



Instituto Superior Universitario Tecnológico del Azuay
Tecnología Superior en Big Data

Taller de ejercicios - Derivadas

Alumno:

Eduardo Mendieta

Materia:

Matemática

Docente:

Lcda. Vilma Duchi, Mgtr.

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Taller de ejercicios - Derivadas

Resolver los siguientes ejercicios:

Actividad N°1:

1)

$$\begin{aligned}f(x) &= (x^3 + 2x)e^x \\f'(x) &= (3x^2 + 2)e^x + (x^3 + 2x)e^x \\f'(x) &= (3x^2e^x + 2e^x) + (x^3e^x + 2xe^x) \\f'(x) &= x^3e^x + 3x^2e^x + 2xe^x + 2e^x\end{aligned}$$

2)

$$\begin{aligned}y &= \frac{x}{e^x} \\y' &= \frac{e^x - xe^x}{(e^x)^2} \\y' &= \frac{e^x(1 - x)}{(e^x)^2} \\y' &= \frac{1 - x}{e^x}\end{aligned}$$

3)

$$\begin{aligned}g(x) &= \frac{1 + 2x}{3 - 4x} \\g'(x) &= \frac{2(3 - 4x) - (1 + 2x)(-4)}{(3 - 4x)^2} \\g'(x) &= \frac{6 - 8x + 4 + 8x}{(3 - 4x)^2} \\g'(x) &= \frac{10}{(3 - 4x)^2}\end{aligned}$$

4)

$$\begin{aligned}H(u) &= (u - \sqrt{u})(u + \sqrt{u}) = (u - u^{1/2})(u + u^{1/2}) \\H'(u) &= \left(1 - \frac{u^{-1/2}}{2}\right)(u + u^{1/2}) + (u - u^{1/2})\left(1 + \frac{u^{-1/2}}{2}\right) \\H'(u) &= \left(1 - \frac{1}{2\sqrt{u}}\right)(u + \sqrt{u}) + (u - \sqrt{u})\left(1 + \frac{1}{2\sqrt{u}}\right) \\H'(u) &= u - \frac{u}{2\sqrt{u}} + \sqrt{u} - \frac{1}{2} + u + \frac{u}{2\sqrt{u}} - \sqrt{u} - \frac{1}{2} \\H'(u) &= 2u - 1\end{aligned}$$

5)

$$J(v) = (v^3 - 2v)(v^{-4} + v^{-2}) = v^{-1} + v - 2v^{-3} - 2v^{-1} = -v^{-1} + v - 2v^{-3}$$

$$J'(v) = v^{-2} + 1 + 6v^{-4}$$

$$J'(v) = \frac{6}{v^4} + \frac{1}{v^2} + 1$$

6)

$$F(y) = \left(\frac{1}{y^2} - \frac{3}{y^4} \right) (y + 5y^3) = \frac{1}{y} + 5y - \frac{3}{y^3} - \frac{15}{y} = 5y - \frac{3}{y^3} - \frac{14}{y}$$

$$F(y) = 5y - 3y^{-3} - 14y^{-1}$$

$$F'(y) = 5 + 9y^{-4} + 14y^{-2}$$

$$F'(y) = \frac{9}{y^4} + \frac{14}{y^2} + 5$$

7)

$$f(z) = (1 - e^z)(z + e^z) = z + e^z - ze^z - e^{2z}$$

$$f'(z) = 1 + e^z - (e^z + ze^z) - (2e^{2z})$$

$$f'(z) = 1 + e^z - e^z - ze^z - 2e^{2z}$$

$$f'(z) = -2e^{2z} - ze^z + 1$$

8)

$$y = \frac{x^3}{1 - x^2}$$

$$y' = \frac{3x^2(1 - x^2) - x^3(-2x)}{(1 - x^2)^2}$$

$$y' = \frac{3x^2 - 3x^4 + 2x^4}{(1 - x^2)^2}$$

$$y' = \frac{3x^2 - x^4}{(1 - x^2)^2}$$

9)

$$y = \frac{t^2 + 2}{t^4 - 3t^2 + 1}$$

$$y' = \frac{2t(t^4 - 3t^2 + 1) - (t^2 + 2)(4t^3 - 6t)}{(t^4 - 3t^2 + 1)^2}$$

$$y' = \frac{2t^5 - 6t^3 + 2t - (4t^5 - 6t^3 + 8t^3 - 12t)}{(t^4 - 3t^2 + 1)^2}$$

$$y' = \frac{2t^5 - 6t^3 + 2t - 4t^5 + 6t^3 - 8t^3 + 12t}{(t^4 - 3t^2 + 1)^2}$$

$$y' = \frac{-2t^5 - 8t^3 + 14t}{(t^4 - 3t^2 + 1)^2}$$

10)

