Accuracy Assessment of Cartesian (X, Y, Z) to Geodetic Coordinates (φ, λ, h) Transformation Procedures in Precise 3D Coordinate Transformation – A Case Study of Ghana Geodetic Refe...

Article · January 2016

DOI: 10.12691/jgg-4-1-1

2 authors:

Bernard Kumi-Boateng
University of Mines and Technology

Yao Yevenyo Ziggah
University of Mines and Technology

Journal of Geosciences and Geomatics, 2016, Vol. 4, No. 1, 1-7
Available online at http://pubs.sciepub.com/jgg/4/1/1

DOI:10.12691/jgg-4-1-1

Accuracy Assessment of Cartesian (X, Y, Z) to Geodetic Coordinates (φ, λ, h) Transformation Procedures in Precise 3D Coordinate Transformation – A Case Study of Ghana Geodetic Reference Network

Bernard Kumi-Boateng[1,*], Yao Yevenyo Ziggah[1,2]

[1]Department of Geomatic Engineering, University of Mines and Technology, Tarkwa, Ghana
[2]Department of Surveying and Mapping, China University of Geosciences, Wuhan, P.R. China
*Corresponding author: kumi@umat.edu.gh

Abstract Ghana a developing country still adopt the non-geocentric ellipsoid known as the War Office 1926 as its horizontal datum for all surveying and mapping activities. Currently, the Survey and Mapping Division of Lands Commission in Ghana has adopted the satellite positioning technology such as Global Positioning System based on a geocentric ellipsoid (World Geodetic System 1984 (WGS84)) for its geodetic surveys. It is therefore necessary to establish a functional relationship between these two different reference frames. To accomplish this task, the Bursa-Wolf transformation model was applied in this study to obtain seven transformation parameters namely; three translations, three rotations and a scale factor. These parameters were then used to transform the WGS84 data into the War office system. However, Ghana's national coordinate system is a projected grid coordinate and thus the new War Office coordinates (X, Y, Z) obtained are not applicable. There is therefore the need to project these coordinates onto the transverse Mercator of Ghana. To do this, the new war office data (X, Y, Z) attained must first be transformed into geodetic coordinates. The reverse conversion from cartesian (X, Y, Z) to its corresponding geodetic coordinate (φ, λ, *h*) is computation intensive with respect to the estimation of geodetic latitude and height. This study aimed at evaluating the performance of seven methods in transforming from cartesian coordinates to geodetic coordinates within the Ghana Geodetic Reference Network. The seven reverse techniques considered are Simple Iteration, Bowring Inverse equation, method of successive substitution, Paul's method, Lin and Wang, Newton Raphson and Borkowski's method. The obtained results were then projected onto the transverse Mercator projection to get the new projected grid coordinates in the Ghana national coordinate system. These results were compared with the existing coordinates to assess their performance. The authors proposed the Paul's method to be a better fit for the Ghana geodetic reference network based on statistical indicators used to evaluate the reverse methods performance.

Keywords: *bursa-wolf model, coordinate transformation, geodetic coordinates, geocentric coordinates*

Cite This Article: Bernard Kumi-Boateng, and Yao Yevenyo Ziggah, "Accuracy Assessment of Cartesian (X, Y, Z) to Geodetic Coordinates (φ, λ, h) Transformation Procedures in Precise 3D Coordinate Transformation – A Case Study of Ghana Geodetic Reference Network." *Journal of Geosciences and Geomatics*, vol. 4, no. 1 (2016): 1-7. doi: 10.12691/jgg-4-1-1.

