Two-country DSGE model for Scotland and the rest of the UK

Closely follows Galí (2015) and Gali and Monacelli (2005)

1. Home country: Scotland (notation - X)
2. Foreign country: rUK (notation - X^*)
3. Notation for UK-wide variables - X^{UK}
4. Scotland and rUK are SOEs
   (a) They trade, take world output, inflation, and consumption as given and cannot influence it
5. Calvo staggered prices, no capital/investment
6. Scotland and RUK are assumed to be symmetrical in market structure and preferences
7. Monetary Union: There is a population-weighted UK-wide interest rate in place, and all four nations within the UK purchase government-issued bonds at this rate
   (a) We also consider counterfactual scenarios, i.e. where both Holyrood and Westminister can issue bonds at country-specific interest rate
8. The government spending can be financed via a lump-sum tax, a labour (income) tax, and borrowing.
9. We consider 4 scenarios:
   (a) Two governments funded by lump-sum tax, <u>two</u> of which can issue bonds
   (b) Two governments funded by lump-sum tax, <u>one</u> of which can issue bonds
   (c) Two governments funded by an income tax, <u>two</u> of which can issue bonds
   (d) Two governments funded by an income tax, <u>one</u> of which can issue bonds

	S HBC:	$P_tC_t + \mathbb{E}_t[R_{t+1}^{-1}B_{t+1}] = B_t + W_tN_t + T_t$
	rUK HBC:	$P_t^*C_t^* + \mathbb{E}_t[R_{t+1}^{*-1}B_{t+1}^*] = B_t^* + W_t^*N_t^* + T_t^*$
	S GBC:	$\mathbb{E}_t[R_{t+1}^{-1}B_{t+1}] + T_t = P_tG_t + B_t$
	rUK GBC:	$\mathbb{E}_t[R_{t+1}^{*-1}B_{t+1}^*] + T_t^* = P_t^*G_t^* + B_t^*$
	S TR:	$T_t = P_tG_t$
Scenario 1	rUK TR:	$T_t^* = P_tG_t^*$
(G: 2, $\tau:0$)	S Debt s.:	$T_t/P_t = \phi_g^*G_Yg_t + \phi_b^*(B_t/P_t); \quad G_Y = 0$
	rUK Debt s.:	$T_t^*/P_t^* = \phi_g^*G_Y^*g_t^* + \phi_b^*(B_t^*/P_t^*); \quad G_Y = 0$
	S Bonds:	$(B_{t+1}/P_t)/R_{t+1} = (1-\phi_g)G_Yg_t + (1-\phi_b)(B_t/P_t); \quad G_Y = 0$
	rUK Bonds:	$(B_{t+1}^*/P_t^*)/R_{t+1}^* = (1-\phi_g^*)G_Y^*g_t^* + (1-\phi_b^*)(B_t^*/P_t^*); \quad G_Y^* = 0$
	S RC:	$Y_t = C_t + G_t + X_t$
	rUK RC:	$Y_t^* = C_t^* + G_t^* + X_t^*$
	S HBC:	$P_tC_t + \mathbb{E}_t[R_{t+1}^{UK-1}B_{t+1}^{UK}] = B_t^{UK} + W_tN_t + \varpi T_t^{UK}$
	rUK HBC:	$P_t^*C_t^* + \mathbb{E}_t[R_{t+1}^{UK-1}B_{t+1}^{UK}] = B_t^{UK} + W_t^*N_t^* + (1-\varpi)T_t^{UK}$
	S GBC:	N/A
	rUK GBC:	$\mathbb{E}_t[R_{t+1}^{UK-1}B_{t+1}^{UK}] + T_t^{UK} = P_t^{UK}G_t^{UK} + B_t^{UK}$
	S TR:	N/A
Scenario 2	rUK TR:	$T_t^{UK} = P_t^{UK}G_t^{UK}$
(G: 1, $\tau:0$)	S Debt s.:	N/A
	rUK Debt s.:	$T_t^{UK}/P_t^{UK} = \phi_g^{UK}G_Y^{UK}g_t^{UK} + \phi_b^{UK}(B_{t+1}^{UK}/P_t^{UK}); \quad G_Y^{UK} = 0$
	S Bonds:	N/A
	rUK Bonds:	$(B_t^{UK}/P_t^{UK})/R_{t+1}^{UK} = (1-\phi_g^{UK})G_Y^{UK}g_t^{UK} + (1-\phi_b^{UK})(B_t^{UK}/P_t^{UK}); \quad G_Y^{UK} = 0$
	S RC:	$Y_t = C_t + \varpi G_t^{UK} + X_t$
	rUK RC:	$Y_t^* = C_t^* + (1-\varpi)G_t^{UK} + X_t^*$
	S HBC:	$P_tC_t + \mathbb{E}_t[R_{t+1}^{-1}B_{t+1}] = B_t + (1-\tau_t)W_tN_t + T_t$
	rUK HBC:	$P_t^*C_t^* + \mathbb{E}_t[R_{t+1}^{*-1}B_{t+1}^*] = B_t^* + (1-\tau_t^*)W_t^*N_t^* + T_t^*$
	S GBC:	$\mathbb{E}_t[R_{t+1}^{-1}B_{t+1}] + T_t = P_tG_t + B_t$
	rUK GBC:	$\mathbb{E}_t[R_{t+1}^{*-1}B_{t+1}^*] + T_t^* = P_t^*G_t^* + B_t^*$
	S TR:	$T_t = \tau_tW_tN_t$
Scenario 3	rUK TR:	$T_t^* = \tau_t^*W_t^*N_t^*$
(G: 2, $\tau:1$)	S Debt s.:	$T_t/P_t = \phi_g^*G_Yg_t + \phi_b^*(B_t/P_t); \quad G_Y = \tau$
	rUK Debt s.:	$T_t^*/P_t^* = \phi_g^*G_Y^*g_t^* + \phi_b^*(B_t^*/P_t^*); \quad G_Y = \tau^*$
	S Bonds:	$(B_{t+1}/P_t)/R_{t+1} = (1-\phi_g)G_Yg_t + (1-\phi_b)(B_t/P_t); \quad G_Y = \tau$
	rUK Bonds:	$(B_{t+1}^*/P_t^*)/R_{t+1}^* = (1-\phi_g^*)G_Y^*g_t^* + (1-\phi_b^*)(B_t^*/P_t^*); \quad G_Y^* = \tau^*$
	S RC:	$Y_t = C_t + G_t + X_t$
	rUK RC:	$Y_t^* = C_t^* + G_t^* + X_t^*$
	S HBC:	$P_tC_t + \mathbb{E}_t[R_{t+1}^{UK-1}B_{t+1}^{UK}] = B_t^{UK} + (1-\tau_t^{UK})W_tN_t + \varpi T_t^{UK}$
	rUK HBC:	$P_t^*C_t^* + \mathbb{E}_t[R_{t+1}^{UK-1}B_{t+1}^{UK}] = B_t^{UK} + (1-\tau_t^{UK})W_t^*N_t^* + (1-\varpi)T_t^{UK}$
	S GBC:	N/A
	rUK GBC:	$\mathbb{E}_t[R_{t+1}^{UK-1}B_{t+1}^{UK}] + T_t^{UK} = P_t^{UK}G_t^{UK} + B_t^{UK}$
	S TR:	N/A
Scenario 4	rUK TR:	$T_t^{UK} = \varpi\tau_t^{UK}W_tN_t + (1-\varpi)\tau_t^{UK}W_t^*N_t^*$
(G: 1, $\tau:1$)	S Debt s.:	N/A
	rUK Debt s.:	$T_t^{UK}/P_t^{UK} = \phi_g^{UK}G_Y^{UK}g_t^{UK} + \phi_b^{UK}(B_t^{UK}/P_t^{UK}); \quad G_Y^{UK} = \tau^{UK}$
	S Bonds:	N/A
	rUK Bonds:	$(B_{t+1}^{UK}/P_t^{UK})/R_{t+1}^{UK} = (1-\phi_g^{UK})G_Y^{UK}g_t^{UK} + (1-\phi_b^{UK})(B_t^{UK}/P_t^{UK}); \quad G_Y^{UK} = \tau^{UK}$
	S RC:	$Y_t = C_t + \varpi G_t^{UK}$
	rUK RC:	$Y_t^* = C_t^* + (1-\varpi)G_t^{UK}$

