



Instituto Superior Universitario Tecnológico del Azuay
Tecnología Superior en Big Data

Taller de ejercicios - Límites

Alumno:

Eduardo Mendieta

Materia:

Matemática

Docente:

Lcda. Vilma Duchi, Mgtr.

Ciclo:

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Taller de ejercicios - Límites

Resolver los siguientes ejercicios:

1. Estime el valor del límite haciendo una tabla de valores, compruebe su trabajo con una gráfica:

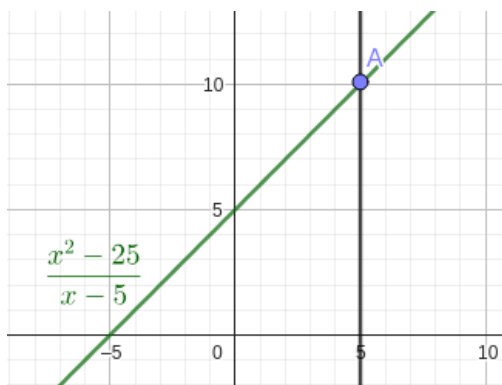
1)

$$\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5} \approx 10$$

■ $\frac{4,9^2 - 25}{4,9 - 5} = 9,9$ $\frac{4,99^2 - 25}{4,99 - 5} = 9,99$ $\frac{4,999^2 - 25}{4,999 - 5} = 9,999$

■ $\frac{5,001^2 - 25}{5,001 - 5} = 10,001$ $\frac{5,01^2 - 25}{5,01 - 5} = 10,01$ $\frac{5,1^2 - 25}{5,1 - 5} = 10,1$

| x | 4.9 | 4.99 | 4.999 | 5 | 5.001 | 5.01 | 5.1 |
|--------|-----|------|-------|----------|--------|-------|------|
| $f(x)$ | 9.9 | 9.99 | 9.999 | 10 | 10.001 | 10.01 | 10.1 |



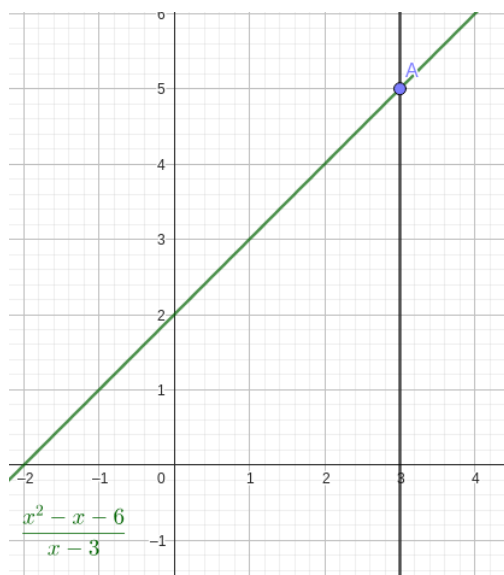
2)

$$\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x - 3} \approx 5$$

■ $\frac{2,9^2 - 2,9 - 6}{2,9 - 3} = 4,9$ $\frac{2,99^2 - 2,99 - 6}{2,99 - 3} = 4,99$ $\frac{2,999^2 - 2,999 - 6}{2,999 - 3} = 4,999$

■ $\frac{3,001^2 - 3,001 - 6}{3,001 - 3} = 5,001$ $\frac{3,01^2 - 3,01 - 6}{3,01 - 3} = 5,01$ $\frac{3,1^2 - 3,1 - 6}{3,1 - 3} = 5,1$

| x | 2.9 | 2.99 | 2.999 | 3 | 3.001 | 3.01 | 3.1 |
|--------|-----|------|-------|----------|-------|------|-----|
| $f(x)$ | 4.9 | 4.99 | 4.999 | 5 | 5.001 | 5.01 | 5.1 |



