An Unpleasant Coincidence for Monetary Policy: Risky Assets and Fiscal Limits

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Abstract

What if the government can default on monetary policy assets? This paper studies how sovereign default risk transmits to monetary policy depending on the policy instrument's credit risk. We lay out a two-agent New-Keynesian (TANK) model with endogenous fiscal limits and then calibrate it to a large emerging economy, Brazil. We find that endogenous expectations of a large recession in case of default push inflation, real, and nominal policy rates higher in the equilibrium distribution of the model. By ignoring risk in the policy asset, the central bank reinforces that unpleasant coincidence. Additionally, we find that pushing the economy toward the fiscal limit, or stabilizing the debt near it, reduces welfare for both Ricardian and hand-to-mouth agents. From a policy perspective, these results raise concerns on the evaluation of monetary policy stance in default-risky economies, while arguably solve a long-standing conundrum of the Brazilian economy.

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1 Introduction

What does change once we assume that the government can default on assets targeted by the central bank? This is the exact opposite assumption to the one adopted by usual monetary policy models. Although government debt is sometimes modeled as risky, the monetary authority is mostly supposed to have risk-free nominal targets which it implements with risk-free instruments. After all, central banks with their deep pockets can always issue more money on demand. This is basically the essence of monetary policy in canonical New-Keynesian models (i.e., Galí (2015); Woodford (2011)). In this paper, we flip that assumption up-side-down and model the policy instrument as a defaultable bond carrying the same risk of default as that of the government. The motivation for doing so is straight-forward. For one, in most countries, the central bank is part of the general government, enjoying different degrees of *de facto* independence. For two, emerging economy central banks often go to the open market with a portfolio of assets issued by the federal government. For three, and perhaps most important, domestic defaults including those of debt denominated in domestic currency have happened in the past, as Reinhart and Rogoff (2009) and Beers and de Leon-Manlagnit (2019) have built databases of such cases.[1]

We develop a two-agent New-Keynesian (TANK) model with distortionary taxes and endogenous fiscal limits, à la Bi (2012), conjugated with a central bank that targets the rate of government defaultable bonds. We model both Ricardian and non-Ricardian (hand-to-mouth) agents to reproduce the environment of emerging economies, in which access to financial assets, including those directly influenced by the central bank's rate, may be quite limited. This heterogeneity may have important implications for welfare, since only the first type of agent suffers a direct wealth loss in case of default, while both types share the updated tax burden for the new debt trajectory as well as the indirect costs of default which we model as a recessionary shock.[2] Additionally, we assume a positive spill-over of govern-

[1]As Reinhart and Rogoff (2009, p. 111) put: "In fact, our reading of the historical record is that overt de jure defaults on domestic public debt, though less common than external defaults, are hardly rare. Our data set includes more than 70 cases of overt default (compared to 250 defaults on external debt) since 1800. These de jure defaults took place via a potpourri of mechanisms, ranging from forcible conversions to lower coupon rates to unilateral reduction of principal (sometimes in conjuction with a currency conversion) to suspension of payments".

[2]Although an economic recession increases the likelihood that default happens, defaulting by itself is as-

ment expenses to productivity as a way to obtain empirically-motivated positive correlation between the first and output. Our nominal frictions are two as they come from the fact that the government issues exclusively nominal debt at the same time that prices are sticky. Inflation works as a wealth tax on Ricardian agents, who detain all sovereign bonds, at the same time that it reduces expected taxation levied on all agents in the future. Furthermore, since prices are sticky, real interest rates in this model may be different from *natural* ones, their flexible-price counterparts. We study two policy rules: (1) targeting the real risk-free rate using risky assets; and (2) targeting the risk-adjusted rate using risky assets. Finally, since our economy contains two non-excluding nonlinearities for the government, taxing at the peak of the Laffer curve and reaching the fiscal limit (ultimately partially defaulting on debt), we adopt Maih (2015)'s regime-switching approach, in which transition probabilities are allowed to be endogenous.

After calibrating the model to a large emerging economy perceived as having not irrelevant default risk, Brazil[3], and which credibly abides by the inflation-targeting regime in the present, despite having defaulted on local-currency-denominated liabilities in the past, we confirm that ignoring such a risk when embedded in the monetary policy asset has potential to put in check the central bank's mandate in many ways.

First, we find that this attitude reinforces the unpleasant coincidence of high real and nominal policy rates while it delivers inflation above the target in the ergodic distribution of the model.[4] A long-standing discussion in Brazil revolves around why the country experi-

sociated with an additional negative shock. Both stylized facts are captured by our model and are empirically validated by Reinhart and Rogoff (2009, p. 129)'s estimations of an average 8% cumulative decline of output in the three-year run-up of a domestic default crisis, in addition to a 4% average decline on the very year of the default event. From a theoretical point of view, as D'Erasmo and Mendoza (2016) show, default must be costly for the government to warrant why it most often than not repays its debt.

[3]Brazil's indicators of default risk are sizable no matter you measure it using CDS (credit default swap) rates, EMBI+ spreads, or using Du and Schreger (2016)'s measure of credit risk on local-currency denominated sovereign debt. Meanwhile, it has accumulated more than 20 years of experience with inflation targeting, being arguably successful on that goal for it has actually delivered inflation inside the previously communicated target range in most of these years. Despite the stability of the last decades, Reinhart and Rogoff (2009, p. 116) point out that Brazil has defaulted on domestic debt in, at least, two occasions: 1986-1987 and 1990. In the first episode, debt previously issued as inflation-indexed forcibly lost the previously agreed indexation, and in the second episode, the government partially froze private financial assets in the context of the Collor Plan.

[4]We consider rates to be empirically high when they are so compared to the rest of the world at the same period. In the model, when they exceed their deterministic-steady-state values in the benchmark regime.

enced that coincidence after the end of its hyperinflation period, as can be seen in Figure 1.[5] Since the Real Plan of July 1994, inflation has been mostly moderate for an emerging economy, but at the expense of considerably high nominal policy rates, resulting in equivalently high ex-post real interest rates.[6][7] The successful monetary reform was preceded four years before by the Collor Plan, in which the government partially froze private financial assets to contain what a recently-elected government considered excessive liquidity in the economy and blamed for the out-of-control inflationary process.[8] In the middle panel of Figure 1, one can note that ex-post real interest rates turned out very negative right after the confiscation amid a pronounced recession (-4.3% YoY GDP in 1990). However, some time before the Real Plan it had already converged to a higher level than before the Collor Plan, around the same level it maintained in the first years after the Real Plan, until the implementation of the inflation-targeting regime in June 1999. That last regime change also contemplated the floating of the exchange rate and the adoption of annual fiscal surplus targets, ultimately setting up the so-called "macroeconomic tripod".[9] In the middle and bottom panels of Figure 1, one can see that ex-post real interest rates, nominal interest rates, and inflation engage together in a sluggish downward trend after the Real Plan, which arguably reflects the slow improvement of Brazil's fundamentals in addition to the fading memory of its government's attempts to break tacit and legal contracts. Our modeling approach of default as the partial confiscation of Ricardian agents' financial wealth conjugated with a severe recession guards some resemblance with what happened in Brazil during the Collor Plan at the same time that it points out to a potential source of misspecification in emerging-economy models which

[5] Franco (2017, p. 726-745) exposes a detailed account of that discussion.

[6] Bacha (2010) associates the persistence of ex-post real interest rates in Brazil with some inherited attributes from the high-inflation period such as high levels of price indexation, net debt to output, earmarked credit, as well as limitations to currency convertibility. He also points out to the unrelenting political tension that stems out from that discrepancy w.r.t the rest of the world and which blurs the government's strategy to repay its debt.

[7] Ex-ante real interest rates calculated with inflation expectations drawn from professional survey or from financial market data are available for the inflation targeting period and exhibit a similar pattern, BCB (2017).

[8] Pastore (1991) points out that 80% of the M4 (M1 plus all financial assets), approximately 30% of the Brazilian GDP at that time, became unavailable to their holders for 18 months, after which they were released in up to 12 installments with interest and inflation correction. The confiscation threshold was set at financial resources above NCz$ 50,000, which as of July 31st 2021 amounts to around BRL 19,500 or USD 3,800 inflation-adjusted.

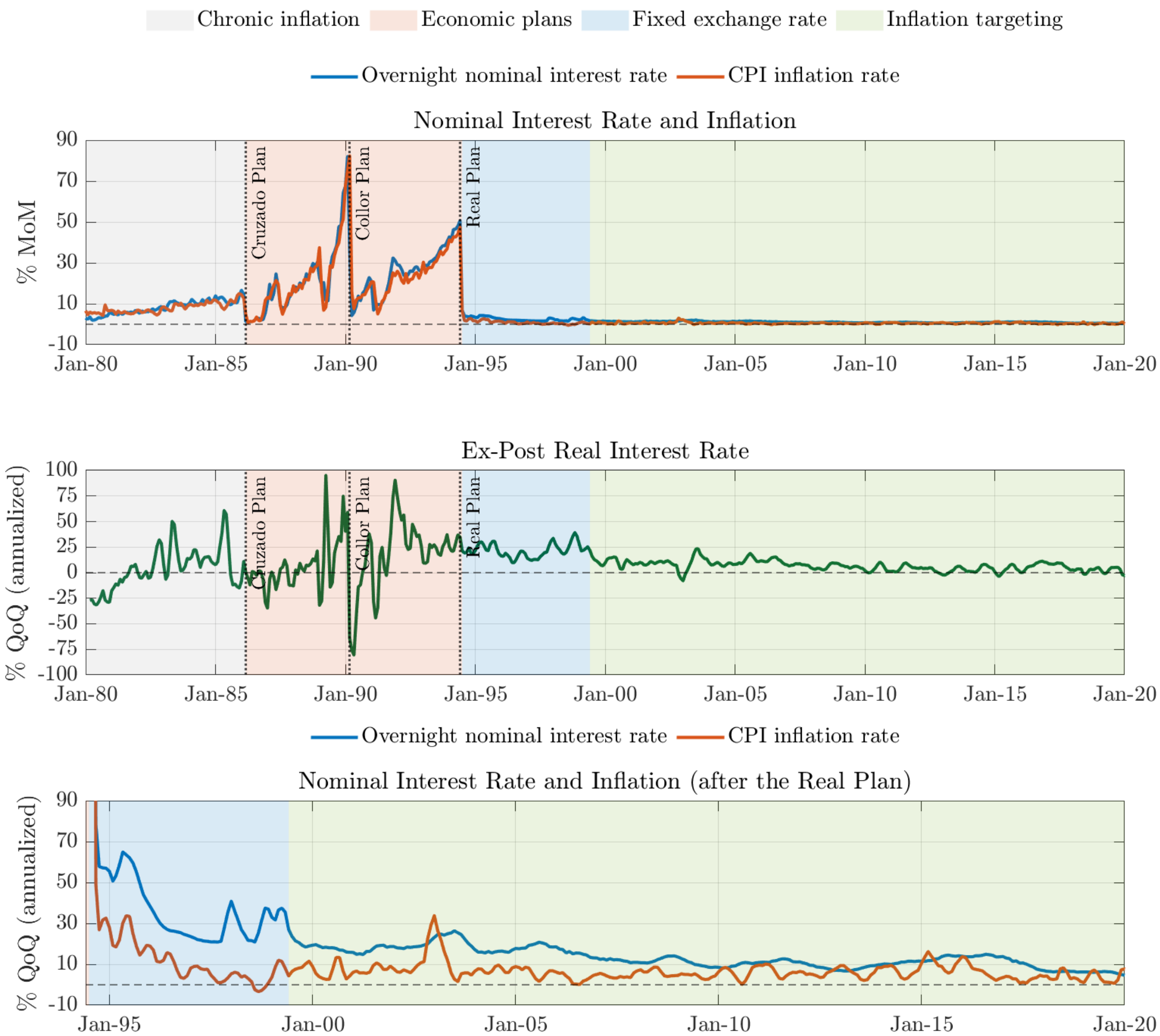
[9] Bevilaqua and Loyo (2005) give a detailed account of this transition and the early years of the inflation-targeting regime in Brazil.

neglect policy-asset default risk. While default by itself is a bad deal for the bondholders, the negative repercussions that it entails is what really drives real interest rates significantly up in the model, even at low levels of expected default probability. Whether the central bank accommodates or not that risk in its policy rule is what, eventually, pushes inflation above the target. From a quantitative point of view, we do not try to exactly replicate the empirical moments of the post-Real period, but rather to demonstrate that default risk is able to generate some of its still intriguing features.

Second, in our set-up, compensating default risk in the policy rule can increase welfare of both Ricardian and hand-to-mouth agents even though inflating the debt implies lower distortionary taxes in the future. This result raises concerns on non-obvious distributional effects of monetary policy and is found for optimal calibrations of monetary and fiscal policy parameters in a subset of rules near the fiscal limit. Nonetheless, pushing the economy toward the fiscal limit, or stabilizing the debt-to-output ratio near it, reduces welfare of all agents. This result contributes to the policy debate on whether the domestic debt level of a country matters. As long as the latter is perceived as risky, the model developed in this paper says it does, even if domestic debt is denominated only in domestic currency.

Overall, our simple dynamic stochastic general equilibrium (DSGE) model provides a new closed-economy explanation for why nominal interest rates, and real interest rates, of emerging economies may be observed as higher than their advanced economies counterparts. By ignoring the risk in monetary policy's underlying asset, central banks miss the adequate intercept of the policy rule, generating undesirable price fluctuations today and expected into the future. In that sense, the model rationalizes an argument in favor of that, in case of, say, a fiscal or political crisis, the central bank should raise the nominal policy rate or reduce it by less than if its instrument was risk-free, considering the adverse effects of the crisis over the economic activity. Needless to say that such a type of reaction is commonly seen in practice, even though most often reasoned as an attempt to contain capital flights to abroad. Moreover, a central bank that credibly commits itself to always update the policy rule intercept to changes in default risk is able to stabilize inflation on the target, at the same time that it may deliver lower nominal and real interest rates throughout time.

In a companion paper, we expand Wicksell (1898)'s partial-equilibrium flexible-price monetary model as interpreted by Woodford (2011) with defaultable bonds being used as monetary policy assets. There, we find that, in such a setting, monetary policy power w.r.t.



Note: The four periods identified are dated as follows: Chronic inflation (Jan-1980 to Feb-1986), Economic plans (Mar-1986 to Jun-1994), Fixed exchange rate (Jul-1994 to May-1999), and Inflation targeting (Jun-1999 onward). The Cruzado Plan is dated as of Mar-1986; the Collor Plan as of Mar-1990; and the Real Plan as of Jun-1994. Constructed with data from IPEADATA. Overnight nominal policy rate is series "Taxa de juros - Over / Selic - acumulada no mês". CPI inflation rate is series "Preços - IPCA - geral". The ex-post real interest rate series is calculated from the previous ones.

Figure 1: Brazilian historical data

prices is reduced, and the central bank ends up raising nominal interest rates more than it would if policy assets were risk-free. This outcome can only be undone in case the central bank adopts, with credibility, a particular nominal interest rate rule with a time-varying intercept that adjusts itself to default-risk variations underlying the policy asset. Moreover, by ignoring the riskiness of its own asset, the policy rule will generate a positive correlation between expected default risk and inflation, a testable implication of the model consistent with empirical data collected from both (risk-free) advanced and (risky) emerging economies.

This paper proceeds as follows. Section 2 presents the related literature. Section 3 describes our model with risky assets and fiscal limits, as well as explains the solution method. Section 4 calibrates and simulates the model to the Brazilian economy. Section 5 makes considerations about welfare. Finally, Section 6 concludes. Appendix A derives the steady state; Appendix B lists the log-linearized equations; Appendix C describes the estimation of some parameters; Appendix D tests the stability of the model; Appendix E plots impulse response functions; Appendix F provides additional analyses; and Appendix G gives technical details on the endogenous regime-switching solution method.

2 Related literature

In the literature, this paper is more closely related to Bi et al. (2018), which investigates for the case of a closed economy with fiscal limits how the inflation-targeting regime is affected when the monetary policy rule targets the interest rate on default-risky sovereign bonds, instead of on risk-free assets. The authors employ a New-Keynesian model with fixed-intercept Taylor rules, and find that, on the one hand, if the central bank targets the risk-free rate, it can still manage to bring inflation to the target after a contractionary monetary shock, although persistently depressing output. On the other hand, if the central bank targets the default-risky interest rate, it would be as if it had lifted the inflation target.[10] Differently from Bi et al. (2018), in this paper, we allow the intercept of the policy rules to vary in order to reflect the evolution of default risk. Additionally, we specify both Ricardian and non-Ricardian agents, for they will be affected differently by an episode of default, in a set-up more similar to an emerging economy. We also take a different approach to solving the

[10]Reis (2018) points out that adjusting the intercept of the policy rule to the risk premium embedded in the risky rate brings inflation back to the target.

model, as well as explore the ergodic distribution induced by each policy rule, in addition to sketching some welfare considerations.

Still in the realm of New-Keynesian models with fiscal limits, Battistini et al. (2019) endogenize the interaction of the latter's distribution with different policy rules and the zero-lower-bound. Bi et al. (2020), by their turn, keep monetary policy conditional to a given fiscal limit distribution, as we do in this paper, but introduce capital and long-term debt to evaluate interest rate normalization on the exit of the zero-lower-bound in the United States.

As this paper deals with the interaction between inflation and government debt default risk, it can be related to papers which explore that dimension, even though central bank operations are still seen as risk-free by them. Bonam and Lukkezen (2019) look into how pro-cyclical fiscal policy when debt is risky may put in check the inflation-targeting regime inside a closed-economy New-Keynesian framework, while Schabert and Van Wijnbergen (2014) analyze the consequences of having a not very responsive fiscal rule in an open-economy New-Keynesian model. Both papers emphasize that default risk makes it more likely that monetary policy is constrained by fiscal policy. Arellano et al. (2020), by their turn, combine the open-economy New-Keynesian framework of Gali and Monacelli (2005) with the literature on strategic external sovereign default of Arellano (2008), and find that default risk amplifies monetary frictions by bringing tension to monetary policy that, in the end, leads to inflation and nominal interest rates being even more volatile, as well as the latter to positively co-move with default risk. Turning to our model, when the central bank accommodates policy-asset default risk it generates correlations with the same sign still in the realm of a closed economy.

Expanding the literature on strategic default to domestic sovereign debt, D'Erasmo and Mendoza (2016) propose a theory based on distributional incentives across heterogeneous agents to show how positive debt can be sustained in equilibrium when defaulting is costly, a situation we reproduce in this paper under the fiscal limits approach by differentiating Ricardian from non-Ricardian households. Deviating somewhat from default risk, but still facing the question of inflation credibility when the central bank can inflate the debt, a problem latent in this paper that we abstract from by assuming the central bank is credible and always abides by its policy rule (excepting for random monetary shocks), there is Aguiar et al. (2014). The authors propose a model to explain why less developed economies who have joined the Eurosystem enjoyed significantly lower interest rates on their sovereign debt after that event, which the authors rationalize as gains of credibility in the conduction of monetary policy.

In the implementation of government default risk in our model, we follow Bi et al. (2020, 2018, 2016) and Bi (2017), for we adopt the stochastic fiscal limit mechanism proposed in Bi (2012). By this approach, a government subject to a Laffer curve may be forced to default on its debt at any period with a probability inferred from the distribution of future shocks in the economy. Differently from the literature of sovereign default led by Eaton and Gersovitz (1981) and Arellano (2008), the preferences of the government are not explicitly modeled, so that default happens not because the government prefers to, or is strategic doing so, but because the trajectory of debt becomes unsustainable given fiscal limitations, namely, an exogenous process for government expenses combined with an endogenous cap on government revenues. For a more empirical perspective, an attempt to calculate market-priced fiscal limits using CDS time series and a semi-structural model is developed in Pallara and Renne (2019).

A similar approach to modeling fiscal limits, but with less emphasis on its distribution, can also be found in Davig et al. (2010) and Davig et al. (2011), who focus on policy uncertainty stemming from the risk that monetary policy may switch from active to passive, in Leeper (1991)'s definition, when under fiscal stress, allowing for the central bank to inflate the debt. A feature both these works share with the present paper is that regime switching happens with endogenous probability. Nonetheless, the mentioned papers provide global solutions obtained through the monotone map method of Coleman (1991), while in this paper we provide locally-approximated solutions by employing the regime-switching method described in Maih (2015).

Although this work always assumes active monetary policy and passive fiscal policy, it can be marginally correlated to the well-established literature that assumes different combinations, either more in line with the fiscal theory of the price level and fiscal dominance, like in Sargent and Wallace (1981), Sims (1994), Woodford (1995), Cochrane (1998), Blanchard (2004), and Uribe (2006), or with the non-existence of a bounded equilibrium, like in Loyo (1999). Two of them deserve special attention in reason of also taking Brazil as a motivating case, although covering completely different periods. Blanchard (2004) tells us a story of hikes in the policy rate increasing real interest rates and, as a consequence, the stock of default-risky government debt. As the latter is related to significantly higher default probability, a higher policy rate is not enough to make domestic debt more attractive, on the contrary, it leads to a real depreciation of the domestic exchange rate which is further transmitted into higher inflation – a situation of fiscal dominance for which monetary policy has its hands tied to decrease inflation. Differently from Blanchard (2004), in this paper the exchange rate channel is absent and inflation only depends on the central bank's reaction both to it and to default risk – a situation better described as of monetary dominance. Particularly contrasting with Loyo (1999), while he tells a story of accelerating inflation and policy rates, we tell a story about their persistence at high levels. In this paper, the central bank may never go for the rescue of the indebted government while, by committing to always adjust the intercept of the policy rule to the default risk underlying its instrument, it can still deliver inflation on the target. Overall, monetary policy is conditionally active to the government repaying its debt, but still active, while fiscal policy is always passive. If, by any chance, the government has to default, it confiscates part of the Ricardian agents wealth, whereas the economy's TFP suffers a persistent negative shock.

3 Closed economy with default-risky policy assets

We present a DSGE model of a sticky-price closed economy composed of households and firms with both a fiscal and a monetary authority. Households are of two types, Ricardians and non-Ricardians. The first type invests her savings in defaultable nominal government bonds. There is no investment in capital in this economy, for it is assumed fixed across firms at all periods.

The central bank conducts monetary policy through open-market operations with government bonds abiding by a nominal interest rate rule. To allow for default risk in the policy asset of this economy, we impose that the fiscal authority is constricted by private perceptions of its stochastic fiscal limit, as in Bi (2012) and Bi et al. (2018), what implies that at any period t there is a non-zero probability that the government will only repay partially its maturing debt at period $t+1$. Consequently, investors charge a default premium for holding government bonds. Default, in addition to confiscation, coincides with a negative TFP shock, but it is costless otherwise, in the sense that both the fiscal authority and the central bank do not suffer any sanction and still maintain their access to the bond market after defaulting. The fiscal authority levies time-varying distortionary taxes on labor and profit income, at the same time that it makes fixed lump-sum transfers to all households.[11]

To turn the calculation of the fiscal limits computationally faster, we adopt ingredients so as to keep the global solution of the model analytical under flexible prices even after we have added government to it. First, we specialize the one-period utility function to be of the GHH type (Greenwood et al., 1988), eliminating income effects from the labor supply, what facilitates aggregation of Ricardian and non-Ricardian agents. Second, we adopt a Cobb-Douglas production function with constant marginal returns to labor.[12]

3.1 Households

At every period t, there is a continuum of households indexed by $i \in [0,1]$ split into two types: Ricardians (optimizing) and non-Ricardians (rule-of-thumb). γ^{NR} represents the fraction of the latter in the continuum. The aggregate consumption of this economy, C_t, is given by

$$C_t = \left(1-\gamma^{NR}\right) C_t^R + \gamma^{NR} C_t^{NR} \tag{1}$$

where C_t^R and C_t^{NR} are, respectively, the amounts consumed by each Ricardian and non-Ricardian representative household.

3.1.1 Ricardian households

The fraction $\left(1-\gamma^{NR}\right)$ of Ricardian households can be consolidated into a representative Ricardian household, who seeks to maximize her value function (2) subject for all t to her nominal budget constraint (3) and to her solvency constraint (4) in order to avoid Ponzi-schemes. We assume that when the government buys goods in the market, it somehow generates positive utility to households, perhaps through the provision of public goods.[13]

[11] Lump-sum transfers will be particularly useful in our calibration strategy as they will be calculated as to ensure that debt is stable at the steady state given the empirical calibration of other fiscal variables.

[12] The combination of Cobb-Douglas production function in which labor has diminishing marginal returns with traditional separable power utility in a model with government impedes a global analytical solution of the model, as it would require the use of approximation or numerical methods inside each iteration of the calculation of the fiscal limits. Although perfectly doable, we opted here to keep the model as tractable as possible, while still obtaining sensible impulse response functions.

[13] In the calculation of the fiscal limits, the nominal rigidities will be turned off. In that flexible-price scenario, simply introducing government expenses and applying a positive shock to it would crowd out private consumption to such an extent that output would decrease instead of increase in the short-run, contrary to the usual empirical evidence. Mechanisms often introduced to induce short-run positive correlation between

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} \left(C_t^R + \alpha_G G_t - \eta \frac{N_t^{1+\chi}}{1+\chi} \right)^{1-\sigma} \tag{2}$$

$$P_t C_t^R + \frac{P_t B_t}{1+i_t} \leq (1-\delta_t) P_{t-1} B_{t-1} + (1-\tau_t)\left(W_t N_t + P_t D_t\right) + P_t Z_t \tag{3}$$

$$\lim_{T\to\infty} \mathbb{E}_t \left[\beta^{T-t} \frac{\left(C_T^R + \alpha_G G_T - \eta \frac{N_T^{1+\chi}}{1+\chi} \right)^{-\sigma}}{\left(C_t^R + \alpha_G G_t - \eta \frac{N_t^{1+\chi}}{1+\chi} \right)^{-\sigma}} \frac{(1-\delta_T) B_T}{P_T} \right] \geq 0 \tag{4}$$

where $\beta \in (0,1)$ is the subjective discount factor; σ is the inverse of the elasticity of intertemporal substitution; η is a parameter that regulates the disutility of labor; and χ is the inverse of the Frisch elasticity of labor supply. Evaluated all at period t, C_t^R is the amount consumed by the Ricardian household; P_t is the price of the consumption good; N_t is the number of hours worked; W_t is the nominal wage paid for hours worked; and D_t is the dividend paid by the representative firm to the representative household as the latter ultimately owns all firms. Concerning the portfolio of assets, B_t is the amount of the one-period discounted risky nominal bond purchased at the end of period t; and i_t is the net interest rate on the risky one-period nominal bond to be paid at period $t+1$. Concerning the government-related parameters, α_G controls how public consumption provides utility to the households. When $\alpha_G > 0$, government consumption substitutes for private consumption; when $\alpha_G = 1$, it is a perfect substitute; and if $\alpha_G < 0$ it complements.[14] Finally, δ_t is a time-varying function that indicates default according to a rule to be defined, but which is going to be positive in case of default, and zero otherwise.

The optimization of the Ricardian household with respect to C_t^R, N_t, and B_t results, respectively, in conditions (5), (6), and (7)

$$U_{C,t} = \lambda_t P_t \tag{5}$$

ΔG_t and ΔY_t such as nominal rigidities would add a new state variable to our model, turning the fiscal limits calculation computationally more demanding. Defining part of households as hand-to-mouth helps to dump the short-run negative impact on output, but it is not enough to change its sign. We opt, then, to put government consumption inside of the utility function of the households, a specification able to induce empirically validated short-run positive correlations between G_t and other real variables in an environment with nominal rigidities, as shown by Fève and Sahuc (2017), with the advantage of not adding a new state variable. Despite that, what will, eventually, make output to respond positively to a public consumption expansion in the flexible-price version of our model is the modification of the production function described ahead.

[14] For a list of papers which play with this parameter, check Fève and Sahuc (2017).

$$U_{N,t} = -\lambda_t (1-\tau_t) W_t \tag{6}$$

$$\frac{\lambda_t}{\mathbb{E}_t \lambda_{t+1}} = \beta (1+i_t)(1-\mathbb{E}_t \delta_{t+1}) \tag{7}$$

where $U_{C,t}$ and $U_{N,t}$ are, in order, the marginal utility of consumption and of labor in period t; and λ_t is the Lagrange multiplier of the household's budget constraint in the same period. Note that we pick a tax on wages on purpose, because it distorts the optimal choice of the households when compared to their choice when such a tax does not exist. Higher taxes reduce household's after-tax real return per worked hour, discouraging labor supply. This feature will generate a curve of revenues for the government against the tax rate τ_t that is concave in its valid domain of $[0,1]$, the so-called Laffer curve. This will be important further ahead when we define default conditions in the model.

After specializing the one-period utility function adopted, the CPOs of the representative Ricardian household are given by the optimal labor supply (8) in addition to the Euler condition (10), valid for all t. The solvency constraint is binding and takes the form of the transversality condition (11), also valid in all periods. Hereafter, we define the stochastic discount factor of the Ricardian household at period t for a risk-free real flow at period $t+1$, $M_{t,t+1}$, by equation (9), where r_t^{RF} is the net real interest rate on that flow.

$$\frac{W_t}{P_t} = \frac{1}{(1-\tau_t)} \eta N_t^{\chi} \tag{8}$$

$$M_{t,t+1} \equiv \frac{1}{1+r_t^{RF}} = \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}^R + \alpha_G G_{t+1} - \eta \frac{N_{t+1}^{1+\chi}}{1+\chi}}{C_t^R + \alpha_G G_t - \eta \frac{N_t^{1+\chi}}{1+\chi}} \right)^{-\sigma} \right] \tag{9}$$

$$\frac{1}{1+i_t} = \mathbb{E}_t \left[\beta (1-\delta_{t+1}) \left(\frac{C_{t+1}^R + \alpha_G G_{t+1} - \eta \frac{N_{t+1}^{1+\chi}}{1+\chi}}{C_t^R + \alpha_G G_t - \eta \frac{N_t^{1+\chi}}{1+\chi}} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] \tag{10}$$

$$\lim_{T\to\infty} \mathbb{E}_t \left[\beta^{T-t} \frac{\left(C_T^R + \alpha_G G_T - \eta \frac{N_T^{1+\chi}}{1+\chi} \right)^{-\sigma}}{\left(C_t^R + \alpha_G G_t - \eta \frac{N_t^{1+\chi}}{1+\chi} \right)^{-\sigma}} \frac{(1-\delta_T) B_T}{P_T} \right] = 0 \tag{11}$$

3.1.2 Non-Ricardian households

The fraction γ^{NR} of non-Ricardian, rule-of-thumb, households can be consolidated into a representative non-Ricardian household, who consumes all her income at every period since it has no access to assets, bonds, or dividends. She also takes prices and nominal wages as given, while she maximizes the utility flow (12) subject for all t to her nominal budget constraint (13).

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} \left(C_t^{NR} + \alpha_G G_t - \eta \frac{\left(N_t^{NR}\right)^{1+\chi}}{1+\chi} \right)^{1-\sigma} \tag{12}$$

$$P_t C_t^{NR} = (1-\tau_t) W_t N_t^{NR} + P_t Z_t \tag{13}$$

where N_t^{NR} is her number of hours worked. As we assume both household types have the same GHH specification for the utility function, they supply the same amount of labor in equilibrium, $N_t = N_t^{NR}\ \forall t$. One can note this by comparing the non-Ricardian CPO for the labor supply (14) with the Ricardian (8)

$$\frac{W_t}{P_t} = \frac{1}{(1-\tau_t)} \eta N_t^{NR\chi} \tag{14}$$

3.2 Final goods sector

At every period t, there is a continuum of competitive firms indexed by j willing to produce a homogeneous final good composed of differentiated goods produced by a continuum of monopolistic firms indexed by k. The final good aggregation technology has a CES form and it is given by

$$Y_{j,t} = \left(\int_0^1 y_{k,t}^{\frac{\theta-1}{\theta}} \,\mathrm{d}k \right)^{\frac{\theta}{\theta-1}} \tag{15}$$

where θ is the elasticity of demand for each intermediate firm's good.

In the final sector, each firm j minimizes its cost generating the following demand curve for the intermediate good of firm k.

$$Y_{k,t} = \left(\frac{P_{k,t}}{P_t} \right)^{-\theta} Y_t \tag{16}$$

The associated price index to the final good is presented next.

$$P_t = \left(\int_0^1 P_{k,t}^{1-\theta} \,\mathrm{d}k \right)^{\frac{1}{1-\theta}} \tag{17}$$

Finally, the representative final sector firm's output, and this economy's output, is easily obtained by symmetry

$$Y_t = \int_0^1 Y_{j,t} \,\mathrm{d}j = Y_{j,t} \tag{18}$$

3.3 Intermediate goods sector

At every period t, there is a continuum of monopolistic firms indexed by k willing to produce a differentiated good. Each of them hires households in the amount of $N_{k,t}$ working hours at that period. These firms make use of the following technology, (19), where $Y_{k,t}$ is the production of firm k at period t, and A_t represents the common TFP process of all firms also at the same period. We assume that the stock of capital belongs directly to the firms and that its amount is equally fixed across them at all periods at the level of $\overline{K}$.[15] Additionally, we include a capital enhancement effect motivated by public consumption[16], where $\gamma_{G\Psi} \in [0,1)$ is the output elasticity with respect to government expenditures.

$$Y_{k,t} = A_t \left(\frac{G_t}{\overline{G}}\right)^{\gamma_{G\Psi}} \overline{K} N_{k,t} \tag{19}$$

Note that this production function implies constant returns to scale, so all intermediate firms optimize by seeking to employ the same amount of input factors. The TFP, A_t, follows the exogenous process

$$\log\left(\frac{A_t}{\overline{A}}\right) = \rho_A \log\left(\frac{A_{t-1}}{\overline{A}}\right) + \sigma_A \varepsilon_t^A - \underbrace{\mathbb{1}_{B_{t-1}>\mathscr{B}_t}\delta^{\text{TFP}}}_{\text{loss in case of default}} \tag{20}$$

where $\rho_A \in [0,1)$ is a parameter that controls the persistence of the shocks; σ_A is their standard deviation; while ε_t^A is i.i.d. $\mathscr{N}(0,1)$. The last term of the equation, $\mathbb{1}_{B_t>\mathscr{B}_{t+1}}\delta^{\text{TFP}}$, is the product of an indicator function that the government has defaulted on the debt by a productivity loss parameter, δ^{TFP}. At last, $\mathscr{B}_t$ is the fiscal limit drawn at period t, as it will be explained further ahead in this paper.

The marginal cost of the intermediate firm, MC_t, is identical across all firms of that type, and can be obtained by dividing the cost of one unit of labor by its marginal product. Note that since the production function is linear in labor, the marginal product does not depend

[15]We keep the constant in the model because it is going to help us with the steady-state calibration. Otherwise, we would have to find a real-world value for $\overline{A}$, the TFP at the steady state.

[16]Barro (1990) argues that private inputs are not a close substitute for public ones and warrants the specification of nonrival public services in the firm-specific production function in terms that it is indifferent whether the government buys a flow of output from the private sector (infrastructure, defense, etc.), which is later offered to the firms to fulfill their private demand, or that the goverment itself carries on the production as long as they share the same production function. Maršál et al. (2017) adopt a similar idea in a DSGE model under a more complex specification of public expenditures.

on the amount of labor employed. At period t, the marginal cost is expressed by

$$\mathrm{MC}_t = \frac{W_t}{P_t} \frac{1}{A_t \left(\frac{G_t}{\overline{\overline{G}}}\right)^{\gamma_{G\Psi}} \overline{K}} \tag{21}$$

Each period, the intermediate firm k solves the optimization problem (22) through which she maximizes profits subject to sticky prices à la Rotemberg (1982), where ϕ^C is a parameter that calibrates the price adjustment cost, and $\overline{\Pi}$ is the steady-state gross inflation,

$$\max_{P_{k,t}} \sum_{h=0}^{\infty} M_{t,t+h} \left(P_{k,t} Y_{k,t} - \mathrm{MC}_t P_t Y_{k,t} - \frac{\phi^C}{2} \left(\frac{P_{k,t}}{P_{k,t-1}} \frac{1}{\overline{\overline{\Pi}}} - 1 \right)^2 P_t Y_t \right) \tag{22}$$

subject to (16), taking P_t, W_t and Y_t as given. After imposing symmetry, the CPO of the representative intermediate firm is the non-linear New-Keynesian Phillips curve under Rotemberg prices, which is given by

$$(1-\theta) + \theta \mathrm{MC}_t - \phi \left(\frac{\Pi_t}{\overline{\overline{\Pi}}} - 1 \right) \frac{\Pi_t}{\overline{\overline{\Pi}}} + \phi^C M_{t,t+1} \left(\frac{\Pi_{t+1}}{\overline{\overline{\Pi}}} - 1 \right) \frac{\Pi_{t+1}}{\overline{\overline{\Pi}}} \frac{Y_{t+1}}{Y_t} = 0 \tag{23}$$

Monopoly profit at any period t is immediately transferred as dividends, D_t, to the Ricardian households, who own the shares of the firms.

$$D_t \equiv Y_t - \mathrm{MC}_t Y_t - \frac{\phi^C}{2} \left(\frac{\Pi_t}{\overline{\overline{\Pi}}} - 1 \right)^2 Y_t \tag{24}$$

3.4 Government

The government is limited by the following budget constraint

$$\frac{P_t B_t}{1+i_t} + \tau_t (W_t N_t + P_t D_t) \geq (1-\delta_t) P_{t-1} B_{t-1} + P_t G_t + P_t Z_t \tag{25}$$

where B_t is real debt at the end of period t; τ_t is a time-varying tax rate applied to labor and dividend income; G_t is real government expenditure; and Z_t is real lump-sum government transfers to the households. The default discount, δ_t, shall be interpreted here not as a partial default in the stock of bonds, but as a partial default on the promised nominal return of all of them, for we impose that the government cannot discriminate bondholders. The government constraint can be rewritten in real terms resulting in

$$\frac{B_t}{(1+i_t)} + T_t \geq (1-\delta_t) \frac{B_{t-1}}{\Pi_t} + G_t + Z_t \tag{26}$$

where $T_t \equiv \tau_t \left(\frac{W_t}{P_t} N_t + D_t\right)$ is real tax revenue. Government expenditure, by its turn, follows an exogenous AR(1) process that also depends on lagged output:[17]

$$\log\left(\frac{G_t}{\overline{G}}\right) = \rho_{GG} \log\left(\frac{G_{t-1}}{\overline{G}}\right) + \rho_{GY} \log\left(\frac{Y_{t-1}}{\overline{Y}}\right) + \sigma_G \varepsilon_t^g \tag{27}$$

where $|\rho_{GG}| < 1$; $|\rho_{GY}| < 1$; σ_G is the standard deviation of the shocks; and $\varepsilon_t^g \sim$ i.i.d. $N(0,1)$.[18] Additionally, we fix transfers at all periods such that $Z_t = \overline{Z}$. To restrain indebtedness, fiscal policy abides by a debt-target rule[19]

$$\log\left(\frac{\tau_t}{\overline{\tau}}\right) = \rho_\tau \log\left(\frac{\tau_{t-1}}{\overline{\tau}}\right) + \gamma_\tau \log\left(\frac{B_{t-1}}{\overline{B}} \frac{\overline{Y}}{Y_{t-1}}\right) \tag{28}$$

where $|\rho_\tau| < 1$; $\gamma_\tau > 0$ is a policy parameter that measures the sensitivity of tax rates to the debt level at the previous period – the higher its value, the more passive fiscal policy should be.

From the government budget constraint holding with equality in equilibrium, we get the law of movement of real debt at the end of period t.

$$B_t = (1 + i_t)\left((1 - \delta_t)\frac{B_{t-1}}{\Pi_t} + G_t + Z_t - T_t\right) \tag{29}$$

3.5 Defaultable bonds

Here, we extend the work of Duffie and Singleton (1999) on defaultable bonds to our economy. We define the net interest rate demanded in equilibrium for holding government bonds that carry default risk, the *net risky interest rate*, as follows.

