

1 CONSUMER

1.1 Optimisation problem

$$\max_{K_t^s, C_t, L_t^s, I_t} U_t = \beta E_t [U_{t+1}] + (1 - \eta)^{-1} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{1-\eta} \quad (1.1)$$

s.t. :

$$C_t + I_t = \pi_t + K_{t-1}^s r_t + L_t^s W_t \quad \left(\lambda_t^{\text{CONSUMER}^1} \right) \quad (1.2)$$

$$K_t^s = I_t + K_{t-1}^s (1 - \delta) \quad \left(\lambda_t^{\text{CONSUMER}^2} \right) \quad (1.3)$$

1.2 First order conditions

$$-\lambda_t^{\text{CONSUMER}^2} + \beta \left((1 - \delta) E_t \left[\lambda_{t+1}^{\text{CONSUMER}^2} \right] + E_t \left[\lambda_{t+1}^{\text{CONSUMER}^1} r_{t+1} \right] \right) = 0 \quad (K_t^s) \quad (1.4)$$

$$-\lambda_t^{\text{CONSUMER}^1} + \mu C_t^{-1+\mu} (1 - L_t^s)^{1-\mu} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (C_t) \quad (1.5)$$

$$\lambda_t^{\text{CONSUMER}^1} W_t + (-1 + \mu) C_t^\mu (1 - L_t^s)^{-\mu} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (L_t^s) \quad (1.6)$$

$$-\lambda_t^{\text{CONSUMER}^1} + \lambda_t^{\text{CONSUMER}^2} = 0 \quad (I_t) \quad (1.7)$$

2 FIRM

2.1 Optimisation problem

$$\max_{K_t^d, L_t^d, Y_t} \pi_t = Y_t - L_t^d W_t - r_t K_t^d \quad (2.1)$$

s.t. :

$$Y_t = Z_t K_t^{d\alpha} L_t^{d1-\alpha} \quad \left(\lambda_t^{\text{FIRM}^1} \right) \quad (2.2)$$

2.2 First order conditions

$$-r_t + \alpha \lambda_t^{\text{FIRM}^1} Z_t K_t^{\text{d}^{-1+\alpha}} L_t^{\text{d}^{1-\alpha}} = 0 \quad (K_t^{\text{d}}) \quad (2.3)$$

$$-W_t + \lambda_t^{\text{FIRM}^1} Z_t (1 - \alpha) K_t^{\text{d}^\alpha} L_t^{\text{d}^{-\alpha}} = 0 \quad (L_t^{\text{d}}) \quad (2.4)$$

$$1 - \lambda_t^{\text{FIRM}^1} = 0 \quad (Y_t) \quad (2.5)$$

2.3 First order conditions after reduction

$$-r_t + \alpha Z_t K_t^{\text{d}^{-1+\alpha}} L_t^{\text{d}^{1-\alpha}} = 0 \quad (K_t^{\text{d}}) \quad (2.6)$$

$$-W_t + Z_t (1 - \alpha) K_t^{\text{d}^\alpha} L_t^{\text{d}^{-\alpha}} = 0 \quad (L_t^{\text{d}}) \quad (2.7)$$

3 EQUILIBRIUM

3.1 Identities

$$K_t^{\text{d}} = K_{t-1}^{\text{s}} \quad (3.1)$$

$$L_t^{\text{d}} = L_t^{\text{s}} \quad (3.2)$$

4 EXOG

4.1 Identities

$$Z_t = e^{\epsilon_t^Z + \phi \log Z_{t-1}} \quad (4.1)$$

5 Equilibrium relationships (after reduction)

$$-r_t + \alpha Z_t K_{t-1}^{\text{s}^{-1+\alpha}} L_t^{\text{s}^{1-\alpha}} = 0 \quad (5.1)$$

$$-W_t + Z_t (1 - \alpha) K_{t-1}^{\text{s}^\alpha} L_t^{\text{s}^{-\alpha}} = 0 \quad (5.2)$$

$$-Y_t + Z_t K_{t-1}^{\text{s}^\alpha} L_t^{\text{s}^{1-\alpha}} = 0 \quad (5.3)$$

$$-Z_t + e^{\epsilon_t^Z + \phi \log Z_{t-1}} = 0 \quad (5.4)$$

$$\beta \left(\mu \text{E}_t \left[r_{t+1} C_{t+1}^{-1+\mu} (1 - L_{t+1}^{\text{s}})^{1-\mu} \left(C_{t+1}^\mu (1 - L_{t+1}^{\text{s}})^{1-\mu} \right)^{-\eta} \right] + \mu (1 - \delta) \text{E}_t \left[C_{t+1}^{-1+\mu} (1 - L_{t+1}^{\text{s}})^{1-\mu} \left(C_{t+1}^\mu (1 - L_{t+1}^{\text{s}})^{1-\mu} \right)^{-\eta} \right] \right) - \mu C_t^{-1+\mu} (1 - L_t^{\text{s}})^{1-\mu} \left(C_t^\mu (1 - L_t^{\text{s}})^{1-\mu} \right)^{-\eta} = \quad (5.5)$$

