Instituto Superior Universitario Tecnológico del Azuay
Tecnología Superior en Big Data

Taller de ejercicios - Límites

Alumno:

Eduardo Mendieta

Materia:

Matemática

Docente:

Lcda. Vilma Duchi, Mgtr.

Ciclo:

Primer ciclo

Fecha:

09/08/2024

Periodo Académico:

Abril 2024 - Agosto 2024

Taller de ejercicios - Límites

Resolver los siguientes ejercicios:

---

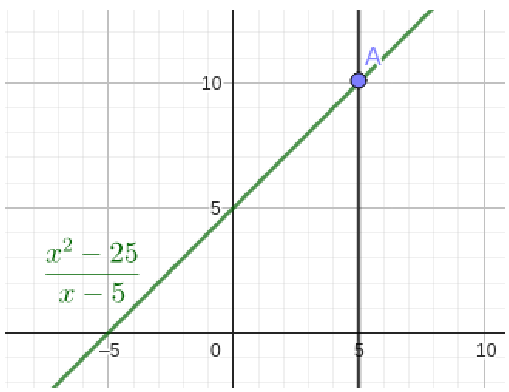
1. **Estime el valor del límite haciendo una tabla de valores, compruebe su trabajo con una gráfica:**

1)

$$\lim_{x \to 5} \frac{x^2 - 25}{x - 5} \approx 10$$

- $\frac{4{,}9^2-25}{4{,}9-5} = 9{,}9$ $\quad \frac{4{,}99^2-25}{4{,}99-5} = 9{,}99$ $\quad \frac{4{,}999^2-25}{4{,}999-5} = 9{,}999$
- $\frac{5{,}001^2-25}{5{,}001-5} = 10{,}001$ $\quad \frac{5{,}01^2-25}{5{,}01-5} = 10{,}01$ $\quad \frac{5{,}1^2-25}{5{,}1-5} = 10{,}1$

x	4.9	4.99	4.999	**5**	5.001	5.01	5.1
$f(x)$	9.9	9.99	9.999	*10*	10.001	10.01	10.1

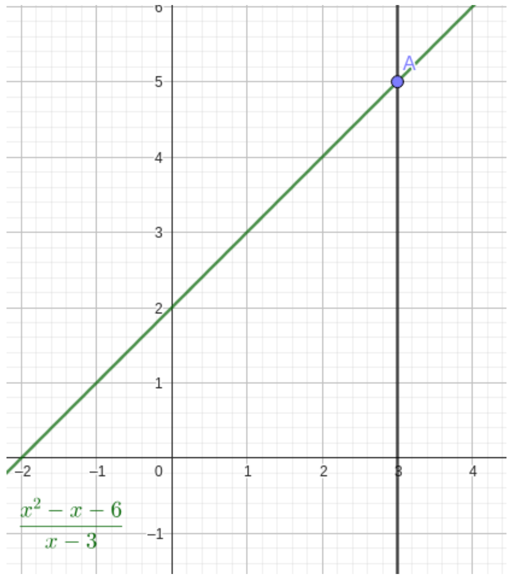


2)

$$\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3} \approx 5$$

- $\frac{2{,}9^2-2{,}9-6}{2{,}9-3} = 4{,}9$ $\quad \frac{2{,}99^2-2{,}99-6}{2{,}99-3} = 4{,}99$ $\quad \frac{2{,}999^2-2{,}999-6}{2{,}999-3} = 4{,}999$
- $\frac{3{,}001^2-3{,}001-6}{3{,}001-3} = 5{,}001$ $\quad \frac{3{,}01^2-3{,}01-6}{3{,}01-3} = 5{,}01$ $\quad \frac{3{,}1^2-3{,}1-6}{3{,}1-3} = 5{,}1$

x	2.9	2.99	2.999	**3**	3.001	3.01	3.1
$f(x)$	4.9	4.99	4.999	*5*	5.001	5.01	5.1



---

2. Complete la tabla de valores (a cinco lugares decimales), y use la tabla para estimar el valor del límite:

1)

$$\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4} \approx 0{,}25$$

- $\frac{\sqrt{3{,}9}-2}{3{,}9-4} = 0{,}252$ $\quad \frac{\sqrt{3{,}99}-2}{3{,}99-4} = 0{,}25$ $\quad \frac{\sqrt{3{,}999}-2}{3{,}999-4} = 0{,}25$
  $\frac{\sqrt{3{,}9999}-2}{3{,}9999-4} = 0{,}25$ $\quad \frac{\sqrt{3{,}99999}-2}{3{,}99999-4} = 0{,}25$
- $\frac{\sqrt{4{,}00001}-2}{4{,}00001-4} = 0{,}25$ $\quad \frac{\sqrt{4{,}0001}-2}{4{,}0001-4} = 0{,}25$ $\quad \frac{\sqrt{4{,}001}-2}{4{,}001-4} = 0{,}25$
  $\frac{\sqrt{4{,}01}-2}{4{,}01-4} = 0{,}25$ $\quad \frac{\sqrt{4{,}1}-2}{4{,}1-4} = 0{,}248$

x	3.9	3.99	3.999	3.9999	3.99999	**4**	4.00001	4.0001	4.001	4.01	4.1
$f(x)$	0.252	0.25	0.25	0.25	0.25	*0.25*	0.25	0.25	0.25	0.25	0.248