[17]Bi et al. (2016) also allow government expenditure to react to output with a lag, which is intended to capture its procyclicality observed in some emerging economies, especially in Latin America, as Gavin and Perotti (1997) find. There is also evidence of fiscal procyclicality at the state level in Brazil, as Sturzenegger and Werneck (2006) and Arena and Revilla (2009) point out.

[18]Having already defined all the required terms, the firm-specific production function can be cast in terms of its labor demand, its autoregressive components, and the shocks as follows: $Y_{k,t} = \overline{A}\,\overline{K}\left[\left(\frac{A_{t-1}}{\overline{A}}\right)^{\rho_A}\left(\frac{G_{t-1}}{\overline{G}}\right)^{\rho_{GG}\gamma_{G\Psi}}\left(\frac{Y_{t-1}}{\overline{Y}}\right)^{\rho_{GY}\gamma_{G\Psi}}\right]\left[e^{\sigma^A\varepsilon_t^A + \gamma_{G\Psi}\sigma_G\varepsilon_t^G - \mathbb{1}_{B_{t-1} > \mathscr{B}_t}\delta^{\text{TFP}}}\right] N_{k,t}$.

[19]We deviate a little from the literature on fiscal limits (i.e. Bi (2012) Bi et al. (2016) and Bi et al. (2018)) in the sense that we maintain the tax rate dependence on B_{t-1} even in the event of a default, instead of replacing it by the effectively repaid debt at that period. We are not aware of systematic empirical evidence that governments reduce taxes at the same period that they default. For instance, in the Collor Plan, a new tax on financial operations (IOF) was introduced, taxes were linked to inflation, and the prices of some public services were raised.

Definition 3.1. *A net risky interest rate, i_t, is a net nominal interest rate that in a market under no-arbitrage hypothesis satisfies the identity:*

$$\mathbb{E}_t^Q(1+i_t) = \left[\left(1-\mathbb{E}_t^Q\mathscr{D}_{t+1}\right)(1+i_t) + \mathbb{E}_t^Q\mathscr{D}_{t+1}(1+i_t)(1-\delta_{t+1})\right] = \left(1-\mathbb{E}_t^Q\mathscr{D}_{t+1}\delta_{t+1}\right)(1+i_t)$$

where $\mathbb{E}_t^Q$ is expectations under a risk-neutral probability measure Q, i_t is the net interest rate in case of non-default; $\mathscr{D}_{t+1}$ is the probability that the risky asset will default at maturity; and δ_{t+1} is the haircut in case of default.

In our model, assuming *no arbitrage opportunities*, and then applying the definition of net risky interest rate (3.1), we have that

$$(1+i_t^{RF}) = \mathbb{E}_t^Q(1+i_t) = Pr^Q(B_t \le \mathscr{B}_{t+1})(1+i_t) + Pr^Q(B_t > \mathscr{B}_{t+1})\left[(1-\delta_{t+1})(1+i_t)\right] \tag{30}$$

where i_t^{RF} is the net risk-free nominal interest rate, $Pr^Q(X)$ represents the risk-neutral probability that the event X will happen. Moreover, we can define $\mathscr{D}_{t+1} \equiv \mathbb{1}_{(B_t > \mathscr{B}_{t+1})}$. With these definitions, we can rewrite (30),

$$\mathbb{E}_t^Q(1+i_t) = \left(1-\mathbb{E}_t^Q\mathscr{D}_{t+1}\delta_{t+1}\right)(1+i_t) \quad \Rightarrow \quad \Phi_t \equiv \frac{1}{\left(1-\mathbb{E}_t^Q\mathscr{D}_{t+1}\delta_{t+1}\right)} = \frac{(1+i_t)}{\mathbb{E}_t^Q(1+i_t)} \tag{31}$$

where Φ_t is the time-varying gross default premium demanded for holding the risky government bond, or, equivalently, the greatest premium the Ricardian agent is willing to pay at period t to avoid default risk.

3.6 Monetary policy

The central bank targets the gross inflation $\overline{\Pi}$, abiding by a nominal interest rate rule (32), which it implements through open-market operations with government risky bonds. All rules react to current period inflation deviation from the target with coefficient ϕ^π, and to output deviation from the steady state with coefficient ϕ^Y, in addition to a monetary shock process, $\mathscr{M}_t$. We also allow the central bank to smooth out the interest rate trajectory by adding a 1-period lag component with coefficient ϕ^i. Finally, what distinguishes each rule is the choice for a different stochastic process for the time-varying intercept, $\bar{\imath}_t$.[20]

$$i_t = (1 + i_{t-1})^{\phi^i} \left(\left(1 + \bar{\imath}_t\right) \overline{\Pi} \left(\frac{\Pi_t}{\overline{\Pi}} \right)^{\phi^\pi} \left(\frac{Y_t}{\overline{Y}} \right)^{\phi^Y} \right)^{1-\phi^i} e^{\mathscr{M}_t} - 1 \tag{32}$$

where the monetary shock process, $\mathscr{M}_t$, is given by

$$\mathscr{M}_t = \rho_M \mathscr{M}_{t-1} + \sigma_M \varepsilon_t^M \tag{33}$$

such that $\rho_M \in [0,1)$ is a parameter that controls the persistence of the shocks; σ_M is their standard deviation; and ε_t^M is i.i.d. $\mathcal{N}(0,1)$.

When the policy rule is implemented with risk-free assets and the central bank targets a risk-free rate (the canonical case), the rule that always hits the inflation target is the one whose intercept always tracks r_t^{RF}, without smoothing, and no monetary shock. As we assume in this model that policy assets are defaultable, the central bank must, then, also neutralize the evolution of the default premium over the r_t^{RF} to achieve the same result. To understand how operating monetary policy with risky assets compares to the canonical case, it is helpful to start from the static case. After substituting the optimal intercept into the interest rate rule (32), and evaluating it at the deterministic steady state, we obtain the monetary policy rule at that point, which can be decomposed as in equation (34), where $\overline{r^n}$ is the natural interest rate at the steady state, whose value is the same of the risk-free real interest rate at the steady state, and $\overline{\Phi} \approx -\log\left(1 - \overline{\delta}\right)$ is the steady-state gross default premium demanded on the policy asset.

$$\bar{i} = \frac{1}{\beta(1-\overline{\delta})} \overline{\Pi} - 1 = \left(1 + \overline{r^n} + \overline{\Phi}\right) \overline{\Pi} - 1 \tag{34}$$

Note how the existence of policy-default risk adds a policy-default premium to the natural interest rate. Therefore, compared to managing monetary policy with risk-free assets, resorting to risky ones instead results in *higher nominal policy rates at the steady state.* Later, we will show how simulating default risk can amplify this effect far beyond its deterministic steady-state value.

[20]Woodford (2001) discusses the optimality of interest rate rules with time-varying intercepts which closely track the real natural interest rate for the sake of consistency with an equilibrium where both the inflation rate and the output gap are stable. These last two variables show up in the microfounded loss function derived from a simple New-Keynesian model – the variance of inflation is a proxy for the variance of output dispersion across individual firms whereas the output gap represents the deviation of aggregate output from its efficient level.

In the presence of other financial assets in the market, such as government bonds, but not only, the expectations channel shall discipline the interest rate of such assets as well, while at the same time they may be more informative of monetary conditions than an overnight rate available to, perhaps, only some selected agents. Kulish (2007) remembers us that the expectations channel works in both directions, and shows that a central bank that operates and targets the interest rate of long-term assets is still able to bring inflation to the target, as long as it follows an approximate version of the Taylor principle. The reasoning here is similar. Under the no-arbitrage assumption, the expectations channel operates through time but also through the safety dimension of the assets, allowing the CB to bring inflation to the target by operating or targeting a risky asset, as long as it adequately adjusts its policy rule to that fact. Assuming that the policy asset is a defaultable government bond or that the CB implicitly operates safe overnight deposits (not modeled) but adopts an explicit interest rate rule for defaultable government bonds end up being the same here, for we do not explicitly model the balance sheet of the CB, following the tradition of the canonical New-Keynesian model. We present, now, our two monetary policy rules, characterized each by a different process for the intercept, $\{\bar{\imath}_t\}$. Later in the paper, we will allow the natural real rate to be in the intercept, approximating our rules to those in the literature and in actual policy-making.

Rule 1: Risky policy asset, but the CB ignores default-risk dynamics

The policy asset is a defaultable government bond, but the CB ignores the dynamics of that risk. It assumes that the mean level of the default premium is part of the real natural interest rate. Not a far-fetched assumption, we believe, since such a rate is non-observable in the real world due to the prevalence of sticky prices. Remember, the reader, that being right about the mean also means being right about interest rates at the deterministic steady state.

$$\bar{\imath}_t = r_t^{RF} + \overline{\Phi} \tag{35}$$

Rule 2: Risky policy asset, and the CB perfectly tracks default-risk dynamics

The policy asset is a defaultable government bond, and the CB adjusts the intercept of the policy rule to neutralize the dynamics of that risk. It knows that the default premium is not part of the real natural interest rate. This rule may be too much to ask from central banks, since it requires loads of informational content quite challenging to observe: the default-risk premium and the risk-free real interest rate.

$$\bar{\imath}_t = r_t^{RF} + \Phi_t \tag{36}$$

To explain how adjusting the intercept works, we turn off the monetary shock in the model, in addition to setting $\phi^Y = 0$ and $\phi^i = 0$. Then, we plot in Figure 2 the simulated transition from a set-up in which $\overline{\mathscr{D}} \approx 0\%$ (left panels) to a set-up in which $\overline{\mathscr{D}} = 5\%$ (second column of panels), considering that the net inflation target, $\overline{\pi}$, is calibrated to 4.5% (annualized). In this transition, we do not fix the wedge of risk premium, $\overline{\Phi}$, that emerges at the deterministic steady state when the policy asset is risky. The ergodic mean of inflation leaves the (degenerated) inflation target and reaches 5.1% at the same time that a negative output gap, defined as $Y_t^{Gap} \equiv \frac{Y_t - Y_t^n}{Y_t^n}$, where Y_t^n is the flexible-price output, emerges. Now, inflation is spread over a long range, since the intercept is not able to accommodate the effect of remaining stochastic shocks on the evolution of the default premium. In the third column of panels, we fix the wedge at the deterministic steady state as defined in Rule 1. Inflation gets closer to the target, but it is still far from it as the dynamics of default risk are not compensated. This is the adequate fix from a deterministic perspective, since if we turn off the shocks and the risk of regime transition, it would successfully set inflation on the target. The output gap turns a bit more negative, but its relation with inflation is rule-dependent. In the last panel, we assume that the central bank, now, tracks the default-risk premium, Φ_t, as defined in Rule 2. Note that inflation is degenerated at its target while the output gap is zero once again. In this case, monetary shock is the only shock capable of introducing volatility to inflation (and to the output gap). To sum up, the nuts and bolts of our model do not preclude the existence of a *divine coincidence* – a term coined by Blanchard and Galí (2007) – as there is no trade-off between the stabilization of inflation and the stabilization of the welfare-relevant output gap.

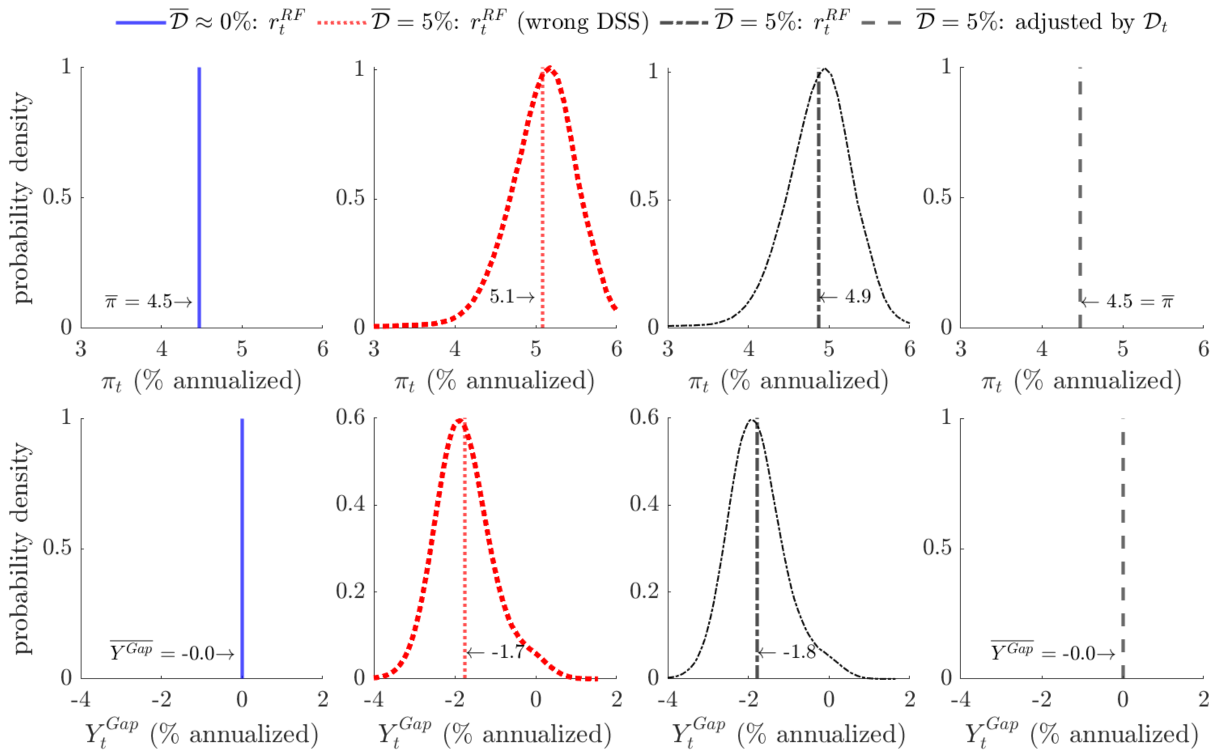


Figure 2: Inflation-targeting with risky assets

3.7 Equilibrium

As we have seen, the policy asset in this economy may actually be a government bond in whose market the central bank intervenes with deep pockets. Nevertheless, bonds are priced by the stochastic discount factor of the Ricardian household, making the latter indifferent about holding them, so we impose that in equilibrium all bonds issued by the government are sold to her.

$$B_t^{\text{demand}} = B_t^{\text{supply}} \quad \forall\, t \tag{37}$$

It is important to note that the Ricardian agent incorporates into her expectations the fact that her and only her will have to save enough to hold all the government's debt in her portfolio. This is an equilibrium selection that, although common in models with fiscal policy, it excludes the possibility that the Ricardian agent refuses to finance the government as long as the debt level to be repaid at the current period is lower or equal to the current draw of the stochastic fiscal limit (described in section 3.9). In case it is not, default happens, what can be understood as either the refusal of the private agents in financing the government, or the refusal of the government to repay its debt, or both. In our specific model, it also excludes the possibility that Ricardian agents behave like non-Ricardian ones even if that behavior would improve their welfare.

The real asset, by its turn, is a private asset available in zero-net-supply and as such its market-clearing condition demands that the net position of the representative Ricardian household on that asset always equals zero by construction.

$$\mathscr{A}_t = 0 \quad \forall t \tag{38}$$

Now, after imposing previous market-clearing conditions, we aggregate the budget constraint of the households (3) with the budget constraint of the government (26) and the dividends definition (24), and then evaluate the resulting constraint in equilibrium to obtain the *resources constraint* of our economy

$$Y_t = C_t + G_t + \frac{\phi^C}{2}\left(\frac{\Pi_t}{\overline{\Pi}} - 1\right)^2 Y_t \tag{39}$$

We have all set to define the recursive competitive equilibrium of this economy.

Definition 3.2. *A recursive competitive equilibrium of this economy is an initial distribution of* $\{B_{i,0}\}$ *across individual households indexed by* $i \in [0,1]$*, a set of exogenous processes* $\{A, G\}$*, a set of prices* $\{r, \frac{W}{P}, P\}$*, a set of policy functions for individual households* $\{C_i, N_i, B_i\}$*, a factor demand policy for individual intermediate firms* $\{N_k\}$ *indexed by* $k \in [0,1]$*, budget-feasible government policies* $\{B, \tau, Z, \delta\}$*, stochastic fiscal limits* $\{\mathscr{B}\}$*, and a monetary policy rule* $\{i\}$ *such that the following conditions hold:*

(a) Aggregation: $Y_t = \int_0^1 Y_{i,t} di$ *and* $C_t = \int_0^1 C_{i,t} di$ *in all* t*, washing out idiosyncratic uncertainty;*

(b) Optimality for the households: solves the representative Ricardian household's optimization problem, and the representative non-Ricardian household's budget constraint is satisfied with equality;

(c) Optimality for intermediate firms: solves the representative intermediate firm's optimization problem;

(d) Optimality for final-sector firms: solves the representative final-sector firm's optimization problem;

(e) Labor-market clearing: $N_t = \int_0^1 N_{i,t} di$ *in all* t*;*

(f) Asset-market clearing: $B_t^{demand} = \int_0^1 B_{i,t} di = B_t^{supply}$ *in all* t*;*

(g) Government's budget constraint is satisfied in all t*;*

(h) Good-market clearing: $Y_t = C_t + G_t + \frac{\phi^C}{2}\left(\frac{\Pi_t}{\overline{\Pi}} - 1\right)^2 Y_t$ *in all* t*.*

3.8 Solution 1st step: the model as a single regime with flexible prices

Our solution strategy is composed of three steps. In this first step, we assume that prices are flexible and solve the model as a function of τ_t and δ_t, the variables directly related to the two nonlinearities in our model, caused by, respectively, the peak of the Laffer curve and the fiscal limit. In the second step, we will describe how the fiscal limit can be reached and how the default probability is calculated from the solution derived in the first step. For such a calculation, we follow Bi et al. (2018), and assume that the central bank always keeps inflation at the target, what eliminates the effect of sticky prices and of explicit monetary policy, warranting the flexible-price solution derived in the first step. Finally, in the third and last step of the algorithm, we segregate the nonlinearities as different regimes of a regime-switching model, where in each regime the sticky-price version of the model is solved after linearization.

Now, we proceed imposing flexible prices, what turns our economy temporarily into a RBC (real business cycle) model. We can obtain a closed expression for the labor quantity in equilibrium by equating the labor supply (8) with the CPO of the firm (23) through eliminating real wages, $\frac{W_t}{P_t}$, after replacing the expression for the marginal cost (21).

$$N_t = \left(\frac{(\theta - 1)}{\theta} \frac{(1 - \tau_t)}{\eta} \overline{K} \left(\frac{G_t}{\overline{G}} \right)^{\gamma_{G\Psi}} A_t \right)^{\frac{1}{\chi}} \tag{40}$$

Real wage in equilibrium is straightforward and given by the CPO of the firm (23), after replacing the expression for the marginal cost (21). Turning to the output in equilibrium, we can obtain it by substituting the labor quantity in equilibrium (40) into the production function (19):

$$Y_t = \left(\frac{(\theta - 1)}{\theta} \frac{(1 - \tau_t)}{\eta} \right)^{\frac{1}{\chi}} \left(\overline{K} \left(\frac{G_t}{\overline{G}} \right)^{\gamma_{G\Psi}} A_t \right)^{1+\frac{1}{\chi}} \tag{41}$$

Aggregate households' consumption of private goods in equilibrium can, now, be obtained by inserting (19) into the resources constraint:

$$C_t = \left(\frac{(\theta - 1)}{\theta} \frac{(1 - \tau_t)}{\eta} \right)^{\frac{1}{\chi}} \left(\overline{K} \left(\frac{G_t}{\overline{G}} \right)^{\gamma_{G\Psi}} A_t \right)^{1+\frac{1}{\chi}} - G_t \tag{42}$$

For obtaining the consumption in equilibrium of each type of household, we proceed as follows. The non-Ricardian household consumes all its income, so her consumption comes directly from her budget constraint.

$$C_t^{NR} = \left(\frac{(\theta - 1)}{\theta} \frac{1}{\eta} \right)^{\frac{1}{\chi}} \left((1 - \tau_t) \overline{K} \left(\frac{G_t}{\overline{G}} \right)^{\gamma_{G\Psi}} A_t \right)^{1+\frac{1}{\chi}} + \overline{Z} \tag{43}$$

The Ricardian type, by her turn, will consume what is left from the aggregate consumption after subtracting the consumption of the non-Ricardian type.

$$
\begin{aligned}
C_t^R = \frac{1}{1-\gamma^{NR}} \Bigg[& \left(\frac{(\theta-1)}{\theta} \frac{(1-\tau_t)}{\eta} \right)^{\frac{1}{\chi}} \left(\overline{K} \left(\frac{G_t}{\overline{G}} \right)^{\gamma_{G\Psi}} A_t \right)^{1+\frac{1}{\chi}} - G_t \\
& -\gamma^{NR} \left(\left(\frac{(\theta-1)}{\theta} \frac{1}{\eta} \right)^{\frac{1}{\chi}} \left((1-\tau_t) \overline{K} \left(\frac{G_t}{\overline{G}} \right)^{\gamma_{G\Psi}} A_t \right)^{1+\frac{1}{\chi}} + \overline{Z} \right) \Bigg]
\end{aligned} \tag{44}
$$

Next, total government revenue in equilibrium is obtained by substituting the equilibrium expressions for N_t, $\frac{W_t}{P_t}$, and D_t into the definition of real taxes.

$$
T_t = \tau_t \left(\frac{(\theta-1)}{\theta} \frac{(1-\tau_t)}{\eta} \right)^{\frac{1}{\chi}} \left(\overline{K} \left(\frac{G_t}{\overline{G}} \right)^{\gamma_{G\Psi}} A_t \right)^{1+\frac{1}{\chi}} \tag{45}
$$

Finally, we can obtain an expression for the real natural interest rate as a function of the state vector and of the tax rate policy rule. We proceed by inserting the Ricardian consumption and the labor expressions, both in equilibrium, into the real bond Euler of the Ricardian household (9).[21]

$$
r_t^n = -1 + \frac{1}{\beta} \mathbb{E}_t \left[\left(\frac{C_{t+1}^R (A_{t+1}, G_{t+1}) + \alpha_G G_{t+1} - \frac{\eta}{1+\chi} \left(\frac{(\theta-1)}{\theta} \frac{(1-\tau_{t+1})}{\eta} \overline{K} \left(\frac{G_{t+1}}{\overline{G}} \right)^{\gamma_{G\Psi}} A_{t+1} \right)^{\frac{1+\chi}{\chi}}}{C_t^R (A_t, G_t) + \alpha_G G_t - \frac{\eta}{1+\chi} \left(\frac{(\theta-1)}{\theta} \frac{(1-\tau_t)}{\eta} \overline{K} \left(\frac{G_t}{\overline{G}} \right)^{\gamma_{G\Psi}} A_t \right)^{\frac{1+\chi}{\chi}}} \right)^{\sigma} \right] \tag{46}
$$

Moreover, we can obtain the *policy asset real interest rate*, r_t in equilibrium, by substituting the market-clearing condition into the Euler of the Ricardian household and then substituting the latter into the Fisher equation. We derive here the more general case, when the policy asset is defaultable and the recovery rate in case of default is allowed to be different from zero. In case the policy asset were risk-free, one would just have to set $\mathbb{E}_t \mathscr{D}_{t+1} = 0 \ \ \forall t$.

$$
\begin{aligned}
r_t = -1 + \frac{1}{\beta} \mathbb{E}_t \Bigg[& \left[1 - \mathscr{D}_{t+1} \overline{\delta} \right]^{-1} \\
& \left(\frac{C_{t+1}^R (A_{t+1}, G_{t+1}) + \alpha_G G_{t+1} - \frac{\eta}{1+\chi} \left(\frac{(\theta-1)}{\theta} \frac{(1-\tau_{t+1})}{\eta} \overline{K} \left(\frac{G_{t+1}}{\overline{G}} \right)^{\gamma_{G\Psi}} A_{t+1} \right)^{\frac{1+\chi}{\chi}}}{C_t^R (A_t, G_t) + \alpha_G G_t - \frac{\eta}{1+\chi} \left(\frac{(\theta-1)}{\theta} \frac{(1-\tau_t)}{\eta} \overline{K} \left(\frac{G_t}{\overline{G}} \right)^{\gamma_{G\Psi}} A_t \right)^{\frac{1+\chi}{\chi}}} \right)^{\sigma} \Bigg]
\end{aligned} \tag{47}
$$

The natural interest rate depends on the fiscal policy, as well as on the technological growth. In addition to that, the government default risk will indirectly interact with r_t^n through its effect on the evolution of indebtedness, which by its turn will affect τ_t under

[21] We keep C^R explicit to avoid cluttering the equation.

the fiscal policy rule adopted. As a consequence, *monetary policy is affected by the risk of default even when it is operated with risk-free assets.* Additionally, what it comes to r_t, it is clear that the expected haircut on the debt, $\mathbb{E}_t \delta_{t+1}$, introduces a wedge over the real natural interest rate compared to when it is nil. Overall, the policy asset real interest rate will be greater than the real natural one whenever there is the perception of default risk ($\mathbb{E}_t \delta_{t+1} > 0$), and the difference between both rates is a default premium.

3.9 Solution 2nd step: fiscal limits and default probability

Having solved the model as a function of τ_t and δ_t in the previous step, we are ready to describe how the fiscal limits and the default probability interact in the model.

At the beginning of every period, an effective fiscal limit $\mathscr{B}$ is drawn from the fiscal limit distribution $\mathscr{B}^* \left(\overline{\mathscr{B}}, \sigma^2_{\mathscr{B}}\right)$, where $\overline{\mathscr{B}}$ is its mean and $\sigma_{\mathscr{B}}$ its standard deviation, and compared to the real debt maturing at that same period, B_{t-1}. If the effective fiscal limit is greater than or equal to the debt level, the government repays the debt in its entirety, so that $\delta_t = 0$. Otherwise, it defaults on a fraction of the outstanding debt, so that $\delta_t = \overline{\delta}$.

$$\delta_t = \begin{cases} \overline{\delta} \in (0,1] & \text{if} \quad B_{t-1} > \mathscr{B}_t \\ 0 & \text{if} \quad B_{t-1} \leq \mathscr{B}_t \end{cases} \tag{48}$$

The fiscal limit adopted here is the private sector's perception of that limit as in Bi (2012), but adapted to the state vector of our model,[22] and under the assumption that the central bank maintains inflation at the target at all times, like in Bi et al. (2018).[23] There are at least two features in favor of that approach against the strategic default framework of Eaton and Gersovitz (1981) and Arellano (2008). First, it makes explicit that debt ratios are not the only factor that affects the private perception of a country's default probability. Present and expected future fiscal policies matter as well as growth prospects, what makes this feature

[22]Bi (2012) specifies a Markov-switching process for transfers Z_t, and estimates δ_t using the empirical distribution of actual default rates – defined as the product of the ratio of defaulted debt to total debt by the haircut on the defaulted debt – of emerging economies from 1998 to 2005 as compiled by Sturzenegger and Zettelmeyer (2008). Bi et al. (2018), who also adopt the fiscal limit approach, include an additional political discount that follows a Markov-switching process aiming to shift the distribution of the fiscal limit to the left and increase its standard-deviation along the debt-to-output axis, so as to match empirical moments of risk premium. We abstract from both features since they are not necessary to our model.

[23]Battistini et al. (2019) turn the fiscal limits endogenous to monetary policy by solving their sticky-price model through numerical methods, since with nominal rigidity there is no analytical solution.

aligned to the empirical fact that sovereign spreads and credit ratings are not linear univariate functions of debt ratios.[24] Second, more than strategic from the point-of-view of a benevolent government, the default decision carries an erratic political component, which goes in favor of modeling it as a stochastic event.[25] Motivated by that, the effective fiscal limit $\mathscr{B}$ of our economy is drawn from the distribution (49).

$$\mathscr{B}^*(A_t, G_t) \sim \sum_{t=0}^{\infty} \underbrace{\beta^t \frac{U_c^{\max}(A_t, G_t)}{U_c^{\max}(A_0, G_0)}}_{\substack{\text{stochastic discount factor} \\ \text{at the peak of the Laffer curve}}} \underbrace{\left(T^{\max}(A_t, G_t) - G_t - Z_t\right)}_{\text{primary balance}} \tag{49}$$

Note that the fiscal limit is the sum of all expected primary balances from today into the future discounted at the peak of the Laffer curve. That specific point, ($\tau_t^{\max}$, $T_t^{\max}$), gives us the tax rate that maximizes tax revenue, $\tau_t^{\max}(A_t, G_t)$, and the maximum revenue that the government can raise, $T_t^{\max}(A_t, G_t)$. The Laffer curve combined with the fiscal rule imply that if government expenses or transfers exhibit an upward trend, they will eventually push the tax rate to reach $\tau_t^{\max}$. This is an example of how the default probability increases under the fiscal limit mechanism. The specification of our model, with its structural parameters and exogenous processes, guarantees a unique mapping between the state vector and the peak of the Laffer curve.

To obtain an expression for $\tau_t^{\max}$, we derive the tax revenue in equilibrium, T_t, with respect to $\tau_t^{\max}$

$$\begin{aligned} T_t &= \tau_t \left(\frac{W_t}{P_t} N_t + D_t\right) = \tau_t \left(\frac{(\theta-1)}{\theta} \frac{(1-\tau_t)}{\eta}\right)^{\frac{1}{\chi}} \left(\overline{K} \left(\frac{G_t}{\overline{G}}\right)^{\gamma_{G\Psi}} A_t\right)^{1+\frac{1}{\chi}} \\ \frac{\partial T_t}{\partial \tau_t} &= \left[\left(\frac{(\theta-1)}{\theta} \frac{(1-\tau_t)}{\eta}\right)^{\frac{1}{\chi}} - \frac{(\theta-1)}{\theta} \frac{1}{\eta} \frac{1}{\chi} \tau_t \left(\frac{(\theta-1)}{\theta} \frac{(1-\tau_t)}{\eta}\right)^{\frac{1}{\chi}-1}\right] = 0 \\ \tau_t^{\max} &= \frac{\chi}{1+\chi} \end{aligned} \tag{50}$$

Therefore, in our model, a government willing to maximize its revenue will always set $\tau_t = \frac{\chi}{1+\chi}$, a result that is independent of the state of the economy. The maximum tax revenue, on

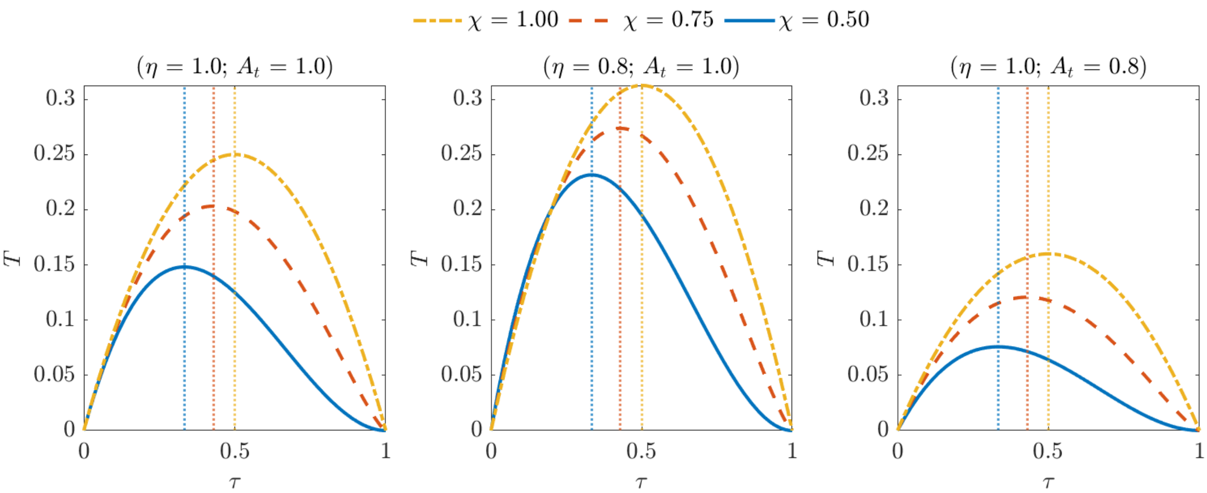
[24]The literature on the determinants of sovereign spreads (in foreign or local-currency-denominated bonds) is as old as at least Edwards (1986). Overall, it finds that determinants can be hardly generalized as each country spread seems to be sensitive to a different set of variables in each time period.

[25]Schabert and Van Wijnbergen (2014) make the case that Bayesian strategies like this one are optimal in some policy game set ups, for which Pastine (2002) is an analogous example of how introducing uncertainty into the central bank decision of when to abandon a fixed-exchange rate can avoid otherwise predictable speculative attacks.

the contrary, does depend on the state of the economy.

$$T_t^{\max}(A_t, G_t) = \frac{\chi}{1+\chi}\left(\frac{(\theta-1)}{\theta}\frac{\left(1-\frac{\chi}{1+\chi}\right)}{\eta}\right)^{\frac{1}{\chi}}\left(\overline{K}\left(\frac{G_t}{\overline{G}}\right)^{\gamma_{G\Psi}}A_t\right)^{1+\frac{1}{\chi}} \tag{51}$$

To give the reader a glimpse of our model's Laffer curve, in Figure 3, we plot the tax revenue against the tax rate for three different values of χ (0.50, 0.75, 1.00) and different values for η and A_t. Additionally, for the only purpose of this exercise, $\overline{K}$ is calibrated to 1.0, whereas $\theta \to \infty$ as in perfect competition, and we set $G_t = \overline{G}$. Vertical-dotted lines cross at the maximum revenue for the Laffer curve with the same color. It is possible to see on all panels that the higher is the Frisch elasticity of labor $\left(\frac{1}{\chi}\right)$, the lower is the maximum capacity of the government to tax this economy. At the same time, comparing the left panel with the middle one, a lower disutility of labor, η, incentivizes work and increases the maximum tax revenue. Finally, comparing the left panel with the right one, lower productivity shrinks fiscal capacity.



Note: vertical-dotted lines cross at the maximum of each curve with the same color.

Figure 3: Stylized Laffer curves

As there is a unique mapping between the state vector and $\tau_t^{\max}$ as well as between the state vector and $T_t^{\max}$, it is possible to obtain the distribution of the fiscal limit, $\mathscr{B}_t^*$, as a function of the state vector through Monte Carlo simulation. Moreover, by comparing such a distribution with the level of debt at the end of any period, we can calculate the conditional default probability, $Pr\left(B_t > \mathscr{B}_{t+1}\right)_t$, at any horizon into the future.

3.10 Solution 3rd step: regime switching with endogenous probabilities

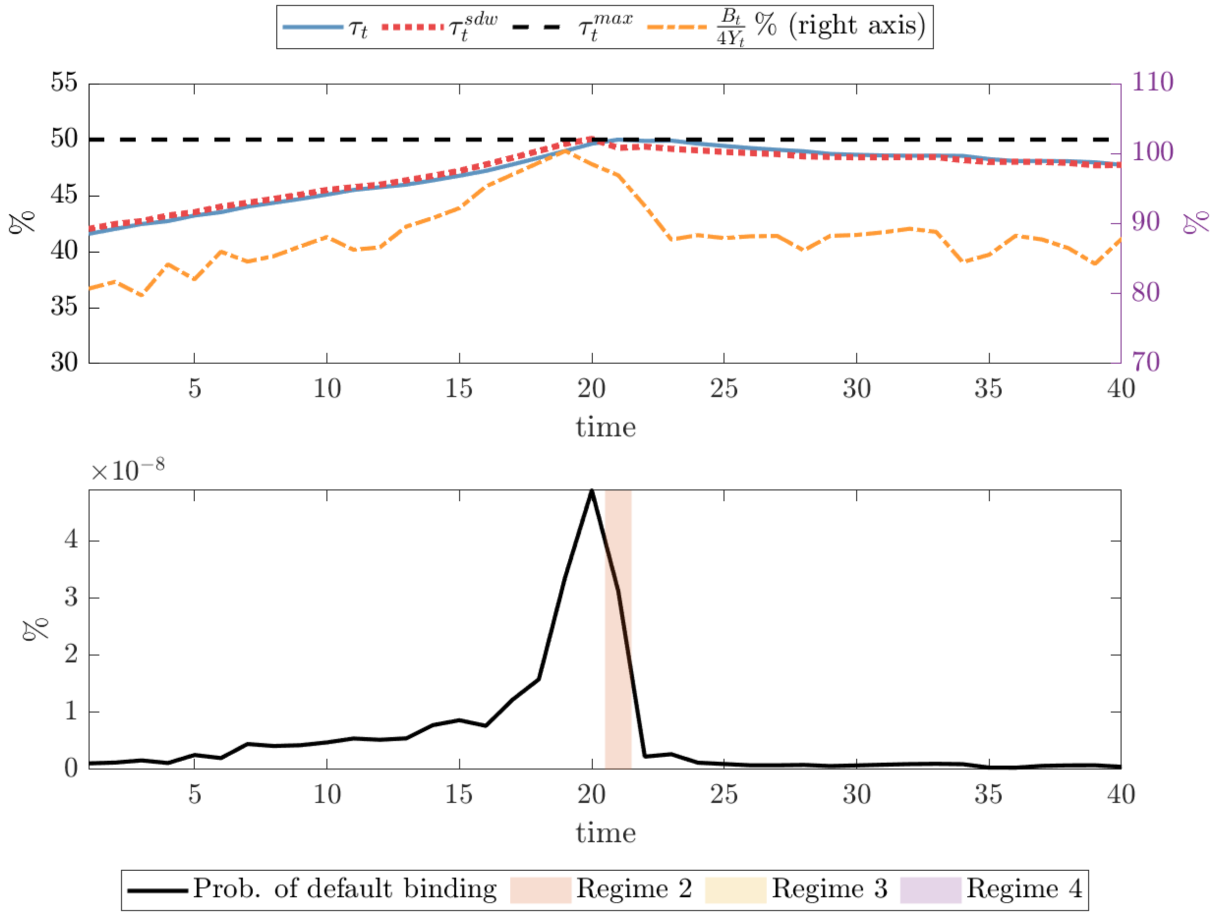
In this last step of our solution algorithm, the model is solved using the endogenous regime-switching method described in Maih (2015).[26] This approach is warranted by the fact that our model contains two nonlinearities, namely, taxing at the peak of the Laffer curve and defaulting on government debt when the fiscal limit is endogenously reached.

The constraint at the peak of the Laffer curve is straightforward. Whenever the prescribed unrestricted tax rule for τ_t, (28), should exceed $\tau_t^{\max}$, we impose that $\tau_t = \tau_t^{\max}$, like an occasionally-binding constraint.[27] As the transition probability for the next period must be set at the end of the period before, we pose a shadow tax rate, τ_t^{sdw}, which anticipates what would be the unrestricted tax rate in the next period were not for the tax constraint. This is the object that will be compared to the rate at the peak of the Laffer curve to define whether the binding probability in the next period is either 1 or 0.[28] In Figure 4, we show an example of situation in which this constraint is binding extracted from a random simulation of the model at the benchmark calibration of section 4.1. Note that the accumulation of debt is matched by successive hikes of the tax rate until the latter reaches the rate that maximizes the government revenue. In the example, debt starts reducing after some time along with the next-period default probability, allowing for the easing of the tax rate.

[26]We use the Matlab toolbox RISE (Rationality In Switching Environments) developed by Junior Maih, to whom we thank immensely for the generosity of making it available and free of charge.

[27]We simplify here by setting $\tau_t^{\max}$ to its flexible-price value at all periods, as in equation (50).

[28]Our modeling device allows for the peak of the Laffer curve to be actually breached after an unexpectedly dire sequence of shocks, but when that happens, in reason of the slow-moving nature of the tax policy rule and the binding constraint, the tax rate will hover around the peak until recede as it faces an "occasionally binding resistance". This kind of mechanism can be seen as is in the same family of other threshold-type mechanisms such as the ones present in Davig et al. (2006) and Chang et al. (2019). For more examples of how to model occasionally binding constraints as regime switches with RISE, check Binning and Maih (2017).