$$(-1 + \mu) C_t^\mu (1 - L_t^{\text{s}})^{-\mu} \left(C_t^\mu (1 - L_t^{\text{s}})^{1-\mu} \right)^{-\eta} + \mu W_t C_t^{-1+\mu} (1 - L_t^{\text{s}})^{1-\mu} \left(C_t^\mu (1 - L_t^{\text{s}})^{1-\mu} \right)^{-\eta} = 0 \quad (5.6)$$

$$-C_t - I_t + Y_t = 0 \quad (5.7)$$

$$I_t - K_t^s + K_{t-1}^s (1 - \delta) = 0 \quad (5.8)$$

$$U_t - \beta E_t [U_{t+1}] - (1 - \eta)^{-1} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{1-\eta} = 0 \quad (5.9)$$

6 Steady state relationships (after reduction)

$$-r_{ss} + \alpha Z_{ss} K_{ss}^{s-1+\alpha} L_{ss}^{s1-\alpha} = 0 \quad (6.1)$$

$$-W_{ss} + Z_{ss} (1 - \alpha) K_{ss}^{s\alpha} L_{ss}^{s-\alpha} = 0 \quad (6.2)$$

$$-Y_{ss} + Z_{ss} K_{ss}^{s\alpha} L_{ss}^{s1-\alpha} = 0 \quad (6.3)$$

$$-Z_{ss} + e^{\phi \log Z_{ss}} = 0 \quad (6.4)$$

$$\beta \left(\mu r_{ss} C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left(C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} + \mu (1 - \delta) C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left(C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} \right) - \mu C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left(C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} = 0 \quad (6.5)$$

$$(-1 + \mu) C_{ss}^\mu (1 - L_{ss}^s)^{-\mu} \left(C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} + \mu W_{ss} C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left(C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} = 0 \quad (6.6)$$

$$-C_{ss} - I_{ss} + Y_{ss} = 0 \quad (6.7)$$

$$I_{ss} - K_{ss}^s + K_{ss}^s (1 - \delta) = 0 \quad (6.8)$$

$$U_{ss} - \beta U_{ss} - (1 - \eta)^{-1} \left(C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{1-\eta} = 0 \quad (6.9)$$

7 Calibrating equations

$$-0.36 Y_{ss} + r_{ss} K_{ss}^s = 0 \quad (7.1)$$

8 Parameter settings

$$\beta = 0.99 \quad (8.1)$$

$$\delta = 0.025 \quad (8.2)$$

$$\eta = 2 \quad (8.3)$$

$$\mu = 0.3 \quad (8.4)$$

$$\phi = 0.95 \quad (8.5)$$

9 Steady-state values

	Steady-state value
r	0.0351
C	0.7422
I	0.2559
K^s	10.2368
L^s	0.2695
U	-136.2372
W	2.3706
Y	0.9981
Z	1

10 The solution of the 1st order perturbation

Matrix P

$$\begin{matrix} K_t^s \\ Z_t \end{matrix} \begin{pmatrix} K_{t-1}^s & Z_{t-1} \\ 0.9631 & 0.0962 \\ 0 & 0.95 \end{pmatrix}$$

Matrix Q

$$\begin{matrix} K^s \\ Z \end{matrix} \begin{pmatrix} \epsilon^Z \\ 0.1012 \\ 1 \end{pmatrix}$$

Matrix R

$$\begin{matrix} r_t \\ C_t \\ I_t \\ L_t^s \\ U_t \\ W_t \\ Y_t \end{matrix} \begin{pmatrix} K_{t-1}^s & Z_{t-1} \\ -0.7559 & 1.3521 \\ 0.4919 & 0.4921 \\ -0.4745 & 3.8461 \\ -0.181 & 0.6282 \\ 0.0418 & 0.0644 \\ 0.4252 & 0.7238 \\ 0.2441 & 1.3521 \end{pmatrix}$$

Matrix S

$$\begin{matrix} r \\ C \\ I \\ L^s \\ U \\ W \\ Y \end{matrix} \begin{pmatrix} \epsilon^Z \\ 1.4232 \\ 0.518 \\ 4.0485 \\ 0.6613 \\ 0.0678 \\ 0.7619 \\ 1.4232 \end{pmatrix}$$

11 Model statistics

11.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
r	0.0351	0.1893	0.0358	Y
C	0.7422	0.0711	0.0051	Y
I	0.2559	0.5284	0.2792	Y
K^s	10.2368	0.0469	0.0022	Y
L^s	0.2695	0.0867	0.0075	Y
U	-136.2372	0.009	0.0001	Y
W	2.3706	0.1011	0.0102	Y
Y	0.9981	0.1857	0.0345	Y
Z	1	0.1303	0.017	Y

