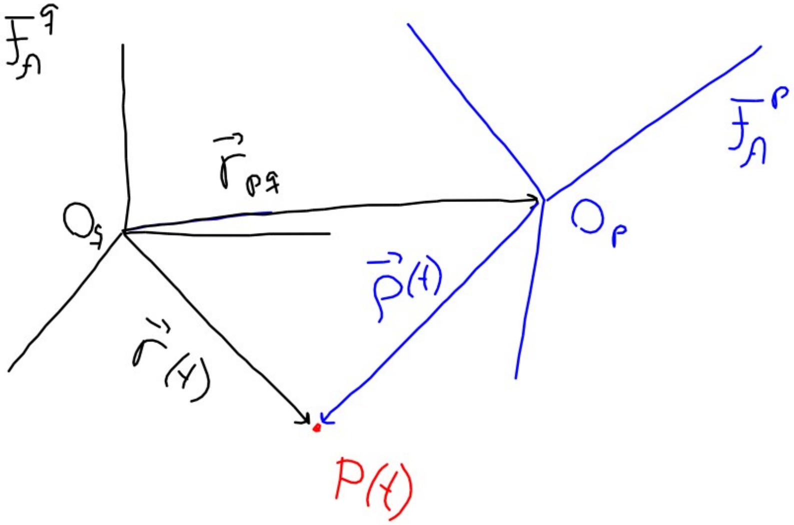
F9/

A.5.4 Derivative of points motion in affine space



We will look at the relation between velocity and accelleration of point P seen from two frames that have a relative motion between them (translation and rotation)

We can either calculate the equations using the geometrical vectors and then for the algebraic vectors, or just calculate for either the geometrical or algebraic vectors, and then use the formulas giving the relation between algebraic and geometrical vectors.

From the figure:

$$\vec{r}(t) = \vec{r}_{qp}(t) + \vec{\rho}(t)$$

$$\vec{r}(t) = P(t) - O_q$$

$$\vec{\rho}(t) = P(t) - O_p$$

Find: $\dot{P}^q(t)$ and $\ddot{P}^q(t)$ $\left(= \ddot{P}^{qq}(t)\right)$

$\iff$ $\dot{\vec{r}}^q(t)$ and $\ddot{\vec{r}}^q(t)$ $\left(= \ddot{\vec{r}}^{qq}(t)\right)$

Formulas: $\dot{\vec{c}}^q = \dot{\vec{c}}^p + \vec{\omega}^q_p \times \vec{c}$

$$\dot{\underline{c}}^{qq} = R^q_p \underline{\dot{c}}^{pp} + S(\underline{\omega}^{qq}_p) R^q_p \underline{c}^p$$

Notation:

$\vec{v}^q = \dot{\vec{r}}^q$: P's velocity seen from $\mathcal{F}^q$

$\vec{v}^p = \dot{\vec{\rho}}^p$: P's velocity seen from $\mathcal{F}^p$

$\vec{a}^q = \dot{\vec{v}}^{qq} = \ddot{\vec{r}}^{qq}$: Acc. seen from $\mathcal{F}^q$

$\vec{a}^p = \dot{\vec{v}}^{pp} = \ddot{\vec{\rho}}^{pp}$: Acc. seen from $\mathcal{F}^p$

Want to use Newton's 2. law:

$\vec{f} = m \cdot \vec{a}^i$, $\mathcal{F}^i$ - inertial frame

In inertial navigation (INS) we measure $\underline{a}^{ib}$ (b: body frame)

Calculation using geometrical equations:

$$\boxed{\begin{aligned} \vec{r} &= \vec{r}_{qp} + \vec{\rho} \\ \vec{v}^q = \dot{\vec{r}}^q &= \dot{\vec{r}}_{qp}^q + \overbrace{\vec{v}^p + \vec{\omega}_p^q \times \vec{\rho}}^{\dot{\vec{\rho}}^q} \end{aligned}}$$

$$\vec{a}^q = \ddot{\vec{r}}^{qq} = \ddot{\vec{r}}_{qp}^{qq} + \underbrace{\dot{\vec{v}}^{pp} + \omega_p^q \times \vec{v}^p}_{\dot{\vec{v}}^{pq}} + \dot{\vec{\omega}}_p^{qq} \times \vec{\rho} + \vec{\omega}_p^q \times (\vec{v}^p + \vec{\omega}_p^q \times \vec{\rho})$$