Note: prob. of default binding refers to default happening the next period. No filling in the bottom graph indicates Regime 1.

Figure 4: Example of peak of the Laffer curve binding in the simulated model

The fiscal limit constraint, by its turn, is imposed through an approximation. We regress separately the distributions for the mean and for the standard deviation of the fiscal limits, both obtained in the second step of the solution, on their respective state-vector deviations from the steady state. Note that the state variables that the fiscal limit depends on have the same steady-state value across all regimes. By proceeding this way, we obtain one reduced form equation for the mean and one for the standard deviation of the fiscal limit. The latter, then, can be incorporated into the regime-switching model at any period as a stochastic draw of a normal distribution whose mean and standard deviation are given by their respective reduced form equation.[29] At any period, the probability of reaching the fiscal limit is endogenously updated by the equation (52), where γ_0, γ_b, γ_a, and γ_g are parameters cal-

[29]The standard-deviation reduced-form equation is not actually necessary in the model. We append it just for completeness.

ibrated from the fiscal limits calculated at the second step of the solution method. Figure 5 exhibits the goodness of fit of that approximation.

$$Pr\left(B_t > \mathscr{B}_{t+1}\right)_t = \frac{1}{1+\exp\left(\gamma_0+\gamma_b\left(B_t-\mathbb{E}_t\mathscr{B}_{t+1}\right)+\gamma_a\left(\mathbb{E}_t A_{t+1}|_{\text{(no default)}}-\overline{A}\right)+\gamma_g\left(\mathbb{E}_t G_{t+1}-\overline{G}\right)\right)} \tag{52}$$

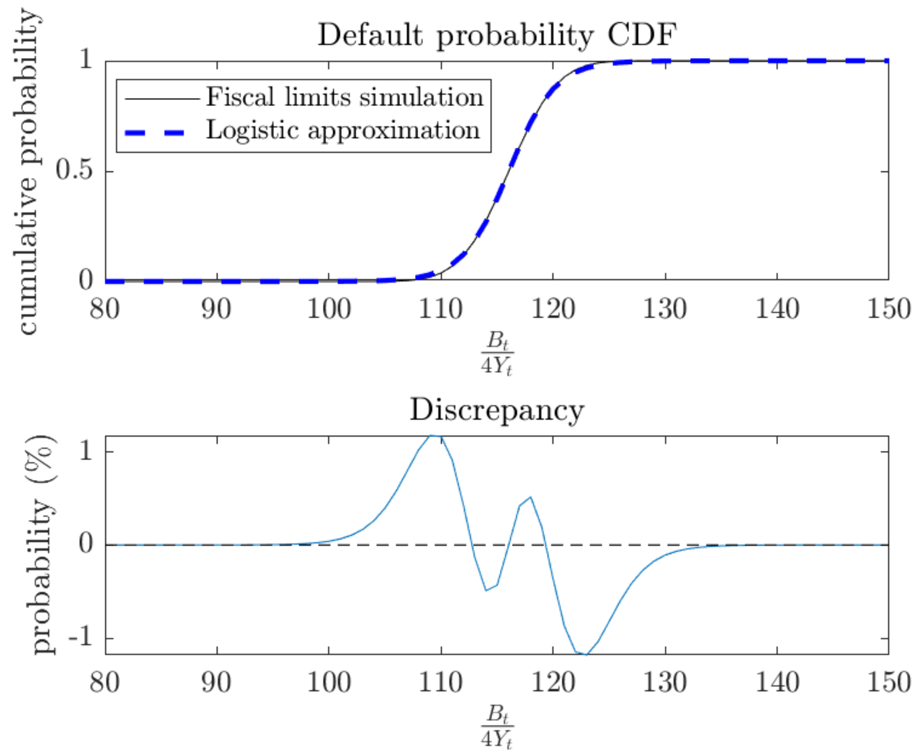


Figure 5: Goodness of fit of the logistic approximation

Note: top panel compares the cumulative probability distribution of the fiscal limits simulation against the logistic approximation; bottom panel plots the arithmetic difference between approximation and simulation values.

The combination of the two nonlinearities generates four regimes in our model, as exposed in Table 1.

	Below $\tau^{\max}$	**At $\tau^{\max}$**
Below fiscal limit	Regime 1	Regime 2
Reached fiscal limit	Regime 3	Regime 4

Table 1: Model regimes

Regime 1 is our starting point for solving and simulating the model, since fiscal limits clearly have not been reached for Brazil in the recent period. No default has happened, and the country can finance itself through debt with relative ease, relying both on domestic and foreign investors (not modeled here). Besides, efforts for increasing the tax burden do not seem to have been preempted by the idea that it will reduce total government revenue, but mostly for political reasons.

Regime 2 takes place when the peak of the Laffer curve is reached. This will happen when the stock of debt grows way beyond its steady-state level. The tax rate is, then, constrained to be equal to the rate that maximizes the government revenue, so that the debt can follow a trajectory that reduces the likelihood of default. Note that although monetary policy still has en effect on the trajectory of the debt in this regime, it is not able to indirectly affect the tax rate, and, through that channel, affect other real variables such as output, consumption and the employment level.[30]

Regime 3 occurs when the government defaults on the debt before its taxing capacity is exhausted. Although an unlikely event if the steady state is calibrated far from the fiscal limit, this may still happen in the model due to the stochastic character of this restriction. There is always a chance that a very large shock will be inflicted in that economy. The tax rate, which reacts with a lag to the inherited stock of debt, may suffer a large drop in the next period if the debt to be repaid in the future becomes much lower. As we calibrate the steady-state debt closer to the fiscal limit, this regime becomes more frequent.

Finally, Regime 4 represents default when the economy has already reached its taxation capacity. This is the case when a sequence of adverse shocks results in a large accumulation of debt. At some (stochastic) point, the government has to default. After that event, in reason of the lower debt level, the tax rate will be adjusted with a lag to below the peak of the Laffer curve, unless the debt level is so high that even after the haircut its sustainability still requires maximum taxation. An extremely high debt-to-output level – for the calibrated economy – would lead to a succession of default events happening near the peak of the Laffer curve until debt became sustainable again; a case of "debt intolerance" resulting in a "serial defaulteer", to employ the language of Reinhart et al. (2003).[31]

[30]Despite that, as long as this regime is not absorbing, monetary policy still affects these variables through the agents' expectation of regime-switching in the future, a channel that exists even under flexible prices. Since prices are sticky in our model, two real effects of monetary policy coexist in this regime: the price adjustment costs, and the regime-switching expectations.

[31]The same observations about the real effects of monetary policy made for Regime 2 also apply here.

The regime-switching solution strategy consists of first log-linearizing the model around each regime-specific non-stochastic steady state, and then applying the algorithm exposed in Maih (2015) and summarized in Appendix G. The list of log-linearized equations is available in Appendix B.

4 Calibration and simulation of the model

We calibrate and simulate the model for Brazil, a country whose sovereign premium on its debt has been considerably high throughout the years. This fact makes that country an adequate candidate for our policy-default-risky model.[32] Furthermore, the Central Bank of Brazil adopted an explicit inflation-targeting regime in June 1999, and, since then, it conducts monetary policy mainly through repo operations involving federal government securities. With respect to fiscal policy, also since 1999 the country has pursued primary surplus targets, but with somewhat less success than it has met its inflation ones. As our tax rule establishes a debt-to-output target instead, we leave a more detailed representation of the Brazilian fiscal policy for further research.

In our calibration strategy described ahead, we partially estimate the flexible-price single-regime version of the model with Bayesian methods for calibrating the fiscal limits. In Appendix D, we test the stability of the model with that calibration under each policy rule and for different levels of default probability at the deterministic steady-state. Additionally, in Appendix E, we analyze the impulse response functions of the model, and in Appendix F we discuss how the policy rule can entail correlations with different signs between inflation and default probability.

4.1 Calibration

We proceed with the parameterization of our model, relying on the calibration and estimation (posterior mean) of De Castro et al. (2015), unless otherwise stated. Our calibration is summarized in Table 2 and Table 3. Estimated parameters are listed in Table 4, while more details on the estimation procedure are available in Appendix C. The steady state is derived in Appendix A.

[32] For more on that choice, we redirect the reader to footnote 3.

Parameter	Description	Value
β	time discount factor	0.989
η	disutility of labor	varies
χ	inverse of the Frisch elasticity of labor	1.0
$\overline{N}$	steady-state labor supply	1/3
k_Y	capital-to-quarterly-output ratio	18.0
γ^{NR}	fraction of non-Ricardian households	0.40
γ_τ	tax-rate elasticity to the debt level	0.108
$\overline{\delta}_{\delta_t>0}$	debt haircut in case of default	0.05
$\overline{\Pi}$	gross inflation target	1.011
δ^{TFP}	TFP loss at the impact of default	0.0238
θ	elasticity of substitution between intermediate goods	11
ϕ^C	price adjustment cost	100
ρ_τ	tax-rate autoregressive coefficient	0.862

Table 2: Calibration of parameters

The time discount factor, β, is set to 0.989; the inverse of the Frisch elasticity of labor, χ, is set to 1.0.[33] The disutility of labor, η, is calibrated so that labor supply equals 1/3 at the steady state, as is common in the literature, while k_Y, the fraction $\frac{\overline{K}}{\overline{Y}}$, is calibrated to 18.0, calculated from the Penn World Table database, whose methodology is described in Feenstra et al. (2015).[34]. Moreover, the fraction of non-Ricardian households is calibrated to $\gamma^{NR} = 0.40$. Since the model in De Castro et al. (2015) does not have defaultable bonds, we resort to other sources. We calibrate the haircut in case of default, $\overline{\delta}_{\delta_t>0}$, to 0.05 (20% annually) like Bi et al. (2018) to emphasize that even small haircuts can generate quantitatively relevant results. Additionally, we calibrate the TFP loss in case of default, δ^{TFP}, so it is consistent with a 4.3% annual output loss, which is the yearly contraction of Brazilian real GDP in 1990, the year of the Collor plan, obtained from the IBGE (Brazilian Institute of Geography and Statistics).[35] Finally, we calibrate the elasticity of substitution between

[33]In De Castro et al. (2015) this parameter is not identified in the estimation, so they pick 1.0 as it is inside the range of values in the literature.

[34]The 1999-2017 average value of annual capital stock divided by 1/4 of annual real GDP, both measured at constant national prices in millions of 2011 U.S. Dollars.

[35]From the log-linearized version of the equation of output in equilibrium (41), we compute the size of the

intermediate goods, θ, to 11, and the price adjustment cost parameter, ϕ^C, to 100.[36]

Turning to the steady-state parameters, we calculate $\overline{A}$ so that $\overline{Y} = 1.0$. To calibrate fiscal variables, we use annual Brazilian data from the Government Finance Statistics (GFS) of the IMF, available only for the period 2006-2018 (Figure 6 and 7). Total government expenses averaged 42.4% of GDP in the period. Of this total, the sum of compensation of employees, the use of goods and services, the consumption of fixed capital, and other expenses averaged 20.6% of GDP, which we use to calibrate $\frac{\overline{G}}{\overline{Y}}$. The sum of subsidies, grants expense, and social benefits averaged 14.2% of GDP in the same period, which we use to calibrate the value of government transfers at the steady state, $\frac{\overline{R}}{\overline{Y}}$. The remaining difference is given to interest expenses, which averaged 8.1% in that time range. Steady-state tax rate when maximum tax rate restriction is not binding, $\overline{\tau}_{\tau_t<\tau_t^{\max}}$, is set to 39.1%, which is the mean fraction of total government revenues to GDP in the period. Finally, debt to output ratio at the steady state, $\frac{\overline{B}}{\overline{Y}}$, is calibrated to 248% of quarterly output, or 61.9% of annual output using data for gross debt from the Central Bank of Brazil (BCB).[37] As our model is a closed economy, we assume that international reserves are either nil or that they cannot be used for the purpose of debt repayment.[38] With BCB and GFS data, we estimate equation (28), using a linear regression, and we get $\gamma_\tau = 0.108$ and $\rho_\tau = (0.552)^{\frac{1}{4}} = 0.862$.[39]

TFP shock at period 1 that $\sum_{t=1}^{4}\left(1+\frac{1}{\chi}\right)\left(\log(A_t)-\log(\overline{A})\right) = (1-0,043)$.

[36]De Castro et al. (2015) adopts Calvo pricing and estimates the frequency of non-optimizing firms at any period to be 0.74 for freely-set prices. Using Keen and Wang (2007)'s first-order equivalence between Calvo and Rotemberg pricing, our calibration would imply a frequency of 0.73.

[37]In 2007, the methodology for calculating the gross debt of the general government published by the Central Bank of Brazil changed. While the old series is available in the sample from 2001M12 to 2019M12, the new series covers 2006M12 onward. We opt to interpolate both series by estimating values from 2001M12 to 2006M11 through adding to the old series the mean difference between then.

[38]Calibrating for net debt instead of gross debt would not affect our results qualitatively as the debt level does not enter in the calculation of the fiscal limits. One can easily interpret B_t as the net debt level and assume that international reserves are fixed at any specific level. In fact, as we will show that the calibrated steady-state gross debt is already compatible with virtually zero default probability, substituting it for the lower steady-state net debt would not entail any additional dynamics to the model. Moreover, the tax rate elasticity to the net debt, calculated with IMF World Economic Outlook data, is $\gamma_\tau = 0.109$, roughly the same as for the gross debt. At least for the estimated period, adopting net debt instead of gross debt is just a negative level shift in the default probability associated with any value of B_t.

[39]Bi et al. (2016) estimate $\gamma_\tau = 0.047$ for Argentina using Bayesian methods in a sample covering 2004Q1:2015Q2. Ramirez and Wright (2017) calculate fiscal limits for 18 economies in Central America and the Caribbean while calibrating γ_τ between 0.26 (Guatemala) and 0.69 (Belize). Overall, we assess that our

Parameter	Description	Value
$\overline{Z}$	steady-state government transfers	$0.142\overline{Y}$
$\overline{G}$	steady-state government expenses	$0.206\overline{Y}$
$\overline{\tau}_{\tau_t<\tau_t^{\max}}$	steady-state tax rate	0.391
$\overline{Y}$	steady-state output	1.0
$\overline{B}$	steady-state debt	2.48
$\overline{P}$	steady-state price level	1.0

Table 3: Calibration of steady-state parameters

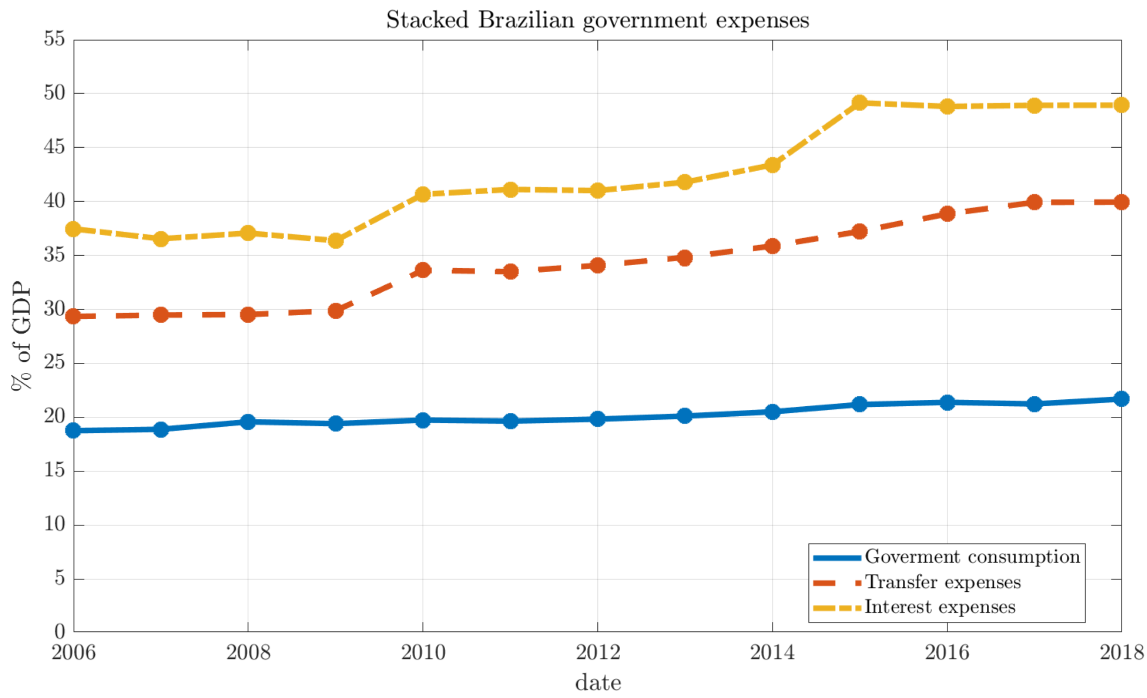


Figure 6: Actual stacked Brazilian government expenses

calibration is in line with the literature, although it should be taken with a grain of salt.

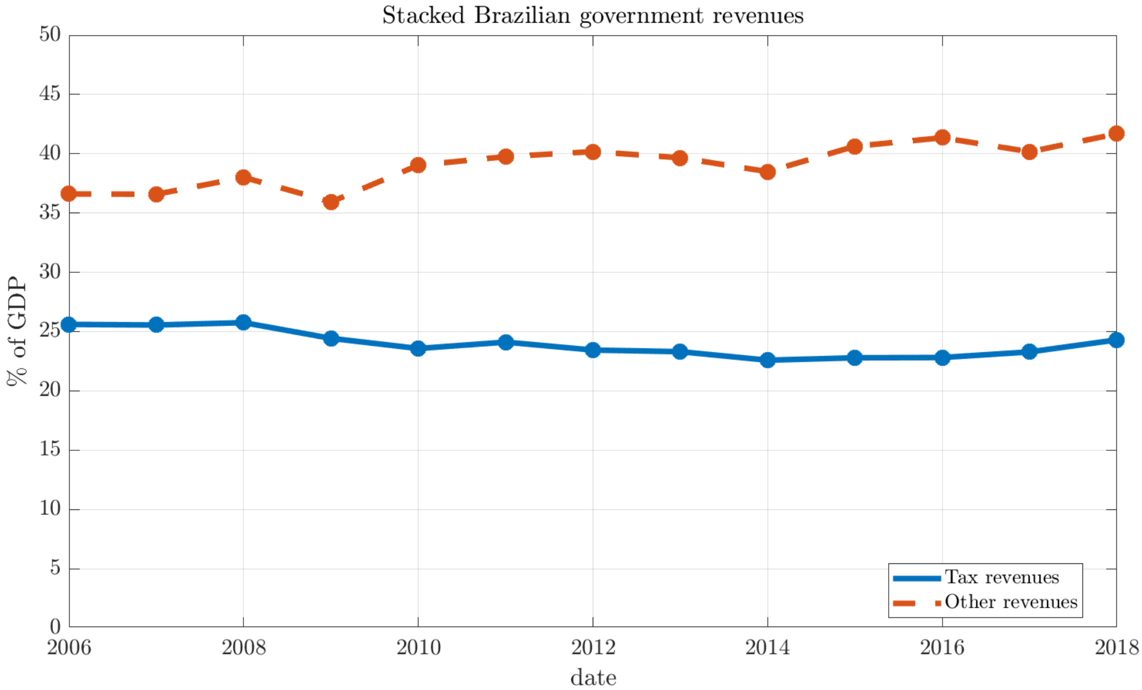


Figure 7: Actual stacked Brazilian government revenues

We conduct a Bayesian estimation, as described in Appendix C, of some parameters of the flexible-price single-regime version of the model, that is, supposing that its two non-linearities are never binding nor expected to bind.[40] Specifically for estimation, we opt for allowing time preferences to evolve according to an AR(1) stochastic process,[41]

$$\log\left(\frac{\beta_t}{\beta}\right) = \rho^\beta \log\left(\frac{\beta_{t-1}}{\beta}\right) + \sigma^\beta \varepsilon_t^\beta \tag{53}$$

where $\rho^\beta \in [0, 1)$ measures its persistence; σ^β is the standard deviation of the shocks; and ε_t^β is i.i.d. $\mathcal{N}(0, 1)$.

The posterior mean of the inverse of the elasticity of intertemporal substitution, σ, is estimated to be 2.132, larger than the 1.3 estimated by De Castro et al. (2015). We, as them, find that this parameter is poorly identified in the data.

Concerning the externalities of public expenditure in the model, we obtain $\alpha_G = 0.551$,

[40]A full Bayesian estimation of the regime-switching model would require more complex estimation techniques, such as the ones employed by Bi and Traum (2012) and Bi and Traum (2014).

[41]There is a noticeable linear trend in the data for nominal interest rates. As in our estimation we prefer to demean it instead of applying an HP filter, so as not to introduce spurious cycles, this preferences process helps us to capture that trend.

implying substitutability between government and private consumption,[42] and $\gamma_{G,\Psi} = 0.160$, so that a positive spill-over to firms' total factor productivity guarantees that the public expenditure multiplier is larger than 1 in the short-run.

In the realm of monetary policy, we estimate the central bank reaction to deviations from the inflation target, ϕ^{π}, to be 2.965. Additionally, $\overline{\Pi}$ is calibrated to 1.011, which results roughly in an annual net inflation target of 4.5%, whereas ϕ^{Y} and ϕ^{i} are estimated to be 0.020 and 0.783, respectively.

In the specific estimation of the remaining exogenous processes, we obtain from posterior means $\rho^{A} = 0.933$ and $\sigma_{A} = 0.005$.[43] Concerning the preference shock, we find $\rho_{\beta} = 0.966$ and $\sigma_{\beta} = 0.004$. For the monetary shock process, we obtain $\rho_{\mathcal{M}} = 0.232$ and $\sigma_{\mathcal{M}} = 0.004$ (annualized 0.016). At last, for the government expenses shock, we obtain $\rho_{GG} = 0.795$, $\rho_{GY} = 0.132$, and $\sigma_{G} = 0.013$.[44]

[42] Fève and Sahuc (2017) estimates it to be -1.51 (posterior mean) for the euro area.

[43] This calibration implies that the likelihood of a TFP shock at least as bad as the one associated with a default event is of 1.72%, an otherwise rare disaster indeed.

[44] If our estimation had found that G_t significantly reacts to output deviation from the steady state with a lag, it could reduce to a large extent the parametric space for which the model has a solution. This shows how sensitive the specification of government expenses can be to models with fiscal limits.

Parameter	Prior Dist.	CI Min	CI Max	% CI	Post. Mode	Post. Mean	Post. Std.	Post. 5%	Post. 95%
σ	gamma	1.500	3.500	99.0%	2.227	2.132	0.259	1.684	2.586
α_G	normal	-1.000	1.000	95.0%	0.551	0.542	0.108	0.376	0.717
$\gamma_{G\Psi}$	normal	-0.250	0.500	95.0%	0.160	0.163	0.028	0.117	0.208
ϕ^π	normal	2.000	3.500	95.0%	2.891	2.965	0.163	2.683	3.199
ϕ^Y	normal	0.000	1.000	95.0%	0.017	0.020	0.011	0.005	0.039
ϕ^i	beta	0.500	0.990	95.0%	0.787	0.783	0.037	0.714	0.834
ρ^A	beta	0.250	0.750	90.0%	0.934	0.933	0.014	0.909	0.955
ρ^β	beta	0.250	0.750	90.0%	0.971	0.966	0.012	0.944	0.984
ρ^{GY}	beta	0.250	0.750	90.0%	0.143	0.132	0.026	0.090	0.176
ρ^{GG}	beta	0.250	0.750	90.0%	0.777	0.795	0.039	0.729	0.858
$\rho^{\mathscr{M}}$	beta	0.250	0.750	90.0%	0.216	0.232	0.054	0.145	0.321
σ^A	inv. gamma	0.000	0.035	99.9%	0.005	0.005	$3.8e^{-4}$	0.004	0.005
σ^β	inv. gamma	0.000	0.020	99.9%	0.004	0.004	$3.6e^{-4}$	0.003	0.004
σ^G	inv. gamma	0.000	0.035	99.9%	0.013	0.013	0.001	0.011	0.015
σ^M	inv. gamma	0.000	0.020	99.9%	0.004	0.004	$8.6e^{-4}$	0.003	0.006
$\sigma^{me,Y}$	inv. gamma	0.000	0.040	99.9%	0.009	0.009	$6.8e^{-4}$	0.008	0.010

Table 4: Summary of parameters estimation

4.2 Simulation of the fiscal limits

Like in Bi (2012), we explore how economic fundamentals affect the fiscal limit by finding the conditional distribution of that limit under different assumptions. The specification of the shocks as processes with infinite support (normal distributions) allows for the default probability to be always present and different from zero, as the possibility that an extremely large shock inflicts our model economy is always positive. It is important to note, though, that the fiscal limits computed here are *extremely sensitive to our parameterization*. For instance, small changes to the elasticity of labor supply, from 1 to 1/1.3 could shift the distribution to the right by more than 100% of output. Besides, we opt for turning off the preferences shock, as we had already done when of the definition of the fiscal limit (49), for such a stochastic shock harshly increases both the mean and the standard deviation of the fiscal limit distribution, leading to implausible results. These are critical limitations of our approach to modeling fiscal limits, and therefore, we prefer to focus on their qualitative features.

The algorithm can be summarized as follows. We approximate the possible values of each exogenous variable with a discrete state vector of size S obtained using the Tauchen (1986) method. Then, for each possible combination of states, we conduct $N = 150,000$ Monte Carlo simulations. In each simulation, we randomly draw as many vectors of size $(T_{FL} + T_{\text{burn-in}}, 1)$ as the number of shock processes we have; where $T_{FL} = 200$ is the number of periods ahead that we use in the summation of the fiscal limit expression, and $T_{\text{burn-in}} = 200$ is the number of initial shocks that we discard. Next, like in Bi et al. (2018), we approximate the empirical distribution of fiscal limits by a normal distribution, finding its respective mean and standard deviation, which we attribute to each specific combination of states. As the fiscal limit is, by definition, the maximum level of debt that the government can support without defaulting, in each iteration the model is solved assuming that at every period $\delta_t = 0$ and $\tau_t = \tau_t^{max} = \tau^{max}$.

Our calibration indicates that the deterministic steady state of the Brazilian economy at the recent period was situated far from the fiscal limit until at least 2019.[45] Figure 8 depicts at

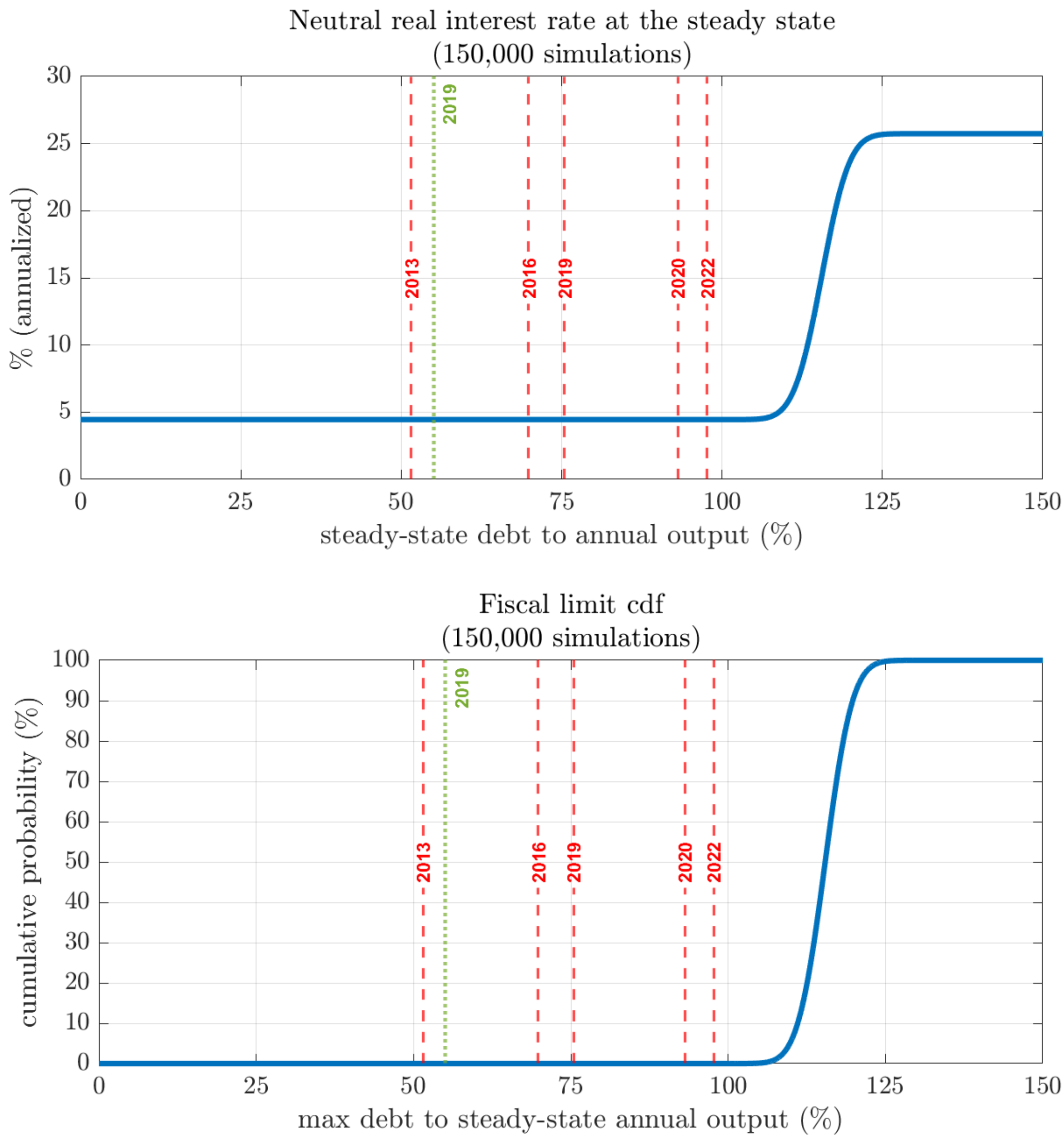
[45] Some important caveats should be raised here. First, our approach to calibration is agnostic w.r.t the source of risk premium embedded on Brazilian interest rates, and we do not try to directly match its respective moments, except for β, which summarizes the deterministic discount of the future. Second, the calibrated period exhibits a pronounced downward trend for interest rates, which we interpret generically as slow improvement of fundamentals, but that we do not try to replicate in this paper. Third, one way of shifting to the left the

the top panel how the annualized neutral real interest rate of steady state changes as we pick a higher debt-to-output ratio. Vertical-dashed lines indicate actual and projected ratios at the end of the respective labeled years (2013, 2016, 2019, 2020, 2022)[46]. Vertical-dotted lines indicate the same but for net debt. The curve is steady until around 110%, but it grows exponentially after that mark, reaching a top plateau after 125% at the time-discounted recovery rate of steady state. The graph at the bottom panel exhibits the cumulative probability distribution of the fiscal limit being lower or equal to the same range of debt-to-output ratios of the top panel. Default probability remains neglectful until around 105%, but grows quickly after it.[47] As a consequence of that switching nature, even a numerically low default probability can levy a heavy interest rate burden on such an economy. It is important to remember that the steady state we calculate here depends on the calibration of the model, especially of our choice for β. It could very well be that, during the period analyzed, the Brazilian economy already presented significant default risk in the eyes of the marginal investor, so our value for β is biased and actually includes some default premium. This fragility of our calibration only reinforces that we should focus on the qualitative aspects of the results presented here.

distribution of the fiscal limits at the steady state along the debt-to-output axis is by assuming that the government is more impatient than households, perhaps in reason of political motives, a common assumption in the sovereign default literature.

[46]2013, 2016, and 2019 are actual values. 2020 and 2022 are projections made by the IFI (Instituto Fiscal Independente) on 17th of November 2020.

[47]Bi (2017) calibrates a simple RBC model with fiscal limits to Brazil and finds that default probability starts growing meaningfully after debt-to-output ratio reaches 110% (by visual inspection of her plotted results). Not such a different result from ours.



Note: Vertical-dashed lines indicate actual and projected ratios of the gross debt at the end of the respective labeled years, where 2013, 2016, and 2019 are actual values, while 2020 and 2022 are projections made by the IFI (Instituto Fiscal Independente) on 17th of November 2020. Vertical-dotted lines indicate actual net debt at the end of the respective label years.

Figure 8: The neutral real interest rate and the fiscal limit cdf

To understand how the fiscal limit is affected by state variables, we plot its distribution at different steady-state values in Figure 9. On the left panels of the figure, we plot the PDFs, and, on the right panels, we plot the respective CDFs. It is possible to see on the top panels that increasing the steady-state value of productivity, $\overline{A}$, amplifies the fiscal space of the government as it turns less likely that the latter will have to default on its debt. Under our calibration, an increase of 1% of $\overline{A}$, ceteris paribus, increases the median of the fiscal limit distribution by 0.15 $\left(4\overline{Y}\right)$, while a symmetrical reduction of $\overline{A}$ reduces the same measure by 0.14 $\left(4\overline{Y}\right)$. This shows how during an economic recession the fiscal limit shrinks. On the second row, by its turn, we can see that the higher the steady-state level of government expenses, the lower is its fiscal space, what is represented by a shift to the left of the density function. Under our calibration, a 1 p.p. increase of $\frac{\overline{G}}{\overline{Y}}$, ceteris paribus, is enough to reduce the median of the distribution by 0.18 $\left(4\overline{Y}\right)$. This large effect reflects the fact that, as the tax rate is already set at the peak of the Laffer curve, augmenting further government expenses severely endangers fiscal sustainability. Finally, at the bottom panel, we plot how changing the discount factor of the households can have a huge impact on fiscal limits. Reducing the steady-state discount factor by 0.25 quarterly p.p. (1 p.p. annually) shifts the distribution to the left by 0.17 $\left(4\overline{Y}\right)$. The effect is clearly non-linear in the value of the parameter, as increasing the discount factor by the same amount shifts the distribution to the right by 0.39 $\left(4\overline{Y}\right)$. The coexistence of very low real interest rates with large debt-to-output ratios in low-risk developed economies is, thus, consistently predicted by the fiscal limits methodology.

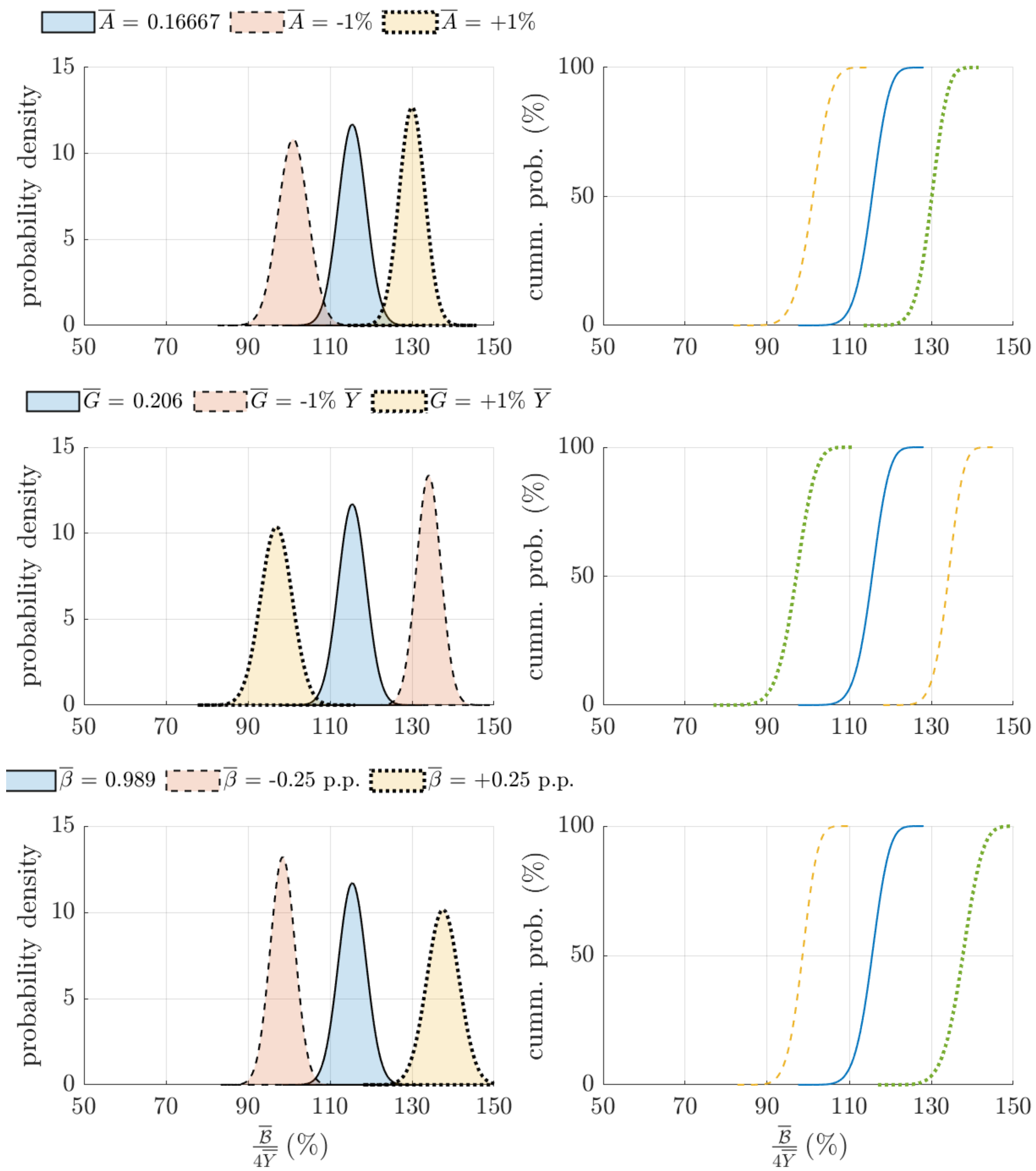


Figure 9: Fiscal limits at different steady states: PDFs (left panels) and CDFs (right panels)

In Figures 10 and 11, we expand the previous analysis to different values of the parameters that govern either the persistence or the standard deviation of the shocks. On the top panels of both figures, the effect of raising any of ρ_A or σ_A over the density distribution is that it gets more spread, what means that default becomes more likely at lower debt-to-output ratios. On the remaining panels, we can see that due to the role that government expenses play in our model, changes in the persistence or the size of a shock to it will only affect the fiscal limit in case the correlation with lagged output changes. The more pro-cyclical public expenditure is, the lower is the risk of default. Changing the role performed by government expenses in the model may overturn our results, so that is a very sensitive feature of the model.

Finishing the sensitivity analysis of our model's fiscal limits, in Figure 12, we plot how they change when we calibrate different values for some selected parameters: $\gamma_{G\Psi}$, α_G, and γ^{NR}. As one can see, a higher spillover from government expenses to productivity shifts the fiscal limit distribution to the left at the same time that it flattens it. Remember that a negative shock to government expenses also reduces the effective productivity of that economy, making it harder for the government to collect enough taxes to repay the debt. Concerning α_G, a negative value of this parameter implies complementarity between the consumption of private and public goods. This tends to shift the fiscal limit distribution to the left. The opposite happens when that parameter is positive, for now consumption of private and public goods are substitutes. We interpret changes in α_G as agents becoming satisfied with less consumption in the second scenario, and, therefore, more willing to postpone consumption. Changes in γ^{NR}, by its turn, regulates the fraction of agents able to finance the government. When $\gamma^{NR} = 0$, all agents are Ricardian, what shifts the fiscal limits distribution to the right. The opposite happens as γ^{NR} gets larger.

In Table 5 we list the mean/median and the standard deviation of all distributions plotted in Figures 9, 10, and 11.

