Economic Heterogeneity in a Small Open Economy Framework

A Two-Country DSGE Model for Scotland and the Rest of the UK

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Two-country DSGE model for Scotland and the rest of the UK

My model. Closely follows Galí (2015) and Gali and Monacelli (2005)

1. Home country: *Scotland*
2. Foreign country: *rUK*
3. Scotland and rUK are SOEs
   (a) They trade, take world output, inflation, and consumption as given and cannot influence it
4. Calvo staggered prices, no capital/investment
5. Scotland and RUK are assumed to be symmetrical in market structure and preferences
6. Monetary Union: rUK sets the interest rate based on a weighted inflation and weighted deviation from the steady state;
7. Trade linkage: output gap in Scotland “today” heavily depends on the expected output gap in rUK “tomorrow”
8. Significant price pass-through: inflation in Scotland “today” heavily depends on the expected inflation in rUK “tomorrow”; also nominal exchange rate equal to zero

Ricci (2019) two-country DSGE model for Scotland and the rest of the UK

1. Home country: *Scotland*
2. Foreign country: *rUK*
3. Two governments: Westminister & Holyrood
4. Barnett formula
5. Oil sector
6. Calvo staggered prices and wages, considers capital and investment
7. Does not consider migration (they say it would make the model too complicated)
8. They assume that Scotland and rUK jointly are a small open economy, whereas I model both economies as SOE

Key equations. Non-asterisk terms refer to Scotland, and asterisk terms refer to the rest of the UK.

NK Phillips Curves:

$$\pi_t = \chi_\pi \, \mathbb{E}_t\{\pi_{t+1}\} + (1-\chi_\pi) \, \mathbb{E}_t\{\pi^*_{t+1}\} + \kappa \tilde{y}_t$$
$$\pi^*_t = \chi^*_\pi \, \mathbb{E}_t\{\pi^*_{t+1}\} + (1-\chi^*_\pi) \, \mathbb{E}_t\{\pi_{t+1}\} + \kappa^* \tilde{y}^*_t$$

Dynamic IS Curves:

$$\tilde{y}_t = \chi_y \, \mathbb{E}_t\{\tilde{y}_{t+1}\} + (1-\chi_y) \, \mathbb{E}_t\{\tilde{y}^*_{t+1}\} - \sigma^{-1}(i^*_t - (\chi_\pi \, \mathbb{E}_t\{\pi_{t+1}\} + (1-\chi_\pi) \, \mathbb{E}_t\{\pi^*_{t+1}\}) - i^{nat}_t)$$
$$\tilde{y}^*_t = \chi^*_y \, \mathbb{E}_t\{\tilde{y}^*_{t+1}\} + (1-\chi^*_y) \, \mathbb{E}_t\{\tilde{y}_{t+1}\} - \sigma^{*-1}(i^*_t - (\chi^*_\pi \, \mathbb{E}_t\{\pi^*_{t+1}\} + (1-\chi^*_\pi) \, \mathbb{E}_t\{\pi_{t+1}\}) - i^{*nat}_t)$$

Monetary Policy Rule

$$i^*_t = \rho_i i^*_{t-1} + (1-\rho_i)(\phi_\pi(\varpi \pi_t + (1-\varpi)\pi^*_t) + \phi_y(\varpi \tilde{y}_t + (1-\varpi)\tilde{y}^*_t)) + \epsilon_{i,t}$$

Estimated parameters

$$\chi_\pi \sim \text{Beta}(\varpi, 0.03) \qquad \chi^*_\pi \sim \text{Beta}((1-\varpi), 0.03)$$
$$\chi_y \sim \text{Beta}(\varpi, 0.03) \qquad \chi^*_y \sim \text{Beta}((1-\varpi), 0.03)$$

Weights are given by:

$$\varpi = \frac{\sum_{t=1998Q1}^{2007Q4} \{\text{Population aged 14-65 in Scotland}\}_t}{\sum_{t=1998Q1}^{2007Q4} \{\text{Population aged 14-65 in the UK}\}_t} = 0.0862$$

OR

Estimated parameters

$$\chi_\pi \sim \text{Beta}(\varpi, 0.03) \qquad \chi^*_\pi \sim \text{Beta}(\varpi^*, 0.03)$$
$$\chi_y \sim \text{Beta}(\varpi, 0.03) \qquad \chi^*_y \sim \text{Beta}(\varpi^*, 0.03)$$

Weights are given by:

$$\varpi = \frac{1}{T} \sum_{t=1998Q1}^{T=2007Q4} \text{Scotland's import from rUK}$$
$$\varpi^* = \frac{1}{T} \sum_{t=1998Q1}^{T=2007Q4} \text{rUK's import from Scotland}$$

"No. of governments": Number of governments that can issue bonds and accrue debt to fund public services. In all scenarios, government spending is assumed to be an AR(1) exogenous process.

1. Scenario 1
   (a) No. of governments: 1
   (b) Public expenses funded by (wasteful) <u>lump-sum tax</u>
2. Scenario 2
   (a) No. of governments: 2
   (b) Public expenses funded by (wasteful) <u>lump-sum tax</u>
3. Scenario 3
   (a) No. of governments: 1
   (b) Public expenses funded by (distortionary) <u>income tax</u>
4. Scenario 4
   (a) No. of governments: 2
   (b) Public expenses funded by (distortionary) <u>income tax</u>

S HBC	Household (Scotland) budget constraint
rUK HBC	Household (rUK) budget constraint
S GBC	Government (Holyrood) budget constraint
rUK GBC	Government (Westminister) budget constraint
S TR	Government (Holyrood) tax revenue
rUK TR	Government (Westminister) tax revenue
S RC	Resource (Scotland) constraint
rUK RC	Resource (rUK) constraint

$$\max_{C_t, N_t, B_t/B_t^*} \mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \mathcal{U}(C_t, N_t) \right\} \tag{1}$$

$s.t.$

	S HBC:	$P_tC_t + \mathbb{E}_t[R_{t+1}^{-1}B_{t+1}] = B_t + W_tN_t + T_t$
	rUK HBC:	$P_t^*C_t^* + \mathbb{E}_t[R_{t+1}^{*-1}B_{t+1}^*] = B_t^* + W_t^*N_t^* + T_t^*$
	S GBC:	$\mathbb{E}_t[R_{t+1}^{-1}B_{t+1}] + T_t = P_tG_t + B_t$
Scenario 1	rUK GBC:	$\mathbb{E}_t[R_{t+1}^{*-1}B_{t+1}^*] + T_t^* = P_t^*G_t^* + B_t^*$
(G: 2, $\tau_n : 0$)	S TR:	$T_t = G_t$
	rUK TR:	$T_t^* = G_t^*$
	S RC:	$Y_t = C_t + G_t$
	rUK RC:	$Y_t^* = C_t^* + G_t^*$
	S HBC:	$P_tC_t + \mathbb{E}_t[R_{t+1}^{*-1}B_{t+1}^*] = B_t^* + W_tN_t + \varpi T_t^*$
	rUK HBC:	$P_t^*C_t^* + \mathbb{E}_t[R_{t+1}^{*-1}B_{t+1}^*] = B_t^* + W_t^*N_t^* + (1-\varpi)T_t^*$
	S GBC:	N/A
Scenario 2	rUK GBC:	$\mathbb{E}_t[R_{t+1}^{*-1}B_{t+1}^*] + T_t^* = P_t^*G_t^* + B_t^*$
(G: 1, $\tau_n : 0$)	S TR:	N/A
	rUK TR:	$T_t^* = P_t^*G_t^*$
	S RC:	$Y_t = C_t + \varpi G_t^*$
	rUK RC:	$Y_t^* = C_t^* + (1-\varpi)G_t^*$
	S HBC:	$P_tC_t + \mathbb{E}_t[R_{t+1}^{-1}B_{t+1}] = B_t + (1-\tau_n)W_tN_t + T_t$
	rUK HBC:	$P_t^*C_t^* + \mathbb{E}_t[R_{t+1}^{*-1}B_{t+1}^*] = B_t^* + (1-\tau_n)W_t^*N_t^* + T_t^*$
	S GBC:	$\mathbb{E}_t[R_{t+1}^{-1}B_{t+1}] + T_t = P_tG_t + B_t$
Scenario 3	rUK GBC:	$\mathbb{E}_t[R_{t+1}^{*-1}B_{t+1}^*] + T_t^* = P_t^*G_t^* + B_t^*$
(G: 2, $\tau_n : 1$)	S TR:	$T_t = \tau_nW_tN_t$
	rUK TR:	$T_t^* = \tau_nW_t^*N_t^*$
	S RC:	$Y_t = C_t + G_t$
	rUK RC:	$Y_t^* = C_t^* + G_t^*$
	S HBC:	$P_tC_t + \mathbb{E}_t[R_{t+1}^{*-1}B_{t+1}^*] = B_t^* + (1-\tau_n)W_tN_t + \varpi T_t$
	rUK HBC:	$P_t^*C_t^* + \mathbb{E}_t[R_{t+1}^{*-1}B_{t+1}^*] = B_t^* + (1-\tau_n)W_t^*N_t^* + (1-\varpi)T_t^*$
	S GBC:	N/A
Scenario 4	rUK GBC:	$\mathbb{E}_t[R_{t+1}^{*-1}B_{t+1}^*] + T_t^* = P_t^*G_t^* + B_t^*$
(G: 1, $\tau_n : 1$)	S TR:	N/A
	rUK TR:	$T_t^* = \varpi\tau_nW_tN_t + (1-\varpi)\tau_nW_t^*N_t^*$
	S RC:	$Y_t = C_t + \varpi G_t^*$
	rUK RC:	$Y_t^* = C_t^* + (1-\varpi)G_t^*$

$$T_t^* = \varpi \tau_t W_t N_t + (1-\varpi)\tau_t W_t^* N_t^* \tag{2}$$

$$T^* = \varpi \tau W N + (1-\varpi)\tau W^* N^* \quad \text{(Steady state)} \tag{3}$$

$$\text{Using Uhlig's (1999) method, } X_t Y_t \approx XY\mathbf{e}^{\tilde{X}_t + \tilde{Y}_t}: \tag{4}$$

$$T\mathbf{e}^{\tilde{T}_t^*} = \varpi \tau W N \mathbf{e}^{\tilde{\tau}_t + \tilde{W}_t + \tilde{N}_t} + (1-\varpi)\tau W^* N^* \mathbf{e}^{\tilde{\tau}_t + \tilde{W}_t^* + \tilde{N}_T^*} \tag{5}$$

$$\text{Using } \mathbf{e}^{\tilde{X}_t} \approx 1 + \tilde{X}_t: \tag{6}$$

$$T^*(1+\tilde{T}_t^*) = \varpi \tau W N (1 + \tilde{\tau}_t + \tilde{W}_t + \tilde{N}_t) + (1-\varpi)\tau W^* N^* (1 + \tilde{\tau}_t + \tilde{W}_t^* + \tilde{N}_T^*) \tag{7}$$

$$\text{Subtract (3):} \tag{8}$$

$$T^*(1+\tilde{T}_t^*) - T^* = \varpi \tau W N (1 + \tilde{\tau}_t + \tilde{W}_t + \tilde{N}_t) - \varpi \tau W N + (1-\varpi)\tau W^* N^* (1 + \tilde{\tau}_t + \tilde{W}_t^* + \tilde{N}_T^*) - (1-\varpi)\tau W^* N^* \tag{9}$$

$$T^*[(1+\tilde{T}_t^*) - 1] = \varpi \tau W N [(1 + \tilde{\tau}_t + \tilde{W}_t + \tilde{N}_t) - 1] + (1-\varpi)\tau W^* N^* [(1 + \tilde{\tau}_t + \tilde{W}_t^* + \tilde{N}_T^*) - 1] \tag{10}$$

$$T^* \tilde{T}_t^* = \varpi \tau W N (\tilde{\tau}_t + \tilde{W}_t + \tilde{N}_t) + (1-\varpi)\tau W^* N^* (\tilde{\tau}_t + \tilde{W}_t^* + \tilde{N}_T^*) \tag{11}$$

$$\text{Divide by } T^*: \tag{12}$$

$$\tilde{T}_t^* = \frac{\tau W N}{T^*} \times \varpi (\tilde{\tau}_t + \tilde{W}_t + \tilde{N}_t) + \frac{\tau W^* N^*}{T^*} \times (1-\varpi)(\tilde{\tau}_t + \tilde{W}_t^* + \tilde{N}_t^*) \tag{13}$$

$$C_t^\sigma N_t^\varphi = \frac{W_t}{P_t}(1-\tau_t) \tag{14}$$

$$C_t^\sigma N_t^\varphi = \frac{W_t}{P_t} - \frac{W_t}{P_t}\tau_t \tag{15}$$

$$C^\sigma N^\varphi = \frac{W}{P} - \frac{W}{P}\tau \quad \text{(Steady state)} \tag{16}$$

$$\text{Using Uhlig's (1999) method, } X_t Y_t \approx XY\mathbf{e}^{\tilde{X}_t + \tilde{Y}_t}: \tag{17}$$

$$C^\sigma N^\varphi \mathbf{e}^{\sigma c_t + \varphi n_t} = \frac{W}{P}\mathbf{e}^{w_t - p_t} - \frac{W}{P}\tau \mathbf{e}^{w_t - p_t + \tilde{\tau}_t} \tag{18}$$

$$\text{Using } \mathbf{e}^{\tilde{X}_t} \approx 1 + \tilde{X}_t: \tag{19}$$

$$C^\sigma N^\varphi (1 + \sigma c_t + \varphi n_t) = \frac{W}{P}(1 + w_t - p_t) - \frac{W}{P}\tau(1 + w_t - p_t + \tilde{\tau}_t) \tag{20}$$

$$\text{Subtract (16):} \tag{21}$$

$$C^\sigma N^\varphi (\sigma c_t + \varphi n_t) = \frac{W}{P}(w_t - p_t) - \frac{W}{P}\tau(w_t - p_t + \tilde{\tau}_t) \tag{22}$$

$$C^\sigma N^\varphi (\sigma c_t + \varphi n_t) = \frac{W}{P}[(w_t - p_t) - \tau(w_t - p_t + \tilde{\tau}_t)] \tag{23}$$

$$C^\sigma N^\varphi (\sigma c_t + \varphi n_t) = \frac{W}{P}[(w_t - p_t) - \tau(w_t - p_t) - \tau\tilde{\tau}_t] \tag{24}$$

$$C^\sigma N^\varphi (\sigma c_t + \varphi n_t) = \frac{W}{P}[(1-\tau)(w_t - p_t) - \tau\tilde{\tau}_t] \tag{25}$$

$$C^\sigma N^\varphi \frac{P}{W}(\sigma c_t + \varphi n_t) = [(1-\tau)(w_t - p_t) - \tau\tilde{\tau}_t] \tag{26}$$

$$(1-\tau)(\sigma c_t + \varphi n_t) = [(1-\tau)(w_t - p_t) - \tau\tilde{\tau}_t] \tag{27}$$

$$\sigma c_t + \varphi n_t = w_t - p_t - \frac{\tau}{1-\tau}\tilde{\tau}_t \tag{28}$$

$$\tag{29}$$

$$
\begin{align}
y_t &= a_t + (1-\alpha)n_t \tag{30}\\
n_t &= \frac{1}{1-\alpha}(y_t - a_t) \tag{31}\\
\varpi &= \sigma\eta + (1-\nu)(\sigma\eta - 1) \tag{32}\\
\sigma c_t + \varphi n_t &= w_t - p_t - \frac{1}{1-\tau}\tau_t \tag{33}\\
y_t &= (1-\nu)c_t + \nu_t(2-\nu)\eta s_t + \nu y_t^* \tag{34}\\
c_t &= y_t^* + \frac{1}{\sigma}z_t + \frac{1-\nu}{\sigma}s_t \tag{35}\\
\implies c_t &= y_t - \nu s_t\left((2-\nu)\eta + \frac{1-\nu}{\sigma}\right) + \frac{\nu}{\sigma}z_t \tag{36}
\end{align}
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\begin{align}
\mu_t &= -(\sigma c_t + \varphi n_t + \frac{1}{1-\tau}\tau_t) - \nu s_t + a_t - \alpha n_t \tag{37}\\
&= -\sigma c_t - \varphi n_t - \frac{1}{1-\tau}\tau_t - \nu s_t + a_t - \alpha n_t \tag{38}\\
&= -\sigma c_t - \frac{1}{1-\tau}\tau_t - \nu s_t + a_t - n_t(\varphi + \alpha) \tag{39}\\
& \tag{40}
\end{align}
$$

$$
\begin{align}
-n_t(\varphi+\alpha) = -\frac{1}{1-\alpha}(y_t - a_t)(\varphi+\alpha) \tag{41}\\
-n_t(\varphi+\alpha) = -\frac{\varphi+\alpha}{1-\alpha}(y_t - a_t) \tag{42}\\
-n_t(\varphi+\alpha) = -\frac{\varphi+\alpha}{1-\alpha}y_t + \frac{\varphi+\alpha}{1-\alpha}a_t \tag{43}\\
\tag{44}
\end{align}
$$

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\begin{align}
\mu_t &= -\sigma c_t - \frac{1}{1-\tau}\tau_t - \nu s_t + a_t - \frac{\varphi+\alpha}{1-\alpha}y_t + \frac{\varphi+\alpha}{1-\alpha}a_t \tag{45}\\
\mu_t &= -\sigma c_t - \frac{1}{1-\tau}\tau_t - \nu s_t + \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)a_t - \frac{\varphi+\alpha}{1-\alpha}y_t \tag{46}\\
\mu_t &= -\sigma\left(y_t - \nu s_t\left((2-\nu)\eta + \frac{1-\nu}{\sigma}\right) + \frac{\nu}{\sigma}z_t\right) - \frac{1}{1-\tau}\tau_t - \nu s_t + \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)a_t - \frac{\varphi+\alpha}{1-\alpha}y_t \tag{47}\\
\mu_t &= -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)y_t + \sigma\nu s_t\left((2-\nu)\eta + \frac{1-\nu}{\sigma}\right) - \nu z_t - \frac{1}{1-\tau}\tau_t - \nu s_t + \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)a_t \tag{48}\\
\mu_t &= -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)y_t + \nu s_t\left(\sigma\left((2-\nu)\eta + \frac{1-\nu}{\sigma}\right) - 1\right) - \frac{1}{1-\tau}\tau_t + \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)a_t - \nu z_t \tag{49}\\
\mu_t &= -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)y_t + \nu s_t\left((2-\nu)\sigma\eta + 1 - \nu\right) - 1 - \frac{1}{1-\tau}\tau_t + \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)a_t - \nu z_t \tag{50}\\
\mu_t &= -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)y_t + \nu s_t\left((2-\nu)\sigma\eta - \nu\right) - \frac{1}{1-\tau}\tau_t + \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)a_t - \nu z_t \tag{51}\\
\mu_t &= -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)y_t + \nu s_t\left(2\sigma\eta - \nu\sigma\eta - \nu\right) - \frac{1}{1-\tau}\tau_t + \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)a_t - \nu z_t \tag{52}\\
\mu_t &= -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)y_t + \nu(\varpi - 1)s_t - \frac{1}{1-\tau}\tau_t + \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)a_t - \nu z_t \tag{53}
\end{align}
$$

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\begin{align}
\pi_{H,t} &= \beta\,\mathbb{E}\{\pi_{H,t+1}\} + \lambda\hat{\mu}_t \implies \pi_{H,t} = \beta\,\mathbb{E}\{\pi_{H,t+1}\} - \lambda\mu_t \tag{54}\\
\pi_{H,t} &= \beta\,\mathbb{E}\{\pi_{H,t+1}\} + \lambda\left(\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)y_t - \nu(\varpi - 1)s_t + \frac{1}{1-\tau}\tau_t - \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)a_t + \nu z_t\right) \tag{55}\\
\pi_{H,t} &= \beta\,\mathbb{E}\{\pi_{H,t+1}\} + \kappa y_t + \lambda\left(-\nu(\varpi - 1)s_t + \frac{1}{1-\tau}\tau_t - \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)a_t + \nu z_t\right) \tag{56}\\
\pi_{H,t} &= \beta\,\mathbb{E}\{\pi_{H,t+1}\} + \kappa y_t - \lambda\nu(\varpi - 1)s_t + \lambda\frac{1}{1-\tau}\tau_t - \lambda\left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)a_t + \lambda\nu z_t \tag{57}\\
& \tag{58}
\end{align}
$$

$$
\begin{aligned}
\mu_t &= -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n + \nu(\varpi-1)s_t^n - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right) a_t - \nu z_t \\
\mu_t - \left(\nu(\varpi-1)s_t^n - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right) a_t - \nu z_t\right) &= -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n \\
\nu(\varpi-1)s_t^n - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right) a_t - \nu z_t - \mu_t &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n \\
\nu(\varpi-1)s_t^n - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right) a_t - \nu z_t - \mu_t &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n \\
\nu(\varpi-1)(\sigma_\nu(y_t^n - y_t^*) - (1-\nu)\Phi z_t - \frac{G}{Y}\sigma_\nu g_t) - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right) a_t - \nu z_t - \mu_t &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n \\
(-1+1+\nu(\varpi-1))(\sigma_\nu(y_t^n - y_t^*) - (1-\nu)\Phi z_t - \frac{G}{Y}\sigma_\nu g_t) - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right) a_t - \nu z_t - \mu_t &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n \\
(-1+\Phi^{-1})(\sigma_\nu(y_t^n - y_t^*) - (1-\nu)\Phi z_t - \frac{G}{Y}\sigma_\nu g_t) - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right) a_t - \nu z_t - \mu_t &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n \\
(-1+\Phi^{-1})(\sigma_\nu y_t^n - \sigma_\nu y_t^* - (1-\nu)\Phi z_t - \frac{G}{Y}\sigma_\nu g_t) - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right) a_t - \nu z_t - \mu_t &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n \\
(-1+\Phi^{-1})\sigma_\nu y_t^n + (1-\Phi^{-1})\sigma_\nu y_t^* + (1-\Phi^{-1})(1-\nu)\Phi z_t + (1-\Phi^{-1})\frac{G}{Y}\sigma_\nu g_t - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right) a_t - \nu z_t - \mu_t &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n \\
(-1+\Phi^{-1})\sigma\Phi y_t^n + (1-\Phi^{-1})\sigma_\nu y_t^* + (1-\Phi^{-1})(1-\nu)\Phi z_t + (1-\Phi^{-1})\frac{G}{Y}\sigma_\nu g_t - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right) a_t - \nu z_t - \mu_t &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n \\
-\sigma\Phi y_t^n + \sigma y_t^n + (1-\Phi^{-1})\sigma_\nu y_t^* + (1-\Phi^{-1})(1-\nu)\Phi z_t + (1-\Phi^{-1})\frac{G}{Y}\sigma_\nu g_t - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right) a_t - \nu z_t - \mu_t &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n \\
(1-\Phi)\sigma y_t^n + (1-\Phi^{-1})\sigma_\nu y_t^* + (1-\Phi^{-1})(1-\nu)\Phi z_t + (1-\Phi^{-1})\frac{G}{Y}\sigma_\nu g_t - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right) a_t - \nu z_t - \mu_t &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n \\
(1-\Phi^{-1})\sigma_\nu y_t^* + (1-\Phi^{-1})(1-\nu)\Phi z_t + (1-\Phi^{-1})\frac{G}{Y}\sigma_\nu g_t - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right) a_t - \nu z_t - \mu_t &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n - (1-\Phi)\sigma y_t^n \\
(1-\Phi^{-1})\sigma_\nu y_t^* + (1-\Phi^{-1})(1-\nu)\Phi z_t + (1-\Phi^{-1})\frac{G}{Y}\sigma_\nu g_t - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right) a_t - \nu z_t - \mu_t &= \left(\frac{\sigma(1-\alpha)+\varphi+\alpha-(1-\Phi)\sigma(1-\alpha)}{1-\alpha}\right) y_t^n \\
(1-\Phi^{-1})\sigma_\nu y_t^* + (1-\Phi^{-1})(1-\nu)\Phi z_t + (1-\Phi^{-1})\frac{G}{Y}\sigma_\nu g_t - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right) a_t - \nu z_t - \mu_t &= \left(\frac{\sigma(1-\alpha)+\varphi+\alpha-(\sigma(1-\alpha)-\sigma\Phi(1-\alpha))}{1-\alpha}\right) y_t^n \\
(1-\Phi^{-1})\sigma_\nu y_t^* + (1-\Phi^{-1})(1-\nu)\Phi z_t + (1-\Phi^{-1})\frac{G}{Y}\sigma_\nu g_t - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right) a_t - \nu z_t - \mu_t &= \left(\frac{-\sigma_\nu(1-\alpha)+\varphi+\alpha}{1-\alpha}\right) y_t^n \\
\left(\frac{-\sigma_\nu(1-\alpha)+\varphi+\alpha}{1-\alpha}\right)^{-1}\left((1-\Phi^{-1})\sigma_\nu y_t^* + (1-\Phi^{-1})(1-\nu)\Phi z_t - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right) a_t - \nu z_t - \mu_t\right) &= y_t^n \\
\Gamma_* y_t^* + \left(\frac{-\sigma_\nu(1-\alpha)+\varphi+\alpha}{1-\alpha}\right)^{-1}\left((1-\Phi^{-1})(1-\nu)\Phi z_t + (1-\Phi^{-1})\frac{G}{Y}\sigma_\nu g_t - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right) a_t - \nu z_t - \mu_t\right) &= y_t^n \\
\Gamma_* y_t^* + \left(\frac{-\sigma_\nu(1-\alpha)+\varphi+\alpha}{1-\alpha}\right)^{-1}\left(\nu\varpi\Phi z_t + (1-\Phi^{-1})\frac{G}{Y}\sigma_\nu g_t - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right) a_t - \mu_t\right) &= y_t^n \\
\Gamma_* y_t^* + \Gamma_z z_t + \Gamma_a a_t + \left(\frac{-\sigma_\nu(1-\alpha)+\varphi+\alpha}{1-\alpha}\right)^{-1}\left((1-\Phi^{-1})\frac{G}{Y}\sigma_\nu g_t - \frac{1}{1-\tau}\tau_t - \mu_t\right) &= y_t^n \\
\Gamma_* y_t^* + \Gamma_z z_t + \Gamma_a a_t + \Gamma_g g_t + \left(\frac{-\sigma_\nu(1-\alpha)+\varphi+\alpha}{1-\alpha}\right)^{-1}\left(-\frac{1}{1-\tau}\tau_t\right) &= y_t^n \\
\Gamma_* y_t^* + \Gamma_z z_t + \Gamma_a a_t + \Gamma_g g_t + \Gamma_\tau \tau_t &= y_t^n
\end{aligned}
$$

