Economic Heterogeneity in a Small Open Economy Framework

A Two-Country DSGE Model for Scotland and the Rest of the UK

B204335

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Two-country DSGE model for Scotland and the rest of the UK

1. Home country: *Scotland*
2. Foreign country: *rUK*
3. Scotland and rUK are SOEs
    (a) They take world output, inflation, and consumption as given and cannot influence it
4. Scotland and RUK are assumed to be symmetrical in market structure and preferences
5. Shocks in Scotland, RUK, and in the World Economy are assumed to be correlated
6. Sticky nominal prices: Calvo Fairy
    (a) Here, I would prefer to use Rotemburg as it is more intuitive and less popular than Calvo Fairy
7. For simplicity:
    (a) No nontradeable goods
    (b) No trading costs
    (c) No possibility of international policy coordination
    (d) No cost-push shocks
    (e) No nominal wage rigidities
    (f) No international financial assets

Structure. Max words: 10000

1. Introduction: **<u>2000 words</u>**
   (a) From RBC to NK DSGE: **1000 words**
   (b) Why Scotland and the rest of the UK? (NIESR policy-related question): **700 words**
   (c) Introduce essay's structure: **300 words**
2. Theoretical DSGE model: **<u>3000 words</u>**
   (a) Households
   (b) Firms
   (c) Equilibrium
3. Application: **<u>1500 words</u>**
   (a) Data: **500 words**
   (b) Estimation (MCMC): **1000 words**
4. Results: **<u>2500 words</u>**
   (a) Analysis of IRFs: **1000 words**
   (b) Other insights (NIESR policy-related question): **1500 words**
5. Conclusion: **<u>1000 words</u>**

0.1 Introduction

0.2 Theoretical DSGE model

The model is primarily based on the work of Galí (2015), Gali and Monacelli (2005), and Ricci (2019). Galí (2015) was used to derive a baseline NK DSGE with small open economy features, while Gali and Monacelli (2005) was used to scale the model to a two-country DSGE model. Finally, a few Scotland-specific DSGE extensions were taken from Ricci (2019) who were the first ones to build a DSGE model for Scotland and the rest of the UK (rUK). Where applicable, the notation follows Galí (2015). Given that Scotland and the rest of the UK are modelled as symmetrical, we denote variables without an asterisk as Scotland-specific variables, e.g. C_t, while variables with an asterisk, e.g. C_t^* refer to rUK variables. Finally, the rest of the World variables are denoted with two asterisks, e.g. C_t^{**}. It is also because the economies are assumed to be symmetrical, the following section only lists model equations for Scotland.

0.2.1 Households

This model assumes that there is infinitely many households in the economy represented by a unit interval. All households are assumed to be be symmetric, i.e. have the same preferences and behave identically. Below, we consider a representative household that wants to maximise their lifetime utility:

$$\mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \mathcal{U}(C_t, C_{t-1}, N_t, \varepsilon_{Z,t}) \right\} \tag{1}$$

$$\mathcal{U}(C_t, C_{t-1} N_t, \varepsilon_{Z,t}) = \left(\frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \varepsilon_{N,t} \frac{N_t^{1+\varphi}}{1+\varphi} \right) \varepsilon_{Z,t} \tag{2}$$

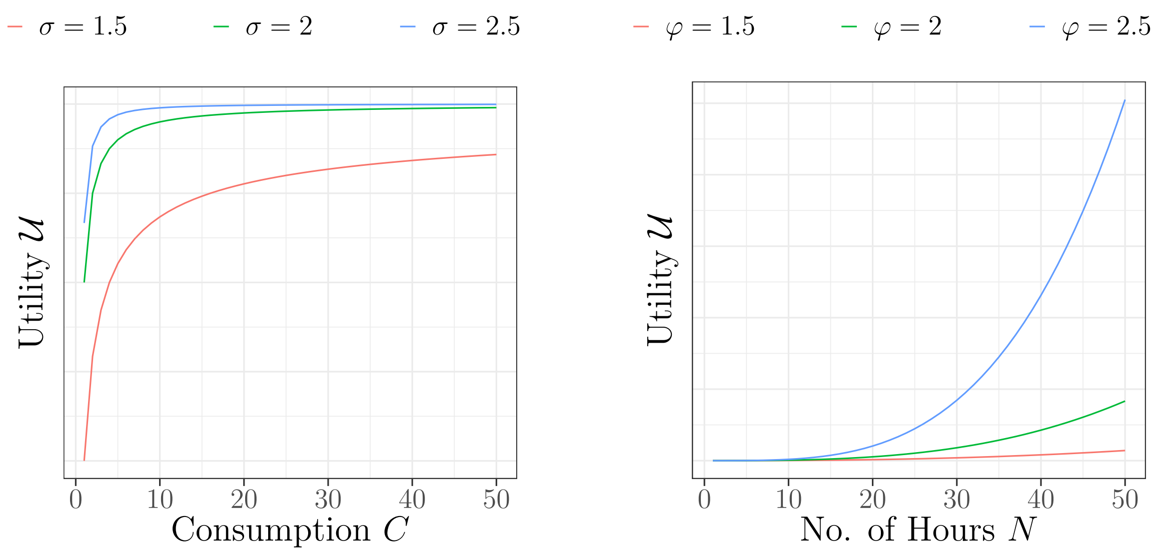
The household's utility depends on consumption C_t and hours worked N_t. As seen from the utility function (Equation (2)), the model assumes the household's utility to be (decreasingly) increasing in consumption C_t and (increasingly) decreasing in hours worked N_t. $\beta \in (0,1)$ is the discount factor, which can be thought of as an opportunity cost or an impatience rate, i.e. a unit of consumption C today will be worth $\beta * C > C$ tomorrow. Using parameter h, we also take into account household's habit formation in terms of consumption, which is found to improve model's fit to empirial macroeconomic data (?, 20??). We also introduce a preference shifter $\varepsilon_{Z,t}$ (Galí, 2015: 225), as well as a shock to the number of hours worked $\varepsilon_{N,t}$ [1] (Kolasa, 2009). The shocks are assumed to follow autoregressive process of order 1:

$$\log(\varepsilon_{Z,t}) = \rho_z \log(\varepsilon_{Z,t-1}) + \epsilon_t^z \tag{3}$$

$$\log(\varepsilon_{N,t}) = \rho_n \log(\varepsilon_{N,t-1}) + \epsilon_t^n \tag{4}$$

The parameters $\sigma \geq 0$ and $\varphi \geq 0$ determine the curvature of the utility of consumption and disutility of labour, respectively (Gali, 2015: 20). Finally, $\mathbb{E}_t\{*\}$ is the expectational operator, conditional on all information available at period t.

[1] While these specific shocks are not relevant to the research question, they help prevent stochastic singularity Pfeifer, 2021 and allows parameter estimation with a greater number of macroeconomic data series, see Section 0.3.



To allow goods differentiation between domestic and foreign, the model assumes that C_t is a composite consumption index defined by:

$$C_t = \left[(1-\nu)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \nu^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{5}$$

Where $C_{H,t}$ and $C_{F,t}$ are indices of consumption of home produced and imported goods, respectively. The parameter $\nu \in [0,1]$ reflects economy's openness for trading, while η denotes household's willingness to substitute domestic good with a foreign good. Our economy is assumed to be open for trading with the rest of the world (ROW), which itself is made of a continuum of infinitely many small economies i represented by a unit interval. Therefore, $C_{F,t}$ is a sum of indices of the quantity of goods imported from all countries i. In a similar fashion, if we denote j as a single variety of goods from a continuum of goods represented by a unit interval, we can express each consumption index as follows:

$$C_{H,t} = \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \qquad \text{Index of consumption of home produced goods}$$

$$C_{i,t} = \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \qquad \text{Index of consumption of country } i\text{'s produced goods}$$

$$C_{F,t} = \left(\int_0^1 C_{i,t}^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}} \qquad \text{Index of consumption of imported goods}$$

Notice that all three indices take the form of *Constant Elasticity Substitution* (*CES*) form, with parameters ε (without subscripts) and γ representing the degree of substitutability between varieties of goods and countries, respectively. Optimal allocation of each variety of goods:

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}; \quad C_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\varepsilon} C_{i,t}; \quad C_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\gamma} C_{F,t} \quad (6)$$

$$P_{H,t} = \left(\int_0^1 P_{H,t}(j)^{1-\varepsilon}\, dj\right)^{\frac{1}{1-\varepsilon}} \qquad \text{Domestic Price Index} \quad (7)$$

$$P_{i,t} = \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon}\, dj\right)^{\frac{1}{1-\varepsilon}} \qquad \text{Price Index of goods produced by country } i \quad (8)$$

$$P_{F,t} = \left(\int_0^1 P_{i,t}^{1-\gamma}\, dj\right)^{\frac{1}{1-\gamma}} \qquad \text{Price Index of Imported goods} \quad (9)$$

$$\int_0^1 P_{H,t}(j) C_{H,t}(j)\, dj = P_{H,t} C_{H,t} \qquad \int_0^1 P_{i,t}(j) C_{i,t}(j)\, dj = P_{i,t} C_{i,t} \quad (10)$$

Using (6) and (9) implies

$$\int_0^1 P_{i,t} C_{i,t} = P_{F,t} C_{F,t} \quad (11)$$

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \qquad C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \quad (12)$$

$$P_t = \left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha (P_{F,t})^{1-\eta}\right]^{\frac{1}{1-\eta}} \qquad \text{Consumption Price Index} \quad (13)$$

A special case $\eta = 1$:

$$P_t = (P_{H,t})^{1-\alpha} \times (P_{F,t})^{\alpha} \qquad C_t = \frac{1}{(1-\alpha)^{(1-\alpha)} \alpha^{\alpha}} (C_{H,t})^{(1-\alpha)} (C_{F,t})^{\alpha} \quad (14)$$

Total consumption expenditures are:

$$P_{H,t} C_{H,t} + P_{F,t} C_{F,t} = P_t C_t \quad (15)$$

So the budget constraint is:

$$P_t C_t + \mathbb{E}_t[Q_{t,t+1} D_{t+1}] \leq D_t + W_t N_t + T_t \quad (16)$$

$$\begin{aligned} \mathcal{L} = {} & \mathbb{E}_0 \sum_{t=0}^{\infty} \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right) \\ & + \lambda_t \left\{D_t + W_t N_t + T_t - P_t C_t - \mathbb{E}_t[Q_{t,t+1} D_{t+1}]\right\} \end{aligned} \quad (17)$$

$$\frac{\partial L}{\partial C_t} = \beta^t C_t^{-\sigma} - \lambda_t P_t = 0; \quad \Rightarrow \quad \beta^t C_t^{-\sigma} P_t^{-1} = \lambda_t$$

$$\frac{\partial L}{\partial N_t} = -\beta^t N_t^{\varphi} + \lambda_t N_t = 0; \quad \Rightarrow \quad \beta^t N_t^{\varphi} W_t^{-1} = \lambda_t$$

$$\begin{aligned}
\frac{\partial L}{\partial C_t} = \frac{\partial L}{\partial N_t} : &\beta^t C_t^{-\sigma} P_t^{-1} = \beta^t N_t^{\varphi} W_t^{-1} \\
\Rightarrow &C_t^{-\sigma} P_t^{-1} = N_t^{\varphi} W_t^{-1} \\
\Rightarrow &C_t^{-\sigma} N_t^{-\varphi} = W_t^{-1} P_t
\end{aligned}$$

$$\Rightarrow \qquad C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \qquad \text{Intratemporal Optimality Condition} \tag{18}$$

$$\frac{\partial L}{\partial C_t} = \beta^t C_t^{-\sigma} P_t^{-1} = \lambda_t; \quad \Rightarrow \quad \mathbb{E}_t[\beta^{t+1} C_{t+1}^{-\sigma} P_{t+1}^{-1}] = \mathbb{E}_t[\lambda_{t+1}]$$

$$\frac{\partial L}{\partial D_{t+1}} = -\lambda_t \, \mathbb{E}_t[Q_{t,t+1}] + \mathbb{E}_t[\lambda_{t+1}] = 0; \quad \Rightarrow \quad \mathbb{E}_t[Q_{t,t+1}] = \frac{\lambda_{t+1}}{\lambda_t}$$

$$\begin{aligned}
\mathbb{E}_t \left[\frac{\beta^t C_t^{-\sigma} P_t^{-1}}{\beta^{t+1} C_{t+1}^{-\sigma} P_{t+1}^{-1}} \right] &= \mathbb{E}_t \left[\frac{\lambda_t}{\lambda_{t+1}} \right] \\
\mathbb{E}_t \left[\frac{\beta^t C_t^{-\sigma} P_t^{-1}}{\beta^{t+1} C_{t+1}^{-\sigma} P_{t+1}^{-1}} \right] &= \mathbb{E}_t \left[\frac{1}{Q_{t,t+1}} \right] \\
\mathbb{E}_t \left[\frac{1}{\beta} \left(\frac{C_t}{C_{t+1}} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right)^{-1} \right] &= \mathbb{E}_t \left[\frac{1}{Q_{t,t+1}} \right]
\end{aligned}$$

$$\beta \, \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right] = Q_t \qquad \text{Euler equation} \tag{19}$$

