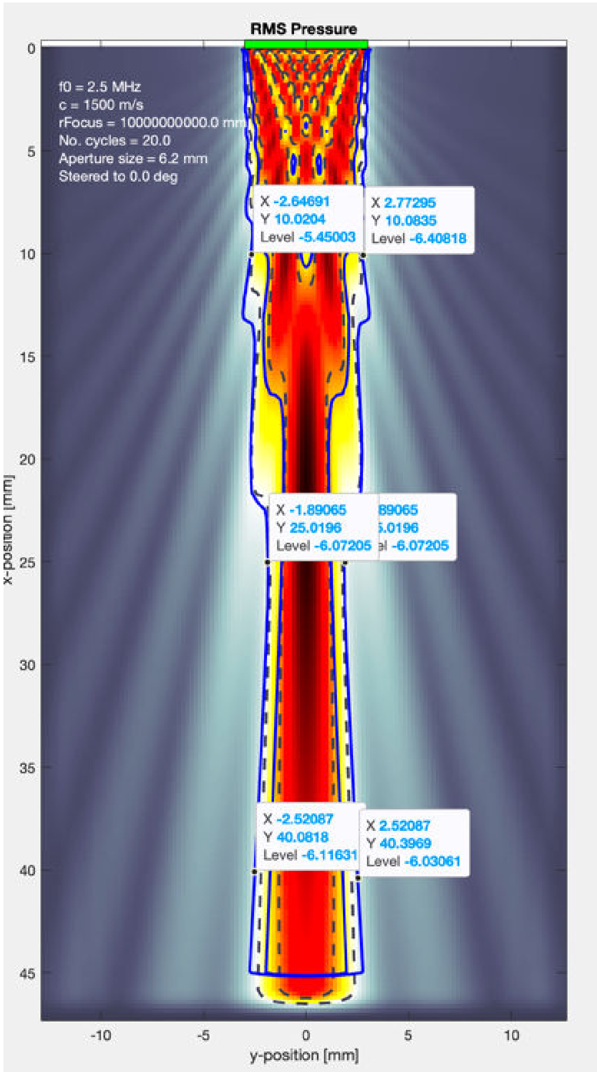
Exercise 1:

Measuring the -6dB beam widths at depths $X \in \{10\text{mm}, 25\text{mm}, 40\text{mm}\}$:



Task I

Observed 6dB Beam widths:

10mm: $[-2.65, 2.77] \Rightarrow 5.42[\text{mm}]$

25mm: $[-1.89, 1.89] \Rightarrow 3.78[\text{mm}]$

40mm: $[-2.5, 2.5] \Rightarrow 5.0[\text{mm}]$

Next, we compare the observed beam widths with the estimated ones:

Estimating the beam width using equation (1.8):

$$y_{res} = x\theta_{6dB} = x\frac{1.21\lambda}{D}$$

First, calculating the wave length λ:

$$\lambda = \frac{c_0}{f} = \frac{1500\text{m/s}}{2.5 * 10^6 s^{-1}} = 6 * 10^{-4} m$$

$$D = 6.2\text{mm} = 6.2 * 10^{-3}\text{m}$$

So, given a depth x, we can estimate the spatial lateral resolution using:

$$y_{res} = x\frac{1.21 * (6 * 10^{-4}\text{m})}{6.2 * 10^{-3}\text{m}} = 0.1171x$$

Plugging in the selected depths $x \in 10mm, 25mm, 40mm$:

$$y_{res}(10\text{mm}) = 0.1171 * (10 * 10^{-3}\text{m}) = \underline{\underline{1.2\text{mm}}}$$

$$y_{res}(25\text{mm}) = 0.1171 * (25 * 10^{-3}\text{m}) = \underline{\underline{2.93\text{mm}}}$$

$$y_{res}(40\text{mm}) = 0.1171 * (40 * 10^{-3}\text{m}) = \underline{\underline{4.68\text{mm}}}$$