$$y_t^n = \Gamma_* y_t^* + \Gamma_z z_t + \Gamma_a a_t + \Gamma_\tau \tau_t \tag{59}$$

$$\Gamma_* = -\frac{\nu(\varpi-1)\sigma_\nu(1-\alpha)}{\sigma_\nu(1-\alpha)+\varphi+\alpha} \tag{60}$$

$$\Gamma_z = -\frac{\nu\varpi\Phi(1-\alpha)}{\sigma_\nu(1-\alpha)+\varphi+\alpha} \tag{61}$$

$$\Gamma_a = \frac{1+\varpi}{\sigma_\nu(1-\alpha)+\varphi+\alpha} \tag{62}$$

$$\Gamma_g = -\frac{\nu(\varpi-1)\sigma_\nu(1-\alpha)}{\sigma_\nu(1-\alpha)+\varphi+\alpha} \times \frac{G}{Y} \tag{63}$$

$$\Gamma_\tau = \frac{(1-\tau)(1-\alpha)}{\sigma_\nu(1-\alpha)+\varphi+\alpha} \tag{64}$$

$$c_t = \mathbb{E}\{c_{t+1}\} - \frac{1}{\sigma}(i_t - \mathbb{E}\{\pi_{t+1}\} - \rho) + \frac{1}{\sigma}(1-\rho_z)z_t \tag{65}$$

$$\pi_t = \pi_{H,t} + \nu \Delta s_t \tag{66}$$

$$\implies c_t = \mathbb{E}\{c_{t+1}\} - \frac{1}{\sigma}(i_t - \mathbb{E}\{\pi_{H,t+1}\} - \rho) + \frac{\nu}{\sigma}\mathbb{E}\{\Delta s_{t+1}\} + \frac{1}{\sigma}(1-\rho_z)z_t \tag{67}$$

$$c_t = y_t^* + \frac{1}{\sigma}z_t + \left(\frac{1-\nu}{\sigma}\right)s_t \tag{68}$$

$$Y_t(i) = C(i)_t + X(i)_t + G(i)_t \tag{69}$$

$$= \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon}\left[(1-\nu)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t + \nu\mathcal{S}_t^{\eta}Y_t^* + G_t\right] \tag{70}$$

Given that

$$Y_t = \left(\int_0^1 (Y_t(i))^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{71}$$

$$Y_t = (1-\nu)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t + \nu\mathcal{S}_t^{\eta}Y_t^* + G_t \tag{72}$$

$$Y\mathbf{e}^{y_t} = (1-\nu)\left(\frac{P}{P_H}\right)^{\eta}C\mathbf{e}^{-\eta p_{H,t}+\eta p_t+c_t} + \nu S^{\eta}Y^*\mathbf{e}^{\eta s_t+y_t^*} + G\mathbf{e}^{g_t} \tag{73}$$

$$Y(1+y_t) = (1-\nu)\left(\frac{P}{P_H}\right)^{\eta}C(1-\eta p_{H,t}+\eta p_t+c_t) + \nu S^{\eta}Y^*(1+\eta s_t+y_t^*) + G(1+g_t) \tag{74}$$

$$Yy_t = (1-\nu)\left(\frac{P}{P_H}\right)^{\eta}C(-\eta p_{H,t}+\eta p_t+c_t) + \nu S^{\eta}Y^*(\eta s_t+y_t^*) + Gg_t \tag{75}$$

$$\text{Using } P/P_H = \nu\mathcal{S},\ \mathcal{S}=1, \text{ and } C = Y^* : \tag{76}$$

$$y_t = (1-\nu)(-\eta p_{H,t}+\eta p_t+c_t) + \nu(\eta s_t+y_t^*) + \frac{G}{Y}g_t \tag{77}$$

$$\text{Using } p_t - p_{H,t} = \nu s_t : \tag{78}$$

$$y_t = (1-\nu)(\eta\nu s_t+c_t) + \nu(\eta s_t+y_t^*) + \frac{G}{Y}g_t \tag{79}$$

$$y_t = (1-\nu)c_t + (1-\nu)\eta\nu s_t + \nu\eta s_t + \nu y_t^* + \frac{G}{Y}g_t \tag{80}$$

$$y_t = (1-\nu)c_t + \nu(2-\nu)\eta s_t + \nu y_t^* + \frac{G}{Y}g_t \tag{81}$$

$$\tag{82}$$

$$c_t = y_t^* + \frac{1}{\sigma} z_t + \frac{1-\nu}{\sigma} s_t \tag{83}$$

$$y_t = (1-\nu) c_t + \nu(2-\nu)\eta s_t + \nu y_t^* + \frac{G}{Y} g_t \tag{84}$$

$$\implies y_t = (1-\nu)\left(y_t^* + \frac{1}{\sigma} z_t + \frac{1-\nu}{\sigma} s_t \right) + \nu(2-\nu)\eta s_t + \nu y_t^* + \frac{G}{Y} g_t \tag{85}$$

$$y_t = (1-\nu) y_t^* + \frac{1-\nu}{\sigma} z_t + (1-\nu)\frac{1-\nu}{\sigma} s_t + \nu(2-\nu)\eta s_t + \nu y_t^* + \frac{G}{Y} g_t \tag{86}$$

$$y_t = y_t^* + \frac{1-\nu}{\sigma} z_t + (1-\nu)\frac{1-\nu}{\sigma} s_t + \nu(2-\nu)\eta s_t + \frac{G}{Y} g_t \tag{87}$$

$$y_t = y_t^* + \frac{1-\nu}{\sigma} z_t + \left((1-\nu)\frac{1-\nu}{\sigma} + \nu(2-\nu)\eta \right) s_t + \frac{G}{Y} g_t \tag{88}$$

$$\left((1-\nu)\frac{1-\nu}{\sigma} + \nu(2-\nu)\eta \right) s_t = y_t - y_t^* - \frac{1-\nu}{\sigma} z_t - \frac{G}{Y} g_t \tag{89}$$

$$\left(\frac{(1-\nu)(1-\nu) + \sigma\nu(2-\nu)\eta}{\sigma} \right) s_t = y_t - y_t^* - \frac{1-\nu}{\sigma} z_t - \frac{G}{Y} g_t \tag{90}$$

$$\left(\frac{1-2\nu+\nu^2 + \sigma\nu(2-\nu)\eta}{\sigma} \right) s_t = y_t - y_t^* - \frac{1-\nu}{\sigma} z_t - \frac{G}{Y} g_t \tag{91}$$

$$\left(\frac{1-\nu(2-\nu) + \sigma\nu(2-\nu)\eta}{\sigma} \right) s_t = y_t - y_t^* - \frac{1-\nu}{\sigma} z_t - \frac{G}{Y} g_t \tag{92}$$

$$\left(\frac{1-\nu(2-\nu)(1-\sigma\eta)}{\sigma} \right) s_t = y_t - y_t^* - \frac{1-\nu}{\sigma} z_t - \frac{G}{Y} g_t \tag{93}$$

$$\left(\frac{1+\nu(2-\nu)(\sigma\eta-1)}{\sigma} \right) s_t = y_t - y_t^* - \frac{1-\nu}{\sigma} z_t - \frac{G}{Y} g_t \tag{94}$$

$$\left(\frac{1+\nu(2\sigma\eta - \nu\sigma\eta - 2 + \nu)}{\sigma} \right) s_t = y_t - y_t^* - \frac{1-\nu}{\sigma} z_t - \frac{G}{Y} g_t \tag{95}$$

$$\left(\frac{1+\nu(\varpi-1)}{\sigma} \right) s_t = y_t - y_t^* - \frac{1-\nu}{\sigma} z_t - \frac{G}{Y} g_t \tag{96}$$

$$\sigma^{-1}\Phi^{-1} s_t = y_t - y_t^* - \frac{1-\nu}{\sigma} z_t - \frac{G}{Y} g_t \tag{97}$$

$$s_t = \sigma\Phi(y_t - y_t^*) - (1-\nu)\Phi z_t - \frac{G}{Y}\sigma\Phi g_t \tag{98}$$

$$\begin{aligned} s_t &= \sigma_\nu(y_t - y_t^*) - (1-\nu)\Phi z_t - \frac{G}{Y}\sigma_\nu g_t \qquad (99) \\ & \qquad (100) \end{aligned}$$

$$\Phi = \frac{1}{1+\nu(\varpi-1)} \tag{101}$$

$$\varpi = \sigma\eta + (1-\nu)(\sigma\eta-1) = 2\sigma\eta - \nu\sigma\eta - 1 + \nu \tag{102}$$