However, Gali uses a different approach to derive the Euler equation, which introduces Arrow securities:

$$\frac{V_{t,t+1}}{P_t} C_t^{-\sigma} = \xi_{t,t+1} \beta C_{t+1}^{-\sigma} \frac{1}{P_{t+1}} \tag{20}$$

Where $V_{t,t+1}$ is an Arrow security and $\xi_{t,t+1}$ is the probability that the Arrow security will yield a payoff next period. Interpretation: LHS is paying for the security (expenses) in terms of consumption at P_t prices. RHS is the payoff if the Arrow security yields a payoff. The consumer will only be willing to pay LHS if the payoff is at least as big on the RHS. Given that:

$$Q_{t,t+1} = \frac{V_{t,t+1}}{\xi_{t,t+1}} \tag{21}$$

$$\frac{V_{t,t+1}}{P_t} C_t^{-\sigma} = \xi_{t,t+1} \beta C_{t+1}^{-\sigma} \frac{1}{P_{t+1}} \tag{22}$$

$$Q_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \tag{23}$$

$$\mathbb{E}_t[Q_{t,t+1}] = \beta \, \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] \tag{24}$$

$$\beta \, \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right] = Q_t \qquad \text{Euler equation} \tag{25}$$

Log-linearising (18):

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t}; \quad \Rightarrow \quad w_t - p_t = \sigma c_t + \varphi n_t \tag{26}$$

Log-linearising (25):

$$\begin{aligned}
\beta \, \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right] &= Q_t \\
\ln \beta - \mathbb{E}_t[\sigma c_{t+1}] + \sigma c_t + p_t - \mathbb{E}_t[p_{t+1}] &= \ln Q_t \\
\sigma c_t = \ln Q_t - \ln \beta + \mathbb{E}_t[\sigma c_{t+1}] - p_t &+ \mathbb{E}_t[p_{t+1}] \\
c_t = \mathbb{E}_t[c_{t+1}] - \frac{1}{\sigma}(- \ln Q_t - \rho &- \mathbb{E}_t[\pi_{t+1}]) \\
c_t = \mathbb{E}_t[c_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] &- \rho) \qquad (27) \\
\text{where } i_t = - \log Q_t, \ \rho = - \log \beta, \ \pi_t &= p_t - p_{t-1}
\end{aligned}$$

$$S_{i,t} = \frac{P_{i,t}}{P_{H,t}} \qquad \text{Bilateral terms of trade} \tag{28}$$

$$S_t = \frac{P_{F,t}}{P_{H,t}} = \left(\int_0^1 (S_{i,t} \, di)^{1-\gamma} \right)^{\frac{1}{1-\gamma}} \qquad \text{Effective terms of trade} \tag{29}$$

$$s_t = p_{F,t} - p_{H,t} = \left(\int_0^1 s_{i,t} \, di \right) \qquad \text{(log) Effective terms of trade} \tag{30}$$

Recall that when $\eta = 1$, then CPI is $P_t = (P_{H,t})^{1-\alpha} \times (P_{F,t})^{\alpha}$, which can be log-linearised to:

$$p_t = (1 - \alpha) p_{H,t} + \alpha p_{F,t} = p_{H,t} + \alpha s_t \tag{31}$$

Equations (30) and (31) hold when $\gamma = 1$ and $\eta = 1$, respectively.

$$\pi_{H,t} = p_{H,t+1} - p_{H,t} \qquad \text{Domestic Inflation} \tag{32}$$

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t \qquad \text{CPI Inflation} \tag{33}$$

The gap between domestic inflation and CPI inflation is only due to percentage change in the terms of trade.

$$P_{i,t}(j) = \mathcal{E}_{i,t} P^i_{i,t}(j) \qquad \text{Law of One Price (LOP)} \tag{34}$$

$$P_{i,t} = \mathcal{E}_{i,t} P^i_{i,t} \qquad \text{Law of One Price (LOP)} \tag{35}$$

$$p_{i,t} = e_{i,t} + p^i_{i,t} \qquad \text{(Log) Law of One Price (LOP)} \tag{36}$$

$$p_{F,t} = \int_0^1 (e_{i,t} + p^i_{i,t})\, di = e_t + p^\star_t \qquad \text{(Log)Price index of Imported Goods} \tag{37}$$

