

Section 5.5 Permutations and combinations

Math 1300

Fall 2019

Example 1:

1. Consider the letters in the set $\{a, b, c\}$. How many different *strings* of two *distinct* letters can be formed?
2. A construction crew has three members with names A , B , and C . How many *different* two-person teams can be formed from this crew?

Solution:

In this example, we have to consider two different kinds of situations: the case when order matters and the case when order does not matter.

Permutations

A of n objects taken r at a time is an arrangement of r of the n objects in a specific order. We will denote the number of permutations by $P(n, r)$.

Example 2:

Consider a baseball team of 9 players forming lines of 3, 6, and 9 players. How many different ways can each of these types of lines be formed?

Solution:

Permutation formula

The number of permutations of n objects taken r at a time, $P(n, r)$, is given by

$$P(n, r) = n(n - 1)(n - 2) \cdots (n - r + 1) = \frac{n!}{(n - r)!}.$$

Example 3:

Compute the following:

1. $P(100, 2)$
2. $P(6, 4)$
3. $P(5, 5)$

Solution:

Putting this product into the calculator is very tedious! So of course there is a way to plug it in faster.

Permutations on a calculator

You can find the number of permutations, $P(n, r)$, using a calculator by the sequence of keys

$$\boxed{n} \boxed{nPr} \boxed{r} \boxed{=}$$

Then for $P(9, 4)$, you would press

$$\boxed{9} \boxed{nPr} \boxed{4} \boxed{=}$$

to get 3024.

Note: When asked how many ways you can order n objects, the answer is

$$P(n, n) = n!$$

Combinations

A of n objects taken r at a time is a selection of r objects from among the n , with order disregarded. The number of combinations is given by $C(n, r)$. The formula is given by

Combination formula

The number of combinations of n objects taken r at a time is

$$C(n, r) = \frac{P(n, r)}{r!} = \frac{n(n-1)(n-2) \cdots (n-r+1)}{r!}$$

or

$$C(n, r) = \frac{n!}{r!(n-r)!}.$$

Example 4:

Compute the following:

1. $C(100, 2)$
2. $C(6, 4)$
3. $C(5, 5)$

Solution:

Yet again there is a way to calculate this in one move:

Combinations on a calculator

You can find the number of permutations, $C(n, r)$, using a calculator by the sequence of keys

$$\boxed{n} \boxed{nCr} \boxed{r} \boxed{=}$$

Applying the permutation and combination formulas

Example 5:

A high school student decided to apply to four of the eight Ivy League colleges. In how many possible ways can the four colleges be selected?

Solution:

Example 6:

A board of directors has 10 members.

1. In how many ways can a committee of 3 be chosen?
2. In how many ways can a chairperson, vice chairperson, and secretary be chosen?

Solution:

Example 7:

If 8 horses are entered in a horse race how many different 1st, 2nd, 3rd place finishes are possible?

Solution:

Example 8:

A political pollster wishes to draw a sample of 1500 individuals from among a population of 5,000,000 individuals.

Solution:

Example 9:

Three couples go to a movie together. In how many ways can they be seated in 6 seats so that each couple is seated together?

Solution:

Example 10:

At a benefit concert, 6 bands have volunteered to perform but there is only enough time for four of the bands to play. How many lineups are possible?

Solution: