1 Household

1.1 Utility Function

$$U_{j,t} = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left[\quad \frac{\varepsilon_{d,t+k}}{1-\sigma} \left(C_{j,t+k} - hC_{j,t+k-1}\right)^{1-\sigma} \quad - \quad \frac{\varepsilon_{l,t+k}}{1-\psi} L_{j,t+k}^{1+\psi} \quad \right] \tag{1}$$

Expression	**Long name**	**Value**
$U_{j,t}$	Utility of Household j at period t	
$\mathbb{E}_t$	Expectation operator cond. on info. at time t	
β	Discount rate	$\beta = 0.99$
σ	Inverse of the Elasticity of Intertemporal Substitution	
$C_{j,t}$	Consumption bundle	
h	External Habit Persistence parameter	
ϕ	Inverse of the Frisch elasticity of labour supply	
$L_{j,t}$	Labour effort	
$\varepsilon_{d,t+k}$	Consumption preference shock	
$\varepsilon_{l,t+k}$	Labour supply shock	

1.2 Budget constraint

$$P_{C,t}C_{j,t} + P_{I,t}I_{j,t} + \mathbb{E}_t \left[\Upsilon_{t,t+1} B_{j,t+1}\right] \tag{2}$$
$$= B_{j,t} + w_{j,t}L_{j,t} + R_{K,t}K_{j,t} + \Pi_{j,H,t} + \Pi_{j,N,t} + T_{j,t} \tag{3}$$

Expression	**Long name**	**Notes**
$P_{C,t}$	Price of consumption	
$P_{I,t}$	Price of investment	
$I_{j,t}$	Investment goods	
$B_{j,t}$	Nominal payoff of the portfolio	
$w_{j,t}$	Nominal wage	
$R_{K,t}$	Household's income from renting capital K_t	
T_t	Lump sum taxes	
$\Pi_{j,H,t}$	Dividends from tradeable goods	
$\Pi_{j,N,t}$	Dividends from non-tradeable goods	
Υ_t	Stochastic discount factor	$\mathbb{E}\left[\Upsilon_{t,t+1}\right] = R_t^{-1}$

1.3 Consumption FOCs

$$0 = B_t + w_t L_t + R_{K,t} K_t + \Pi_{H,t} + \Pi_{N,t} + T_t - P_{C,t} C_t - P_{I,t} I_t - \mathbb{E}_t[\Upsilon_{t,t+1} B_{t+1}] \tag{4}$$

$$\mathcal{L} = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left[\frac{\varepsilon_{d,t+k}}{1-\sigma} \left(C_{j,t+k} - hC_{j,t+k-1}\right)^{1-\sigma} - \frac{\varepsilon_{l,t+k}}{1-\psi} L_{j,t+k}^{1+\psi} \right] \tag{5}$$

$$+ \lambda \times (B_t + w_t L_t + R_{K,t} K_t + \Pi_{H,t} + \Pi_{N,t} + T_t - P_{C,t} C_t - P_{I,t} I_t - \mathbb{E}_t[\Upsilon_{t,t+1} B_{t+1}]) \tag{6}$$

$$\frac{\partial \mathcal{L}}{\partial B_{t+1}} = -\lambda_t R_t^{-1} + \mathbb{E}[\lambda_{t+1}] = 0 \tag{7}$$

$$\Rightarrow \mathbb{E}\left[\frac{\lambda_t}{\lambda_{t+1}}\right] = R_t \tag{8}$$

$$\frac{\partial \mathcal{L}}{\partial C_t} = \tag{9}$$

$$1.\ \mathbb{E}_t[\beta^k (1-\sigma) \frac{\varepsilon_{d,t}}{1-\sigma} (C_{j,t} - hC_{j,t-1})^{-\sigma} - \lambda_t P_{c,t}] = 0 \tag{10}$$

