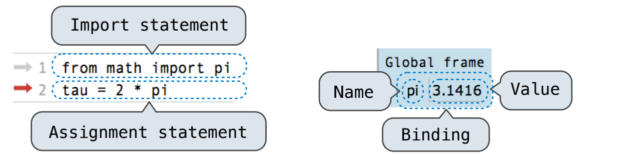
CS 61A Midterm 1 Study Guide – Page 1



```
from math import pi
tau = 2 * pi
```

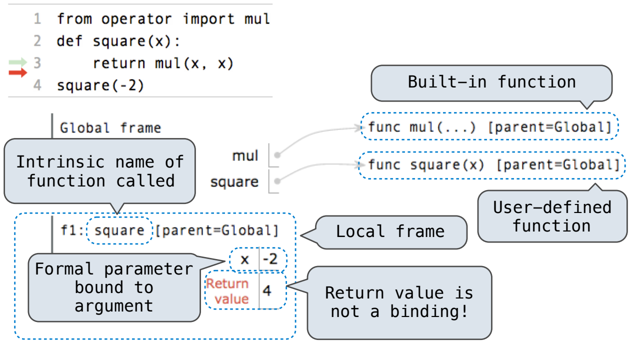
Code (left):

Statements and expressions
Red arrow points to next line.
Gray arrow points to the line just executed

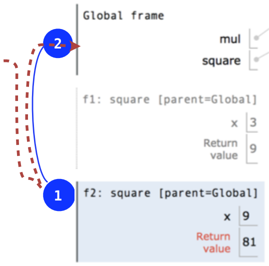
Frames (right):

A name is bound to a value

In a frame, there is at most one binding per name


Built-in function
Global frame
mul
square
func mul(...) [parent=Global]
func square(x) [parent=Global]
Intrinsic name of function called
User-defined function
f1: square [parent=Global]
Local frame
Formal parameter bound to argument
x
-2
Return value
4
Return value is not a binding!

```
from operator import mul
def square(x):
    return mul(x, x)
square(-2)
```



```
from operator import mul
def square(x):
    return mul(x, x)
square(square(3))
```

A name evaluates to the value bound to that name in the earliest frame of the current environment in which that name is found.

Evaluation rule for call expressions:

1.Evaluate the operator and operand subexpressions.
2.Apply the function that is the value of the operator subexpression to the arguments that are the values of the operand subexpressions.

Applying user-defined functions:

1.Create a new local frame with the same parent as the function that was applied.
2.Bind the arguments to the function's formal parameter names in that frame.
3.Execute the body of the function in the environment beginning at that frame.

Execution rule for def statements:

1.Create a new function value with the specified name, formal parameters, and function body.
2.Its parent is the first frame of the current environment.
3.Bind the name of the function to the function value in the first frame of the current environment.

Execution rule for assignment statements:

1.Evaluate the expression(s) on the right of the equal sign.
2.Simultaneously bind the names on the left to those values, in the first frame of the current environment.

Execution rule for conditional statements:

Each clause is considered in order.
1.Evaluate the header's expression.
2.If it is a true value, execute the suite, then skip the remaining clauses in the statement.

Evaluation rule for or expressions:

1.Evaluate the subexpression <left>.
2.If the result is a true value v, then the expression evaluates to v.
3.Otherwise, the expression evaluates to the value of the subexpression <right>.

Evaluation rule for and expressions:

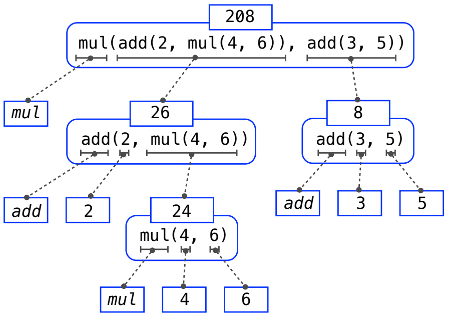
1.Evaluate the subexpression <left>.
2.If the result is a false value v, then the expression evaluates to v.
3.Otherwise, the expression evaluates to the value of the subexpression <right>.

Evaluation rule for not expressions:

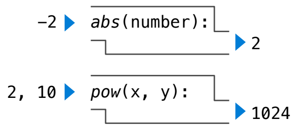
1.Evaluate <exp>; The value is True if the result is a false value, and False otherwise.

Execution rule for while statements:

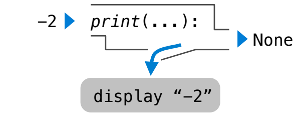
1. Evaluate the header’s expression.
2. If it is a true value, execute the (*whole*) suite, then return to step 1.

