Instituto Superior Universitario Tecnológico del Azuay

Tecnología Superior en Big Data

Taller de ejercicios - Derivadas

Alumno:

Eduardo Mendieta

Materia:

Matemática

Docente:

Lcda. Vilma Duchi, Mgtr.

Ciclo:

Primer ciclo

Fecha:

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Taller de ejercicios - Derivadas

Resolver los siguientes ejercicios:

Actividad N°1:

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1)

$$\boldsymbol{f(x) = (x^3 + 2x)e^x}$$
$$f'(x) = (3x^2 + 2)e^x + (x^3 + 2x)e^x$$
$$f'(x) = (3x^2 e^x + 2e^x) + (x^3 e^x + 2xe^x)$$
$$f'(x) = x^3 e^x + 3x^2 e^x + 2xe^x + 2e^x$$

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2)

$$\boldsymbol{y = \frac{x}{e^x}}$$
$$y' = \frac{e^x - xe^x}{(e^x)^2}$$
$$y' = \frac{e^x(1 - x)}{(e^x)^2}$$
$$y' = \frac{1 - x}{e^x}$$

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3)

$$\boldsymbol{g(x) = \frac{1 + 2x}{3 - 4x}}$$
$$g'(x) = \frac{2(3 - 4x) - (1 + 2x)(-4)}{(3 - 4x)^2}$$
$$g'(x) = \frac{6 - 8x + 4 + 8x}{(3 - 4x)^2}$$
$$g'(x) = \frac{10}{(3 - 4x)^2}$$

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4)

$$\boldsymbol{H(u) = (u - \sqrt{u})(u + \sqrt{u})} = (u - u^{1/2})(u + u^{1/2})$$
$$H'(u) = \left(1 - \frac{u^{-1/2}}{2}\right)(u + u^{1/2}) + (u - u^{1/2})\left(1 + \frac{u^{-1/2}}{2}\right)$$
$$H'(u) = \left(1 - \frac{1}{2\sqrt{u}}\right)(u + \sqrt{u}) + (u - \sqrt{u})\left(1 + \frac{1}{2\sqrt{u}}\right)$$
$$H'(u) = u - \frac{u}{2\sqrt{u}} + \sqrt{u} - \frac{1}{2} + u + \frac{u}{2\sqrt{u}} - \sqrt{u} - \frac{1}{2}$$
$$H'(u) = 2u - 1$$

5)

$$\boldsymbol{J(v) = (v^3 - 2v)(v^{-4} + v^{-2})} = v^{-1} + v - 2v^{-3} - 2v^{-1} = -v^{-1} + v - 2v^{-3}$$

$$J'(v) = v^{-2} + 1 + 6v^{-4}$$

$$J'(v) = \frac{6}{v^4} + \frac{1}{v^2} + 1$$

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6)

$$\boldsymbol{F(y) = \left(\frac{1}{y^2} - \frac{3}{y^4}\right)(y + 5y^3)} = \frac{1}{y} + 5y - \frac{3}{y^3} - \frac{15}{y} = 5y - \frac{3}{y^3} - \frac{14}{y}$$

$$F(y) = 5y - 3y^{-3} - 14y^{-1}$$

$$F'(y) = 5 + 9y^{-4} + 14y^{-2}$$

$$F'(y) = \frac{9}{y^4} + \frac{14}{y^2} + 5$$

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7)

$$\boldsymbol{f(z) = (1 - e^z)(z + e^z)} = z + e^z - ze^z - e^{2z}$$

$$f'(z) = 1 + e^z - (e^z + ze^z) - (2e^{2z})$$

$$f'(z) = 1 + e^z - e^z - ze^z - 2e^{2z}$$

$$f'(z) = -2e^{2z} - ze^z + 1$$

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8)

$$\boldsymbol{y = \frac{x^3}{1 - x^2}}$$

$$y' = \frac{3x^2(1 - x^2) - x^3(-2x)}{(1 - x^2)^2}$$

$$y' = \frac{3x^2 - 3x^4 + 2x^4}{(1 - x^2)^2}$$

$$y' = \frac{3x^2 - x^4}{(1 - x^2)^2}$$

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9)

