Time Series Analysis of the US Housing Market and the Nasdaq-100 Stock Market Index

Erik Duus - STAT619, Spring 2021

Abstract

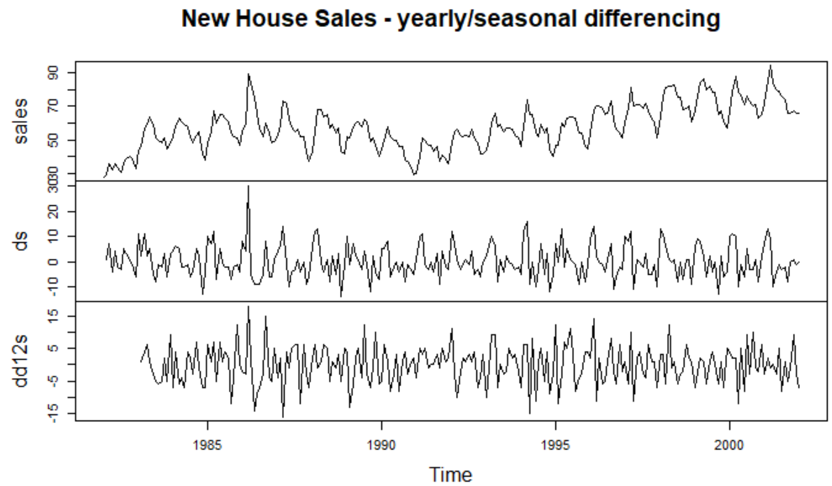
New single-family house sales are an essential measure of US economic activity. The Federal Economic Reserve Data (FRED) service generates a monthly time series of new house sales for public use. This data series is downloaded and used to develop a pair of time series models, the first using ARIMA and the second using regression with ARMA errors. The predictive performance of the models is assessed and contrasted. A second measure of economic conditions is the performance of US stock markets, which is tracked via stock market indices. A second pair of time series models is developed using seasonal ARIMA and LSTM to predict the future prices of the Nasdaq-100 ETF. The performance of the models is again assessed, and the models are compared.

Introduction: US New Housing Sales

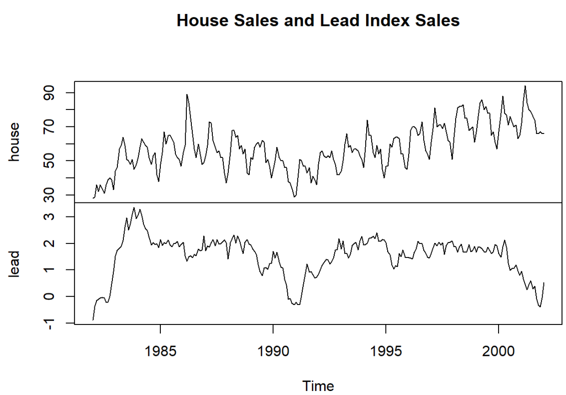
FRED provides an extensive catalog of data series related to economic conditions in the US. A significant data series is New One Family Houses Sold: United States, a monthly series dating back to 1963 that is not seasonally adjusted. Another important series is the Leading Index for the United States, which predicts the change in economic activity over the ensuing six months. It is also a monthly series, dating back to 1979. This analysis will apply to the 20 years 1982-2001, and predictions will be compared to the first 7 months of 2002.

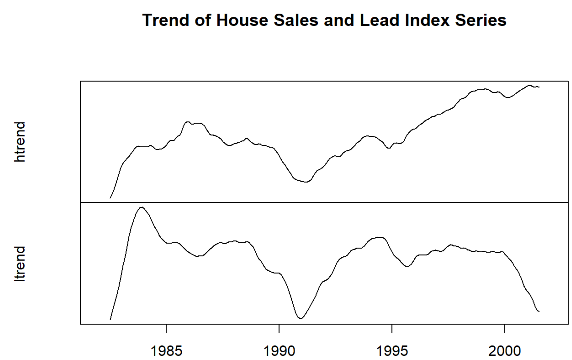
Exploratory Data Analysis

The time series plot of monthly housing sales shows both trend and annual seasonality. Monthly differencing is applied to detrend the series, followed by annual differencing to remove the seasonality. A plot of the resulting series shows a constant mean around zero and close to constant variance.



