# ENM 360: Introduction to Data-driven Modeling

## *Lecture #7: Bayesian linear regression*

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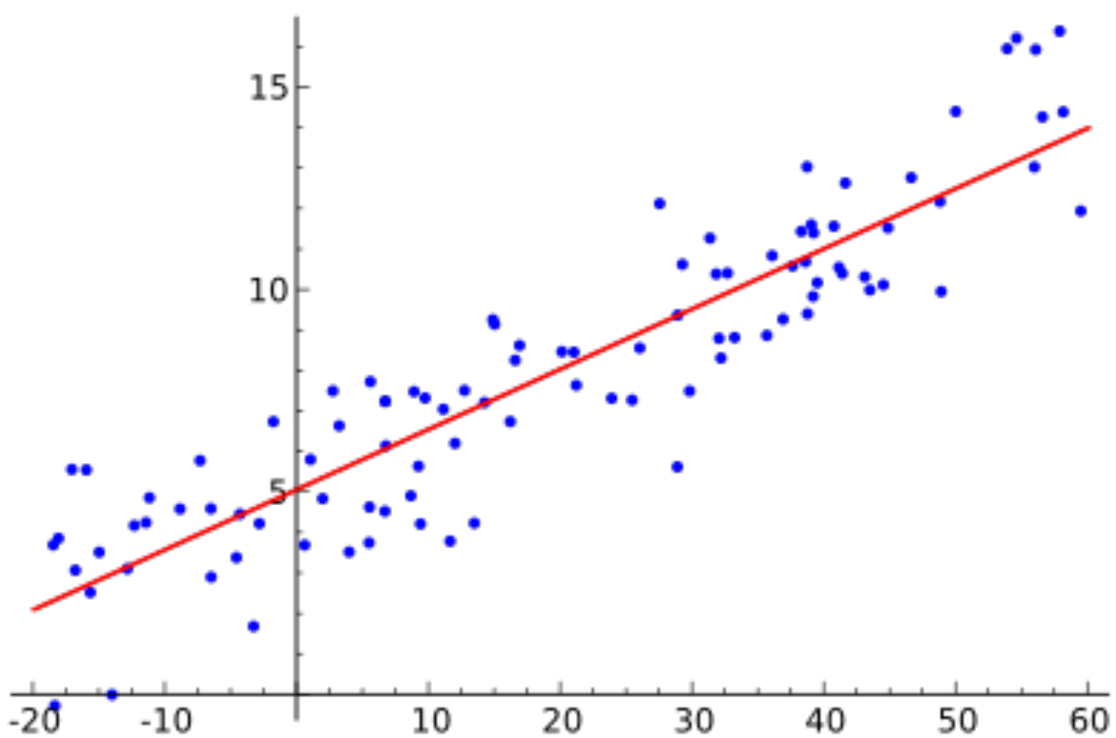
# Linear regression

$$f : \mathcal{X} \rightarrow \mathcal{Y}$$

$$\mathcal{D} = \{\boldsymbol{x}, \boldsymbol{y}\},\ \boldsymbol{x} \in \mathcal{X},\ \boldsymbol{y} \in \mathcal{Y}$$

$$\boldsymbol{y} = f(\boldsymbol{x}) + \epsilon$$

$$f(\boldsymbol{x}) = w^T \boldsymbol{x}$$




*"It's <u>not</u> just about lines and planes!"*

# Linear regression with basis functions

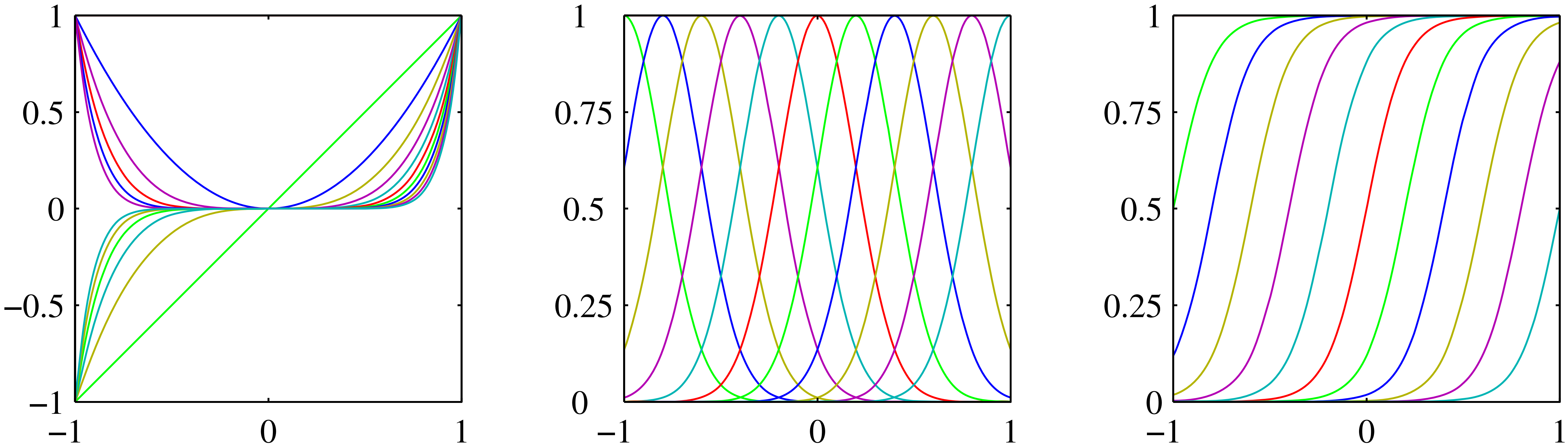
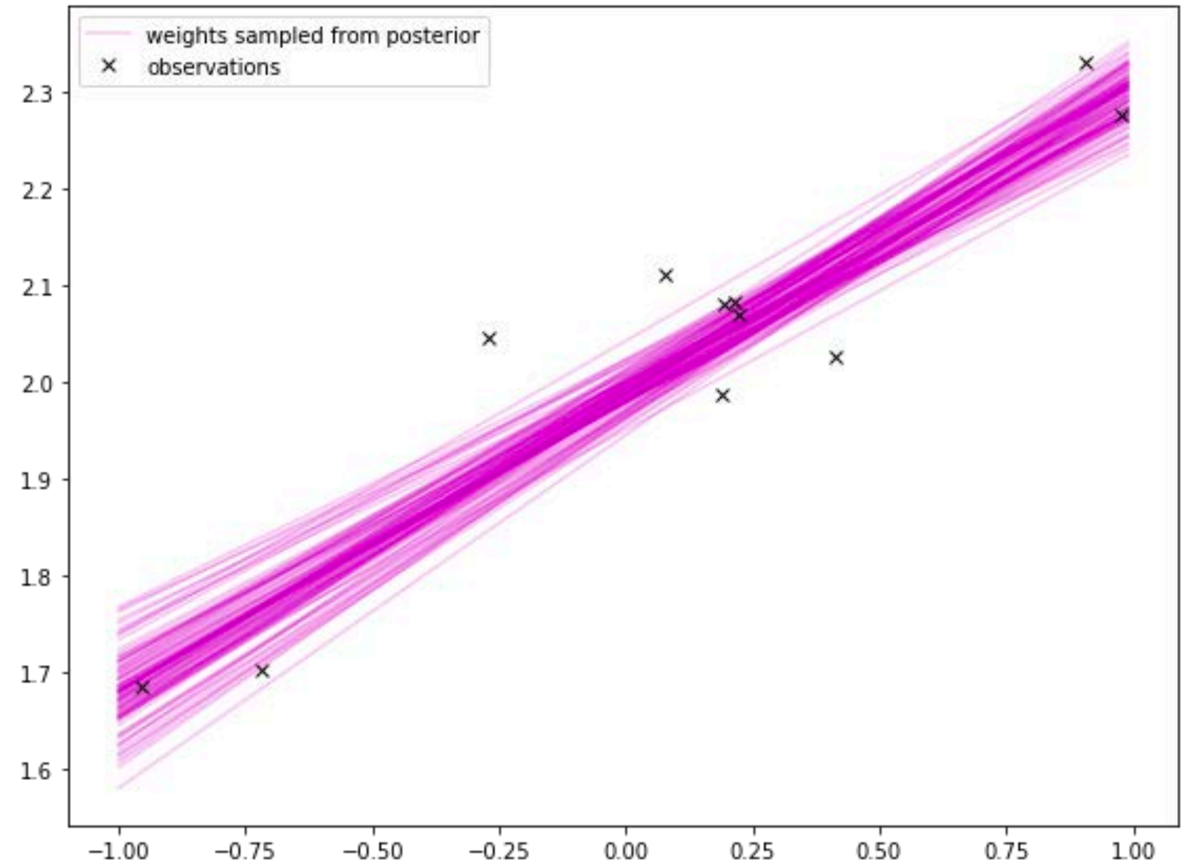
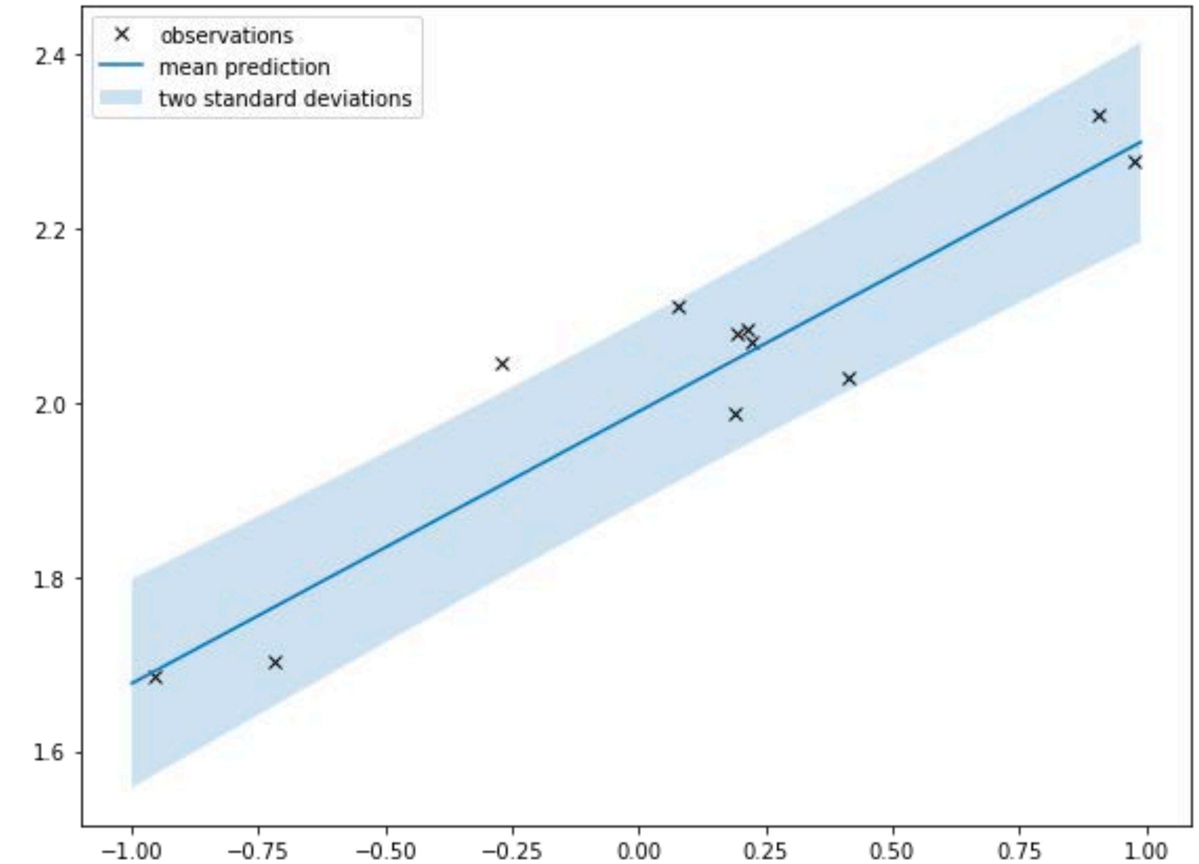
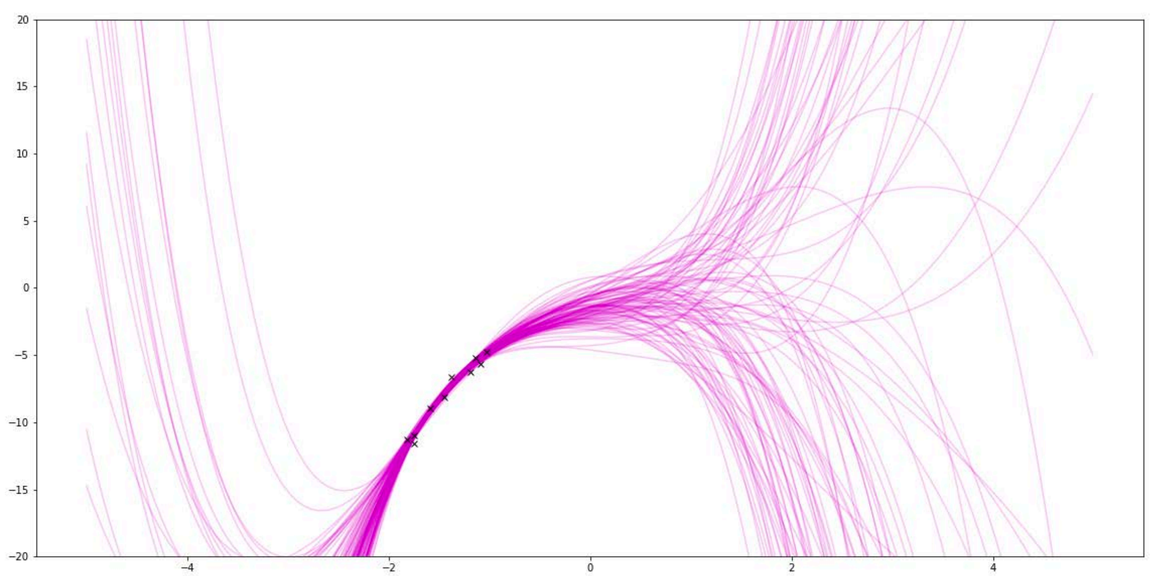


