Y D14	CT975717 CMAI Curncy
Slovakia	SLOVAK CDS USD SR 5Y D14	CSLVK1U5 CMAI Curncy
Slovenia	SLOVEN CDS USD SR 5Y D14	CT354156 CMAI Curncy
Tunisia	BTUN CDS USD SR 5Y D14	CTUN1U5 CMAI Curncy
Morocco	MOROC CDS USD SR 5Y D14	CY002690 CMAI Curncy
Serbia	SERBIA CDS USD SR 5Y D14	CT355197 CMAI Curncy
Estonia	ESTONI CDS USD SR 5Y D14	CT354249 CMAI Curncy
Iceland	ICELND CDS USD SR 5Y D14	CX855635 CMAI Curncy
Algeria	ZZALGR CDS USD SR 5Y D14	CT393750 CMAI Curncy
Latvia	LATVIA CDS USD SR 5Y D14	CT354225 CMAI Curncy
Lithuania	LITHUN CDS USD SR 5Y D14	CT354237 CMAI Curncy
Angola	ANGOL CDS USD SR 5Y D14	CY341739 CMAI Curncy
Iraq	IRAQ CDS USD SR 5Y D14	CT394067 CMAI Curncy

Table 8: CDS data source: part 3

Country	Ticker Description	Bloomberg Ticker
Kenya	KENYA CDS USD SR 5Y D14	CY341643 CMAI Curncy
Cyprus	CYPRUS CDS USD SR 5Y D14	CT412021 CMAI Curncy
Senegal	SENEGAL CDS USD SR 5Y D14	CY342027 CMAI Curncy
Rwanda	RWAND CDS USD SR 5Y D14	CY341931 CMAI Curncy
Cameroon	REPUBLICOFCAMEROON CDS USD SR	CY341835 CMAI Curncy
Japan	JGB CDS USD SR 5Y D14	CJGB1U5 CMAI Curncy
Australia	AUSTLA CDS USD SR 5Y D14	CT855561 CBIN Curncy
New Zealand	NZ CDS USD SR 5Y D14	CT778495 CMAI Curncy
South Korea	KOREA CDS USD SR 5Y D14	CKREA1U5 CMAI Curncy
Indonesia	INDON CDS USD SR 5Y D14	CINO1U5 CMAI Curncy
China	CHINAGOV CDS USD SR 5Y D14	CCHIN1U5 CBIN Curncy
India	INDIA CDS USD SR 5Y D14	CIGB1U5 CMAI Curncy
Malaysia	MALAYS CDS USD SR 5Y D14	CMLAY1U5 CBIN Curncy
Philippines	PHILIP CDS USD SR 5Y D14	CPHIL1U5 CBIN Curncy
Thailand	THAI CDS USD SR 5Y D14	CTHAI1U5 CMAI Curncy
Vietnam	VIETNM CDS USD SR 5Y D14	CX355151 CMAI Curncy
Hong Kong	HONGK CDS USD SR 5Y D14	CHKS1U5 CMAI Curncy
Pakistan	PKSTAN CDS USD SR 5Y D14	CPKT1U5 CMAI Curncy

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