$$\mathbf{y} = \mathbf{e}^p(\mathbf{p} + \mathbf{p}\sqrt{\mathbf{p}}) = e^p(p + pp^{1/2}) = e^p(p + p^{3/2})$$

$$y' = e^p(p + p^{3/2}) + e^p \left(1 + \frac{3p^{1/2}}{2} \right)$$

$$y' = e^p p + e^p p^{3/2} + e^p + \frac{3e^p p^{1/2}}{2}$$

$$y' = e^p p + e^p \sqrt{p^3} + \frac{3e^p \sqrt{p}}{2} + e^p$$

11)

$$\mathbf{y} = \frac{\mathbf{v}^3 - 2\mathbf{v}\sqrt{\mathbf{v}}}{\mathbf{v}} = \frac{v(v^2 - 2v^{1/2})}{v} = v^2 - 2v^{1/2}$$

$$y' = 2v - \frac{1}{\sqrt{v}}$$

12)

$$\mathbf{f}(\mathbf{t}) = \frac{2\mathbf{t}}{2 + \sqrt{\mathbf{t}}}$$

$$f'(t) = \frac{2(2 + t^{1/2}) - 2t(\frac{t^{-1/2}}{2})}{(2 + \sqrt{t})^2}$$

$$f'(t) = \frac{4 + 2t^{1/2} - t^{1/2}}{(2 + \sqrt{t})^2}$$

$$f'(t) = \frac{4 + t^{1/2}}{(2 + \sqrt{t})^2}$$

$$f'(t) = \frac{4 + \sqrt{t}}{(2 + \sqrt{t})^2}$$

13)

$$\mathbf{f}(\mathbf{x}) = \frac{\mathbf{A}}{\mathbf{B} + \mathbf{C}\mathbf{e}^{\mathbf{x}}}$$

$$f'(x) = \frac{-A(Ce^x)}{(B + Ce^x)^2}$$

$$f'(x) = \frac{-ACe^x}{(B + Ce^x)^2}$$

14)

$$\begin{aligned}g(x) &= \sqrt{x}e^x \\g'(x) &= \left(\frac{x^{-1/2}}{2}e^x\right) + x^{1/2}e^x \\g'(x) &= \frac{e^x}{2\sqrt{x}} + e^x\sqrt{x}\end{aligned}$$

15)

$$\begin{aligned}y &= \frac{e^x}{1 - e^x} \\y' &= \frac{e^x(1 - e^x) - e^x(-e^x)}{(1 - e^x)^2} \\y' &= \frac{e^x - e^{2x} + e^{2x}}{(1 - e^x)^2} \\y' &= \frac{e^x}{(1 - e^x)^2}\end{aligned}$$

16)

$$\begin{aligned}G(x) &= \frac{x^2 - 2}{2x + 1} \\G'(x) &= \frac{2x(2x + 1) - 2(x^2 - 2)}{(2x + 1)^2} \\G'(x) &= \frac{4x^2 + 2x - 2x^2 + 4}{(2x + 1)^2} \\G'(x) &= \frac{2x^2 + 2x + 4}{(2x + 1)^2}\end{aligned}$$

17)

$$\begin{aligned}y &= \frac{x + 1}{x^3 + x - 2} \\y' &= \frac{x^3 + x - 2 - (x + 1)(3x^2 + 1)}{(x^3 + x - 2)^2} \\y' &= \frac{x^3 + x - 2 - (3x^3 + x + 3x^2 + 1)}{(x^3 + x - 2)^2} \\y' &= \frac{x^3 + x - 2 - 3x^3 - x - 3x^2 - 1}{(x^3 + x - 2)^2} \\y' &= \frac{-2x^3 - 3x^2 - 3}{(x^3 + x - 2)^2}\end{aligned}$$

18)

$$\begin{aligned}y &= \frac{t}{(t-1)^2} \\y' &= \frac{(t-1)^2 - t(2t-2)}{((t-1)^2)^2} \\y' &= \frac{t^2 - 2t + 1 - 2t^2 + 2t}{(t-1)^4} \\y' &= \frac{-t^2 + 1}{(t-1)^4} \\y' &= \frac{-(t^2 - 1)}{(t-1)^4} \\y' &= \frac{-(t+1)(t-1)}{(t-1)^4} \\y' &= \frac{-t-1}{(t-1)^3}\end{aligned}$$

19)

$$\begin{aligned}y &= \frac{1}{s + ke^s} \\y' &= \frac{-(1 + ke^s)}{(s + ke^s)^2} \\y' &= \frac{-ke^s - 1}{(s + ke^s)^2}\end{aligned}$$