2. Complete la tabla de valores (a cinco lugares decimales), y use la tabla para estimar el valor del límite:

1)

$$\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4} \approx 0,25$$

$$\begin{array}{lll} \blacksquare \frac{\sqrt{3,9}-2}{3,9-4} = 0,252 & \frac{\sqrt{3,99}-2}{3,99-4} = 0,25 & \frac{\sqrt{3,999}-2}{3,999-4} = 0,25 \\ \frac{\sqrt{3,9999}-2}{3,9999-4} = 0,25 & \frac{\sqrt{3,99999}-2}{3,99999-4} = 0,25 & \\ \blacksquare \frac{\sqrt{4,00001}-2}{4,00001-4} = 0,25 & \frac{\sqrt{4,0001}-2}{4,0001-4} = 0,25 & \frac{\sqrt{4,001}-2}{4,001-4} = 0,25 \\ \frac{\sqrt{4,01}-2}{4,01-4} = 0,25 & \frac{\sqrt{4,1}-2}{4,1-4} = 0,248 & \end{array}$$

| | | | | | | | | | | | |
|--------|-------|------|-------|--------|---------|------|---------|--------|-------|------|-------|
| x | 3.9 | 3.99 | 3.999 | 3.9999 | 3.99999 | 4 | 4.00001 | 4.0001 | 4.001 | 4.01 | 4.1 |
| $f(x)$ | 0.252 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.248 |

2)

$$\lim_{x \rightarrow 2} \frac{x - 2}{x^2 + x - 6} \approx 0,2$$

$$\begin{array}{lll} \blacksquare \frac{1,9-2}{1,9^2+1,9-6} = 0,204 & \frac{1,99-2}{1,99^2+1,99-6} = 0,2 & \frac{1,999-2}{1,999^2+1,999-6} = 0,2 \\ \frac{1,9999-2}{1,9999^2+1,9999-6} = 0,2 & \frac{1,99999-2}{1,99999^2+1,99999-6} = 0,2 & \\ \blacksquare \frac{2,00001-2}{2,00001^2+2,00001-6} = 0,2 & \frac{2,0001-2}{2,0001^2+2,0001-6} = 0,2 & \frac{2,001-2}{2,001^2+2,001-6} = 0,2 \\ \frac{2,01-2}{2,01^2+2,01-6} = 0,2 & \frac{2,1-2}{2,1^2+2,1-6} = 0,196 & \end{array}$$

| | | | | | | | | | | | |
|--------|-------|------|-------|--------|---------|----------|---------|--------|-------|------|-------|
| x | 1.9 | 1.99 | 1.999 | 1.9999 | 1.99999 | 2 | 2.00001 | 2.0001 | 2.001 | 2.01 | 2.1 |
| $f(x)$ | 0.204 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.196 |

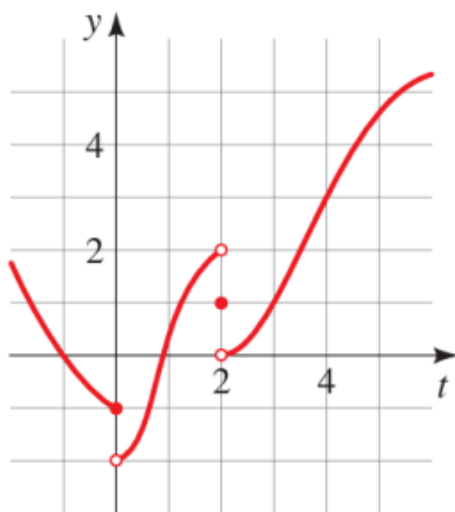
3)

$$\lim_{x \rightarrow 1} \frac{x-1}{x^3-1} \approx 0,333$$

$$\begin{aligned} \blacksquare \quad & \frac{0,9-1}{0,9^3-1} = 0,369 & \frac{0,99-1}{0,99^3-1} = 0,337 & \frac{0,999-1}{0,999^3-1} = 0,334 \\ & \frac{0,9999-1}{0,9999^3-1} = 0,333 & \frac{0,99999-1}{0,99999^3-1} = 0,333 & \\ \blacksquare \quad & \frac{1,00001-1}{1,00001^3-1} = 0,333 & \frac{1,0001-1}{1,0001^3-1} = 0,333 & \frac{1,001-1}{1,001^3-1} = 0,333 \\ & \frac{1,01-1}{1,01^3-1} = 0,33 & \frac{1,1-1}{1,1^3-1} = 0,302 & \end{aligned}$$

| | | | | | | | | | | | |
|--------|-------|-------|-------|--------|---------|----------|---------|--------|-------|------|-------|
| x | 0.9 | 0.99 | 0.999 | 0.9999 | 0.99999 | 1 | 1.00001 | 1.0001 | 1.001 | 1.01 | 1.1 |
| $f(x)$ | 0.369 | 0.337 | 0.334 | 0.333 | 0.333 | 0.333 | 0.333 | 0.333 | 0.333 | 0.33 | 0.302 |

3. Para la función f cuya gráfica nos dan, exprese el valor de la cantidad dada si existe; si no existe, explique por qué:



a.

$$\lim_{t \rightarrow 0^-} g(t) \approx -1$$

b.

$$\lim_{t \rightarrow 0^+} g(t) \approx -2$$

c.

$$\lim_{t \rightarrow 0} g(t)$$

El límite no existe porque

$$\lim_{t \rightarrow 0^-} g(t) \neq \lim_{t \rightarrow 0^+} g(t)$$

d.

$$\lim_{t \rightarrow 2^-} g(t) \approx 2$$

e.

$$\lim_{t \rightarrow 2^+} g(t) \approx 0$$

f.

$$\lim_{t \rightarrow 2} g(t)$$

El límite no existe porque

$$\lim_{t \rightarrow 2^-} g(t) \neq \lim_{t \rightarrow 2^+} g(t)$$

g.

$$g(2) = 1$$

h.

$$\lim_{t \rightarrow 4} g(t) \approx 3$$

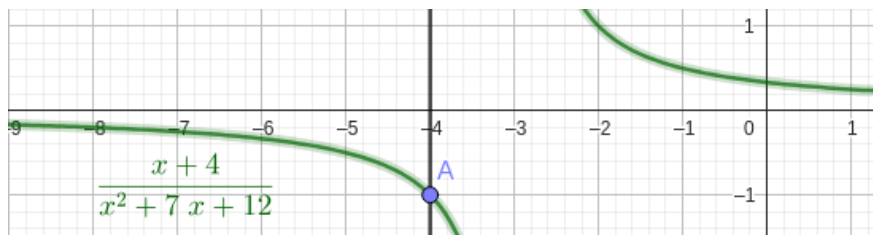
4. Use la tabla de valores para estimar el valor del límite. A continuación, use una calculadora gráfica para confirmar gráficamente sus resultados:

1)

$$\lim_{x \rightarrow -4} \frac{x + 4}{x^2 + 7x + 12} \approx -1$$

$$\begin{aligned} \blacksquare \quad & \frac{-4,1+4}{(-4,1)^2+7(-4,1)+12} = -0,909 & \frac{-4,01+4}{(-4,01)^2+7(-4,01)+12} = -0,99 \\ & \frac{-4,001+4}{(-4,001)^2+7(-4,001)+12} = -0,999 \\ \blacksquare \quad & \frac{-3,999+4}{(-3,999)^2+7(-3,999)+12} = -1,001 & \frac{-3,99+4}{(-3,99)^2+7(-3,99)+12} = -1,01 \\ & \frac{-3,9+4}{(-3,9)^2+7(-3,9)+12} = -1,111 \end{aligned}$$

| | | | | | | | |
|--------------------------|--------|-------|--------|-----------|--------|-------|--------|
| x | -4.1 | -4.01 | -4.001 | -4 | -3.999 | -3.99 | -3.9 |
| $f(x)$ | -0.909 | -0.99 | -0.999 | <i>-1</i> | -1.001 | -1.01 | -1.111 |