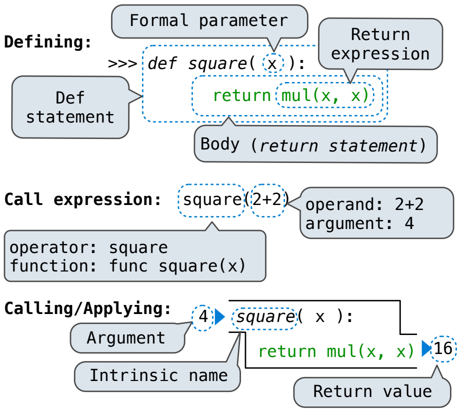


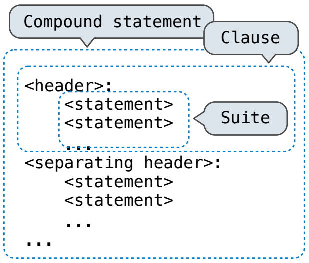
Pure Functions



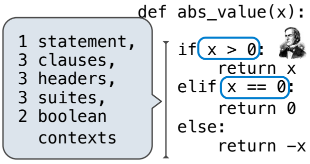
Non-Pure Functions



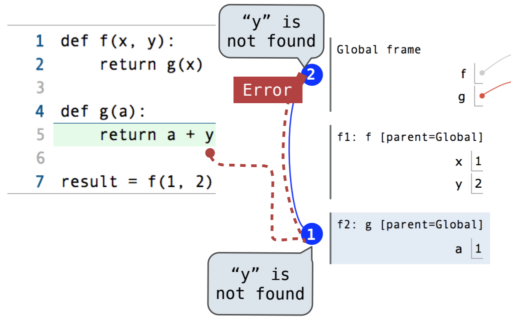




```
<header>:
    <statement>
    <statement>
    ...
<separating header>:
    <statement>
    <statement>
    ...
...
```

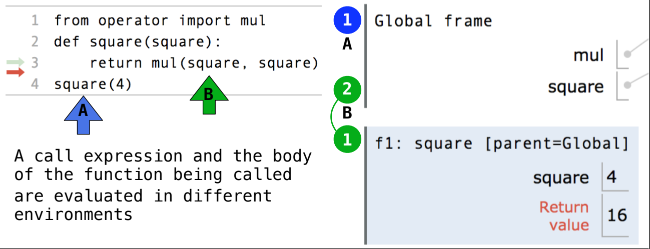


```
def abs_value(x):
    if x > 0:
        return x
    elif x == 0:
        return 0
    else:
        return -x
```



```
def f(x, y):
    return g(x)

def g(a):
    return a + y

result = f(1, 2)
```

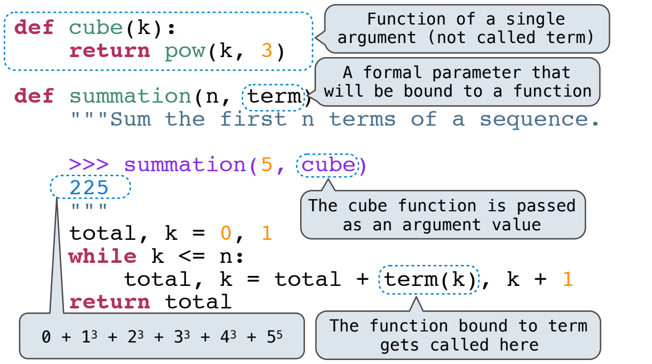
- An environment is a sequence of frames
- An environment for a non-nested function (no def within def) consists of one local frame, followed by the global frame



```
from operator import mul
def square(square):
    return mul(square, square)
square(4)
```

A call expression and the body of the function being called are evaluated in different environments

```
def fib(n):
    """Compute the nth Fibonacci number, for N >= 1."""
    pred, curr = 0, 1   # Zeroth and first Fibonacci numbers
    k = 1               # curr is the kth Fibonacci number
    while k < n:
        pred, curr = curr, pred + curr
        k = k + 1
    return curr
```


Function of a single argument (not called term)
A formal parameter that will be bound to a function
The cube function is passed as an argument value
$0 + 1^3 + 2^3 + 3^3 + 4^3 + 5^5$
The function bound to term gets called here

```
def cube(k):
    return pow(k, 3)

def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
    225
    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
    return total
```

Higher-order function: A function that takes a function as an argument value or returns a function as a return value

Nested def statements: Functions defined within other function bodies are bound to names in the local frame

```
square = lambda x,y: x * y
```

Evaluates to a function. No "return" keyword!