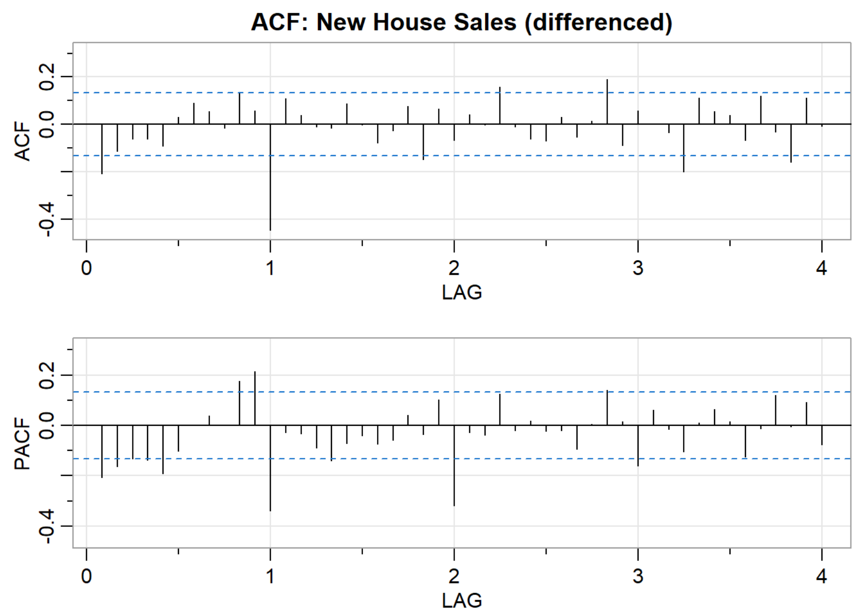
Examination of the US leading index series plot reveals some correlation with the US house sales series, indicating that it may be a useful correlate for a linear regression model. Decomposing both series and examining their trends confirms correlation through most of the series, although the trends decouple in the last years of the analysis period.