1. Introduction

Surveying with the advancement of modern science and technology has undergone an epoch-making transformation to break the spatial limitations of classical surveys, and to enter a new stage of development of modern surveying. One of the most important discovery that science and technology has provided the geo-scientific community due to its global coverage and free access is the Global Navigation Satellite Systems (GNSS) [1]. This GNSS technology such as Global Positioning System (GPS) furnishes the principal technology for geomatic and geodetic activities for both develop and developing countries. As an example, it is well known that in Ghana the GPS has been adopted as a viable tool for the majority of geodetic surveys due to its numerous advantages over classical methods of surveying. It is worth stating that the GPS ellipsoid of reference is the World Geodetic System 1984 (WGS84) and thus provides coordinates in latitude, longitude and ellipsoidal height. It is pertinent to note that it was indicated in [2] that assimilating GPS data into the mapping system of a country like Ghana cannot be done straightforwardly without appropriate mathematical conversions to determine transformation parameters. In an effort to have such integration into the mapping system of Ghana, the initial step is to convert the geodetic coordinates to the cartesian coordinate system. The method of directly converting geodetic coordinates to cartesian coordinates can be carried out using Bowring forward equation [3,4]. Conversely, it was emphasized by [5] that the reverse transformation is more complex considering the latitude and the geodetic height. This problem has been attributed to the inability of the latitude to be separated from the radius of curvature in the prime vertical when carrying out the reverse approach. Thus, creating a lot of research interests among geodesist and mathematicians. It is worth mentioning that several methods have been developed in estimating the latitude and geodetic height respectively. These methods as stated in [6] could be divided into three categories namely; Exact or closed form approaches, Approximation methods, and Iterative methods.

A comparative study of the above mentioned techniques have been carried out by researchers from various countries. For instance, [7] tested two direct and four indirect methods on a region covering Australia and found to give acceptable results for the computation of ϕ and h. It was concluded in their study that Lin and Wang's method was appreciably faster than the other five methods and was recommended for use in Australia. Reference [8] reviewed exact transformation formulas and compared with the approximation methods by considering their computational complexity and sensitivity to computer round off error. The author concluded that the exact transformation formulas produced negligible errors in coordinate transformation and thus recommend to be used in practice. Reference [9] evaluated four non-iterative and four iterative methods in transforming cartesian coordinates to its corresponding geodetic coordinates. It was found that the iterative methods were faster than the non-iterative and converges to a sub millimetre accuracy exceeding the requirements of any practical application. In [10] a general overview of ten iterative methods and one direct method were presented. The author concluded that the direct procedure is fairly simple and straightforward compared with the iterative methods that requires an approximation to get at the desired solution. Reference [11] demonstrated that the Bowring's inverse method in terms of computational speed was faster than Borkowski. However, it was shown that Lin and Wang method is faster than Bowring's method [12].

Several authors have proposed new techniques for transforming geocentric coordinates to geodetic coordinates. For example, [13] introduced a closed-form algebraic technique that is applicable globally including the poles regardless of the eccentricity value of the ellipsoids. The proposed method produced refine values when compared with Bowring Inverse equation. In [14] an exact and relatively simple analytical transform of the rectangular coordinates to the geodetic coordinates was presented. The method was based on one solution of the quartic equation. Reference [15] developed a more robust algorithm that yields more accurate results than most of the published algorithms. A significant attribute of the procedure is that the processing time is nearly a constant. A vector-based algorithm to transform cartesian coordinate to geodetic coordinates was introduced by [16] and was further extended to triaxial ellipsoids [17]. Both algorithms showed identical result in accuracy, similar computer processing unit requirements, and also worked well for celestial bodies having significantly greater flattening than the Earth. Reference [18] compared computational intelligence algorithms with the conventional methods in transforming geocentric coordinates into geodetic coordinates. Among all the methods, the differential search algorithm applicable to numerical optimization problems yielded a very high level of accuracy.

It should be known that the situation in Ghana is different as researchers have only utilized the Bowring inverse equation in their datum transformation. For instance, in [19] the Bowring Inverse equation was applied in datum transformation between WGS84 and War Office datum. The same inverse method was implemented also in [2] and [20]. In view of the above development, it can be seen that such a comparative study on the applicability of the reverse methods within the Ghana geodetic reference network has not been fully investigated. This study thus compares seven of the reverse techniques developed by researchers namely; Simple Iteration, Bowring Inverse equation, Lin and Wang method, Paul's method, method of Successive Substitution, Newton Raphson approach, and Borkowski's method within the Ghana geodetic reference network. It is worth stating that that these are not the only techniques for computing geodetic latitude (ϕ) from cartesian coordinate (X, Y, Z). Hence, this study is not definitive since only a small selection of the published techniques are chosen. However, the choice of the methods was influenced by its gaining popularity in usage within the last two decades by researchers.