2)

$$\lim_{x \to 2} \frac{x - 2}{x^2 + x - 6} \approx 0{,}2$$

- $\frac{1{,}9-2}{1{,}9^2+1{,}9-6} = 0{,}204$ $\quad \frac{1{,}99-2}{1{,}99^2+1{,}99-6} = 0{,}2$ $\quad \frac{1{,}999-2}{1{,}999^2+1{,}999-6} = 0{,}2$
  $\frac{1{,}9999-2}{1{,}9999^2+1{,}9999-6} = 0{,}2$ $\quad \frac{1{,}99999-2}{1{,}99999^2+1{,}99999-6} = 0{,}2$
- $\frac{2{,}00001-2}{2{,}00001^2+2{,}00001-6} = 0{,}2$ $\quad \frac{2{,}0001-2}{2{,}0001^2+2{,}0001-6} = 0{,}2$ $\quad \frac{2{,}001-2}{2{,}001^2+2{,}001-6} = 0{,}2$
  $\frac{2{,}01-2}{2{,}01^2+2{,}01-6} = 0{,}2$ $\quad \frac{2{,}1-2}{2{,}1^2+2{,}1-6} = 0{,}196$

x	1.9	1.99	1.999	1.9999	1.99999	**2**	2.00001	2.0001	2.001	2.01	2.1
$f(x)$	0.204	0.2	0.2	0.2	0.2	*0.2*	0.2	0.2	0.2	0.2	0.196

3)

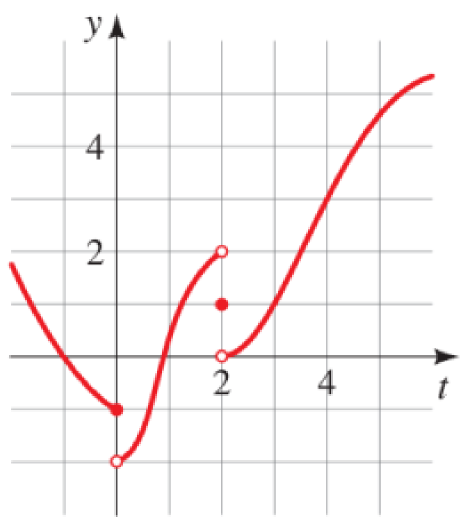
$$\lim_{x\to 1} \frac{x-1}{x^3-1} \approx 0{,}333$$

- $\frac{0{,}9-1}{0{,}9^3-1} = 0{,}369$ $\frac{0{,}99-1}{0{,}99^3-1} = 0{,}337$ $\frac{0{,}999-1}{0{,}999^3-1} = 0{,}334$
  $\frac{0{,}9999-1}{0{,}9999^3-1} = 0{,}333$ $\frac{0{,}99999-1}{0{,}99999^3-1} = 0{,}333$
- $\frac{1{,}00001-1}{1{,}00001^3-1} = 0{,}333$ $\frac{1{,}0001-1}{1{,}0001^3-1} = 0{,}333$ $\frac{1{,}001-1}{1{,}001^3-1} = 0{,}333$
  $\frac{1{,}01-1}{1{,}01^3-1} = 0{,}33$ $\frac{1{,}1-1}{1{,}1^3-1} = 0{,}302$

x	0.9	0.99	0.999	0.9999	0.99999	**1**	1.00001	1.0001	1.001	1.01	1.1
$f(x)$	0.369	0.337	0.334	0.333	0.333	*0.333*	0.333	0.333	0.333	0.33	0.302

---

3. Para la función f cuya gráfica nos dan, exprese el valor de la cantidad dada si existe; si no existe, explique por qué:



a.

$$\lim_{t\to 0^-} g(t) \approx -1$$

b.

$$\lim_{t\to 0^+} g(t) \approx -2$$

c.

$$\lim_{t\to 0} g(t)$$

El límite no existe porque

$$\lim_{t\to 0^-} g(t) \neq \lim_{t\to 0^+} g(t)$$

d.