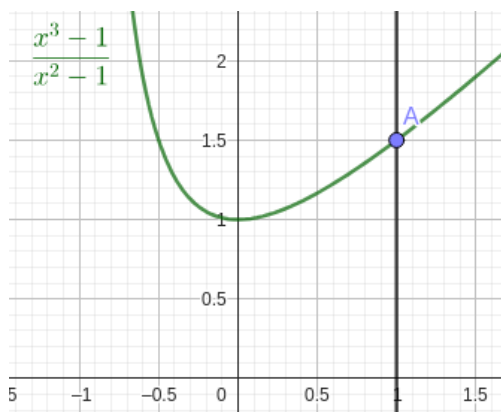


2)

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1} \approx 1,5$$

$$\begin{array}{lll} \blacksquare \frac{0,9^3 - 1}{0,9^2 - 1} = 1,426 & \frac{0,99^3 - 1}{0,99^2 - 1} = 1,493 & \frac{0,999^3 - 1}{0,999^2 - 1} = 1,499 \\ \blacksquare \frac{1,001^3 - 1}{1,001^2 - 1} = 1,501 & \frac{1,01^3 - 1}{1,01^2 - 1} = 1,508 & \frac{1,1^3 - 1}{1,1^2 - 1} = 1,576 \end{array}$$

| x | 0.9 | 0.99 | 0.999 | 1 | 1.001 | 1.01 | 1.1 |
|--------|-------|-------|-------|-----|-------|-------|-------|
| $f(x)$ | 1.426 | 1.493 | 1.499 | 1.5 | 1.501 | 1.508 | 1.576 |

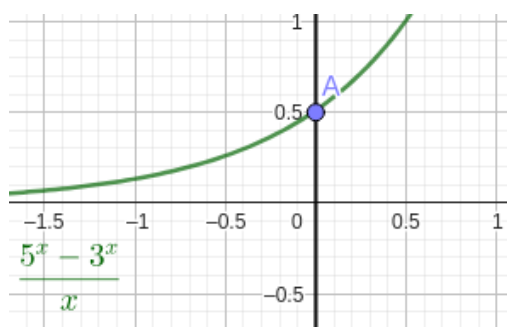


3)

$$\lim_{x \rightarrow 0} \frac{5^x - 3^x}{x} \approx 0,5$$

$$\begin{array}{lll} \blacksquare \frac{5^{-0,1} - 3^{-0,1}}{-0,1} = 0,446 & \frac{5^{-0,01} - 3^{-0,01}}{-0,01} = 0,504 & \frac{5^{-0,001} - 3^{-0,001}}{-0,001} = 0,51 \\ \blacksquare \frac{5^{0,001} - 3^{0,001}}{0,001} = 0,512 & \frac{5^{0,01} - 3^{0,01}}{0,01} = 0,518 & \frac{5^{0,1} - 3^{0,1}}{0,1} = 0,585 \end{array}$$

| | | | | | | | |
|--------|-------|-------|--------|----------|-------|-------|-------|
| x | -0.1 | -0.01 | -0.001 | 0 | 0.001 | 0.01 | 0.1 |
| $f(x)$ | 0.446 | 0.504 | 0.51 | 0.5 | 0.512 | 0.518 | 0.585 |

