# F1/ TEK 4040 - Matematisk modellering av dynamiske systemer

References:

- O. Hallingstad: Matematisk modellering av dynamiske systemer
- John J. Craig: Robotics (ch 1-6)
- Peter H. Zipfel: Modelling and simulation of Aerospace Vehicle Systems

Anders Rødningsby
anders.rodningsby@ffi.no
63807295

We want to simulate:

- Orientatio of satellites in space
- Aerplanes
- Robots - mechanics
- Inertial navigation systems (INS)
  - Trajector generator (gives position and orientation) => velocity, acc, angular velocity derivatives
  - Navigation equations
  - Only work with deterministic equations
  - Noise is included in TEK4050 Stokastiske systeme

We want to describe the motion of objects by:

Enough for particles
- Position (centre of mass)
- Velocity
- Accelleration

Extended objects
- Attitude / Orientation
- Angular velocity
- Angular accelleration

## DYNAMICS

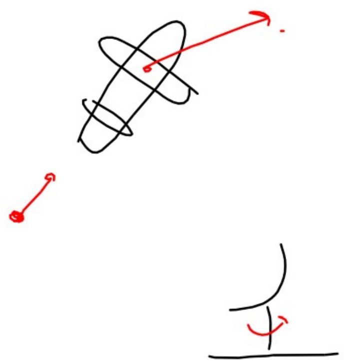
1) Kinematics: Describing the motion } Part A

2) Kinetics: Describe the relation between the motion and the forces creating the motion. } Part B

A rigid bodys motion can be put together by translation and rotation.

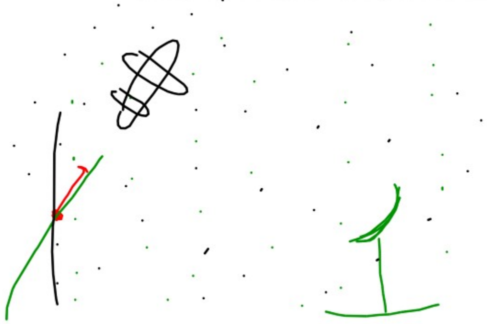
## Procedure of modelling

### 1. Define the physical system, choose objects



Vi need to assume the objects can be described with sufficient accuracy as particles (point with mass) and rigid bodies (the molecules have a fixed position relative to each other)

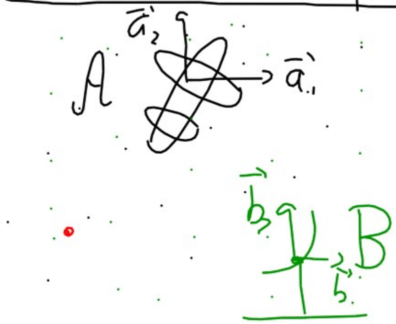
### 2. Define reference space or inertial space



We get the reference space of an object by expanding the object by points that fills the space and that are fixed to the object.

NB! Reference space can be defined different in other text books.

## 3. Define affine spaces (modell of reference space)




$\mathcal{A}$ : affine space

Objects : points + vectors

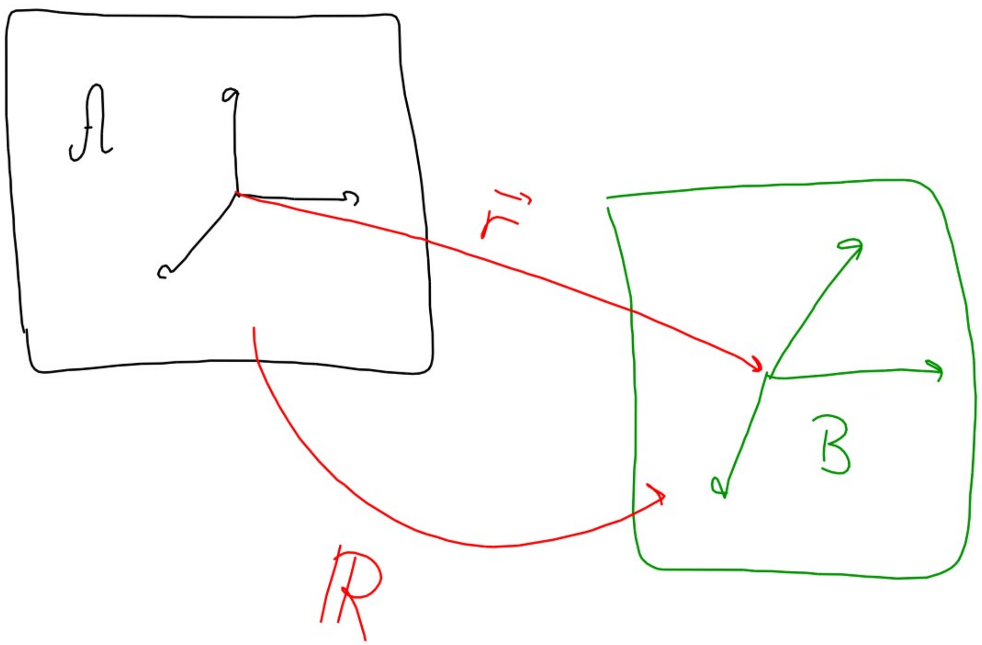
Operations: $P = Q + \vec{v}$ , $\vec{v} = P - Q$