2. Study Area

Ghana is a country located at the Western part of Africa sharing boarders with Cote D'Ivoire to the West, Togo to the East, Burkina Faso to the North and the Gulf of Guinea to the South. The country has a 239,460 km^2 land mass generally consisting of low plains [21] with 2,093 km of international land borders. Figure 1 shows the study area.

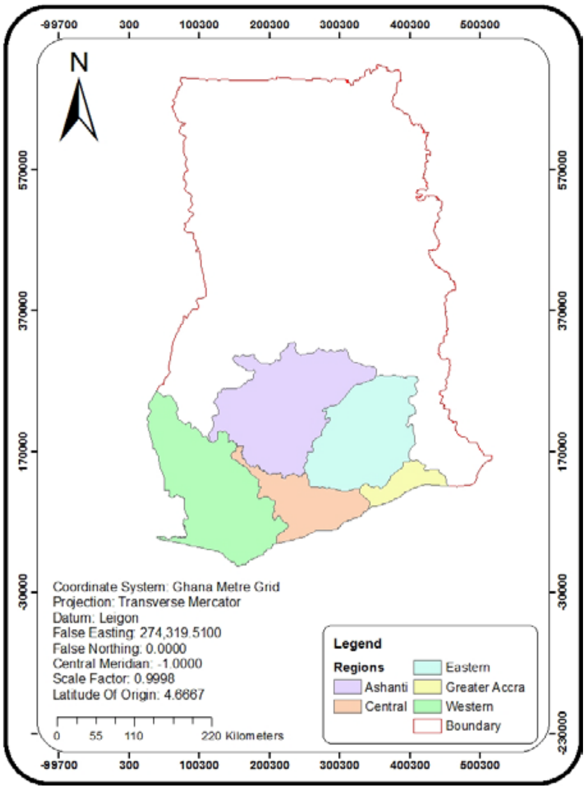

Figure 1. Map of Ghana showing the study area

This study covers the first phase of the ongoing project by the Ghana Survey and Mapping Division of Lands Commission in establishing a new geodetic reference network base on the International Terrestrial Reference System (ITRS) [1]. The choice of the five out of ten administrative regions for the first phase of the project was due to the following reasons; almost all the natural resources such as gold, bauxite, manganese, oil, timber, cocoa, diamond and many others found in the country are situated in these regions. Hence, contributing significantly to the economic growth of Ghana.

3. Methods

3.1. Transformation from Geodetic Coordinates (ϕ,λ,h) to Cartesian Coordinates (X,Y,Z)

Secondary data of common points in both WGS84 and War Office 1926 geodetic coordinates for the new and old Ghana geodetic reference network were obtained from the Survey and Mapping Division of Lands Commission in Ghana. The Bowring forward equation was first used to transform the geodetic coordinates into cartesian coordinates because it is a prerequisite in most 3D datum transformation parameter determination. The direct transformation of the geodetic coordinates (ϕ,λ,h) above a reference ellipsoid to the cartesian coordinates (X, Y, Z) could be carried out using the Bowring forward equation. This equation is expressed mathematically as:

$$X = (N+h)\cos\phi\cos\lambda$$

$$Y = (N+h)\cos\phi\sin\lambda$$

$$Z = (N(1-e^2)+h)\sin\phi$$

Where; φ = Latitude, λ = longitude, f = flattening , e = eccentricity. The radius of curvature in the prime vertical plane N, and e^2 the first eccentricity are given by

$$N = \frac{a}{\sqrt{1-e^2\sin^2\phi}}$$

$$e^2 = f(2-f).$$

3.2. Bursa-Wolf Transformation Model

Figure 2 shows the geometry of the Bursa–Wolf transformation model. The X, Y, Z axes of system 1 are rotated by very small angles $\varepsilon_X, \varepsilon_Y, \varepsilon_Z$ from the X, Y, Z axes of system 2, and the origins of the two systems are displaced by translations t_X, t_Y, t_Z in the directions of the X, Y, Z axes of system 2 [22]. I_1 and I_2 are vectors of coordinates in both systems and ***t*** is a vector of translations.

The mathematical relationship between coordinates in both systems can be written in the form of a vector equation [22]); $I_2 = t_2 + (1+ds)\, R_S I_1$.