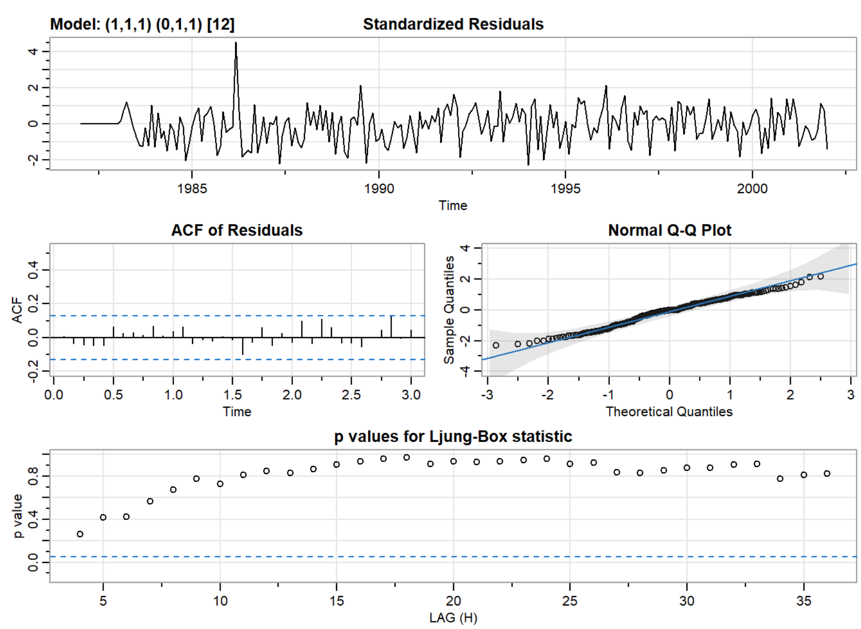


Statistical Analysis: ARIMA

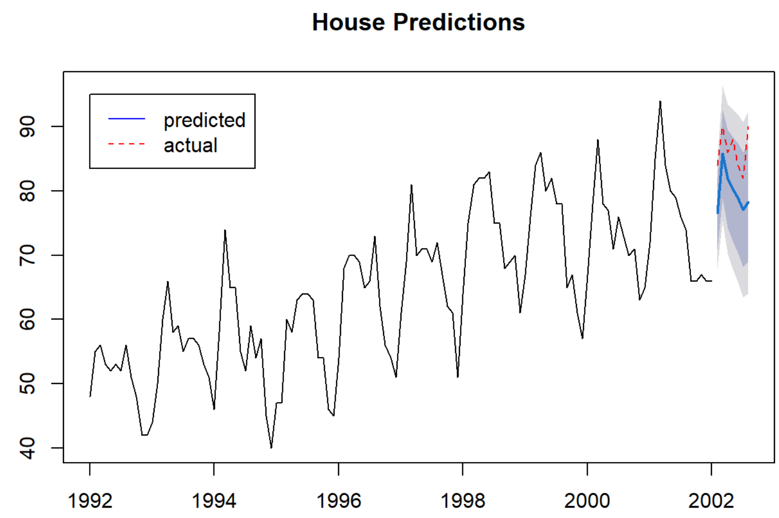
Dickey-Fuller and Phillips-Perron tests of the differenced sales series indicate no unit root, indicating a stationary mean. Examination of the ACF and PACF of the sales series shows decay, also confirming stationarity. Spikes are visible at seasonal lags, with the PACF appearing to tail off and the ACF cutting off after 1 lag, suggesting a seasonal MA(1) model. Examination of between season lags shows both the ACF and PACF tailing off, indicating a within-season ARMA(1,1) model and a final ARIMA(1,1,1)x$(0,1,1)_{12}$ model.



`auto.arima` suggests a slightly different ARIMA(1,1,1)x(0,1,2)$_{12}$ model, so both are evaluated. `sarima` is first used to fit the ARIMA(1,1,1)x(0,1,2)$_{12,}$ but the model is discarded since the second seasonal MA coefficient is not statistically significant. ARIMA(1,1,1)x(0,1,1)$_{12}$ produces a model with significant coefficients. Examination of diagnostic plots shows that residuals are normally distributed and white noise. The Ljung-Box statistics also suggest the residuals are white noise, leading to the conclusion that the model is adequate.



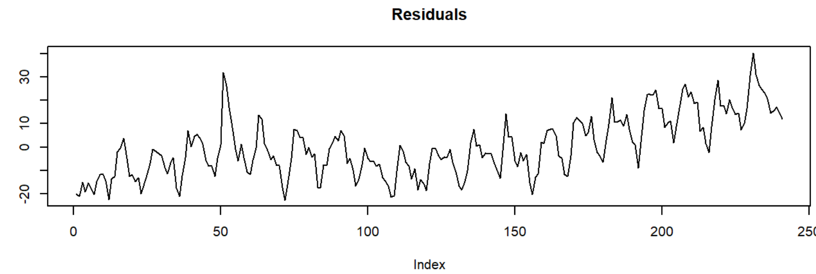
The fitted model is used to predict the first 7 months of 2002, and the predictions are compared to the true values from the FRED sales series.

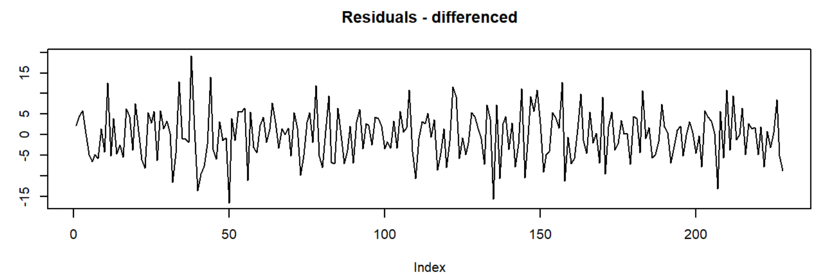


The model's predictions follow the trend and seasonality of the series, and although the actual values are higher than the predictions, they lie within the confidence interval. RMSE is computed to be 6.96.