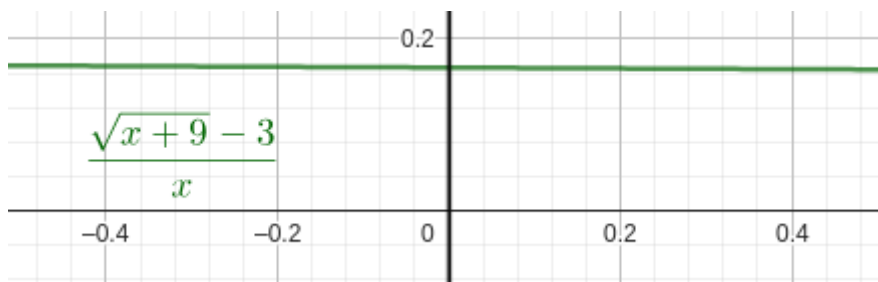


4)

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+9} - 3}{x} \approx 0,17$$

$$\begin{aligned} \blacksquare \frac{\sqrt{-0,1+9}-3}{-0,1} &= 0,167 & \frac{\sqrt{-0,01+9}-3}{-0,01} &= 0,167 & \frac{\sqrt{-0,001+9}-3}{-0,001} &= 0,167 \\ \blacksquare \frac{\sqrt{0,001+9}-3}{0,001} &= 0,167 & \frac{\sqrt{0,01+9}-3}{0,01} &= 0,167 & \frac{\sqrt{0,1+9}-3}{0,1} &= 0,166 \end{aligned}$$

| | | | | | | | |
|--------|-------|-------|--------|----------|-------|-------|-------|
| x | -0.1 | -0.01 | -0.001 | 0 | 0.001 | 0.01 | 0.1 |
| $f(x)$ | 0.167 | 0.167 | 0.167 | 0.17 | 0.167 | 0.167 | 0.166 |



5. Evalúe el límite y justifique cada paso al indicar las leyes de límites apropiadas:

1)

$$\lim_{x \rightarrow 4} (5x^2 - 2x + 3)$$

Por la ley de la suma/resta:

$$\lim_{x \rightarrow 4} 5x^2 - \lim_{x \rightarrow 4} 2x + \lim_{x \rightarrow 4} 3$$

Evaluando los límites:

$$= 5(4)^2 - 2(4) + 3 = 75$$

2)

$$\lim_{x \rightarrow 3} (x^3 + 2)(x^2 - 5x)$$

Por la ley del producto:

$$\lim_{x \rightarrow 3} x^3 + 2 \cdot \lim_{x \rightarrow 3} x^2 - 5x$$

Por la ley de la suma/resta:

$$\left(\lim_{x \rightarrow 3} x^3 + \lim_{x \rightarrow 3} 2 \right) \cdot \left(\lim_{x \rightarrow 3} x^2 - \lim_{x \rightarrow 3} 5x \right)$$

Evaluando los límites:

$$= ((3)^3 + 2) \cdot ((3)^2 - 5(3)) = -174$$

3)

$$\lim_{x \rightarrow -1} \frac{x - 2}{x^2 + 4x - 3}$$

Por la ley del cociente:

$$\frac{\lim_{x \rightarrow -1} x - 2}{\lim_{x \rightarrow -1} x^2 + 4x - 3}$$

Por la ley de la suma/resta:

$$\frac{\lim_{x \rightarrow -1} x - \lim_{x \rightarrow -1} 2}{\lim_{x \rightarrow -1} x^2 + \lim_{x \rightarrow -1} 4x - \lim_{x \rightarrow -1} 3}$$

Evaluando los límites:

$$= \frac{(-1) - 2}{(-1)^2 + 4(-1) - 3} = 0,5$$

4)

$$\lim_{x \rightarrow 1} \left(\frac{x^4 + x^2 - 6}{x^4 + 2x + 3} \right)^2$$

Por la ley de la potencia:

$$\left(\lim_{x \rightarrow 1} \frac{x^4 + x^2 - 6}{x^4 + 2x + 3} \right)^2$$

Por la ley del cociente:

$$\left(\frac{\lim_{x \rightarrow 1} x^4 + \lim_{x \rightarrow 1} x^2 - \lim_{x \rightarrow 1} 6}{\lim_{x \rightarrow 1} x^4 + \lim_{x \rightarrow 1} 2x + \lim_{x \rightarrow 1} 3} \right)^2$$