$$\boldsymbol{y = \frac{t^2 + 2}{t^4 - 3t^2 + 1}}$$

$$y' = \frac{2t(t^4 - 3t^2 + 1) - (t^2 + 2)(4t^3 - 6t)}{(t^4 - 3t^2 + 1)^2}$$

$$y' = \frac{2t^5 - 6t^3 + 2t - (4t^5 - 6t^3 + 8t^3 - 12t)}{(t^4 - 3t^2 + 1)^2}$$

$$y' = \frac{2t^5 - 6t^3 + 2t - 4t^5 + 6t^3 - 8t^3 + 12t}{(t^4 - 3t^2 + 1)^2}$$

$$y' = \frac{-2t^5 - 8t^3 + 14t}{(t^4 - 3t^2 + 1)^2}$$

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10)

$$\boldsymbol{y = e^p(p + p\sqrt{p})} = e^p(p + pp^{1/2}) = e^p(p + p^{3/2})$$

$$y' = e^p(p + p^{3/2}) + e^p\left(1 + \frac{3p^{1/2}}{2}\right)$$

$$y' = e^p p + e^p p^{3/2} + e^p + \frac{3e^p p^{1/2}}{2}$$

$$y' = e^p p + e^p\sqrt{p^3} + \frac{3e^p\sqrt{p}}{2} + e^p$$

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11)

$$\boldsymbol{y = \frac{v^3 - 2v\sqrt{v}}{v}} = \frac{v(v^2 - 2v^{1/2})}{v} = v^2 - 2v^{1/2}$$

$$y' = 2v - \frac{1}{\sqrt{v}}$$

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12)

$$\boldsymbol{f(t) = \frac{2t}{2 + \sqrt{t}}}$$

$$f'(t) = \frac{2(2 + t^{1/2}) - 2t(\frac{t^{-1/2}}{2})}{(2 + \sqrt{t})^2}$$

$$f'(t) = \frac{4 + 2t^{1/2} - t^{1/2}}{(2 + \sqrt{t})^2}$$

$$f'(t) = \frac{4 + t^{1/2}}{(2 + \sqrt{t})^2}$$

$$f'(t) = \frac{4 + \sqrt{t}}{(2 + \sqrt{t})^2}$$

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13)

$$\boldsymbol{f(x) = \frac{A}{B + Ce^x}}$$

$$f'(x) = \frac{-A(Ce^x)}{(B + Ce^x)^2}$$

$$f'(x) = \frac{-ACe^x}{(B + Ce^x)^2}$$

14)

$$\boldsymbol{g(x) = \sqrt{x}e^{x}}$$

$$g'(x) = \left(\frac{x^{-1/2}}{2}e^{x}\right) + x^{1/2}e^{x}$$

$$g'(x) = \frac{e^{x}}{2\sqrt{x}} + e^{x}\sqrt{x}$$

15)

$$\boldsymbol{y = \frac{e^{x}}{1 - e^{x}}}$$

$$y' = \frac{e^{x}(1 - e^{x}) - e^{x}(-e^{x})}{(1 - e^{x})^{2}}$$

$$y' = \frac{e^{x} - e^{2x} + e^{2x}}{(1 - e^{x})^{2}}$$

$$y' = \frac{e^{x}}{(1 - e^{x})^{2}}$$

16)

$$\boldsymbol{G(x) = \frac{x^{2} - 2}{2x + 1}}$$

$$G'(x) = \frac{2x(2x + 1) - 2(x^{2} - 2)}{(2x + 1)^{2}}$$

$$G'(x) = \frac{4x^{2} + 2x - 2x^{2} + 4}{(2x + 1)^{2}}$$

$$G'(x) = \frac{2x^{2} + 2x + 4}{(2x + 1)^{2}}$$

17)

$$\boldsymbol{y = \frac{x + 1}{x^{3} + x - 2}}$$

$$y' = \frac{x^{3} + x - 2 - (x + 1)(3x^{2} + 1)}{(x^{3} + x - 2)^{2}}$$

$$y' = \frac{x^{3} + x - 2 - (3x^{3} + x + 3x^{2} + 1)}{(x^{3} + x - 2)^{2}}$$

$$y' = \frac{x^{3} + x - 2 - 3x^{3} - x - 3x^{2} - 1}{(x^{3} + x - 2)^{2}}$$

$$y' = \frac{-2x^{3} - 3x^{2} - 3}{(x^{3} + x - 2)^{2}}$$

18)