where:

S HBC	Household (Scotland) budget constraint
rUK HBC	Household (rUK) budget constraint
S GBC	Government (Holyrood) budget constraint
rUK GBC	Government (Westminister) budget constraint
S TR	Government (Holyrood) tax revenue
rUK TR	Government (Westminister) tax revenue
S RC	Resource (Scotland) constraint
rUK RC	Resource (rUK) constraint

Tax Revenue

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\begin{aligned}
T_t^* &= \varpi \tau_t W_t N_t + (1-\varpi)\tau_t W_t^* N_t^* \\
T^* &= \varpi \tau W N + (1-\varpi)\tau W^* N^* \quad \text{(Steady state)} \\
&\text{Using Uhlig's (1999) method, } X_t Y_t \approx XY\mathbf{e}^{\tilde{X}_t + \tilde{Y}_t}\text{:} \\
T\mathbf{e}^{\tilde{T}_t^*} &= \varpi \tau W N \mathbf{e}^{\tilde{\tau}_t + \tilde{W}_t + \tilde{N}_t} + (1-\varpi)\tau W^* N^* \mathbf{e}^{\tilde{\tau}_t + \tilde{W}_t^* + \tilde{N}_T^*} \\
&\text{Using } \mathbf{e}^{\tilde{X}_t} \approx 1 + \tilde{X}_t\text{:} \\
T^*(1+\tilde{T}_t^*) &= \varpi \tau W N(1+\tilde{\tau}_t + \tilde{W}_t + \tilde{N}_t) + (1-\varpi)\tau W^* N^*(1+\tilde{\tau}_t + \tilde{W}_t^* + \tilde{N}_T^*) \\
&\text{Subtract (\textbf{??}):} \\
T^*(1+\tilde{T}_t^*) - T^* &= \varpi \tau W N(1+\tilde{\tau}_t + \tilde{W}_t + \tilde{N}_t) - \varpi\tau W N + (1-\varpi)\tau W^* N^*(1+\tilde{\tau}_t + \tilde{W}_t^* + \tilde{N}_T^*) - (1-\varpi)\tau W^* N^* \\
T^*[(1+\tilde{T}_t^*) - 1] &= \varpi \tau W N[(1+\tilde{\tau}_t + \tilde{W}_t + \tilde{N}_t) - 1] + (1-\varpi)\tau W^* N^*[(1+\tilde{\tau}_t + \tilde{W}_t^* + \tilde{N}_T^*) - 1] \\
T^*\tilde{T}_t^* &= \varpi \tau W N(\tilde{\tau}_t + \tilde{W}_t + \tilde{N}_t) + (1-\varpi)\tau W^* N^*(\tilde{\tau}_t + \tilde{W}_t^* + \tilde{N}_T^*) \\
&\text{Divide by } T^*\text{ :}
\end{aligned}
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\boxed{\tilde{T}_t^* = \tau \tfrac{WN}{T^*} \times \varpi(\tilde{\tau}_t + \tilde{W}_t + \tilde{N}_t) + \tau \tfrac{W^* N^*}{T^*} \times (1-\varpi)(\tilde{\tau}_t + \tilde{W}_t^* + \tilde{N}_t^*)} \tag{1}
$$

