Task 2

Analysis of piloted airplaine stability.

Contents

- Transfer function
- Step response and stability
- Propotional control transfer function
- P-control stability
- Kp-stable values
- Control system step response

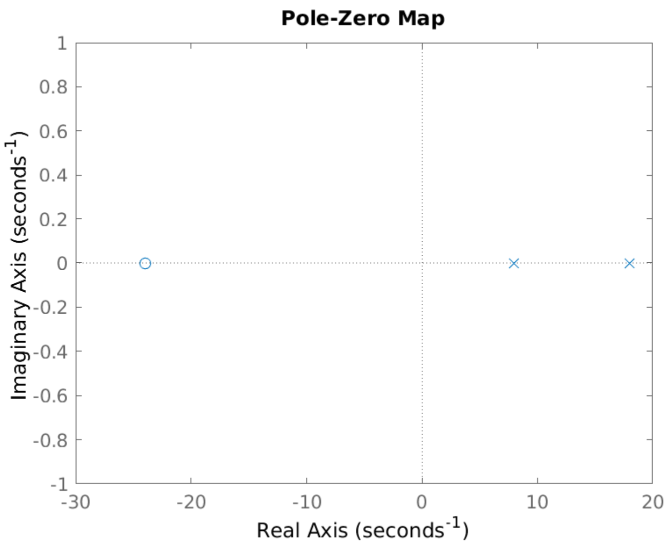
Transfer function

the transfer function, with canard deflection as input and pitch altitude as output, is given as:

$$\frac{\theta}{\delta_c} = \frac{s+24}{(s-8)(s-18)}$$

This system is clearly unstable, as both poles are positive real numbers (8 and 18) \\ We can verify this by using pzplot and looking at the poles:

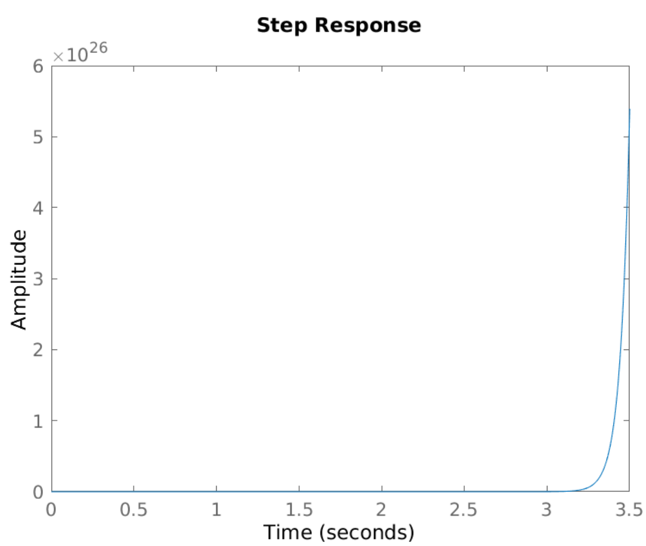
```
s = tf('s');
sys = (s+24)/((s-8)*(s-18));
pzplot(sys);
```



As we can see, both poles have positive real part, so the system must be unstable

Step response and stability

```
close;
step(sys);
```



The output of our system blows up to infinity, so it is clearly unstable.

The open-loop system does not satisfy BIBO, and requires closed loop control to become stable.

Propotional control transfer function

We add a proportional control and find an equivalent transfer function for the whole system. I use the block diagram to help express the new system.

```
close;

s = tf('s');
H = (s + 24)/((s-8)*(s-18));  % The plant
Kp = -5;  % The proportional gain

closed_loop = (Kp*s+Kp*24)/(s^2+(Kp-26)*s+(144+24*Kp));
pole(closed_loop)
```

```
ans =

   30.2054
    0.7946
```

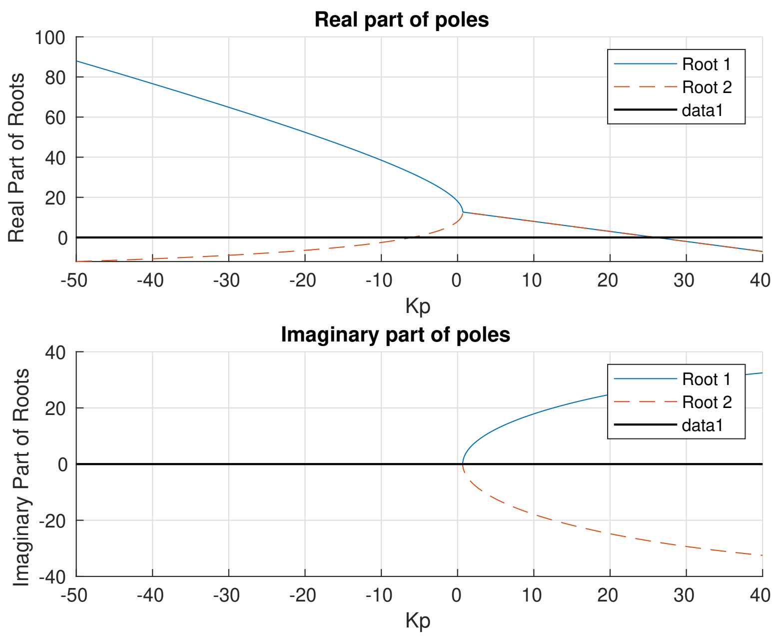
P-control stability

We can plot the poles of this system as a function of the proportional gain we need to plot the roots of the polynomial:

$$s^2 + (Kp - 26)s + (144 + 24)Kp$$

```
close;

Kp = -50:0.05:40;
s_1 = (-(Kp-26)+sqrt((Kp-26).^2-4*(144+24.*Kp)))/2;
s_2 = (-(Kp-26)-sqrt((Kp-26).^2-4*(144+24.*Kp)))/2;

figure;

subplot(2,1,1);
grid on;
hold on;
```

```
plot(Kp, real(s_1), 'DisplayName', 'Root 1', 'LineStyle', '-');
plot(Kp, real(s_2), 'DisplayName', 'Root 2', 'LineStyle', '--');
plot(Kp, zeros(size(Kp)), 'k', 'LineWidth', 1);  % x-axis
xlabel('Kp'); ylabel('Real Part of Roots');
title('Real part of poles');
legend;

subplot(2,1,2);
grid on;
hold on;
plot(Kp, imag(s_1), 'DisplayName', 'Root 1', 'LineStyle', '-');
plot(Kp, imag(s_2), 'DisplayName', 'Root 2', 'LineStyle', '--');
plot(Kp, zeros(size(Kp)), 'k', 'LineWidth', 1);  % x-axis
xlabel('Kp'); ylabel('Imaginary Part of Roots');
title('Imaginary part of poles');
legend;
```



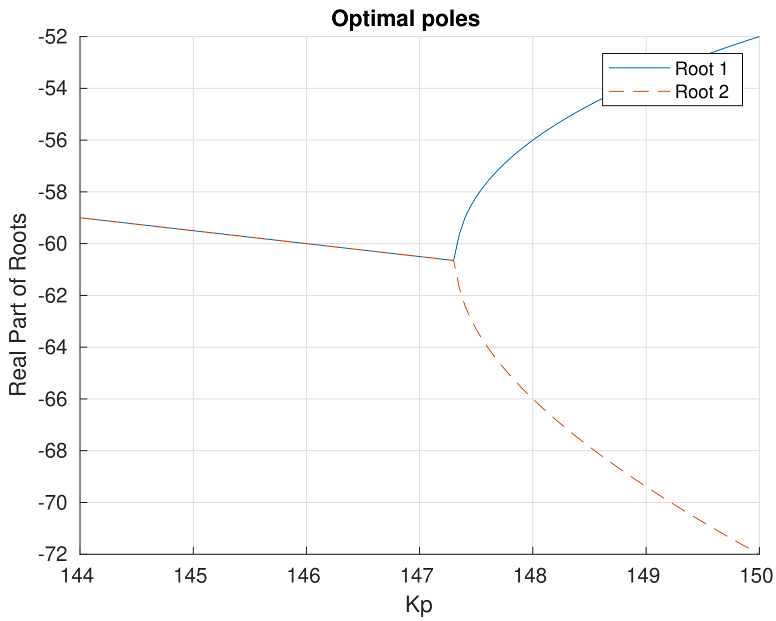
Kp-stable values

From inspecting the plot we see that both poles have negative real part when $K_p > 26$.

We can also see that $Kp \approx 147$ gives us optimal poles.

```
close;
```

```
figure;
grid on;
hold on;

Kp = 144:0.05:150;
s_1 = (-(Kp-26)+sqrt((Kp-26).^2-4*(144+24.*Kp)))/2;
s_2 = (-(Kp-26)-sqrt((Kp-26).^2-4*(144+24.*Kp)))/2;
plot(Kp, real(s_1), 'DisplayName', 'Root 1', 'LineStyle', '-');
plot(Kp, real(s_2), 'DisplayName', 'Root 2', 'LineStyle', '--');
xlabel('Kp'); ylabel('Real Part of Roots');
title('Optimal poles');
legend;
```



Verifying that Kp = 26 gives zero-poles

```
close;
Kp = 26;
closed_loop = (Kp*s+Kp*24)/(s^2+(Kp-26)*s+(144+24*Kp));
real(pole(closed_loop))
```

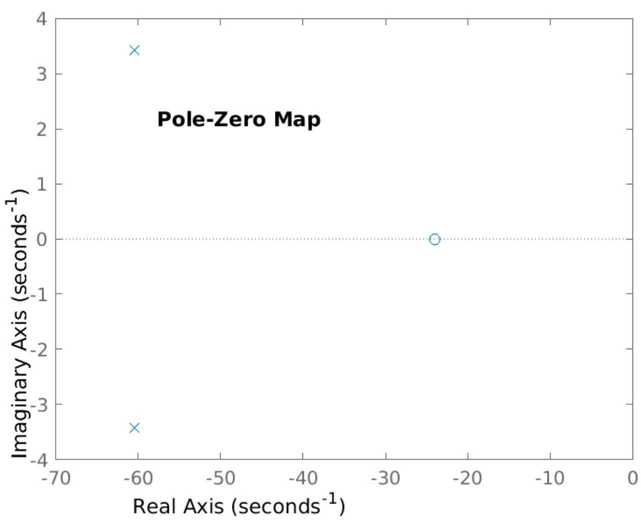
```
ans =

     0
     0
```

looking at the pole values for our new controller with $Kp = 147$

```
Kp = 147;
closed_loop = (Kp*s+Kp*24)/(s^2+(Kp-26)*s+(144+24*Kp));
real(pole(closed_loop))
```

```
ans =

  -60.5000
  -60.5000
```

```
close;
pzplot(closed_loop);
```



Control system step response

We have verified that the poles of our new system are all negative real part. This means the system stable, and we can verify this by viewing the step response of the closed loop system:

```
[y,t]=step(closed_loop); %save the output values to check steady state
SS_error = abs(1-y(end))
```

```
SS_error =

    0.0389
```

Thus we have verified that our control system is stable and reaches a steady state error of 3.89% We can also check that our simulink model agrees with our matlab simulations (see images).