Where e_t is (Log) Effective Nominal Exchange Rate, $p^\star_t$ is the World Price Index.

$$s_t = p_{F,t} - p_{H,t} = e_t + p^\star_t - p_{H,t} \quad \text{Terms of trade but with the World Price Index} \tag{38}$$

$$\begin{aligned}
\mathcal{Q}_{i,t} &= \frac{\mathcal{E}_{i,t} P^i_t}{P_t} && \text{Bilateral Exchange Rate} && (39)\\
q_t &= \int_0^1 \log\left(\frac{\mathcal{E}_{i,t} P^i_t}{P_t}\right) di && && (40)\\
&= \int_0^1 (e_{i,t} + p^i_{i,t} - p_t)\, di && && (41)\\
&= e_t + p^\star_t - p_t && \text{using (37)} && (42)\\
&= s_t + p_{H_t} - p_t && \text{using (38)} && (43)\\
&= (1-\alpha) s_t && \text{using (31)} && (44)
\end{aligned}$$

International Risk-Sharing Equation (20) for country i can be rewritten as:

$$\frac{V_{t,t+1}}{\mathcal{E}^i_t P^i_t} (C^i_t)^{-\sigma} = \xi_{t,t+1} \beta (C^i_{t+1})^{-\sigma} \frac{1}{\mathcal{E}^i_{t+1} P^i_{t+1}} \tag{45}$$

Given that:

$$Q_{t,t+1} = \frac{V_{t,t+1}}{\xi_{t,t+1}} \tag{46}$$

$$\frac{V_{t,t+1}}{\mathcal{E}^i_t P^i_t} (C^i_t)^{-\sigma} = \xi_{t,t+1} \beta (C^i_{t+1})^{-\sigma} \frac{1}{\mathcal{E}^i_{t+1} P^i_{t+1}} \tag{47}$$

$$Q_{t,t+1} = \beta \left(\frac{C^i_{t+1}}{C^i_t}\right)^{-\sigma} \left(\frac{P^i_t}{P^i_{t+1}}\right) \left(\frac{\mathcal{E}^i_t}{\mathcal{E}^i_{t+1}}\right) \tag{48}$$

Recall that:

$$\begin{aligned}
\frac{\partial L}{\partial C_t} &= \beta^t C_t^{-\sigma} P_t^{-1} = \lambda_t\\
\frac{\partial L}{\partial D_{t+1}} &= -\lambda_t \mathbb{E}_t[Q_{t,t+1}] = -\mathbb{E}_t[\lambda_{t+1}]\\
&= -\beta^t C_t^{-\sigma} P_t^{-1} \mathbb{E}_t[Q_{t,t+1}] = -\mathbb{E}_t[\lambda_{t+1}] \qquad (49)
\end{aligned}$$

Which is symmetrical for country i:

$$\frac{\partial L^i}{\partial C_t^i} = \beta^t (C_t^i)^{-\sigma} (\mathcal{E}_{i,t} P_t^i)^{-1} = \lambda_t^i$$
$$\begin{aligned}\frac{\partial L^i}{\partial D_{t+1}^i} &= -\lambda_t^i \, \mathbb{E}_t[Q_{t,t+1}] = -\mathbb{E}_t[\lambda_{t+1}^i] \\ &= -\beta^t (C_t^i)^{-\sigma} (\mathcal{E}_{i,t} P_t^i)^{-1} \, \mathbb{E}_t[Q_{t,t+1}] = -\mathbb{E}_t[\lambda_{t+1}^i] \qquad (50)\end{aligned}$$

$$\frac{(49)}{(50)} : \frac{-\beta^t C_t^{-\sigma} P_t^{-1} \, \mathbb{E}_t[Q_{t,t+1}]}{-\beta^t (C_t^i)^{-\sigma} (\mathcal{E}_{i,t} P_t^i)^{-1} \, \mathbb{E}_t[Q_{t,t+1}]} = \frac{-\mathbb{E}_t[\lambda_{t+1}]}{-\mathbb{E}_t[\lambda_{t+1}^i]}$$