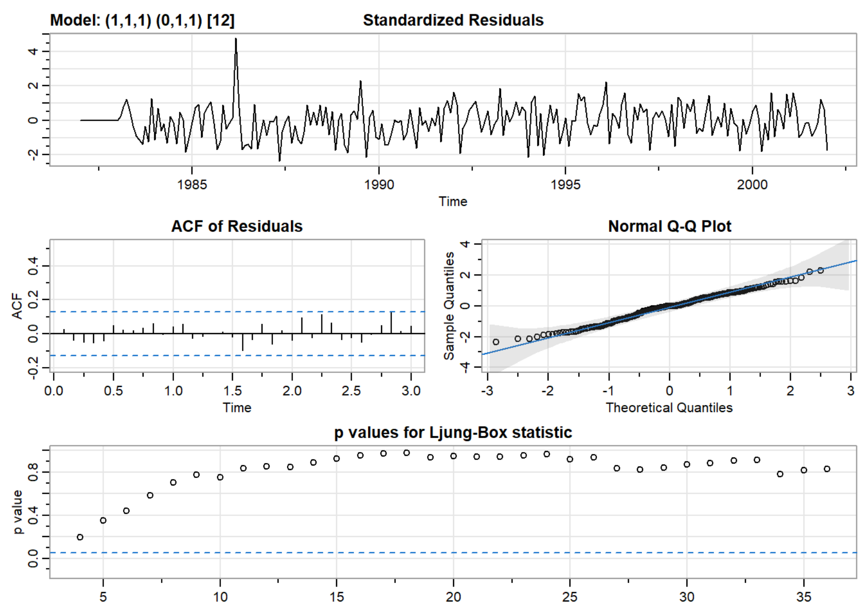
Statistical Analysis: Regression with ARIMA errors

A second model is constructed by fitting a linear regression between house sales and the FRED leading index. The model coefficients are all significant, but the adjusted R-squared is 0.05, indicating that the model explains only a small portion of the variance. Examination of the residuals shows they are not white noise; trend and annual seasonality are visible, as in the main sales series. Application of annual and monthly differencing produces a plot that appears to be white noise. This suggests an ARIMA error model should be added to the linear regression.

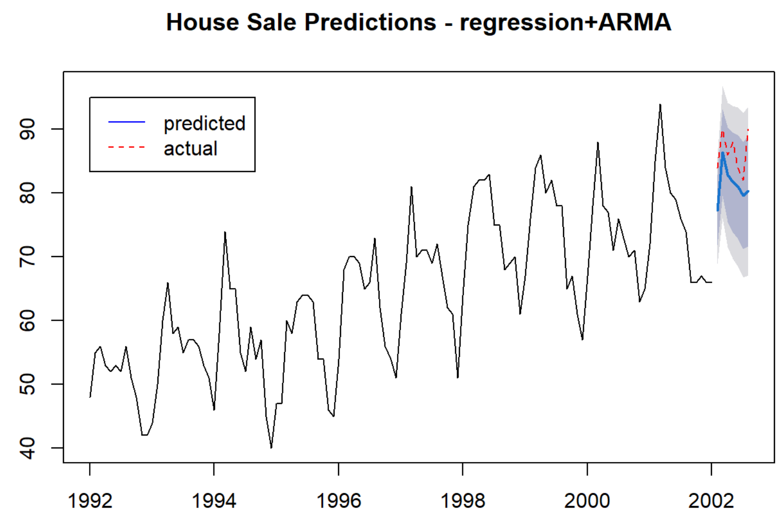




Analysis of the ACF and PCF of the differenced residuals suggests an ARIMA(1,1,1)x(0,1,1)$_{12}$ model for the errors. Fitting a new ARIMA+regression model using `sarima` produces a model with significant coefficients. The diagnostic plots show the residuals are normal and appear to be white noise. The Ljung-Box statistics and ACF plot also confirm that the residuals are white noise, and it can be concluded that the model is adequate.



As before, the model is used to predict the first 7 months of 2002 and the predictions compared to the actual values from the house sales series.