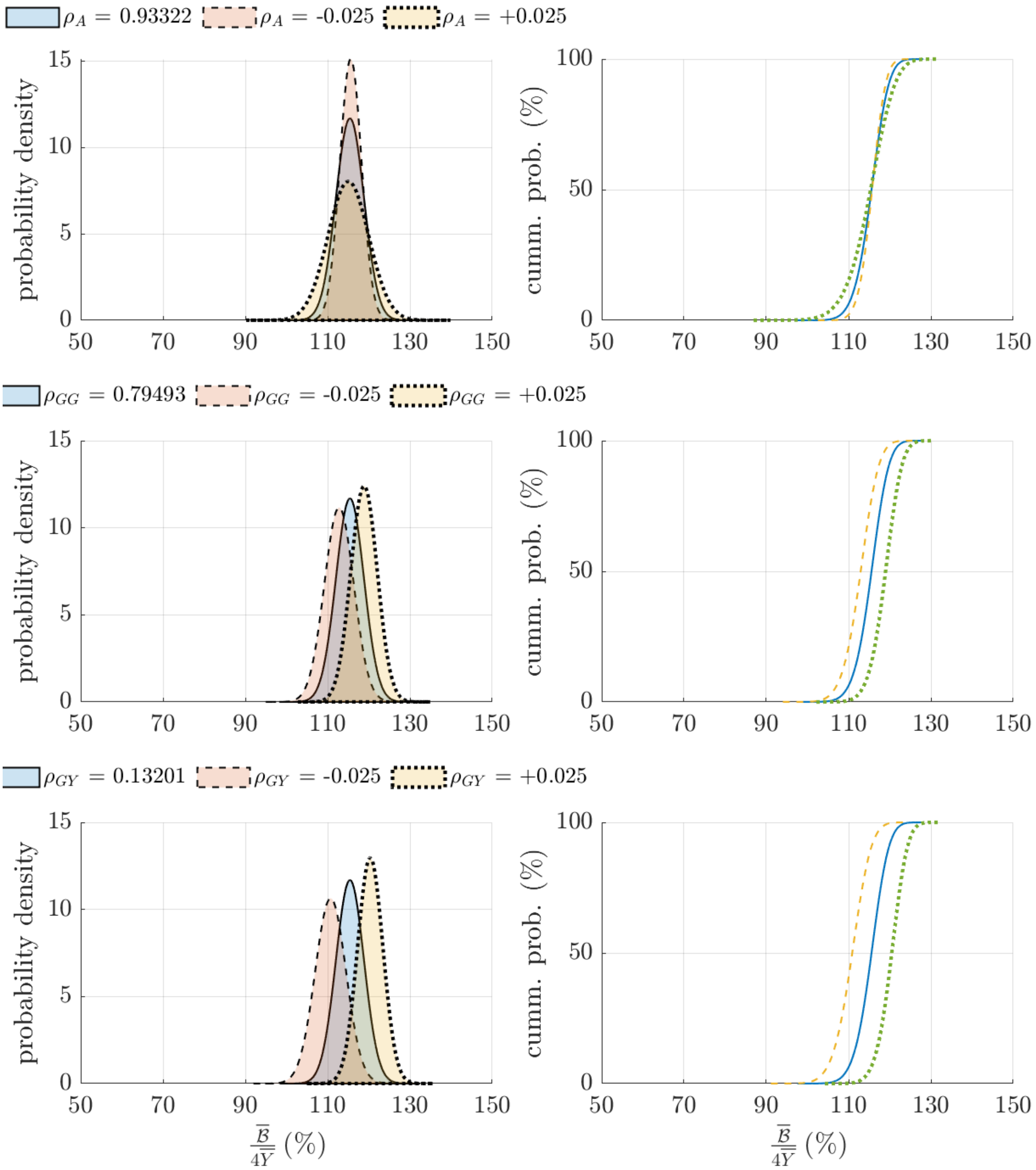


Figure 10: Fiscal limits at different shock persistence coefficients: PDFs (left panels) and CDFs (right panels)

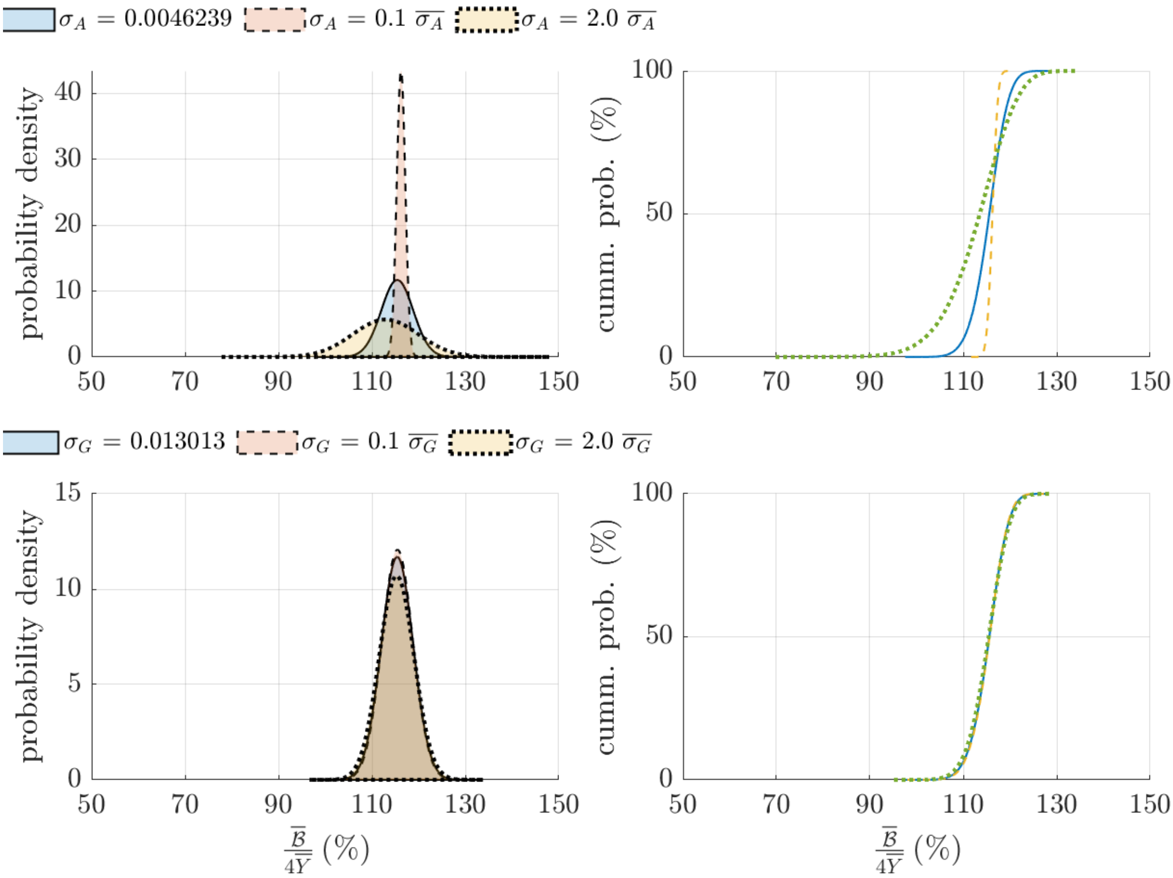


Figure 11: Fiscal limits at different shock volatility coefficients: PDFs (left panels) and CDFs (right panels)

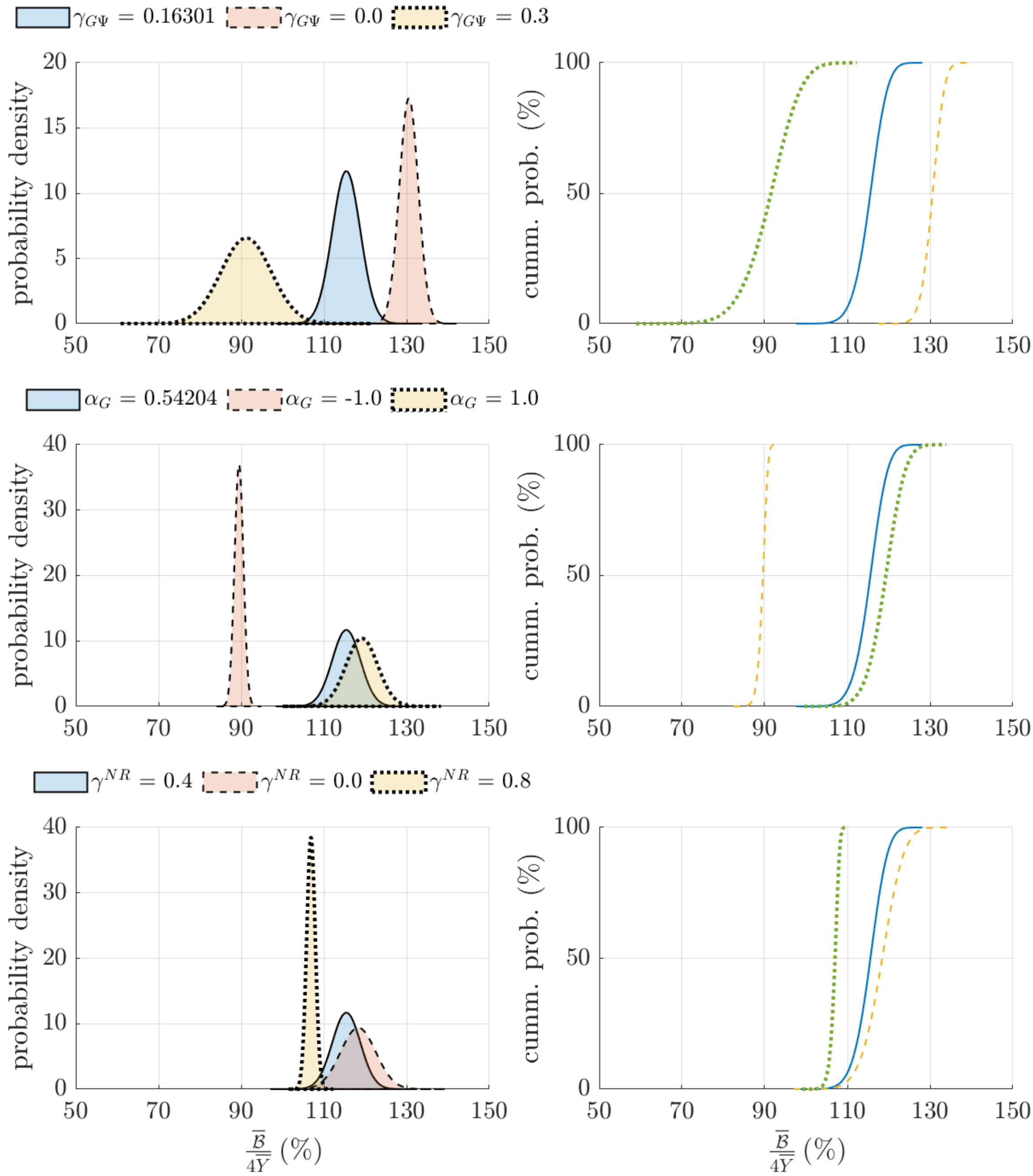


Figure 12: Fiscal limits after changing selected parameters: PDFs (left panels) and CDFs (right panels)

	Medium Param.	Low Param.	High Param.	Medium μ $(\% 4\overline{Y})$	Low μ $(\% 4\overline{Y})$	High μ $(\% 4\overline{Y})$	Medium σ $(\% 4\overline{Y})$	Low σ $(\% 4\overline{Y})$	High σ $(\% 4\overline{Y})$
$\overline{A}$	0.167	0.165	0.168	115.6	101.2	130.1	3.4	3.7	3.1
$\overline{G}$	0.206	0.196	0.216	115.6	134.2	97.1	3.4	3.0	3.8
$\overline{\beta}$	0.989	0.987	0.991	115.6	98.6	137.7	3.4	3.0	3.9
ρ_A	0.933	0.908	0.958	115.6	115.7	115.3	3.4	2.6	5.0
ρ_{GG}	0.795	0.770	0.820	115.6	112.9	119.0	3.4	3.6	3.2
ρ_{GY}	0.132	0.107	0.157	115.6	110.9	120.5	3.4	3.8	3.1
σ_A	0.005	0.000	0.009	115.6	116.2	113.5	3.4	0.9	7.0
σ_G	0.013	0.001	0.026	115.6	115.6	115.4	3.4	3.3	3.7
$\gamma_{G\Psi}$	0.163	0.000	0.300	115.6	130.6	91.5	3.4	2.3	6.1
α_G	0.542	-1.000	1.000	115.6	89.6	119.3	3.4	1.1	3.8
γ^{NR}	0.400	0.000	0.800	115.6	118.4	107.0	3.4	4.3	1.0

Table 5: Fiscal limits: sensitivity analysis

4.3 Equilibrium distribution

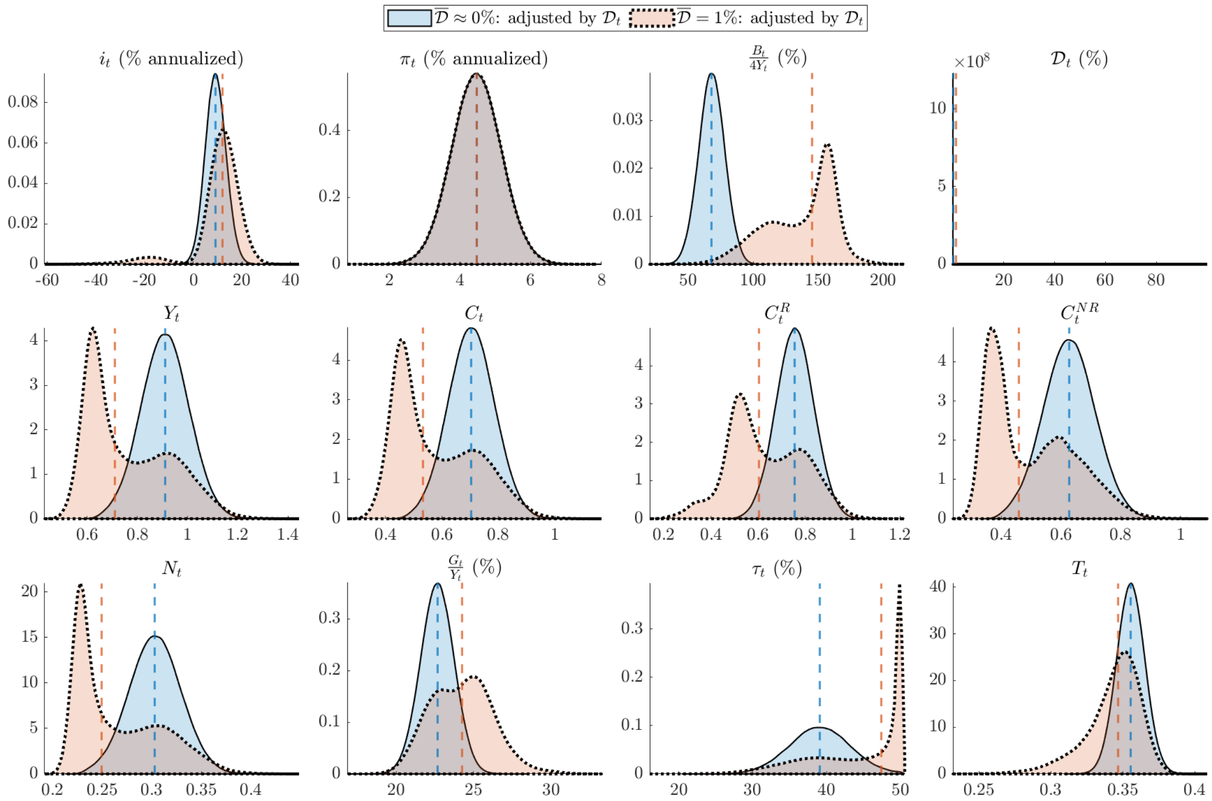
We analyze, here, the equilibrium distribution of the model by drawing stochastic shocks for 4 Markov chains with 330,000 periods each, where the first 30,000 periods are discarded. The unconditional distributions are obtained from piling up all periods of all chains. To make clear the differences between the monetary policy rules, we simplify them by turning off the interest rate smoothing and the dependency on the output gap. In all simulation periods, the model is solved around the current regime-specific non-stochastic steady-state.

Still, one important caveat must be raised in the interpretation of the following results. As we have simplified the policy rules, they will not necessarily induce similar distributions as their fully specified versions. Besides, as our calibration relies on the Bayesian estimation of the (default-risk-free) single-regime flexible-price version of the model, the estimated parameters may be biased, and, therefore, may not be very close approximations to their equivalent values had we estimated the full regime-switching model. Despite all that, we believe the qualitative results we present shall remain valid under more precise calibrations.

4.3.1 Variables distribution

One of the questions we seek to answer in this paper is whether the existence of default risk underlying the policy asset influences the distribution of inflation and nominal interest rates in a given economy. It is an empirical fact that emerging (risky) economies tend to adopt higher nominal policy rates at the same time that their inflation process is perceived as more volatile in comparison with advanced (risk-free) economies. Our approach sheds some light on that topic.

First, in Figure 13, we plot the unconditional ergodic distribution of selected variables under the risk-adjusted policy rule far from the fiscal limit, at our benchmark calibration, and near the fiscal limit, at a steady-state debt level consistent with 1% default risk as calculated from the fiscal limits. In addition to the common monetary dominance, some differences stand out. For one, the higher $\frac{\overline{B}}{Y}$ makes taxation at the peak of the Laffer curve much more likely. *The transition between regimes leads output, consumption and employment to exhibit multimodality, higher volatility, and lower means, three undesirable features of pushing the economy towards its fiscal limit.* Despite the higher average tax rate, government revenue is lower in reason of the depressed tax base, at the same time that $\mathbb{E}\,\frac{G_t}{Y_t}$ is higher in reason of the lower output. It is easy to conclude that, in this model, *welfare reduces near the fiscal limit, what warrants an important policy recommendation against expanding the debt level near it.* Note that, as we plot default-adjusted policy rule environments, inflation successfully averages around the central bank's target no matter the presence or not of significant default risk, where deviations from the target are due to unexpected monetary shocks. The policy rate, by its turn, exhibits both higher mean and volatility when default risk is significant. Note also that negative interest rates become more likely in that case, as we have not imposed a zero or effective lower bound for the policy rate in the model.

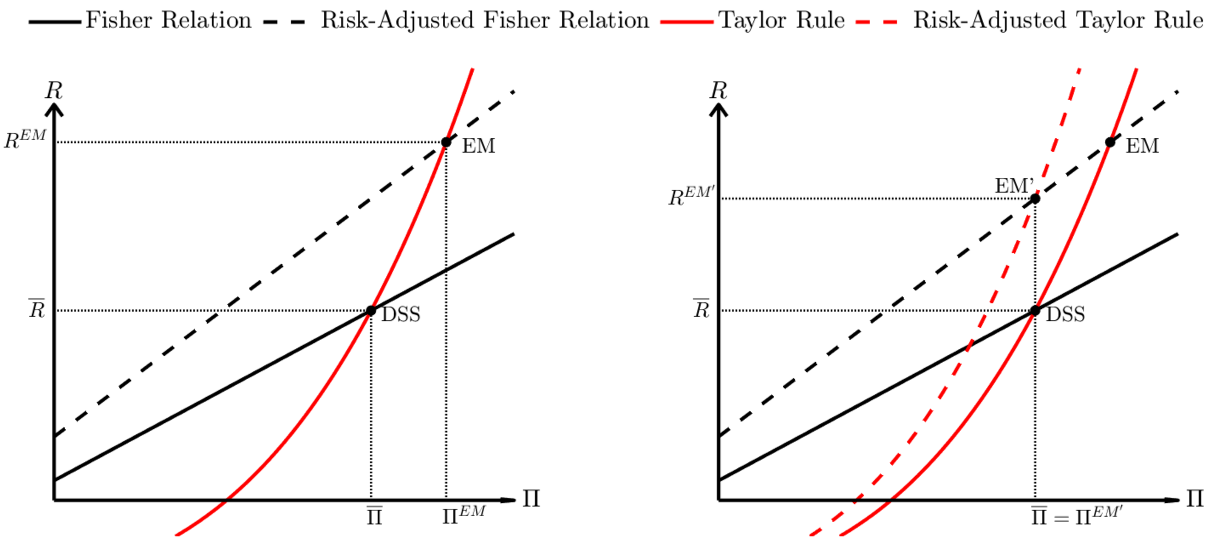


Note: Each vertical line is at the median of the distribution with the same color. Horizontal axis indicates the values of the variables, while the vertical axis indicates the probability density of each value. Density functions are estimated with the Epanechnikov kernel.

Figure 13: Ergodic distribution of selected endogenous variables far and near the fiscal limit

4.3.2 Rules comparison

We move to the comparison across the policy rules. In Figure 15, we plot both of them under the benchmark calibration. As one can observe, the policy rules are hardly distinguishable in that case. Then, in Figure 16, we plot the same variables near the fiscal limit (2% default probability). The multimodality of some variables stand out, but switching rules impact more the distributions of the policy rate and of inflation. In Figure 17, we separate theses variables and plot the consequence of pushing the economy even closer to the fiscal limit, at a debt level consistent with 5% default probability. *When the central bank ignores default risk, both π_t and i_t are higher compared to when it decides to fully offset default risk.* Under our calibration, conducting monetary policy with risky assets and accommodating default risk results in the average annualized policy rate being 0.2 p.p. higher (0.6 p.p. excluding negative policy rates) than when the central bank offsets default risk. At the same time, annualized inflation is 0.2 p.p. higher (0.3 excluding negative policy rates) than the inflation target. Remember that, in the non-stochastic steady state, all rules hit the target with success. It is only when shocks are activated and regime-switching is allowed (default and taxation risk) that such divergences between the studied rules appear. Figure 14 plots the aforementioned result using stylized curves for the Fisher relation and the Taylor rule both in their risk-free and default-risk adjusted versions. The left panel represents Rule 1, when the central bank ignores the policy asset default risk, whereas the right panel represents Rule 2, when the central bank adjusts its policy rule to that risk. We have not specified an effective-lower-bound for the Taylor rules as risky economies are less likely to visit that constraint, but if we had the graphs would change in two ways. First, there would be also a deflationary equilibrium in which the effective-lower-bound is binding. Second, the inflationary bias of the policy-asset default risk would be to some extent offset by the deflationary bias of the effective-lower-bound, where the last bias is described in Hills et al. (2019) as existing at both the risky steady state and at the unconditional mean.



Note: Left panel represents Policy Rule 1, when the central bank ignores the policy asset default risk, whereas right panel represents a switch from that rule to Policy Rule 2, when the central bank adjusts its policy rule to that risk. R is the gross nominal policy rate and Π is gross inflation. Variables with an overbar represent targets or deterministic values. DSS stands for deterministic steady state, while EM stands for ergodic mean.

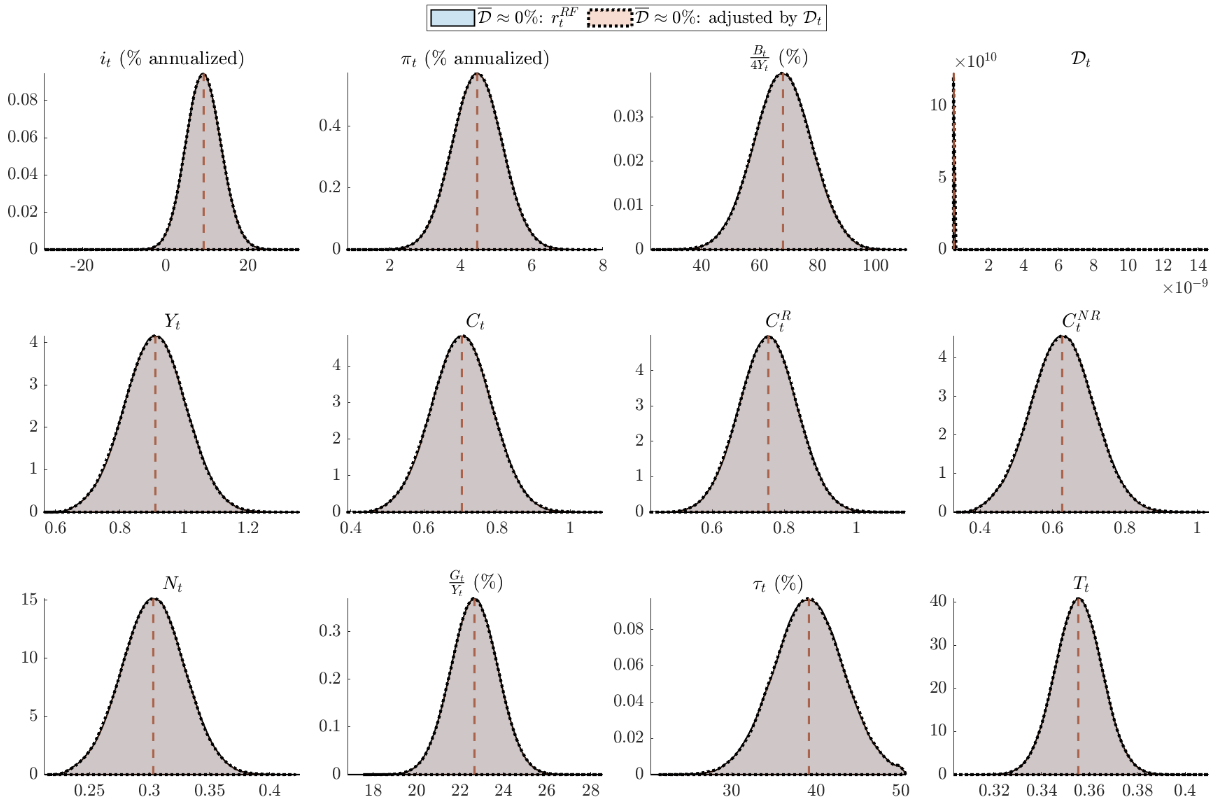
Figure 14: Stylized representation of the effect of policy-asset default risk on equilibrium

The summary of our comparison is tabulated in Table 6, while Table 7 shows that, with inflation under control, real interest rates (r_t and r_t^{Gov}) are the amplified variables. Important to note that, as we raise the default probability, both the risk-free and the default-risky real interest rate demanded in equilibrium increase. This result is engendered by consumption smoothing under incomplete markets of the type modeled here, where only part of the households are able to buy (partial) insurance against income fluctuations.[48] Our regime-switching set-up in which defaulting triggers nasty recessions drives its magnitude, as will be shown in Section 4.4.

Above all, our results suggest that assuming default risk in the underlying policy asset may have numerically relevant implications for monetary policy. Since our solution method includes the linearization of the model, we rely to some extent on certainty equivalence[49], what excludes excessive risk premia due to the risk-aversion of the agents. Having this in mind gives us a glimpse that, in periods of financial distress, the divergence between the policy rules should be even larger.

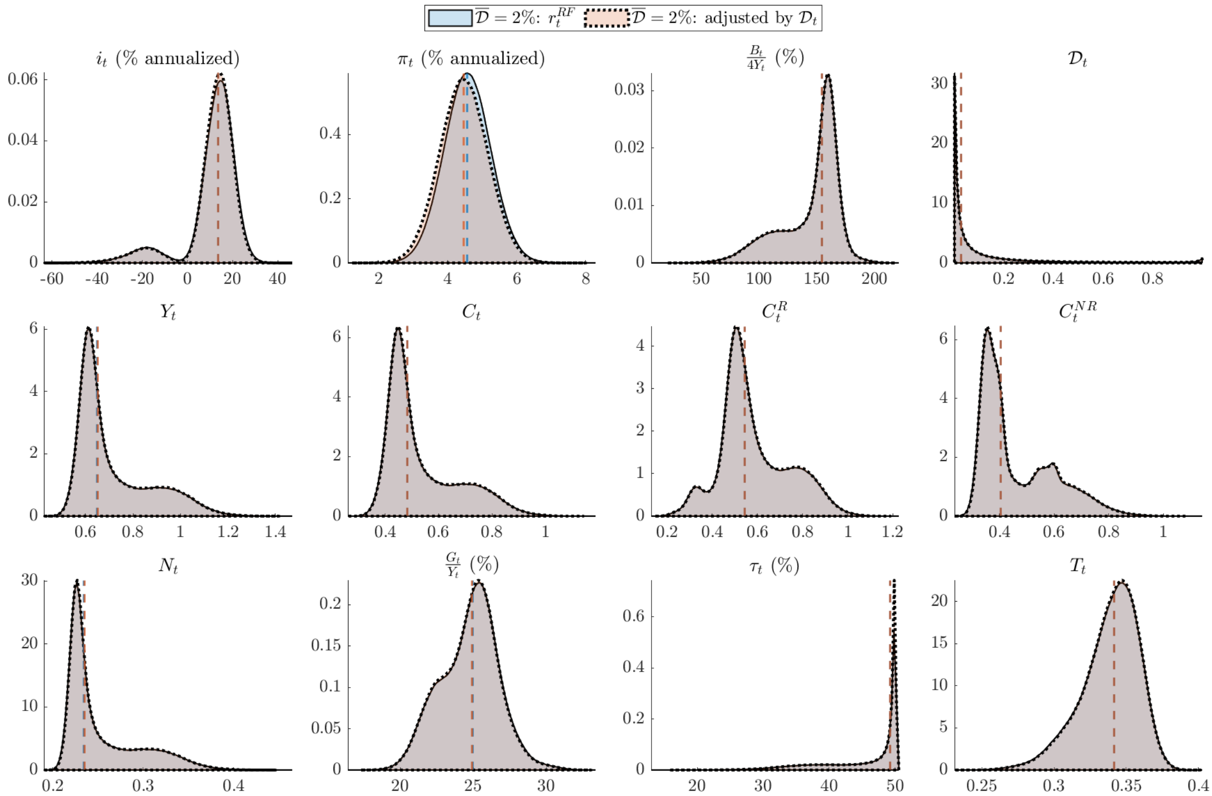
[48] Kocherlakota (2015) singles out a similar result in the incomplete market class of models of Bewley (1986) and Aiyagari (1994) – solved with global methods – where additional supply of public debt is not completely offset by corresponding additional demand due to borrowing constraints, resulting in the increase of the long-run neutral real interest rate.

[49] Regime-switching models are not usually certainty-equivalence models since the decision rule that solves the stochastic problem may differ from the one that solves the nonstochastic problem. Specifically, this paper's model violates the property that $\mathbb{E}(\varepsilon_{t+1}|x_t) = 0$, where x_t is the state vector. For instance, being close to the fiscal limit increases the likelihood of a negative TFP shock $\left(\delta^{TFP} > 0\right)$.



Note: Each vertical line is at the median of the distribution with the same color. Horizontal axis indicates the values of the variables, while the vertical axis indicates the probability density of each value. Density functions are estimated with the Epanechnikov kernel.

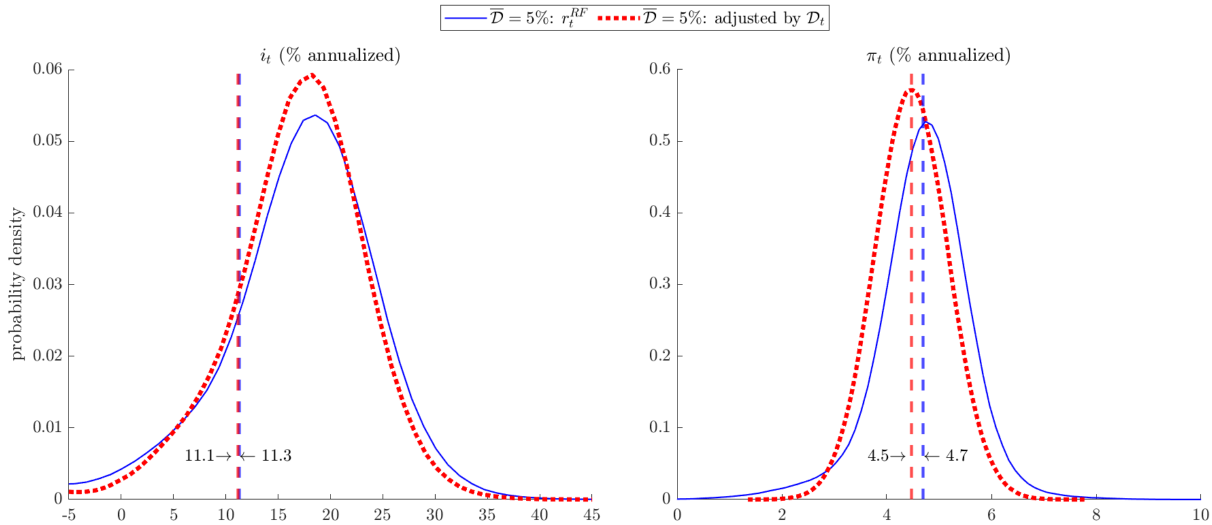
Figure 15: Ergodic distribution of selected endogenous variables under different monetary policy rules far from the fiscal limit



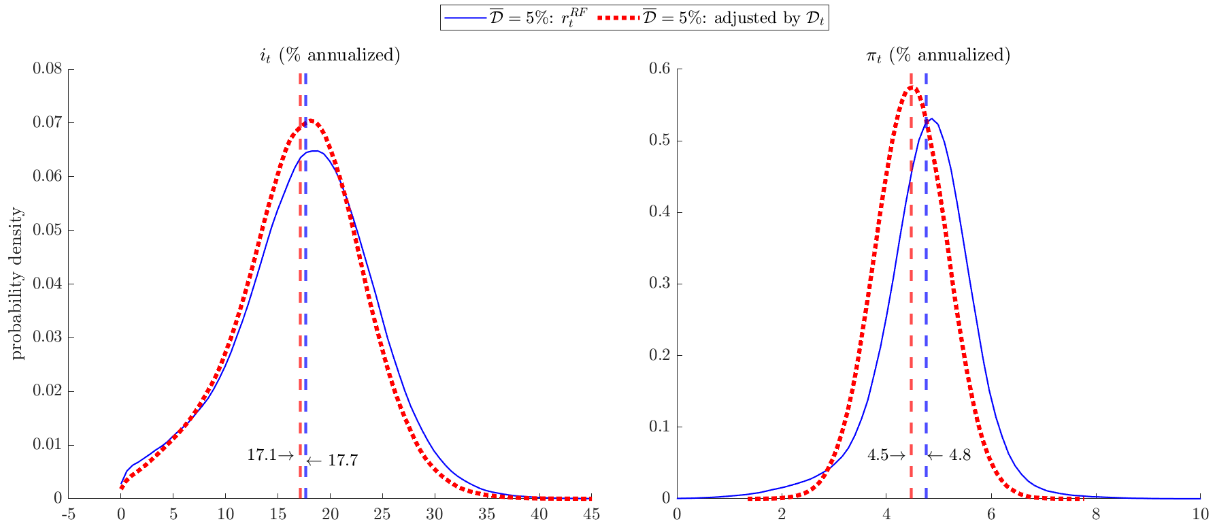
Note: Each vertical line is at the median of the distribution with the same color. Horizontal axis indicates the values of the variables, while the vertical axis indicates the probability density of each value. Density functions are estimated with the Epanechnikov kernel.

Figure 16: Ergodic distribution of selected endogenous variables under different monetary policy rules near the fiscal limit

All observations



Only observations in which the policy rate is above zero



Note: Each vertical line is at the mean of the distribution with the same color. Horizontal axis indicates the values of the variables, while the vertical axis indicates the probability density of each value. Density functions are estimated with the Epanechnikov kernel.

Figure 17: Ergodic distribution of the policy rate and of inflation under different monetary policy rules near the fiscal limit

All observations

	i_t (% annualized)			π_t (% annualized)		
	DSS	EM	Bias	DSS	EM	Bias
$\overline{\mathscr{D}} = 5\%$: r_t^{RF}	9.7	11.3	1.6	4.5	4.7	0.2
$\overline{\mathscr{D}} = 5\%$: adjusted by $\mathscr{D}_t$	9.7	11.1	1.4	4.5	4.5	0

Only observations in which the policy rate is above zero

	i_t (% annualized)			π_t (% annualized)		
	DSS	EM	Bias	DSS	EM	Bias
$\overline{\mathscr{D}} = 5\%$: r_t^{RF}	9.7	17.7	8	4.5	4.8	0.3
$\overline{\mathscr{D}} = 5\%$: adjusted by $\mathscr{D}_t$	9.7	17.1	7.4	4.5	4.5	0

Note: DSS stands for deterministic steady state (or non-stochastic) and the ones displayed belong to Regime 1; EM stands for the ergodic mean of a Monte Carlo simulation with 4 Markov Chains including 330,000 periods each, where the first 30,000 periods are discarded as burn-in; while Bias is calculated as the difference between EM and DSS. Displayed values are the means of annualized rates.

Table 6: Bias comparison: i_t and π_t

All observations

	r_t (% annualized)			r_t^{Gov} (% annualized)		
	DSS	EM	Bias	DSS	EM	Bias
$\overline{\mathscr{D}} = 5\%$: r_t^{RF}	5	6.1	1.1	5	6.1	1.1
$\overline{\mathscr{D}} = 5\%$: adjusted by $\mathscr{D}_t$	5	6.4	1.4	5	6.4	1.4

Only observations in which the policy rate is above zero

	r_t (% annualized)			r_t^{Gov} (% annualized)		
	DSS	EM	Bias	DSS	EM	Bias
$\overline{\mathscr{D}} = 5\%$: r_t^{RF}	5	12.2	7.2	5	12.2	7.2
$\overline{\mathscr{D}} = 5\%$: adjusted by $\mathscr{D}_t$	5	12.1	7.1	5	12.1	7.1

Note: DSS stands for deterministic steady state (or non-stochastic) and the ones displayed belong to Regime 1; EM stands for the ergodic mean of a Monte Carlo simulation with 4 Markov Chains including 330,000 periods each, where the first 30,000 periods are discarded as burn-in; while Bias is calculated as the difference between EM and DSS. Displayed values are the means of annualized rates.

Table 7: Bias comparison: r_t and r_t^{Gov}

4.4 Bringing inflation back to the model

So far, we have analyzed rules in which a time-varying intercept mostly neutralizes the (dis)inflationary impact of the other shocks of this economy but the monetary one, namely, the TFP and the government expenditure shocks. In reason of that, these rules represent a monetary policy's commitment with keeping inflation close to the target at every period, preventing long and lasting deviations. Not very realistic, indeed, as real-life central banks tend to have commitments with longer horizons, what gives them some margin to smooth out policy rate adjustments, including their reaction to temporary changes of the equilibrium real rate.[50]

To loose our policy rules so as to unleash inflation, we proceed in two ways. First, we re-

[50]The Brazilian Central Bank targets annual inflation measured at the end of each calendar year. Other central banks, such as the Bank of England, pursue a rolling 12-month inflation target.

activate smoothing in the policy rate by varying its parameter, ϕ^i, from 0.0 (no smoothing) to 0.9 (high smoothing). Second, we replace the risk-free real rate, r_t^{RF}, by the risk-free natural real rate, r_t^n, in the time-varying intercept. This last change brings our policy rules closer to some of the rules in the literature as the natural rate is the one consistent with flexible prices.[51]

On the importance of the natural rate, Edge et al. (2008) estimate a New-Keynesian model for the U.S. economy between 1984 and 2004 using Bayesian methods, and compare the story told by the estimated output and interest rate gaps to the actual policy narrative of the period. They find that some high-frequency fluctuations of the business cycle perceived by policymakers as inefficient in the period were, in fact, efficient responses to movements of the estimated natural interest rate. In comparison with Laubach and Williams (2003), they find the natural rate to be even more volatile, what can be explained by its short-term definition in DSGE models. Moreover, the authors find that actual market real interest rates seem to be less volatile than the natural one, what could be explained by the central banks' preference for Taylor rules with smooth adjustment. In our exercise, we opt to introduce the natural rate by appending to the model the counterfactual flexible-price equations (40), (41), (44), and (46), as well as duplicating (27), while renaming each variable with a n superscript.[52] This way, while we do not ignore the effect of sticky prices in the simulated trajectory of the variables, to some extent we are able to clean the reaction of the central bank to price stickiness. Therefore, temporary deviations from the inflation target emerge

[51]The FED lists five interest rate rules on its website that according to it are illustrative of the many policy rules that have received attention in the academic literature. Among them, four adopt time-varying long-run real interest rates in the intercept, r_t^{LR}, while the other one follows a first-difference process that should eventually converge to the same intercept. r_t^{LR} is defined as "the level of the neutral inflation-adjusted federal funds rate in the longer run that, on average, is expected to be consistent with sustaining inflation at 2 percent and output at its full resource utilization level". In a move to take these rules to the numbers, the FED conducts an exercise in which it calculates that intercept as "the difference between the linearly interpolated quarterly average values of the long-term forecast for the three-month Treasury bill rate and the long-term forecast for inflation of the implicit GDP price deflator from Blue Chip Economic Indicators". This measure is supposed to be a slow-mover, contrasting with the behavior of the natural rate simulated in DSGE models. Available on https://www.federalreserve.gov/monetarypolicy/policy-rules-and-how-policymakers-use-them.htm as of July 17 2021.