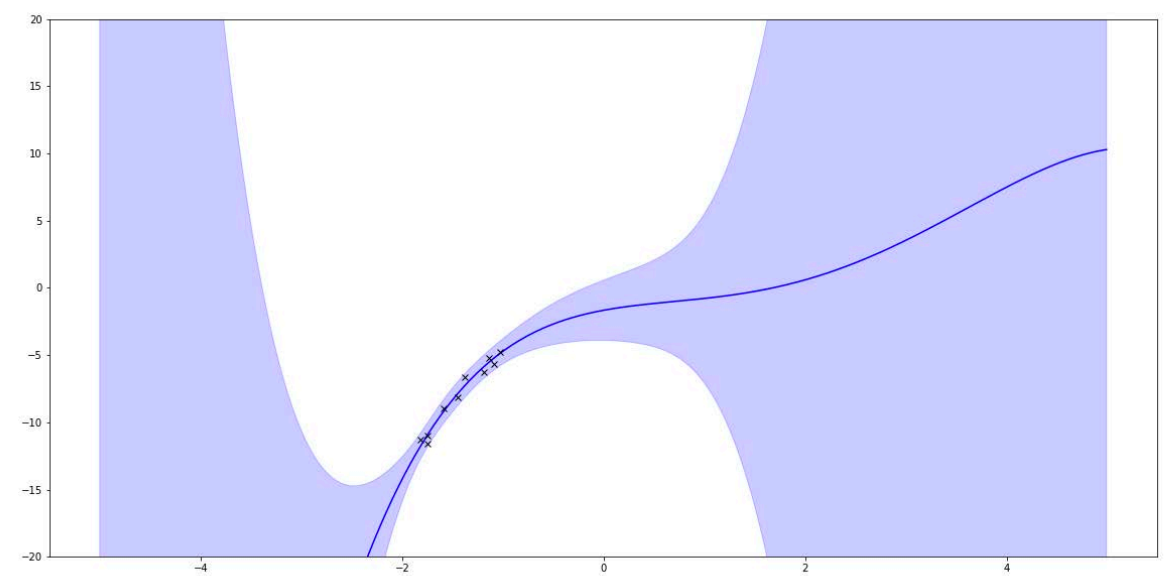

**Figure 3.1** Examples of basis functions, showing polynomials on the left, Gaussians of the form (3.4) in the centre, and sigmoidal of the form (3.5) on the right.

# Bayesian linear regression





# Bayesian linear regression

```python
class BayesianLinearRegression:
  """
    Linear regression model: y = (w.T)*x + \epsilon
    w ~ N(0,beta^(-1)I)
    P(y|x,w) ~ N(y|(w.T)*x,alpha^(-1)I)
  """
  def __init__(self, X, y, alpha = 1.0, beta = 1.0):

      self.X = X
      self.y = y

      self.alpha = alpha
      self.beta = beta

      self.jitter = 1e-8


  def fit_MLE(self):
      xTx_inv = np.linalg.inv(np.matmul(self.X.T,self.X) + self.jitter)
      xTy = np.matmul(self.X.T, self.y)
      w_MLE = np.matmul(xTx_inv, xTy)

      self.w_MLE = w_MLE

      return w_MLE

  def fit_MAP(self):
      Lambda = np.matmul(self.X.T,self.X) + \
               (self.beta/self.alpha)*np.eye(self.X.shape[1])
      Lambda_inv = np.linalg.inv(Lambda)
      xTy = np.matmul(self.X.T, self.y)
      mu = np.matmul(Lambda_inv, xTy)

      self.w_MAP = mu
      self.Lambda_inv = Lambda_inv

      return mu, Lambda_inv

  def predictive_distribution(self, X_star):
      mean_star = np.matmul(X_star, self.w_MAP)
      var_star = 1.0/self.alpha + \
                 np.matmul(X_star, np.matmul(self.Lambda_inv, X_star.T))
      return mean_star, var_star
```