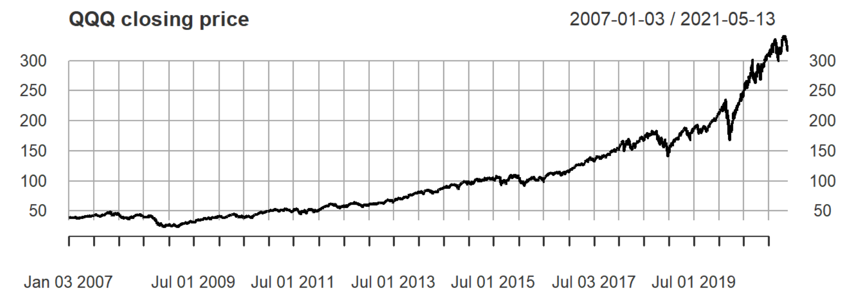
This model also seems to capture the seasonality and general trend of the series, and the actual values fall within the confidence interval of the predictions. The RMSE of the predictions is 5.54, which is an improvement over the first model's performance.

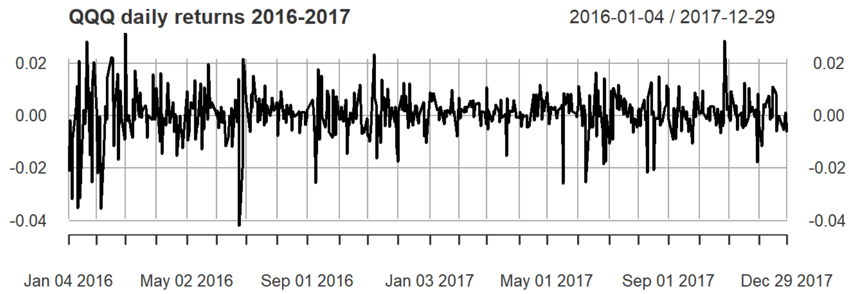
Introduction: Nasdaq-100 ETF

The Nasdaq-100 is a stock market index of the 100 largest-capitalized stocks that trade on the Nasdaq stock exchange. A good proxy for the price activity of the index is the Nasdaq-100 ETF (symbol: QQQ) which is an exchange-traded fund comprised of the constituent companies of the index. Daily historical prices for QQQ dating back to 1999 are publicly available for download from Yahoo Finance. This analysis is performed on the 2-year period 2016-2017, and predictions are compared to the actual prices from the first 14 days of 2018.

Exploratory Data Analysis

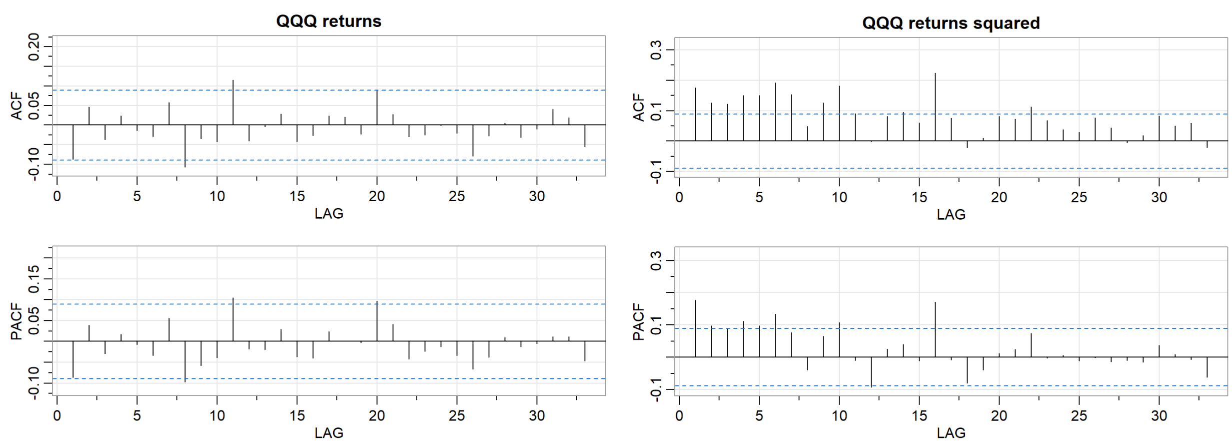
The historical price data series is downloaded as a CSV. Examination of the data reveals a handful of missing values that are backfilled via simple interpolation. A plot of the series shows exponential growth in the values, violating assumptions of stationarity. A log transformation followed by daily differencing transforms the prices into a series of daily returns. A plot of the resulting series shows a constant mean but non-constant variance, again violating stationarity. The returns appear to go through higher and lower volatility periods, suggesting that a GARCH model is required.