$$\boxed{\vec{a}^q = \ddot{\vec{r}}_{qp}^q + \vec{a}^p + \dot{\vec{\omega}}_p^q \times \vec{\rho} + \vec{\omega}_p^q \times (\vec{\omega}_p^q \times \vec{\rho}) + 2\,\vec{\omega}_p^q \times \vec{v}^p}$$

$$\begin{aligned}
\underline{r}^q &= \underline{r}_{qp}^q + R_p^q \underline{\rho}^p \\
\underline{v}^q &= \dot{\underline{r}}_{qp}^q + R_p^q \left(\underline{v}^p + \underline{\omega}_p^{qp} \times \underline{\rho}^p\right) \\
&= \dot{\underline{r}}_{qp}^q + R_p^q \underline{v}^p + \underline{\omega}_p^q \times R_p^q \underline{\rho}^p \\
\underline{a}^q &= \ddot{\underline{r}}_{qp}^q + R_p^q \left(\underline{a}^p + \dot{\underline{\omega}}_p^{qpp} \times \underline{\rho}^p + \underline{\omega}_p^{qp} \times \left(\underline{\omega}_p^{qp} \times \underline{\rho}^p\right) + 2\underline{\omega}_p^{qp} \times \underline{v}^p\right) \\
&= \ddot{\underline{r}}_{qp}^q + R_p^q \dot{\underline{v}}^p + \dot{\underline{\omega}}_p^q \times R_p^q \underline{\rho}^p + \underline{\omega}_p^q \times \left(\underline{\omega}_p^q \times R_p^q \underline{\rho}^p\right) + 2\underline{\omega}_p^q \times R_p^q \underline{v}^p
\end{aligned}$$

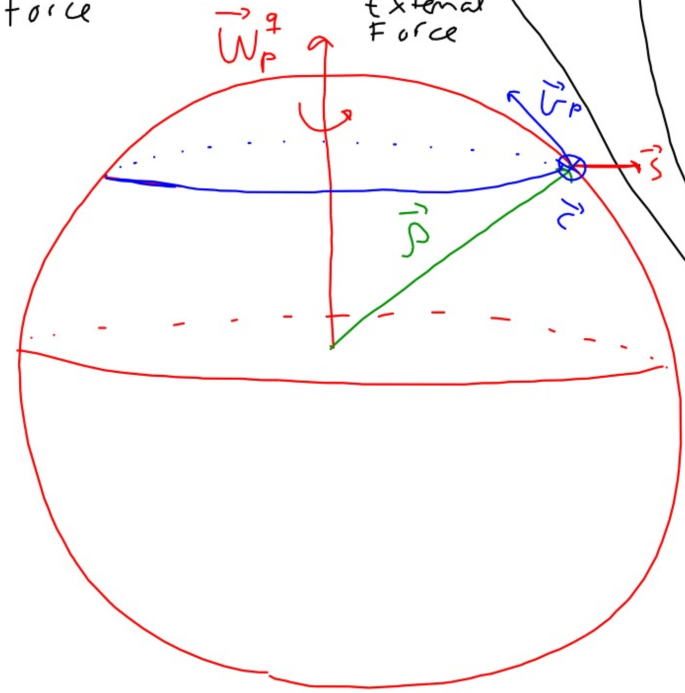
$$\vec{f} = m\,\vec{a}^q \tag{A- 119}$$

Ved å uttrykke akselerasjonen $\vec{a}^q$ vha ledda på høgre sida, får vi de kreftene som må innføres i et ikke-inertial system. Likninga ovenfor blir nå :

$$m\vec{a}^q = m\left(\ddot{\vec{r}}^{\,q}_{qp} + \vec{a}^p + \dot{\vec{\omega}}^q_p \times \vec{\rho} + \vec{\omega}^q_p \times (\vec{\omega}^q_p \times \vec{\rho}) + 2\vec{\omega}^q_p \times \vec{v}^p\right) \tag{A- 120}$$