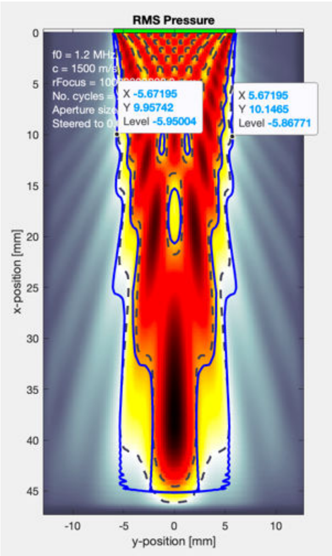
From these results, we see that the estimated lateral resolution is not valid in the near-field of the array. However, as we approach the far field, the error in the estimate decreases.

For this setup, the softest far field is:

$$R = \frac{D^2}{k\lambda} = \frac{(6.2 * 10^{-3}\text{m})^2}{4 * (6 * 10^{-4}\text{m})} = 1.6 * 10^{-3}\text{m} = \underline{\underline{16\text{mm}}}$$

Task II

Performing the same experiment with $f_0 = 1.25MHz$:



Beam widths:

10mm => 5.54mm

25mm => 6.3mm

40mm => 9.82mm

Recomputing the wavelength λ:

$$\lambda = \frac{c_0}{f} = \frac{1500\text{m/s}}{1.25 * 10^6 s^{-1}} = 1.2 * 10^{-3} m$$

Estimating the beam widths:

$$y_{res}(x) = 0.2342x$$

$$y_{res}(10\text{mm}) = 0.2342(10 * 10^{-3}\text{m}) = \underline{\underline{2.3\text{mm}}}$$

$$y_{res}(25\text{mm}) = 0.2342(25 * 10^{-3}\text{m}) = \underline{\underline{5.9\text{mm}}}$$

$$y_{res}(40\text{mm}) = 0.2342(40 * 10^{-3}\text{m}) = \underline{\underline{9.4\text{mm}}}$$

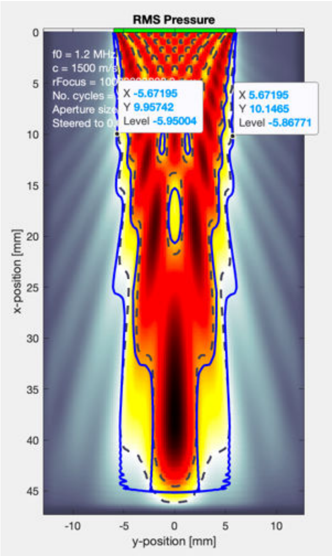
The softest far field is now

$$R = \frac{D^2}{k\lambda} = \frac{(6.2 * 10^{-3}\text{m})^2}{4 * (1.2 * 10^{-3}\text{m})} = 8\text{mm}$$

Halving the frequency doubles the wave-length, which in turn halves the distance to the far-field, as the far-field distance is inversely proportional to the wavelength λ. This reduces the errors in our beam width estimates.

Task III

Performing the same experiment with $f_0 = 1.25MHz$ and 61 sensors:



Beam widths:

$10\text{mm} \Rightarrow 11.34\text{mm}$

$25\text{mm} \Rightarrow 12\text{mm}$

$40\text{mm} \Rightarrow 10.8\text{mm}$

Recomputing the wavelength λ and aperture D:

$$\lambda = \frac{c_0}{f} = \frac{1500\text{m/s}}{1.25 * 10^6 s^{-1}} = 1.2 * 10^{-3}\text{m}$$

$$D = 12.2 * 10^{-3}\text{m}$$

Estimating the beam widths:

$$y_{res}(x) = 0.1190x$$

$$y_{res}(10\text{mm}) = 0.1190(10 * 10^{-3}\text{m}) = \underline{\underline{1.2\text{mm}}}$$

$$y_{res}(25\text{mm}) = 0.1190(25 * 10^{-3}\text{m}) = \underline{\underline{3.0\text{mm}}}$$

$$y_{res}(40\text{mm}) = 0.1190(40 * 10^{-3}\text{m}) = \underline{\underline{4.8\text{mm}}}$$

The softest far field is now

$$R = \frac{D^2}{k\lambda} = \frac{(6.2 * 10^{-3}\text{m})^2}{4 * (1.2 * 10^{-3}\text{m})} = 31.0\text{mm}$$

By (approximately) doubling the aperture, the far-field distance is scaled by a factor of 4, as the far-field distance is proportional to the square of the aperture size.