$$\sigma_\nu = \sigma\Phi \tag{103}$$

$$
\begin{aligned}
& y_t = (1-\nu)c_t + \nu(2-\nu)\eta s_t + \nu y_t^* + G_Y g_t && (104)\\
\implies\; & c_t = (1-\nu)^{-1}(y_t - \nu(2-\nu)\eta s_t - \nu y_t^* - G_Y g_t) && (105)\\
& c_t = \mathbb{E}\{c_{t+1}\} - \frac{1}{\sigma}(i_t - \mathbb{E}\{\pi_{t+1}\} - \rho) + \frac{\nu}{\sigma}\mathbb{E}\{\Delta s_{t+1}\} + \frac{1}{\sigma}(1-\rho_z)z_t && (106)\\
& s_t = \sigma_\nu(y_t - y_t^*) - (1-\nu)\Phi z_t - \frac{G}{Y}\sigma_\nu g_t && (107)\\
& && (108)
\end{aligned}
$$

$$
\begin{aligned}
0 &= \mathbb{E}\{\Delta c_{t+1}\} - \frac{1}{\sigma}(i_t - \mathbb{E}\{\pi_{H,+1}\} - \rho) + \frac{\nu}{\sigma}\mathbb{E}\{\Delta s_{t+1}\} + \frac{1}{\sigma}(1-\rho_z)z_t\\
0 &= \mathbb{E}\{\Delta y_{t+1}\} - \nu(2-\nu)\eta\,\mathbb{E}\{\Delta s_{t+1}\} - \nu\,\mathbb{E}\{\Delta y_{t+1}^*\} - G_Y\Delta g_{t+1} - \frac{1-\nu}{\sigma}(i_t - \mathbb{E}\{\pi_{H,+1}\} - \rho) + \frac{(1-\nu)\nu}{\sigma}\mathbb{E}\{\Delta s_{t+1}\} + \frac{1-\nu}{\sigma}(1-\rho_z)z_t\\
0 &= \mathbb{E}\{\Delta y_{t+1}\} + \left(\frac{(1-\nu)\nu}{\sigma} - \nu(2-\nu)\eta\right)\mathbb{E}\{\Delta s_{t+1}\} - \nu\,\mathbb{E}\{\Delta y_{t+1}^*\} - G_Y\Delta g_{t+1} - \frac{1-\nu}{\sigma}(i_t - \mathbb{E}\{\pi_{H,+1}\} - \rho) + \frac{1-\nu}{\sigma}(1-\rho_z)z_t\\
0 &= \mathbb{E}\{\Delta y_{t+1}\} - \left(\frac{\nu\varpi}{\sigma}\right)\mathbb{E}\{\Delta s_{t+1}\} - \nu\,\mathbb{E}\{\Delta y_{t+1}^*\} - G_Y\Delta g_{t+1} - \frac{1-\nu}{\sigma}(i_t - \mathbb{E}\{\pi_{H,+1}\} - \rho) + \frac{1-\nu}{\sigma}(1-\rho_z)z_t\\
0 &= \mathbb{E}\{\Delta y_{t+1}\} - \left(\frac{\nu\varpi}{\sigma}\right)\mathbb{E}\left(\sigma_\nu(\Delta y_{t+1} - \Delta y_{t+1}^*) - (1-\nu)\Phi(\Delta z_{t+1}) - G_Y\sigma_\nu\Delta g_{t+1}\right) - \nu\,\mathbb{E}\{\Delta y_{t+1}^*\} - \frac{1-\nu}{\sigma}(i_t - \mathbb{E}\{\pi_{H,+1}\} - \rho) + \frac{1-\nu}{\sigma}(1-\rho_z)z_t\\
0 &= \mathbb{E}\{\Delta y_{t+1}\} - \nu\varpi\Phi\Delta y_{t+1} + \nu\varpi\Phi\Delta y_{t+1}^* - \nu\,\mathbb{E}\{\Delta y_{t+1}^*\} + \frac{1-\nu}{\sigma}(1-\rho_z)z_t + \nu\varpi\Phi\frac{1-\nu}{\sigma}(\rho_z-1)z_t + \nu\varpi\Phi G_Y\Delta g_{t+1} - G_Y\Delta g_{t+1} - \frac{1-\nu}{\sigma}(i_t - \mathbb{E}\{\pi_{H,+1}\\
0 &= \Phi^{-1}\,\mathbb{E}\{\Delta y_{t+1}\} - \nu\varpi\Delta y_{t+1} + \nu\varpi\Delta y_{t+1}^* - \nu\Phi^{-1}\,\mathbb{E}\{\Delta y_{t+1}^*\}\\
&\quad + \Phi^{-1}\frac{1-\nu}{\sigma}(1-\rho_z)z_t + \nu\varpi\frac{1-\nu}{\sigma}(\rho_z-1)z_t + \nu\varpi G_Y\Delta g_{t+1} - G_Y\Delta g_{t+1}\\
&\quad - \frac{1-\nu}{\sigma_\nu}(i_t - \mathbb{E}\{\pi_{H,+1}\} - \rho)\\
0 &= (\Phi^{-1} - \nu\varpi)\Delta y_{t+1} + \nu(\varpi - \Phi^{-1})\,\mathbb{E}\{\Delta y_{t+1}^*\}\\
&\quad - (\Phi^{-1} - \nu\varpi)\frac{1-\nu}{\sigma}(\rho_z-1)z_t + (\nu\varpi-1)G_Y\Delta g_{t+1}\\
&\quad - \frac{1-\nu}{\sigma_\nu}(i_t - \mathbb{E}\{\pi_{H,+1}\} - \rho)\\
0 &= \Delta y_{t+1} + \frac{\nu(\varpi - \Phi^{-1})}{\Phi^{-1} - \nu\varpi}\,\mathbb{E}\{\Delta y_{t+1}^*\}\\
&\quad - \frac{1-\nu}{\sigma}(\rho_z-1)z_t + \frac{(\nu\varpi-1)}{\Phi^{-1} - \nu\varpi}G_Y\Delta g_{t+1}\\
&\quad - \frac{1-\nu}{\Phi^{-1} - \nu\varpi}\frac{1}{\sigma_\nu}(i_t - \mathbb{E}\{\pi_{H,+1}\} - \rho)\\
y_t &= y_{t+1} - \frac{1-\nu}{\sigma}(\rho_z-1)z_t\\
&\quad + \frac{(\nu\varpi-1)}{\Phi^{-1} - \nu\varpi}G_Y\Delta g_{t+1} + \frac{\nu(\varpi - \Phi^{-1})}{\Phi^{-1} - \nu\varpi}\,\mathbb{E}\{\Delta y_{t+1}^*\} - \frac{1-\nu}{\Phi^{-1} - \nu\varpi}\frac{1}{\sigma_\nu}(i_t - \mathbb{E}\{\pi_{H,+1}\} - \rho)\\
y_t &= y_{t+1} - \frac{1-\nu}{\sigma}(\rho_z-1)z_t - \frac{1-\nu}{1-\nu}\frac{1}{\sigma_\nu}(i_t - \mathbb{E}\{\pi_{H,+1}\} - \rho) + \frac{(1-\nu)(\nu\varpi-\nu)}{1-\nu}\,\mathbb{E}\{\Delta y_{t+1}^*\} + \frac{\nu\varpi-1}{1-\nu}G_Y\Delta g_{t+1}\\
y_t &= y_{t+1} - \frac{1}{\sigma_\nu}(i_t - \mathbb{E}\{\pi_{H,+1}\} - \rho) + \nu(\varpi-1)\,\mathbb{E}\{\Delta y_{t+1}^*\} + \frac{1-\nu}{\sigma}(1-\rho_z)z_t + \frac{\nu\varpi-1}{1-\nu}G_Y\Delta g_{t+1}
\end{aligned}
$$

$$
\begin{aligned}
y_t^n &= y_{t+1}^n - \frac{1}{\sigma_\nu}(r_t^n - \rho) + \nu(\varpi-1)\,\mathbb{E}\{\Delta y_{t+1}^*\} + \frac{1-\nu}{\sigma}(1-\rho_z)z_t + \frac{\nu\varpi-1}{1-\nu}G_Y\Delta g_{t+1} && (109)\\
0 &= \Delta y_{t+1}^n - \frac{1}{\sigma_\nu}(r_t^n - \rho) + \nu(\varpi-1)\,\mathbb{E}\{\Delta y_{t+1}^*\} + \frac{1-\nu}{\sigma}(1-\rho_z)z_t + \frac{\nu\varpi-1}{1-\nu}G_Y\Delta g_{t+1} && (110)\\
0 &= \sigma_\nu\Delta y_{t+1}^n - (r_t^n - \rho) + \sigma_\nu\nu(\varpi-1)\,\mathbb{E}\{\Delta y_{t+1}^*\} + \Phi(1-\nu)(1-\rho_z)z_t + \sigma_\nu\frac{\nu\varpi-1}{1-\nu}G_Y\Delta g_{t+1} && (111)\\
r_t^n &= \sigma_\nu\Delta y_{t+1}^n + \rho + \sigma_\nu\nu(\varpi-1)\,\mathbb{E}\{\Delta y_{t+1}^*\} + \Phi(1-\nu)(1-\rho_z)z_t + \sigma_\nu\frac{\nu\varpi-1}{1-\nu}G_Y\Delta g_{t+1} && (112)\\
& && (113)
\end{aligned}
$$

$$
\begin{aligned}
0 &= \sigma_\nu\Delta y_{t+1} - (i_t - \mathbb{E}\{\pi_{H,+1}\}) + \rho + \sigma_\nu\nu(\varpi-1)\,\mathbb{E}\{\Delta y_{t+1}^*\} + \Phi(1-\nu)(1-\rho_z)z_t + \sigma_\nu\frac{\nu\varpi-1}{1-\nu}G_Y\Delta g_{t+1} && (114)\\
&\quad - (\sigma_\nu\Delta y_{t+1}^n - (r_t^n - \rho) + \sigma_\nu\nu(\varpi-1)\,\mathbb{E}\{\Delta y_{t+1}^*\} + \Phi(1-\nu)(1-\rho_z)z_t + \sigma_\nu\frac{\nu\varpi-1}{1-\nu}G_Y\Delta g_{t+1}) && (115)\\
0 &= \sigma_\nu\Delta\tilde{y}_{t+1} - (i_t - \mathbb{E}\{\pi_{H,+1}\} - r_t^n) && (116)\\
\tilde{y}_t &= \tilde{y}_{t+1} - \frac{1}{\sigma_\nu}(i_t - \mathbb{E}\{\pi_{H,+1}\} - r_t^n) \quad \text{Dynamic IS Curve} && (117)\\
& && (118)
\end{aligned}
$$

$$r_t^n = \sigma_\nu \Delta y_{t+1}^n + \rho + \sigma_\nu \nu(\varpi - 1)\,\mathbb{E}\{\Delta y_{t+1}^*\} + \Phi(1-\nu)(1-\rho_z)z_t + \sigma_\nu \frac{\nu\varpi - 1}{1-\nu} G_Y \Delta g_{t+1} \tag{119}$$

$$\sigma_\nu \Delta y_{t+1}^n = \sigma_\nu(\Gamma_* \Delta y_{t+1}^* + \Gamma_z \Delta z_{t+1} + \Gamma_a \Delta a_{t+1} + \Gamma_g \Delta g_{t+1} + \Gamma_\tau \Delta \tau_{t+1}) \tag{120}$$

$$y_t^* : \sigma_\nu \Gamma_* \Delta y_{t+1}^* + \sigma_\nu \nu(\varpi - 1)\Delta y_{t+1}^* = \sigma_\nu(\Gamma_* + \nu(\varpi-1))\Delta y_{t+1}^* = \Psi_* y_{t+1}^* \tag{121}$$

$$z_t : \sigma_\nu \Gamma_z \Delta z_{t+1} + \Phi(1-\rho_z)z_t = \sigma_\nu \Gamma_z(\rho_z - 1)z_t + \Phi(1-\nu)(1-\rho_z)z_t \tag{122}$$

$$= \sigma_\nu \Gamma_z(\rho_z - 1)z_t + \Phi(1-\nu)(1-\rho_z)z_t = (\Phi(1-\nu) - \sigma_\nu \Gamma_z)(1-\rho_z)z_t = \Psi_z(1-\rho_z)z_t \tag{123}$$

$$a_t : \sigma_\nu \Gamma_a \Delta a_{t+1} = \sigma_\nu \Gamma_a(\rho_a - 1)a_t = -\sigma_\nu \Gamma_a(1-\rho_a)a_t \tag{124}$$

$$g_t : \sigma_\nu \Gamma_g G_Y \Delta g_{t+1} + \sigma_\nu \frac{\nu\varpi - 1}{1-\nu} G_Y \Delta g_{t+1} = \left(\sigma_\nu \left(\Gamma_g + \frac{\nu\varpi - 1}{1-\nu}\right)\right) G_Y \Delta g_{t+1} = -\Psi_g G_Y(1-\rho_g)g_t \tag{125}$$

$$\tau_t : \sigma_\nu \Gamma_\tau \Delta \tau_{t+1} \tag{126}$$

$$r_t^n = \rho + \Psi_* y_{t+1}^* + \Psi_z(1-\rho_z)z_t - \sigma_\nu \Gamma_a(1-\rho_a)a_t - \Psi_g G_Y(1-\rho_g)g_t + \sigma_\nu \Gamma_\tau \Delta \tau_{t+1} \tag{127}$$