$$\boldsymbol{y = \frac{t}{(t-1)^2}}$$

$$y' = \frac{(t-1)^2 - t(2t-2)}{((t-1)^2)^2}$$

$$y' = \frac{t^2 - 2t + 1 - 2t^2 + 2t}{(t-1)^4}$$

$$y' = \frac{-t^2 + 1}{(t-1)^4}$$

$$y' = \frac{-(t^2 - 1)}{(t-1)^4}$$

$$y' = \frac{-(t+1)(t-1)}{(t-1)^4}$$

$$y' = \frac{-t-1}{(t-1)^3}$$

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19)

$$\boldsymbol{y = \frac{1}{s + ke^s}}$$

$$y' = \frac{-(1 + ke^s)}{(s + ke^s)^2}$$

$$y' = \frac{-ke^s - 1}{(s + ke^s)^2}$$

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20)

$$\boldsymbol{z = w^{3/2}(w + ce^w)} = w^{5/2} + cw^{3/2}e^w$$

$$z' = \frac{5w^{3/2}}{2} + c\left(\frac{3w^{1/2}e^w}{2} + w^{3/2}e^w\right)$$

$$z' = \frac{5\sqrt{w^3}}{2} + \frac{3ce^w\sqrt{w}}{2} + ce^w\sqrt{w^3}$$

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21)

$$\boldsymbol{g(t) = \frac{t - \sqrt{t}}{t^{1/3}}}$$

$$g'(t) = \frac{\left(1 - \frac{t^{-1/2}}{2}\right)t^{1/3} - (t - t^{1/2})\left(\frac{t^{-2/3}}{3}\right)}{(t^{1/3})^2}$$

$$g'(t) = \frac{\left(t^{1/3} - \frac{t^{-1/6}}{2}\right) - \left(\frac{t^{1/3}}{3} - \frac{t^{-1/6}}{3}\right)}{t^{2/3}}$$

$$g'(t) = \frac{t^{1/3} - \frac{t^{-1/6}}{2} - \frac{t^{1/3}}{3} + \frac{t^{-1/6}}{3}}{t^{2/3}}$$

$$g'(t) = \frac{\frac{2t^{1/3}}{3} - \frac{1}{6t^{1/6}}}{t^{2/3}}$$

$$g'(t) = \frac{\frac{4t^{1/2}-1}{6t^{1/6}}}{t^{2/3}}$$

$$g'(t) = \frac{6t^{5/6}}{4t^{1/2} - 1}$$

$$g'(t) = \frac{6\sqrt[6]{t^5}}{4\sqrt{t} - 1}$$

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22)

$$\boldsymbol{f(x) = \frac{1 - xe^x}{x + e^x}}$$

$$f'(x) = \frac{-(e^x + xe^x)(x + e^x) - (1 - xe^x)(1 + e^x)}{(x + e^x)^2}$$

$$f'(x) = \frac{-xe^x - e^{2x} - x^2e^x - xe^{2x} - (1 + e^x - xe^x - xe^{2x})}{(x + e^x)^2}$$

$$f'(x) = \frac{-xe^x - e^{2x} - x^2e^x - xe^{2x} - 1 - e^x + xe^x + xe^{2x}}{(x + e^x)^2}$$

$$f'(x) = \frac{-e^x x^2 - e^{2x} - e^x - 1}{(x + e^x)^2}$$

Actividad N°2:

1)

$$\boldsymbol{f(x) = 3x^2 - 2\cos x}$$

$$f'(x) = 6x + 2\operatorname{sen} x$$

2)

$$\boldsymbol{f(x) = \sqrt{x}\operatorname{sen} x}$$

$$f'(x) = \frac{1}{2\sqrt{x}}\operatorname{sen} x + \sqrt{x}\cos x$$

3)

$$\boldsymbol{f(x) = \operatorname{sen} x + \frac{1}{2}\cot x}$$

$$f'(x) = \cos x - \frac{\csc^2 x}{2}$$

4)

$$\boldsymbol{y = 2\sec x - \csc x}$$

$$y' = 2\sec x\tan x + \csc x\cot x$$

5)

$$\boldsymbol{y = \sec x\tan x}$$

$$y' = \sec x\tan x\tan x + \sec x\sec^2 x$$

$$y' = \sec x\tan^2 x + \sec^3 x$$

6)