[52]Alternatively, the natural rate could have been obtained from the simulation of the flexible-price version of the regime-switching model.

more frequently in the model.[53]

We proceed by conducting a Monte Carlo simulation with 4 Markov Chains in which each chain includes 33,000 periods where the first 3,000 are excluded as burn-in. Table 8 displays the simulated means for the policy rate, the real interest rate, and inflation, all annualized, in a set-up with $\mathscr{D} = 2\%$ and r_t^n in the intercept. Different combinations of ϕ^π and ϕ^i are simulated. Note that $\mathbb{E}\, i_t$ and $\mathbb{E}\, r_t$ are always higher than their value at the deterministic steady-state, while $\mathbb{E}\pi_t$ increases exponentially with smoothing.[54] This result shows that even low levels of default risk can lead to a reality of high policy rates, high real rates, and moderately high inflation. Table 9 excludes observations of all regimes but Regime 1 to allow us to make a closer comparison with single-regime models. One can note how the deviation from the deterministic steady state is sizable, which highlights the importance of "hidden" regimes to explain observed levels of variables. In Table 10, we present the results of the same set-up, but with the model solved in second order around the deterministic steady state of each regime. Now, there is more than expected loss and regime-switching engendering the unpleasant coincidence, there is also risk-aversion from the households amplifying the phenomenon.

[53]This approach is adopted in Neiss and Nelson (2003) and has the benefit of making the real natural rate an exogenous object, but at the cost of, as Woodford (2011, sec. 3.4 of ch. 5) criticizes, reducing its connection with equilibrium determination in the sticky-price economy.

[54]There is usually no closed-form relation between the ergodic mean of variables in an endogenous regime-switching model and deterministic steady states. Each non-linearity in the model, including asymmetrical shock distributions, may bias equilibrium values to one direction or the other.

i_t (% annualized, DSS $\bar{i}$ = 9.7 in Regime 1)

	ϕ^i=0.000	ϕ^i=0.150	ϕ^i=0.300	ϕ^i=0.450	ϕ^i=0.600	ϕ^i=0.750	ϕ^i=0.900
ϕ^π=1.00	28.2	27	23.8	21.3	18.1	14.7	10.8
ϕ^π=1.25	11.7	11.4	11.5	11.4	11.5	11.8	11.1
ϕ^π=1.50	11	10.9	10.8	10.8	11	11.1	11.1
ϕ^π=1.75	10.7	10.7	10.7	10.8	10.8	11	11
ϕ^π=2.00	10.7	10.7	10.7	10.8	10.8	11	11.1
ϕ^π=2.25	10.8	10.7	10.7	10.7	10.7	11.1	11.1
ϕ^π=2.50	10.7	10.7	10.7	10.7	10.8	11	11.1
ϕ^π=2.75	10.8	10.6	10.7	10.7	10.8	11	11.2
ϕ^π=3.00	10.7	10.8	10.7	10.8	10.8	11	11.1

r_t (% annualized, DSS $\bar{r}$ = 5.0 in Regime 1)

	ϕ^i=0.000	ϕ^i=0.150	ϕ^i=0.300	ϕ^i=0.450	ϕ^i=0.600	ϕ^i=0.750	ϕ^i=0.900
ϕ^π=1.00	6.5	6.5	6.3	6.3	6.2	6.2	5.6
ϕ^π=1.25	5.8	5.7	5.7	5.7	5.8	6	5.4
ϕ^π=1.50	6	5.9	5.9	5.9	5.9	5.9	5.4
ϕ^π=1.75	5.9	5.9	6	6	5.9	5.9	5.4
ϕ^π=2.00	5.9	6	5.9	6	5.9	5.9	5.4
ϕ^π=2.25	6	6	5.9	5.9	5.9	6	5.3
ϕ^π=2.50	6	6	6	6	6	5.9	5.3
ϕ^π=2.75	6.1	5.9	6	5.9	5.9	5.9	5.3
ϕ^π=3.00	6	6	5.9	6	5.9	5.8	5.3

π_t (% annualized, DSS $\bar{\pi}$ = 4.5 in Regime 1)

	ϕ^i=0.000	ϕ^i=0.150	ϕ^i=0.300	ϕ^i=0.450	ϕ^i=0.600	ϕ^i=0.750	ϕ^i=0.900
ϕ^π=1.00	20.6	19.5	16.7	14.4	11.5	8.5	5.8
ϕ^π=1.25	5.3	5.3	5.4	5.4	5.6	5.9	6.3
ϕ^π=1.50	4.6	4.6	4.7	4.7	4.9	5.3	6.3
ϕ^π=1.75	4.4	4.5	4.5	4.6	4.8	5.2	6.2
ϕ^π=2.00	4.4	4.4	4.5	4.6	4.8	5.2	6.3
ϕ^π=2.25	4.4	4.4	4.5	4.6	4.8	5.2	6.3
ϕ^π=2.50	4.4	4.4	4.5	4.6	4.8	5.2	6.3
ϕ^π=2.75	4.4	4.4	4.5	4.6	4.8	5.2	6.3
ϕ^π=3.00	4.4	4.5	4.5	4.7	4.8	5.2	6.3

Table 8: Simulated means: $\overline{\mathcal{D}}$ = 2%; r_t^n in the intercept

i_t (% annualized, DSS $\bar{i}$ = 9.7 in Regime 1)

	ϕ^i=0.000	ϕ^i=0.150	ϕ^i=0.300	ϕ^i=0.450	ϕ^i=0.600	ϕ^i=0.750	ϕ^i=0.900
ϕ^π=1.00	28.7	27	24.2	21.8	17.8	14.8	10.6
ϕ^π=1.25	14.9	14.3	13.8	13.3	13	12.6	11.3
ϕ^π=1.50	14.2	13.6	13.2	12.5	12.2	12.1	11.4
ϕ^π=1.75	13.8	13.3	12.7	12.5	12.1	12	11.6
ϕ^π=2.00	13.7	13.4	12.8	12.5	12.2	12.1	11.6
ϕ^π=2.25	13.8	13.1	12.8	12.6	12.4	12.1	11.8
ϕ^π=2.50	13.6	13.4	12.9	12.7	12.4	12.2	12.1
ϕ^π=2.75	13.7	13.3	13.1	12.7	12.4	12.1	12
ϕ^π=3.00	13.8	13.2	12.9	12.6	12.3	12.1	12.1

r_t (% annualized, DSS $\bar{r}$ = 5.0 in Regime 1)

	ϕ^i=0.000	ϕ^i=0.150	ϕ^i=0.300	ϕ^i=0.450	ϕ^i=0.600	ϕ^i=0.750	ϕ^i=0.900
ϕ^π=1.00	9.8	9.3	8.7	8.3	7.6	7.5	6.8
ϕ^π=1.25	8.9	8.4	8	7.6	7.4	7.3	7
ϕ^π=1.50	9	8.5	8.1	7.6	7.3	7.2	6.9
ϕ^π=1.75	8.8	8.4	7.9	7.7	7.3	7.2	7
ϕ^π=2.00	8.8	8.5	8	7.8	7.5	7.3	7
ϕ^π=2.25	8.9	8.2	8	7.8	7.6	7.2	7
ϕ^π=2.50	8.7	8.5	8.1	7.9	7.6	7.4	7.2
ϕ^π=2.75	8.9	8.4	8.3	7.9	7.6	7.2	7
ϕ^π=3.00	8.9	8.3	8.1	7.8	7.5	7.2	7.1

π_t (% annualized, DSS $\bar{\pi}$ = 4.5 in Regime 1)

	ϕ^i=0.000	ϕ^i=0.150	ϕ^i=0.300	ϕ^i=0.450	ϕ^i=0.600	ϕ^i=0.750	ϕ^i=0.900
ϕ^π=1.00	17.3	16.3	14.5	12.7	9.7	7.2	4.3
ϕ^π=1.25	5.4	5.3	5.3	5.3	5.4	5.4	4.9
ϕ^π=1.50	4.7	4.7	4.7	4.7	4.8	4.9	5
ϕ^π=1.75	4.5	4.5	4.5	4.6	4.7	4.9	5.1
ϕ^π=2.00	4.5	4.5	4.5	4.6	4.7	4.9	5.2
ϕ^π=2.25	4.5	4.5	4.5	4.6	4.7	4.9	5.4
ϕ^π=2.50	4.5	4.5	4.5	4.6	4.7	5	5.5
ϕ^π=2.75	4.5	4.5	4.5	4.6	4.7	5	5.5
ϕ^π=3.00	4.5	4.5	4.5	4.6	4.7	5	5.6

Table 9: Simulated means (Regime 1): $\overline{\mathscr{D}} = 2\%$; r_t^n in the intercept

i_t (% annualized, DSS $\bar{i}$ = 9.7 in Regime 1)

	ϕ^i=0.000	ϕ^i=0.150	ϕ^i=0.300	ϕ^i=0.450	ϕ^i=0.600	ϕ^i=0.750	ϕ^i=0.900
ϕ^π=1.00	3051977.4	969350.4	252996.6	35574.6	3155.1	4323.7	176639.4
ϕ^π=1.25	16.5	16.2	16.1	16.3	16.9	18.9	32.1
ϕ^π=1.50	13.5	13.2	13.1	13	13.4	14.6	21.4
ϕ^π=1.75	12.7	12.5	12.2	12.2	12.4	13.3	17.9
ϕ^π=2.00	12.4	12.2	11.9	12	12	12.7	16.1
ϕ^π=2.25	12.3	12	11.8	11.7	11.9	12.4	15.1
ϕ^π=2.50	12.2	11.9	11.7	11.7	11.8	12.1	14.6
ϕ^π=2.75	12.2	11.8	11.7	11.6	11.7	12.1	14.1
ϕ^π=3.00	12.1	11.8	11.6	11.6	11.7	12	13.7

r_t (% annualized, DSS $\bar{r}$ = 5.0 in Regime 1)

	ϕ^i=0.000	ϕ^i=0.150	ϕ^i=0.300	ϕ^i=0.450	ϕ^i=0.600	ϕ^i=0.750	ϕ^i=0.900
ϕ^π=1.00	413.3	170.4	63.7	16.9	6.1	5.3	10.6
ϕ^π=1.25	6.9	6.6	6.5	6.4	6.5	6.9	7.9
ϕ^π=1.50	6.9	6.6	6.5	6.4	6.5	6.8	7.7
ϕ^π=1.75	7	6.7	6.5	6.5	6.4	6.7	7.4
ϕ^π=2.00	7	6.8	6.6	6.5	6.5	6.6	7.3
ϕ^π=2.25	7	6.8	6.5	6.5	6.4	6.5	7.1
ϕ^π=2.50	7.1	6.8	6.6	6.5	6.5	6.5	7
ϕ^π=2.75	7.1	6.8	6.6	6.5	6.5	6.5	7
ϕ^π=3.00	7.1	6.8	6.6	6.6	6.5	6.5	6.8

π_t (% annualized, DSS $\bar{\pi}$ = 4.5 in Regime 1)

	ϕ^i=0.000	ϕ^i=0.150	ϕ^i=0.300	ϕ^i=0.450	ϕ^i=0.600	ϕ^i=0.750	ϕ^i=0.900
ϕ^π=1.00	2858858.7	914789.3	241008.9	34430.9	3045.8	4123.1	169819.4
ϕ^π=1.25	8.5	8.7	8.8	9.3	10.1	12	24.5
ϕ^π=1.50	5.8	5.9	6.1	6.2	6.8	8.1	14.6
ϕ^π=1.75	5.1	5.2	5.3	5.5	5.9	6.9	11.6
ϕ^π=2.00	4.9	4.9	5	5.2	5.6	6.4	10
ϕ^π=2.25	4.7	4.8	4.9	5.1	5.4	6.1	9.1
ϕ^π=2.50	4.7	4.7	4.8	5	5.3	5.9	8.6
ϕ^π=2.75	4.6	4.7	4.8	5	5.2	5.8	8.2
ϕ^π=3.00	4.6	4.7	4.8	4.9	5.2	5.8	7.9

Table 10: Simulated means (2$^{\text{nd}}$ Order): $\overline{\mathscr{D}} = 2\%$; r_t^n in the intercept

So, what does drive these results? In Table 11, we repeat the exercise, but this time eliminating the recession shock in case of default, $\delta^{TFP} = 0$. Note how we practically return to the deterministic steady state of Regime 1 for all combinations of ϕ^{π} and ϕ^{i}. This sheds light on why default risk is so powerful as to generate previous results. Confiscating resources from the Ricardian households may be (morally) bad, but the recession that it elicits is (economically) much worse. Although rare, it is a disaster, and one which increases the likelihood of drawing an extremely negative TFP shock from the latter's, otherwise normal, distribution. In traditional models without regime-switching, that distribution is usually assumed to be symmetrical. This explains why they cannot generate the unpleasant coincidence discussed in this paper. Overall, Ricardian agents suffer a financial wealth loss at the exact time that they need it the most for smoothing their consumption out: at a severe recession.

From the Euler equation of the policy asset (10) evaluated at the ergodic mean, one variable that helps to set the wedge w.r.t the deterministic steady state is the expected default probability, whose ergodic mean value increases with the size of δ^{TFP}, since more profound recessions deteriorate by more the fiscal position of the government and also increase the likelihood of reaching the peak of the Laffer curve. In decomposition (54),[55] we make explicit that the covariance between the Ricardian's household one-period utility growth and the expected default probability for the next period should be negative due to default-enacted recession, which allows us to infer that default-risky real interest rates are higher at the ergodic mean in comparison with at the deterministic steady state, since utility at a default event is highly likely to be lower than at the period before of it.[56] Besides, in reason that default risk makes a regime switch more likely, the volatility of endogenous variables increase, as we have seen in Figure 16, what also helps to push upward interest rates at the ergodic mean, making the wedge sizable.

$$\mathbb{E}^{EM}\left[\frac{1}{1+r_t}\right] = \mathbb{E}^{EM}\left[\underbrace{\beta\left(\frac{C^R_{t+1}+\alpha_G G_{t+1}-\eta\frac{N_{t+1}^{1+\chi}}{1+\chi}}{C^R_t+\alpha_G G_t-\eta\frac{N_t^{1+\chi}}{1+\chi}}\right)^{-\sigma}}_{\Lambda_U}\underbrace{(1-\delta_{t+1})}_{\Lambda_D}\right] \tag{54}$$

[55]$\mathbb{E}^{EM}$ and COV^{EM} stand for, respectively, the unconditional expectation and the unconditional covariance operators for the ergodic distribution.

[56]For low values of δ^{TFP}, this relation is less obvious given the distribution of non-TFP shocks.

$$= \underbrace{\mathbb{E}^{EM}[\Lambda_U]\,\mathbb{E}^{EM}[\Lambda_D] + \underbrace{\mathrm{COV}^{EM}[\Lambda_U, \Lambda_D]}_{<0}}_{\mathbb{E}^{DSS}[r_t] < \mathbb{E}^{EM}[r_t]}$$

So, how important is that regimes switch endogenously? It can be seen in Table 12 what happens if the default probability is kept fixed at its deterministic steady-state value (exogenous switching).[57] Both nominal and real policy rates as well as inflation shrink in comparison with Table 8. Taking the difference between both tables results in the endogenous component of the total *expectations-formation effect*, following the terminology of Leeper and Zha (2003) and Chang et al. (2019). That effect is induced by changes in the agents' beliefs about regimes. The fact that it is sizable in the model highlights not only the importance of the Lucas (1976) critique, but also the importance of allowing transitions matrices to evolve with the state vector.

$$\begin{aligned} \text{Total effect} &\equiv \mathbb{E}^{EM}[X_t] - \mathbb{E}^{EM}\left[X_t | \text{Regime}_t = 1\,\forall\, t\right] \\ \text{Endogenous effect} &\equiv \mathbb{E}^{EM}[X_t] - \mathbb{E}^{EM}\left[X_t | \text{Exogenous switching}\right] \\ \text{Exogenous effect} &\equiv \text{Total effect} - \text{Endogenous effect} \end{aligned}$$

Finally, what about the heterogeneity of our households? In Table 13, we repeat the exercise setting $\gamma^{NR} = 0$, so that all households are now Ricardian.[58] We are, basically, back to similar results as the ones displayed in Table 8. This shows that even a representative agent New-Keynesian (RANK) model with regime-switching would be able to reproduce the high levels of policy rate, real interest rate and inflation we find.

[57] We maintain the peak of the Laffer curve constraint endogenous in reason that it is modeled like an occasionally binding constraint.

[58] A more rigorous exercise would include recalculating the fiscal limits, which we have showed in this paper it will be displaced to the right along the debt-to-output axis, turning default less likely at any level of debt.

i_t (% annualized, DSS $\bar{i}$ = 9.7 in Regime 1)

	ϕ^i=0.000	ϕ^i=0.150	ϕ^i=0.300	ϕ^i=0.450	ϕ^i=0.600	ϕ^i=0.750	ϕ^i=0.900
ϕ^π=1.00	9.3	9.5	9.4	9.3	9.3	9.4	9.1
ϕ^π=1.25	9.3	9.2	9.2	9.1	9.2	9.2	8.9
ϕ^π=1.50	9.3	9.3	9.3	9.3	9.3	9.2	9
ϕ^π=1.75	9.3	9.3	9.5	9.3	9.3	9.2	8.9
ϕ^π=2.00	9.3	9.3	9.2	9.4	9.3	9.2	9
ϕ^π=2.25	9.4	9.3	9.4	9.3	9.3	9.3	9
ϕ^π=2.50	9.4	9.3	9.3	9.3	9.3	9.3	9
ϕ^π=2.75	9.4	9.3	9.3	9.3	9.3	9.3	9.1
ϕ^π=3.00	9.3	9.4	9.3	9.4	9.4	9.3	9.1

r_t (% annualized, DSS $\bar{r}$ = 5.0 in Regime 1)

	ϕ^i=0.000	ϕ^i=0.150	ϕ^i=0.300	ϕ^i=0.450	ϕ^i=0.600	ϕ^i=0.750	ϕ^i=0.900
ϕ^π=1.00	4.6	4.7	4.7	4.7	4.6	4.7	4.4
ϕ^π=1.25	4.7	4.6	4.6	4.6	4.6	4.6	4.3
ϕ^π=1.50	4.7	4.7	4.6	4.7	4.6	4.6	4.4
ϕ^π=1.75	4.6	4.6	4.8	4.7	4.6	4.6	4.4
ϕ^π=2.00	4.6	4.7	4.6	4.8	4.6	4.6	4.4
ϕ^π=2.25	4.7	4.6	4.7	4.7	4.6	4.7	4.4
ϕ^π=2.50	4.7	4.7	4.6	4.7	4.6	4.7	4.4
ϕ^π=2.75	4.7	4.7	4.6	4.6	4.7	4.7	4.5
ϕ^π=3.00	4.7	4.7	4.6	4.7	4.7	4.6	4.5

π_t (% annualized, DSS $\bar{\pi}$ = 4.5 in Regime 1)

	ϕ^i=0.000	ϕ^i=0.150	ϕ^i=0.300	ϕ^i=0.450	ϕ^i=0.600	ϕ^i=0.750	ϕ^i=0.900
ϕ^π=1.00	4.4	4.5	4.5	4.4	4.4	4.4	4.6
ϕ^π=1.25	4.4	4.4	4.4	4.3	4.4	4.4	4.5
ϕ^π=1.50	4.4	4.4	4.4	4.4	4.4	4.4	4.4
ϕ^π=1.75	4.4	4.4	4.5	4.4	4.4	4.4	4.4
ϕ^π=2.00	4.4	4.5	4.4	4.5	4.4	4.4	4.4
ϕ^π=2.25	4.5	4.5	4.5	4.5	4.4	4.4	4.4
ϕ^π=2.50	4.5	4.5	4.5	4.5	4.5	4.4	4.5
ϕ^π=2.75	4.5	4.5	4.5	4.5	4.5	4.5	4.5
ϕ^π=3.00	4.5	4.5	4.5	4.5	4.5	4.5	4.5

Table 11: Simulated means: $\overline{\mathscr{D}} = 2\%$; r_t^n in the intercept; and $\delta^{TFP} = 0$

i_t (% annualized, DSS $\bar{i}$ = 9.7 in Regime 1)

	ϕ^i=0.000	ϕ^i=0.150	ϕ^i=0.300	ϕ^i=0.450	ϕ^i=0.600	ϕ^i=0.750	ϕ^i=0.900
ϕ^π=1.00	14.2	13.5	13	12.1	11.7	10.7	9.4
ϕ^π=1.25	10.5	10.4	10.7	10.4	10.3	10.1	9.5
ϕ^π=1.50	10	10.3	10.1	10	10.2	10.1	9.5
ϕ^π=1.75	10.2	10.2	10.1	10.1	10	10.1	9.5
ϕ^π=2.00	10.1	10.2	10	10	10	9.9	9.5
ϕ^π=2.25	10.1	10	9.9	10	9.9	9.9	9.4
ϕ^π=2.50	10	9.9	10	10	9.9	9.9	9.5
ϕ^π=2.75	10	10	10	10.1	10	9.9	9.5
ϕ^π=3.00	9.9	10	9.9	10	10	9.9	9.5

r_t (% annualized, DSS $\bar{r}$ = 5.0 in Regime 1)

	ϕ^i=0.000	ϕ^i=0.150	ϕ^i=0.300	ϕ^i=0.450	ϕ^i=0.600	ϕ^i=0.750	ϕ^i=0.900
ϕ^π=1.00	5.3	5.2	5.2	5.2	5.2	5.1	4.6
ϕ^π=1.25	5.2	5.2	5.3	5.1	5.1	5.1	4.6
ϕ^π=1.50	5.1	5.2	5.1	5.1	5.2	5.1	4.6
ϕ^π=1.75	5.2	5.3	5.2	5.2	5.1	5.2	4.7
ϕ^π=2.00	5.2	5.3	5.2	5.1	5.1	5.1	4.7
ϕ^π=2.25	5.2	5.2	5.1	5.2	5.1	5.1	4.6
ϕ^π=2.50	5.2	5.1	5.2	5.2	5.1	5.2	4.7
ϕ^π=2.75	5.2	5.2	5.2	5.3	5.2	5.2	4.7
ϕ^π=3.00	5.1	5.2	5.2	5.2	5.2	5.1	4.8

π_t (% annualized, DSS $\bar{\pi}$ = 4.5 in Regime 1)

	ϕ^i=0.000	ϕ^i=0.150	ϕ^i=0.300	ϕ^i=0.450	ϕ^i=0.600	ϕ^i=0.750	ϕ^i=0.900
ϕ^π=1.00	8.4	7.8	7.4	6.6	6.2	5.3	4.8
ϕ^π=1.25	5.1	5	5.1	5	4.9	4.8	4.8
ϕ^π=1.50	4.7	4.8	4.7	4.7	4.7	4.7	4.7
ϕ^π=1.75	4.7	4.7	4.7	4.7	4.6	4.7	4.8
ϕ^π=2.00	4.6	4.6	4.6	4.6	4.6	4.6	4.7
ϕ^π=2.25	4.6	4.6	4.6	4.6	4.6	4.6	4.7
ϕ^π=2.50	4.6	4.5	4.6	4.6	4.6	4.6	4.7
ϕ^π=2.75	4.5	4.5	4.5	4.6	4.6	4.6	4.6
ϕ^π=3.00	4.5	4.5	4.5	4.5	4.6	4.6	4.6

Table 12: Simulated means (exogenous regime-switching): $\overline{\mathscr{D}} = 2\%$; r_t^n in the intercept

i_t (% annualized, DSS $\bar{i}$ = 9.7 in Regime 1)

	ϕ^i=0.000	ϕ^i=0.150	ϕ^i=0.300	ϕ^i=0.450	ϕ^i=0.600	ϕ^i=0.750	ϕ^i=0.900
ϕ^π=1.00	22.7	21.5	20.1	18.7	16.4	13.9	10.5
ϕ^π=1.25	12.5	12.4	12.6	12.4	12.1	11.9	10.7
ϕ^π=1.50	11.6	11.7	11.5	11.6	11.6	11.4	10.8
ϕ^π=1.75	11.5	11.3	11.4	11.2	11.2	11.2	10.6
ϕ^π=2.00	11.4	11.3	11.1	11.2	11.1	11.2	10.8
ϕ^π=2.25	11.2	11.1	11.2	11.1	11.1	11.2	10.6
ϕ^π=2.50	11.1	11.2	11	11	11	11	10.7
ϕ^π=2.75	11.1	11.1	11.2	11.2	11	11.1	10.7
ϕ^π=3.00	11	11	11.1	11.1	11.1	11.1	10.7

r_t (% annualized, DSS $\bar{r}$ = 5.0 in Regime 1)

	ϕ^i=0.000	ϕ^i=0.150	ϕ^i=0.300	ϕ^i=0.450	ϕ^i=0.600	ϕ^i=0.750	ϕ^i=0.900
ϕ^π=1.00	6.6	6.6	6.5	6.5	6.4	6.3	5.5
ϕ^π=1.25	6.2	6.2	6.3	6.2	6.1	6.1	5.3
ϕ^π=1.50	6.2	6.2	6.1	6.2	6.2	6	5.4
ϕ^π=1.75	6.3	6.1	6.2	6.1	6.1	6	5.2
ϕ^π=2.00	6.3	6.2	6.1	6.2	6.1	6	5.4
ϕ^π=2.25	6.2	6.2	6.2	6.1	6.1	6	5.3
ϕ^π=2.50	6.2	6.3	6.1	6.1	6.1	5.9	5.3
ϕ^π=2.75	6.2	6.1	6.3	6.2	6	6.1	5.4
ϕ^π=3.00	6.1	6.1	6.2	6.2	6.1	6.1	5.4

π_t (% annualized, DSS $\bar{\pi}$ = 4.5 in Regime 1)

	ϕ^i=0.000	ϕ^i=0.150	ϕ^i=0.300	ϕ^i=0.450	ϕ^i=0.600	ϕ^i=0.750	ϕ^i=0.900
ϕ^π=1.00	15.3	14.2	12.9	11.6	9.6	7.4	5.3
ϕ^π=1.25	5.9	5.9	5.9	5.9	5.7	5.7	5.6
ϕ^π=1.50	5.1	5.1	5.1	5.2	5.2	5.3	5.6
ϕ^π=1.75	4.9	4.9	4.9	4.9	5	5.1	5.6
ϕ^π=2.00	4.8	4.8	4.8	4.8	4.9	5	5.6
ϕ^π=2.25	4.7	4.7	4.7	4.8	4.8	5	5.5
ϕ^π=2.50	4.6	4.7	4.7	4.7	4.8	4.9	5.5
ϕ^π=2.75	4.6	4.6	4.7	4.7	4.8	4.9	5.5
ϕ^π=3.00	4.6	4.6	4.6	4.7	4.8	4.9	5.4

Table 13: Simulated means: $\overline{\mathscr{D}} = 2\%$; r_t^n in the intercept; and $\gamma^{NR} = 0$

5 Welfare

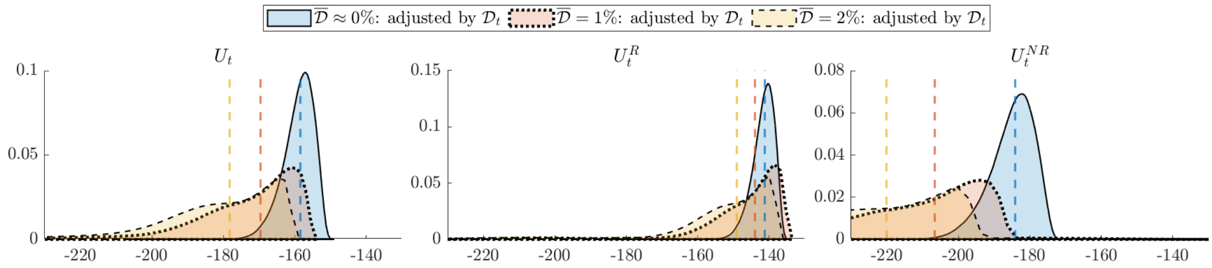
In our model, we have both Ricardian and non-Ricardian households, who will enjoy differently the benefits and downsides of inflating or defaulting the debt. On the one hand, higher inflation in the present means lower taxes in the future, what benefits both types of agents. On the other hand, inflation works as a wealth tax on the portfolio of Ricardian households. Moreover, when the government defaults on the debt, while it reduces the wealth of the Ricardian type by reducing the debt level, what, *coeteris paribus*, reduces the expected tax rates levied on both types of agents, it also enacts a negative TFP shock in the economy, which alone is enough to worsen the trajectory of the debt by reducing expected government revenues.

To compare welfare under different policy rules, we first turn off both smoothing and the dependency on the output gap and then we simulate the model resorting to 4 Markov chains with 330,000 periods each (30,000 excluded as burn-in)[59]. We plot the result for the risk-adjusted rule in Figure 18. The graphs for Rule 1 are omitted here because both rules exhibit the same pattern to the extent that they are not individually distinguishable by visual inspection. As our measure of welfare, we adopt the second-order Taylor expansion of the discounted sum of expected utility flows (2) around the non-stochastic steady state of each regime. Assuming that all individual households have the same weight in the welfare arithmetic, and recollecting that both types supply the same amount of labor at any period, while γ^{NR} is the fraction of non-Ricardian households in the whole population, the welfare equations can be recursively written as in (55) and (56). They allow us to answer the question on which policy rule is best in the sense that it maximizes either type-specific or aggregate welfare.

$$U_t^i = \frac{\left(C_t^i + \alpha_G G_t - \eta \frac{N_t^{1+\chi}}{1+\chi}\right)^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t U_{t+1}^i \quad i \in \{R, NR\} \tag{55}$$

$$U_t = (1-\gamma^{NR})U_t^R + \gamma^{NR} U_t^{NR} \tag{56}$$

[59]Although not originally meant for regime-switching models, we resort to the pruning method of Kim et al. (2008) to exclude explosive paths.



Note: Each vertical line is at the median of the distribution with the same color. Horizontal axis indicates the values of the variables, while the vertical axis indicates the frequency of each value. Density functions are estimated with the Epanechnikov kernel.

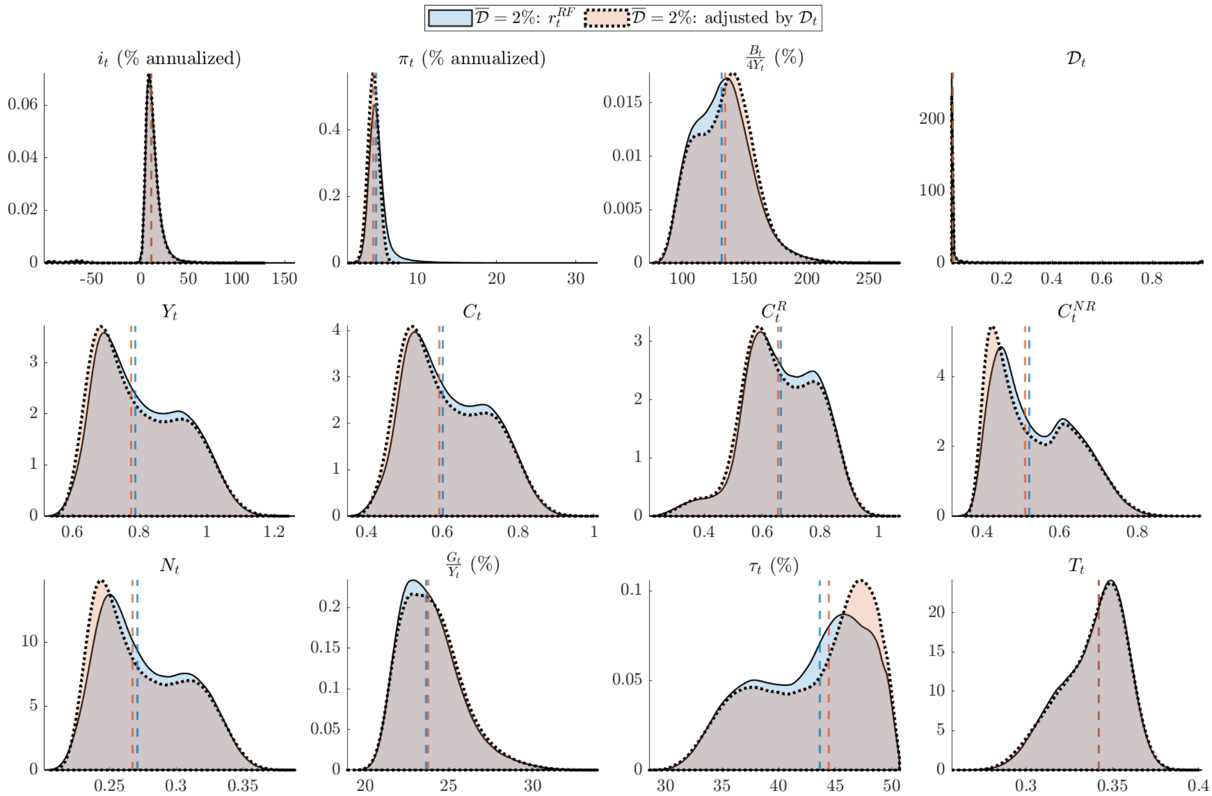
Figure 18: Welfare comparison

Analyzing the results of the simulations, we find that the policy rule that adjusts the intercept to policy-default risk, despite reducing inflation and the policy rate in equilibrium, comes with a small welfare cost under our calibration and for up to low default probabilities. Figure 19 shows that *accommodating default risk may improve welfare for it keeps the debt level to some extent lower, what, by our fiscal rule, induces lower taxes in this economy. The result is the same no matter the household is Ricardian or not,* what will be shown next and indicates alignment of interests between these two types of agents in the specification of the monetary policy rule.

Our finding that a central bank that avoids adjusting the intercept of the policy rule to the evolution of policy-default risk maximizes welfare is only valid under our calibration and in the subset of our tested rules. It is still possible that untested rules dominate our rule of choice in terms of welfare, or that different calibrations result in other policy recommendations. To investigate this, we conduct a sensitivity analysis of the policy parameters $\phi^\pi \in [1.00 : 0.25 : 3.00]$ and $\gamma_\tau \in [0.100 : 0.025 : 0.200]$, running for each combination of them 4 Markov chains with 33,000 periods each where we exclude the first 3,000 periods as burn-in. In the Tables 14, 15, and 16, we list the aggregate welfare under monetary policy rules 1 and 2, varying these parameters, and with a debt level consistent with ≈ 0%, 2%, and 5% steady-state default probability, respectively. In the Appendix F, we discriminate the welfare of the Ricardian household and of the non-Ricardian one in Tables 18, 19, 20, 21, 22, and 23.

This sensitivity analysis suggests that, at the benchmark calibration ($\phi^\pi = 2.965$ and $\gamma_\tau = 0.108$), the policy parameters are already consistent with the maximization of welfare under both monetary policy rules for all types of agents. With $\overline{\mathscr{D}} = 1\%$, the ϕ^π that maximizes welfare should be quite low, near 1.0, or around 3.0 under Rule 1, while it should be close to 3.0 under Rule 2. The non-linearity under Rule 1 reflects gains of the non-Ricardian agent in detriment of the Ricardian type as the latter seems to be negatively affected by the proximity with the indeterminacy region, which by itself would recommend keeping ϕ^{π} at the high level region. In this scenario, welfare would also increase with a somewhat more passive stance of fiscal policy, by rising γ_{τ} to the 0.150-0.175 region, a result that shows up even with just a remarkably small default probability. Beyond that, welfare deteriorates. With $\overline{\mathscr{D}}$ = 2%, the same recommendations are warranted. Finally, with $\overline{\mathscr{D}}$ = 5%, the parameter combinations that maximize welfare narrow down to γ_{τ} = 0.200 under both rules, while ϕ^{π} = 3.0 is optimal under Rule 1 and ϕ^{π} = 2.0 under Rule 2. For the first time, a clear preference for offsetting policy-default risk appears.[60] Nonetheless, and most important, *pushing the economy's steady state toward its fiscal limit, or stabilizing the debt near it, consistently deteriorates welfare across the board.* The coincidence of moderate-to-high levels of inflation in addition to high real and nominal policy rates observed in some emerging economies may, in fact, reflect an unpleasant reality underneath.

[60] Our tests showed that this specific preference for offsetting default risk instead of accommodating it seems to be sensitive to the relative frequency of the regimes. Inducing a relative higher prevalence of default events can make accommodating default risk preferable to offsetting it.



Note: Each vertical line is at the median of the distribution with the same color. Horizontal axis indicates the values of the variables, while the vertical axis indicates the frequency of each value. Density functions are estimated with the Epanechnikov kernel.

Figure 19: Ergodic distribution of selected endogenous variables under different monetary policy rules near the fiscal limit (2$^{\text{nd}}$ order approximation)

Rule 1: Risky policy asset, but the CB ignores default-risk dynamics

	γ_τ=0.100	γ_τ=0.125	γ_τ=0.150	γ_τ=0.175	γ_τ=0.200
ϕ^π=1.00	∞	NaN	-∞	NaN	-∞
ϕ^π=1.25	-162	-163	-163	-164	-164
ϕ^π=1.50	-161	-161	-162	-162	-162
ϕ^π=1.75	-160	-160	-161	-161	-161
ϕ^π=2.00	-160	-160	-160	-161	-161
ϕ^π=2.25	-160	-160	-160	-160	-160
ϕ^π=2.50	-159	-160	-160	-160	-160
ϕ^π=2.75	-159	-159	-160	-160	-160
ϕ^π=3.00	-159	-159	-159	-160	-160

Rule 2: Risky policy asset, and the CB perfectly tracks default-risk dynamics

	γ_τ=0.100	γ_τ=0.125	γ_τ=0.150	γ_τ=0.175	γ_τ=0.200
ϕ^π=1.00	NaN	NaN	NaN	NaN	NaN
ϕ^π=1.25	-162	-162	-163	-164	-164
ϕ^π=1.50	-161	-161	-162	-162	-162
ϕ^π=1.75	-160	-160	-161	-161	-161
ϕ^π=2.00	-160	-160	-160	-161	-161
ϕ^π=2.25	-160	-160	-160	-160	-160
ϕ^π=2.50	-159	-160	-160	-160	-160
ϕ^π=2.75	-159	-160	-160	-160	-160
ϕ^π=3.00	-159	-159	-159	-160	-160

Note: NaN means that no solution was found under that calibration. Shaded cells indicate the parameter values combinations that generate the highest welfare in the table.