Alternatively, the Bursa–Wolf transformation may be written as

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_2 = (1+ds)\begin{bmatrix} 1 & \varepsilon_Z & -\varepsilon_Y \\ -\varepsilon_Z & 1 & \varepsilon_X \\ \varepsilon_Y & -\varepsilon_X & 1 \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_1 + \begin{bmatrix} t_X \\ t_Y \\ t_Z \end{bmatrix}_2 .$$

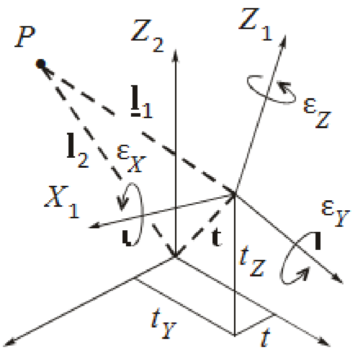

Figure 2. Geometry of Bursa-Wolf model (adopted from [22])

3.3. Transforming Cartesian Coordinates to Geodetic Coordinates

The new War office rectangular coordinates obtained after applying the Bursa-Wolf parameters determined need to be transformed back into geodetic coordinates (latitude, longitude, ellipsoidal heights) to enable projection of the coordinate onto the transverse Mercator that is utilized in Ghana. This will also enable the coordinates to be expressed in the Ghana national projected grid coordinate system (Easting, Northing). The reverse transformation was carried out using seven inverse equation models. The various models applied are described in the subsequent section.

3.3.1. Simple Iteration

An approximate latitude value ϕ_0 was computed from the equation

$$\tan\phi_0 = \frac{Z\left(1+e'^2\right)}{P}$$

where $P = \sqrt{X^2+Y^2}$, second eccentricity is given us $e'^2 = \frac{a^2-b^2}{b^2} = \frac{e^2}{1-e^2}$ with **a** and **b** being the semi-major and minor axis of the reference ellipsoid respectively. The approximate value, ϕ_0 was then used in the right hand side (RHS) of the equation

$$\tan\phi = \frac{Z+Ne^2\sin\phi}{P}$$

to evaluate $\tan\phi_1$, on the left hand side (LHS). The new value, ϕ_1 was then applied in the right hand side (RHS) to give the next value $\tan\phi_2$. This procedure was repeated until the difference between successive LHS values, $\tan\phi_i$, $\tan\phi_{i+1}$ reaches an acceptable limit thus, the iteration converges to a solution of $\tan\phi$.

3.3.2. Bowring Inverse Equation

The starting value, ψ_0 was obtained from the relationship between the geocentric and parametric latitude using

$$\tan\psi_0 \approx \frac{aZ}{bP}.$$

The required latitude was then computed from

$$\tan\phi = \frac{Z + be'^2 \sin^3\psi}{P - ae^2\cos^3\psi}$$

Where ψ is the parametric latitude; **a** is the semi-major axis of the reference ellipsoid; $P = \sqrt{X^2 + Y^2}$ is the perpendicular distance from the rotational axis; $e^2 = f(2-f)$ is the first eccentricity and e'^2 is the second eccentricity expressed as $e'^2 = \frac{a^2 - b^2}{b^2} = \frac{e^2}{1-e^2}$.

3.3.3. Lin and Wang

This iterative method uses Newton Raphson Iteration to evaluate a scalar multiplier q of the normal vector to the ellipsoid [23]. Once q was calculated, simple relationships between cartesian coordinates of P and its normal projection Q onto the ellipsoid at P were used to evaluate the cartesian coordinates of Q. The initial approximation of the latitude was computed using;

$$q_0 = \frac{ab\left(a^2Z^2 + b^2{P_P}^2\right)^{3/2} - a^2b^2\left(a^2{Z_P}^2 + b^2{P_P}^2\right)}{2\left(a^4{Z_P}^2 + b^4{P_P}^2\right)}.$$

The latitude, ϕ and height, h were finally estimated from the equations below;

$$\tan\phi = \left(\frac{Z_Q}{(1-f)^2 P_Q}\right) = \left(\frac{Z_Q}{\left(1-e^2\right)P_Q}\right)$$

$$h = \pm\sqrt{\left(P_P - P_Q\right)^2 + (Z_P - Z_Q)^2}$$

The P_Q and Z_Q were obtained from

$$P_Q = \frac{a^2 P_P}{a^2 + 2q_0},\ Z_Q = \frac{b^2 Z_P}{b^2 + 2q_0}$$

It should be noted that h is negative if $P_P + |Z_P|$ is less than $P_Q + |Z_Q|$.