$$\lim_{t\to 2^-} g(t) \approx 2$$

e.

$$\lim_{t\to 2^+} g(t) \approx 0$$

f.

$$\lim_{t\to 2} g(t)$$

El límite no existe porque

$$\lim_{t\to 2^-} g(t) \neq \lim_{t\to 2^+} g(t)$$

g.

$$g(2) = 1$$

h.

$$\lim_{t\to 4} g(t) \approx 3$$

---

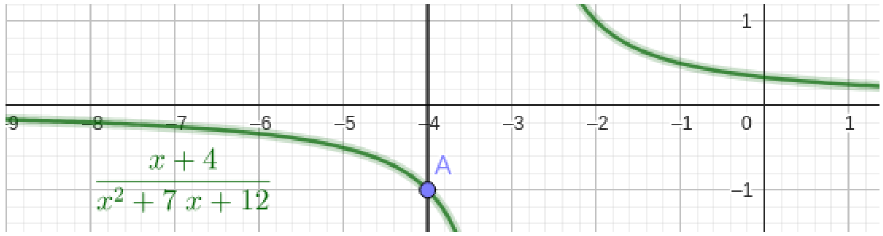
4. Use la tabla de valores para estimar el valor del límite. A continuación, use una calculadora gráfica para confirmar gráficamente sus resultados:

1)

$$\lim_{x\to -4} \frac{x+4}{x^2+7x+12} \approx -1$$

- $\frac{-4{,}1+4}{(-4{,}1)^2+7(-4{,}1)+12} = -0{,}909$ $\quad\frac{-4{,}01+4}{(-4{,}01)^2+7(-4{,}01)+12} = -0{,}99$ $\quad\frac{-4{,}001+4}{(-4{,}001)^2+7(-4{,}001)+12} = -0{,}999$
- $\frac{-3{,}999+4}{(-3{,}999)^2+7(-3{,}999)+12} = -1{,}001$ $\quad\frac{-3{,}99+4}{(-3{,}99)^2+7(-3{,}99)+12} = -1{,}01$ $\quad\frac{-3{,}9+4}{(-3{,}9)^2+7(-3{,}9)+12} = -1{,}111$

x	-4.1	-4.01	-4.001	**-4**	-3.999	-3.99	-3.9
$f(x)$	-0.909	-0.99	-0.999	*-1*	-1.001	-1.01	-1.111

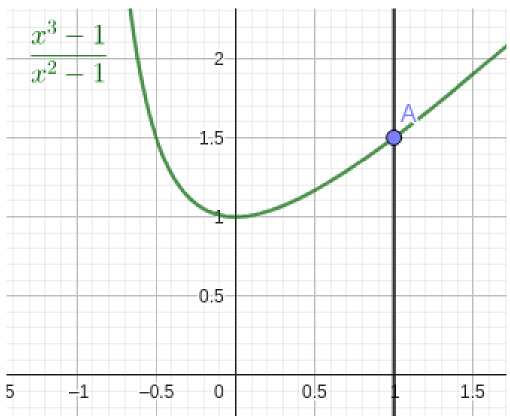


2)

$$\lim_{x\to 1}\frac{x^3-1}{x^2-1}\approx 1{,}5$$

- $\frac{0{,}9^3-1}{0{,}9^2-1}=1{,}426$ $\quad\frac{0{,}99^3-1}{0{,}99^2-1}=1{,}493$ $\quad\frac{0{,}999^3-1}{0{,}999^2-1}=1{,}499$
- $\frac{1{,}001^3-1}{1{,}001^2-1}=1{,}501$ $\quad\frac{1{,}01^3-1}{1{,}01^2-1}=1{,}508$ $\quad\frac{1{,}1^3-1}{1{,}1^2-1}=1{,}576$

x	0.9	0.99	0.999	**1**	1.001	1.01	1.1
$f(x)$	1.426	1.493	1.499	*1.5*	1.501	1.508	1.576

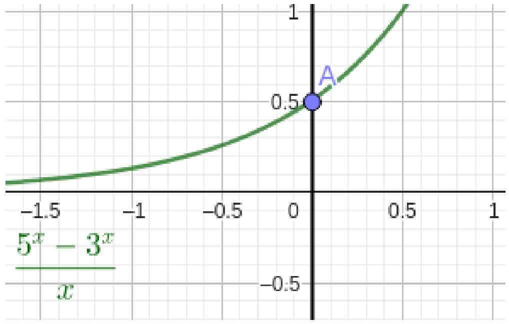


3)

$$\lim_{x\to 0}\frac{5^x-3^x}{x}\approx 0{,}5$$

- $\frac{5^{-0{,}1}-3^{-0{,}1}}{-0{,}1}=0{,}446$ $\quad\frac{5^{-0{,}01}-3^{-0{,}01}}{-0{,}01}=0{,}504$ $\quad\frac{5^{-0{,}001}-3^{-0{,}001}}{-0{,}001}=0{,}51$
- $\frac{5^0{,}001-3^0{,}001}{0{,}001}=0{,}512$ $\quad\frac{5^0{,}01-3^0{,}01}{0{,}01}=0{,}518$ $\quad\frac{5^0{,}1-3^0{,}1}{0{,}1}=0{,}585$

x	-0.1	-0.01	-0.001	**0**	0.001	0.01	0.1
$f(x)$	0.446	0.504	0.51	*0.5*	0.512	0.518	0.585