Løser likninga mhp $m\vec{a}^p$:

$$\underbrace{m\vec{a}^p}_{\text{Apparent Force}} = \underbrace{m\vec{a}^q}_{\text{External Force}} \underbrace{- m\ddot{\vec{r}}^{\,q}_{qp}}_{\text{Force due to acc. of } O_p \text{ seen from } \mathcal{F}^q} \underbrace{- m\dot{\vec{\omega}}^q_p \times \vec{\rho}}_{\text{Angular acc. term (tangential force)}} \underbrace{- m\vec{\omega}^q_p \times (\vec{\omega}^q_p \times \vec{\rho})}_{\text{Sentrifugal force } (\vec{s})} \underbrace{- m2\vec{\omega}^q_p \times \vec{v}^p}_{\text{Coriollis force } (\vec{c})} \tag{A- 121}$$



PartB: DYNAMICS

Dynamics is:

1) Kinematics:

   - Describe the motion using mathmatics (Part A)

2) Kinetic:

   - The relation between the motion of an object and the forces creating the motion (mathematics + physics)
     e.g. Newtons 2. law.

Terms

Reference space – Inertial space

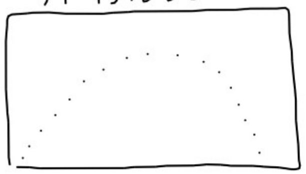
- Is connected to a physical system
- Coordinate systems (frame, units, function)
- Particles (modeled by points and mass)
- Pos., vel., acc. (modeled by vectors)
- Rigid body (modeled by frame and mass)
- Attitude/orientation (modeled by frames)

Affine space

- Mathematical model of reference space
- $\mathcal{F}_A^a : \{O_a, \vec{a}_1, \vec{a}_2, \dots, \vec{a}_n\}$
- P : points
- $\vec{v}$: vectors

By your own: "Grunnleggende prinsipper i klassisk mekanikk"