3.3.4. Paul's Method

Paul's method is direct in so far as $\tan\phi$ is obtained from a simple closed equation but only after several intermediate variables have been evaluated. Thus, having X, Y, Z for a point related to an ellipsoid, the latitude, ϕ was obtained by computing the following variables ($\alpha, \beta, q, u_1, t_1, \varsigma$) in order from the following equations;

$$\alpha = \frac{P^2 + a^2e^4}{1-e^2}, \beta = \frac{P^2 - a^2e^4}{1-e^2}, q = 1 + \frac{27Z^2\left(\alpha^2 - \beta\right)}{2\left(\beta + Z^2\right)^3},$$

$$u_1 = 1/2\left\{\sqrt[3]{q + \sqrt{q^2 - 1}} + \sqrt[3]{q - \sqrt{q^2 - 1}}\right\},$$

$$t = \left(\frac{\beta + Z^2}{6}\right)u + \frac{Z^2}{12} - \frac{\beta}{6},$$

$$\varsigma = \sqrt{t_1} + \sqrt{\frac{Z^2}{4} - \frac{\beta}{2} - t_1 + \frac{aZ}{4\sqrt{t_1}}}.$$

It should be noted that all square roots in ς have the same sign as Z .The geodetic latitude, ϕ was estimated from

$$P\tan\phi = \varsigma + \frac{Z}{2}.$$

Detailed description of the Paul's method can be found in [7].

3.3.5. Method of Successive Substitution

The method of Successive Substitution is popular because of its programming simplicity. This method is comparable to that of simple iteration. A starting value ϕ_0 was first calculated from the relationship

$$\tan\phi_0 = \frac{Z}{P(1-f)^2}$$

This ϕ_o was used on the RHS of the equation below

$$P\tan\phi = Z + Ne'^2\sin\phi$$

to evaluate $P\tan\phi_1$ (and hence ϕ_1) on the LHS. This new value, ϕ_1 was then applied on the RHS to give the next value, $P\tan\phi_2$ (and hence ϕ_2). The procedure was repeated until the difference between successive LHS values ϕ_n , ϕ_{n+1} reached an acceptable limit. Thus, the iteration converges to a solution for $P\tan\phi$ and hence φ.

3.3.6. Newton Raphson

The Newton Raphson iteration can improve the rate of convergence for ϕ in successive substitution solution for real roots of equation $f(\phi) = 0$ given in the form of an iterative equation as

$$\phi_{n+1} = \phi_n - \frac{f(\phi_n)}{f'(\phi_n)}$$

where n represents n^{th} iteration and the function $f(\phi)$ is expressed as;

$$f(\phi) = Z + Ne^2\sin\phi - P\tan\phi.$$

The derivative of the function $f(\phi)$ is given as;

$$f'(\phi) = \frac{c}{V^3}e'^2\cos\phi - \frac{P}{\cos^2\phi}$$

where, $V^2 = 1 + e'^2\cos^2\phi$.

A starting latitude value, ϕ_0 was obtained from

$$\tan\phi_0 = \frac{Z}{P(1-f)^2}.$$

The iteration continued until the correction factor $\Delta\phi_n = \left|\frac{f(\phi)}{f'(\phi_n)}\right|$ in ϕ_{n+1} reached an acceptable small magnitude.

3.3.7. Borkowski's Method

Reference [7] indicated that Borkowski's method is an indirect method that uses Newton's iterative technique to solve for the parametric latitude, ψ . The ψ is expressed as

$$\psi_{n+1} = \psi_n - \frac{f(\psi)}{f'(\psi)}$$

where the function and its derivative are expressed as;

$$f(\psi) = 2\sin(\psi_n - \Omega) - c\sin 2\psi_n$$

and $f'(\psi) = 2(\cos(\psi_n - \Omega) - c\cos 2\psi_n)$. ψ was obtained by iteration after calculating the constants Ω , $\boldsymbol{q}$ and $\boldsymbol{c}$ from