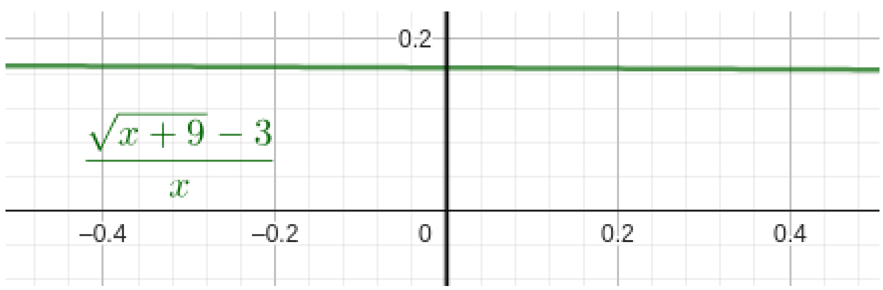


4)

$$\lim_{x \to 0} \frac{\sqrt{x+9}-3}{x} \approx 0{,}17$$

- $\frac{\sqrt{-0{,}1+9}-3}{-0{,}1} = 0{,}167$ $\quad$ $\frac{\sqrt{-0{,}01+9}-3}{-0{,}01} = 0{,}167$ $\quad$ $\frac{\sqrt{-0{,}001+9}-3}{-0{,}001} = 0{,}167$
- $\frac{\sqrt{0{,}001+9}-3}{0{,}001} = 0{,}167$ $\quad$ $\frac{\sqrt{0{,}01+9}-3}{0{,}01} = 0{,}167$ $\quad$ $\frac{\sqrt{0{,}1+9}-3}{0{,}1} = 0{,}166$

x	-0.1	-0.01	-0.001	**0**	0.001	0.01	0.1
$f(x)$	0.167	0.167	0.167	*0.17*	0.167	0.167	0.166



---

5. Evalúe el límite y justifique cada paso al indicar las leyes de límites apropiadas:

1)

$$\lim_{x \to 4} (5x^2 - 2x + 3)$$

Por la ley de la suma/resta:

$$\lim_{x \to 4} 5x^2 - \lim_{x \to 4} 2x + \lim_{x \to 4} 3$$

Evaluando los límites:

$$= 5(4)^2 - 2(4) + 3 = 75$$

2)

$$\boldsymbol{\lim_{x\to 3}(x^3+2)(x^2-5x)}$$

Por la ley del producto:

$$\lim_{x\to 3} x^3+2 \cdot \lim_{x\to 3} x^2-5x$$

Por la ley de la suma/resta:

$$\left(\lim_{x\to 3} x^3 + \lim_{x\to 3} 2\right) \cdot \left(\lim_{x\to 3} x^2 - \lim_{x\to 3} 5x\right)$$

Evaluando los límites:

$$= ((3)^3+2)\cdot((3)^2-5(3)) = -174$$

3)

$$\boldsymbol{\lim_{x\to -1} \frac{x-2}{x^2+4x-3}}$$

Por la ley del cociente:

$$\frac{\lim_{x\to -1} x-2}{\lim_{x\to -1} x^2+4x-3}$$

Por la ley de la suma/resta:

$$\frac{\lim_{x\to -1} x - \lim_{x\to -1} 2}{\lim_{x\to -1} x^2 + \lim_{x\to -1} 4x - \lim_{x\to -1} 3}$$

Evaluando los límites:

$$= \frac{(-1)-2}{(-1)^2+4(-1)-3} = 0{,}5$$

4)

$$\boldsymbol{\lim_{x\to 1}\left(\frac{x^4+x^2-6}{x^4+2x+3}\right)^2}$$

Por la ley de la potencia:

$$\left(\lim_{x\to 1}\frac{x^4+x^2-6}{x^4+2x+3}\right)^2$$

Por la ley del cociente:

$$\left(\frac{\lim_{x\to 1} x^4+x^2-6}{\lim_{x\to 1} x^4+2x+3}\right)^2$$

Por la ley de la suma/resta:

$$\left(\frac{\lim_{x\to 1} x^4 + \lim_{x\to 1} x^2 - \lim_{x\to 1} 6}{\lim_{x\to 1} x^4 + \lim_{x\to 1} 2x + \lim_{x\to 1} 3}\right)^2$$