11.2 Correlation matrix

	r	C	I	K^s	L^s	U	W	Y	Z
r	1	0.868	0.989	0.084	0.996	0.918	0.926	0.969	0.982
C		1	0.931	0.567	0.909	0.994	0.991	0.964	0.946
I			1	0.228	0.998	0.966	0.971	0.995	0.999
K^s				1	0.174	0.472	0.454	0.328	0.269
L^s					1	0.95	0.956	0.987	0.995
U						1	1	0.988	0.976
W							1	0.991	0.98
Y								1	0.998
Z									1

11.3 Cross correlations with the reference variable (Y)

	$\sigma[\cdot]$ rel. to $\sigma[Y]$	Y_{t-5}	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Y_{t+5}
r_t	1.019	0.116	0.234	0.378	0.548	0.745	0.969	0.623	0.343	0.121	-0.048	-0.172
C_t	0.383	-0.14	-0.013	0.156	0.372	0.64	0.964	0.77	0.592	0.431	0.29	0.169
I_t	2.845	0.044	0.168	0.323	0.511	0.735	0.995	0.684	0.426	0.216	0.05	-0.077
K_t^s	0.253	-0.474	-0.415	-0.313	-0.162	0.049	0.328	0.513	0.62	0.665	0.662	0.624
L_t^s	0.467	0.072	0.194	0.345	0.527	0.741	0.987	0.662	0.395	0.18	0.013	-0.113
U_t	0.049	-0.086	0.042	0.21	0.42	0.679	0.988	0.755	0.55	0.372	0.221	0.096
W_t	0.544	-0.076	0.052	0.219	0.428	0.685	0.991	0.752	0.541	0.361	0.208	0.083
Y_t	1	-0.008	0.119	0.28	0.479	0.718	1	0.718	0.479	0.28	0.119	-0.008
Z_t	0.702	0.023	0.149	0.306	0.499	0.729	0.998	0.699	0.448	0.242	0.078	-0.049

11.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
r	0.71	0.466	0.265	0.103	-0.022
C	0.76	0.545	0.357	0.196	0.063
I	0.711	0.467	0.267	0.105	-0.02
K^s	0.96	0.862	0.727	0.571	0.407
L^s	0.709	0.465	0.264	0.102	-0.023
U	0.739	0.512	0.319	0.158	0.028
W	0.736	0.507	0.312	0.152	0.022
Y	0.718	0.479	0.28	0.119	-0.008
Z	0.713	0.471	0.271	0.11	-0.016

12 Impulse response functions

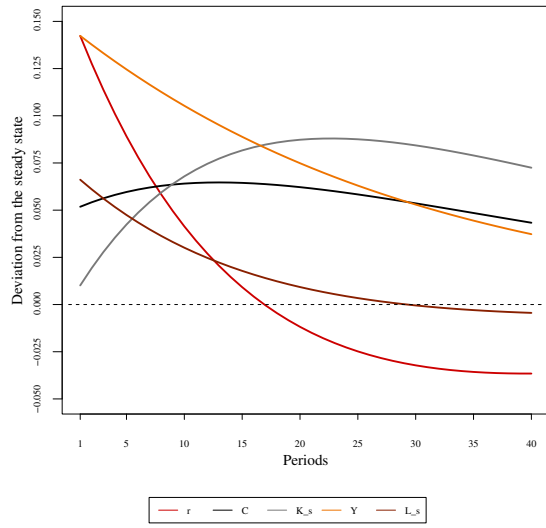


Figure 1: Impulse responses (r, C, K^s, Y, L^s) to ϵ^Z shock

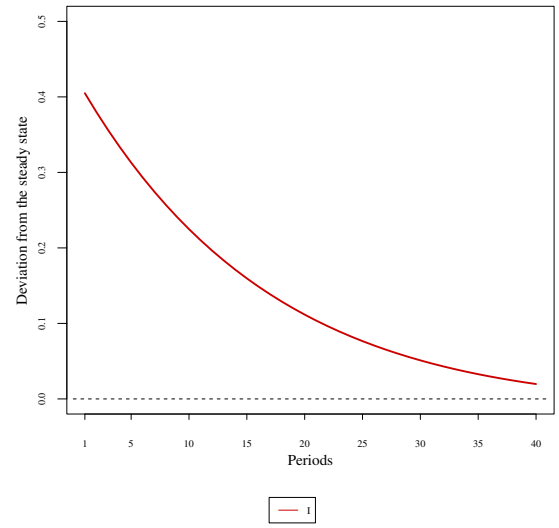


Figure 2: Impulse response (I) to ϵ^Z shock