

# 1 Recursion and Tree Recursion

## Questions

- 1.1 What are three things you find in every recursive function?
- 1.2 When you write a Recursive function, you seem to call it before it has been fully defined. Why doesn't this break the Python interpreter?

- 1.3 The **domain** is the type of data that a function takes in as argument. The **range** is the type of data that a function returns. For example, the domain of the function **square** is numbers. The range is numbers.  
Below is a Python function that computes the nth Fibonacci number. What's its domain and range? Also identify the three things it contains as a recursive function (from 1.1).

```
def fib(n):  
    if n == 0:  
        return 0  
    elif n == 1:  
        return 1  
    else:  
        return fib(n-1) + fib(n-2)
```

- 1.4 With the definition of the Fibonacci function above, draw out a diagram of the recursive calls made when **fib(4)** is called.

1.5 What does the following function **cascade2** do? What is its domain and range?

```
def cascade2(n):  
    print(n)  
    if n >= 10:  
        cascade2(n//10)  
    print(n)
```

1.6 What does each of the the following functions do?

```
def mystery(n):  
    if n == 0:  
        return 0  
    else:  
        return n + mystery(n - 1)
```

```
def foo(n):  
    if n <= 1:  
        return n  
    return foo(n - 2) + foo(n - 1)
```

```
def fooply(n):  
    if n < 0:  
        return 0  
    return foo(n) + fooply(n - 1)
```

## 2 Higher Order Functions

### Questions

- 2.1 What do lambda expressions do? Can we write all functions as lambda expressions?  
In what cases are lambda expressions useful?

- 2.2 Determine if each of the following will error:

```
>>> 1/0
```

```
>>> boom = lambda: 1/0
```

```
>>> boom()
```

- 2.3 Express the following lambda expression using a **def** statement, and the **def** statement using a lambda expression.

```
pow = lambda x, y: x**y
```

```
def foo(x):
    def f(y):
        def g(z):
            return x + y * z
        return g
    return f
```

2.4 Draw Environment Diagrams for the following lines of code

```
square = lambda x: x * x
```

```
higher = lambda f: lambda y: f(f(y))
```

```
higher(square)(5)
```

```
a = (lambda f, a: f(a))(lambda b: b * b, 2)
```

- 2.5 Write **make\_skipper**, which takes in a number *n* and outputs a function. When this function takes in a number *x*, it prints out all the numbers between 0 and *x*, skipping every *n*th number (meaning skip any value that is a multiple of *n*).

```
def make_skipper(n):
    """
    >>> a = make_skipper(2)
    >>> a(5)
    1
    3
    5
    """
```

- 2.6 Write **make\_alternator** which takes in two functions, *f* and *g*, and outputs a function. When this function takes in a number *x*, it prints out all the numbers between 1 and *x*, applying the function *f* to every odd-indexed number and *g* to every even-indexed number before printing.

```
def make_alternator(f, g):
    """
    >>> a = make_alternator(lambda x: x * x, lambda x: x + 4)
    >>> a(5)
    1
    6
    9
    8
    25
    >>> b = make_alternator(lambda x: x * 2, lambda x: x + 2)
    >>> b(4)
    2
    4
    6
    6
    """
```