$$
\begin{aligned}
C_t^\sigma N_t^\varphi &= \frac{W_t}{P_t}(1-\tau_t) && (2) \\
C_t^\sigma N_t^\varphi &= \frac{W_t}{P_t} - \frac{W_t}{P_t}\tau_t && (3) \\
C^\sigma N^\varphi &= \frac{W}{P} - \frac{W}{P}\tau \quad \text{(Steady state)} && (4) \\
&\text{Using Uhlig's (1999) method, } X_t Y_t \approx XY\mathbf{e}^{\tilde{X}_t + \tilde{Y}_t}, && (5) \\
&\text{...and since } \tau_t \text{ is tax rate (percentage), we do not need to take logs of it:} && (6) \\
C^\sigma N^\varphi \mathbf{e}^{\sigma c_t + \varphi n_t} &= \frac{W}{P}\mathbf{e}^{w_t - p_t} - \frac{W}{P}\tau \mathbf{e}^{w_t - p_t} - \tilde{\tau}_t && (7) \\
&\text{Using } \mathbf{e}^{\tilde{X}_t} \approx 1 + \tilde{X}_t\text{:} && (8) \\
C^\sigma N^\varphi(1 + \sigma c_t + \varphi n_t) &= \frac{W}{P}(1 + w_t - p_t) - \frac{W}{P}\tau(1 + w_t - p_t) - \tilde{\tau}_t && (9) \\
&\text{Subtract (4):} && (10) \\
C^\sigma N^\varphi(\sigma c_t + \varphi n_t) &= \frac{W}{P}(w_t - p_t) - \frac{W}{P}\tau(w_t - p_t) - \tilde{\tau}_t && (11) \\
C^\sigma N^\varphi(\sigma c_t + \varphi n_t) &= \frac{W}{P}\left[(w_t - p_t) - \tau(w_t - p_t) - \tilde{\tau}_t\right] && (12) \\
C^\sigma N^\varphi(\sigma c_t + \varphi n_t) &= \frac{W}{P}\left[(1-\tau)(w_t - p_t) - \tilde{\tau}_t\right] && (13) \\
C^\sigma N^\varphi \frac{P}{W}(\sigma c_t + \varphi n_t) &= \left[(1-\tau)(w_t - p_t) - \tilde{\tau}_t\right] && (14) \\
(1-\tau)(\sigma c_t + \varphi n_t) &= \left[(1-\tau)(w_t - p_t) - \tilde{\tau}_t\right] && (15) \\
\sigma c_t + \varphi n_t &= w_t - p_t - \frac{1}{1-\tau}\tilde{\tau}_t && (16) \\
& && (17)
\end{aligned}
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$$y_t = a_t + (1-\alpha)n_t \tag{18}$$
$$n_t = \frac{1}{1-\alpha}(y_t - a_t) \tag{19}$$
$$\varpi = \sigma\eta + (1-\upsilon)(\sigma\eta - 1) \tag{20}$$
$$\sigma c_t + \varphi n_t = w_t - p_t - \frac{1}{1-\tau}\tau_t \tag{21}$$
$$y_t = (1-\upsilon)c_t + \upsilon_t(2-\upsilon)\eta s_t + \upsilon y_t^* \tag{22}$$
$$c_t = y_t^* + \frac{1}{\sigma}z_t + \frac{1-\upsilon}{\sigma}s_t \tag{23}$$
$$\implies c_t = y_t - \upsilon s_t\left((2-\upsilon)\eta + \frac{1-\upsilon}{\sigma}\right) + \frac{\upsilon}{\sigma}z_t \tag{24}$$

$$\mu_t = -(\sigma c_t + \varphi n_t + \frac{1}{1-\tau}\tau_t) - \upsilon s_t + a_t - \alpha n_t \tag{25}$$
$$= -\sigma c_t - \varphi n_t - \frac{1}{1-\tau}\tau_t - \upsilon s_t + a_t - \alpha n_t \tag{26}$$
$$= -\sigma c_t - \frac{1}{1-\tau}\tau_t - \upsilon s_t + a_t - n_t(\varphi + \alpha) \tag{27}$$
$$\tag{28}$$

$$-n_t(\varphi+\alpha) = -\frac{1}{1-\alpha}(y_t - a_t)(\varphi+\alpha) \tag{29}$$
$$-n_t(\varphi+\alpha) = -\frac{\varphi+\alpha}{1-\alpha}(y_t - a_t) \tag{30}$$
$$-n_t(\varphi+\alpha) = -\frac{\varphi+\alpha}{1-\alpha}y_t + \frac{\varphi+\alpha}{1-\alpha}a_t \tag{31}$$
$$\tag{32}$$

$$\mu_t = -\sigma c_t - \frac{1}{1-\tau}\tau_t - \upsilon s_t + a_t - \frac{\varphi+\alpha}{1-\alpha}y_t + \frac{\varphi+\alpha}{1-\alpha}a_t \tag{33}$$
$$\mu_t = -\sigma c_t - \frac{1}{1-\tau}\tau_t - \upsilon s_t + \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)a_t - \frac{\varphi+\alpha}{1-\alpha}y_t \tag{34}$$
$$\mu_t = -\sigma\left(y_t - \upsilon s_t\left((2-\upsilon)\eta + \frac{1-\upsilon}{\sigma}\right) + \frac{\upsilon}{\sigma}z_t\right) - \frac{1}{1-\tau}\tau_t - \upsilon s_t + \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)a_t - \frac{\varphi+\alpha}{1-\alpha}y_t \tag{35}$$
$$\mu_t = -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)y_t + \sigma\upsilon s_t\left((2-\upsilon)\eta + \frac{1-\upsilon}{\sigma}\right) - \upsilon z_t - \frac{1}{1-\tau}\tau_t - \upsilon s_t + \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)a_t \tag{36}$$
$$\mu_t = -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)y_t + \upsilon s_t\left(\sigma\left((2-\upsilon)\eta + \frac{1-\upsilon}{\sigma}\right) - 1\right) - \frac{1}{1-\tau}\tau_t + \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)a_t - \upsilon z_t \tag{37}$$
$$\mu_t = -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)y_t + \upsilon s_t\left((2-\upsilon)\sigma\eta + 1 - \upsilon\right) - 1 - \frac{1}{1-\tau}\tau_t + \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)a_t - \upsilon z_t \tag{38}$$
$$\mu_t = -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)y_t + \upsilon s_t\left((2-\upsilon)\sigma\eta - \upsilon\right) - \frac{1}{1-\tau}\tau_t + \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)a_t - \upsilon z_t \tag{39}$$
$$\mu_t = -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)y_t + \upsilon s_t\left(2\sigma\eta - \upsilon\sigma\eta - \upsilon\right) - \frac{1}{1-\tau}\tau_t + \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)a_t - \upsilon z_t \tag{40}$$
$$\mu_t = -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)y_t + \upsilon(\varpi - 1)s_t - \frac{1}{1-\tau}\tau_t + \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)a_t - \upsilon z_t \tag{41}$$

$$\pi_{H,t} = \beta\,\mathbb{E}\{\pi_{H,t+1}\} + \lambda\hat{\mu}_t \implies \pi_{H,t} = \beta\,\mathbb{E}\{\pi_{H,t+1}\} - \lambda\mu_t \tag{42}$$
$$\pi_{H,t} = \beta\,\mathbb{E}\{\pi_{H,t+1}\} + \lambda\left(\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)y_t - \upsilon(\varpi-1)s_t + \frac{1}{1-\tau}\tau_t - \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)a_t + \upsilon z_t\right) \tag{43}$$
$$\pi_{H,t} = \beta\,\mathbb{E}\{\pi_{H,t+1}\} + \kappa y_t + \lambda\left(-\upsilon(\varpi-1)s_t + \frac{1}{1-\tau}\tau_t - \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)a_t + \upsilon z_t\right) \tag{44}$$
$$\pi_{H,t} = \beta\,\mathbb{E}\{\pi_{H,t+1}\} + \kappa y_t - \lambda\upsilon(\varpi-1)s_t + \lambda\frac{1}{1-\tau}\tau_t - \lambda\left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)a_t + \lambda\upsilon z_t \tag{45}$$
$$\tag{46}$$