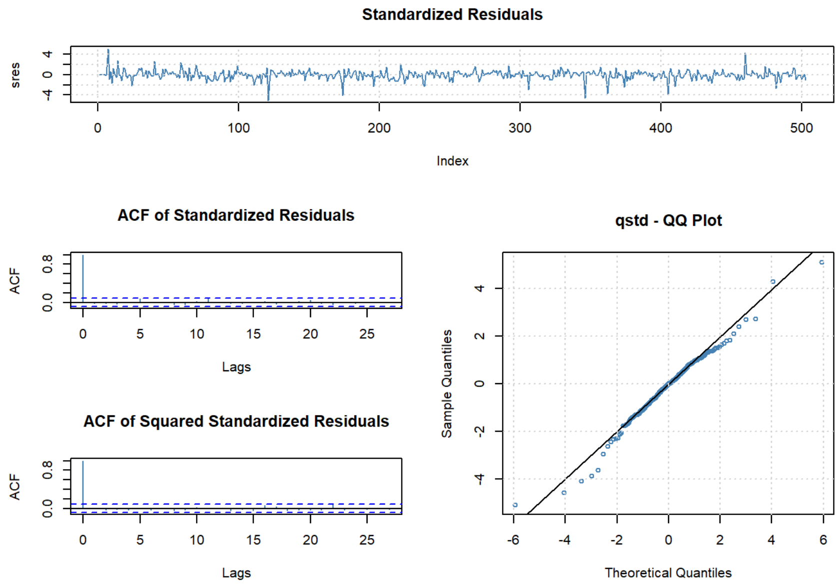


Statistical Analysis: ARIMA

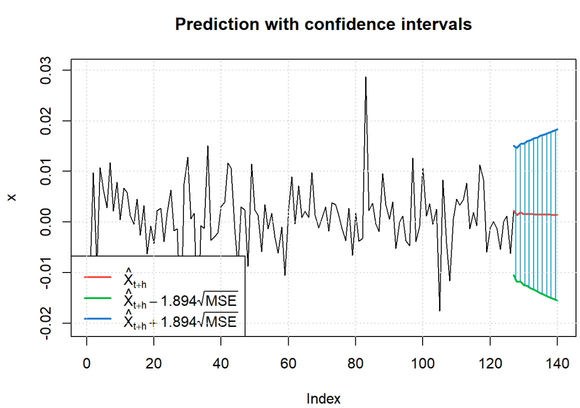
Dickey-Fuller and Phillips-Perron tests indicate no unit root, confirming a stationary mean. The ACF and PACF of the returns show a small amount of auto-correlation, suggesting either an ARMA model or white noise. However, the ACF and PACF of the squared returns show significant auto-correlation, confirming the need for a GARCH model. Both ACF and PACF decay, suggesting GARCH(1,1).

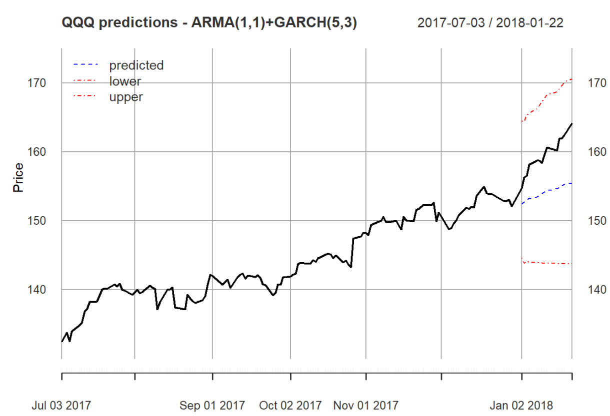


Multiple ARMA+GARCH models are fitted to the series, up to ARMA(5,3)+GARCH(1,1). This model's coefficients are significant and the QQ-plot shows nearly normally distributed residuals. Diagnostic plots and Ljung-Box statistics suggest the model's residuals are white noise. Of the models considered, ARMA(5,3)+GARCH(1,1) also had the lowest AIC score, leading to the conclusion that this model is adequate.