$$\begin{aligned} C_t^{-\sigma} (C_t^i)^{\sigma} \frac{\mathcal{E}_{i,t} P_t^i}{P_t} &= 1 \\ C_t^{-\sigma} (C_t^i)^{\sigma} \mathcal{Q}_{i,t} &= 1 \\ C_t^{-\sigma} (C_t^i)^{\sigma} &= \frac{1}{\mathcal{Q}_{i,t}} \\ C_t^{-\sigma} &= \frac{1}{\mathcal{Q}_{i,t}} (C_t^i)^{-\sigma} \\ C_t^{\sigma} &= \mathcal{Q}_{i,t} (C_t^i)^{\sigma} \end{aligned}$$

$$\Rightarrow C_t = C_t^i \mathcal{Q}_{i,t}^{\frac{1}{\sigma}} \qquad (51)$$

Log-linearising (51) yields:

$$c_t = c_t^i + \frac{1}{\sigma} q_{i,t} \qquad (52)$$

Integrating both sides over i:

$$\begin{aligned} c_t &= c_t^\star + \frac{1}{\sigma} q_t && (53) \\ &= c_t^\star + \left(\frac{1-\alpha}{\sigma} \right) s_t \qquad \text{using } q_t = (1-\alpha) s_t && (54) \end{aligned}$$

$c_t^\star$ is the log world consumption. Equation (54) is the link between the domestic consumption and the world consumption.

0.2.2 Firms

$$\begin{aligned} Y_t(j) &= A_t N_t(j) && (55) \\ \log A_t &= \alpha_t && (56) \\ \alpha_t &= \rho_a \alpha_{t-1} + \varepsilon_t && (57) \end{aligned}$$

$$L = P_t(j)Y_t(j) - W_t(j)N_t(j) \tag{58}$$
$$\Rightarrow L = P_tY_t - W_tN_t \tag{59}$$
$$\Rightarrow L = P_tA_tN_t - W_tN_t \tag{60}$$
$$\tag{61}$$

$$\frac{\partial L}{\partial N_t} = P_tA_t - W_t = 0 \qquad \Rightarrow W_t - P_tA_t = 0 \tag{62}$$
$$MC_t = W_t - P_tA_t \tag{63}$$
$$mc_t = w_t - p_t - a_t \tag{64}$$
$$mc_t = -\nu + w_t - p_t - a_t \tag{65}$$
$$mc_t = -\nu + w_t - p_{H,t} - a_t \tag{66}$$
$$\tag{67}$$

$\nu = -(\log(1-\tau))$, where τ is the employment subsidy, introduced later. $p_{H,t}$ because this is for domestic firms.

Firms that get to reset their price, do it using the following problem:

$$p_{\bar{H},t} = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k\,\mathbb{E}_t[mc_{t+k} + p_{H,t+k}] \tag{68}$$

$$p_{\bar{H},t} \qquad \text{Is the (log) new price} \tag{69}$$
$$\mu \qquad \text{Is the (log) markup in the steady state} \tag{70}$$

0.2.3 Equilibrium

$$Y_t(j) = C_{H,t}(j) + \int_0^1 C^i_{H,t}(j)\,di \tag{71}$$
$$= \left(\frac{P_{H,t}(j)}{P_{H,t}}^{-\varepsilon}\right)\left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t + \alpha\int_0^1\left(\frac{P_{H_t}}{\mathcal{E}_{i,t}P^i_{F,t}}\right)^{-\gamma}\left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta}C^i_t\,di\right] \tag{72}$$

Given that

$$Y_t = \left(\int_0^1 (Y_t(j))^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{73}$$

$$Y_t = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t + \alpha\int_0^1\left(\frac{P_{H_t}}{\mathcal{E}_{i,t}P^i_{F,t}}\right)^{-\gamma}\left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta}C^i_t\,di \tag{74}$$
$$= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t\left[(1-\alpha) + \alpha\int_0^1 (S^i_tS_{i,t})^{\gamma-\eta}\mathcal{Q}_{i,t}^{\eta-\frac{1}{\sigma}}\,di\right] \tag{75}$$

Which can be log-linearised to:

$$y_t = c_t + \alpha\gamma s_t + \alpha(\eta - \frac{1}{\sigma})q_t \tag{76}$$
$$= c_t + \frac{\alpha w}{\sigma} s_t \tag{77}$$
$$w_t = \sigma\gamma + (1-\alpha)(\sigma\eta - 1) \tag{78}$$

