CS 61A Recursion and Tree Recursion, HOFs
Spring 2019 Guerrilla Section 1: February 23, 2019 Solutions

1 Recursion and Tree Recursion

Questions

1.1 What are three things you find in every recursive function?

```
1) Base Case(s)
2) Way(s) to reduce the problem into a smaller problem of the same type
3) Recursive case(s) that uses the solution of the smaller problem to solve the original (large) problem
```

1.2 When you write a Recursive function, you seem to call it before it has been fully defined. Why doesn't this break the Python interpreter?

When you define a function, Python does not evaluate the body of the function.

1.3 The **domain** is the type of data that a function takes in as argument. The **range** is the type of data that a function returns. For example, the domain of the function **square** is numbers. The range is numbers.
Below is a Python function that computes the nth Fibonacci number. What's its domain and range? Also identify the three things it contains as a recursive function (from 1.1).

```python
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-1) + fib(n-2)
```

```
Domain is integers, range is integers.
Base Cases: if n == 0: ..., elif n == 1: ...
Finding Smaller Problems: finding fib(n - 1), fib(n - 2)
Recursive Case: when n is neither 0 nor 1, add together the fib(n - 1) and fib(n - 2) to find fib(n)
```

1.4 With the definition of the Fibonacci function above, draw out a diagram of the recursive calls made when **fib(4)** is called.

```
                fib(4)
                /    \
            fib(3)   fib(2)
           /    |      |    \
      fib(2) fib(1) fib(1) fib(0)
     /    |
```

```
fib(1)  fib(0)
```

1.5 What does the following function **cascade2** do? What is its domain and range?

```python
def cascade2(n):
    print(n)
    if n >= 10:
        cascade2(n//10)
        print(n)
```

Domain is integers, range is None. It takes in a number n and prints out n, then prints out n excluding the ones digit, then prints n excluding the hundreds digit, and so on, then back up to the full number.

1.6 What does each of the the following functions do?

```python
def mystery(n):
    if n == 0:
        return 0
    else:
        return n + mystery(n - 1)
```

sums positive integers up to n (1 + 2 + ... + n)

```python
def foo(n):
    if n <= 1:
        return n
    return foo(n - 2) + foo(n - 1)
```

returns the nth Fibonacci number

```python
def fooply(n):
    if n < 0:
        return 0
    return foo(n) + fooply(n - 1)
```

returns the sum of the first n Fibonacci numbers.

2 Higher Order Functions

Questions

2.1 What do lambda expressions do? Can we write all functions as lambda expressions? In what cases are lambda expressions useful?

Lambda expressions create functions. When a lambda expression is evaluated, it produces a function. We often use lambdas to create short anonymous functions that we won?t need for too long.
We can?t write all functions as lambda expressions because lambda functions all have to have ?return? statements and they can?t contain very complex multi-line expressions.

2.2 Determine if each of the following will error:

```
>>> 1/0
```

```
Error
```

```
>>> boom = lambda: 1/0
```

```
No error, since we don't evaluate the body of the lambda when we define
    it.
```

```
>>> boom()
```

```
Error
```

2.3 Express the following lambda expression using a **def** statement, and the **def** statement using a lambda expression.

```
pow = lambda x, y: x**y
```

```
def pow(x, y):
    return x**y
```

```
def foo(x):
    def f(y):
        def g(z):
            return x + y * z
        return g
    return f
```

foo = lambda x: lambda y: lambda z: x + y * z

2.4 Draw Environment Diagrams for the following lines of code

```
square = lambda x: x * x
higher = lambda f: lambda y: f(f(y))
higher(square)(5)
```

Solution: https://goo.gl/LATqV9

```
a = (lambda f, a: f(a))(lambda b: b * b, 2)
```

Solution: https://goo.gl/TyriuP

2.5 Write **make_skipper**, which takes in a number n and outputs a function. When this function takes in a number x, it prints out all the numbers between 0 and x, skipping every nth number (meaning skip any value that is a multiple of n).

```
def make_skipper(n):
    """
    >>> a = make_skipper(2)
    >>> a(5)
    1
    3
    5
    """

    def skipper(x):
        for i in range(x + 1):
            if i % n != 0:
                print(i)
    return skipper
```

2.6 Write **make_alternator** which takes in two functions, f and g, and outputs a function. When this function takes in a number x, it prints out all the numbers between 1 and x, applying the function f to every odd-indexed number and g to every even-indexed number before printing.

```
def make_alternator(f, g):
    """
    >>> a = make_alternator(lambda x: x * x, lambda x: x + 4)
    >>> a(5)
    1
    6
    9
    8
    25
    >>> b = make_alternator(lambda x: x * 2, lambda x: x + 2)
    >>> b(4)
    2
    4
    6
    6
    """

    def alternator(n):
        i = 1
        while i <= n:
            if i % 2 == 1:
                print(f(i))
            else:
                print(g(i))
```

```
        i += 1
    return alternator
```