$$\tan\Omega = \frac{bZ}{aP}, q = \sqrt{(aP)^2 + (bZ)^2},$$

$$c = \frac{a^2 - b^2}{q} = \frac{a^2 - b^2}{\sqrt{(aP)^2 + (bZ)^2}}.$$

An initial approximation, ψ_0 was estimated using the relation;

$$\tan\psi_0 = \frac{aZ}{bP}.$$

After solving for ψ , ϕ was computed from;

$$\tan\phi = \frac{a}{b}\tan\psi.$$

4. Results and Discussion

4.1. Bursa-Wolf Transformation Model

The derived parameters for transforming data from WGS 84 to War Office datum with their associated standard deviations using Bursa-Wolf transformation model are presented in Table 1 below. It was observed from the results of the translation parameters $(\Delta X, \Delta Y, \Delta Z)$ in Table 1 that, from the origin of the War Office to the origin of WGS 84, the X-axes that intersects the Greenwich Meridian and the Equator has a negative displacement. On the other hand, the Y-axes created to have a right angle to the X, Z-axes and the Z-axes passing through the Earth instantaneous pole has a positive displacement between the two reference systems (War Office and WGS 84 datum). The individual standard deviations for the obtained transformation parameters are also shown in Table 1. The calculated reference standard deviation and reference adjustment variance were also estimated to check the precision for the overall observations (Table 1).

Table 1. Summary of derived parameters results

BURSA-WOLF MODEL		
PARAMETER	VALUES	STANDARD DEVIATION (SD)
ΔX	-151.1891	10.1714
ΔY	31.5931	16.9151
ΔZ	327.1767	16.8742
Rx	0.4451400785	0.0000016043
Ry	-0.00581646132	0.0000026521
Rz	0.02198989116	0.0000026397
Sc	-7.1677	0.0000015827
Reference SD	0.59297	
Reference Adjustment Variance	0.35162	

The translation parameter, ΔX revealing the existence of a negative displacement from the geocenter is because both X-axes in the two reference ellipsoids are moving in opposite directions. Conversely, ΔY and ΔZ translation parameters from the geocenter evident from Table 1 above shows that, the axes of both reference ellipsoids move in the same direction. In Table 1, the standard deviations indicate the spread of the data about the mean measured in the same units as the data. They also indicate the limits of the error bound within which the most probable value (MPV) of the mean lies. Since the individual standard deviations computed for each translation parameter are small, it implies that the data points are closer to the MPV value of the mean which indicates that the parameters response will be fairly uniform when applied to the observation data within the study area. In addition, the smaller standard deviation obtained signifies a steep bell-shaped on the normal distribution curve.

4.2. Reverse Techniques Performance Evaluation

The reverse transformation methods were evaluated based on the residuals generated from new projected grid coordinates and the existing projected coordinates within the Ghana geodetic reference network. The following statistical indicators were used as a performance criteria index (PCI) namely: Mean Square error (MSE), Root Mean Square error (RMSE), Nash-Sutcliffe Efficiency Index (N) and Modified Index of Agreement (D). Their mathematical expressions are given below:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(O_i - P_i)^2, RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(O_i - P_i)^2},$$

$$N = 1 - \frac{\sum_{i=1}^{n}(O_i - P_i)^2}{\sum_{i=1}^{n}(O_i - \bar{O}_i)^2}, MID = 1 - \frac{\sum_{i=1}^{n}|O_i - P_i|}{\sum_{i=1}^{n}\left(|P_i - \bar{O}| + |O_i - \bar{O}|\right)}.$$

Where n is the total number of observations used in the Bursa-Wolf model, O and P are the existing projected coordinates and predicted projected coordinates from the reverse techniques used. $\overline{O}$ is the mean of the existing projected coordinates. To assess the quality of the transformation results the MSE, RMSE, N and MID were computed as shown in Table 2 and Table 3.