Aristoteles



Absolute points

Newton

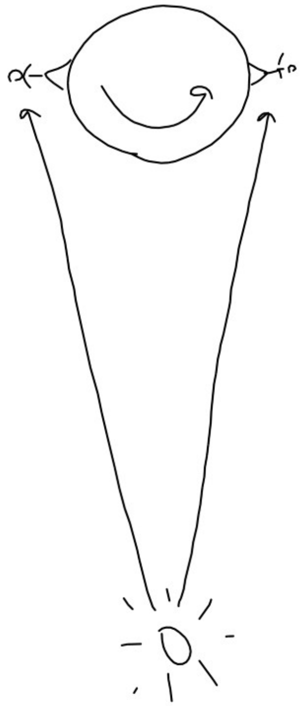
$$\vec{f} = \frac{d}{dt}(m\vec{v})$$

$$\vec{v} = \vec{v}_{ball} + \vec{v}_{train}$$

Einstein

$$\|\vec{v}\| \leq c \text{ (speed of light)}$$

$$\vec{v} \neq \vec{v}_{ball} + \vec{v}_{train}$$

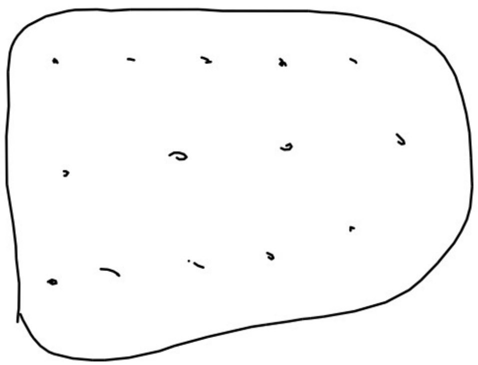


Part B. Dynamics

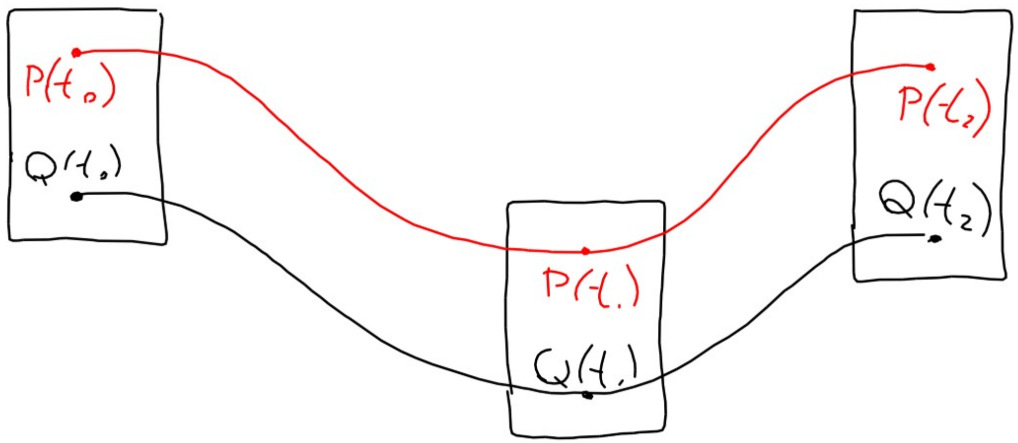
B.1. Kinematic

B.1.1 Kinematic description of particles

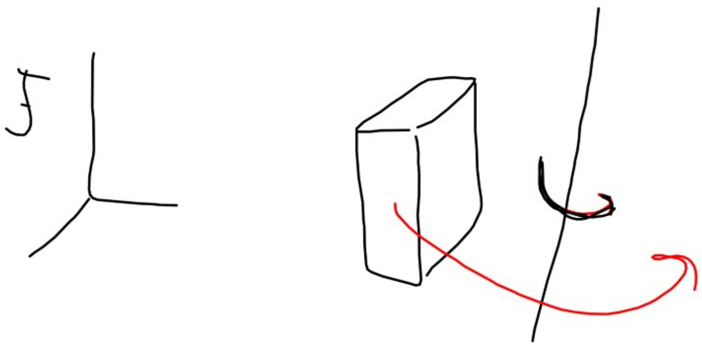
Def. B.1 Rigid body:



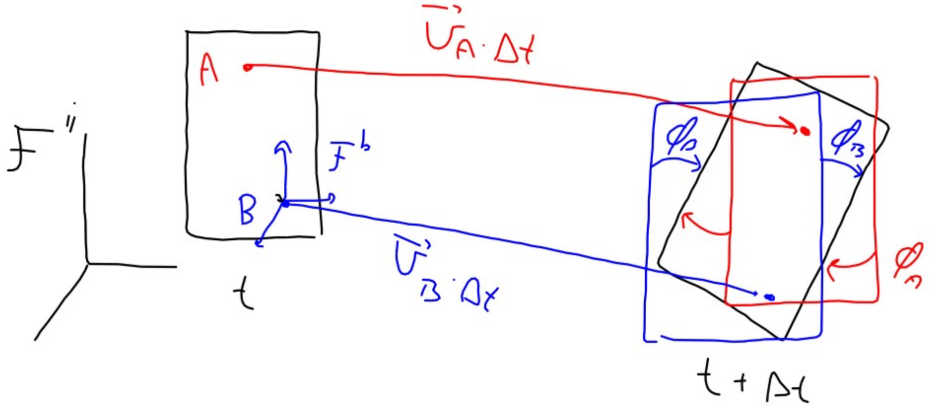
Def B.2 Pure translation of a rigid body



Def B.3 Rotation of a rigid body



Teorem B.1 (Chasley's teorem) ***Dekomponering i translasjon og rotasjon***
Bevegelsen av et stivt legeme relativt et k.s. kan settes sammen av translasjon og rotasjon. Dette kan gjøres på følgende måte :
1) Velg et punkt A (B) i legemet. Anta at alle punktene i legemet har samme hastighet, $\vec{v}_A\,(\vec{v}_B)$, hvor $\vec{v}_A\,(\vec{v}_B)$ er hastigheten relativt vårt k.s.
2) Superponer en ren rotasjon om punktet A med vinkelhastighet $\vec{\omega}$ relativt vårt k.s. (NB : $\vec{\omega} = \vec{\omega}_A = \vec{\omega}_B$, mens generelt er $\vec{v}_A \neq \vec{v}_B$ ($\vec{v}_A = \lim_{\Delta t \to 0}(\Delta \vec{r}_A/\Delta t)$).)