$$\Rightarrow \beta^k \, \mathbb{E}_t[\varepsilon_{d,t} (C_{j,t} - hC_{j,t-1})^{-\sigma}] - \mathbb{E}[\lambda_t P_{c,t}] = 0 \tag{11}$$

$$\Rightarrow \beta^{k+1} \, \mathbb{E}_t[\varepsilon_{d,t+1} (C_{j,t+1} - hC_{j,t})^{-\sigma}] - \mathbb{E}[\lambda_{t+1} P_{c,t+1}] = 0 \tag{12}$$

$$\frac{1.}{2.} = \frac{\beta^k}{\beta^{k+1}} \, \mathbb{E}_t \left[\frac{\varepsilon_{d,t} (C_{j,t} - hC_{j,t-1})^{-\sigma}}{\varepsilon_{d,t+1} (C_{j,t+1} - hC_{j,t})^{-\sigma}} \right] - \mathbb{E}_t \left[\frac{\lambda_t}{\lambda_{t+1}} \right] \mathbb{E} \left[\frac{P_{c,t}}{P_{c,t+1}} \right] = 0 \tag{13}$$

$$\frac{\beta^k}{\beta^{k+1}} \, \mathbb{E}_t \left[\frac{\varepsilon_{d,t} (C_{j,t} - hC_{j,t-1})^{-\sigma}}{\varepsilon_{d,t+1} (C_{j,t+1} - hC_{j,t})^{-\sigma}} \right] - R_t \, \mathbb{E} \left[\frac{P_{c,t}}{P_{c,t+1}} \right] = 0 \tag{14}$$

$$\frac{\beta^k}{\beta^{k+1}} \, \mathbb{E}_t \left[\frac{\varepsilon_{d,t} (C_{j,t} - hC_{j,t-1})^{-\sigma}}{\varepsilon_{d,t+1} (C_{j,t+1} - hC_{j,t})^{-\sigma}} \right] = R_t \, \mathbb{E} \left[\frac{P_{c,t}}{P_{c,t+1}} \right] \tag{15}$$

$$\frac{1}{\beta} \, \mathbb{E}_t \left[\frac{\varepsilon_{d,t} (C_{j,t} - hC_{j,t-1})^{-\sigma}}{\varepsilon_{d,t+1} (C_{j,t+1} - hC_{j,t})^{-\sigma}} \frac{P_{c,t+1}}{P_{c,t}} \right] = R_t \tag{16}$$

$$\frac{1}{\beta R_t} \, \mathbb{E}_t \left[\frac{\varepsilon_{d,t} (C_{j,t} - hC_{j,t-1})^{-\sigma}}{\varepsilon_{d,t+1} (C_{j,t+1} - hC_{j,t})^{-\sigma}} \frac{P_{c,t+1}}{P_{c,t}} \right] = 1 \tag{17}$$

$$\beta R_t \, \mathbb{E}_t \left[\frac{\varepsilon_{d,t+1}}{\varepsilon_{d,t}} \left(\frac{C_{j,t+1} - hC_{j,t}}{C_{j,t} - hC_{j,t-1}} \right)^{-\sigma} \times \frac{P_{c,t}}{P_{c,t+1}} \right] = 1 \tag{18}$$

Euler equation:

$$\beta R_t \, \mathbb{E}_t \left[\frac{\varepsilon_{d,t+1}}{\varepsilon_{d,t}} \left(\frac{C_{j,t+1} - hC_{j,t}}{C_{j,t} - hC_{j,t-1}} \right)^{-\sigma} \times \frac{P_{c,t}}{P_{c,t+1}} \right] = 1 \tag{19}$$

Consumption is made of tradable and nontradable goods

$$C_t = \frac{C_{T,t}^{\gamma_c} C_{N,t}^{1-\gamma_c}}{\gamma_c^{\gamma_c} (1-\gamma_c)^{1-\gamma_c}} \tag{20}$$