Table 14: Sensitivity analysis of aggregate welfare under $\overline{\mathscr{D}} \approx 0\%$

Rule 1: Risky policy asset, but the CB ignores default-risk dynamics

	γ_τ=0.100	γ_τ=0.125	γ_τ=0.150	γ_τ=0.175	γ_τ=0.200
ϕ^π=1.00	-522	-2241	-398	-547	-876
ϕ^π=1.25	-232	-196	-189	-188	-189
ϕ^π=1.50	-219	-189	-183	-182	-183
ϕ^π=1.75	-212	-185	-179	-179	-180
ϕ^π=2.00	-206	-182	-177	-176	-177
ϕ^π=2.25	-203	-180	-175	-175	-176
ϕ^π=2.50	-202	-178	-174	-174	-175
ϕ^π=2.75	-199	-177	-173	-173	-174
ϕ^π=3.00	-198	-176	-172	-172	-173

Rule 2: Risky policy asset, and the CB perfectly tracks default-risk dynamics

	γ_τ=0.100	γ_τ=0.125	γ_τ=0.150	γ_τ=0.175	γ_τ=0.200
ϕ^π=1.00	NaN	NaN	NaN	NaN	NaN
ϕ^π=1.25	-234	-198	-189	-188	-190
ϕ^π=1.50	-220	-189	-183	-182	-183
ϕ^π=1.75	-213	-185	-179	-179	-180
ϕ^π=2.00	-207	-182	-177	-176	-178
ϕ^π=2.25	-204	-180	-175	-175	-176
ϕ^π=2.50	-202	-178	-174	-174	-175
ϕ^π=2.75	-200	-177	-173	-173	-174
ϕ^π=3.00	-199	-176	-172	-172	-173

Note: NaN means that no solution was found under that calibration. Shaded cells indicate the parameter values combinations that generate the highest welfare in the table.

Table 15: Sensitivity analysis of aggregate welfare under $\overline{\mathscr{D}} = 2\%$

Rule 1: Risky policy asset, but the CB ignores default-risk dynamics

	γ_τ=0.100	γ_τ=0.125	γ_τ=0.150	γ_τ=0.175	γ_τ=0.200
ϕ^π=1.00	3718	-5271	NaN	NaN	NaN
ϕ^π=1.25	NaN	-2192	-949	-594	-465
ϕ^π=1.50	NaN	-2227	-1000	-627	-479
ϕ^π=1.75	NaN	-2206	-1009	-632	-482
ϕ^π=2.00	NaN	-2243	-1005	-634	-473
ϕ^π=2.25	NaN	-2012	-1011	-634	-435
ϕ^π=2.50	NaN	-1047	-1007	-616	-401
ϕ^π=2.75	NaN	-466	-859	-591	-429
ϕ^π=3.00	NaN	-780	-775	-549	-303

Rule 2: Risky policy asset, and the CB perfectly tracks default-risk dynamics

	γ_τ=0.100	γ_τ=0.125	γ_τ=0.150	γ_τ=0.175	γ_τ=0.200
ϕ^π=1.00	NaN	NaN	NaN	NaN	NaN
ϕ^π=1.25	NaN	-2475	-1103	-693	-535
ϕ^π=1.50	NaN	-2428	-1081	-683	-458
ϕ^π=1.75	NaN	-2268	-1000	-576	-267
ϕ^π=2.00	NaN	-2172	-984	-587	-201
ϕ^π=2.25	NaN	-1177	-879	-430	-337
ϕ^π=2.50	NaN	-1081	-804	-291	-276
ϕ^π=2.75	NaN	-614	-596	-300	-255
ϕ^π=3.00	NaN	-210	-607	-279	-256

Note: NaN means that no solution was found under that calibration. Shaded cells indicate the parameter values combinations that generate the highest welfare in the table.

Table 16: Sensitivity analysis of aggregate welfare under $\overline{\mathscr{D}} = 5\%$

6 Conclusion

The highly frequent idea that governments can, and will, always print money to repay its debt denominated in domestic currency before choosing to default on it is at odds with economic history, what should raise concerns for monetary policy. In this paper, we sought to answer the question of whether conducting monetary policy with such default-risky assets, instead of usually conceived risk-free ones, should matter for a central bank. By employing a relatively simple TANK model with government, nominal debt, and endogenous fiscal limits, we find that not only inflation dynamics is affected, but also monetary policy simultaneously generates higher inflation, real, and nominal policy rates unless the central bank updates the intercept of its policy rule in response to changes in the default-risk premium. This result sheds new light on a long-standing discussion on why Brazil, whose economy we use as reference for calibrating our model, has displayed that unpleasant coincidence, which we explain as motivated by endogenous expectations of severe recessions in episodes of default. The slow attenuation of the unpleasant coincidence since the Real Plan is supposed to reflect a similarly slow transition between higher and lower long-run default risk perceptions of the agents.

Moreover, we find that, when the default probability is relevant, welfare of both Ricardian and hand-to-mouth households may be higher if fiscal policy is somewhat more passive, but not by much, as inflating the debt to some extent can mean lower distortionary taxes and, consequently, higher output and consumption in the future for all. Choosing to systematically ignore default risk can be optimal for at least low-to-moderate levels of the latter, but that may change otherwise. This shows that monitoring policy-asset default risk to distinguish its movements from changes in the unobserved real risk-free rate can help central banks improve welfare and reduce inflation, as they may be able to consciously opt to accommodate that risk or not to some extent.

It is important to note, though, that pushing the economy toward its fiscal limit, or stabilizing the debt near it, reduces welfare across the board, not only because default is modeled as a negative productivity shock, but also because it demands more distortionary taxation along the way. Contrary to the current observation of high debt accumulation in advanced economies which seems to suggest the opposite, the level of debt-to-output ratio of a country does matter, and the coincidence of high levels of inflation and nominal policy rate may be the symptom of an unpleasant reality indeed. The stance of monetary policy suggested by emerging economy models which neglect this source of misspecification may, therefore, be misleading.

Overall, this paper turns upside-down an assumption prevalent in monetary models and shows that, contrary to Milton Friedman's observation that "inflation is always and everywhere a monetary phenomenon"[61], it is sometimes simply the lack of confidence in the government (or in the central bank).

[61]In 1970, Milton Friedman gave a now famous lecture named “The Counter-Revolution in Monetary Theory”, in which he declared: "It follows from the propositions I have so far stated that inflation is always and everywhere a monetary phenomenon in the sense that it is and can be produced only by a more rapid increase in the quantity of money than in output. However, there are many different possible reasons for monetary growth, including gold discoveries, financing of government spending, and financing of private spending" Friedman (1970).

A Steady state of the model

In this section, we derive regime-specific non-stochastic steady states for this paper's model. Note that we define variables with an overline as the steady-state values of respective variables, and hereafter we make thorough use of our calibration in Table 3.

We start by defining two indicator functions that will promote the switching behavior across the four regime-specific steady states.

$$\mathbb{1}_{\tau_t > \tau_t^{\max} \in \{0,1\}} \tag{57}$$

$$\mathbb{1}_{B_{t-1} > \mathscr{B}_t \in \{0,1\}} \tag{58}$$

The first indicator function will regulate the tax rate at the steady state. Equation 50 and Table 3 give us the values for $\overline{\tau}_{\tau_t = \tau_t^{\max}}$ and $\overline{\tau}_{\tau_t < \tau_t^{\max}}$, respectively, so we obtain $\overline{\tau^{\max}}$ and $\overline{\tau}$:

$$\overline{\tau^{\max}} = \frac{\chi}{1+\chi} = \overline{\tau}_{\tau_t = \tau_t^{\max}} \tag{59}$$

$$\overline{\tau} = \left(1 - \mathbb{1}_{\tau_t > \tau_t^{\max} \in \{0,1\}}\right) \overline{\tau}_{\tau_t < \tau_t^{\max}} + \mathbb{1}_{\tau_t > \tau_t^{\max} \in \{0,1\}} \overline{\tau}_{\tau_t = \tau_t^{\max}} \tag{60}$$

The second indicator function will regulate the value of $\overline{\delta}$:

$$\overline{\delta} = \mathbb{1}_{B_{t-1} > \mathscr{B}_t \in \{0,1\}} \overline{\delta}_{\delta_t > 0} \tag{61}$$

We proceed, then, by stabilizing the endogenous variables that are affected by the shocks in a first pass. From our calibration, we opt to normalize $\overline{Y} = 1$ at the no-binding regime, what allows us to calibrate $\overline{A}$ as a function of η and θ:

$$\overline{Y} = \left(\frac{(\theta-1)}{\theta} \frac{\left(1-\overline{\tau}\right)}{\eta}\right)^{\frac{1}{\chi}} \left(\overline{KA}\right)^{1+\frac{1}{\chi}} \quad \Rightarrow \quad \overline{A} = \frac{1}{\overline{K}} \left(\frac{\overline{Y}}{\left(\frac{(\theta-1)}{\theta}\frac{(1-\overline{\tau})}{\eta}\right)^{\frac{1}{\chi}}}\right)^{\left(\frac{1}{1+\frac{1}{\chi}}\right)} \quad \Rightarrow \quad \overline{A} = \frac{1}{\overline{K}} \left(\frac{1}{\left(\frac{(\theta-1)}{\theta}\frac{(1-\overline{\tau})}{\eta}\right)^{\frac{1}{\chi}}}\right)^{\left(\frac{1}{1+\frac{1}{\chi}}\right)} \tag{62}$$

To finally pin down $\overline{A}$ and η, we simultaneously solve a system with equations (62) and (63), imposing that $\overline{N} = 1/3$.

$$\overline{N} = \frac{1}{3} = \left(\frac{(\theta-1)}{\theta} \frac{\left(1-\overline{\tau}\right)}{\eta} \overline{KA}\right)^{\frac{1}{\chi}} \tag{63}$$

Then we obtain, in order:

$$\overline{W} = \overline{KA} \tag{64}$$

$$\overline{C} = \overline{Y} - \overline{G} = 1 - \overline{G} \tag{65}$$

$$\overline{C^{NR}} = \left(\frac{(\theta-1)}{\theta}\frac{1}{\eta}\right)^{\frac{1}{\chi}} \left((1-\overline{\tau})\overline{KA}\right)^{1+\frac{1}{\chi}} + \overline{Z} \tag{66}$$

$$\overline{C^{R}} = \frac{1}{1-\gamma^{NR}}\left(1-\overline{G}-\gamma^{NR}\overline{C^{NR}}\right) \tag{67}$$

$$\overline{U_c} = \left(\overline{C^R} + \alpha_G\overline{G} - \eta\frac{\overline{N}^{1+\chi}}{1+\chi}\right)^{-\sigma} \tag{68}$$

$$\overline{T} = \overline{\tau}\left(\frac{(\theta-1)}{\theta}\frac{\left(1-\overline{\tau}\right)}{\eta}\right)^{\frac{1}{\chi}} \left(\overline{KA}\right)^{1+\frac{1}{\chi}} \tag{69}$$

Given the inflation target, $\overline{\Pi}$, and $\overline{\mathscr{M}}$ we can define the steady-state expressions for our model's interest rates.

$$\overline{r^{RF}} = \overline{r^n} = -1 + \frac{1}{\beta} \tag{70}$$

$$\bar{i} = -1 + \overline{\Pi}\frac{1}{\beta} \tag{71}$$

Next, we obtain the steady-state intercept of each policy rule by adding or subtracting the steady-state risk premium, $\overline{\Phi} = \left(\overline{r} - \overline{r^n}\right)$, when necessary.[62] The nominal policy interest rate will depend on the policy rule as follows:

If central bank operated a risk-free asset and targeted a risk-free rate:

$$\overline{r} = -1 + \frac{1}{\beta} \tag{72}$$

$$\overline{\imath} = \overline{r} \tag{73}$$

$$\overline{i} = (1+\overline{r})\overline{\Pi} - 1 \tag{74}$$

If central bank operates a risky asset and ignores risk:

$$\overline{r} = -1 + \frac{1}{\beta(1-\overline{\delta})} \tag{75}$$

$$\overline{\imath} = \overline{r^n} + \left(\overline{r} - \overline{r^n}\right) = \overline{r} \tag{76}$$

$$\overline{i} = (1+\overline{r})\overline{\Pi} - 1 \tag{77}$$

If central bank operates a risky asset and adjusts by default risk:

$$\overline{r} = -1 + \frac{1}{\beta(1-\overline{\delta})} \tag{78}$$

[62] One way of interpreting this modeling device is that the central bank employs the stabilizing intercept on average, but the way that it deals with fluctuations on expected default risk is what affects differently each policy rule.

$$\bar{\imath} = \bar{r} \tag{79}$$

$$\bar{i} = (1 + \bar{r})\overline{\Pi} - 1 \tag{80}$$

With our calibration for $\overline{G}$ from Table 3, we can find the actual repaid debt, $\overline{B^d}$, the fiscal limit, $\overline{\mathscr{B}}$, and the policy default probability, $\overline{\mathscr{D}}$, all evaluated at the steady state as a function of $\overline{B}$:

$$\overline{B^d} = \left(1 - \overline{\delta}\right)\overline{B} \tag{81}$$

$$\overline{\mathscr{B}} = \mathscr{B}\left(\overline{A}, \overline{G}\right) \tag{82}$$

$$\overline{\mathscr{D}} = Pr\left(\overline{B} > \overline{\mathscr{B}}\right) = \frac{1}{1 + \exp\left(\gamma_0 + \gamma_b\left(\overline{B} - \overline{\mathscr{B}}\right)\right)} \tag{83}$$

With our calibration for $\overline{B}$ from Table 3, we can certainly find the steady-state value of previous variables that we left as a function of $\overline{B}$. However, due to different levels of distortionary taxation and default risk, each regime would have its own steady-state debt level. We fix this by simultaneously solving the equation for the intercept of the policy rule at the steady state, in addition to (83), and (84)

$$\overline{B} = \frac{\left((1 + \bar{\imath})\overline{\Pi}\right)\left(\overline{G} + \overline{Z} + \overline{T_{LS}} - \overline{T}\right)}{\left(1 - (1 + \bar{\imath})(1 - \delta)\right)} \tag{84}$$

where we introduce a regime-specific fixed lump-sum tax, $\overline{T_{LS}}$, to impose stationarity on the debt trajectory.

In the calculation of fiscal limits, variables are evaluated assuming the tax rate constraint is binding while the default constraint is not. To obtain the steady-state values in that regime just remake previous steps accordingly.

B Log-linearized model

In the third step of the solution algorithm, the model is approximated in first order around each regime-specific steady state for the regime-switching method. Here, we present the log-linearized equations used in that approximation. Note that for any variable X_t, $\widetilde{x}_t \equiv \log(X_t) - \log(\overline{X}) \approx \frac{X_t - \overline{X}}{\overline{X}}$. To simplify notation, though, we define $\widetilde{w}_t \equiv \log\left(\frac{W_t}{P_t}\right) - \log\left(\frac{\overline{W}}{\overline{P}}\right)$, and, for variables denoted with greek letters, we do not replace them by their respective lower case versions. Moreover, we define $\widetilde{R^i}_t \equiv \log(1+i_t) - \log\left(1+\overline{i}\right)$, $\widetilde{R^{\bar{\imath}}}_t \equiv \log\left(1+\bar{\imath}_t\right) - \log\left(1+\bar{\bar{\imath}}\right)$, $\widetilde{R^{RF}}_t \equiv \log\left(1+r^{RF}_t\right) - \log\left(1+\overline{r^{RF}}\right)$, and $\widetilde{\Phi}_t \equiv \log(1-\delta_t) - \log\left(1-\overline{\delta}\right)$. Finally, we define an auxiliary constant, the steady-state fiscal deficit, given by $\Upsilon \equiv \left(1-\overline{\delta}\right)\frac{\overline{B}}{\overline{\Pi}} + \overline{G} + \overline{Z} - \overline{T}$. Beware that $\overline{\delta}$ is regime-specific.

B.1 Equations

We start by listing the exogenous processes.

$$\widetilde{a}_t = \rho_A \widetilde{a}_{t-1} + \sigma_A \varepsilon^A_t - \mathbb{1}_{B_{t-1} > \mathscr{B}_t} \delta^{\text{TFP}} \tag{85}$$

$$\widetilde{g}_t = \rho_{GG} \widetilde{g}_{t-1} + \rho_{GY} \widetilde{y}_{t-1} + \sigma_G \varepsilon^G_t \tag{86}$$

$$\widetilde{\mathscr{M}}_t = \rho_{\mathscr{M}} \widetilde{\mathscr{M}}_{t-1} + \sigma_{\mathscr{M}} \varepsilon^{\mathscr{M}}_t \tag{87}$$

$$\widetilde{\beta}_t = \rho_\beta \widetilde{\beta}_{t-1} + \sigma_\beta \varepsilon^\beta_t \tag{88}$$

Now, we list the endogenous equations for any given monetary policy rule.

$$\widetilde{\text{mc}}_t = \widetilde{w}_t - \widetilde{a}_t - \gamma^{G\Psi} \widetilde{g}_t \tag{89}$$

$$\widetilde{w}_t = \frac{\overline{\tau}}{1-\overline{\tau}} \widetilde{\tau}_t + \chi \widetilde{n}_t \tag{90}$$

$$\widetilde{y}_t = \widetilde{a}_t + \gamma_{G\Psi} \widetilde{g}_t + \widetilde{n}_t \tag{91}$$

$$\widetilde{\Pi}_t = \frac{\theta}{\phi^C} \widetilde{\text{mc}}_t + \beta \mathbb{E}_t \widetilde{\Pi}_{t+1} \tag{92}$$

$$\widetilde{y}_t = \frac{\overline{C}}{\overline{Y}} \widetilde{c}_t + \frac{\overline{G}}{\overline{Y}} \widetilde{g}_t \tag{93}$$

$$\widetilde{m}_{t,t+1} = -\widetilde{R}^{RF}_t \tag{94}$$

$$\frac{1}{\sigma} \widetilde{R^{RF}}_t = \frac{\overline{C^R}}{\Upsilon} \left(\mathbb{E}_t \widetilde{c^R}_{t+1} - \widetilde{c^R}_t\right) + \alpha_G \frac{\overline{G}}{\Upsilon} \left(\mathbb{E}_t \widetilde{g}_{t+1} - \widetilde{g}_t\right) - \eta \frac{\overline{N}^{\chi+1}}{\Upsilon} \left(\mathbb{E}_t \widetilde{n}_{t+1} - \widetilde{n}_t\right) \tag{95}$$

$$\widetilde{R^{Gov}}_t = \widetilde{R^{RF}}_t + \mathbb{E}_t \widetilde{\Pi}_{t+1} + \mathbb{E}_t \widetilde{\Phi}_{t+1} \tag{96}$$

$$\widetilde{R^i}_t = \phi^i \widetilde{R^i}_{t-1} + \left(1-\phi^i\right) \widetilde{R^{\bar{\imath}}}_t + \left(1-\phi^i\right) \phi^\pi \widetilde{\Pi}_t + \left(1-\phi^i\right) \phi^y \widetilde{y}_t + \mathscr{M}_t \tag{97}$$

$$\widetilde{b}_t = \widetilde{R^i}_t + \frac{\left(1-\overline{\delta}\right)\overline{B}}{\Upsilon\overline{\Pi}}\left((1-\delta_t)\,\widetilde{b}_{t-1} - \widetilde{\Pi}_t\right) + \frac{\overline{G}}{\Upsilon}\widetilde{g}_t + \frac{\overline{Z}}{\Upsilon}\widetilde{z}_t + \frac{\overline{T}}{\Upsilon}\widetilde{t}_t \tag{98}$$

$$\delta_t = \mathbb{1}_{B_{t-1}>\mathscr{B}_t}\overline{\delta} \tag{99}$$

$$\widetilde{\tau}_t = \mathbb{1}_{\mathbb{E}_t\tau_{t+1}>\tau^{\max}}\tau^{\max} + \left(1-\mathbb{1}_{\mathbb{E}_t\tau_{t+1}>\tau^{\max}}\right)\left[\rho_\tau\widetilde{\tau}_{t-1} + \gamma_\tau\left(\widetilde{b}_{t-1} - \widetilde{y}_{t-1}\right)\right] \tag{100}$$

$$\widetilde{t}_t = \widetilde{\tau}_t + \widetilde{y}_t \tag{101}$$

$$\widetilde{z}_t = 0 \tag{102}$$

$$\frac{\overline{C^{NR}}}{\overline{C^{NR}}-\overline{Z}}\widetilde{c^{NR}}_t = \frac{\overline{C^{NR}}}{\overline{C^{NR}}-\overline{Z}}\widetilde{z}_t - \frac{\overline{\tau}}{1-\overline{\tau}}\widetilde{\tau}_t + \widetilde{w}_t + \widetilde{n}_t \tag{103}$$

$$\widetilde{c}_t = \left(1-\gamma^{NR}\right)\frac{\overline{C^R}}{\overline{C}}\widetilde{c^R}_t + \gamma^{NR}\frac{\overline{C^{NR}}}{\overline{C}}\widetilde{c^{NR}}_t \tag{104}$$

The intercepts of the four monetary policy rules are approximated, in order, as follows. Note that $\bar{\bar{\imath}}$ varies, as it is specific to each rule.

$$\text{Rule 1:}\quad \widetilde{R^{\bar{\imath}}}_t = \frac{\overline{r^{RF}}}{\bar{\bar{\imath}}}\widetilde{R^{RF}}_t \tag{105}$$

$$\text{Rule 2:}\quad \widetilde{R^{\bar{\imath}}}_t = \frac{\overline{r^{RF}}}{\bar{\bar{\imath}}}\widetilde{R^{RF}}_t + \frac{\overline{\Phi}}{\bar{\bar{\imath}}}\mathbb{E}_t\widetilde{\Phi}_{t+1} \tag{106}$$

$$\text{Rule 3:}\quad \widetilde{R^{\bar{\imath}}}_t = \frac{\overline{r^{RF}}}{\bar{\bar{\imath}}}\widetilde{R^{RF}}_t \tag{107}$$

$$\text{Rule 4:}\quad \widetilde{R^{\bar{\imath}}}_t = \frac{\overline{r^{RF}}}{\bar{\bar{\imath}}}\widetilde{R^{RF}}_t + \frac{\overline{\Phi}}{\bar{\bar{\imath}}}\mathbb{E}_t\widetilde{\Phi}_{t+1} \tag{108}$$

As the policy asset may be defaultable or not depending on the policy rule, the relation between $\widetilde{R^i}_t$ and $\widetilde{R^{Gov}}_t$ also depends on the policy rule.

$$\text{Rules 1, 2 and 4:}\quad \widetilde{R^i}_t = \widetilde{R^{Gov}}_t \tag{109}$$

$$\text{Rule 3:}\quad \widetilde{R^i}_t = \widetilde{R^{RF}}_t + \mathbb{E}_t\widetilde{\Pi}_{t+1} \tag{110}$$

Finally, we express the default premium, $\widetilde{\Phi}_{t+1}$, as a linear function of the probability of reaching the fiscal limit, which, by its turn, is expressed as a linear function of the state variables that go into the fiscal limits computation and $\widetilde{b}_t$, where $\beta_{\mu a}$, $\beta_{\mu g}$, $\beta_{\sigma a}$, $\beta_{\sigma g}$, β_{p0}, β_{pb}, β_{pa}, and β_{pg} are parameters calibrated from the estimated fiscal limits. To simplify notation, we define an auxiliary constant $\Upsilon^{\mathrm{FL}} \equiv \frac{1}{1+e^{-\beta_{p0}+\beta_{pb}\left(\overline{B}-\overline{\mu^{\mathrm{FL}}}\right)}}$.

$$\widetilde{\Phi}_{t+1} = \overline{\delta}\widetilde{Pr}\left(B_t > \mathscr{B}_{t+1}\right)_t \tag{111}$$

$$\widetilde{\mu^{\mathrm{FL}}}_t = \beta_{\mu a}\widetilde{a}_t + \beta_{\mu g}\widetilde{g}_t \tag{112}$$

$$\widetilde{\sigma^{\mathrm{FL}}}_t = \beta_{\sigma a}\widetilde{a}_t + \beta_{\sigma g}\widetilde{g}_t \tag{113}$$

$$\widetilde{Pr}\left(B_t > \mathscr{B}_{t+1}\right)_t = -\Upsilon^{\mathrm{FL}}\left(\beta_{pb}\left(\widetilde{b}_t - \widetilde{\mu^{\mathrm{FL}}}_t\right) + \beta_{pa}\widetilde{a}_t + \beta_{pg}\widetilde{g}_t\right) \tag{114}$$

C Estimation of the single-regime model

For calibrating the shock parameters of our model, we conduct a Bayesian estimation of its single-regime linearized version with only risk-free policy assets (non-linearities do not and are never expected to bind) and flexible prices. Hereafter, for any variable X, X^{obs} is its observed time series. The data sample covers the period between 1999Q3 and 2019Q4, encompassing 82 observation periods.

For $Y^{obs}, C^{obs}, G^{obs}$, we use the log-difference of the quarterly deseasonalized real series calculated by the IBGE. For π^{obs}, we turn the IPCA MoM% series from the IBGE into an index, aggregate it quarterly, and then subtract the fourth root of each year's yearly gross inflation target subtracted by 1.[63] For i^{obs}, we aggregate the quarterly mean of both the daily Selic series (from the BCB) and the daily 3-month Swap Pre x DI series (from the B3). Then, we splice both series, using the latter whenever available. Once we have collected our 5 series, $\left(Y^{obs}, C^{obs}, G^{obs}, \pi^{obs}, i^{obs}\right)$, we demean all of them by the sample mean.

Table 17 displays the summary statistics of the sample, and Figure 20 exhibits the series before demeaning. Moreover, (115) shows the measurement equations that are appended to the model, where, for any variable X, X^{det} is the respective detrended series. We incorporate a measurement error of output, $\varepsilon_t^{me,Y}$, to mitigate the misspecification problem of modeling an open-economy as a closed one. For estimation, we turn on the structural shocks ε_t^A, ε^β, ε^G, and ε^M.

$$\begin{pmatrix} Y_t^{obs} \\ C_t^{obs} \\ G_t^{obs} \\ \pi_t^{obs} \\ i_t^{obs} \end{pmatrix} \xrightarrow{\text{det}} \begin{pmatrix} Y_t^{det} \\ C_t^{det} \\ G_t^{det} \\ \pi_t^{det} \\ i_t^{det} \end{pmatrix} = \begin{pmatrix} \log(Y_t/Y_{t-1}) \\ \log(C_t/C_{t-1}) \\ \log(G_t/G_{t-1}) \\ \pi_t \\ (1+i_t)^4 - 1 \end{pmatrix} + \begin{pmatrix} \varepsilon_t^{me,Y} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{115}$$

[63]Net inflation targets: in 1999, 8%; in 2000, 6%; in 2001, 4%; in 2002, 3.5%; in 2003, 4%; in 2004, 5.5%; from 2005 to 2018, 4.5%; and in 2019, 4.25%. For 2003 and 2004, the inflation targets originally announced were amended in the year before they should become effective, so we use the amended values. Source: https://www.bcb.gov.br/controleinflacao/historicometas

	Y_t^{obs}	C_t^{obs}	G_t^{obs}	π_t^{obs}	i_t^{obs}
Mean (%)	0.58	0.68	0.44	1.54	13.12
Std. (%)	1.15	1.13	1.3	0.91	4.71
AR(1)	0.4	0.36	-0.31	0.53	0.96
Number of obs.	82	82	82	82	82
Start date	1999Q3	1999Q3	1999Q3	1999Q3	1999Q3
End date	2019Q4	2019Q4	2019Q4	2019Q4	2019Q4

Table 17: Summary of observables' statistics before demeaning

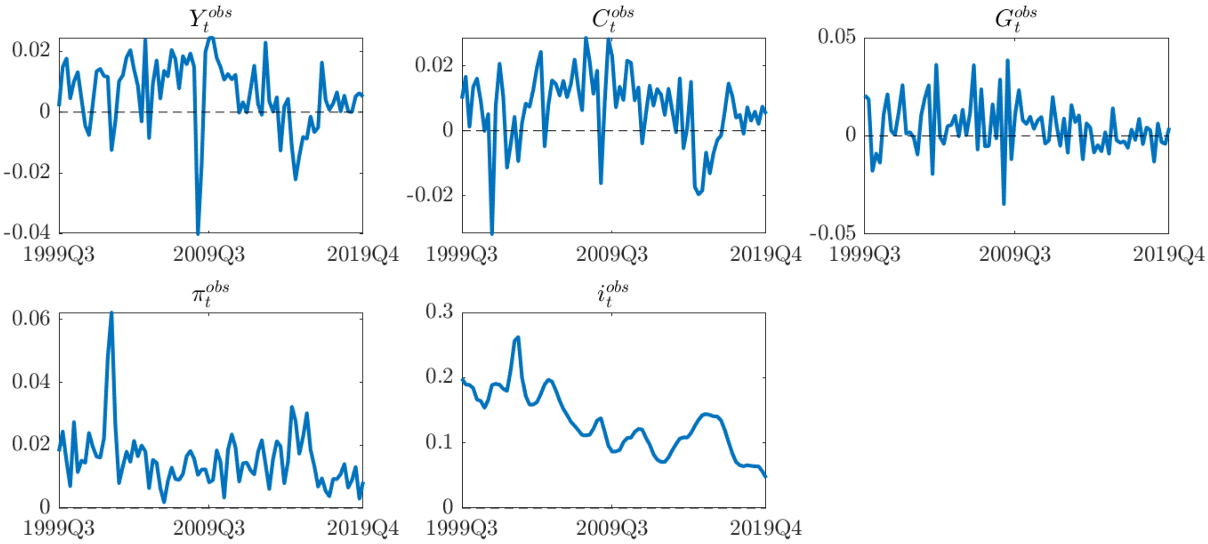


Figure 20: Data before demeaning used for estimation

We pick for estimation 16 parameters associated with either the utility function, the spill-over of government expenses to total factor productivity, the monetary policy rule, the shock processes, or the measurement errors: σ, α_G, $\gamma_{G\Psi}$, ϕ^{π}, ϕ^{Y}, ϕ^{i}, ρ^{A}, ρ^{β}, ρ^{GY}, ρ^{GG}, $\rho^{\mathscr{M}}$, σ^{A}, σ^{β}, σ^{M}, σ^{G}, $\sigma^{me,Y}$. Consistent with recommendations in Herbst and Schorfheide (2015), we adopt beta distributions for parameters whose domain is exclusively $[0,1]$; inverse gamma distributions for the standard deviation of the shocks, and gamma distributions for other non-negative parameters; as well as normal distributions for the remaining ones. Each prior is identified by the tuple: distribution, lower bound, upper bound, and the fraction of that distribution contained between these bounds.

Before estimating the model, we linearize it, and only then we apply the Metropolis-Hastings algorithm to sample the posterior distribution of the parameters. We produce 8

Markov chains containing 200,000 draws each, where the first 20,000 draws of every chain have been dropped as burn-in.[64] In Table 4, we summarized the estimation specification for the priors and the characteristics of the posterior we obtained.

Figure 21 plots the prior (blue full line) vs. the posterior (red dotted line) distribution of the estimated parameters, as well as the mode (green vertical line) and the mean (black vertical line) of each posterior distribution.

Figure22 plots the curvature at the mode of the posterior distribution of the estimated parameters vs. their respective priors. Blue-full lines are the priors, while red-dotted ones are the posteriors.

Figure 23 shows the convergence of the estimated parameters according to the Potential Scale Reduction Factor (PSRF) of Gelman et al. (1992). This metric makes a comparison of within-chain and between-chain variances, where a large deviation between them indicates non-convergence. A PSRF larger than 1 indicates that the between-chain variance is substantially greater than the within-chain variance, and, therefore, longer simulations would be needed. On the contrary, if a PSRF is close to 1, then the chains are likely to have converged.

Figure 24 exhibits the smoothed exogenous shocks after the estimation. Figure 25 contrasts the observed against the smoothed series of the model obtained after estimation. Figure 26 plots the variance decomposition of the observed variables into each exogenous shock contribution according to the estimated model. In each panel, the decomposition is carried on for the first 40 periods and for the long-run.

[64]The optimization algorithm is MATLAB's fmincon. All Markov chains have acceptance ratios near 35%.

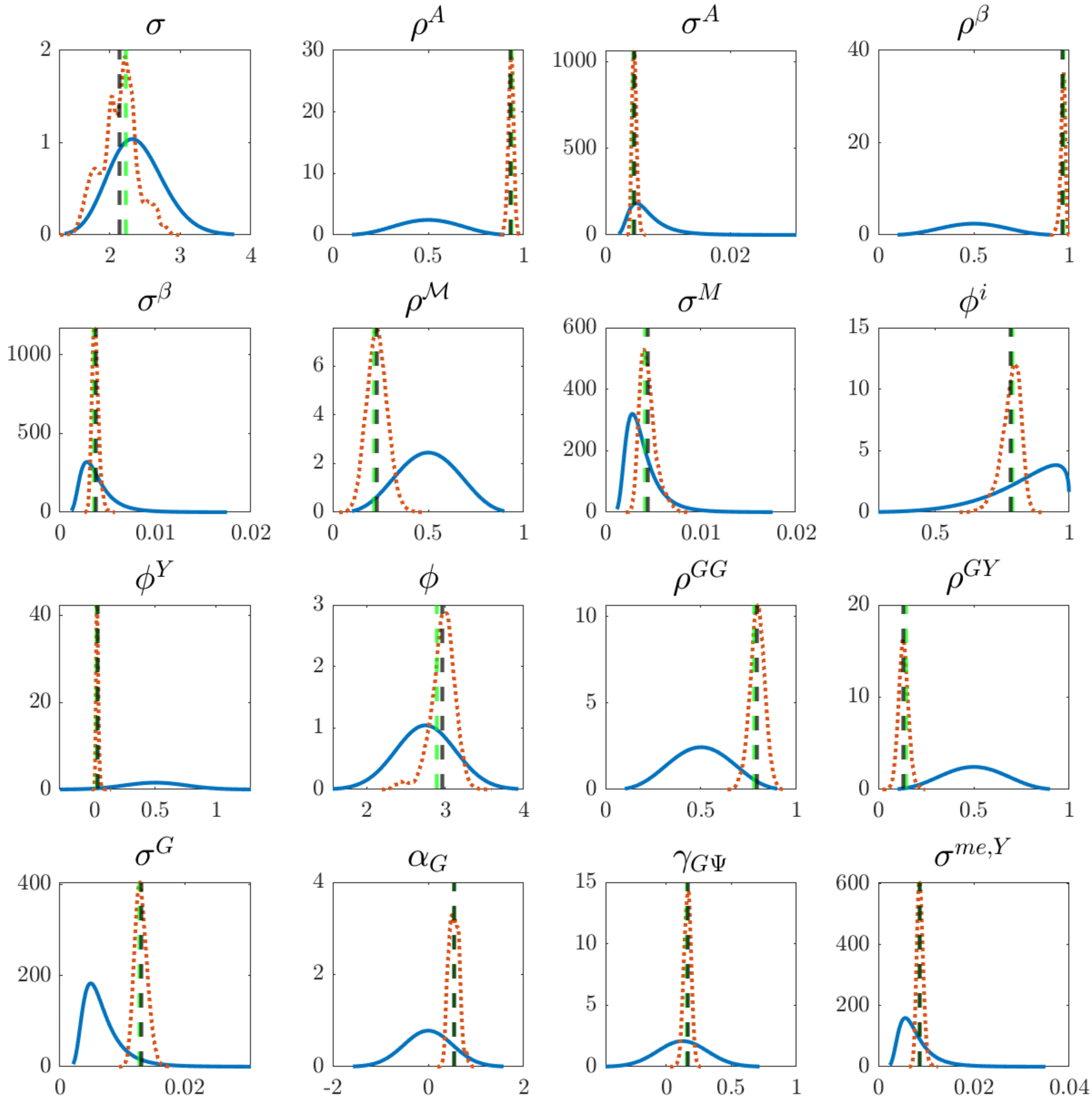


Figure 21: Prior and posterior distributions of estimated parameters

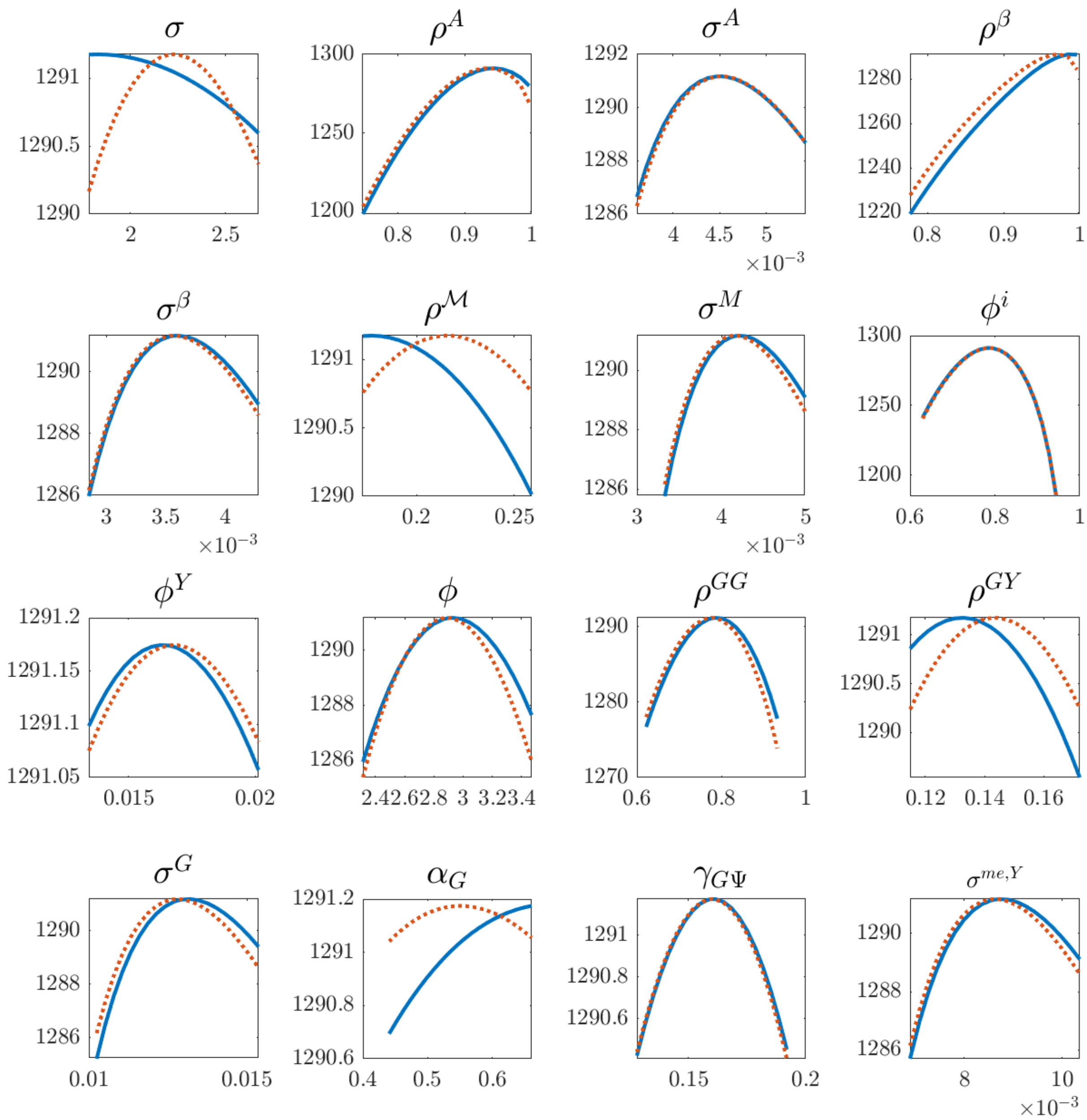


Figure 22: Curvature at the mode of the posterior distribution of the estimated parameters