Evaluando los límites:

$$= \left(\frac{(1)^4+(1)^2-6}{(1)^4+2(1)+3}\right)^2 \approx 0{,}4444$$

6. Evalúe el límite si existe:

1)

$$\lim_{\boldsymbol{x\to 2}} \frac{\boldsymbol{x^2+x-6}}{\boldsymbol{x-2}} = \lim_{x\to 2} \frac{(x+3)(x-2)}{x-2} = \lim_{x\to 2} x+3 = (2)+3 = 5$$

2)

$$\lim_{\boldsymbol{x\to -4}} \frac{\boldsymbol{x^2+5x+4}}{\boldsymbol{x^2+3x-4}} = \lim_{x\to -4} \frac{(x+4)(x+1)}{(x+4)(x-1)} = \lim_{x\to -4} \frac{x+1}{x-1} = \frac{(-4)+1}{(-4)-1} = \frac{-3}{-5} = \frac{3}{5}$$

3)

$$\lim_{\boldsymbol{x\to 2}} \frac{\boldsymbol{x^2-x+6}}{\boldsymbol{x+2}} = \frac{(2)^2-(2)+6}{(2)+2} = \frac{8}{4} = 2$$

4)

$$\lim_{\boldsymbol{x\to 1}} \frac{\boldsymbol{x^3-1}}{\boldsymbol{x^2-1}} = \lim_{x\to 1} \frac{(x-1)(x^2+x+1)}{(x+1)(x-1)} = \lim_{x\to 1} \frac{x^2+x+1}{x+1} = \frac{(1)^2+(1)+1}{(1)+1} = \frac{3}{2}$$

5)

$$\lim_{\boldsymbol{t\to -3}} \frac{\boldsymbol{t^2-9}}{\boldsymbol{2t^2+7t+3}} = \lim_{t\to -3} \frac{(t-3)(t+3)}{(2t+1)(t+3)} = \lim_{t\to -3} \frac{t-3}{2t+1} = \frac{(-3)-3}{2(-3)+1} = \frac{-6}{-5} = \frac{6}{5}$$

6)

$$\lim_{\boldsymbol{h\to 0}} \frac{\boldsymbol{\sqrt{1+h}-1}}{\boldsymbol{h}} = \lim_{h\to 0} \frac{\sqrt{1+h}-1}{h} \cdot \frac{\sqrt{1+h}+1}{\sqrt{1+h}+1} = \lim_{h\to 0} \frac{1+h-1}{h(\sqrt{1+h}+1)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{1+h}+1} = \frac{1}{\sqrt{1+(0)}+1} = \frac{1}{2}$$

7)

$$\lim_{\boldsymbol{h\to 0}} \frac{\boldsymbol{(2+h)^3-8}}{\boldsymbol{h}} = \lim_{h\to 0} \frac{2^3+3(2^2)(h)+3(2)(h^2)+h^3-8}{h}$$

$$= \lim_{h\to 0} \frac{12h+6h^2+h^3}{h} = \lim_{h\to 0} \frac{h(12+6h+h^2)}{h} = \lim_{h\to 0} h^2+6h+12 = (0)^2+6(0)+12 = 12$$

8)

$$\lim_{\boldsymbol{x\to 2}} \frac{\boldsymbol{x^4-16}}{\boldsymbol{x-2}} = \lim_{x\to 2} \frac{(x^2+4)(x^2-4)}{x-2} = \lim_{x\to 2} \frac{(x^2+4)(x+2)(x-2)}{x-2}$$

$$= \lim_{x\to 2}(x^2+4)(x+2) = ((2)^2+4)\cdot((2)+2) = 32$$

9)

$$\lim_{x\to 7}\frac{\sqrt{x+2}-3}{x-7}=\lim_{x\to 7}\frac{\sqrt{x+2}-3}{x-7}\cdot\frac{\sqrt{x+2}+3}{\sqrt{x+2}+3}=\lim_{x\to 7}\frac{x+2-9}{(x-7)(\sqrt{x+2}+3)}$$

$$=\lim_{x\to 7}\frac{x-7}{(x-7)(\sqrt{x+2}+3)}=\lim_{x\to 7}\frac{1}{\sqrt{x+2}+3}=\frac{1}{\sqrt{(7)+2}+3}=\frac{1}{6}$$

