University of South-Eastern Norway

EXAMINATION INFORMATION PAGE

Home exam

General information about the exam:

Subject code: TSE2190

Subject name: Reguleringsteknikk

Responsible course manager: Fabio Andrade

Campus: Vestfold

Faculty: TNM

Assignment given (date and time): 08.05.2023 12:00

Submission time (date and time): 22.05.2023 12:00

Number of assignments: 3

Number of attachments: 1

Number of pages (including front page and attachments): 7

Important information for all types of home exams:

A. Examination support material and cooperation:

Permitted examination support material: All examination support material permitted.

Examination form (tick off): Individual ☒ Group ☐

In an individual home exam, it is not allowed to collaborate with others on how to write the answer. The text, method for solving the task, academic reflections, etc. (your answer) that is submitted for the exam should be your own work (or the group's if it is a group exam). It is not allowed to have other persons or tools/functions (such as artificial intelligence) write the exam answer for you.

We would like to remind you that plagiarism control will be conducted on all answers, and unnatural similarities, illegal use of aids, etc. - or if you have not written the answer yourself - will result in USN creating a case for cheating. Cheating can result in the annulment of the answer, loss of the right to take exams at USN and other universities and colleges, or temporary suspension from USN. Description of individual exams and illegal collaboration can be found on the USN intranet.

Criteria for the answer paper:

Font: Times New Roman

Font size: 12

Line spacing: 1,5

Number of words (minimum/maximum): Not Applicable

Max number of pages (excluding first page and attachments,): 10

Reference style: Not Applicable

Exam assignments:

You have to deliver a written report which answers all items of the following 3 questions plus a 10 minutes presentation about your answers. The presentation will be followed by an oral examination.

Exam tips:

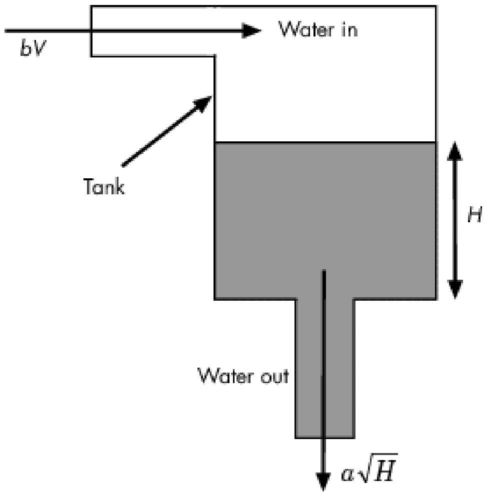
- Remember to always include the units in the graphs and answers.

- The graphs can be directly copied from MATLAB or Simulink if you wish.

- Remember to include all the steps that lead to the final answer.

Question 1

Consider a water tank like the figure below. Water enters the tank from the top at a rate proportional to the voltage, V, applied to the pump. The water leaves through an opening in the tank base at a rate that is proportional to the square root of the water height, H, in the tank. The presence of the square root in the water flow rate results in a nonlinear plant.

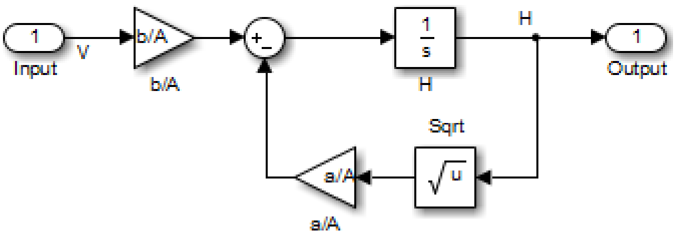


The differential equation of the system is: $\frac{d}{dt}Vol = A\frac{dH}{dt} = bV - a\sqrt{H}$

The following table describes the variables, parameters, states, inputs, and outputs of the Water-Tank System.

Variables	*H* is the height of water in the tank in [m]. *Vol* is the volume of water in the tank in [m^3] *V* is the voltage applied to the pump in [V].
Parameters	*A* is the cross-sectional area of the tank in [m^2]. *b* is a constant related to the flow rate into the tank in [m^3/(s.V)]. *a* is a constant related to the flow rate out of the tank in [m^(5/2)/s].
States	H
Inputs	V
Outputs	H

The Water-Tank System can be implemented in Simulink as the image below for simulation purposes.



Given that your USN email is XXYYZZ@usn.no, the parameters of the water tank will be:

A (cross-sectional area of the tank) = XX

b (constant related to the flow rate into the tank) = YY

a (constant related to the flow rate out of the tank) = ZZ

For example, for an email 627328@usn.no, A = 62 m^2, b = 73 m^3/(s.V) and a = 28 m^(5/2)/s.