Structure. Max words: 10 000. Currently: 1691

1. Introduction: **1000 words**
2. Literature Review **2000 words**
   (a) From RBC to NK DSGE: **1200 words**
   (b) Why Scotland and the rest of the UK? (NIESR policy-related question): **800 words**
3. Theoretical DSGE model: **2000 words**
   (a) Households
   (b) Firms
   (c) Equilibrium
4. Application: **1500 words**
   (a) Data: **500 words**
   (b) Estimation (MCMC): **1000 words**
5. Results: **2500 words**
   (a) Analysis of IRFs: **1000 words**
   (b) Other insights (NIESR policy-related question): **1500 words**
6. Conclusion: **1000 words**

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Sum count: 1691
Words in text: 1518
Words in headers: 31
Words outside text (captions, etc.): 44
Number of headers: 13
Number of floats/tables/figures: 2
Number of math inlines: 42
Number of math displayed: 56
Files: 13
Subcounts:
  text+headers+captions (#headers/#floats/#inlines/#displayed)
  4+9+0 (7/0/0/0) File: main.tex
  99+11+0 (1/0/0/0) Included file: ././model.tex
  65+6+0 (1/0/0/0) Included file: ././structure.tex
  591+2+0 (1/0/0/0) Included file: ././Introduction/literature_review.tex
  155+0+0 (0/0/3/0) Included file: ././Theoretical DSGE model/model.tex
  557+1+33 (1/0/36/41) Included file: ././Theoretical DSGE model/households.tex
  26+1+0 (1/0/3/5) Included file: ././Theoretical DSGE model/firms.tex
  20+1+0 (1/0/0/10) Included file: ././Theoretical DSGE model/equilibrium.tex
  1+0+0 (0/0/0/0) Included file: ././final_wordcount.txt
  0+0+11 (0/1/0/0) Included file: ././Graphs/timeline.tex
  0+0+0 (0/1/0/0) Included file: ././Graphs/sigma_varphi.tex
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1 Introduction

1.1 Literature Review

On the 10th of May, 2023, the Monetary Policy Committee at the Bank of England gathered to discuss the latest international and domestic data on economic activity. Even though the Committee has a 2% CPI target, the UK's economy had undergone a sequence of very large and unexpected shocks and disturbances, resulting in twelve-month CPI inflation above 10%. The majority of the Committee members (78%) believed that an increase in interest rate "was warranted" (BoE, 2023: 4), while the remaining members believed that the CPI inflation will "fall sharply in 2023" (BoE, 2023: 5) as a result of the economy naturally adjusting to the effects of the energy price shocks. They feared that the preceding increases in the interest rate have not yet been internalised and raising the interest rate any further could result in a reduction of inflation "well below the target" (BoE, 2023: 5). This is an excellent illustration of the "informal dimension of the monetary policy process", that (Galsí and Gertler, 2007: 26) referred to in their work explaining modern macro models and new frameworks. According to them, while the informal dimension cannot be removed, we can build formal and rigorous models that would help the Committee and institutions-alike to understand "objectives of the monetary policy and how the latter should be conducted in order to attain those objectives" (Galí, 2015: 2). This task is not straightforward and has been central (albeit - fruitful) to most macroeconomic research in the past decades. The following section of the literature review will present a brief evolution of the study of business cycles and monetary theory. It will be followed by an overview of large macroeconomic models adopted by central banks and international organisations to illustrate the relevance of this research. The final part of the literature review will discuss {NIESR policy question, also Scotland-UK deltas} and the latest research on the issue.

Blanchard (2000) offers a compact description of macroeconomic research in the twentieth century. In their panegyric and optimistic essay, the researcher argues that the century can be divided into three epochs based on the prevailing beliefs about the economy and frameworks of the time: Pre-1940, From 1940 to 1980, and Post-1980.

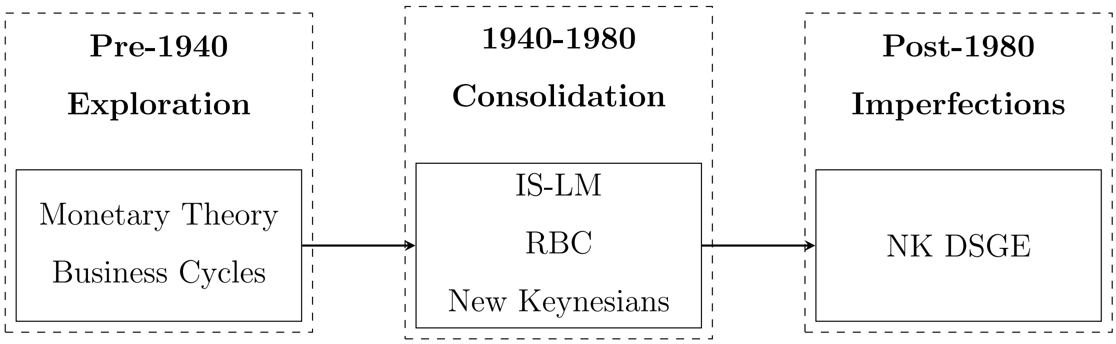


Figure 1: Timeline of macroeconomic research, according to Blanchard (2000). Authors own illustration.

According to them, the pre-1940 epoch was the epoch of exploration, with economists primarily concerned with the Monetary Theory ("Why does money affect output?") and Business Cycles ("What are the major shocks that affect output?"). Even though both of those puzzles fall under the study of "macroeconomics", the term did not appear in the literature until the mid-1940s (Blanchard, 2000). The Monetary Theory ideas of that time did not differ drastically from what is believed today, e.g. short-run money non-neutrality and long-run neutrality, but the economic models were "incomplete and partial equilibrium in nature" (Blanchard, 2000: 1377). The business cycles were attributed to "real factors", such as technological innovations (Blanchard, 2000), and this belief persisted until more data became available and more sophisticated time series methods were applied in the early 2000s (Galí, 2015: 3). The subsequent epoch was "the golden age of macroeconomics" (Blanchard, 2000: 1379). Hicks (1937) formalised the *IS-LM* framework TBC

In the 1980s, Kydland and Prescott (1982) and Prescott (1986) published seminal papers on the Real Business Cycles (RBC) theory. According to Galí (2015: 2), frameworks presented in the papers "provided the main reference" and firmly established the use of dynamic stochastic general equilibrium (DSGE) models as crucial tools for macroeconomic analysis. The models allow quantitative analysis and incorporation of data either via calibration or estimation of parameters. TBC

International Monetary Fund (IMF)

2 Theoretical DSGE model

The model is primarily based on the work of Galí (2015), Gali and Monacelli (2005), and Ricci (2019). Galí (2015) was used to derive a baseline NK DSGE with small open economy features, while Gali and Monacelli (2005) was used to scale the model to a two-country DSGE model. Finally, a few Scotland-specific DSGE extensions were taken from Ricci (2019) who were the first ones to build a DSGE model for Scotland and the rest of the UK (rUK). Where applicable, the notation follows Galí (2015). Given that Scotland and the rest of the UK are modelled as symmetrical, we denote variables without an asterisk as Scotland-specific variables, e.g. C_t, while variables with an asterisk, e.g. C_t^* refer to rUK variables. Finally, the rest of the World variables are denoted with two asterisks, e.g. C_t^{**}. It is also because the economies are assumed to be symmetrical, the following section only lists model equations for Scotland.

2.1 Households

This model assumes that there is infinitely many households in the economy represented by a unit interval. All households are assumed to be be symmetric, i.e. have the same preferences and behave identically. Below, we consider a representative household that wants to maximise their lifetime utility:

$$\mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \mathcal{U}(C_t, C_{t-1}, N_t, \varepsilon_{Z,t}) \right\} \tag{128}$$

$$\mathcal{U}(C_t, C_{t-1} N_t, \varepsilon_{Z,t}) = \left(\frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \varepsilon_{N,t} \frac{N_t^{1+\varphi}}{1+\varphi} \right) \varepsilon_{Z,t} \tag{129}$$

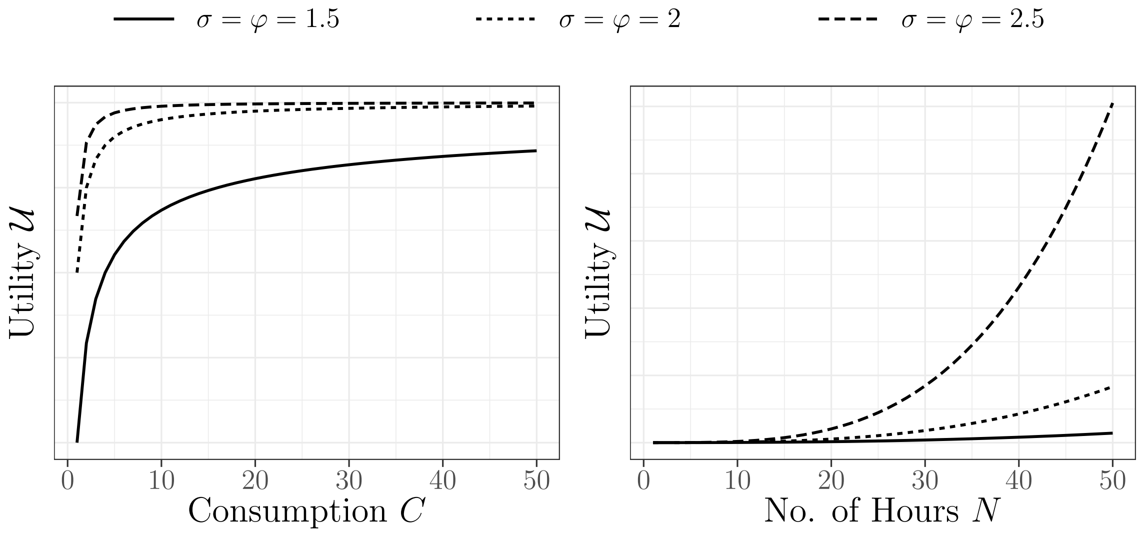
The household's utility depends on consumption C_t and hours worked N_t. As seen from the utility function (Equation (129)), the model assumes the household's utility to be (decreasingly) increasing in consumption C_t and (increasingly) decreasing in hours worked N_t. $\beta \in (0, 1)$ is the discount factor, which can be thought of as an opportunity cost or an impatience rate, i.e. a unit of consumption C today will be worth $\beta * C > C$ tomorrow. Using parameter h, we also take into account household's habit formation in terms of consumption, which is found to improve model's fit to empirial macroeconomic data (?, 20??). We also introduce a preference shifter $\varepsilon_{Z,t}$ (Galí, 2015: 225), as well as a shock to the number of hours worked $\varepsilon_{N,t}$ [1] (Kolasa, 2009). The shocks are assumed to follow autoregressive process of order 1:

$$\log(\varepsilon_{Z,t}) = \rho_z \log(\varepsilon_{Z,t-1}) + \epsilon_t^z \tag{130}$$

$$\log(\varepsilon_{N,t}) = \rho_n \log(\varepsilon_{N,t-1}) + \epsilon_t^n \tag{131}$$

The parameters $\sigma \geq 0$ and $\varphi \geq 0$ determine the curvature of the utility of consumption and disutility of labour, respectively (Gali, 2015: 20). Finally, $\mathbb{E}_t\{*\}$ is the expectational operator, conditional on all information available at period t.