$$
\begin{aligned}
\mu_t &= -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n + \upsilon(\varpi - 1)s_t^n - \frac{1}{1-\tau}\tau_t + \left(1 + \frac{\varphi+\alpha}{1-\alpha}\right) a_t - \upsilon z_t \\
\mu_t - \left(\upsilon(\varpi-1)s_t^n - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right)a_t - \upsilon z_t\right) &= -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n \\
\upsilon(\varpi-1)s_t^n - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right)a_t - \upsilon z_t - \mu_t &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n \\
\upsilon(\varpi-1)s_t^n - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right)a_t - \upsilon z_t - \mu_t &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n \\
\upsilon(\varpi-1)(\sigma_\upsilon(y_t^n - y_t^*) - (1-\upsilon)\Phi z_t - G_Y\sigma_\upsilon g_t) - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right)a_t - \upsilon z_t - \mu_t &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n \\
(-1+1+\upsilon(\varpi-1))(\sigma_\upsilon(y_t^n - y_t^*) - (1-\upsilon)\Phi z_t - G_Y\sigma_\upsilon g_t) - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right)a_t - \upsilon z_t - \mu_t &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n \\
(-1+\Phi^{-1})(\sigma_\upsilon(y_t^n - y_t^*) - (1-\upsilon)\Phi z_t - G_Y\sigma_\upsilon g_t) - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right)a_t - \upsilon z_t - \mu_t &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n \\
(-1+\Phi^{-1})(\sigma_\upsilon y_t^n - \sigma_\upsilon y_t^* - (1-\upsilon)\Phi z_t - G_Y\sigma_\upsilon g_t) - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right)a_t - \upsilon z_t - \mu_t &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n \\
(-1+\Phi^{-1})\sigma_\upsilon y_t^n + (1-\Phi^{-1})\sigma_\upsilon y_t^* + (1-\Phi^{-1})(1-\upsilon)\Phi z_t + (1-\Phi^{-1})G_Y\sigma_\upsilon g_t - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right)a_t - \upsilon z_t - \mu_t &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n \\
(-1+\Phi^{-1})\sigma\Phi y_t^n + (1-\Phi^{-1})\sigma_\upsilon y_t^* + (1-\Phi^{-1})(1-\upsilon)\Phi z_t + (1-\Phi^{-1})G_Y\sigma_\upsilon g_t - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right)a_t - \upsilon z_t - \mu_t &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n \\
-\sigma\Phi y_t^n + \sigma y_t^n + (1-\Phi^{-1})\sigma_\upsilon y_t^* + (1-\Phi^{-1})(1-\upsilon)\Phi z_t + (1-\Phi^{-1})G_Y\sigma_\upsilon g_t - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right)a_t - \upsilon z_t - \mu_t &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n \\
(1-\Phi)\sigma y_t^n + (1-\Phi^{-1})\sigma_\upsilon y_t^* + (1-\Phi^{-1})(1-\upsilon)\Phi z_t + (1-\Phi^{-1})G_Y\sigma_\upsilon g_t - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right)a_t - \upsilon z_t - \mu_t &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n \\
(1-\Phi^{-1})\sigma_\upsilon y_t^* + (1-\Phi^{-1})(1-\upsilon)\Phi z_t + (1-\Phi^{-1})G_Y\sigma_\upsilon g_t - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right)a_t - \upsilon z_t - \mu_t &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n - (1-\Phi)\sigma y_t^n \\
(1-\Phi^{-1})\sigma_\upsilon y_t^* + (1-\Phi^{-1})(1-\upsilon)\Phi z_t + (1-\Phi^{-1})G_Y\sigma_\upsilon g_t - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right)a_t - \upsilon z_t - \mu_t &= \left(\frac{\sigma(1-\alpha) + \varphi + \alpha - (1-\Phi)\sigma(1-\alpha)}{1-\alpha}\right) y_t^n \\
(1-\Phi^{-1})\sigma_\upsilon y_t^* + (1-\Phi^{-1})(1-\upsilon)\Phi z_t + (1-\Phi^{-1})G_Y\sigma_\upsilon g_t - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right)a_t - \upsilon z_t - \mu_t &= \left(\frac{\sigma(1-\alpha) + \varphi + \alpha - (\sigma(1-\alpha) - \sigma\Phi(1-\alpha))}{1-\alpha}\right) y_t^n \\
(1-\Phi^{-1})\sigma_\upsilon y_t^* + (1-\Phi^{-1})(1-\upsilon)\Phi z_t + (1-\Phi^{-1})G_Y\sigma_\upsilon g_t - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right)a_t - \upsilon z_t - \mu_t &= \left(\frac{-\sigma_\upsilon(1-\alpha) + \varphi + \alpha}{1-\alpha}\right) y_t^n \\
\left(\frac{-\sigma_\upsilon(1-\alpha)+\varphi+\alpha}{1-\alpha}\right)^{-1}\left((1-\Phi^{-1})\sigma_\upsilon y_t^* + (1-\Phi^{-1})(1-\upsilon)\Phi z_t - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right)a_t - \upsilon z_t - \mu_t\right) &= y_t^n \\
\Gamma_* y_t^* + \left(\frac{-\sigma_\upsilon(1-\alpha)+\varphi+\alpha}{1-\alpha}\right)^{-1}\left((1-\Phi^{-1})(1-\upsilon)\Phi z_t + (1-\Phi^{-1})G_Y\sigma_\upsilon g_t - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right)a_t - \upsilon z_t - \mu_t\right) &= y_t^n \\
\Gamma_* y_t^* + \left(\frac{-\sigma_\upsilon(1-\alpha)+\varphi+\alpha}{1-\alpha}\right)^{-1}\left(\upsilon\varpi\Phi z_t + (1-\Phi^{-1})G_Y\sigma_\upsilon g_t - \frac{1}{1-\tau}\tau_t + \left(1+\frac{\varphi+\alpha}{1-\alpha}\right)a_t - \mu_t\right) &= y_t^n \\
\Gamma_* y_t^* + \Gamma_z z_t + \Gamma_a a_t + \left(\frac{-\sigma_\upsilon(1-\alpha)+\varphi+\alpha}{1-\alpha}\right)^{-1}\left((1-\Phi^{-1})G_Y\sigma_\upsilon g_t - \frac{1}{1-\tau}\tau_t - \mu_t\right) &= y_t^n \\
\Gamma_* y_t^* + \Gamma_z z_t + \Gamma_a a_t + \Gamma_g g_t + \left(\frac{-\sigma_\upsilon(1-\alpha)+\varphi+\alpha}{1-\alpha}\right)^{-1}\left(-\frac{1}{1-\tau}\tau_t\right) &= y_t^n \\
\Gamma_* y_t^* + \Gamma_z z_t + \Gamma_a a_t + \Gamma_g g_t + \Gamma_\tau \tau_t &= y_t^n
\end{aligned}
$$