The transfer function of the linearized system model around the operating point of 10 m of height is:

$$\frac{H(s)}{V(s)} = \frac{b/A}{s + 0.16a/A}$$

a) What is the state space representation of the linearized system (replacing A, b and a with the values regarding your USN email)?

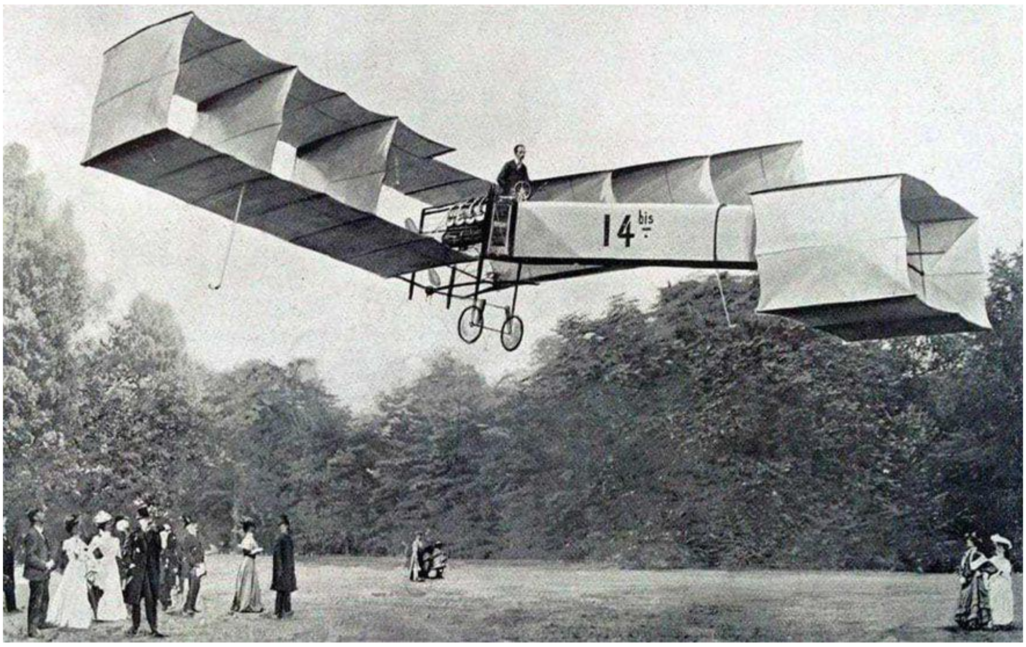
b) Implement both the nonlinear Water-Tank System (given in the previous page) and the linearized system (using the transfer function block with initial outputs) in Simulink. Show a screenshot of the Simulink block diagrams.

c) Design a PID controller for the linearized model to achieve less than 20% of overshoot, less than 5 second of rise time and less than 10 seconds of settling time. What are the values of Kp, Ki and Kd and the achieved rise time, settling time and overshoot?

d) Implement the PID controller in Simulink for both linearized and nonlinear models. Show a screenshot of the Simulink block diagram.

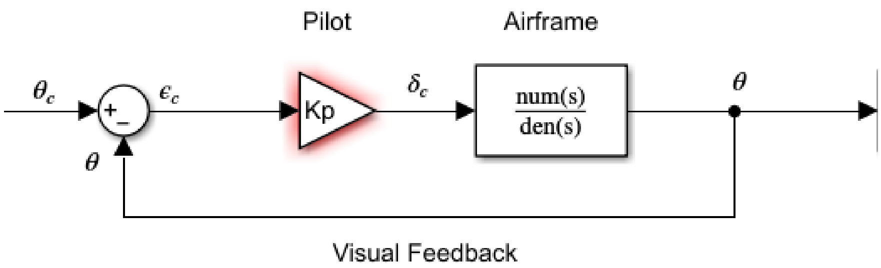
e) Plot the responses of the controlled linearized and nonlinear model simulated in Simulink for a setpoint of 10 meters and initial condition of 9 meters of height (use the “transfer function with initial outputs” block for the linearized model and the “initial output” in the integrator block of the nonlinear model).

Question 2



The 14-bis was a canard-style airplane designed, built and first flown by the Brazilian aviation pioneer Alberto Santos Drummond in 1906 as the first public powered flight in history. In 12 November 1906 he won the Archdeacon Award and the French Aeroclub Award by performing a 220-metre flight in Paris[1]. Although the airplane was difficult to fly, the combination of the pilot and the airplane could be made to be a stable system. In the closed loop block diagram, the pilot (controller) is represented as a pure gain, k_p. The transfer function from the input (canard deflection δ_c) and the output (pitch attitude θ) is given as follow. XX, YY and ZZ are gotten from your USN email XXYYZZ@usn.no.

$$\frac{\theta}{\delta_c} = \frac{s + XX}{(s - YY)(s - ZZ)}$$



[1] «https://www.smithsonianmag.com/air-space-magazine/10-milestone-flights-4056259/». Air and Space Magazine

a) Plot the poles and zeros of the plant (Airframe) in the Laplace plane (s-plane).

b) Plot the step response of the plant.

c) Is the system stable?

d) What is the equivalent transfer function of the control system (with the proportional controller)?

e) For what range of Kp is the system stable?

f) Find a Kp that makes the steady-state error of around 5%.