[1] While these specific shocks are not relevant to the research question, they help prevent stochastic singularity Pfeifer, 2021 and allows parameter estimation with a greater number of macroeconomic data series, see Section 3.



To allow goods differentiation between domestic and foreign, the model assumes that C_t is a composite consumption index defined by:

$$C_t = \left[(1-\nu)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \nu^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{132}$$

Where $C_{H,t}$ and $C_{F,t}$ are indices of consumption of home produced and imported goods, respectively. The parameter $\nu \in [0,1]$ reflects economy's openness for trading, while η denotes household's willingness to substitute domestic good with a foreign good. Our economy is assumed to be open for trading with the rest of the world (ROW), which itself is made of a continuum of infinitely many small economies i represented by a unit interval. Therefore, $C_{F,t}$ is a sum of indices of the quantity of goods imported from all countries i. In a similar fashion, if we denote j as a single variety of goods from a continuum of goods represented by a unit interval, we can express each consumption index as follows:

$$C_{H,t} = \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \qquad \text{Index of consumption of home produced goods}$$

$$C_{i,t} = \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \qquad \text{Index of consumption of country } i\text{'s produced goods}$$

$$C_{F,t} = \left(\int_0^1 C_{i,t}^{\frac{\gamma-1}{\gamma}} \, di \right)^{\frac{\gamma}{\gamma-1}} \qquad \text{Index of consumption of imported goods}$$

Notice that all three indices take the form of *Constant Elasticity Substitution* (*CES*) form, with parameters ε (without subscripts) and γ representing the degree of substitutability between varieties of goods and countries, respectively. Optimal allocation of each variety of goods:

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t}; \qquad C_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}} \right)^{-\varepsilon} C_{i,t}; \qquad C_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}} \right)^{-\gamma} C_{F,t} \tag{133}$$

$$P_{H,t} = \left(\int_0^1 P_{H,t}(j)^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}} \qquad \text{Domestic Price Index} \tag{134}$$

$$P_{i,t} = \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}} \qquad \text{Price Index of goods produced by country } i \tag{135}$$

$$P_{F,t} = \left(\int_0^1 P_{i,t}^{1-\gamma} \, dj \right)^{\frac{1}{1-\gamma}} \qquad \text{Price Index of Imported goods} \tag{136}$$

$$\int_0^1 P_{H,t}(j)C_{H,t}(j)\,dj = P_{H,t}C_{H,t} \qquad \int_0^1 P_{i,t}(j)C_{i,t}(j)\,dj = P_{i,t}C_{i,t} \tag{137}$$

Using (133) and (136) implies

$$\int_0^1 P_{i,t}C_{i,t} = P_{F,t}C_{F,t} \tag{138}$$

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \qquad C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{139}$$

$$P_t = \left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}\right]^{\frac{1}{1-\eta}} \qquad \text{Consumption Price Index} \tag{140}$$

A special case $\eta = 1$:

$$P_t = (P_{H,t})^{1-\alpha} \times (P_{F,t})^{\alpha} \qquad C_t = \frac{1}{(1-\alpha)^{(1-\alpha)}\alpha^{\alpha}}(C_{H,t})^{(1-\alpha)}(C_{F,t})^{\alpha} \tag{141}$$

Total consumption expenditures are:

$$P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_t C_t \tag{142}$$

So the budget constraint is:

$$P_t C_t + \mathbb{E}_t[Q_{t,t+1}D_{t+1}] \leq D_t + W_t N_t + T_t \tag{143}$$

$$\begin{aligned}\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} &\left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right) \\ &+ \lambda_t\left\{D_t + W_t N_t + T_t - P_t C_t - \mathbb{E}_t[Q_{t,t+1}D_{t+1}]\right\}\end{aligned} \tag{144}$$

$$\begin{aligned}\frac{\partial L}{\partial C_t} &= \beta^t C_t^{-\sigma} - \lambda_t P_t = 0; \quad \Rightarrow \quad \beta^t C_t^{-\sigma} P_t^{-1} = \lambda_t \\ \frac{\partial L}{\partial N_t} &= -\beta^t N_t^{\varphi} + \lambda_t N_t = 0; \quad \Rightarrow \quad \beta^t N_t^{\varphi} W_t^{-1} = \lambda_t \\ \frac{\partial L}{\partial C_t} = \frac{\partial L}{\partial N_t} &: \beta^t C_t^{-\sigma} P_t^{-1} = \beta^t N_t^{\varphi} W_t^{-1} \\ &\Rightarrow C_t^{-\sigma} P_t^{-1} = N_t^{\varphi} W_t^{-1} \\ &\Rightarrow C_t^{-\sigma} N_t^{-\varphi} = W_t^{-1} P_t\end{aligned}$$

$$\Rightarrow \qquad C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \qquad \text{Intratemporal Optimality Condition} \tag{145}$$

$$\frac{\partial L}{\partial C_t} = \beta^t C_t^{-\sigma} P_t^{-1} = \lambda_t; \quad \Rightarrow \quad \mathbb{E}_t[\beta^{t+1} C_{t+1}^{-\sigma} P_{t+1}^{-1}] = \mathbb{E}_t[\lambda_{t+1}]$$
$$\frac{\partial L}{\partial D_{t+1}} = -\lambda_t \, \mathbb{E}_t[Q_{t,t+1}] + \mathbb{E}_t[\lambda_{t+1}] = 0; \quad \Rightarrow \quad \mathbb{E}_t[Q_{t,t+1}] = \frac{\lambda_{t+1}}{\lambda_t}$$

$$\mathbb{E}_t\left[\frac{\beta^t C_t^{-\sigma} P_t^{-1}}{\beta^{t+1} C_{t+1}^{-\sigma} P_{t+1}^{-1}}\right] = \mathbb{E}_t\left[\frac{\lambda_t}{\lambda_{t+1}}\right]$$
$$\mathbb{E}_t\left[\frac{\beta^t C_t^{-\sigma} P_t^{-1}}{\beta^{t+1} C_{t+1}^{-\sigma} P_{t+1}^{-1}}\right] = \mathbb{E}_t\left[\frac{1}{Q_{t,t+1}}\right]$$
$$\mathbb{E}_t\left[\frac{1}{\beta}\left(\frac{C_t}{C_{t+1}}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)^{-1}\right] = \mathbb{E}_t\left[\frac{1}{Q_{t,t+1}}\right]$$

$$\beta \, \mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\right] = Q_t \qquad \text{Euler equation} \tag{146}$$

However, Gali uses a different approach to derive the Euler equation, which introduces Arrow securities:

$$\frac{V_{t,t+1}}{P_t} C_t^{-\sigma} = \xi_{t,t+1} \beta C_{t+1}^{-\sigma} \frac{1}{P_{t+1}} \tag{147}$$

Where $V_{t,t+1}$ is an Arrow security and $\xi_{t,t+1}$ is the probability that the Arrow security will yield a payoff next period. Interpretation: LHS is paying for the security (expenses) in terms of consumption at P_t prices. RHS is the payoff if the Arrow security yields a payoff. The consumer will only be willing to pay LHS if the payoff is at least as big on the RHS. Given that:

$$Q_{t,t+1} = \frac{V_{t,t+1}}{\xi_{t,t+1}} \tag{148}$$

$$\frac{V_{t,t+1}}{P_t} C_t^{-\sigma} = \xi_{t,t+1} \beta C_{t+1}^{-\sigma} \frac{1}{P_{t+1}} \tag{149}$$
$$Q_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}} \tag{150}$$
$$\mathbb{E}_t[Q_{t,t+1}] = \beta \, \mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}}\right] \tag{151}$$

$$\beta \, \mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\right] = Q_t \qquad \text{Euler equation} \tag{152}$$

Log-linearising (145):

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t}; \quad \Rightarrow \quad w_t - p_t = \sigma c_t + \varphi n_t \tag{153}$$

Log-linearising (152):

$$\beta\,\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\right] = Q_t$$
$$\ln\beta - \mathbb{E}_t[\sigma c_{t+1}] + \sigma c_t + p_t - \mathbb{E}_t[p_{t+1}] = \ln Q_t$$
$$\sigma c_t = \ln Q_t - \ln\beta + \mathbb{E}_t[\sigma c_{t+1}] - p_t + \mathbb{E}_t[p_{t+1}]$$
$$c_t = \mathbb{E}_t[c_{t+1}] - \frac{1}{\sigma}(-\ln Q_t - \rho - \mathbb{E}_t[\pi_{t+1}])$$
$$c_t = \mathbb{E}_t[c_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] - \rho) \tag{154}$$
$$\text{where } i_t = -\log Q_t,\ \rho = -\log\beta,\ \pi_t = p_t - p_{t-1}$$

$$S_{i,t} = \frac{P_{i,t}}{P_{H,t}} \quad \text{Bilateral terms of trade} \tag{155}$$
$$S_t = \frac{P_{F,t}}{P_{H,t}} = \left(\int_0^1 (S_{i,t}\,di)^{1-\gamma}\right)^{\frac{1}{1-\gamma}} \quad \text{Effective terms of trade} \tag{156}$$
$$s_t = p_{F,t} - p_{H,t} = \left(\int_0^1 s_{i,t}\,di\right) \quad \text{(log) Effective terms of trade} \tag{157}$$

Recall that when $\eta = 1$, then CPI is $P_t = (P_{H,t})^{1-\alpha} \times (P_{F,t})^{\alpha}$, which can be log-linearised to:

$$p_t = (1-\alpha)p_{H,t} + \alpha p_{F,t} = p_{H,t} + \alpha s_t \tag{158}$$

Equations (157) and (158) hold when $\gamma = 1$ and $\eta = 1$, respectively.

$$\pi_{H,t} = p_{H,t+1} - p_{H,t} \quad \text{Domestic Inflation} \tag{159}$$
$$\pi_t = \pi_{H,t} + \alpha\Delta s_t \quad \text{CPI Inflation} \tag{160}$$

The gap between domestic inflation and CPI inflation is only due to percentage change in the terms of trade.

$$P_{i,t}(j) = \mathcal{E}_{i,t}P^i_{i,t}(j) \quad \text{Law of One Price (LOP)} \tag{161}$$
$$P_{i,t} = \mathcal{E}_{i,t}P^i_{i,t} \quad \text{Law of One Price (LOP)} \tag{162}$$
$$p_{i,t} = e_{i,t} + p^i_{i,t} \quad \text{(Log) Law of One Price (LOP)} \tag{163}$$
$$p_{F,t} = \int_0^1 (e_{i,t} + p^i_{i,t})\,di = e_t + p^\star_t \quad \text{(Log)Price index of Imported Goods} \tag{164}$$

Where e_t is (Log) Effective Nominal Exchange Rate, $p^\star_t$ is the World Price Index.

$$s_t = p_{F,t} - p_{H,t} = e_t + p^\star_t - p_{H,t} \quad \text{Terms of trade but with the World Price Index} \tag{165}$$

$$
\begin{aligned}
\mathcal{Q}_{i,t} &= \frac{\mathcal{E}_{i,t} P_t^i}{P_t} && \text{Bilateral Exchange Rate} && (166)\\
q_t &= \int_0^1 \log\left(\frac{\mathcal{E}_{i,t} P_t^i}{P_t}\right) di && && (167)\\
&= \int_0^1 (e_{i,t} + p_{i,t}^i - p_t)\, di && && (168)\\
&= e_t + p_t^\star - p_t && \text{using (164)} && (169)\\
&= s_t + p_{H_t} - p_t && \text{using (165)} && (170)\\
&= (1-\alpha) s_t && \text{using (158)} && (171)
\end{aligned}
$$