$$y_t^n = \Gamma_* y_t^* + \Gamma_z z_t + \Gamma_a a_t + \Gamma_\tau \tau_t \tag{47}$$

$$\Gamma_* = -\frac{\upsilon(\varpi-1)\sigma_\upsilon(1-\alpha)}{\sigma_\upsilon(1-\alpha)+\varphi+\alpha} \tag{48}$$

$$\Gamma_z = -\frac{\upsilon\varpi\Phi(1-\alpha)}{\sigma_\upsilon(1-\alpha)+\varphi+\alpha} \tag{49}$$

$$\Gamma_a = \frac{1+\varpi}{\sigma_\upsilon(1-\alpha)+\varphi+\alpha} \tag{50}$$

$$\Gamma_g = -\frac{\upsilon(\varpi-1)\sigma_\upsilon(1-\alpha)}{\sigma_\upsilon(1-\alpha)+\varphi+\alpha}\times \tag{51}$$

$$\Gamma_\tau = \frac{(1-\tau)(1-\alpha)}{\sigma_\upsilon(1-\alpha)+\varphi+\alpha} \tag{52}$$

$$c_t = \mathbb{E}\{c_{t+1}\} - \frac{1}{\sigma}(i_t - \mathbb{E}\{\pi_{t+1}\} - \rho) + \frac{1}{\sigma}(1-\rho_z)z_t \tag{53}$$

$$\pi_t = \pi_{H,t} + \upsilon\Delta s_t \tag{54}$$

$$\implies c_t = \mathbb{E}\{c_{t+1}\} - \frac{1}{\sigma}(i_t - \mathbb{E}\{\pi_{H,t+1}\} - \rho) + \frac{\upsilon}{\sigma}\mathbb{E}\{\Delta s_{t+1}\} + \frac{1}{\sigma}(1-\rho_z)z_t \tag{55}$$

$$c_t = y_t^* + \frac{1}{\sigma}z_t + \left(\frac{1-\upsilon}{\sigma}\right)s_t \tag{56}$$

$$Y_t(i) = C(i)_t + X(i)_t + G(i)_t \tag{57}$$

$$= \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon}\left[(1-\upsilon)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t + \upsilon\mathcal{S}_t^\eta Y_t^* + G_t\right] \tag{58}$$

Given that

$$Y_t = \left(\int_0^1 (Y_t(i))^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{59}$$

$$Y_t = (1-\upsilon)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t + \upsilon\mathcal{S}_t^\eta Y_t^* + G_t \tag{60}$$

$$Y\mathbf{e}^{y_t} = (1-\upsilon)\left(\frac{P}{P_H}\right)^{\eta}C\mathbf{e}^{-\eta p_{H,t}+\eta p_t+c_t} + \upsilon S^\eta Y^*\mathbf{e}^{\eta s_t + y_t^*} + G\mathbf{e}^{g_t} \tag{61}$$

$$Y(1+y_t) = (1-\upsilon)\left(\frac{P}{P_H}\right)^{\eta}C(1-\eta p_{H,t}+\eta p_t + c_t) + \upsilon S^\eta Y^*(1+\eta s_t + y_t^*) + G(1+g_t) \tag{62}$$

$$Y y_t = (1-\upsilon)\left(\frac{P}{P_H}\right)^{\eta}C(-\eta p_{H,t}+\eta p_t + c_t) + \upsilon S^\eta Y^*(\eta s_t + y_t^*) + G g_t \tag{63}$$

$$\text{Using } P/P_H = \upsilon\mathcal{S},\ \mathcal{S}=1, \text{ and } C = Y^*: \tag{64}$$

$$y_t = (1-\upsilon)(-\eta p_{H,t} + \eta p_t + c_t) + \upsilon(\eta s_t + y_t^*) + G_Y g_t \tag{65}$$

$$\text{Using } p_t - p_{H,t} = \upsilon s_t: \tag{66}$$

$$y_t = (1-\upsilon)(\eta\upsilon s_t + c_t) + \upsilon(\eta s_t + y_t^*) + G_Y g_t \tag{67}$$

$$y_t = (1-\upsilon)c_t + (1-\upsilon)\eta\upsilon s_t + \upsilon\eta s_t + \upsilon y_t^* + G_Y g_t \tag{68}$$

$$y_t = (1-\upsilon)c_t + \upsilon(2-\upsilon)\eta s_t + \upsilon y_t^* + G_Y g_t \tag{69}$$

(70)

$$c_t = y_t^* + \frac{1}{\sigma} z_t + \frac{1-\upsilon}{\sigma} s_t \tag{71}$$

$$y_t = (1-\upsilon)c_t + \upsilon(2-\upsilon)\eta s_t + \upsilon y_t^* + G_Y g_t \tag{72}$$

$$\implies y_t = (1-\upsilon)\left(y_t^* + \frac{1}{\sigma} z_t + \frac{1-\upsilon}{\sigma} s_t\right) + \upsilon(2-\upsilon)\eta s_t + \upsilon y_t^* + G_Y g_t \tag{73}$$

$$y_t = (1-\upsilon)y_t^* + \frac{1-\upsilon}{\sigma} z_t + (1-\upsilon)\frac{1-\upsilon}{\sigma} s_t + \upsilon(2-\upsilon)\eta s_t + \upsilon y_t^* + G_Y g_t \tag{74}$$

$$y_t = y_t^* + \frac{1-\upsilon}{\sigma} z_t + (1-\upsilon)\frac{1-\upsilon}{\sigma} s_t + \upsilon(2-\upsilon)\eta s_t + G_Y g_t \tag{75}$$

$$y_t = y_t^* + \frac{1-\upsilon}{\sigma} z_t + \left((1-\upsilon)\frac{1-\upsilon}{\sigma} + \upsilon(2-\upsilon)\eta\right) s_t + G_Y g_t \tag{76}$$