Table 2. Statistical Indicators results for Easting Coordinate

METHODS	Performance Criteria Index			
	MSE (m)	RMSE (m)	N	MID
Simple Iteration	0.717367	0.467604	1	0.999996
Bowring Inverse	0.717367	0.467604	1	0.999996
Lin and Wang	0.717367	0.467604	1	0.999996
Pauls Method	0.717367	0.467604	1	0.999996
Successive Substitution	0.717367	0.467604	1	0.999996
Newton Raphson	0.717353	0.467599	1	0.999996
Borkowski	0.717367	0.467604	1	0.999996

Table 3. Statistical Indicators results for Northing Coordinate

METHODS	Performance Criteria Index			
	MSE (m)	RMSE (m)	N	MID
Simple Iteration	0.704059	0.839082	1	0.999993
Bowring Inverse	0.703625	0.838824	1	0.999993
Lin and Wang	0.703625	0.838824	1	0.999993
Pauls Method	0.703607	0.838813	1	0.999993
Successive Substitution	0.704059	0.839082	1	0.999993
Newton Raphson	0.770122	0.877566	1	0.999993
Borkowski	0.703626	0.838824	1	0.999993

The MSE values obtained signifies good performance from the various reverse method. This is because it is well known that for 3D datum transformation between non-geocentric and geocentric datums, the acceptable accuracy is within ±1 m due to the distortions in the local network. Also, the MSE further showed the degree of fitness of the reverse methods to the existing data.

The Nash-Sutcliffe Efficiency Index (*N*) was used in this study as a measure of the reverse methods efficiency by comparing the new projected grid coordinates to existing grid coordinates. From Table 2 and Table 3, both methods attained the optimum value of 1. The MID varies from 0 to 1 with higher values indicating a better fit of the model. With this in mind, it can be stated that the reverse methods modelled values (new projected grid coordinates) have better agreement with the existing datasets.

A careful observation of Table 2 and Table 3 reveals that; Borkowski's method, Bowring Inverse method and Lin and Wang have the same range of errors in Eastings and Northings respectively. Thus, both methods achieved identical RMSE value of 0.467604 m and 0.838824 m in both Easting and Northing coordinates. Simple Iteration and method of Successive Substitution were the other two methods having the same RMSE values in both coordinates system. Having compared all the seven methods it was observed from Table 2 that Newton Raphson's method gave the least RMSE in the Eastings compared to the other methods. However, its error in the Northing coordinates was higher than all the other reverse methods used in the assessment. Conversely, Paul's method had the least error in the RMSE in the Northing coordinates. With the exception of the Newton Raphson's method the same RMSE was achieved in the Easting coordinates when compared with the other reverse techniques. Upon critical examination of Tables 2 and 3, the authors proposed the Paul's method as the most viable approach to be used within the Ghana geodetic reference network when precise 3D coordinate transformation values are needed. This is because the RMSE deviation estimated showed that the Pauls method deviate from Newton Raphson by 0.000005 m in the Eastings. In the case of the Northing coordinates, Newton Raphson performed poorly by deviating 0.038753 m from the Paul's method.

5. Conclusion

Seven reverse methods namely Simple Iteration, Bowring Inverse equation, Lin and Wang, Pauls method, Successive Substitution, Newton Raphson and Borkowski has been tested within the Ghana geodetic reference network using Bursa-Wolf transformation model. The results obtained showed that the seven methods produced acceptable results in the coordinate transformation. This further confirms the assertion made by [8] that the reverse methods are not only imperative in theory, but can also be used in practice. To conclude, the authors' based on the results obtained recommend Pauls method as the most practicable technique for precise 3D coordinate transformation work within the Ghana geodetic reference network.

Acknowledgement

The authors wish to express their appreciation to the Ghana Survey and Mapping Division of Lands Commission for making available the data for this research.

Statement of Competing Interests

The authors have no competing interests.

References

[1] Poku-Gyamfi, Y. and Gunter W. H. (2006). Framework for the Establishment of a Nationwide Network of Global Navigation Satellite System (GNSS): A Cost Effective Tool for Land Development in Ghana. *5th FIG Conference*, 1-13.

[2] Ayer, J. and Fosu, C. (2008). Map Coordinate Referencing and the Use of GPS Datasets in Ghana. *Journal of Science and Technology*, Vol. 28, No. 1, 1-18.

[3] Leick, A. (2004). *GPS Satellite Surveying*. John Wiley and Sons, Inc., Hoboken, New Jersey, United States of America.

[4] Sickle, J. V. (2010). *Basic GIS Coordinates*. CRC Press, Taylor and Francis Group, 2nd Ed., 6000 Broken Sound Parkway NW, Suite 300, Boca Raton, FL 33487-2742 USA, 7.