A function

with formal parameters x and y

that returns the value of "x * y"

Must be a single expression

```
square = lambda x: x * x      VS      def square(x):
                                          return x * x
```

- Both create a function with the same domain, range, and behavior.
- Both functions have as their parent the environment in which they were defined.
- Both bind that function to the name square.
- Only the def statement gives the function an intrinsic name.

```
def make_adder(n):
    """Return a function that takes one argument k and returns k + n.

    >>> add_three = make_adder(3)
    >>> add_three(4)
    7
    """
    def adder(k):
        return k + n
    return adder
```

A function that returns a function

The name add_three is bound to a function

A local def statement

Can refer to names in the enclosing function

When a function is defined:

1. Create a **function value**: func <name>(<formal parameters>)
2. Its parent is the current frame.

   f1: make_adder   func adder(k) [parent=f1]

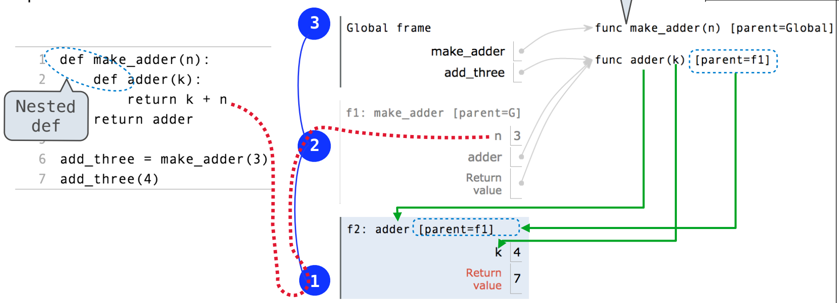
3. Bind <name> to the **function value** in the current frame (which is the first frame of the current environment).

When a function is called:

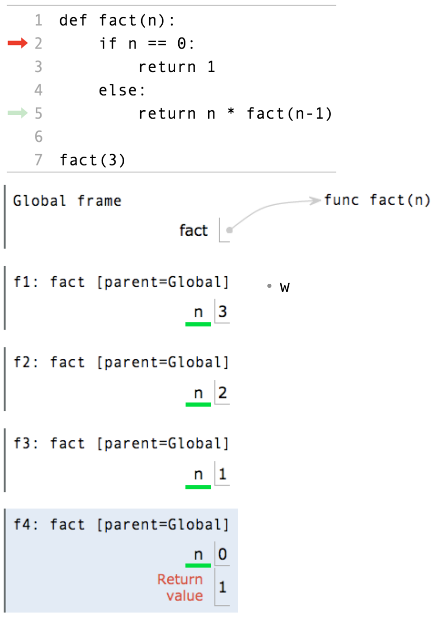
1. Add a **local frame**, titled with the <name> of the function being called.
2. Copy the parent of the function to the **local frame**: [parent=<label>]
3. Bind the <formal parameters> to the arguments in the **local frame**.
4. Execute the body of the function in the environment that starts with the **local frame**.

- Every user-defined **function** has a ***parent frame*** (often global)
- The parent of a **function** is the frame in which it was *defined*
- Every local **frame** has a *parent* ***frame*** (often global)
- The parent of a **frame** is the parent of the function *called*

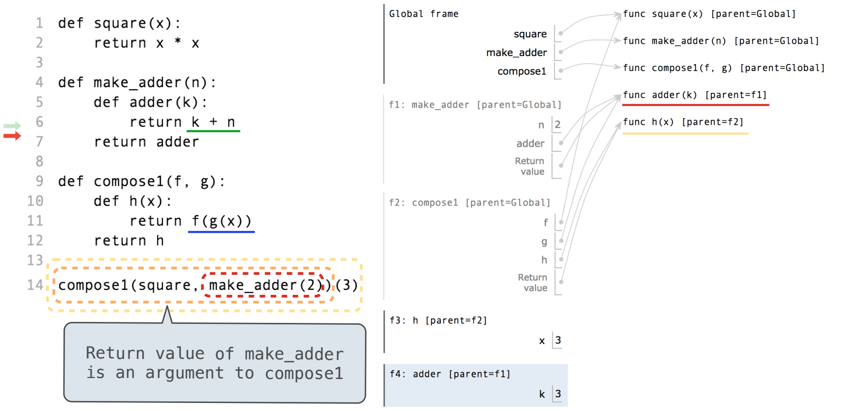
A function's signature has all the information to create a local frame



```
def make_adder(n):
    def adder(k):
        return k + n
    return adder

add_three = make_adder(3)
add_three(4)
```



```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)

fact(3)
```