Long name	**Expression**
Consumption of tradable goods	$C_{T,t}$
Consumption of nontradable goods	$C_{N,t}$
Share of tradable goods consumption in a household	γ_c

Consuming a unit of final tradable good requires ω units of nontradable distribution services $Y_{D,t}$:

$$C_{T,t} = \min \left\{ C_{R,t}; \omega^{-1} Y_{D,t} \right\} \tag{21}$$

$$C_{R,t} = \frac{C_{H,t}^{\alpha} C_{F,t}^{1-\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha}} \tag{22}$$

Long name	**Expression**
Bundle of home-made tradable goods consumed at Home	$C_{H,t}$
Bundle of foreign-made tradable goods consumed at Home	$C_{F,t}$
Share of home-made goods in the home-consumed basked of tradable goods	γ_c

$$C_{N,t} = \left[\left(\frac{1}{n} \right)^{\frac{1}{\phi_N}} \int_0^n C_t(z_N)^{\frac{\phi_N - 1}{\phi_N}} \, dz_N \right]^{\frac{\phi_N}{\phi_N - 1}} \tag{23}$$

$$C_{H,t} = \left[\left(\frac{1}{n} \right)^{\frac{1}{\phi_H}} \int_0^n C_t(z_H)^{\frac{\phi_H - 1}{\phi_H}} \, dz_H \right]^{\frac{\phi_H}{\phi_H - 1}} \tag{24}$$

$$C_{F,t} = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\phi_F}} \int_n^1 C_t(z_F)^{\frac{\phi_F - 1}{\phi_F}} \, dz_F \right]^{\frac{\phi_F}{\phi_F - 1}} \tag{25}$$

Long name	**Expression**
Elasticity of substitution for nontradable goods	ϕ_N
Elasticity of substitution for home-made tradable goods	ϕ_H
Elasticity of substitution for foreign-made tradable goods	ϕ_F

FOCs:

$$C_t(z_N) = \frac{1}{n} (1-\gamma_c) \left(\frac{P_t(z_N)}{P_{N,t}} \right)^{-\phi_N} \left(\frac{P_{N,t}}{P_{C,t}} \right)^{-1} C_t \tag{26}$$

$$C_t(z_H) = \frac{1}{n} \gamma_c \alpha \left(\frac{P_t(z_H)}{P_{H,t}} \right)^{-\phi_H} \left(\frac{P_{H,t}}{P_{R,t}} \right)^{-1} \left(\frac{P_{T,t}}{P_{C,t}} \right)^{-1} C_t \tag{27}$$

$$C_t(z_F) = \frac{1}{1-n} \gamma_c (1-\alpha) \left(\frac{P_t(z_F)}{P_{F,t}} \right)^{-\phi_F} \left(\frac{P_{F,t}}{P_{R,t}} \right)^{-1} \left(\frac{P_{T,t}}{P_{C,t}} \right)^{-1} C_t \tag{28}$$

$$P_{N,t} = \left[\frac{1}{n}\int_0^n P_t\left(z_N\right)^{1-\phi_N}\, dz_N\right]^{\frac{1}{1-\phi_N}} \tag{29}$$

$$P_{H,t} = \left[\frac{1}{n}\int_0^n P_t\left(z_H\right)^{1-\phi_H}\, dz_H\right]^{\frac{1}{1-\phi_H}} \tag{30}$$

$$P_{F,t} = \left[\frac{1}{1-n}\int_n^1 P_t\left(z_F\right)^{1-\phi_F}\, dz_F\right]^{\frac{1}{1-\phi_F}} \tag{31}$$

$$P_{R,t} = P_{H,t}^{\alpha} P_{F,t}^{1-\alpha} \tag{32}$$

$$P_{T,t} = P_{R,t} + \omega P_{N,t} \tag{33}$$

$$P_{C,t} = P_{T,t}^{\gamma_c} P_{N,t}^{1-\gamma_c} \tag{34}$$