International Risk-Sharing Equation (147) for country i can be rewritten as:

$$
\frac{V_{t,t+1}}{\mathcal{E}_t^i P_t^i} (C_t^i)^{-\sigma} = \xi_{t,t+1} \beta (C_{t+1}^i)^{-\sigma} \frac{1}{\mathcal{E}_{t+1}^i P_{t+1}^i} \tag{172}
$$

Given that:

$$
Q_{t,t+1} = \frac{V_{t,t+1}}{\xi_{t,t+1}} \tag{173}
$$

$$
\begin{aligned}
\frac{V_{t,t+1}}{\mathcal{E}_t^i P_t^i} (C_t^i)^{-\sigma} &= \xi_{t,t+1} \beta (C_{t+1}^i)^{-\sigma} \frac{1}{\mathcal{E}_{t+1}^i P_{t+1}^i} && (174)\\
Q_{t,t+1} &= \beta \left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\sigma} \left(\frac{P_t^i}{P_{t+1}^i}\right) \left(\frac{\mathcal{E}_t^i}{\mathcal{E}_{t+1}^i}\right) && (175)
\end{aligned}
$$

Recall that:

$$
\begin{aligned}
\frac{\partial L}{\partial C_t} &= \beta^t C_t^{-\sigma} P_t^{-1} = \lambda_t \\
\frac{\partial L}{\partial D_{t+1}} &= -\lambda_t \, \mathbb{E}_t[Q_{t,t+1}] = -\,\mathbb{E}_t[\lambda_{t+1}] \\
&= -\beta^t C_t^{-\sigma} P_t^{-1} \, \mathbb{E}_t[Q_{t,t+1}] = -\,\mathbb{E}_t[\lambda_{t+1}] && (176)
\end{aligned}
$$

Which is symmetrical for country i:

$$
\begin{aligned}
\frac{\partial L^i}{\partial C_t^i} &= \beta^t (C_t^i)^{-\sigma} (\mathcal{E}_{i,t} P_t^i)^{-1} = \lambda_t^i \\
\frac{\partial L^i}{\partial D_{t+1}^i} &= -\lambda_t^i \, \mathbb{E}_t[Q_{t,t+1}] = -\,\mathbb{E}_t[\lambda_{t+1}^i] \\
&= -\beta^t (C_t^i)^{-\sigma} (\mathcal{E}_{i,t} P_t^i)^{-1} \, \mathbb{E}_t[Q_{t,t+1}] = -\,\mathbb{E}_t[\lambda_{t+1}^i] && (177)
\end{aligned}
$$

$$
\frac{(176)}{(177)} : \frac{-\beta^t C_t^{-\sigma} P_t^{-1} \, \mathbb{E}_t[Q_{t,t+1}]}{-\beta^t (C_t^i)^{-\sigma} (\mathcal{E}_{i,t} P_t^i)^{-1} \, \mathbb{E}_t[Q_{t,t+1}]} = \frac{-\,\mathbb{E}_t[\lambda_{t+1}]}{-\,\mathbb{E}_t[\lambda_{t+1}^i]}
$$

$$
\begin{aligned}
C_t^{-\sigma}(C_t^i)^{\sigma}\frac{\mathcal{E}_{i,t}P_t^i}{P_t} &= 1 \\
C_t^{-\sigma}(C_t^i)^{\sigma}\mathcal{Q}_{i,t} &= 1 \\
C_t^{-\sigma}(C_t^i)^{\sigma} &= \frac{1}{\mathcal{Q}_{i,t}} \\
C_t^{-\sigma} &= \frac{1}{\mathcal{Q}_{i,t}}(C_t^i)^{-\sigma} \\
C_t^{\sigma} &= \mathcal{Q}_{i,t}(C_t^i)^{\sigma}
\end{aligned}
$$

$$
\Rightarrow C_t = C_t^i \mathcal{Q}_{i,t}^{\frac{1}{\sigma}} \tag{178}
$$

Log-linearising (178) yields:

$$
c_t = c_t^i + \frac{1}{\sigma} q_{i,t} \tag{179}
$$

Integrating both sides over i:

$$
c_t = c_t^\star + \frac{1}{\sigma} q_t \tag{180}
$$

$$
= c_t^\star + \left(\frac{1-\alpha}{\sigma}\right) s_t \qquad \text{using } q_t = (1-\alpha)s_t \tag{181}
$$

$c_t^\star$ is the log world consumption. Equation (181) is the link between the domestic consumption and the world consumption.

2.2 Firms

$$
Y_t(j) = A_t N_t(j) \tag{182}
$$

$$
\log A_t = \alpha_t \tag{183}
$$

$$
\alpha_t = \rho_a \alpha_{t-1} + \varepsilon_t \tag{184}
$$

$$
L = P_t(j)Y_t(j) - W_t(j)N_t(j) \tag{185}
$$

$$
\Rightarrow L = P_t Y_t - W_t N_t \tag{186}
$$

$$
\Rightarrow L = P_t A_t N_t - W_t N_t \tag{187}
$$

$$
\tag{188}
$$

$$
\frac{\partial L}{\partial N_t} = P_t A_t - W_t = 0 \qquad \Rightarrow W_t - P_t A_t = 0 \tag{189}
$$

$$
MC_t = W_t - P_t A_t \tag{190}
$$

$$
mc_t = w_t - p_t - a_t \tag{191}
$$

$$
mc_t = -\nu + w_t - p_t - a_t \tag{192}
$$

$$
mc_t = -\nu + w_t - p_{H,t} - a_t \tag{193}
$$

$$
\tag{194}
$$

$\nu = -(\log(1-\tau))$, where τ is the employment subsidy, introduced later. $p_{H,t}$ because this is for domestic firms.

Firms that get to reset their price, do it using the following problem:

$$
p_{\bar{H},t} = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \,\mathbb{E}_t[mc_{t+k} + p_{H,t+k}] \tag{195}
$$

$$p_{\bar{H},t} \qquad \text{Is the (log) new price} \tag{196}$$

$$\mu \qquad \text{Is the (log) markup in the steady state} \tag{197}$$

2.3 Equilibrium

$$Y_t(j) = C_{H,t}(j) + \int_0^1 C^i_{H,t}(j)\, di \tag{198}$$

$$= \left(\frac{P_{H,t}(j)}{P_{H,t}}^{-\varepsilon}\right)\left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H_t}}{\mathcal{E}_{i,t} P^i_{F,t}}\right)^{-\gamma}\left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t\, di\right] \tag{199}$$

Given that

$$Y_t = \left(\int_0^1 (Y_t(j))^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{200}$$

$$Y_t = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H_t}}{\mathcal{E}_{i,t} P^i_{F,t}}\right)^{-\gamma}\left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t\, di \tag{201}$$

$$= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \left[(1-\alpha) + \alpha \int_0^1 (S^i_t S_{i,t})^{\gamma-\eta} \mathcal{Q}_{i,t}^{\eta-\frac{1}{\sigma}}\, di\right] \tag{202}$$

Which can be log-linearised to:

$$y_t = c_t + \alpha\gamma s_t + \alpha(\eta - \frac{1}{\sigma}) q_t \tag{203}$$

$$= c_t + \frac{\alpha w}{\sigma} s_t \tag{204}$$

$$w_t = \sigma\gamma + (1-\alpha)(\sigma\eta - 1) \tag{205}$$

Assuming that country i is symmetric:

$$y^i_t = c^i_t + \frac{\alpha w}{\sigma} s^i_t \tag{206}$$

$$\int_0^1 y^i_t = \int_0^1 c^i_t + 0 = c^\star_t \qquad \text{World Consumption} \tag{207}$$

Using equations (), (), ():

$$y = c_t + \frac{\alpha w}{\sigma} s_t \tag{208}$$

$$y_t = c^\star_t + \frac{1-\alpha}{\sigma} s_t + \frac{\alpha w}{\sigma} s_t \tag{209}$$

$$y_t = y^\star_t + \frac{1-\alpha}{\sigma} s_t + \frac{\alpha w}{\sigma} s_t \tag{210}$$

$$y_t = y^\star_t + \frac{1-\alpha+\alpha w}{\sigma} s_t \tag{211}$$

$$y_t = y^\star_t + \frac{1+\alpha(w-1)}{\sigma} s_t \tag{212}$$

$$\sigma_\alpha = \frac{1+\alpha(w-1)}{\sigma} \tag{213}$$

$$\Rightarrow \quad y_t = y^\star_t + \frac{1}{\sigma_\alpha} s_t \tag{214}$$

Combining Euler equation () and (214) gives :

$$
\begin{aligned}
c_t &= \mathbb{E}_t[c_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] - \rho) \\
y_t - \frac{\alpha w}{\sigma} s_t &= \mathbb{E}_t\left[y_{t-1} - \frac{\alpha w}{\sigma} s_{t+1}\right] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] - \rho) \\
y_t &= \mathbb{E}_t\left[y_{t-1}\right] - \frac{\alpha w}{\sigma}\,\mathbb{E}_t[(s_{t+1} - s_t)] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] - \rho) \\
&= \mathbb{E}_t\left[y_{t-1}\right] - \frac{\alpha w}{\sigma}\,\mathbb{E}_t[\Delta s_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] - \rho) \\
&= \mathbb{E}_t\left[y_{t-1}\right] - \frac{\alpha w}{\sigma}\,\mathbb{E}_t[\Delta s_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \alpha\,\mathbb{E}_t[\Delta s_{t+1}] - \rho) \\
&= \mathbb{E}_t\left[y_{t-1}\right] - \frac{\alpha w}{\sigma}\,\mathbb{E}_t[\Delta s_{t+1}] + \frac{\alpha}{\sigma}\,\mathbb{E}_t[\Delta s_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
&= \mathbb{E}_t\left[y_{t-1}\right] - \frac{\alpha w + \alpha}{\sigma}\,\mathbb{E}_t[\Delta s_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
&= \mathbb{E}_t\left[y_{t-1}\right] + \frac{\alpha - \alpha w}{\sigma}\,\mathbb{E}_t[\Delta s_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
&= \mathbb{E}_t\left[y_{t-1}\right] + \frac{-\alpha(-1 + w)}{\sigma}\,\mathbb{E}_t[\Delta s_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
&= \mathbb{E}_t\left[y_{t-1}\right] - \frac{\alpha(w - 1)}{\sigma}\,\mathbb{E}_t[\Delta s_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
&= \mathbb{E}_t\left[y_{t-1}\right] - \frac{\alpha\Theta}{\sigma}\,\mathbb{E}_t[\Delta s_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
\mathbb{E}_t[\Delta s_{t+1}] &= \sigma_\alpha\,\mathbb{E}_t[\Delta y_{t+1} - \Delta y^\star_{t+1}] \quad \text{from Equation():} \\
&= \mathbb{E}_t\left[y_{t-1}\right] - \frac{\alpha\Theta}{\sigma}\sigma_\alpha\,\mathbb{E}_t[\Delta y_{t+1} - \Delta y^\star_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
&= \mathbb{E}_t\left[y_{t-1}\right] - \frac{\alpha\Theta}{1 + \alpha\Theta}\,\mathbb{E}_t[\Delta y_{t+1} - \Delta y^\star_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
&= \mathbb{E}_t\left[y_{t-1}\right] - \frac{\alpha\Theta}{1 + \alpha\Theta}\,\mathbb{E}_t[\Delta y_{t+1}] + \frac{\alpha\Theta}{1 + \alpha\Theta}\,\mathbb{E}_t[\Delta y^\star_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
&= \mathbb{E}_t\left[y_{t-1}\right] - \frac{\alpha\Theta}{1 + \alpha\Theta}\,\mathbb{E}_t[\Delta y_{t+1}] + \frac{\alpha\Theta}{1 + \alpha\Theta}\,\mathbb{E}_t[\Delta y^\star_{t+1}] - \frac{1}{(1 + \alpha\Theta)\sigma_\alpha}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho)
\end{aligned}
$$