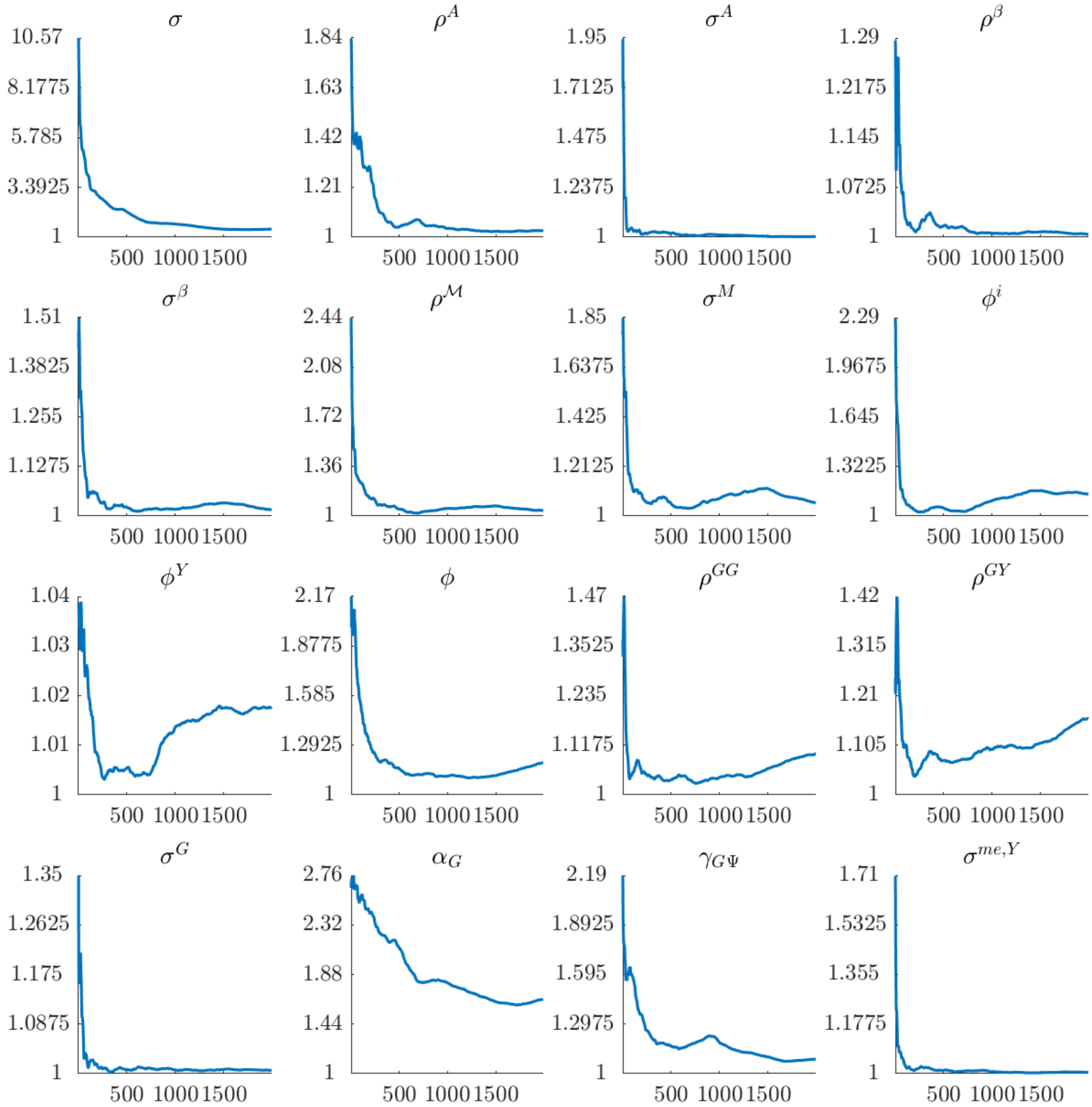


Figure 23: PSRF of the estimated parameters

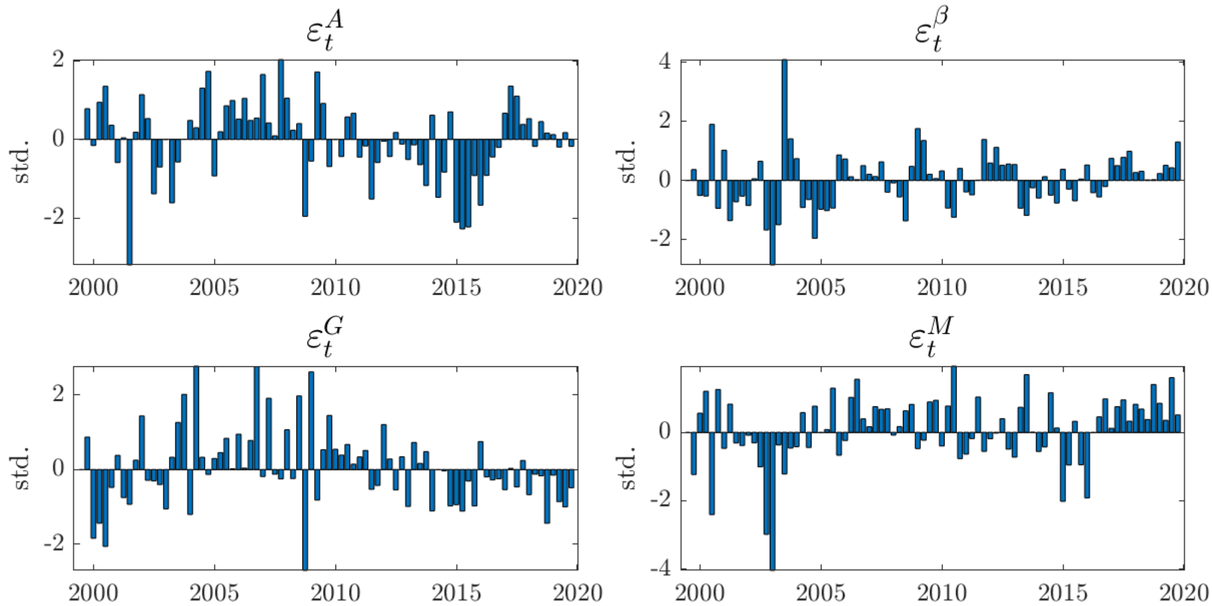


Figure 24: Smoothed shocks from the estimated model

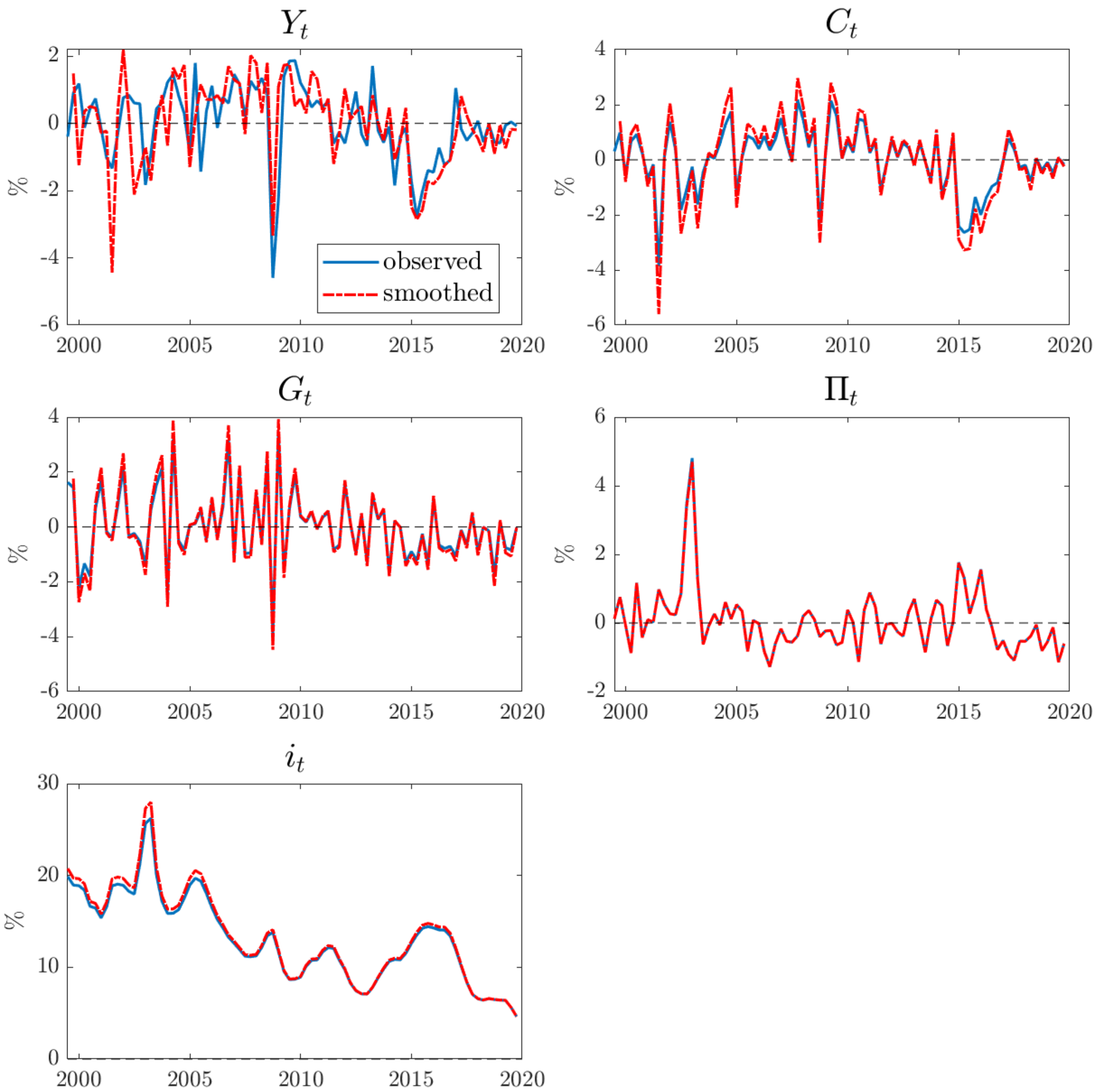


Figure 25: Observed and smoothed variables after estimation

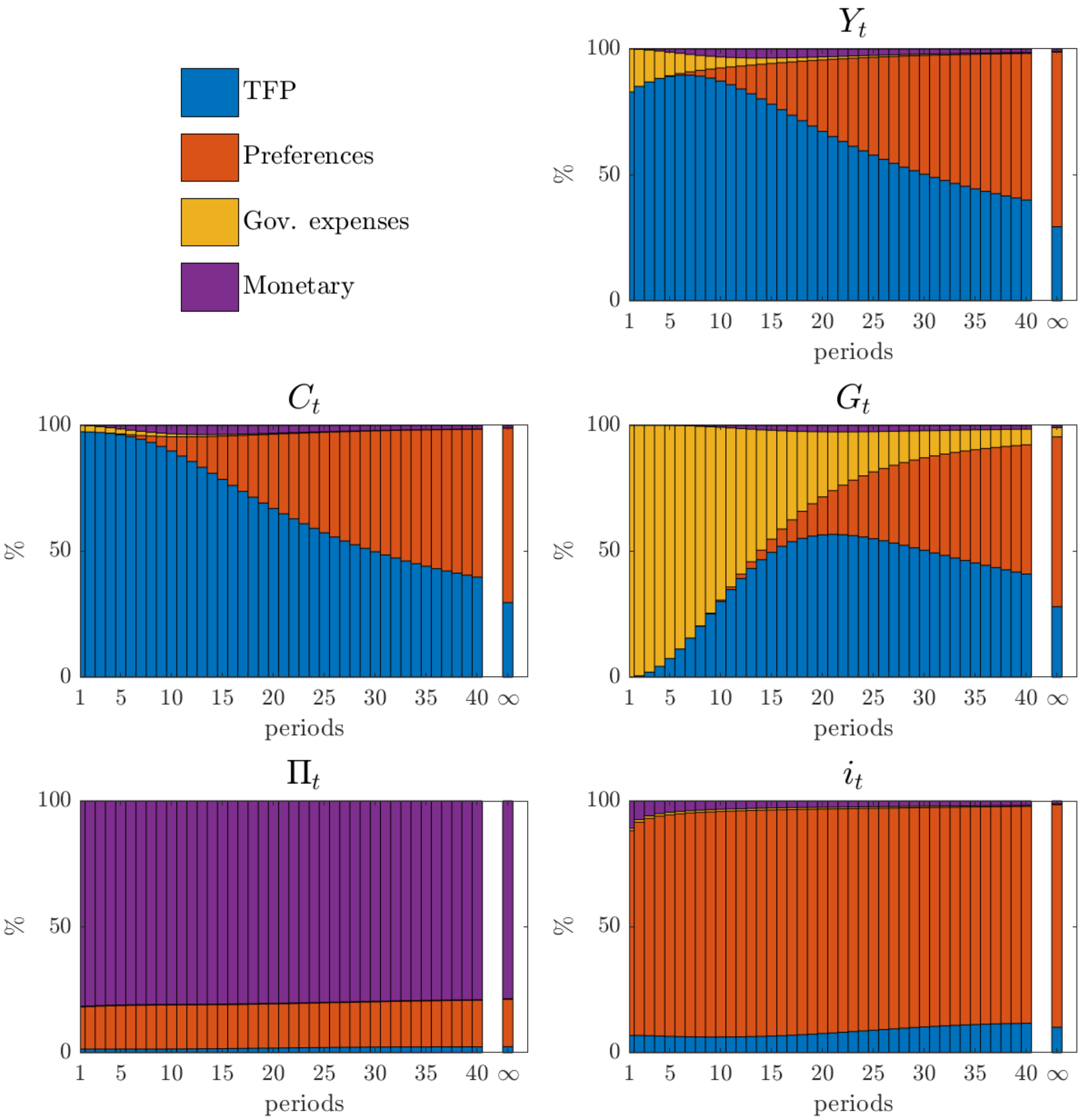


Figure 26: Variance decomposition of the estimated model

D Policy rules and stability

We conduct a numerical exercise so as to obtain the parameter range in which a solution exists. This approach is warranted since the stability of regime-switching models with endogenous transition probabilities is still an open question in the literature. Barthélemy and Marx (2017) provide for conditions under which a unique bounded equilibrium exists in such models, guaranteeing local determinacy, but their approach is limited to the case of small shocks and smooth transition probabilities between regimes. The latter is violated in our model since the peak of the Laffer curve is an occasionally binding constraint whose transition probability is not modeled as a smooth function.

Our approach consists in numerically testing whether the solution algorithm is able to find any solution for different combinations of parameter values for ϕ^{π} and γ_{τ} under each policy rule. To highlight the importance of having risk in the policy asset, we test both the benchmark calibration and an alternative scenario in which the debt level is consistent with a 5% probability of sovereign default at the steady state. Additionally, to simplify the analysis, we turn off the interest rate smoothing and the reaction to output deviations from the steady state in the policy rules. We apply the Mean Squared Stability criterion of Costa et al. (2006) to distinguish between what we call "stable" and "unstable" solutions. Although this is a method that *only applies to regime-switching with constant transition probabilities*, heuristically, we note that this may be helpful in mapping the parameter spaces in which finding a solution is dodgy and the likelihood that simulations diverge increases.[65] Above all, one must remember that finding a solution here is no guarantee that it is unique. Therefore, we leave a rigorous definition of active/passive fiscal and monetary policy under endogenous regime-switching for future research. Hereafter, they will be only loosely located in the parameter space.

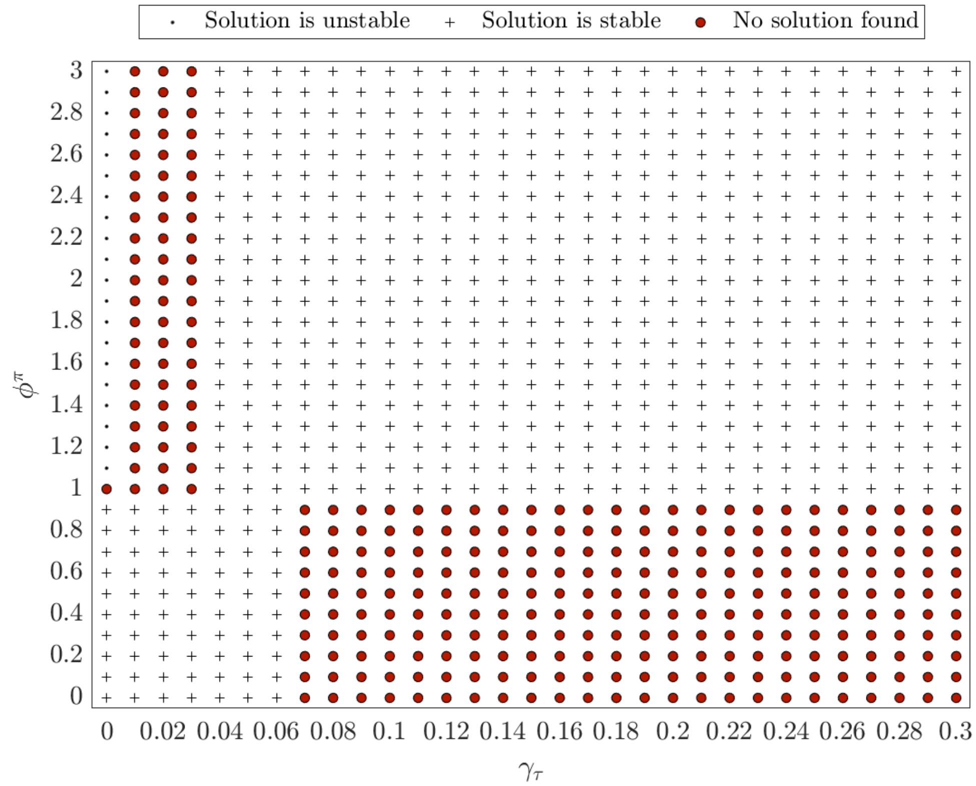
In constant parameter models, solution is stable as long as fiscal and monetary policy stances are mismatched à la Leeper (1991).[66] Fiscal dominance will operate in the third

[65]We thank Junior Maih for sharing with us his considerations on the topic.

[66]As Leeper (1991) defines: "parameters associated with active behavior make policy unresponsive to current budgetary conditions and parameters connected with passive behavior force the authority to use its tax to balance the budget". "Tax" is understood here as inflation in the case of the monetary authority. With respect to this paper's model, contrary to what happens in Leeper (1991), fiscal disturbances may influence equilibrium prices and interest rates depending on the central bank's policy rule, but this happens through default

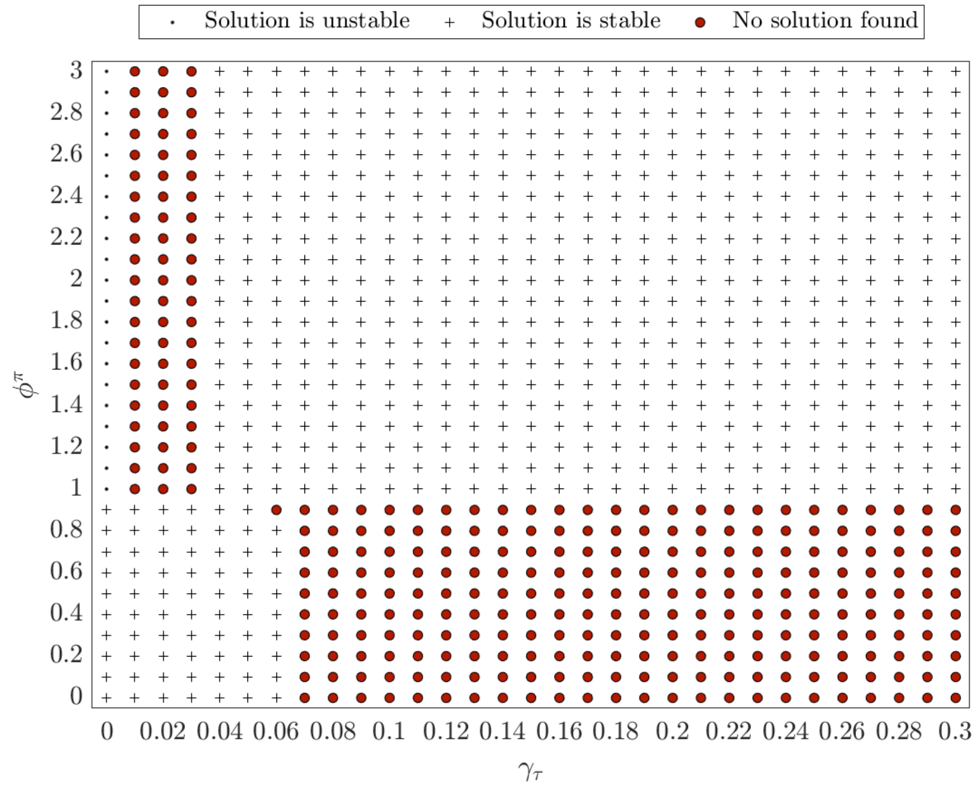
quadrant, while monetary one takes place in the first quadrant. No solution exists when both policies are active because independent variations in both violate the government's budget constraint. Multiple equilibria exist when both policies are passive, though. Figures 27 and 28 plot the two policy rules, ignoring the evolution of default risk and accounting for it, respectively, both under the benchmark calibration, $\overline{\mathscr{D}} \approx 0\%$. Note that the parameter space of monetary dominance is amplified beyond the Taylor Principle and fiscal policy only needs to be slightly passive for the system to be solved. As steady-state default probability rises, stability conditions change dramatically. Figures 29 and 30 plot the same rules under $\overline{\mathscr{D}} = 5\%$. Violating the Taylor principle turns out more critical, while fiscal policy is demanded to react more strongly to deviations from the steady-state debt level, considerably shrinking the parameter space of stability. Overall, we can see in all figures that matching either active monetary policy with passive fiscal policy or passive monetary policy with active fiscal policy is a sufficient condition for the stability of the model under any of the presented rules.

risk, not in reason of the impossibility of stabilization of the debt level otherwise, since we limit our exploration to the combination of active monetary policy and passive fiscal policy. Besides, as default probability grows with the size of the debt, it also helps to rule out, at least some, explosive trajectories for the debt level.



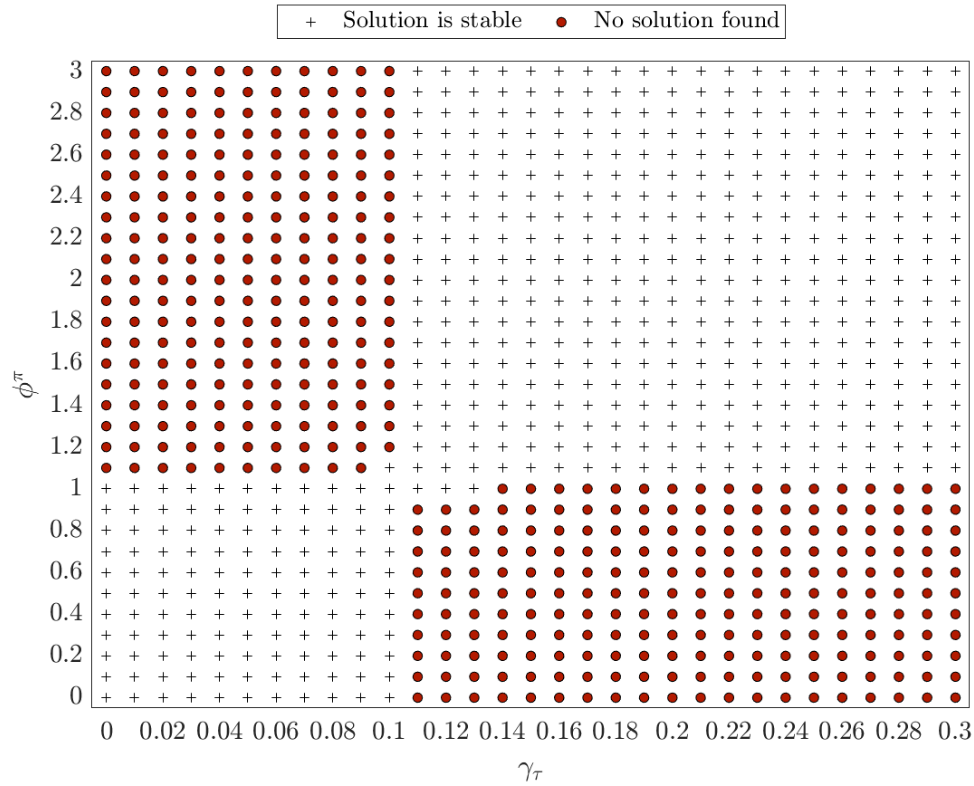
Note: In the lack of an adequate stability criterion for regime-switching models with endogenous transition probabilities, we rely on the Mean Squared Stability criterion of Costa et al. (2006).

Figure 27: Parameter stability: policy instrument is risky, but CB targets r_t^{RF} under $\overline{\mathscr{D}} = 0\%$



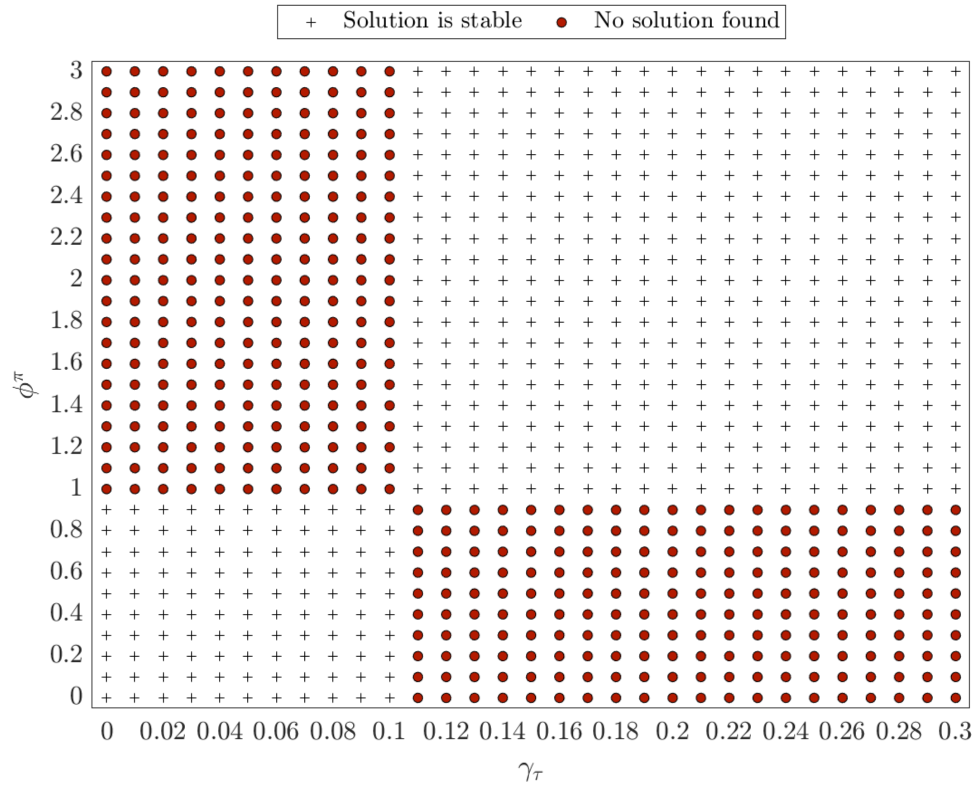
Note: In the lack of an adequate stability criterion for regime-switching models with endogenous transition probabilities, we rely on the Mean Squared Stability criterion of Costa et al. (2006).

Figure 28: Parameter stability: policy instrument is risky, and CB targets the risky rate under $\overline{\mathscr{D}} = 0\%$



Note: In the lack of an adequate stability criterion for regime-switching models with endogenous transition probabilities, we rely on the Mean Squared Stability criterion of Costa et al. (2006).

Figure 29: Parameter stability: policy instrument is risky, but CB targets r_t^{RF} under $\overline{\mathscr{D}} = 5\%$



Note: In the lack of an adequate stability criterion for regime-switching models with endogenous transition probabilities, we rely on the Mean Squared Stability criterion of Costa et al. (2006).

Figure 30: Parameter stability: policy instrument is risky, and CB targets the risky rate under $\overline{\mathscr{D}} = 5\%$

E Impulse response functions

How do the model variables react to the exogenous shocks? To answer that question, we plot in Figures 31 and 32 the regime-specific impulse response functions for the first two regimes of selected variables ($Y_t, C_t, N_t, \frac{B_t}{Y_t}, i_t, r_t, \Pi_t$) to three different shocks ($\varepsilon^A, \varepsilon^G, \varepsilon^{\mathscr{M}}$) under two different environments: (1) our calibrated $\overline{B}$, whose $\overline{\mathscr{D}}$ is low, at ≈ 0% (blue line); and a (2) hypothetical higher steady-state debt level whose default probability is at 1% (red-dotted line). Both environments assume the central bank adjusts the policy rule to default risk. In the figures, all variables start at their respective regime-specific non-stochastic steady-state values, and graphs display deviations from such values. To get a clearer response of monetary policy, we set $\phi^Y = 0$ and $\phi^i = 0$. Since, concerning the regime-specific steady state, crossing the fiscal limit only affects the haircut on the debt, which does not change fundamentally the dynamics of the model when there is no regime-switching occurring, the same graphs for Regime 3 are similar to the ones in Regime 1, whereas the same graphs for Regime 4 are similar to the ones in Regime 2. Additionally, in Figures 33 and 34, we plot the regime-specific impulse response functions of, respectively, inflation and the policy rate in the first two regimes changing the policy rules. We omit here the effect on other variables because for them the difference between the rules is neglectful near the regime-specific non-stochastic steady states, whereas inflation and the policy rate already exhibit the diverging dynamics enacted by each rule. The monetary shock is the main driver of inflation in our model, a result that is explained by our monetary rules that closely track the inflation target.

With the caveat that all happens in general equilibrium, we provide, now, a rationale for the dynamics observed in the model. Under a one-standard-deviation positive TFP shock, the marginal productivity of labor increases the labor demand at the same time that higher real wages increase the labor supply, in which output increases as a result of both. Since government expenditures react only by a little and with a lag, the increment of output becomes mostly consumption for the households, whereas the consumption of the Ricardian type increases by more. The expansion of activity leads to higher tax revenues for the government, which allows that sovereign debt is reduced, what, by its turn, leads to lower tax rates given the fiscal rule of the government, what explains the behavior of Ricardian consumers. The improvement of the government's fiscal position reduces the probability of default at the next period, what reduces the real return demanded by the households to hold government bonds. Turning to prices, the reduction of real interest rates, generated by the fact that households are able to smooth out consumption with lower rates after the shock, decreases expected inflation, what, by its turn, reduces current prices, to which the central bank endogenously reacts by reducing the nominal interest rate according to its monetary policy rule. By comparing the two policy rules in Regime 1, tracking r_t^{RF} leads to convergence to the inflation target from below. These results reverse in Regime 2, as higher output in reason of higher productivity is accompanied of higher government expenditures while the tax rate remains constant in that regime. As expected, adjusting the intercept to default risk is able to stabilize prices at all periods, like in the canonical case. Comparing the two environments, we have that the real sector reacts more strongly and with more persistence to the shock in the riskier set-up in reason of the more benign debt trajectory.

Under a one-standard-deviation positive shock to government expenditures, despite the positive effect over output in the short-run through the positive spill-over to productivity, the higher demand from the government will slowly weaken the latter's fiscal position, as it will lead to both higher debt accumulation and higher expected default probability. Debt growth, by its turn, will raise the tax rate given the fiscal policy rule. Higher tax rates lower the supply of labor while the demand for labor weakens given the lower aggregate demand. With eventually a lower amount of labor in equilibrium, the output must fall. Since government expenditures are mostly exogenous, it also means that less output is left for private consumption, which falls by relatively more than output. Due to the fact that prices are sticky, all this process can coexist with inflation if the central bank tracks r_t^{RF}. Under flexible prices, the results reverse and are numerically even smaller. Adjusting the intercept to default risk remains the stabilizing option for inflation. The real interest rate falls at the time of the shock, then it goes up as it is dominated by the rising debt accumulation and its effect over the default probability. Tax revenue increases, though, helping the government to recover fiscal balance in the long-run.

Finally, a positive monetary shock (a shock to the intercept of the policy rule) depresses the economy, for it has a negative effect on inflation, making it more difficult for the debt to be repaid by the government. Forward-looking agents anticipate, as a consequence, that higher taxes will be enacted by the fiscal authority into the future, leading Ricardian consumers to smooth out consumption by cutting it in the present. Note that under Regime 2, at the peak of the Laffer curve, the monetary shock also depresses Y_t, C_t and N_t, but this is purely due to sticky prices and the real interest rate channel. Under flexible prices, monetary policy is only neutral at the peak of the Laffer curve (Regimes 2 and 4), as in the other regimes its effect over the debt trajectory transmits to the real economy through the fiscal rule. This happens because, even though the policy rate still influences the debt accumulation by the government, the latter does not affect the tax rate in that regime, breaking the link with the incentives of the households to supply labor and their expected income. Monetary policy becomes hawkish for both policy rules, so that disinflation shows up, requiring a negative endogenous reaction of nominal interest rates. When prices are flexible, the endogenous component dominates, resulting in the reduction of i_t, r_t and Π_t. Due to disinflation, it is harder now to roll over the stock of debt, for its real value has jumped up, pushing default probability in the same direction.

In general, the presence of *positive* $\overline{\mathscr{D}}$ *amplifies the effect of the shocks on the debt accumulation process*, what, by its turn, makes it more persistent on the real economy.

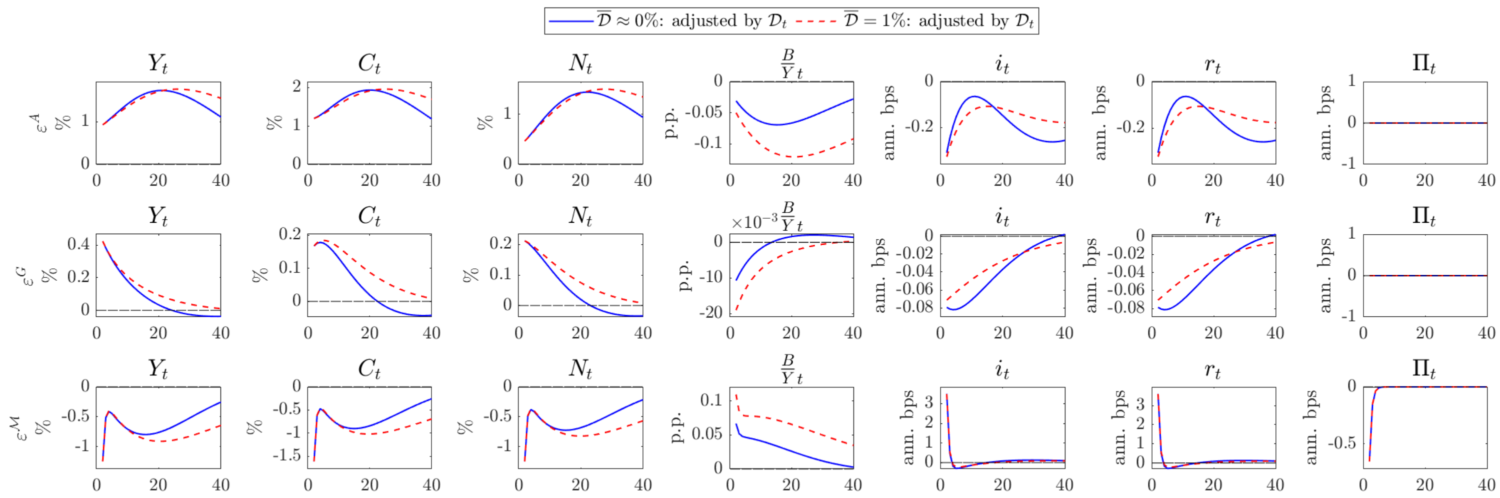


Figure 31: (Regime 1)-specific impulse response function of selected variables to 1 std. shocks

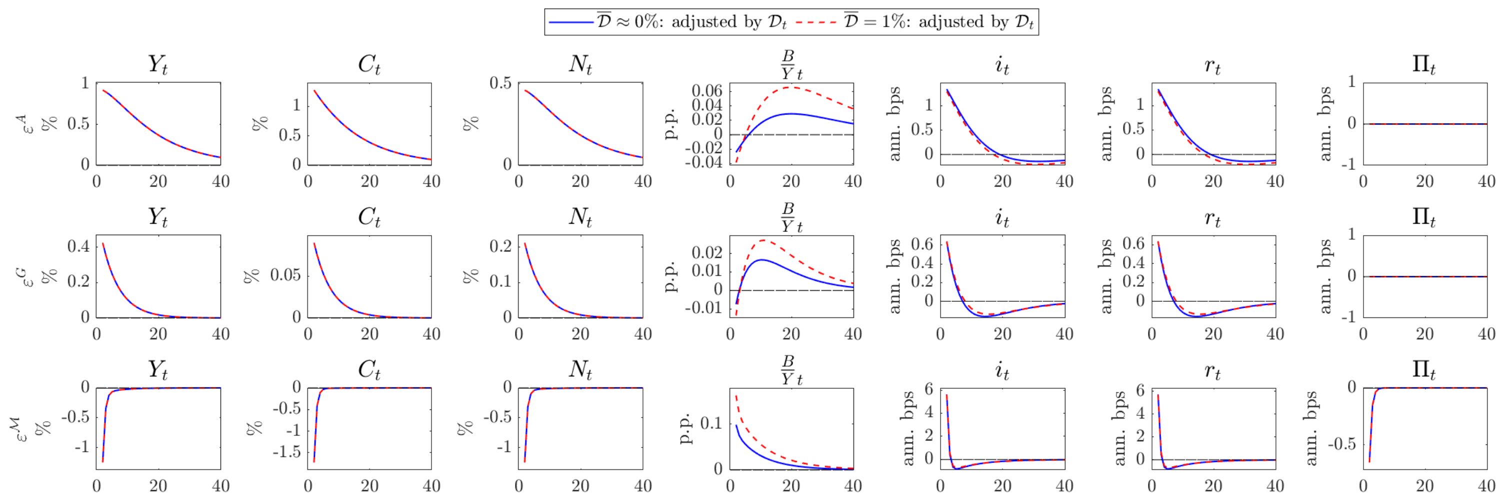


Figure 32: (Regime 2)-specific impulse response function of selected variables to 1 std. shocks

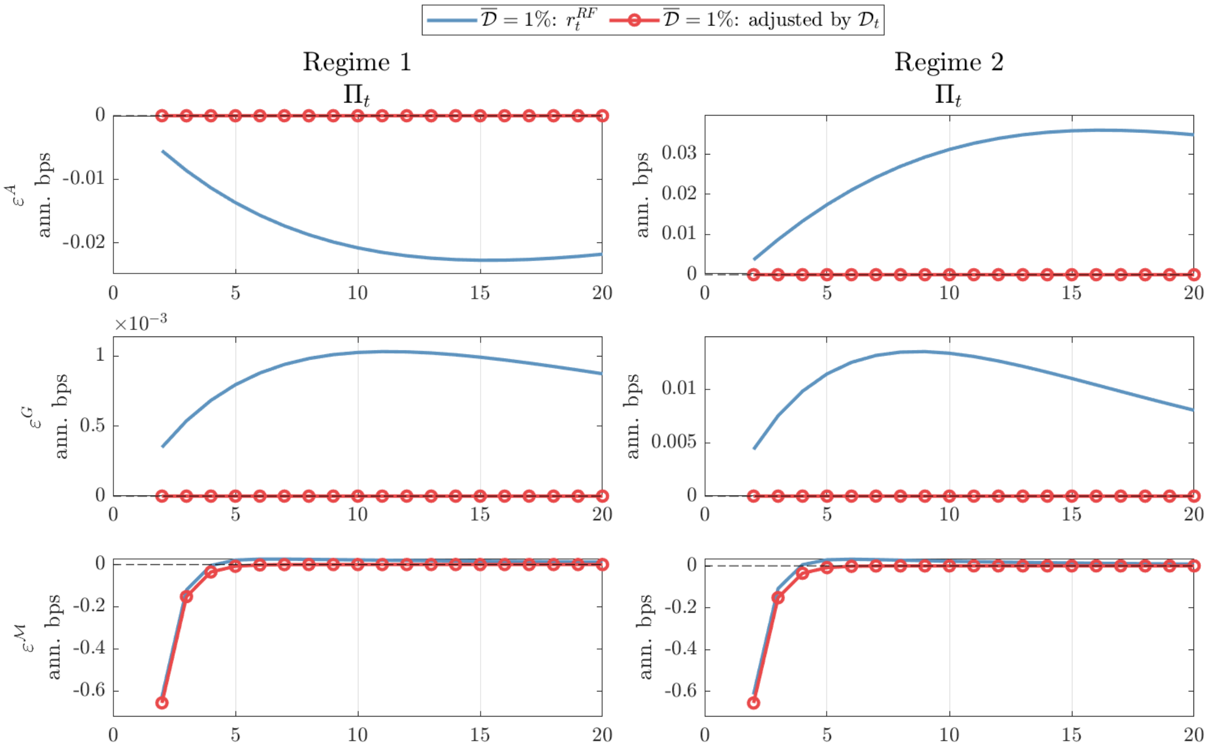


Figure 33: Regime-specific impulse response functions of Π_t to 1 std. shocks

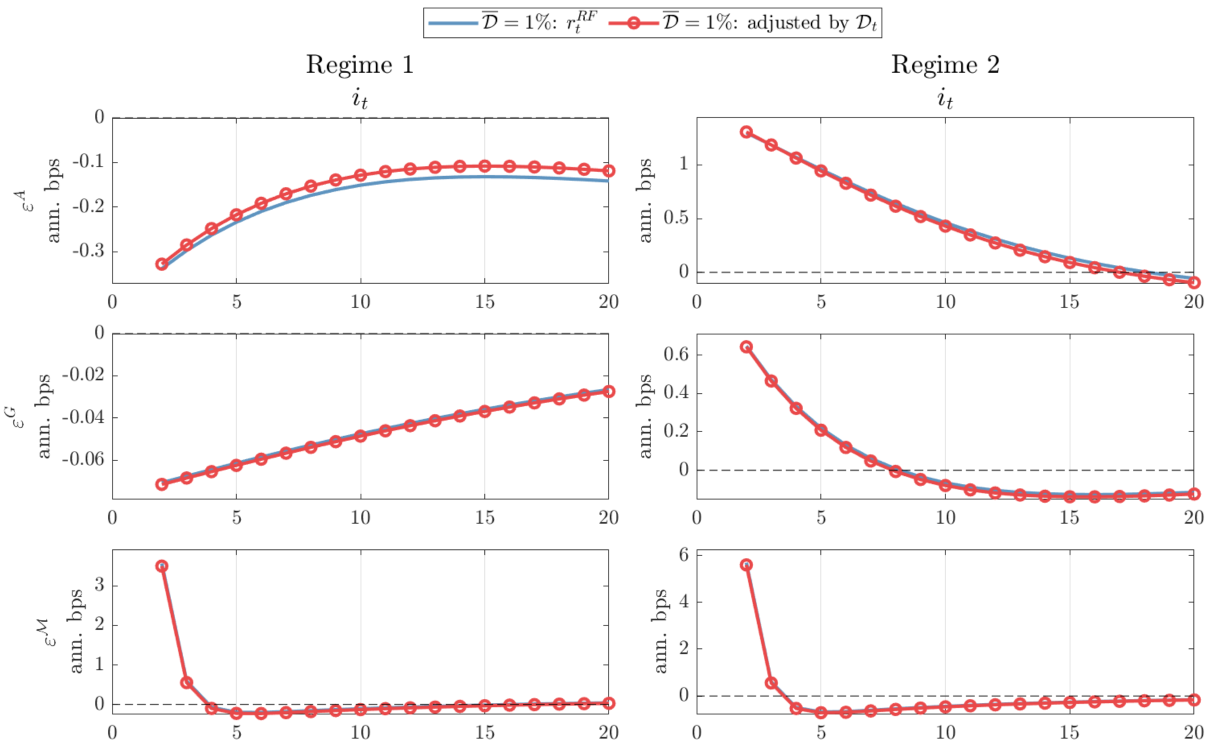


Figure 34: Regime-specific impulse response functions of i_t to 1 std. shocks

F Additional analyses

F.1 Correlation between Π_t and $\mathbb{E}_t \mathscr{D}_{t+1}$

In a companion paper, we find in a partial-equilibrium flexible-price monetary model that no matter the correlation between r_t^n and $\mathbb{E}_t \mathscr{D}_{t+1}$, the correlation between Π_t and $\mathbb{E}_t \mathscr{D}_{t+1}$ would be positive and grow with ϕ^π under either a fixed intercept rule or a rule that tracked r_t^n. In the specific case of a central bank that adjusts its rule to the default probability underlying its policy asset, however, the correlation would be eliminated. We repeat a similar exercise here, simulating the model for 4 Markov chains with 300,000 periods each, and measuring the correlation between inflation and expected default probability for different levels of steady-state default probability and several combinations of values for the policy parameters ϕ^π and γ_τ. Note that the relevant range here will be the one that should, arguably, match active monetary policy with passive fiscal policy, that is, the upper-right quadrant of figures below. Empty squares in them represent parameter value combinations for which the model could not be solved, which happen to occur in the regions usually associated to indeterminacy in New-Keynesian models. To make clear the differences between the monetary policy rules, once again we simplify them by turning off the interest rate smoothing and the dependency on the output gap. Overall, we confirm that our companion paper results remain solid also under sticky prices.