$$\left((1-\upsilon)\frac{1-\upsilon}{\sigma} + \upsilon(2-\upsilon)\eta\right) s_t = y_t - y_t^* - \frac{1-\upsilon}{\sigma} z_t - G_Y g_t \tag{77}$$

$$\left(\frac{(1-\upsilon)(1-\upsilon) + \sigma\upsilon(2-\upsilon)\eta}{\sigma}\right) s_t = y_t - y_t^* - \frac{1-\upsilon}{\sigma} z_t - G_Y g_t \tag{78}$$

$$\left(\frac{1 - 2\upsilon + \upsilon^2 + \sigma\upsilon(2-\upsilon)\eta}{\sigma}\right) s_t = y_t - y_t^* - \frac{1-\upsilon}{\sigma} z_t - G_Y g_t \tag{79}$$

$$\left(\frac{1 - \upsilon(2-\upsilon) + \sigma\upsilon(2-\upsilon)\eta}{\sigma}\right) s_t = y_t - y_t^* - \frac{1-\upsilon}{\sigma} z_t - G_Y g_t \tag{80}$$

$$\left(\frac{1 - \upsilon(2-\upsilon)(1-\sigma\eta)}{\sigma}\right) s_t = y_t - y_t^* - \frac{1-\upsilon}{\sigma} z_t - G_Y g_t \tag{81}$$

$$\left(\frac{1 + \upsilon(2-\upsilon)(\sigma\eta - 1)}{\sigma}\right) s_t = y_t - y_t^* - \frac{1-\upsilon}{\sigma} z_t - G_Y g_t \tag{82}$$

$$\left(\frac{1 + \upsilon(2\sigma\eta - \upsilon\sigma\eta - 2 + \upsilon)}{\sigma}\right) s_t = y_t - y_t^* - \frac{1-\upsilon}{\sigma} z_t - G_Y g_t \tag{83}$$

$$\left(\frac{1 + \upsilon(\varpi - 1)}{\sigma}\right) s_t = y_t - y_t^* - \frac{1-\upsilon}{\sigma} z_t - G_Y g_t \tag{84}$$

$$\sigma^{-1}\Phi^{-1} s_t = y_t - y_t^* - \frac{1-\upsilon}{\sigma} z_t - G_Y g_t \tag{85}$$

$$s_t = \sigma\Phi(y_t - y_t^*) - (1-\upsilon)\Phi z_t - G_Y \sigma\Phi g_t \tag{86}$$

$$s_t = \sigma_\upsilon(y_t - y_t^*) - (1-\upsilon)\Phi z_t - G_Y \sigma_\upsilon g_t \tag{87}$$

$$\tag{88}$$

$$\Phi = \frac{1}{1 + \upsilon(\varpi - 1)} \tag{89}$$

$$\varpi = \sigma\eta + (1-\upsilon)(\sigma\eta - 1) = 2\sigma\eta - \upsilon\sigma\eta - 1 + \upsilon \tag{90}$$

$$\sigma_\upsilon = \sigma\Phi \tag{91}$$

$$
\begin{aligned}
& y_t = (1-\upsilon)c_t + \upsilon(2-\upsilon)\eta s_t + \upsilon y_t^* + G_Y g_t && (92)\\
\implies \; & c_t = (1-\upsilon)^{-1}(y_t - \upsilon(2-\upsilon)\eta s_t - \upsilon y_t^* - G_Y g_t) && (93)\\
& c_t = \mathbb{E}\{c_{t+1}\} - \frac{1}{\sigma}(i_t - \mathbb{E}\{\pi_{t+1}\} - \rho) + \frac{\upsilon}{\sigma}\mathbb{E}\{\Delta s_{t+1}\} + \frac{1}{\sigma}(1-\rho_z)z_t && (94)\\
& s_t = \sigma_\upsilon(y_t - y_t^*) - (1-\upsilon)\Phi z_t - G_Y \sigma_\upsilon g_t && (95)\\
& && (96)
\end{aligned}
$$