As the far-distance increases, our estimates should become worse than before, as we can see in the discrepency between the estimates and observations.

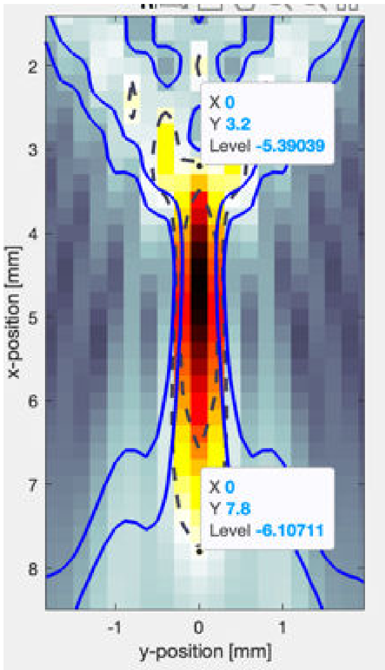
Task IV

5mm => 0.76mm

10mm => 5.54mm

15mm => 11.08mm

To estimate the depth of field, we can look at the dashed lines around the area of focus, and measure the along-axis distance for where the innermost dashed line intersects with the beam axis:



From the plot, the estimated depth of field is :

$7.8\text{mm} - 3.2\text{mm} = \underline{\underline{4.6\text{mm}}}$

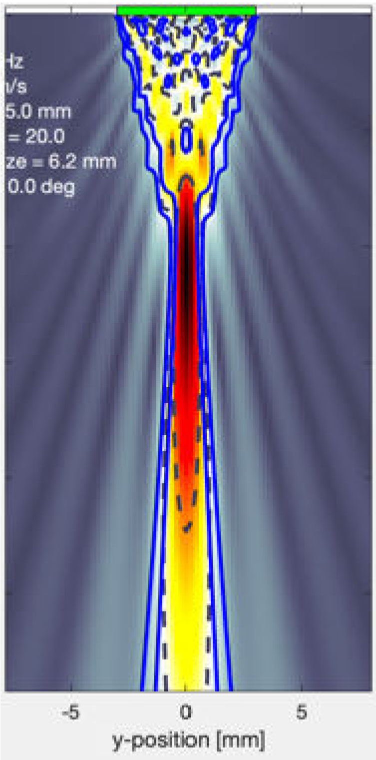
Task V

Beam widths:

5mm => 3.78mm

10mm => 1.26mm

15mm => 1.76mm



From the plot, the estimated depth of field is :

$$35.4\text{mm} - 5.9\text{mm} = \underline{\underline{29.5\text{mm}}}$$

Task VI

Changing the focal range changes the rate of convergence/divergence of the pressure field- or the narrowing/expansion of the beam width.

Lowering the focal length means that the "lens" needs to focus the beam closer to the aperture, which requires a higher rate of convergence, which in turn causes the beam to spread out more beyond the focal point. From the observations, we see that **the change of beam width around the focal zone is much more extreme in the** 5mm **focal length case**, which supports this. Also, the beam width at the focal point itself increases with focal length:

$$W = \frac{1.41\lambda}{2a}F$$

Which is clear in the observed focal widths (0.76mm and 1.76mm)

With a focal length, we also observe an **increase in the Depth of Field** (DoF, the along-axis length of the focal area). This makes sense, as the depth of field is given by:

$$D = 9.7\lambda\left(\frac{F}{2a}\right)^2$$

The increase in DoF is expected, as it is proportional to the square of the focal length. Since the focal point is further away, the beam must converge more slowly, and it is as though the focal zone is *smeared out*.