Por la ley de la suma/resta:

$$\left(\frac{\lim_{x \rightarrow 1} x^4 + \lim_{x \rightarrow 1} x^2 - \lim_{x \rightarrow 1} 6}{\lim_{x \rightarrow 1} x^4 + \lim_{x \rightarrow 1} 2x + \lim_{x \rightarrow 1} 3} \right)^2$$

Evaluando los límites:

$$= \left(\frac{(1)^4 + (1)^2 - 6}{(1)^4 + 2(1) + 3} \right)^2 \approx 0,4444$$

6. Evalúe el límite si existe:

1)

$$\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2}$$

2)

$$\lim_{x \rightarrow -4} \frac{x^2 + 5x + 4}{x^2 + 3x - 4}$$

3)

$$\lim_{x \rightarrow 2} \frac{x^2 - x + 6}{x + 2}$$

4)

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1}$$

5)

$$\lim_{t \rightarrow -3} \frac{t^2 - 9}{2t^2 + 7t + 3}$$

6)

$$\lim_{h \rightarrow 0} \frac{\sqrt{1+h} - 1}{h}$$

7)

$$\lim_{h \rightarrow 0} \frac{(2+h)^3 - 8}{h}$$

8)

$$\lim_{x \rightarrow 2} \frac{x^4 - 16}{x - 2}$$

9)

$$\lim_{x \rightarrow 7} \frac{\sqrt{x+2} - 3}{x - 7}$$

10)

$$\lim_{h \rightarrow 0} \frac{(3+h)^{-1} - 3^{-1}}{h}$$

11)

$$\lim_{x \rightarrow -4} \frac{\frac{1}{4} + \frac{1}{x}}{4 + x}$$

12)

$$\lim_{t \rightarrow 0} \left(\frac{1}{t} - \frac{1}{t^2 + t} \right)$$

7. Encuentre el límite, si existe. Si el límite no existe, explique por qué:

1)

$$\lim_{x \rightarrow -4} |x + 4|$$

2)

$$\lim_{x \rightarrow -4^-} \frac{|x + 4|}{x + 4}$$

3)

$$\lim_{x \rightarrow 2} \frac{|x - 2|}{x - 2}$$

4)

$$\lim_{x \rightarrow 1,5} \frac{2x^2 - 3x}{|2x - 3|}$$

5)

$$\lim_{x \rightarrow 0^-} \left(\frac{1}{x} - \frac{1}{|x|} \right)$$

6)

$$\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{|x|} \right)$$

8. Sea:

$$f(x) = \begin{cases} x - 1 & , \text{ si } x < 2 \\ x^2 - 4x + 6 & , \text{ si } x \geq 2 \end{cases}$$

9. Sea:

$$h(x) = \begin{cases} x & , \text{ si } x < 0 \\ x^2 & , \text{ si } 0 < x \leq 2 \\ 8 - x & , \text{ si } x > 2 \end{cases}$$

10. Resuelva los siguientes límites al infinito:

1)

$$\lim_{x \rightarrow +\infty} \left(\frac{x^3 + 1}{x - 1} - \frac{x}{4} \right)$$

2)

$$\lim_{x \rightarrow +\infty} \left(4x^2 - \sqrt{x^4 + 1} \right)$$

3)

$$\lim_{x \rightarrow +\infty} \left(2x - 1 - \sqrt{4x^2 + 1} \right)$$

4)

$$\lim_{x \rightarrow +\infty} \frac{5x + 8}{-5x + 2}$$

5)

$$\lim_{x \rightarrow -\infty} \frac{x^2 + 3x + 5}{x^4 - x - 6}$$

6)

$$\lim_{x \rightarrow +\infty} \frac{\sqrt[3]{x^7 - 4x^3}}{x^2 + 5x}$$