$$
\begin{aligned}
0 &= \mathbb{E}\{\Delta c_{t+1}\} - \frac{1}{\sigma}(i_t - \mathbb{E}\{\pi_{H,+1}\} - \rho) + \frac{\upsilon}{\sigma}\mathbb{E}\{\Delta s_{t+1}\} + \frac{1}{\sigma}(1-\rho_z)z_t\\
0 &= \mathbb{E}\{\Delta y_{t+1}\} - \upsilon(2-\upsilon)\eta\,\mathbb{E}\{\Delta s_{t+1}\} - \upsilon\,\mathbb{E}\{\Delta y_{t+1}^*\} - G_Y \Delta g_{t+1} - \frac{1-\upsilon}{\sigma}(i_t - \mathbb{E}\{\pi_{H,+1}\} - \rho) + \frac{(1-\upsilon)\upsilon}{\sigma}\mathbb{E}\{\Delta s_{t+1}\} + \frac{1-\upsilon}{\sigma}(1-\rho_z)z_t\\
0 &= \mathbb{E}\{\Delta y_{t+1}\} + \left(\frac{(1-\upsilon)\upsilon}{\sigma} - \upsilon(2-\upsilon)\eta\right)\mathbb{E}\{\Delta s_{t+1}\} - \upsilon\,\mathbb{E}\{\Delta y_{t+1}^*\} - G_Y \Delta g_{t+1} - \frac{1-\upsilon}{\sigma}(i_t - \mathbb{E}\{\pi_{H,+1}\} - \rho) + \frac{1-\upsilon}{\sigma}(1-\rho_z)z_t\\
0 &= \mathbb{E}\{\Delta y_{t+1}\} - \left(\frac{\upsilon\varpi}{\sigma}\right)\mathbb{E}\{\Delta s_{t+1}\} - \upsilon\,\mathbb{E}\{\Delta y_{t+1}^*\} - G_Y \Delta g_{t+1} - \frac{1-\upsilon}{\sigma}(i_t - \mathbb{E}\{\pi_{H,+1}\} - \rho) + \frac{1-\upsilon}{\sigma}(1-\rho_z)z_t\\
0 &= \mathbb{E}\{\Delta y_{t+1}\} - \left(\frac{\upsilon\varpi}{\sigma}\right)\mathbb{E}\left(\sigma_\upsilon(\Delta y_{t+1} - \Delta y_{t+1}^*) - (1-\upsilon)\Phi(\Delta z_{t+1}) - G_Y\sigma_\upsilon \Delta g_{t+1}\right) - \upsilon\,\mathbb{E}\{\Delta y_{t+1}^*\} - \frac{1-\upsilon}{\sigma}(i_t - \mathbb{E}\{\pi_{H,+1}\} - \rho) + \frac{1-\upsilon}{\sigma}(1-\rho_z)z_t\\
0 &= \mathbb{E}\{\Delta y_{t+1}\} - \upsilon\varpi\Phi\Delta y_{t+1} + \upsilon\varpi\Phi\Delta y_{t+1}^* - \upsilon\,\mathbb{E}\{\Delta y_{t+1}^*\} + \frac{1-\upsilon}{\sigma}(1-\rho_z)z_t + \upsilon\varpi\Phi\frac{1-\upsilon}{\sigma}(\rho_z - 1)z_t + \upsilon\varpi\Phi G_Y \Delta g_{t+1} - G_Y \Delta g_{t+1} - \frac{1-\upsilon}{\sigma}(i_t - \mathbb{E}\{\pi_{H,+1}\\
0 &= \Phi^{-1}\mathbb{E}\{\Delta y_{t+1}\} - \upsilon\varpi\Delta y_{t+1} + \upsilon\varpi\Delta y_{t+1}^* - \upsilon\Phi^{-1}\mathbb{E}\{\Delta y_{t+1}^*\}\\
&\quad + \Phi^{-1}\frac{1-\upsilon}{\sigma}(1-\rho_z)z_t + \upsilon\varpi\frac{1-\upsilon}{\sigma}(\rho_z - 1)z_t + \upsilon\varpi G_Y \Delta g_{t+1} - G_Y \Delta g_{t+1}\\
&\quad - \frac{1-\upsilon}{\sigma_\upsilon}(i_t - \mathbb{E}\{\pi_{H,+1}\} - \rho)\\
0 &= (\Phi^{-1} - \upsilon\varpi)\Delta y_{t+1} + \upsilon(\varpi - \Phi^{-1})\mathbb{E}\{\Delta y_{t+1}^*\}\\
&\quad - (\Phi^{-1} - \upsilon\varpi)\frac{1-\upsilon}{\sigma}(\rho_z - 1)z_t + (\upsilon\varpi - 1)G_Y \Delta g_{t+1}\\
&\quad - \frac{1-\upsilon}{\sigma_\upsilon}(i_t - \mathbb{E}\{\pi_{H,+1}\} - \rho)\\
0 &= \Delta y_{t+1} + \frac{\upsilon(\varpi - \Phi^{-1})}{\Phi^{-1} - \upsilon\varpi}\mathbb{E}\{\Delta y_{t+1}^*\}\\
&\quad - \frac{1-\upsilon}{\sigma}(\rho_z - 1)z_t + \frac{(\upsilon\varpi - 1)}{\Phi^{-1} - \upsilon\varpi}G_Y \Delta g_{t+1}\\
&\quad - \frac{1-\upsilon}{\Phi^{-1} - \upsilon\varpi}\frac{1}{\sigma_\upsilon}(i_t - \mathbb{E}\{\pi_{H,+1}\} - \rho)\\
y_t &= y_{t+1} - \frac{1-\upsilon}{\sigma}(\rho_z - 1)z_t\\
&\quad + \frac{(\upsilon\varpi - 1)}{\Phi^{-1} - \upsilon\varpi}G_Y \Delta g_{t+1} + \frac{\upsilon(\varpi - \Phi^{-1})}{\Phi^{-1} - \upsilon\varpi}\mathbb{E}\{\Delta y_{t+1}^*\} - \frac{1-\upsilon}{\Phi^{-1} - \upsilon\varpi}\frac{1}{\sigma_\upsilon}(i_t - \mathbb{E}\{\pi_{H,+1}\} - \rho)\\
y_t &= y_{t+1} - \frac{1-\upsilon}{\sigma}(\rho_z - 1)z_t - \frac{1-\upsilon}{1-\upsilon}\frac{1}{\sigma_\upsilon}(i_t - \mathbb{E}\{\pi_{H,+1}\} - \rho) + \frac{(1-\upsilon)(\upsilon\varpi - \upsilon)}{1-\upsilon}\mathbb{E}\{\Delta y_{t+1}^*\} + \frac{\upsilon\varpi - 1}{1-\upsilon}G_Y \Delta g_{t+1}\\
y_t &= y_{t+1} - \frac{1}{\sigma_\upsilon}(i_t - \mathbb{E}\{\pi_{H,+1}\} - \rho) + \upsilon(\varpi - 1)\mathbb{E}\{\Delta y_{t+1}^*\} + \frac{1-\upsilon}{\sigma}(1-\rho_z)z_t + \frac{\upsilon\varpi - 1}{1-\upsilon}G_Y \Delta g_{t+1}
\end{aligned}
$$

$$
\begin{aligned}
y_t^n &= y_{t+1}^n - \frac{1}{\sigma_\upsilon}(r_t^n - \rho) + \upsilon(\varpi - 1)\mathbb{E}\{\Delta y_{t+1}^*\} + \frac{1-\upsilon}{\sigma}(1-\rho_z)z_t + \frac{\upsilon\varpi - 1}{1-\upsilon}G_Y \Delta g_{t+1} && (97)\\
0 &= \Delta y_{t+1}^n - \frac{1}{\sigma_\upsilon}(r_t^n - \rho) + \upsilon(\varpi - 1)\mathbb{E}\{\Delta y_{t+1}^*\} + \frac{1-\upsilon}{\sigma}(1-\rho_z)z_t + \frac{\upsilon\varpi - 1}{1-\upsilon}G_Y \Delta g_{t+1} && (98)\\
0 &= \sigma_\upsilon\Delta y_{t+1}^n - (r_t^n - \rho) + \sigma_\upsilon\upsilon(\varpi - 1)\mathbb{E}\{\Delta y_{t+1}^*\} + \Phi(1-\upsilon)(1-\rho_z)z_t + \sigma_\upsilon\frac{\upsilon\varpi - 1}{1-\upsilon}G_Y \Delta g_{t+1} && (99)\\
r_t^n &= \sigma_\upsilon\Delta y_{t+1}^n + \rho + \sigma_\upsilon\upsilon(\varpi - 1)\mathbb{E}\{\Delta y_{t+1}^*\} + \Phi(1-\upsilon)(1-\rho_z)z_t + \sigma_\upsilon\frac{\upsilon\varpi - 1}{1-\upsilon}G_Y \Delta g_{t+1} && (100)\\
& && (101)
\end{aligned}
$$