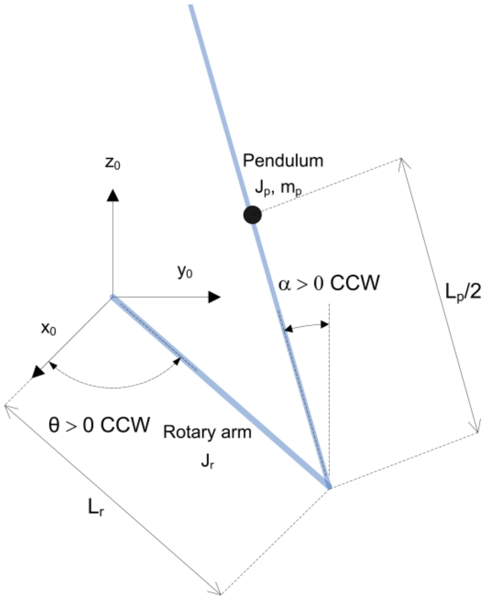
g) Plot the poles and zeros of the transfer function of the equivalent system with the found Kp in the Laplace plane (s-plane).

h) Plot the unit step response. What is the steady state value?

i) Is the new system stable?

Question 3

Given the state space representation of the rotary inverted pendulum:



$$\begin{bmatrix}\dot{\theta}\\ \dot{\alpha}\\ \ddot{\theta}\\ \ddot{\alpha}\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & \dfrac{M_p^2 L_p^2 L_a g}{4J_t} & 0 & 0\\ 0 & \dfrac{M_p L_p g(J_a + M_p L_a^2)}{2J_t} & 0 & 0\end{bmatrix}\begin{bmatrix}\theta\\ \alpha\\ \dot{\theta}\\ \dot{\alpha}\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \dfrac{J_p + \frac{1}{4}M_p L_p^2}{J_t}\\ \dfrac{M_p L_p L_a}{2J_t}\end{bmatrix} T$$

$$\vec{y} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}\theta\\ \alpha\\ \dot{\theta}\\ \dot{\alpha}\end{bmatrix}$$

Where:

g is the gravitational acceleration of 9.81 m/s^2

$J_t = J_a J_p + J_a M_p (\frac{L_p}{2})^2 + J_p M_p L_a^2$,

$J_a = \frac{M_a L_a^2}{12}$,

$J_p = \frac{M_p L_p^2}{12}$,

M_p is the mass of the pendulum in kilograms,

L_a is the length of the rotary arm in meters,

M_a is the mass of the rotary arm of $\frac{L_a}{10} + 0.04$, and

L_p is the length of the pendulum in meters.

If your USN email is XXYYZZ@usn.no, the pendulum length in centimeters is 10+XX*20/30, the rotary arm length in centimeters is 5+YY*25/30, and the weight of the pendulum in centigrams is ZZ.

Example of MATLAB code for XXYYZZ = 627328

```
XX = 62
YY = 73
ZZ = 28
g = 9.81
L_a = (10+XX*20/100)*0.01
L_p = (5+YY*25/100)*0.01
M_a = L_a/10+0.04
M_p = ZZ/100
J_a =  (M_a *L_a^2)/12 ; J_p =  (M_p*L_p^2)/12 ; J_t = J_a*J_p + J_a*M_p*(L_p/2)^2 + J_p*M_p* L_a^2
A = [0 0 1 0 ; 0 0 0 1; 0 M_p^2*L_p^2*L_a*g/(4*J_t) 0 0;0 M_p*L_p*g*(J_a+M_p*L_a^2)/(2*J_t) 0 0]
B = [0;0;(J_p + 1/4*M_p*L_p^2)/J_t ; M_p*L_p*L_a/(2*J_t)]
C = [1 0 0 0; 0 1 0 0 ]
D = [0;0]
sys_ss = ss(A,B,C,D)
```

a) What is the transfer function of the **theta** part (rotary arm angle)?

b) What is the transfer function of the **alpha** part (pendulum angle)?

c) Plot the step responses of the plant.

d) What are the poles of the entire plant? How many poles are there? Is the plant stable?

e) What is the feedback gain vector K that moves all poles to -10?

f) What is the A matrix of the new system?

g) Plot the step responses of the new system.