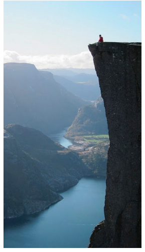
Global frame
square
make_adder
compose1
func square(x) [parent=Global]
func make_adder(n) [parent=Global]
func compose1(f, g) [parent=Global]
func adder(k) [parent=f1]
func h(x) [parent=f2]
f1: make_adder [parent=Global]
n 2
adder
Return value
f2: compose1 [parent=Global]
f
g
h
Return value
f3: h [parent=f2]
x 3
f4: adder [parent=f1]
k 3

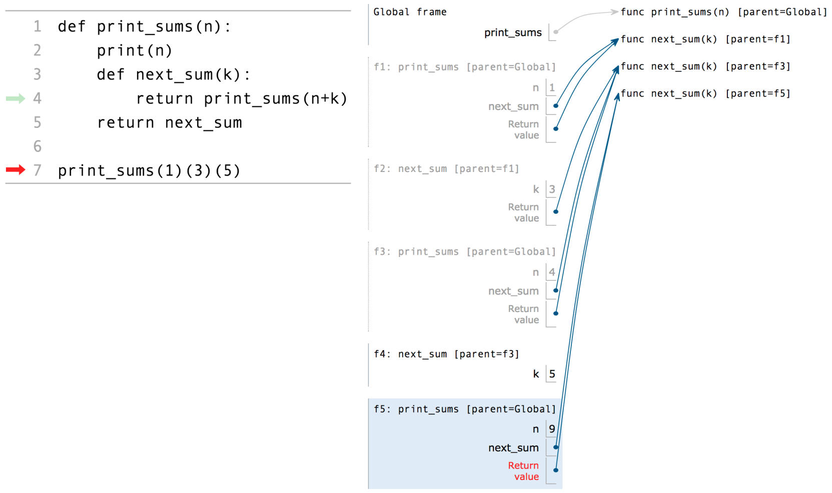
```
def square(x):
    return x * x

def make_adder(n):
    def adder(k):
        return k + n
    return adder

def compose1(f, g):
    def h(x):
        return f(g(x))
    return h

compose1(square, make_adder(2))(3)
```

Return value of make_adder is an argument to compose1

Is fact implemented correctly?

1. Verify the base case.
2. Treat fact as a functional abstraction!
3. Assume that fact(n-1) is correct.
4. Verify that fact(n) is correct, assuming that fact(n-1) correct.





Global frame
print_sums
func print_sums(n) [parent=Global]
func next_sum(k) [parent=f1]
func next_sum(k) [parent=f3]
func next_sum(k) [parent=f5]
f1: print_sums [parent=Global]
n 1
next_sum
Return value
f2: next_sum [parent=f1]
k 3
Return value
f3: print_sums [parent=Global]
n 4
next_sum
Return value
f4: next_sum [parent=f3]
k 5
f5: print_sums [parent=Global]
n 9
next_sum
Return value

```
def print_sums(n):
    print(n)
    def next_sum(k):
        return print_sums(n+k)
    return next_sum

print_sums(1)(3)(5)
```

Anatomy of a recursive function:

- The **def statement header** is similar to other functions
- Conditional statements check for **base cases**
- Base cases are evaluated **without recursive calls**
- Recursive cases are evaluated **with recursive calls**

```
def sum_digits(n):
  """Return the sum of the digits of positive integer n."""
  if n < 10:
      return n
  else:
      all_but_last, last = n // 10, n % 10
      return sum_digits(all_but_last) + last
```

```
from operator import floordiv, mod
def divide_exact(n, d):
    """Return the quotient and remainder of dividing N by D.

    >>> q, r = divide_exact(2012, 10)
    >>> q
    201
    >>> r
    2
    """
    return floordiv(n, d), mod(n, d)
```

Multiple assignment to two names

Two return values, separated by commas

CS 61A Midterm 2 Study Guide — Page 1

Rational implementation using functions:

```
def rational(n, d):
    def select(name):
        if name == 'n':
            return n
        elif name == 'd':
            return d
    return select
def numer(x):
    return x('n')
def denom(x):
    return x('d')
```

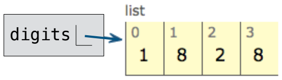
This function represents a rational number

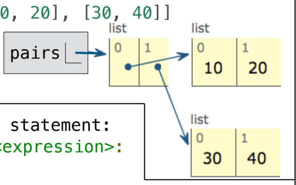
Constructor is a higher-order function

Selector calls x

Lists:

```
>>> digits = [1, 8, 2, 8]
>>> len(digits)
4
>>> digits[3]
8
>>> [2, 7] + digits * 2
[2, 7, 1, 8, 2, 8, 1, 8, 2, 8]
>>> pairs = [[10, 20], [30, 40]]
>>> pairs[1]
[30, 40]
>>> pairs[1][0]
30
```