Figure 35 plots the studied correlation subject to each of the four monetary policy rules when varying the steady-state default probability from virtually 0% (benchmark calibration) up to 5%. Ignoring default risk enacts a positive correlation that increases with the default probability. Adjusting the intercept to that risk or conducting monetary policy with risk-free assets, practically, eliminates the correlation. Reacting to defaut risk while operating risk-free assets, by its turn, entails a negative correlation that increases in size with the steady-state default risk.

Figure 36 is built employing Rule 1, the central bank tracks r_t^{RF}, and varying the policy parameters ϕ^π and γ_τ under a debt level consistent with 5% default probability. As predicted by our partial equilibrium results, correlation is always positive, whenever a model solution is found (NA stands for no solution is available). This tells us that ignoring the evolution of risk in the underlying asset of monetary policy brings about a correlation of the same sign found in empirical data of emerging economies. In the literature, Schabert and Van Wijnbergen (2014) get a positive correlation in their model, because close-to-active fiscal policy in combination with active monetary policy jeopardizes macroeconomic stability. In our model, the explanation starts earlier, as a positive correlation is motivated by higher expected default rates of the policy asset being accommodated with lower policy rates (higher inflation).

Figure 37 shows the results under Rule 2, the policy rule that adjusts for default risk. Since inflation is on the target at all periods except for monetary shocks, there is virtually no correlation, like what was predicted by our companion paper's partial equilibrium model. Remember that near zero, or very low correlation, is what is usually found in empirical data of advanced economies, a result that is perfectly consistent with that same rule in case the default probability is too small.

We, now, introduce two additional policy rules to show that this correlation may turn negative if monetary policy is operated with risk-free assets.

Rule 3: Risk-free policy asset, and the CB perfectly tracks the real risk-free rate

The policy asset is default-risk-free, and the CB wants to indirectly influence the rate of government bonds, which are default-risky, through arbitrage. This is the time-varying version of the canonical case.

$$\bar{\imath}_t = r_t^{RF} \quad \text{and} \quad \mathbb{1}_{\text{Rule}} = 1 \tag{116}$$

Rule 4: Risk-free policy asset, but the CB targets a default-risky rate

The policy asset is default-risk-free, but the CB operates as if its policy instrument shared the same default risk of government bonds. It knows, though, that the mean level of the default premium is not part of the real natural interest rate.

$$\bar{\imath}_t = r_t^{RF} + \Phi_t - \overline{\Phi} \quad \text{and} \quad \mathbb{1}_{\text{Rule}} = 0 \tag{117}$$

Figure 38 shows that when the central bank operates with risk-free assets targeting the risk-free rate, Rule 3, the correlation between inflation and default risk is close to zero. Figure 39, by its turn, plots the case in which the central bank targets a risky rate using risk-free assets, Rule 4. Correlation is, now, negative for all combinations in the upper-right quadrant as the negative impact of default risk on the trajectory of debt takes its toll from the trajectory of consumption (lower inflation). Such a negative correlation between inflation and default risk, despite at odds with real-world data, is also expected to be present in other models in the literature, like Bonam and Lukkezen (2019). In their model, debt-elastic sovereign interest rates under an augment of default risk crowd out consumption, alleviating pressure on inflation, and resulting in lower nominal interest rates.

Under Rule 1, increasing γ_τ brings the correlation between inflation and expected default risk closer to zero, that is, more reactive fiscal rules tend to attenuate such a correlation, approximating it of the optimal rule w.r.t inflation. It is important to note that other policy rules may enact different patterns of correlation.

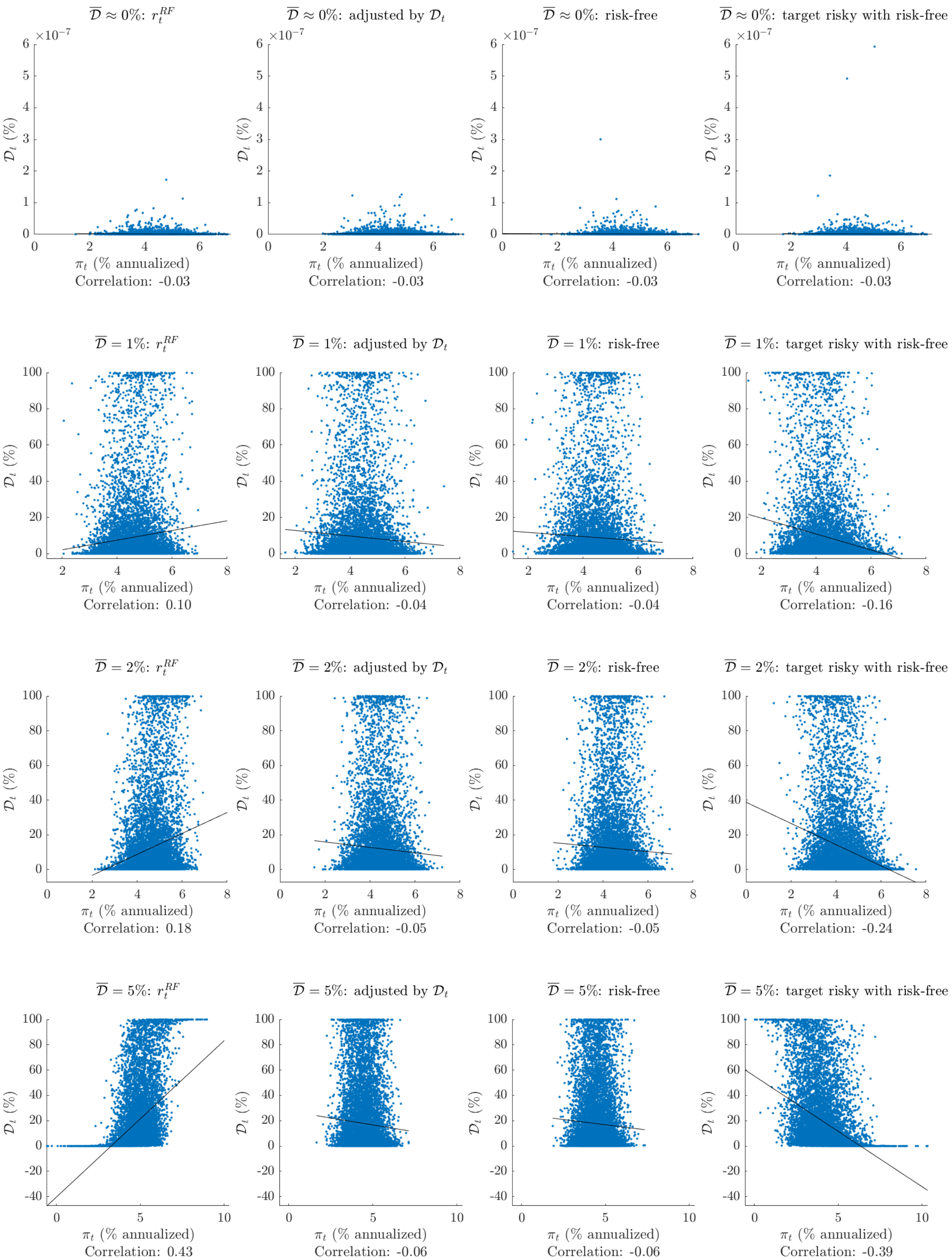


Figure 35: Correlation between Π_t and $\mathbb{E}_t\mathscr{D}_{t+1}$ under different $\overline{\mathscr{D}}$ and policy rules

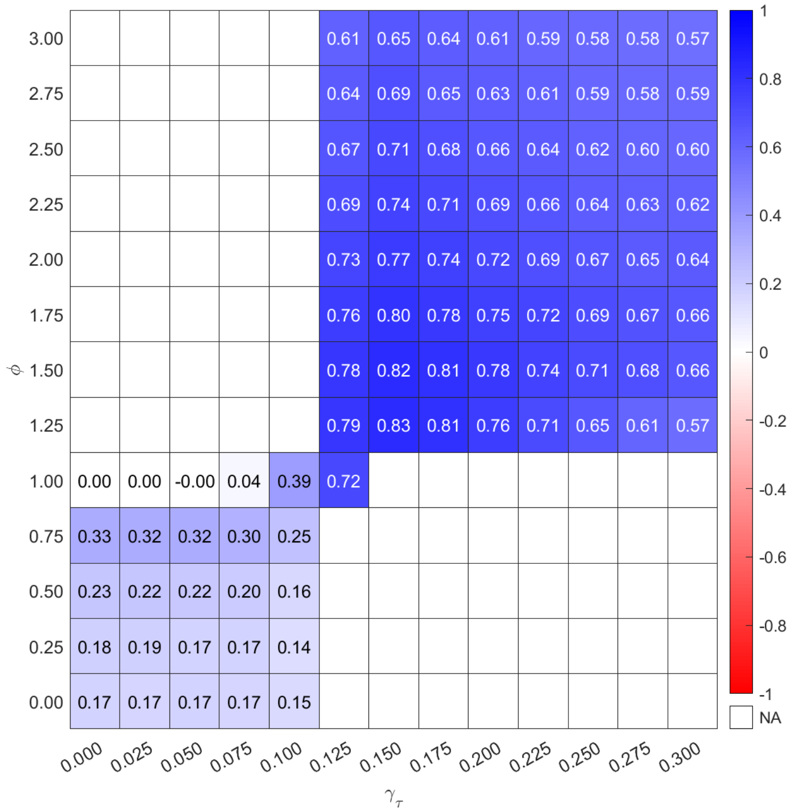


Figure 36: Correlation between Π_t and $\mathbb{E}_t \mathscr{D}_{t+1}$ under Rule 1 with $\overline{\mathscr{D}} = 5\%$

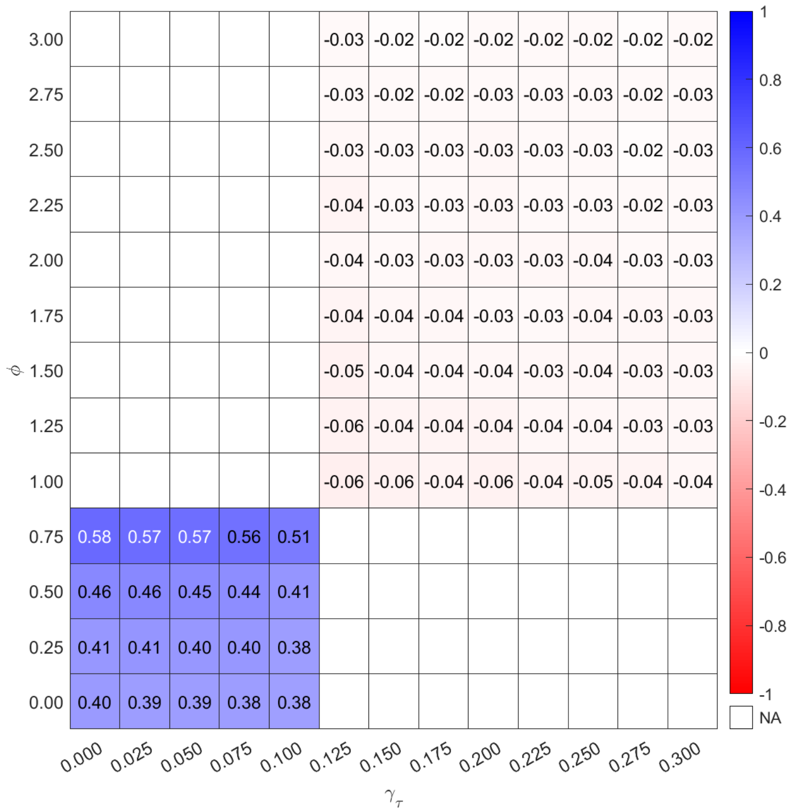


Figure 37: Correlation between Π_t and $\mathbb{E}_t \mathscr{D}_{t+1}$ under Rule 2 with $\overline{\mathscr{D}} = 5\%$

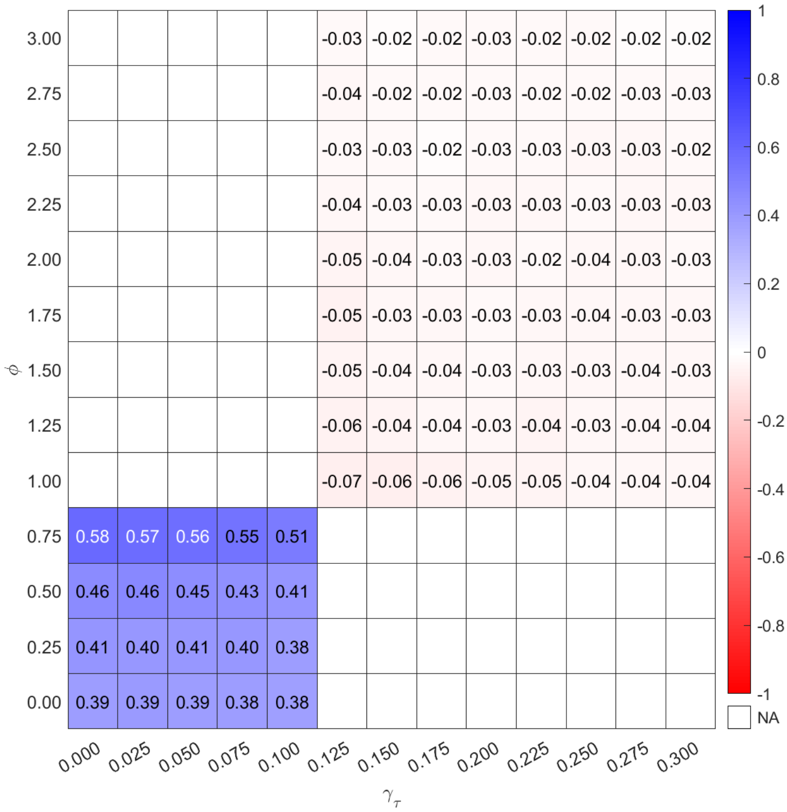


Figure 38: Correlation between Π_t and $\mathbb{E}_t \mathscr{D}_{t+1}$ under Rule 3 with $\overline{\mathscr{D}} = 5\%$

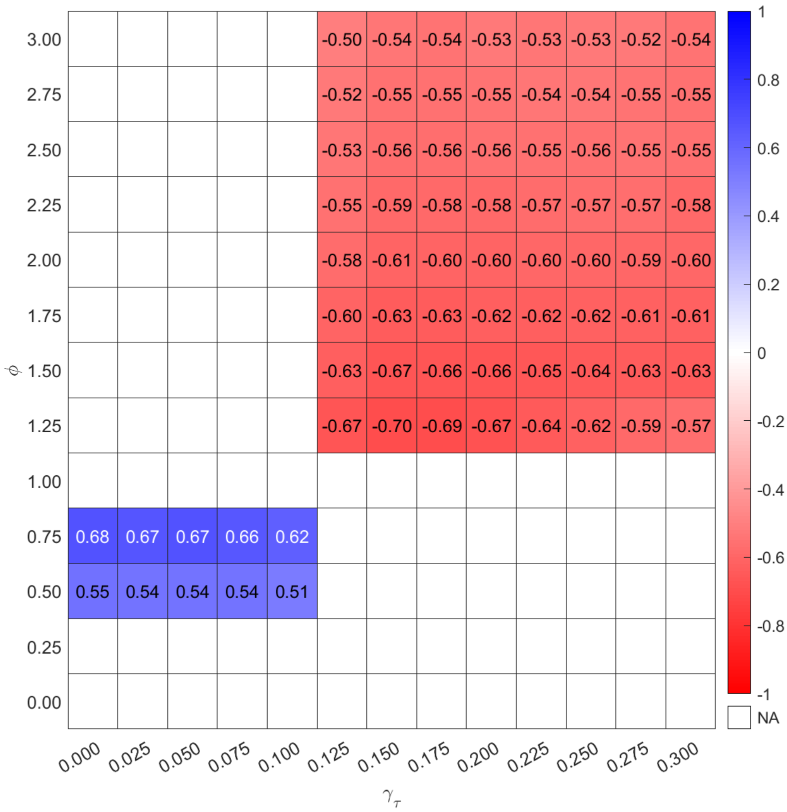


Figure 39: Correlation between Π_t and $\mathbb{E}_t \mathscr{D}_{t+1}$ under Rule 4 with $\overline{\mathscr{D}} = 5\%$

F.2 Welfare of Ricardian and non-Ricardian households

F.2.1 Sensitivity analysis under $\overline{\mathscr{D}} \approx 0\%$

Rule 1: Risky policy asset, but the CB ignores default-risk dynamics

	γ_τ=0.100	γ_τ=0.125	γ_τ=0.150	γ_τ=0.175	γ_τ=0.200
ϕ^π=1.00	∞	NaN	-∞	NaN	-∞
ϕ^π=1.25	-144	-145	-145	-146	-146
ϕ^π=1.50	-143	-144	-144	-144	-145
ϕ^π=1.75	-143	-143	-143	-143	-144
ϕ^π=2.00	-142	-143	-143	-143	-143
ϕ^π=2.25	-142	-142	-143	-143	-143
ϕ^π=2.50	-142	-142	-142	-143	-143
ϕ^π=2.75	-142	-142	-142	-142	-143
ϕ^π=3.00	-142	-142	-142	-142	-142

Rule 2: Risky policy asset, and the CB perfectly tracks default-risk dynamics

	γ_τ=0.100	γ_τ=0.125	γ_τ=0.150	γ_τ=0.175	γ_τ=0.200
ϕ^π=1.00	NaN	NaN	NaN	NaN	NaN
ϕ^π=1.25	-144	-145	-145	-146	-146
ϕ^π=1.50	-143	-144	-144	-144	-145
ϕ^π=1.75	-143	-143	-143	-144	-144
ϕ^π=2.00	-142	-143	-143	-143	-143
ϕ^π=2.25	-142	-142	-142	-143	-143
ϕ^π=2.50	-142	-142	-142	-143	-143
ϕ^π=2.75	-142	-142	-142	-142	-143
ϕ^π=3.00	-142	-142	-142	-142	-142

Note: NaN means that no solution was found under that calibration. Shaded cells indicate the parameter values combinations that generate the highest welfare in the table.

Table 18: Sensitivity analysis of Ricardian welfare under $\overline{\mathscr{D}} \approx 0\%$

Rule 1: Risky policy asset, but the CB ignores default-risk dynamics

	γ_τ=0.100	γ_τ=0.125	γ_τ=0.150	γ_τ=0.175	γ_τ=0.200
ϕ^π=1.00	∞	NaN	-∞	NaN	-∞
ϕ^π=1.25	-189	-189	-190	-190	-191
ϕ^π=1.50	-187	-188	-188	-189	-189
ϕ^π=1.75	-186	-187	-187	-187	-188
ϕ^π=2.00	-186	-186	-187	-187	-187
ϕ^π=2.25	-186	-186	-186	-186	-187
ϕ^π=2.50	-185	-186	-186	-186	-186
ϕ^π=2.75	-185	-186	-186	-186	-186
ϕ^π=3.00	-185	-185	-185	-186	-186

Rule 2: Risky policy asset, and the CB perfectly tracks default-risk dynamics

	γ_τ=0.100	γ_τ=0.125	γ_τ=0.150	γ_τ=0.175	γ_τ=0.200
ϕ^π=1.00	NaN	NaN	NaN	NaN	NaN
ϕ^π=1.25	-189	-189	-190	-191	-191
ϕ^π=1.50	-187	-188	-188	-188	-189
ϕ^π=1.75	-186	-187	-187	-188	-188
ϕ^π=2.00	-186	-186	-187	-187	-187
ϕ^π=2.25	-186	-186	-186	-186	-187
ϕ^π=2.50	-185	-186	-186	-186	-186
ϕ^π=2.75	-185	-186	-186	-186	-186
ϕ^π=3.00	-185	-185	-185	-186	-186

Note: NaN means that no solution was found under that calibration. Shaded cells indicate the parameter values combinations that generate the highest welfare in the table.

Table 19: Sensitivity analysis of Non-Ricardian welfare under $\overline{\mathscr{D}} \approx 0\%$

F.2.2 Sensitivity analysis under $\overline{\mathscr{D}} = 2\%$

Rule 1: Risky policy asset, but the CB ignores default-risk dynamics

	γ_τ=0.100	γ_τ=0.125	γ_τ=0.150	γ_τ=0.175	γ_τ=0.200
ϕ^π=1.00	-722	-1285	-292	-470	-768
ϕ^π=1.25	-186	-164	-159	-159	-160
ϕ^π=1.50	-178	-159	-155	-155	-155
ϕ^π=1.75	-173	-156	-153	-152	-153
ϕ^π=2.00	-169	-154	-151	-150	-151
ϕ^π=2.25	-167	-153	-150	-149	-150
ϕ^π=2.50	-166	-152	-149	-148	-149
ϕ^π=2.75	-165	-151	-148	-148	-148
ϕ^π=3.00	-164	-150	-147	-147	-148

Rule 2: Risky policy asset, and the CB perfectly tracks default-risk dynamics

	γ_τ=0.100	γ_τ=0.125	γ_τ=0.150	γ_τ=0.175	γ_τ=0.200
ϕ^π=1.00	NaN	NaN	NaN	NaN	NaN
ϕ^π=1.25	-187	-165	-160	-159	-160
ϕ^π=1.50	-178	-159	-155	-155	-155
ϕ^π=1.75	-173	-156	-153	-152	-153
ϕ^π=2.00	-170	-154	-151	-150	-151
ϕ^π=2.25	-168	-153	-150	-149	-150
ϕ^π=2.50	-166	-152	-149	-148	-149
ϕ^π=2.75	-165	-151	-148	-148	-148
ϕ^π=3.00	-164	-150	-147	-147	-148

Note: NaN means that no solution was found under that calibration. Shaded cells indicate the parameter values combinations that generate the highest welfare in the table.

Table 20: Sensitivity analysis of Ricardian welfare under $\overline{\mathscr{D}} = 2\%$

Rule 1: Risky policy asset, but the CB ignores default-risk dynamics

	γ_τ=0.100	γ_τ=0.125	γ_τ=0.150	γ_τ=0.175	γ_τ=0.200
ϕ^π=1.00	-222	-3675	-556	-663	-1040
ϕ^π=1.25	-301	-245	-232	-230	-232
ϕ^π=1.50	-282	-234	-224	-223	-225
ϕ^π=1.75	-271	-228	-219	-218	-220
ϕ^π=2.00	-262	-223	-216	-215	-217
ϕ^π=2.25	-258	-220	-214	-213	-215
ϕ^π=2.50	-255	-218	-212	-212	-213
ϕ^π=2.75	-252	-216	-210	-210	-212
ϕ^π=3.00	-249	-215	-209	-209	-211

Rule 2: Risky policy asset, and the CB perfectly tracks default-risk dynamics

	γ_τ=0.100	γ_τ=0.125	γ_τ=0.150	γ_τ=0.175	γ_τ=0.200
ϕ^π=1.00	NaN	NaN	NaN	NaN	NaN
ϕ^π=1.25	-306	-246	-233	-232	-234
ϕ^π=1.50	-284	-234	-224	-223	-225
ϕ^π=1.75	-272	-227	-219	-219	-220
ϕ^π=2.00	-263	-223	-216	-215	-217
ϕ^π=2.25	-259	-220	-214	-213	-215
ϕ^π=2.50	-255	-218	-212	-211	-213
ϕ^π=2.75	-252	-216	-210	-210	-212
ϕ^π=3.00	-251	-215	-209	-209	-211

Note: NaN means that no solution was found under that calibration. Shaded cells indicate the parameter values combinations that generate the highest welfare in the table.

Table 21: Sensitivity analysis of Non-Ricardian welfare under $\overline{\mathscr{D}} = 2\%$

F.2.3 Sensitivity analysis under $\overline{\mathscr{D}} = 5\%$

Rule 1: Risky policy asset, but the CB ignores default-risk dynamics

	γ_τ=0.100	γ_τ=0.125	γ_τ=0.150	γ_τ=0.175	γ_τ=0.200
ϕ^π=1.00	417	-2942	NaN	NaN	NaN
ϕ^π=1.25	NaN	-1325	-640	-445	-377
ϕ^π=1.50	NaN	-1317	-653	-451	-371
ϕ^π=1.75	NaN	-1297	-653	-451	-370
ϕ^π=2.00	NaN	-1314	-649	-451	-362
ϕ^π=2.25	NaN	-1183	-652	-450	-334
ϕ^π=2.50	NaN	-641	-649	-437	-309
ϕ^π=2.75	NaN	-314	-560	-420	-329
ϕ^π=3.00	NaN	-490	-509	-393	-239

Rule 2: Risky policy asset, and the CB perfectly tracks default-risk dynamics

	γ_τ=0.100	γ_τ=0.125	γ_τ=0.150	γ_τ=0.175	γ_τ=0.200
ϕ^π=1.00	NaN	NaN	NaN	NaN	NaN
ϕ^π=1.25	NaN	-1438	-702	-482	-398
ϕ^π=1.50	NaN	-1411	-690	-476	-345
ϕ^π=1.75	NaN	-1322	-642	-408	-210
ϕ^π=2.00	NaN	-1269	-632	-415	-164
ϕ^π=2.25	NaN	-712	-570	-313	-261
ϕ^π=2.50	NaN	-658	-525	-223	-218
ϕ^π=2.75	NaN	-396	-401	-229	-203
ϕ^π=3.00	NaN	-170	-407	-216	-204

Note: NaN means that no solution was found under that calibration. Shaded cells indicate the parameter values combinations that generate the highest welfare in the table.

Table 22: Sensitivity analysis of Ricardian welfare under $\overline{\mathscr{D}} = 5\%$

Rule 1: Risky policy asset, but the CB ignores default-risk dynamics

	γ_τ=0.100	γ_τ=0.125	γ_τ=0.150	γ_τ=0.175	γ_τ=0.200
ϕ^π=1.00	8670	-8765	NaN	NaN	NaN
ϕ^π=1.25	NaN	-3492	-1412	-816	-599
ϕ^π=1.50	NaN	-3591	-1520	-891	-641
ϕ^π=1.75	NaN	-3570	-1541	-904	-650
ϕ^π=2.00	NaN	-3635	-1538	-910	-639
ϕ^π=2.25	NaN	-3256	-1549	-911	-586
ϕ^π=2.50	NaN	-1657	-1544	-884	-539
ϕ^π=2.75	NaN	-694	-1308	-847	-579
ϕ^π=3.00	NaN	-1215	-1173	-783	-400

Rule 2: Risky policy asset, and the CB perfectly tracks default-risk dynamics

	γ_τ=0.100	γ_τ=0.125	γ_τ=0.150	γ_τ=0.175	γ_τ=0.200
ϕ^π=1.00	NaN	NaN	NaN	NaN	NaN
ϕ^π=1.25	NaN	-4032	-1704	-1010	-741
ϕ^π=1.50	NaN	-3953	-1669	-993	-627
ϕ^π=1.75	NaN	-3688	-1538	-829	-352
ϕ^π=2.00	NaN	-3528	-1512	-845	-257
ϕ^π=2.25	NaN	-1875	-1343	-604	-451
ϕ^π=2.50	NaN	-1715	-1222	-393	-364
ϕ^π=2.75	NaN	-940	-888	-406	-333
ϕ^π=3.00	NaN	-270	-907	-375	-334

Note: NaN means that no solution was found under that calibration. Shaded cells indicate the parameter values combinations that generate the highest welfare in the table.

Table 23: Sensitivity analysis of Non-Ricardian welfare under $\overline{\mathscr{D}} = 5\%$

G Endogenous regime-switching DSGE solution strategy

The third step[67] of the solution algorithm of this paper's regime-switching dynamic stochastic general equilibrium (RS-DSGE) model employs the method described in Maih (2015) with the help of the MatLab toolbox RISE[68], developed by the same author. This method embeds switching behavior in perturbation solutions, allowing for the transition probability matrix to be time-varying and determined endogenously, that is, state-to-state probabilities can depend on the state vector of the model. Besides, such a method does not require the existence of a unique steady state across different regimes, differently from methods that first linearize the model to only then impose Markov-switching behavior to some of its parameters – an approach that reduces the capacity of the model of truly reflecting its original non-linearities. Furthermore, compared to so-called "global methods" that discretize the state space into a grid of points, the curse of dimensionality is a much smaller problem here.

The algorithm develops as follows, where we maintain the notation of Maih (2015), but with fewer details.

The problem we solve is

$$\mathbb{E}_t \sum_{r_{t+1=1}}^{h} \pi_{r_t,r_{t+1}}(I_t)\, \tilde{d}_{r_t}(v) = 0 \tag{118}$$

where h is the regime at time t, $\tilde{d}_{r_t} : \Re^{n_v} \rightarrow \Re^{n_d}$ is a n_d x 1 vector of functions with argument v, r_t is the regime at time t, $\pi_{r_t,r_{t+1}}(I_t)$ is the transition probability from regime r_t to r_{t+1} at the next period, and I_t is the information set at time t. The vector v, by its turn, is given by

$$v \equiv \begin{bmatrix} b_{t+1}(r_{t+1})' & f_{t+1}(r_{t+1})' & s_t(r_t)' & p_t(r_t)' & b_t(r_t)' & f_t(r_t)' p'_{t-1} & b'_{t-1} & \varepsilon'_t & \theta'_{r_{t+1}} \end{bmatrix}' \tag{119}$$

- s_t is a vector of static variables, the ones that appear only at time t
- f_t is a vector of forward-looking variables, the ones that appear at times t and $t+1$
- p_t is a vector of predetermined variables, the ones that appear at times t and $t-1$
- b_t is a vector of both variables, the ones that appear at times $t-1$, t, and $t+1$

[67] The first step is the solution of the model as a function of its non-linearities; the second step is the calculation of the fiscal limit distribution using "global methods".

[68] Available on https://github.com/jmaih/RISE_toolbox/ as of August 16 2020.

- ε_t is a vector of shocks such that $\varepsilon_t \sim N\left(0, I_{n_\varepsilon}\right)$[69]

- $\theta_{r_{t+1}}$ is a vector of switching parameters appearing with a lead

where n_g is the length of vector g. Note that the parameters of the model are implicitly attached to the object $\tilde{d}_{r_t}$, so that even the equations in each regime are allowed to differ. Besides, it is required that each transition probability function contains only parameters that are fixed through time, and that input variables have a unique steady state. Both conditions are satisfied in our model. The expected default probability function depends only on stationary exogenous processes and a stationary variable, debt, whose convergence is guaranteed by the tax policy rule. The probability that the peak of the Laffer curve is binding, by its turn, depends only on the shadow tax rate, whose steady state is the same across regimes.

The algorithm assumes that agents may have information about shocks into the future.[70] The state vector is defined as

$$z_t \equiv \begin{bmatrix} p'_{t-1} & b'_{t-1} & \sigma & \varepsilon'_t & \varepsilon'_{t+1} & \dots & \varepsilon'_{t+k} \end{bmatrix}' \tag{120}$$

where σ is the perturbation parameter.

The general solution to our problem takes the form

$$y_t(r_t) \equiv \begin{pmatrix} s_t(r_t) \\ p_t(r_t) \\ b_t(r_t) \\ f_t(r_t) \end{pmatrix} = \mathscr{T}^{r_t}(z_t) \equiv \begin{pmatrix} \mathscr{S}^{r_t}(z_t) \\ \mathscr{P}^{r_t}(z_t) \\ \mathscr{B}^{r_t}(z_t) \\ \mathscr{F}^{r_t}(z_t) \end{pmatrix} \tag{121}$$

where $y_t(r_t)$ is the vector of all endogenous variables, and $\mathscr{T}^{r_t}(z_t)$ is the vector of their respective decision rules.

To help us in obtaining the approximations of the model, we define some selector matrices. First, for all $g \in \{s, p, b, f\}$, $\lambda_x \equiv \begin{pmatrix} \lambda_p \\ \lambda_b \end{pmatrix}$ and $\lambda_{bf} \equiv \begin{pmatrix} \lambda_b \\ \lambda_f \end{pmatrix}$ select $p-b$ and $b-f$ variables, respectively. Second, for all $g \in z_t$, a matrix m_g selects g-type variables in z_t.

We express now forward-looking variables as a function of the current state vector by employing the relation $z_{t+1} = h^{r_t}(z_t) + uz_t$, where

[69]Modeling correlated shocks is possible under this solution algorithm by creating auxiliary variables that are linear combinations of the shocks.

[70]This "anticipated shocks" approach has advantages over the "news shocks" approach, for in the first one shocks are actually structural, suffer horizon-discount, and allow for Bayesian model comparison (Maih (2015)).

$h^{r_t}(z_t) \equiv \left[(\lambda_x \mathscr{T}^{r_t}(z_t))' \quad (m_\sigma z_t)' \quad (m_{\varepsilon,1} z_t)' \quad \ldots \quad (m_{\varepsilon,k} z_t)' \quad (0_{n_\varepsilon} x1)' \right]$

and $u \equiv \begin{bmatrix} 0_{(n_p+n_b+1+k_{n_\varepsilon})\times n_z} \\ \varepsilon_{t+k+1} m_\sigma \end{bmatrix}$

Following the algorithm, we can express v as a function of the state variables, including the switching parameters, for which we postulate that $\theta_{r_{t+1}} = \overline{\theta}_{r_t} + \sigma \hat{\theta}_{r_{t+1}}$,[71] where we adopt in this paper that $\overline{\theta}_{r_t}$ is the steady state of regime 1 (no constraints are binding).

$$v = \begin{pmatrix} \lambda_{bf} \mathscr{T}^{r_{t+1}} (h^{r_t}(z_t) + u z_t) \\ \mathscr{T}^{r_t}(z_t) \\ m_p z_t \\ m_b z_t \\ m_{\varepsilon,0} z_t \\ \overline{\theta}_{r_t} + \hat{\theta}_{r_{t+1}} m_\sigma z_t \end{pmatrix} \tag{122}$$

The logic of choosing a regime-specific steady state is that, once the system is at that regime, it will remain at the same point in the absence of further shocks and of regime changes. Maih (2015) makes the case that the ergodic mean is not necessarily an attractor of the system, and that agents may act as to insure against switching to another regime and not to the ergodic mean. In that last case, unconditional forecasts done with the model could miss the level suggested by the recent history of the data. The regime-specific steady state can be computed by making

$$\tilde{d}_{r_t} \left(b_t(r_t), f_t(r_t), s_t(r_t), p_t(r_t), b_t(r_t), f_t(r_t), p_t(r_t), b_t(r_t), 0, \theta_{r_t} \right) = 0 \tag{123}$$

By now, we are able to take 1$^{\text{st}}$ and 2$^{\text{nd}}$ order (for calculating welfare) Taylor approximations of the objective function around our picked reference point. We redirect the reader to sections 3.2 and 3.3 of Maih (2015), where she can find more details of these approximations, and to section 5.2 of the same paper, where she can find more details about the solution technique employed by RISE. The toolbox uses a functional iteration algorithm that, despite the fact that *it is not able to find all the solutions*, it is still useful for solving large systems. It is important to note that, so far to the best of our knowledge, there is no stability concept in the literature that comprehends solutions of regime-switching models with endogenous (not necessarily smooth) transition probabilities of the like employed in this paper.

[71] Maih (2015) shows that this is the expression that results from a first-order approximation of an endogenous auxiliary variable meant for handling the presence of future switching parameters in the model.

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