$\omega_B = \lim \dfrac{\phi_B}{\Delta t}$ $\quad \vec{v}_A \neq \vec{v}_B$

$\omega_A = \omega_B$

$\omega_A = \lim \dfrac{\phi_A}{\Delta t}$

The motion of a rigid body is created by $\vec{v}_{O_b}^{\,i}(t)$ and $\vec{\omega}_b^{\,i}(t)$, i is the inertial frame

That means we can modell the motion of a rigid body as the motion of the frame $\mathcal{F}^b$ relative to $\mathcal{F}^i$

B.2 Kinetic

Newton's laws for a particle :

Teorem B.2 (Newtons 1.lov) *Dersom en partikkel er langt borte fra innflytelsen fra alle andre partiler i universet, vil den bevege seg med konstant hastighet mht et treghetssystem,* **i** *(kan egentlig utledes fra Newtons 2.lov).* (N.1 is a special case of N.2)

Teorem B.3 (Newtons 2.lov) *Dersom det lineære moment,* $\vec{p}^{\mathbf{i}}$*, for en partikkel i et treghetssystem* **i** *endres med tiden, sies partikkelen å være påvirket av en kraft,* $\vec{f}$*, gitt ved :*

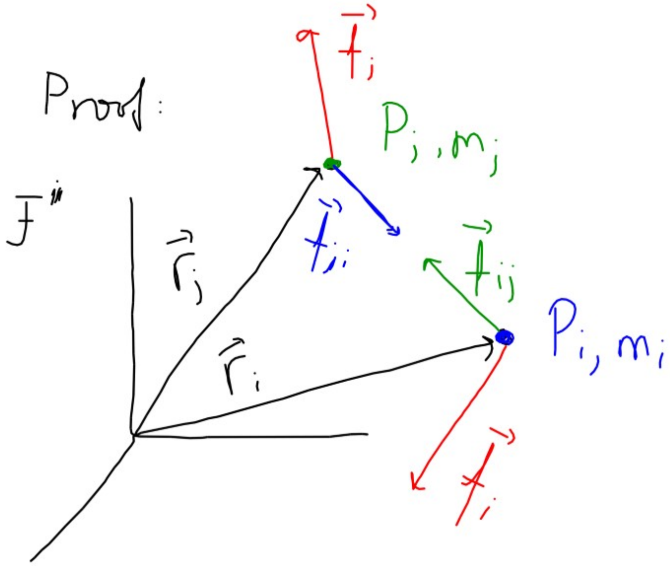
$$\vec{f} = \dot{\vec{p}}^{\mathbf{i}} \quad hvor \quad \vec{p}^{\mathbf{i}} = m\vec{v}^{\mathbf{i}} \tag{B- 139}$$

Teorem B.4 (Newtons 3.lov) *Dersom to isolerte partikler interakterer med hverandre vil den krafta partikkel nr 1 utsetter partikkel nr 2 for være lik i størrelse, men motsatt rettet den krafta partikkel nr 2 utsetter partikkel 1 for. Dvs : aksjon = reaksjon eller kraft = motkraft.*

Teorem B.5 ***Newtons 2. lov for et system av partikler***
Vi antar at Newtons 3. lov gjelder for krafta mellom partiklene, dvs $\vec{f}_{ij} = -\vec{f}_{ji}$. Da vil den totale ytre kraft, $\vec{F}$, være lik total masse, M, ganger med massesenterets akselerasjon, $\vec{a}_c^{\mathbf{i}}$, sett fra treghetsramma :

$$\boxed{\vec{F} = M \frac{d^{\mathbf{i}} d^{\mathbf{i}}}{dt^2} \vec{r}_c = M \vec{a}_c^{\mathbf{i}}} \qquad \text{(B- 140)}$$

Proof:



$\vec{f}_i$: external force

f_{ij} : internal force (satisfy N.3 law)

m_i : mass of particle i

$\vec{r}_i$: position vector of particle i

$\vec{r}_c$: centre of gravity/mass

$$M = \sum_{i=1}^{n} m_i \quad , \quad \vec{F} = \sum_{i=1}^{n} \vec{f}_i$$

$$\vec{r}_c = \frac{1}{M} \sum_{i=1}^{n} m_i \vec{r}_i$$

$$\Downarrow$$

$$M \vec{r}_c = \sum_{i=1}^{n} m_i \vec{r}_i$$