Executing a for statement:

```
for <name> in <expression>:
    <suite>
```

1. Evaluate the header <expression>, which must yield an iterable value (a list, tuple, iterator, etc.)
2. For each element in that sequence, in order:
   A. Bind <name> to that element in the current frame
   B. Execute the <suite>

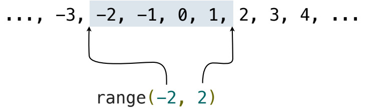
Unpacking in a for statement:

A sequence of fixed-length sequences

```
>>> pairs=[[1, 2], [2, 2], [3, 2], [4, 4]]
>>> same_count = 0
```

A name for each element in a fixed-length sequence

```
>>> for x, y in pairs:
...     if x == y:
...         same_count = same_count + 1
>>> same_count
2
```



Length: ending value - starting value

Element selection: starting value + index

```
>>> list(range(-2, 2))
[-2, -1, 0, 1]
```

List constructor

```
>>> list(range(4))
[0, 1, 2, 3]
```

Range with a 0 starting value

Membership:

```
>>> digits = [1, 8, 2, 8]
>>> 2 in digits
True
>>> 1828 not in digits
True
```

Slicing:

```
>>> digits[0:2]
[1, 8]
>>> digits[1:]
[8, 2, 8]
```

Slicing creates a new object

Functions that aggregate iterable arguments

- **sum**(iterable[, start]) -> value
- **max**(iterable[, key=func]) -> value
  max(a, b, c, ...[, key=func]) -> value
  min(iterable[, key=func]) -> value
  min(a, b, c, ...[, key=func]) -> value
- **all**(iterable) -> bool
  any(iterable) -> bool

List comprehensions:

```
[<map exp> for <name> in <iter exp> if <filter exp>]
```

Short version: `[<map exp> for <name> in <iter exp>]`

A combined expression that evaluates to a list using this evaluation procedure:

1. Add a new frame with the current frame as its parent
2. Create an empty *result list* that is the value of the expression
3. For each element in the iterable value of <iter exp>:
   A. Bind <name> to that element in the new frame from step 1
   B. If <filter exp> evaluates to a true value, then add the value of <map exp> to the result list

The result of calling **repr** on a value is what Python prints in an interactive session

The result of calling **str** on a value is what Python prints using the **print** function

```
>>> 12e12
12000000000000.0
>>> print(repr(12e12))
12000000000000.0
```

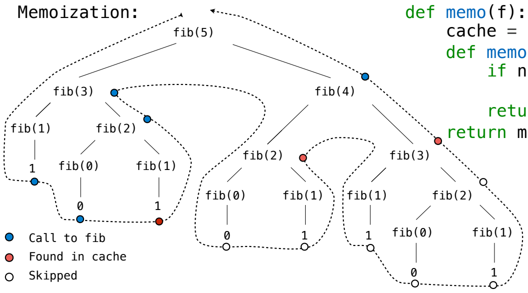
```
>>> print(today)
2014-10-13
```

str and **repr** are both polymorphic; they apply to any object
repr invokes a zero-argument method __repr__ on its argument

```
>>> today.__repr__()
'datetime.date(2014, 10, 13)'
```

```
>>> today.__str__()
'2014-10-13'
```

Memoization:



```
def memo(f):
    cache = {}
    def memoized(n):
        if n not in cache:
            cache[n] = f(n)
        return cache[n]
    return memoized
```

Type dispatching: Look up a cross-type implementation of an operation based on the types of its arguments
Type coercion: Look up a function for converting one type to another, then apply a type-specific implementation.

$R(n) = \Theta(f(n))$ means that there are positive constants k_1 and k_2 such that $k_1 \cdot f(n) \leq R(n) \leq k_2 \cdot f(n)$ for all **n** larger than some **m**