The fitted model is then used to forecast the first 14 days of 2018. Since the model was fitted against daily returns, the predictions are also daily returns, shown below with the prediction interval. The predictions are then converted to prices by inversing the differencing and log transformation. While the actual values are higher than the predictions, they are within the prediction interval, and the predictions do follow the general uptrend of the series. RMSE of the predictions is calculated as 5.68.

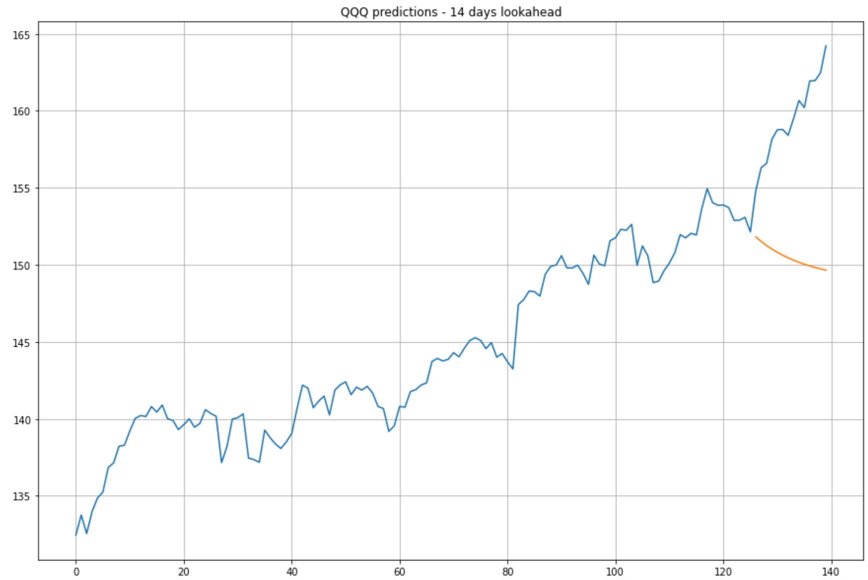




Statistical Analysis: LSTM

As a comparison, an LSTM network is fitted to the daily price series. The LSTM uses the same architecture as presented during class, with 4 LSTM units and a 1-day look back. No analysis of stationarity or model identification is required. The network is simply trained with the price series as an input. Some massaging of the data is required to transform it for input to the LSTM. Network training achieves a minimal loss function value after only a handful of training epochs.

As before, the trained LSTM network is used to predict QQQ prices for the first 14 days of 2018. In this case, the predictions follow the local downtrend rather than the global uptrend. Of note is that the network returns no confidence interval; LSTM simply generates predictions. RMSE is computed as 9.52, higher than the ARMA+GARCH model.



Discussion

Time series analysis of the housing sales series generated an ARIMA model and a regression plus ARIMA errors model. Both models produced similar predictions that matched well with the global trend and seasonal activity of the series. The predictive accuracy of the models was comparable, although the regression model performed slightly better. This is not unexpected as the condition of the housing market generally reflects the prospects of the broader economy.

Analysis of QQQ pricing also generated a pair of models, a classic ARMA+GARCH model and an LSTM network. The predictions from the models were very different, with the GARCH model predictions reflecting more of the global trend and the LSTM predictions capturing the local trend. Experimentation with different look-back parameters for LSTM could perhaps generate models which retain more of the global trend. Also of interest is the fundamental differences between the two models. GARCH is a classic time series technique that produces a descriptive model and consequently can generate a prediction interval. In financial applications, the prediction interval is a vital model output as it reflects the volatility of the price series. LSTM is, to some degree, a black box; capable of generating predictions but not explanations or confidence intervals. Therefore, it requires more extensive backtesting for validation since no model fitting process is performed.