Assuming that country i is symmetric:

$$y_t^i = c_t^i + \frac{\alpha w}{\sigma} s_t^i \tag{79}$$
$$\int_0^1 y_t^i = \int_0^1 c_t^i + 0 = c_t^\star \qquad \text{World Consumption} \tag{80}$$

Using equations (), (), ():

$$y = c_t + \frac{\alpha w}{\sigma} s_t \tag{81}$$
$$y_t = c_t^\star + \frac{1-\alpha}{\sigma} s_t + \frac{\alpha w}{\sigma} s_t \tag{82}$$
$$y_t = y_t^\star + \frac{1-\alpha}{\sigma} s_t + \frac{\alpha w}{\sigma} s_t \tag{83}$$
$$y_t = y_t^\star + \frac{1-\alpha+\alpha w}{\sigma} s_t \tag{84}$$
$$y_t = y_t^\star + \frac{1+\alpha(w-1)}{\sigma} s_t \tag{85}$$
$$\sigma_\alpha = \frac{1+\alpha(w-1)}{\sigma} \tag{86}$$
$$\Rightarrow \quad y_t = y_t^\star + \frac{1}{\sigma_\alpha} s_t \tag{87}$$

Combining Euler equation () and (87) gives :

$$
\begin{aligned}
c_t &= \mathbb{E}_t[c_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] - \rho) \\
y_t - \frac{\alpha w}{\sigma} s_t &= \mathbb{E}_t\left[y_{t-1} - \frac{\alpha w}{\sigma} s_{t+1}\right] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] - \rho) \\
y_t &= \mathbb{E}_t\left[y_{t-1}\right] - \frac{\alpha w}{\sigma}\, \mathbb{E}_t[(s_{t+1} - s_t)] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] - \rho) \\
&= \mathbb{E}_t\left[y_{t-1}\right] - \frac{\alpha w}{\sigma}\, \mathbb{E}_t[\Delta s_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] - \rho) \\
&= \mathbb{E}_t\left[y_{t-1}\right] - \frac{\alpha w}{\sigma}\, \mathbb{E}_t[\Delta s_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \alpha\, \mathbb{E}_t[\Delta s_{t+1}] - \rho) \\
&= \mathbb{E}_t\left[y_{t-1}\right] - \frac{\alpha w}{\sigma}\, \mathbb{E}_t[\Delta s_{t+1}] + \frac{\alpha}{\sigma}\, \mathbb{E}_t[\Delta s_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
&= \mathbb{E}_t\left[y_{t-1}\right] - \frac{\alpha w + \alpha}{\sigma}\, \mathbb{E}_t[\Delta s_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
&= \mathbb{E}_t\left[y_{t-1}\right] + \frac{\alpha - \alpha w}{\sigma}\, \mathbb{E}_t[\Delta s_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
&= \mathbb{E}_t\left[y_{t-1}\right] + \frac{-\alpha(-1 + w)}{\sigma}\, \mathbb{E}_t[\Delta s_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
&= \mathbb{E}_t\left[y_{t-1}\right] - \frac{\alpha(w - 1)}{\sigma}\, \mathbb{E}_t[\Delta s_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
&= \mathbb{E}_t\left[y_{t-1}\right] - \frac{\alpha\Theta}{\sigma}\, \mathbb{E}_t[\Delta s_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
\mathbb{E}_t[\Delta s_{t+1}] &= \sigma_\alpha\, \mathbb{E}_t[\Delta y_{t+1} - \Delta y^\star_{t+1}] \quad \text{from Equation():} \\
&= \mathbb{E}_t\left[y_{t-1}\right] - \frac{\alpha\Theta}{\sigma}\sigma_\alpha\, \mathbb{E}_t[\Delta y_{t+1} - \Delta y^\star_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
&= \mathbb{E}_t\left[y_{t-1}\right] - \frac{\alpha\Theta}{1 + \alpha\Theta}\, \mathbb{E}_t[\Delta y_{t+1} - \Delta y^\star_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
&= \mathbb{E}_t\left[y_{t-1}\right] - \frac{\alpha\Theta}{1 + \alpha\Theta}\, \mathbb{E}_t[\Delta y_{t+1}] + \frac{\alpha\Theta}{1 + \alpha\Theta}\, \mathbb{E}_t[\Delta y^\star_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
&= \mathbb{E}_t\left[y_{t-1}\right] - \frac{\alpha\Theta}{1 + \alpha\Theta}\, \mathbb{E}_t[\Delta y_{t+1}] + \frac{\alpha\Theta}{1 + \alpha\Theta}\, \mathbb{E}_t[\Delta y^\star_{t+1}] - \frac{1}{(1 + \alpha\Theta)\sigma_\alpha}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho)
\end{aligned}
$$