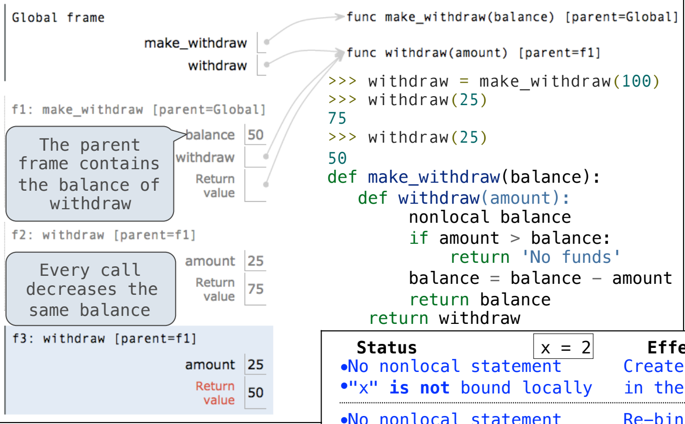
$\Theta(b^n)$ Exponential growth. Recursive fib takes $\Theta(\phi^n)$ steps, where $\phi = \frac{1+\sqrt{5}}{2} \approx 1.61828$
Incrementing the problem scales R(n) by a factor

$\Theta(n^2)$ Quadratic growth. E.g., overlap
Incrementing n increases R(n) by the problem size n

$\Theta(n)$ Linear growth. E.g., factors or exp

$\Theta(\log n)$ Logarithmic growth. E.g., exp_fast
Doubling the problem only increments R(n)

$\Theta(1)$ Constant. The problem size doesn't matter



The parent frame contains the balance of withdraw

Every call decreases the same balance

```
>>> withdraw = make_withdraw(100)
>>> withdraw(25)
75
>>> withdraw(25)
50
def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            return 'No funds'
        balance = balance - amount
        return balance
    return withdraw
```

List & dictionary mutation:

```
>>> a = [10]
>>> b = a
>>> a == b
True
>>> a.append(20)
>>> a == b
True
>>> a
[10, 20]
>>> b
[10, 20]
```

```
>>> a = [10]
>>> b = [10]
>>> a == b
True
>>> b.append(20)
>>> a
[10]
>>> b
[10, 20]
>>> a == b
False
```

```
>>> nums = {'I': 1.0, 'V': 5, 'X': 10}
>>> nums['X']
10
>>> nums['I'] = 1
>>> nums['L'] = 50
>>> nums
{'X': 10, 'L': 50, 'V': 5, 'I': 1}
>>> sum(nums.values())
66
>>> dict([(3, 9), (4, 16), (5, 25)])
{3: 9, 4: 16, 5: 25}
>>> nums.get('A', 0)
0
>>> nums.get('V', 0)
5
>>> {x: x*x for x in range(3,6)}
{3: 9, 4: 16, 5: 25}
```

```
>>> suits = ['coin', 'string', 'myriad']
>>> suits.pop()
'myriad'
>>> suits.remove('string')
>>> suits.append('cup')
>>> suits.extend(['sword', 'club'])
>>> suits[2] = 'spade'
>>> suits
['coin', 'cup', 'spade', 'club']
>>> suits[0:2] = ['diamond']
>>> suits
['diamond', 'spade', 'club']
>>> suits.insert(0, 'heart')
>>> suits
['heart', 'diamond', 'spade', 'club']
```

Remove and return the last element

Remove a value

Add all values

Replace a slice with values

Add an element at an index

Identity:
<exp0> **is** <exp1>
evaluates to True if both <exp0> and <exp1> evaluate to the same object
Equality:
<exp0> == <exp1>
evaluates to True if both <exp0> and <exp1> evaluate to equal values
Identical objects are always equal values

You can **copy** a list by calling the list constructor or slicing the list from the beginning to the end.

Constants: Constant terms do not affect the order of growth of a process
$\Theta(n)$ $\Theta(500 \cdot n)$ $\Theta(\frac{1}{500} \cdot n)$

Logarithms: The base of a logarithm does not affect the order of growth of a process
$\Theta(\log_2 n)$ $\Theta(\log_{10} n)$ $\Theta(\ln n)$

Nesting: When an inner process is repeated for each step in an outer process,multiply the steps in the outer and inner processes to find the total number of steps

```
def overlap(a, b):
    count = 0
    for item in a:
        if item in b:
            count += 1
    return count
```

Outer: length of a

Inner: length of b

If a and b are both length **n**, then overlap takes $\Theta(n^2)$ steps

Lower-order terms: The fastest-growing part of the computation dominates the total
$\Theta(n^2)$ $\Theta(n^2+n)$ $\Theta(n^2+500 \cdot n+\log_2 n+1000)$

iter(iterable):
  Return an iterator over the elements of an iterable value
next(iterator):
  Return the next element

```
>>> s = [3, 4, 5]
>>> t = iter(s)
>>> next(t)
3
>>> next(t)
4
```

```
>>> d = {'one': 1, 'two': 2, 'three': 3}
>>> k = iter(d)
>>> next(k)
'one'
>>> next(k)
'two'
```

```
>>> v = iter(d.values())
>>> next(v)
1
>>> next(v)
2
```