<u>**Eq. (9.22) in Cox and Treeby VS Eq. (4) in Hykes:**</u>

How you define the depth of field or focal zone is somewhat arbitrary. The purpose of having these definitions is to characterize a specific ultrasound configuration, and look at how they are affected by various parameters. They might use different boundaries for the focal zone, maybe **Cox and Treeby define it as the -6dB zone and Hykes uses the -3dB zone?** The important thing here is to consistently use only **one of the definitions**.

Task VII

To determine the number of beams needed for a full sector scan, we need to estimate the width of a single beam, in degrees.

Running the simulation and inspecting the beam width at the focal range (50mm), we observe a beam width of $\underline{2.62\text{mm}}$. We can then find the beam width in angles by:

$$\theta = 2 * \arctan\left(\frac{1.31\text{mm}}{50\text{mm}}\right) = 3°$$

We want to cover a range of $[-30, 30]$, so the number of beams required is $\frac{60°}{3°} = 20$

Next, to estimate the framerate, we calculate the time it takes for 20 beams to fully transmit and return for a 100mm range scan.

For a single beam, the time for the sound to travel 100mm and back is:

$$t = \frac{d}{v} = \frac{2 * 100 * 10^{-3}\text{m}}{1500\text{m/s}} = 133.33\mu s$$

Then, for 20 beams, the total time is $133.33\mu s * 20 = 2.7\text{ms}$

The frame rate is then given by $\frac{1}{T} = \frac{1}{2.7 * 10^{-3}} = 370.37$

The sector scan requires 20 beams and achieves a frame rate of 370fps.

Exercise 2

Task I

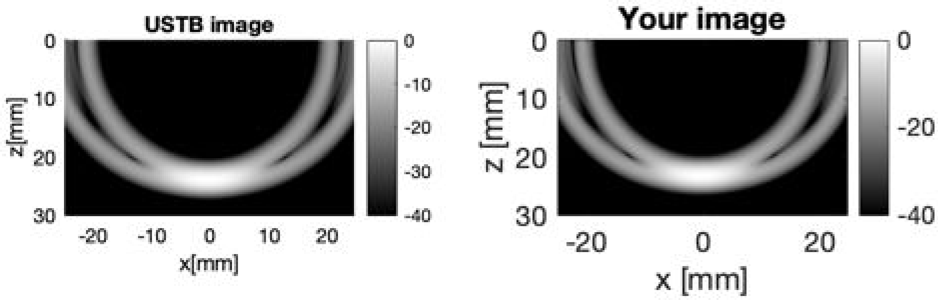
To compute the beamformed image, we need to calculate the receive delays for each pixel in the image, for each array element.

Using the formula: $R(x, z, m) = \sqrt{z^2 + (x - m)^2}$ we can compute the receive distances for each pixel (x, y), for a given element offset m:

```
for rx = 1:channel_data.N_elements
    z_dist = z_pixels-z_element_position(rx);
    x_dist = x_pixels-x_element_position(rx);
    dist = sqrt(z_dist.^2 + x_dist.^2);
    receive_delay(:,:,rx) = dist/1500;
end
```

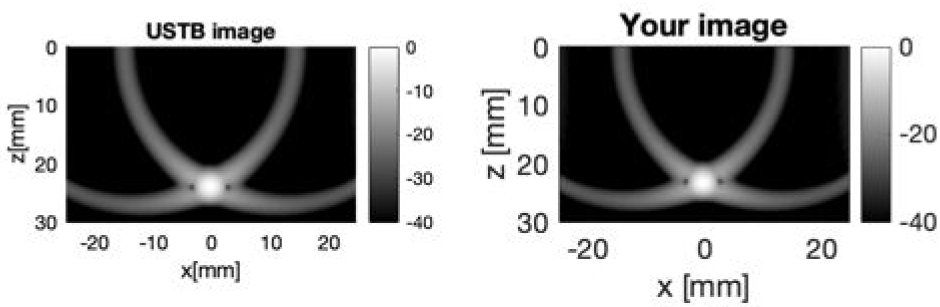
the `dist` object is now an 1054x1054 matrix whose values are the distances from the current array element to the pixel at that index.