10)

$$\lim_{h\to 0}\frac{(3+h)^{-1}-3^{-1}}{h}=\lim_{h\to 0}\frac{(3+h)^{-1}}{h}-\frac{3^{-1}}{h}=\lim_{h\to 0}\frac{1}{h(3+h)}-\frac{1}{3h}$$

$$=\lim_{h\to 0}\frac{1}{3h+h^2}-\frac{1}{3h}=\lim_{h\to 0}\frac{3h-(3h+h^2)}{(3h+h^2)(3h)}=\lim_{h\to 0}\frac{-h^2}{9h^2+3h^3}=\lim_{h\to 0}\frac{-h^2}{3h^2(3+h)}$$

$$\lim_{h\to 0}\frac{-1}{9+3h}=\frac{-1}{9+3(0)}=-\frac{1}{9}$$

11)

$$\lim_{x\to -4}\frac{\frac{1}{4}+\frac{1}{x}}{4+x}=\lim_{x\to -4}\frac{\frac{x+4}{4x}}{4+x}=\lim_{x\to -4}\frac{x+4}{4x(x+4)}=\lim_{x\to -4}\frac{1}{4x}=\frac{1}{4(-4)}=-\frac{1}{16}$$

12)

$$\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2+t}\right)=\lim_{t\to 0}\frac{t^2+t-t}{t(t^2+t)}=\lim_{t\to 0}\frac{t^2}{t^2(t+1)}=\lim_{t\to 0}\frac{1}{t+1}=\frac{1}{(0)+1}=1$$

---

7. Encuentre el límite, si existe. Si el límite no existe, explique por qué:

1)

$$\lim_{x\to -4}|x+4|=|(-4)+4|=|0|=0$$

2)

$$\lim_{x\to -4^-}\frac{|x+4|}{x+4}\approx -1$$

- $\frac{|(-4,1)+4|}{(-4,1)+4}=-1$ $\quad\frac{|(-4,01)+4|}{(-4,01)+4}=-1$ $\quad\frac{|(-4,001)+4|}{(-4,001)+4}=-1$

x	-4.1	-4.01	-4.001	**-4**
$f(x)$	-1	-1	-1	*-1*

3)

$$\lim_{x \to 2} \frac{|x-2|}{x-2}$$

El límite no existe porque:

$$\lim_{x \to 2^-} \frac{|x-2|}{x-2} \neq \lim_{x \to 2^+} \frac{|x-2|}{x-2}$$

- $\frac{|(1,9)-2|}{(1,9)+4} = -1$ $\frac{|(1,99)-2|}{(1,99)-2} = -1$ $\frac{|(1,999)-2|}{(1,999)-2} = -1$
- $\frac{|(2,001)-2|}{(2,001)-2} = 1$ $\frac{|(2,01)-2|}{(2,01)-2} = 1$ $\frac{|(2,1)-2|}{(2,1)-2} = 1$

$\boldsymbol{x}$	1.9	1.99	1.999	**2**	2.001	2.01	2.1
$\boldsymbol{f(x)}$	-1	-1	-1	$\nexists$	1	1	1

4)

$$\lim_{x \to 1,5} \frac{2x^2-3x}{|2x-3|}$$

El límite no existe porque:

$$\lim_{x \to 1,5^-} \frac{2x^2-3x}{|2x-3|} \neq \lim_{x \to 1,5^+} \frac{2x^2-3x}{|2x-3|}$$

- $\frac{2(1,49)^2-3(1,49)}{|2(1,49)-3|} = -1,49$ $\frac{2(1,499)^2-3(1,499)}{|2(1,499)-3|} = -1,499$ $\frac{2(1,4999)^2-3(1,4999)}{|2(1,4999)-3|} = -1,4999$
- $\frac{2(1,5001)^2-3(1,5001)}{|2(1,5001)-3|} = 1,5001$ $\frac{2(1,501)^2-3(1,501)}{|2(1,501)-3|} = 1,501$ $\frac{2(1,51)^2-3(1,51)}{|2(1,51)-3|} = 1,51$

$\boldsymbol{x}$	1.49	1.499	1.4999	**1.5**	1.5001	1.501	1.51
$\boldsymbol{f(x)}$	-1.49	-1.499	-1.499	$\nexists$	1.5001	1.501	1.51

5)

$$\lim_{x \to 0^-} \left(\frac{1}{x} - \frac{1}{|x|}\right) \approx -\infty$$

- $\left(\frac{1}{(-0,1)} - \frac{1}{|-0,1|}\right) = -20$ $\left(\frac{1}{(-0,01)} - \frac{1}{|-0,01|}\right) = -200$ $\left(\frac{1}{(-0,001)} - \frac{1}{|-0,001|}\right) = -2000$