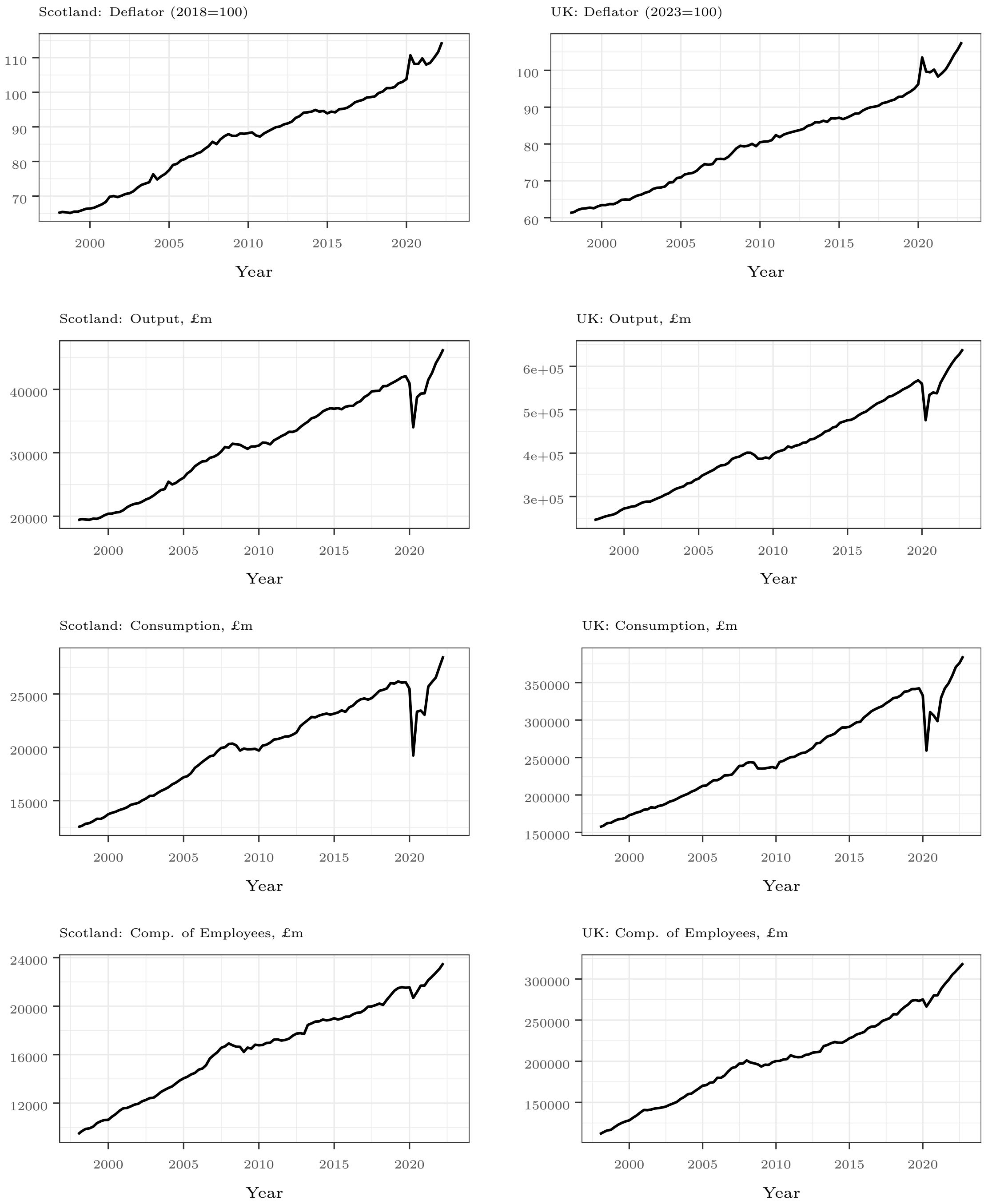
$$
\begin{aligned}
(1+\alpha\Theta)y_t &= (1+\alpha\Theta)\,\mathbb{E}_t\left[y_{t-1}\right] - \alpha\Theta\,\mathbb{E}_t[\Delta y_{t+1}] + \alpha\Theta\,\mathbb{E}_t[\Delta y^{\star}_{t+1}] - \frac{1}{\sigma_\alpha}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
(1+\alpha\Theta)\,\mathbb{E}_t\left[y_t - y_{t-1}\right] &= -\alpha\Theta\,\mathbb{E}_t[\Delta y_{t+1}] + \alpha\Theta\,\mathbb{E}_t[\Delta y^{\star}_{t+1}] - \frac{1}{\sigma_\alpha}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
-(1+\alpha\Theta)\,\mathbb{E}_t\left[y_{t-1} - y_t\right] &= -\alpha\Theta\,\mathbb{E}_t[\Delta y_{t+1}] + \alpha\Theta\,\mathbb{E}_t[\Delta y^{\star}_{t+1}] - \frac{1}{\sigma_\alpha}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
(1+\alpha\Theta)\,\mathbb{E}_t\left[\Delta y_{t+1}\right] &= \alpha\Theta\,\mathbb{E}_t[\Delta y_{t+1}] - \alpha\Theta\,\mathbb{E}_t[\Delta y^{\star}_{t+1}] + \frac{1}{\sigma_\alpha}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
(1+\alpha\Theta)\,\mathbb{E}_t\left[\Delta y_{t+1}\right] - \alpha\Theta\,\mathbb{E}_t[\Delta y_{t+1}] &= -\alpha\Theta\,\mathbb{E}_t[\Delta y^{\star}_{t+1}] + \frac{1}{\sigma_\alpha}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
\mathbb{E}_t\left[\Delta y_{t+1}\right] + \alpha\Theta\,\mathbb{E}_t\left[\Delta y_{t+1}\right] - \alpha\Theta\,\mathbb{E}_t[\Delta y_{t+1}] &= -\alpha\Theta\,\mathbb{E}_t[\Delta y^{\star}_{t+1}] + \frac{1}{\sigma_\alpha}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
\mathbb{E}_t\left[\Delta y_{t+1}\right] &= -\alpha\Theta\,\mathbb{E}_t[\Delta y^{\star}_{t+1}] + \frac{1}{\sigma_\alpha}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
\mathbb{E}_t\left[y_{t+1} - y_t\right] &= -\alpha\Theta\,\mathbb{E}_t[\Delta y^{\star}_{t+1}] + \frac{1}{\sigma_\alpha}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
-y_t &= -\,\mathbb{E}_t\left[y_{t+1}\right] - \alpha\Theta\,\mathbb{E}_t[\Delta y^{\star}_{t+1}] + \frac{1}{\sigma_\alpha}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
\Rightarrow y_t &= \mathbb{E}_t\left[y_{t+1}\right] - \frac{1}{\sigma_\alpha}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) + \alpha\Theta\,\mathbb{E}_t[\Delta y^{\star}_{t+1}] \qquad (215) \\
& \qquad (216)
\end{aligned}
$$

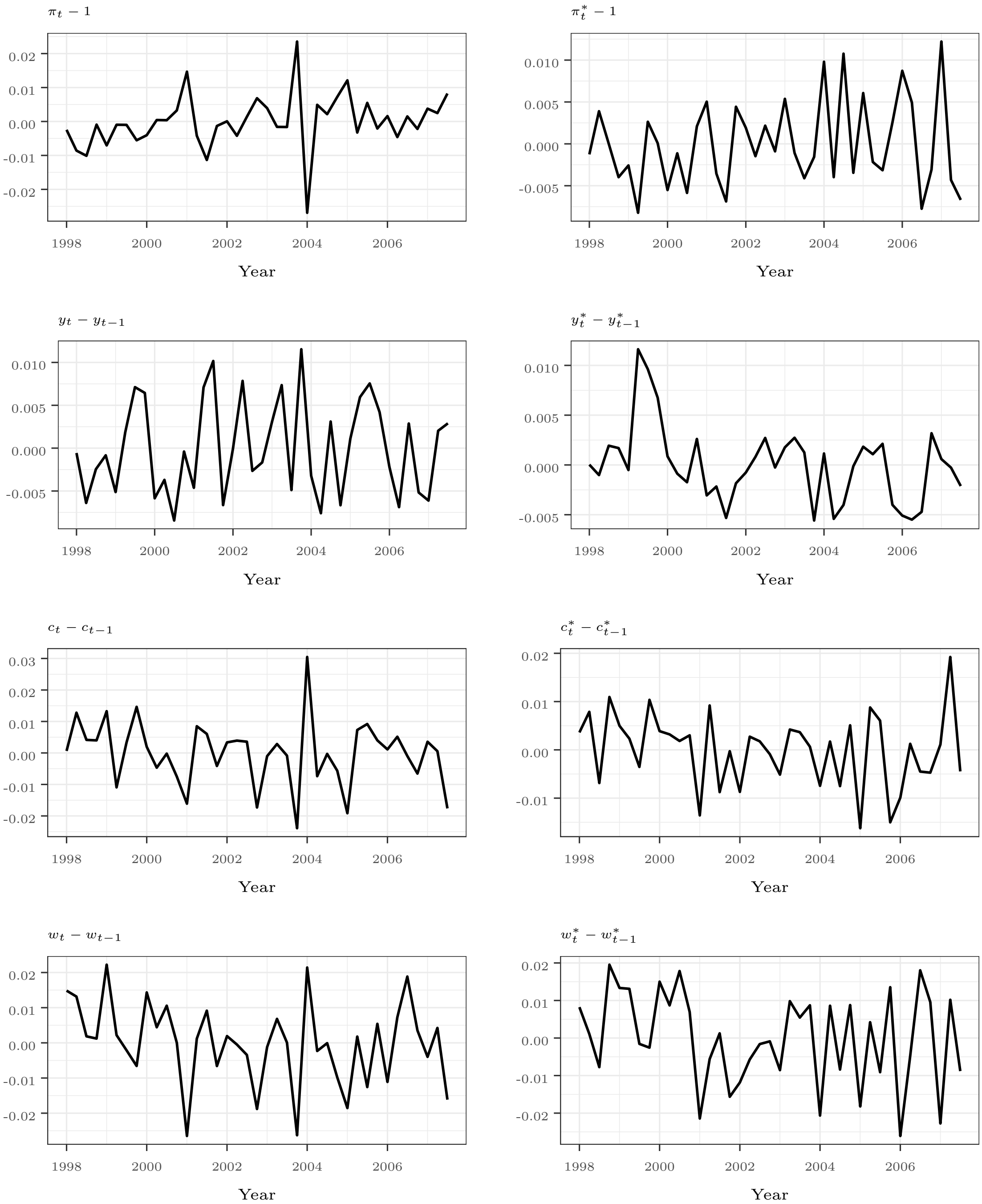
$$\pi_{avg,t} = 0.0816\pi_{H,t}^{scot} + 0.9184\pi_{H,t}^{ruk} \tag{217}$$

$$\tilde{y}_{avg,t} = 0.0816\tilde{y}_{scot,t} + 0.9184\tilde{y}_{ruk,t} \tag{218}$$

$$i_t = \rho_\pi \pi_{avg,t-1} + \rho_{\tilde{y}} \tilde{y}_{avg,t-1} + \Delta\pi_{avg,t-1} + \Delta\tilde{y}_{avg,t-1} + \nu_t \tag{219}$$

3 Application




$\pi_t - 1$
$\pi_t^* - 1$
$y_t - y_{t-1}$
$y_t^* - y_{t-1}^*$
$c_t - c_{t-1}$
$c_t^* - c_{t-1}^*$
$w_t - w_{t-1}$
$w_t^* - w_{t-1}^*$
Year

4 Results

5 Conclusion

6 Bibliography

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7 Appendix