$$
\begin{aligned}
(1+\alpha\Theta)y_t &= (1+\alpha\Theta)\,\mathbb{E}_t\left[y_{t-1}\right] - \alpha\Theta\,\mathbb{E}_t[\Delta y_{t+1}] + \alpha\Theta\,\mathbb{E}_t[\Delta y^\star_{t+1}] - \frac{1}{\sigma_\alpha}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
(1+\alpha\Theta)\,\mathbb{E}_t\left[y_t - y_{t-1}\right] &= -\alpha\Theta\,\mathbb{E}_t[\Delta y_{t+1}] + \alpha\Theta\,\mathbb{E}_t[\Delta y^\star_{t+1}] - \frac{1}{\sigma_\alpha}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
-(1+\alpha\Theta)\,\mathbb{E}_t\left[y_{t-1} - y_t\right] &= -\alpha\Theta\,\mathbb{E}_t[\Delta y_{t+1}] + \alpha\Theta\,\mathbb{E}_t[\Delta y^\star_{t+1}] - \frac{1}{\sigma_\alpha}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
(1+\alpha\Theta)\,\mathbb{E}_t\left[\Delta y_{t+1}\right] &= \alpha\Theta\,\mathbb{E}_t[\Delta y_{t+1}] - \alpha\Theta\,\mathbb{E}_t[\Delta y^\star_{t+1}] + \frac{1}{\sigma_\alpha}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
(1+\alpha\Theta)\,\mathbb{E}_t\left[\Delta y_{t+1}\right] - \alpha\Theta\,\mathbb{E}_t[\Delta y_{t+1}] &= -\alpha\Theta\,\mathbb{E}_t[\Delta y^\star_{t+1}] + \frac{1}{\sigma_\alpha}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
\mathbb{E}_t\left[\Delta y_{t+1}\right] + \alpha\Theta\,\mathbb{E}_t\left[\Delta y_{t+1}\right] - \alpha\Theta\,\mathbb{E}_t[\Delta y_{t+1}] &= -\alpha\Theta\,\mathbb{E}_t[\Delta y^\star_{t+1}] + \frac{1}{\sigma_\alpha}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
\mathbb{E}_t\left[\Delta y_{t+1}\right] &= -\alpha\Theta\,\mathbb{E}_t[\Delta y^\star_{t+1}] + \frac{1}{\sigma_\alpha}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
\mathbb{E}_t\left[y_{t+1} - y_t\right] &= -\alpha\Theta\,\mathbb{E}_t[\Delta y^\star_{t+1}] + \frac{1}{\sigma_\alpha}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
-y_t &= -\,\mathbb{E}_t\left[y_{t+1}\right] - \alpha\Theta\,\mathbb{E}_t[\Delta y^\star_{t+1}] + \frac{1}{\sigma_\alpha}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) \\
\Rightarrow y_t &= \mathbb{E}_t\left[y_{t+1}\right] - \frac{1}{\sigma_\alpha}(i_t - \mathbb{E}_t[\pi_{H,t+1}] - \rho) + \alpha\Theta\,\mathbb{E}_t[\Delta y^\star_{t+1}] \qquad (88)
\end{aligned}
$$

$$(89)$$

$$\pi_{avg,t} = 0.0816\pi_{H,t}^{scot} + 0.9184\pi_{H,t}^{ruk} \tag{90}$$

$$\tilde{y}_{avg,t} = 0.0816\tilde{y}_{scot,t} + 0.9184\tilde{y}_{ruk,t} \tag{91}$$

$$i_t = \rho_\pi \pi_{avg,t-1} + \rho_{\tilde{y}} \tilde{y}_{avg,t-1} + \Delta\pi_{avg,t-1} + \Delta\tilde{y}_{avg,t-1} + \nu_t \tag{92}$$

0.3 Application

0.4 Results

0.5 Conclusion

0.6 Bibliography

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0.7 Appendix