## 4. Introduce (reference) frames in the affine space

$$\mathcal{A} : \mathcal{F}_{\mathcal{A}}^{a} = \{ O_a , \vec{a}_1 , \vec{a}_2 , \vec{a}_3 \} = \{a\}$$

$$\mathcal{B} : \mathcal{F}_{\mathcal{B}}^{b} = \{ O_b , \vec{b}_1 , \vec{b}_2 , \vec{b}_3 \} = \{b\}$$

## 5. Define the relation of objects in different affine spaces




The original rigid bodies are now described by points and frames in affine spaces.

If we have changes as function of time:

$\vec{r}(t)$, $\mathbb{R}(t)$

6. Define the transformation from affine space to $\mathbb{R}^n$ (n axis of real numbers)

The transformation is done by decomposing vectors ($\vec{r}$) and operators ($\mathbb{S}$) using the basis vector set (frame)

$$\underbrace{\vec{y} = \mathbb{S}\,\vec{x}}_{\text{Affine space}} \quad \underset{\mathcal{F}^a}{\Longleftrightarrow} \quad \underbrace{\underline{y}^a = \underline{S}^a \underline{x}^a}_{\mathbb{R}^n}$$

$\underline{x}^a, \underline{y}^a$ is column matrices (column vectors)

$\underline{S}^a$ is matrix

7. Introduce time dependent vectors and operator. Define differentiation and integration

$$\vec{r}(t), \mathbb{S}(t) \Longleftrightarrow \underline{r}^a(t), \underline{S}^a(t)$$

$$\dot{\vec{r}}^b(t), \dot{\mathbb{S}}^b(t) \Longleftrightarrow \underline{\dot{r}}^{ba}(t), \dot{\underline{S}}^{ba}(t)$$

Now we have all mathematics to describe the kinematics

## 8. Kinetic: Find the relation between forces and the motion

Forces: modeled by vectors

$$\vec{f} \in A(V) \iff \underline{f}^a \in \mathbb{R}^n$$

Torque: modelled by vectors

$$\vec{n} \in A(V) \iff \underline{n}^a \in \mathbb{R}^n$$

$i$: inertial frame

Mass of a particle or a rigid body: $m$

Newton's 2. law: $\vec{f} = m\vec{a}^{\,ii} \iff \underline{f}^i = m\,\underline{a}^{\,iii}$

Inertial matrix for rigid bodies (corresponds to mass for particles in rotating objects)

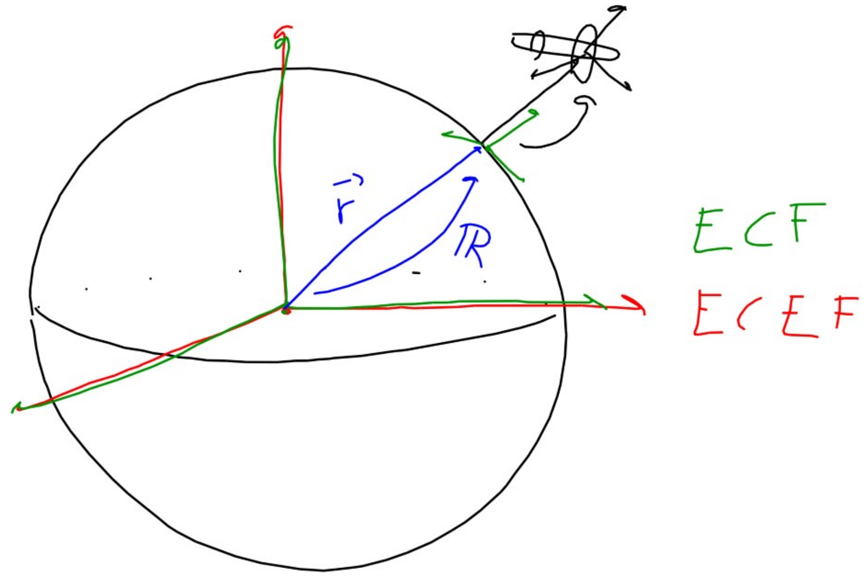
$$\vec{n} = \vec{h}^{\,i} \iff \underline{n}^i = T\,\underline{\omega}^{\,iii}$$

$\vec{h}$: angular momentum, $T$ is the inertial matrix.

## Comments:

Our equations become:

- algebraic
- ordinary vector differential equations (state space equations)



$\vec{r}$
$\mathbb{R}$
ECF
ECEF