*A generator function is a function that **yield**s values instead of **return**ing them.*

```
>>> def plus_minus(x):
...     yield x
...     yield -x
```

```
>>> t = plus_minus(3)
>>> next(t)
3
>>> next(t)
-3
```

```
def a_then_b(a, b):
    yield from a
    yield from b
>>> list(a_then_b([3, 4], [5, 6]))
[3, 4, 5, 6]
```

x = 2

Status	Effect
•No nonlocal statement •"x" **is not** bound locally	Create a new binding from name "x" to number 2 in the first frame of the current environment
•No nonlocal statement •"x" **is** bound locally	Re-bind name "x" to object 2 in the first frame of the current environment
•nonlocal x •"x" **is** bound in a non-local frame	Re-bind "x" to 2 in the first non-local frame of the current environment in which "x" is bound
•nonlocal x •"x" **is not** bound in a non-local frame	SyntaxError: no binding for nonlocal 'x' found
•nonlocal x •"x" **is** bound in a non-local frame •"x" also bound locally	SyntaxError: name 'x' is parameter and nonlocal

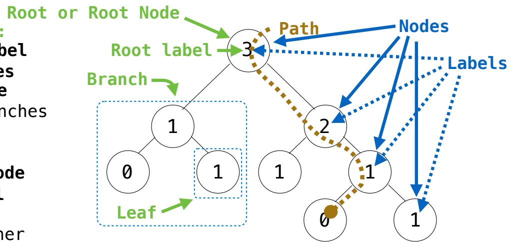
CS 61A Midterm 2 Study Guide — Page 2

Recursive description:

- A **tree** has a root **label** and a list of **branches**
- Each branch is a **tree**
- A tree with zero branches is called a **leaf**

Relative description:

- Each location is a **node**
- Each **node** has a **label**
- One node can be the **parent/child** of another

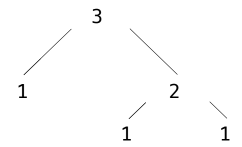


```
def tree(label, branches=[]):
    for branch in branches:
        assert is_tree(branch)
    return [label] + list(branches)
def label(tree):
    return tree[0]
def branches(tree):
    return tree[1:]
def is_tree(tree):
    if type(tree) != list or len(tree) < 1:
        return False
    for branch in branches(tree):
        if not is_tree(branch):
            return False
    return True
def is_leaf(tree):
    return not branches(tree)
def leaves(t):
    """The leaf values in t.
    >>> leaves(fib_tree(5))
    [1, 0, 1, 0, 1, 1, 0, 1]
    """
    if is_leaf(t):
        return [label(t)]
    else:
        return sum([leaves(b) for b in branches(t)], [])
```

Verifies the tree definition

Creates a list from a sequence of branches

Verifies that tree is bound to a list



```
>>> tree(3, [tree(1),
...          tree(2, [tree(1),
...                   tree(1)])])
[3, [1], [2, [1], [1]]]
```

```
def fib_tree(n):
    if n == 0 or n == 1:
        return tree(n)
    else:
        left = fib_tree(n-2),
        right = fib_tree(n-1)
        fib_n = label(left) + label(right)
        return tree(fib_n, [left, right])
```

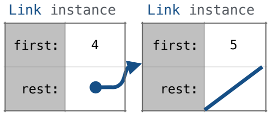
```
class Tree:
    def __init__(self, label, branches=[]):
        self.label = label
        for branch in branches:
            assert isinstance(branch, Tree)
        self.branches = list(branches)

    def is_leaf(self):
        return not self.branches

def leaves(tree):
    "The leaf values in a tree."
    if tree.is_leaf():
        return [tree.label]
    else:
        return sum([leaves(b) for b in tree.branches], [])
```

Built-in isinstance function: returns True if branch has a class that *is* **or** *inherits from* Tree

```
def fib_Tree(n):
    if n == 0 or n == 1:
        return Tree(n)
    else:
        left = fib_Tree(n-2)
        right = fib_Tree(n-1)
        fib_n = left.label+right.label
        return Tree(fib_n,[left, right])
```

```
class Link:
    empty = ()

    def __init__(self, first, rest=empty):
        assert rest is Link.empty or isinstance(rest, Link)
        self.first = first
        self.rest = rest

    def __repr__(self):
        if self.rest:
            rest = ', ' + repr(self.rest)
        else:
            rest = ''
        return 'Link('+repr(self.first)+rest+')'

    def __str__(self):
        string = '<'
        while self.rest is not Link.empty:
            string += str(self.first) + ', '
            self = self.rest
        return string + str(self.first) + '>'
```