[5] Jekeli, C. (2012). *Geometric Reference Systems in Geodesy*. Division of Geodetic Science, School of Earth Sciences Ohio State University, USA, 15.

[6] Shu, C. and Li, F. (2010). An Iterative Algorithm to Compute Geodetic Coordinates. *Computer and Geosciences*, Vol. 36, 1145-1149.

[7] Gerdan, G. P. and Deakin, R. E. (1999). Transforming Cartesian Coordinates (X, Y, Z) to Geographical Coordinates (φ, λ, h). *The Australian Surveyor*, Vol. 44, No. 1, 55-63.

[8] Zhu, J. (1994). Conversion of Earth-Centered Earth-Fixed Coordinates to Geodetic Coordinates. *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 30, No. 3, 957-961.

[9] Fok, S. H. and Iz, H. B. (2003). A Comparative Analysis of the Performance of Iterative and Non-iterative Solutions to the Cartesian to Geodetic Coordinate Transformation. *Journal of Geospatial Engineering*, Vol. 5, No.2, 61-74.

[10] Burtch, R. (2006). A Comparison of Methods Used in Rectangular to Geodetic Coordinate Transformations, *ACSM Annual Conference and Technology Exhibition*, *Orlando, FL*, 1-25.

[11] Laskowski, P. (1991). Is Newton's iteration faster than simple iteration for transformation between geocentric and geodetic coordinates? *Bull Geod,* 65:14–17.

[12] Lin K.C. and Wang, J. (1995). Transformation from geocentric to geodetic coordinates using Newton's iteration. *Bull Geod,* 69:14-17.

[13] Vermeille, H. (2002). Direct transformation from geocentric coordinates to geodetic coordinates, Journal of Geodesy 76: 451-454.

[14] Borkowski, K. M. (1989). Accurate Algorithms to Transform Geocentric to Geodetic Coordinates. Turon Radio Astronomy Observatory*; Bull Geoid*, Vol. 63, pp. 50-56.

[15] Toms, R. (1996). An Improved Algorithm for Geocentric to Geodetic Coordinate Conversion. *Fourteenth Workshop on Standards for Distributed Interactive Simulations Orlando*, Lawrence Livermore National Laboratory, University of California, W-7405-Eng-48.

[16] Feltens, J. (2007). Vector methods to compute azimuth, elevation, ellipsoidal normal, and the Cartesian (X, Y, Z) to geodetic (φ, λ, h) transformation. *Journal of Geodesy*, Vol. 82, Issue 8, 493-504.

[17] Feltens, J. (2009). Vector method to compute the Cartesian (X, Y, Z) to geodetic (φ, λ, h) transformation on a triaxial ellipsoid. *Journal of Geodesy*, Vol. 83, Issue 2, 129-137.

[18] Civicioglu, P. (2012). Transforming Geocentric Cartesian Coordinates to Geodetic Coordinates by using Differential Search Algorithm. *Computers & Geosciences*, Vol. 46, 229-247.

[19] Ziggah, Y.Y., Youjian, H., Odutola, A.C. and Nguyen, T.T. (2013). Accuracy Assessment of Centroid Computation Methods in Precise GPS Coordinates Transformation Parameters Determination: A Case Study, Ghana, *European Scientific Journal*, Vol. 9, No. 15, 1-12.

[20] Dzidefo, A. (2011). Determination of Transformation Parameters between the World Geodetic System 1984 and the Ghana Geodetic Network. *Master's Thesis*, Department of Civil and Geomatic Engineering, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana.

[21] Baabereyir, A. (2009). Urban environmental problems in Ghana: case study of social and environmental injustice in solid waste management in Accra and Sekondi-Takoradi. *Thesis submitted to the Department of Geography, University of Nottingham for the Degree of Doctor of Philosophy*, UK, 97.

[22] Deakin, R.E. (2006). A Note on the Bursa-Wolf and Molodensky-Badekas Transformations. School of Mathematical and Geospatial Sciences, RMIT University, 1-21.

[23] Deakin, R. E. and Hunter, M. N. (2010). Geometric Geodesy - Part A. *School of Mathematical and Geospatial Sciences,* RMIT University, Melbourne, Australia, 146.