# Instituto Superior Universitario Tecnológico del Azuay
# Tecnología Superior en Big Data

## Taller de ejercicios - Derivadas implicitas


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# Taller de ejercicios - Derivadas implicitas

**Suponiendo que en los problemas del 1 al 12 cada ecuación define una función derivable de $x$, encuentre $D_x y$ por medio de la derivación implícita:**

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**1)**

$$\boldsymbol{y^2 - x^2 = 1}$$

$$2yy' - 2x = 0$$

$$2yy' = 2x$$

$$y' = \frac{x}{y}$$

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**2)**

$$\boldsymbol{9x^2 + 4y^2 = 36}$$

$$18x + 8yy' = 0$$

$$8yy' = -18x$$

$$y' = -\frac{9x}{4y}$$

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**3)**

$$\boldsymbol{xy = 1}$$

$$y + xy' = 0$$

$$xy' = -y$$

$$y' = -\frac{y}{x}$$

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**4)**

$$\boldsymbol{x^2 + \alpha^2 y^2 = 4\alpha^2}$$

Donde $\alpha$ es una constante.

$$2x + 2\alpha^2 yy' = 0$$

$$2\alpha^2 yy' = -2x$$

$$y' = -\frac{x}{\alpha^2 y}$$

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**5)**

$$\boldsymbol{xy^2 = x - 8}$$

$$y^2 + 2xyy' = 1$$

$$2xyy' = 1 - y^2$$

$$y' = \frac{-y^2 + 1}{2xy}$$

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**6)**

$$\boldsymbol{x^2 + 2x^2y + 3xy = 0}$$
$$2x + 4xy + 2x^2y' + 3y + 3xy' = 0$$
$$2x^2y' + 3xy' = -2x - 4xy - 3y$$
$$y'(2x^2 + 3x) = -2x - 4xy - 3y$$
$$y' = \frac{-2x - 4xy - 3y}{2x^2 + 3x}$$

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**7)**

$$\boldsymbol{4x^3 + 7xy^2 = 2y^3}$$
$$12x^2 + 7y^2 + 14xyy' = 6y^2y'$$
$$14xyy' - 6y^2y' = -12x^2 - 7y^2$$
$$y'(14xy - 6y^2) = -12x^2 - 7y^2$$
$$y' = \frac{-12x^2 - 7y^2}{14xy - 6y^2}$$

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**8)**

$$\boldsymbol{x^2y = 1 + y^2x}$$
$$2xy + x^2y' = 2yy'x + y^2$$
$$x^2y' - 2yy'x = y^2 - 2xy$$
$$y'(x^2 - 2yx) = y^2 - 2xy$$
$$y' = \frac{y^2 - 2xy}{x^2 - 2yx}$$

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**9)**

$$\boldsymbol{\sqrt{5xy} + 2y = y^2 + xy^3}$$
$$\frac{(5xy)^{-1/2}}{2}(5y + 5xy') + 2y' = 2yy' + y^3 + 3xy^2y'$$
$$\frac{5y}{2(5xy)^{1/2}} + \frac{5xy'}{2(5xy)^{1/2}} + 2y' = 2yy' + y^3 + 3xy^2y'$$
$$\frac{5xy'}{2(5xy)^{1/2}} + 2y' - 2yy' - 3xy^2y' = y^3 - \frac{5y}{2(5xy)^{1/2}}$$
$$y'\left(\frac{5x}{2(5xy)^{1/2}} + 2 - 2y - 3xy^2\right) = y^3 - \frac{5y}{2(5xy)^{1/2}}$$
$$y'\left(\frac{5x + 4(5xy)^{1/2} - 4y(5xy)^{1/2} - 6xy^2(5xy)^{1/2}}{2(5xy)^{1/2}}\right) = \frac{2y^3(5xy)^{1/2} - 5y}{2(5xy)^{1/2}}$$
$$y' = \frac{2y^3(5xy)^{1/2} - 5y}{5x + 4(5xy)^{1/2} - 4y(5xy)^{1/2} - 6xy^2(5xy)^{1/2}}$$
$$y' = \frac{2y^3\sqrt{5xy} - 5y}{5x + 4\sqrt{5xy} - 4y\sqrt{5xy} - 6xy^2\sqrt{5xy}}$$

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**10)**

$$\boldsymbol{x\sqrt{y+1} = xy + 1}$$

$$(y+1)^{1/2} + \frac{x}{2(y+1)^{1/2}}y' = y + xy'$$

$$\frac{x}{2(y+1)^{1/2}}y' - xy' = y - (y+1)^{1/2}$$

$$y'\left(\frac{x}{2(y+1)^{1/2}} - x\right) = y - (y+1)^{1/2}$$

$$y'\left(\frac{x - 2x(y+1)^{1/2}}{2(y+1)^{1/2}}\right) = y - (y+1)^{1/2}$$

$$y' = \frac{(y - (y+1)^{1/2})(2(y+1)^{1/2})}{x - 2x(y+1)^{1/2}}$$

$$y' = \frac{2y\sqrt{y+1} + 2y + 2}{x - 2x\sqrt{y+1}}$$

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**11)**

$$\boldsymbol{xy + \operatorname{sen}(xy) = 1}$$

$$y + xy' + \cos(xy)(y + xy') = 0$$

$$y + xy' + y\cos(xy) + x\cos(xy)y' = 0$$

$$xy' + x\cos(xy)y' = -y - y\cos(xy)$$

$$y'(x + x\cos(xy)) = -y - y\cos(xy)$$

$$y' = \frac{-y - y\cos(xy)}{x + x\cos(xy)}$$

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**12)**

$$\boldsymbol{\cos(xy^2) = y^2 + x}$$

$$-\operatorname{sen}(xy^2)(y^2 + 2xyy') = 2yy' + 1$$

$$-y^2\operatorname{sen}(xy^2) - 2xy\operatorname{sen}(xy^2)y' = 2yy' + 1$$

$$-2xy\operatorname{sen}(xy^2)y' - 2yy' = 1 + y^2\operatorname{sen}(xy^2)$$

$$y'(-2xy\operatorname{sen}(xy^2) - 2y) = 1 + y^2\operatorname{sen}(xy^2)$$

$$y' = \frac{1 + y^2\operatorname{sen}(xy^2)}{-2xy\operatorname{sen}(xy^2) - 2y}$$