$\boldsymbol{x}$	-0.1	-0.01	-0.001	**0**
$\boldsymbol{f(x)}$	-20	-200	-2000	$-\infty$

6)

$$\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{|x|}\right)\approx 0$$

- $\left(\frac{1}{(0{,}001)}-\frac{1}{|0{,}001|}\right)=0$ $\qquad\left(\frac{1}{(0{,}01)}-\frac{1}{|0{,}01|}\right)=0$

  $\left(\frac{1}{(0{,}1)}-\frac{1}{|0{,}1|}\right)=0$

x	**0**	0.001	0.01	0.1
$f(x)$	*0*	0	0	0

---

8. Sea:

$$f(x)=\begin{cases} x-1 & \text{, si } x<2 \\ x^2-4x+6 & \text{, si } x\geq 2\end{cases}$$

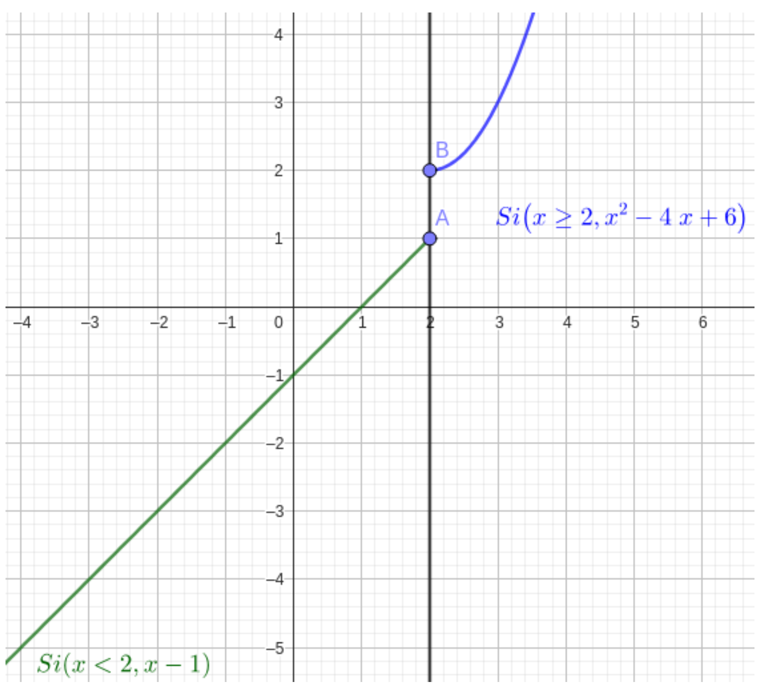
(a) **Encuentre:**

$$\lim_{x\to 2^-} f(x)=\lim_{x\to 2^-} x-1=(2)-1=1$$

$$\lim_{x\to 2^+} f(x)=\lim_{x\to 2^+} x^2-4x+6=(2)^2-4(2)+6=2$$

(b) **¿Existe el $\lim_{x\to 2} f(x)$?** El límite no existe porque $lim_{x\to 2^-}f(x)\neq lim_{x\to 2^+}f(x)$

(c) **Trace la gráfica de f**



9. Sea:

$$\boldsymbol{h(x)} = \begin{cases} \boldsymbol{x} & , \text{ si } \boldsymbol{x < 0} \\ \boldsymbol{x^2} & , \text{ si } \boldsymbol{0 < x \leq 2} \\ \boldsymbol{8 - x} & , \text{ si } \boldsymbol{x > 2} \end{cases}$$

(a) Evalúe cada límite si existe:

(i)

$$\lim_{\boldsymbol{x \to 0^+}} \boldsymbol{h(x)} = \lim_{x \to 0^+} x^2 = (0)^2 = 0$$

(ii)

$$\lim_{\boldsymbol{x \to 0}} \boldsymbol{h(x)} \approx 0$$

$$\lim_{x \to 0^-} x = (0) = 0$$

$$\lim_{x \to 0^-} h(x) = \lim_{x \to 0^+} h(x) = \lim_{x \to 0} h(x) = 0$$

(iii)

$$\lim_{\boldsymbol{x \to 1}} \boldsymbol{h(x)} = \lim_{x \to 1} x^2 = (1)^2 = 1$$

(iv)

$$\lim_{\boldsymbol{x \to 2^-}} \boldsymbol{h(x)} = \lim_{x \to 2^-} x^2 = (2)^2 = 4$$

(v)