$$\boldsymbol{g(x) = e^x(\tan x - x)}$$

$$g'(x) = e^x(\tan x - x) + e^x(\sec^2 x - 1)$$

$$g'(x) = e^x\tan x - e^x x + e^x\sec^2 x - e^x$$

7)

$$\boldsymbol{y = c\cos t + t^2\operatorname{sen} t}$$

$$y' = -c\operatorname{sen} t + 2t\operatorname{sen} t + t^2\cos t$$

$$y' = t^2\cos t + 2t\operatorname{sen} t - c\operatorname{sen} t$$

8)

$$\boldsymbol{f(t) = \frac{\cot t}{e^t}}$$

$$f'(t) = \frac{-\csc^2 te^t - \cot te^t}{(e^t)^2}$$

$$f'(t) = \frac{-e^t\csc^2 t - e^t\cot t}{e^{2t}}$$

9)

$$\boldsymbol{y = \frac{x}{2 - \tan x}}$$

$$y' = \frac{2 - \tan x - x(-\sec^2 x)}{(2 - \tan x)^2}$$

$$y' = \frac{2 - \tan x + x\sec^2 x}{(2 - \tan x)^2}$$

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10)

$$\boldsymbol{y = \operatorname{sen} x \cos x}$$

$$y' = \cos x \cos x - \operatorname{sen} x \operatorname{sen} x$$

$$y' = \cos^2 x - \operatorname{sen}^2 x$$

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11)

$$\boldsymbol{f(x) = \frac{\sec x}{1 + \sec x}}$$

$$f'(x) = \frac{\sec x \tan x(1 + \sec x) - \sec x \sec x \tan x}{(1 + \sec x)^2}$$

$$f'(x) = \frac{\sec x \tan x + \sec^2 x \tan x - \sec^2 x \tan x}{(1 + \sec x)^2}$$

$$f'(x) = \frac{\sec x \tan x}{(1 + \sec x)^2}$$

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12)

$$\boldsymbol{y = \frac{\cos x}{1 - \operatorname{sen} x}}$$

$$y' = \frac{-\operatorname{sen} x(1 - \operatorname{sen} x) + \cos x \cos x}{(1 - \operatorname{sen} x)^2}$$

$$y' = \frac{-\operatorname{sen} x + \operatorname{sen}^2 x + \cos^2 x}{(1 - \operatorname{sen} x)^2}$$

$$y' = \frac{1 - \operatorname{sen} x}{(1 - \operatorname{sen} x)^2}$$

$$y' = \frac{1}{1 - \operatorname{sen} x}$$

13)

$$\boldsymbol{y = \frac{t \operatorname{sen} t}{1+t}}$$

$$y' = \frac{(\operatorname{sen} t + t\cos t)(1+t) - t\operatorname{sen} t}{(1+t)^2}$$

$$y' = \frac{\operatorname{sen} t + t\operatorname{sen} t + t\cos t + t^2\cos t - t\operatorname{sen} t}{(1+t)^2}$$

$$y' = \frac{t^2\cos t + t\cos t + \operatorname{sen} t}{(1+t)^2}$$

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14)

$$\boldsymbol{y = \frac{1-\sec x}{\tan x}}$$

$$y' = \frac{-\sec x\tan x\tan x - (1-\sec x)\sec^2 x}{\tan^2 x}$$

$$y' = \frac{-\sec x\tan^2 x - \sec^2 x + \sec^3 x}{\tan^2 x}$$

$$y' = \frac{\sec^3 x - \sec^2 x - \sec x\tan^2 x}{\tan^2 x}$$

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15)

$$\boldsymbol{f(x) = xe^x \csc x}$$

$$f'(x) = (e^x + xe^x)\csc x - xe^x\csc x\cot x$$

$$f'(x) = e^x\csc x + xe^x\csc x - xe^x\csc x\cot x$$

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16)

$$\boldsymbol{y = x^2 \operatorname{sen} x \tan x}$$

$$y' = (2x\operatorname{sen} x + x^2\cos x)\tan x + x^2\operatorname{sen} x\sec^2 x$$

$$y' = \frac{2x\operatorname{sen}^2 x}{\cos x} + \frac{x^2\operatorname{sen} x\cos x}{\cos x} + x^2\operatorname{sen} x\sec^2 x$$

$$y' = \frac{2x\operatorname{sen}^2 x}{\cos x} + x^2\operatorname{sen} x + x^2\operatorname{sen} x\sec^2 x$$