To compute the receive delay, we simply divide by the assumed speed of sound. Using this receive delay, the result appears identical to the one constructed through USTB:



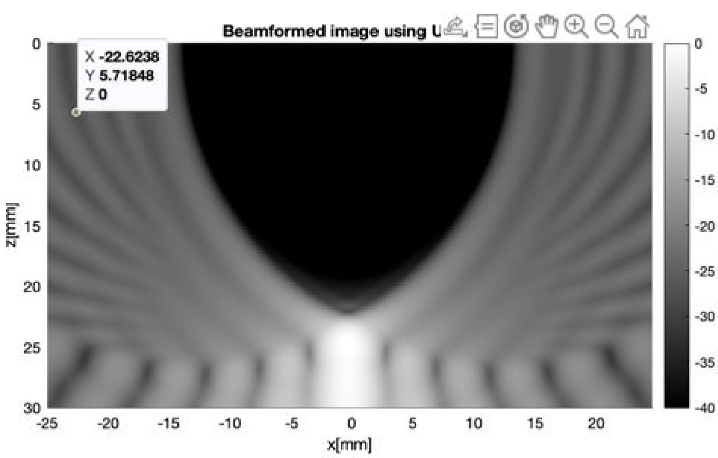
Task II

Changing the element count from 4 to 16 results in the following beamformed images:



The position of the scatterer is more apparent now (higher resolution), and the pattern of the image has changed.

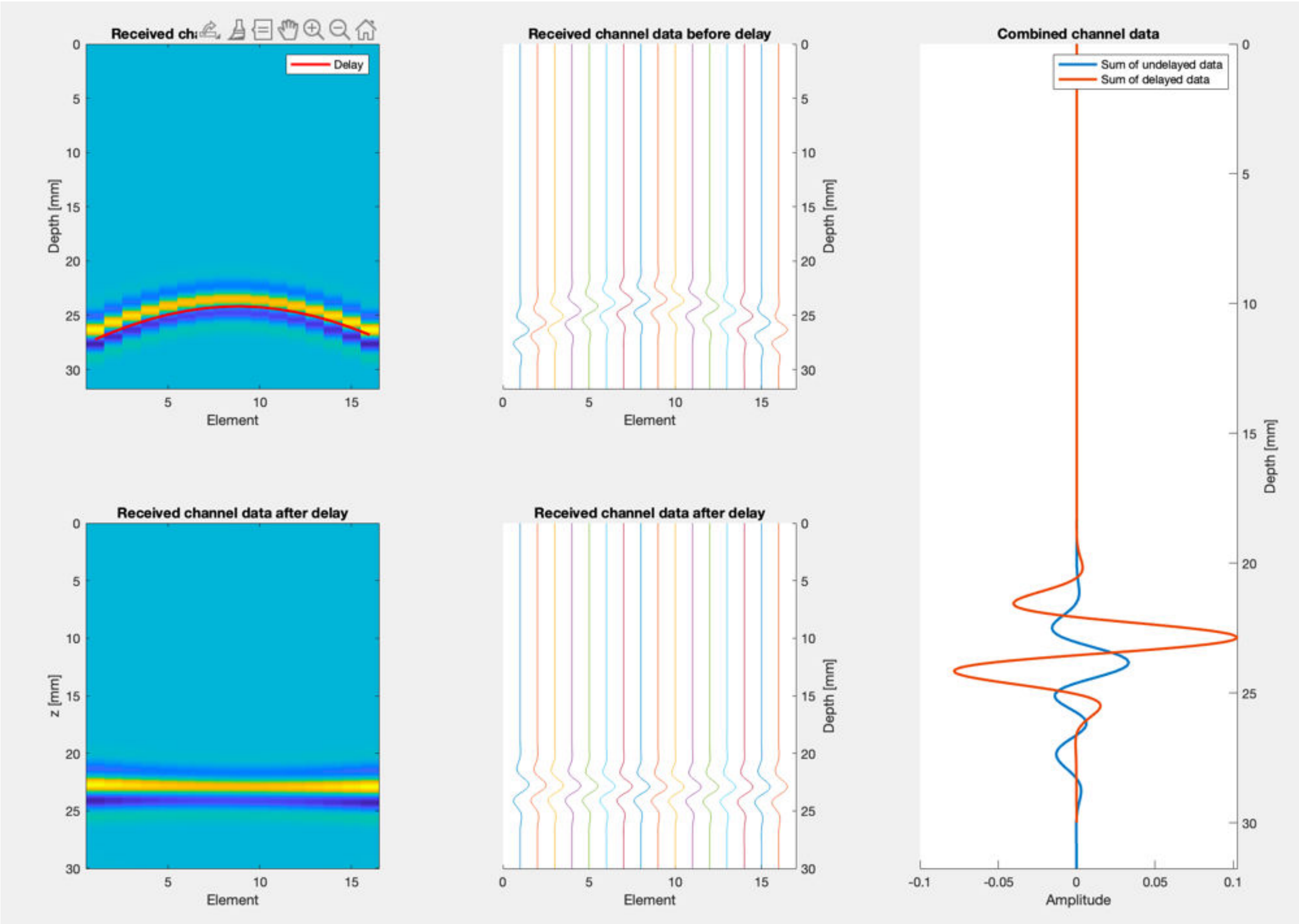
Using a sinusoid instead of a gaussian pulse:



This gives us a smeared out result with low resolution. Both the axial and the lateral resolution are much worse.

The bandwidth and frequency (wavelength) appear in the equations for estimating axial and lateral resolution (axial resolution is inversely proportional to bandwidth, and lateral resolution is proportional to wavelength).

Task III



The plot shows that the estimated position of the scatterer is accurate, since we have managed to align the received channel data quite well.

In the raw received data, there is clearly a misalignment in timing. This is because the array elements have different positions, and the signal transmitted from the scatterer consequently arrives at different times. By delaying the data appropriately for each element/channel, we align it such that we end up with a non-warped, useful beamformed image.

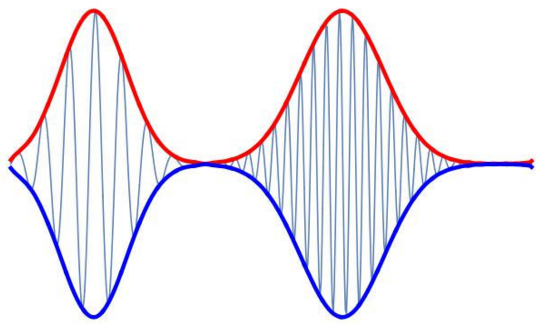
The plot on the right shows that the summed delayed signal is much stronger than the summed raw signal, as the waves no longer destructively interfere with each other.

Task IV

Figure 13 shows the image representations of the signals for each individual element. The final image is constructed as the average of these images, where some parts from different elements cancel out and others amplify through "digital interference". By adding these images together, we are able to pinpoint the position of the scatterer.

Figure 14

Instead of plotting the signal directly, we can plot the envelope of the signal instead:



Before envelope detection, the colors are mapped directly from the signal strength, so the parts that we expect to be completely black are grey, and overall the image is harder to interpret, because we are including all the small variations of the signal. By taking the envelope, we get a more intuitive and low-frequency representation of the signal strength, giving us a nicer image to look analyze.