Some zero length sequence



```
>>> s = Link(4, Link(5))
>>> s
Link(4, Link(5))
>>> s.first
4
>>> s.rest
Link(5)
>>> print(s)
<4, 5>
>>> print(s.rest)
<5>
>>> s.rest.rest is Link.empty
True
```

Python built-in sets:

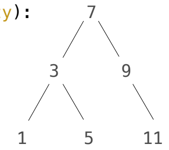
```
>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}
```

```
>>> 3 in s
True
>>> len(s)
4
```

```
>>> s.union({1, 5})
{1, 2, 3, 4, 5}
>>> s.intersection({6, 5, 4, 3})
{3, 4}
```

A binary search tree is a binary tree where each root is **larger** than all values in its left branch and **smaller** than all values in its right branch

```
class BTree(Tree):
    empty = Tree(None)
    def __init__(self, label, left=empty, right=empty):
        Tree.__init__(self, label, [left, right])
    @property
    def left(self):
        return self.branches[0]
    @property
    def right(self):
        return self.branches[1]
```



Python object system:

Idea: All bank accounts have a **balance** and an account **holder**; the **Account** class should add those attributes to each of its instances

A new instance is created by calling a class

```
>>> a = Account('Jim')
>>> a.holder
'Jim'
>>> a.balance
0
```

An account instance

balance: 0   holder: 'Jim'

When a class is called:

1. A new instance of that class is created:
2. The `__init__` method of the class is called with the new object as its first argument (named `self`), along with any additional arguments provided in the call expression.

```
class Account:
    def __init__(self, account_holder):
        self.balance = 0
        self.holder = account_holder
    def deposit(self, amount):
        self.balance = self.balance + amount
        return self.balance
    def withdraw(self, amount):
        if amount > self.balance:
            return 'Insufficient funds'
        self.balance = self.balance - amount
        return self.balance
```

`__init__` is called a constructor

self should always be bound to an instance of the Account class or a subclass of Account

Function call: all arguments within parentheses

```
>>> type(Account.deposit)
<class 'function'>
>>> type(a.deposit)
<class 'method'>
```

Method invocation: One object before the dot and other arguments within parentheses

```
>>> Account.deposit(a, 5)
10
>>> a.deposit(2)
12
```

Call expression

Dot expression

<expression> . <name>

The <expression> can be any valid Python expression. The <name> must be a simple name.

Evaluates to the value of the attribute looked up by <name> in the object that is the value of the <expression>.

To evaluate a dot expression:

1. Evaluate the <expression> to the left of the dot, which yields the object of the dot expression
2. <name> is matched against the instance attributes of that object; if an attribute with that name exists, its value is returned
3. If not, <name> is looked up in the class, which yields a class attribute value
4. That value is returned unless it is a function, in which case a bound method is returned instead

Assignment statements with a dot expression on their left-hand side affect attributes for the object of that dot expression

- If the object is an instance, then assignment sets an instance attribute
- If the object is a class, then assignment sets a class attribute

Account class attributes

interest: ~~0.02~~ ~~0.04~~ 0.05
(withdraw, deposit, `__init__`)

Instance attributes of jim_account

balance: 0
holder: 'Jim'
interest: 0.08

Instance attributes of tom_account

balance: 0
holder: 'Tom'

```
>>> jim_account = Account('Jim')
>>> tom_account = Account('Tom')
>>> tom_account.interest
0.02
>>> jim_account.interest
0.02
>>> Account.interest = 0.04
>>> tom_account.interest
0.04
>>> jim_account.interest
0.04
```

```
>>> jim_account.interest = 0.08
>>> jim_account.interest
0.08
>>> tom_account.interest
0.04
>>> Account.interest = 0.05
>>> tom_account.interest
0.05
>>> jim_account.interest
0.08
```

```
class CheckingAccount(Account):
    """A bank account that charges for withdrawals."""
    withdraw_fee = 1
    interest = 0.01
    def withdraw(self, amount):
        return Account.withdraw(self, amount + self.withdraw_fee)
                                          or
        return super().withdraw(      amount + self.withdraw_fee)
```

To look up a name in a class:

1. If it names an attribute in the class, return the attribute value.
2. Otherwise, look up the name in the base class, if there is one.

```
>>> ch = CheckingAccount('Tom')  # Calls Account.__init__
>>> ch.interest     # Found in CheckingAccount
0.01
>>> ch.deposit(20)  # Found in Account
20
>>> ch.withdraw(5)  # Found in CheckingAccount
14
```