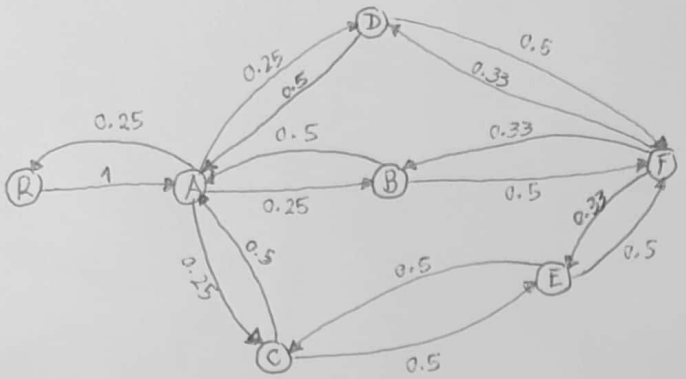
HW1

Eduardo Miranda 95569
Rodrigo Pinto 95666

(a) state space: R, A, B, C, D, E, F

Transition probabilities:



$$P = \begin{array}{c} \\ R \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{ccccccc} R & A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0.25 & 0 & 0.25 & 0.25 & 0.25 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 & 0.5 \\ 0 & 0.5 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0.33 & 0 & 0.33 & 0.33 & 0 \end{bmatrix} \end{array}$$

(b) $X_0 = R$

$$P^2 = \mathbb{P}[X_2 = y \mid X_0 = x]$$

$$\mu_0 = [1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]$$

$$\mu_0 P^2 = [1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0.25 & 0 & 0.25 & 0.25 & 0.25 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 & 0.5 \\ 0 & 0.5 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0.33 & 0 & 0.33 & 0.33 & 0 \end{bmatrix}^2 = [0.25 \ 0 \ 0.25 \ 0.25 \ 0.25 \ 0 \ 0]$$

c) $\begin{cases} t_R = 30 + t_A \\ t_A = 1/4 \times 30 + 1/4 \times (40 + t_B) + 1/4 \times (55 + t_C) + 1/4 \times (70 + t_D) \\ t_B = 1/2 \times (40 + t_A) + 1/2 \times (80 + t_F) \\ t_C = 1/2 \times (55 + t_A) + 1/2 \times (55 + t_E) \\ t_D = 1/2 \times (70 + t_A) + 1/2 \times (70 + t_F) \\ t_E = 1/2 \times (55 + t_C) + 1/2 \times (20 + t_F) \\ t_F = 1/3 \times (80 + t_B) + 1/3 \times (70 + t_D) + 1/3 \times (20 + t_E) \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} t_R = 30 + t_A \\ t_A = 1/4 \times (195 + t_B + t_C + t_D) \\ t_B = 1/2 \times (120 + t_A + t_F) \\ t_C = 1/2 \times (110 + t_A + t_E) \\ t_D = 1/2 \times (140 + t_A + t_F) \\ t_E = 1/2 \times (75 + t_C + t_F) \\ t_F = 1/3 \times (170 + t_B + t_D + t_E) \end{cases}$

Se pegarmos na equação t_D temos:

$2t_D = 140 + t_A + t_F \Leftrightarrow$

$\Leftrightarrow 2t_D - 20 = 120 + t_A + t_F \Leftrightarrow$

$\Leftrightarrow t_D - 10 = 1/2 \times (120 + t_A + t_F) \Leftrightarrow$

$\Leftrightarrow t_D - 10 = t_B$

Logo podemos eliminar a variável t_B substituindo-a por $t_D - 10$. $\Rightarrow$

$\Rightarrow \begin{cases} t_R = 30 + t_A \\ t_A = 1/4 \times (185 + t_C + 2t_D) \\ t_C = 1/2 \times (110 + t_A + t_E) \\ t_D = 1/2 \times (140 + t_A + t_F) \\ t_E = 1/2 \times (75 + t_C + t_F) \\ t_F = 1/3 \times (160 + 2t_D + t_E) \end{cases}$

Pegando na equação t_F e substituindo t_D temos:

$3t_F = 160 + 2 \times \frac{1}{2}(140 + t_A + t_F) + t_E \Leftrightarrow$

$\Leftrightarrow 3t_F - t_F = 300 + t_A + t_E \Leftrightarrow$

$\Leftrightarrow 2t_F - 190 = 110 + t_A + t_E \Leftrightarrow$

$\Leftrightarrow t_F - 95 = 1/2 \times (110 + t_A + t_E) \Leftrightarrow$

$\Leftrightarrow t_F - 95 = t_C \Leftrightarrow t_F = t_C + 95 \Rightarrow$

Agora eliminamos t_F

$\Rightarrow \begin{cases} t_R = 30 + t_A \\ t_A = 1/4 \times (185 + t_C + 2t_D) \\ t_C = 1/2 \times (110 + t_A + t_E) \\ t_D = 1/2 \times (235 + t_A + t_C) \\ t_E = 1/2 \times (170 + 2t_C) \end{cases}$

$\Rightarrow t_E = \frac{1}{2} \times (170 + 2t_C) \Leftrightarrow t_E = 85 + t_C$

agora eliminamos t_E $\Rightarrow$

$\Rightarrow \begin{cases} t_R = 30 + t_A \\ t_A = 1/4 \times (185 + t_C + 2t_D) \\ t_C = 1/2 \times (195 + t_A + t_C) \\ t_D = 1/2 \times (235 + t_A + t_C) \end{cases}$

Pegando na equação t_D temos:

$\Rightarrow 2t_D = 235 + t_A + t_C \Leftrightarrow 2t_D - 40 = 195 + t_A + t_C \Leftrightarrow$

$\Leftrightarrow t_D - 20 = \frac{1}{2} \times (195 + t_A + t_C) \Leftrightarrow t_D - 20 = t_C \Leftrightarrow$

$\Leftrightarrow t_D = t_C + 20$ Agora eliminamos t_D $\Rightarrow$

$$\Rightarrow \begin{cases} t_R = 30 + t_A \\ t_A = 1/4 \times (225 + 3t_c) \\ t_c = 1/2 \times (195 + t_A + t_c) \end{cases}$$

Pegando em t_c temos:

$$\Rightarrow 2t_c - t_c = 195 + t_A \Leftrightarrow t_c = 195 + t_A$$

Agora eliminamos t_c $\Rightarrow$

$$\Rightarrow \begin{cases} t_R = 30 + t_A \\ t_A = 1/4 \times (225 + 3 \times (195 + t_A)) \end{cases} \Leftrightarrow \begin{cases} t_R = 30 + t_A \\ 4t_A - 3t_A = 225 + 585 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} t_R = 30 + 810 \\ t_A = 810 \end{cases} \Rightarrow t_R = 840 \text{ minutos} = 14 \text{ horas}$$

If the truck leaves the recycling plant at 10am on Monday it is expected to return at 24 pm on Monday.