20)

$$\begin{aligned}z &= w^{3/2}(w + ce^w) = w^{5/2} + cw^{3/2}e^w \\z' &= \frac{5w^{3/2}}{2} + c \left(\frac{3w^{1/2}e^w}{2} + w^{3/2}e^w \right) \\z' &= \frac{5\sqrt{w^3}}{2} + \frac{3ce^w\sqrt{w}}{2} + ce^w\sqrt{w^3}\end{aligned}$$

21)

$$\begin{aligned}g(t) &= \frac{t - \sqrt{t}}{t^{1/3}} \\g'(t) &= \frac{\left(1 - \frac{t^{-1/2}}{2}\right)t^{1/3} - (t - t^{1/2})\left(\frac{t^{-2/3}}{3}\right)}{(t^{1/3})^2} \\g'(t) &= \frac{\left(t^{1/3} - \frac{t^{-1/6}}{2}\right) - \left(\frac{t^{1/3}}{3} - \frac{t^{-1/6}}{3}\right)}{t^{2/3}}\end{aligned}$$

$$g'(t) = \frac{t^{1/3} - \frac{t^{-1/6}}{2} - \frac{t^{1/3}}{3} + \frac{t^{-1/6}}{3}}{t^{2/3}}$$

$$g'(t) = \frac{\frac{2t^{1/3}}{3} - \frac{1}{6t^{1/6}}}{t^{2/3}}$$

$$g'(t) = \frac{\frac{4t^{1/2}-1}{6t^{1/6}}}{t^{2/3}}$$

$$g'(t) = \frac{6t^{5/6}}{4t^{1/2} - 1}$$

$$g'(t) = \frac{6\sqrt[6]{t^5}}{4\sqrt{t} - 1}$$

22)

$$f(x) = \frac{1 - xe^x}{x + e^x}$$

$$f'(x) = \frac{-(e^x + xe^x)(x + e^x) - (1 - xe^x)(1 + e^x)}{(x + e^x)^2}$$

$$f'(x) = \frac{-xe^x - e^{2x} - x^2e^x - xe^{2x} - (1 + e^x - xe^x - xe^{2x})}{(x + e^x)^2}$$

$$f'(x) = \frac{-xe^x - e^{2x} - x^2e^x - xe^{2x} - 1 - e^x + xe^x + xe^{2x}}{(x + e^x)^2}$$

$$f'(x) = \frac{-e^xx^2 - e^{2x} - e^x - 1}{(x + e^x)^2}$$

Actividad N°2:

1)

$$f(x) = 3x^2 - 2 \cos x$$

$$f'(x) = 6x + 2 \operatorname{sen} x$$

2)

$$f(x) = \sqrt{x} \operatorname{sen} x$$

$$f'(x) = \frac{1}{2\sqrt{x}} \operatorname{sen} x + \sqrt{x} \cos x$$

3)

$$f(x) = \operatorname{sen} x + \frac{1}{2} \cot x$$

$$f'(x) = \cos x - \frac{\csc^2 x}{2}$$

4)

$$y = 2 \sec x - \csc x$$

$$y' = 2 \sec x \tan x + \csc x \cot x$$

5)

$$y = \sec x \tan x$$

$$y' = \sec x \tan x \tan x + \sec x \sec^2 x$$

$$y' = \sec x \tan^2 x + \sec^3 x$$

6)

$$g(x) = e^x (\tan x - x)$$

$$g'(x) = e^x (\tan x - x) + e^x (\sec^2 x - 1)$$

$$g'(x) = e^x \tan x - e^x x + e^x \sec^2 x - e^x$$

7)

$$y = c \cos t + t^2 \operatorname{sen} t$$

$$y' = -c \operatorname{sen} t + 2t \operatorname{sen} t + t^2 \cos t$$

$$y' = t^2 \cos t + 2t \operatorname{sen} t - c \operatorname{sen} t$$

8)

$$f(t) = \frac{\cot t}{e^t}$$

$$f'(t) = \frac{-\csc^2 t e^t - \cot t e^t}{(e^t)^2}$$

$$f'(t) = \frac{-e^t \csc^2 t - e^t \cot t}{e^{2t}}$$

9)

$$y = \frac{x}{2 - \tan x}$$
$$y' = \frac{2 - \tan x - x(-\sec^2 x)}{(2 - \tan x)^2}$$
$$y' = \frac{2 - \tan x + x \sec^2 x}{(2 - \tan x)^2}$$

10)

$$y = \sin x \cos x$$
$$y' = \cos x \cos x - \sin x \sin x$$
$$y' = \cos^2 x - \sin^2 x$$

11)

$$f(\theta) = \frac{\sec \theta}{1 + \sec \theta}$$

12)

$$y = \frac{\cos x}{1 - \sin x}$$

13)

$$y = \frac{t \sin t}{1 + t}$$

14)

$$y = \frac{1 - \sec x}{\tan x}$$

15)

$$f(x) = x e^x \csc x$$

16)

$$y = x^2 \sin x \tan x$$