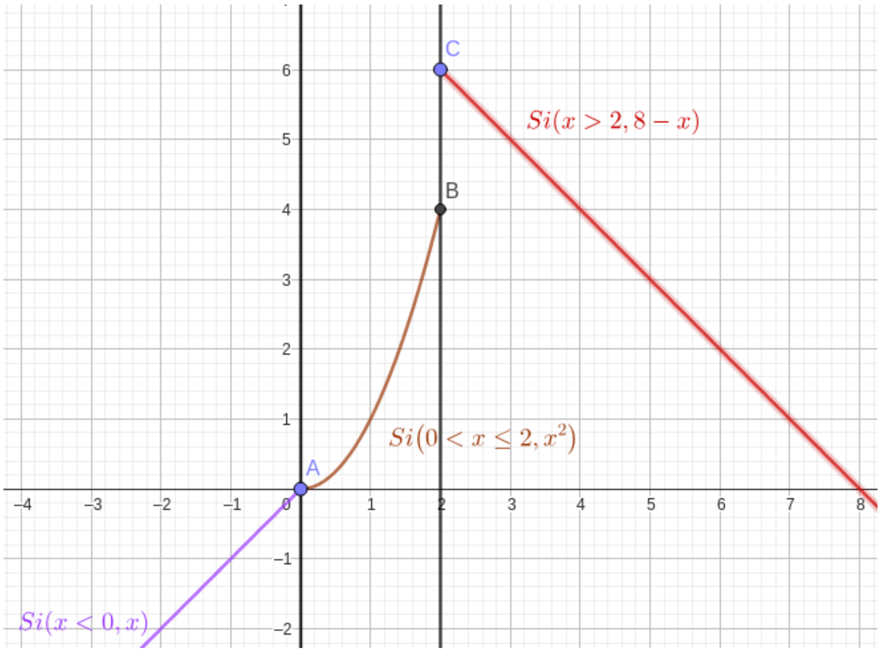
$$\lim_{\boldsymbol{x \to 2^+}} \boldsymbol{h(x)} = \lim_{x \to 2^+} 8 - x = 8 - (2) = 6$$

(vi)

$$\lim_{\boldsymbol{x \to 2}} \boldsymbol{h(x)}$$

El límite no existe porque $lim_{x \to 2^-} h(x) \neq lim_{x \to 2^+} h(x)$

(b) **Trace la gráfica de h**



---

10. Resuelva los siguientes límites al infinito:

1)

$$\lim_{x\to+\infty}\left(\frac{x^3+1}{x-1}-\frac{x}{4}\right)=\lim_{x\to+\infty}\frac{4(x^3+1)-x(x-1)}{4(x-1)}=\lim_{x\to+\infty}\frac{4x^3-x^2+x+4}{4x-4}$$

$$=\frac{4(\infty)^3}{4(\infty)}=\infty$$

2)

$$\lim_{x\to+\infty}\left(4x^2-\sqrt{x^4+1}\right)=\lim_{x\to+\infty}\left(4x^2-\sqrt{x^4+1}\right)\cdot\frac{4x^2+\sqrt{x^4+1}}{4x^2+\sqrt{x^4+1}}$$

$$=\lim_{x\to+\infty}\frac{16x^4-(x^4+1)}{4x^2+\sqrt{x^4+1}}=\lim_{x\to+\infty}\frac{15x^4-1}{4x^2+\sqrt{x^4+1}}=\frac{15(\infty)^4}{4(\infty)^2}=\infty$$

3)

$$\lim_{x\to+\infty}\left(2x-1-\sqrt{4x^2+1}\right)=\lim_{x\to+\infty}-1+2x-\sqrt{4x^2+1}$$

$$=\lim_{x\to+\infty}(-1)+\lim_{x\to+\infty}(2x-\sqrt{4x^2+1})=-1+\lim_{x\to+\infty}\frac{(2x-\sqrt{4x^2+1})(2x+\sqrt{4x^2+1})}{2x+\sqrt{4x^2+1}}$$

$$=-1+\lim_{x\to+\infty}\frac{4x^2-(4x^2+1)}{2x+\sqrt{4x^2+1}}=-1+\lim_{x\to+\infty}-\frac{1}{2x+\sqrt{4x^2+1}}=-1-\frac{1}{\infty}=-1-0=-1$$

4)

$$\lim_{x\to+\infty} \frac{\boldsymbol{5x+8}}{\boldsymbol{-5x+2}} = \frac{5(\infty)}{-5(\infty)} = -1$$

5)

$$\lim_{x\to-\infty} \frac{\boldsymbol{x^2+3x+5}}{\boldsymbol{x^4-x-6}} = \frac{(-\infty)^2}{(-\infty)^4} = 0$$

6)

$$\lim_{x\to+\infty} \frac{\boldsymbol{\sqrt[3]{x^7-4x^3}}}{\boldsymbol{x^2+5x}} = \frac{(\infty)^{\frac{7}{3}}}{(\infty)^2} = \infty$$