$$
\begin{aligned}
0 &= \sigma_\upsilon\Delta y_{t+1} - (i_t - \mathbb{E}\{\pi_{H,+1}\}) + \rho + \sigma_\upsilon\upsilon(\varpi - 1)\mathbb{E}\{\Delta y_{t+1}^*\} + \Phi(1-\upsilon)(1-\rho_z)z_t + \sigma_\upsilon\frac{\upsilon\varpi - 1}{1-\upsilon}G_Y \Delta g_{t+1} && (102)\\
&\quad - (\sigma_\upsilon\Delta y_{t+1}^n - (r_t^n - \rho) + \sigma_\upsilon\upsilon(\varpi - 1)\mathbb{E}\{\Delta y_{t+1}^*\} + \Phi(1-\upsilon)(1-\rho_z)z_t + \sigma_\upsilon\frac{\upsilon\varpi - 1}{1-\upsilon}G_Y \Delta g_{t+1}) && (103)\\
0 &= \sigma_\upsilon\Delta\tilde{y}_{t+1} - (i_t - \mathbb{E}\{\pi_{H,+1}\} - r_t^n) && (104)\\
\tilde{y}_t &= \tilde{y}_{t+1} - \frac{1}{\sigma_\upsilon}(i_t - \mathbb{E}\{\pi_{H,+1}\} - r_t^n) \quad \text{Dynamic IS Curve} && (105)\\
& && (106)
\end{aligned}
$$

$$r_t^n = \sigma_\upsilon \Delta y_{t+1}^n + \rho + \sigma_\upsilon \upsilon(\varpi - 1)\,\mathbb{E}\{\Delta y_{t+1}^*\} + \Phi(1-\upsilon)(1-\rho_z)z_t + \sigma_\upsilon \frac{\upsilon\varpi - 1}{1-\upsilon} G_Y \Delta g_{t+1} \tag{107}$$

$$\sigma_\upsilon \Delta y_{t+1}^n = \sigma_\upsilon(\Gamma_* \Delta y_{t+1}^* + \Gamma_z \Delta z_{t+1} + \Gamma_a \Delta a_{t+1} + \Gamma_g \Delta g_{t+1} + \Gamma_\tau \Delta \tau_{t+1}) \tag{108}$$

$$y_t^* : \sigma_\upsilon \Gamma_* \Delta y_{t+1}^* + \sigma_\upsilon \upsilon(\varpi - 1)\Delta y_{t+1}^* = \sigma_\upsilon(\Gamma_* + \upsilon(\varpi - 1))\Delta y_{t+1}^* = \Psi_* y_{t+1}^* \tag{109}$$

$$z_t : \sigma_\upsilon \Gamma_z \Delta z_{t+1} + \Phi(1-\rho_z)z_t = \sigma_\upsilon \Gamma_z(\rho_z - 1)z_t + \Phi(1-\upsilon)(1-\rho_z)z_t \tag{110}$$

$$= \sigma_\upsilon \Gamma_z(\rho_z - 1)z_t + \Phi(1-\upsilon)(1-\rho_z)z_t = (\Phi(1-\upsilon) - \sigma_\upsilon\Gamma_z)(1-\rho_z)z_t = \Psi_z(1-\rho_z)z_t \tag{111}$$

$$a_t : \sigma_\upsilon \Gamma_a \Delta a_{t+1} = \sigma_\upsilon \Gamma_a(\rho_a - 1)a_t = -\sigma_\upsilon \Gamma_a(1-\rho_a)a_t \tag{112}$$

$$g_t : \sigma_\upsilon \Gamma_g G_Y \Delta g_{t+1} + \sigma_\upsilon \frac{\upsilon\varpi - 1}{1-\upsilon} G_Y \Delta g_{t+1} = \left(\sigma_\upsilon \left(\Gamma_g + \frac{\upsilon\varpi - 1}{1-\upsilon}\right)\right) G_Y \Delta g_{t+1} = -\Psi_g G_Y(1-\rho_g)g_t \tag{113}$$

$$\tau_t : \sigma_\upsilon \Gamma_\tau \Delta \tau_{t+1} \tag{114}$$

$$r_t^n = \rho + \Psi_* y_{t+1}^* + \Psi_z(1-\rho_z)z_t - \sigma_\upsilon \Gamma_a(1-\rho_a)a_t - \Psi_g G_Y(1-\rho_g)g_t + \sigma_\upsilon \Gamma_\tau \Delta \tau_{t+1} \tag{115}$$

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Structure. Max words: 10 000. Currently: 1691

1. Introduction: **<u>1000 words</u>**
2. Literature Review **<u>2000 words</u>**
   (a) From RBC to NK DSGE: **1200 words**
   (b) Why Scotland and the rest of the UK? (NIESR policy-related question): **800 words**
3. Theoretical DSGE model: **<u>2000 words</u>**
   (a) Households
   (b) Firms
   (c) Equilibrium
4. Application: **<u>1500 words</u>**
   (a) Data: **500 words**
   (b) Estimation (MCMC): **1000 words**
5. Results: **<u>2500 words</u>**
   (a) Analysis of IRFs: **1000 words**
   (b) Other insights (NIESR policy-related question): **1500 words**
6. Conclusion: **<u>1000 words</u>**

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Sum count: 1691
Words in text: 1518
Words in headers: 31
Words outside text (captions, etc.): 44
Number of headers: 13
Number of floats/tables/figures: 2
Number of math inlines: 42
Number of math displayed: 56
Files: 13
Subcounts:
  text+headers+captions (#headers/#floats/#inlines/#displayed)
  4+9+0 (7/0/0/0) File: main.tex
  99+11+0 (1/0/0/0) Included file: ././model.tex
  65+6+0 (1/0/0/0) Included file: ././structure.tex
  591+2+0 (1/0/0/0) Included file: ././Introduction/literature_review.tex
  155+0+0 (0/0/3/0) Included file: ././Theoretical DSGE model/model.tex
  557+1+33 (1/0/36/41) Included file: ././Theoretical DSGE model/households.tex
  26+1+0 (1/0/3/5) Included file: ././Theoretical DSGE model/firms.tex
  20+1+0 (1/0/0/10) Included file: ././Theoretical DSGE model/equilibrium.tex
  1+0+0 (0/0/0/0) Included file: ././final_wordcount.txt
  0+0+11 (0/1/0/0) Included file: ././Graphs/timeline.tex
  0+0+0 (0/1/0/0) Included file: ././Graphs/sigma_varphi.tex
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