Network Economics of Political Repression: Maximizing Centrality Through Node Removal

Eduard Airapetian
New Economic School

April 19, 2025

Abstract

We model autocratic purges as a network-design problem. A ruler first chooses which elite nodes to remove, paying a legitimacy cost proportional to each target's Katz-Bonacich centrality; the remaining agents then play a linear-quadratic network game in which equilibrium influence equals their centrality. The ruler's payoff is his own post-purge centrality minus removal costs. Deleting any single elite never lowers — and on connected graphs strictly raises — the ruler's centrality. If legitimacy costs are low enough, removing every rival (forming a star) is uniquely optimal. Higher network density or stronger spillovers increase the size of the optimal purge, while higher legitimacy costs temper it. The framework yields clear, testable predictions about who gets purged and provides tractable tools for simulating counterfactual repression scenarios.

1 Introduction and Motivation

Authoritarian leaders often consolidate power by eliminating influential elites in their regime. Historically, Joseph Stalin's purges offer a vivid example: he systematically targeted high-ranking party members to weaken alternative power centers and solidify his control. By removing key figures, Stalin increased his own prominence in the Soviet political network, effectively maximizing his central position. These observations motivate a formal model in which a dictator strategically "prunes" a network of elites to enhance his network centrality (a proxy for political power), while weighing the legitimacy costs of eliminating prominent individuals.

We develop a two-stage game to capture this dynamic of political repression via strategic node removal. In Stage 1, the ruler (planner) chooses which elite nodes to repress (remove) from the network, incurring a cost that increases with each target's influence or centrality. In Stage 2, the remaining agents (including the ruler) engage in a network interaction game that determines their equilibrium levels of political activity or influence. The dictator's payoff is increasing in his own resulting centrality or influence in this post-purge network. This setup endogenizes the trade-off between eliminating rivals and maintaining legitimacy: removing a highly connected elite can greatly boost the ruler's relative network position, but comes at a steep legitimacy cost (due to public backlash or lost expertise) that grows with the victim's importance.

Our approach bridges network theory and political economy. On the network side, it builds on the idea that a player's action or influence in equilibrium can be characterized by their Bonacich or eigenvector centrality (Ballester et al., 2006). On the political economy side, it formalizes the classic "divide-and-rule" strategy of autocrats (Acemoglu et al., 2004). By selectively "cutting out" links among elites, the dictator fragments the opposition network, ensuring that "no one challenges the ruler because of the threat of divide-and-rule." (Acemoglu et al., 2004) In equilibrium, remaining elites are less able to coordinate or rival the dictator's influence. However, consistent with Egorov and Sonin's (Egorov and Sonin, 2011) insight that dictators often sacrifice competence for loyalty , our model shows that a rational autocrat will tolerate some powerful elites if the cost of removing them outweighs the benefit.

We proceed by laying out the model assumptions and structure, deriving the equilibrium of the stage-2 network game, and then analyzing the ruler's optimal repression strategy in stage 1. We highlight conditions for equilibrium existence, provide comparative statics on how network structure and initial power of the ruler affect the optimal repression policy, and illustrate the results with simple examples. The goal is an elegant, tractable model that yields sharp intuition: a dictator maximizes influence by becoming the unique central hub of the network, but only insofar as legitimacy constraints allow. This illuminates why real-world autocrats often target the most connected opponents yet stop short of indiscriminately eliminating all rivals.

Unlike the "key-player" problem of Ballester, Calvó-Armengol and Zenou (Ballester et al., 2006)—where a planner removes the node that maximally reduces total equilibrium activity—our planner is partisan. He seeks to increase the influence of a single distinguished node (the ruler) rather than depress output system-wide. This changes the objective from minimizing a global sum to maximizing one coordinate of the Katz vector, overturning many comparative-static predictions. It also departs from standard network-interdiction models, which usually disable nodes or links to minimise flow between two exogenous terminals. Here the "terminal" is endogenous: the ruler's centrality rises precisely because rival-to-rival paths are severed.

Our focus therefore complements recent empirical work that treats elite networks as strategic assets. Bueno de Mesquita and Smith (2017) show how co-optation shapes cabinet turnover, while Cruz, Labonne and Querubín (2020) document how Philippine mayors leverage kinship ties to survive. Those papers take the network as given and study payoffs; our model endogenises the network itself, offering a theoretical basis for when leaders purge bridge elites versus rival hubs and providing micro-foundations for the survival patterns they document.

2 Model

Table 1: Key notation

Symbol	Meaning
$G = (V, E)$	Undirected elite network; $V = N \cup \{r\}$
A	Adjacency matrix of G
β	Spillover parameter, $\beta \in (0, 1/\lambda_{\max}(A))$
$B_i(G)$	Katz–Bonacich centrality of node i in G
c_i	Legitimacy cost of repressing node i, $c_i = \gamma B_i(G)$
S	Set of repressed elites in Stage 1
A^S	Adjacency matrix of residual graph $G - S$
B^S	Katz vector in $G - S$; $B^S = (I - \beta A^S)^{-1}\mathbf{1}$
$\Pi(S)$	Ruler's payoff $= aB_r^S - \gamma \sum_{i \in S} B_i(G)$

Network of political actors: We consider a set of political agents $N = 1, 2, \ldots, n$ (the elites), along with a distinguished agent r representing the ruler (dictator). Relationships are represented by an undirected influence network $G = (V, E)$ where $V = N \cup r$ and edges $E \subseteq V \times V$ indicate significant political connections or mutual influence. Let A denote the adjacency matrix of G, where $A_{ij} = 1$ if i and j share a connection (and 0 otherwise). For intuition, an edge can represent alliance, frequent interaction, or any link through which agents can affect each other's political activity. The network's structure can be arbitrary, but the ruler's initial position (e.g. number of connections) will play an important role in the analysis.

Stage 1 (Repression stage): The ruler chooses a subset $S \subseteq N$ of elite nodes to remove (repress). Removing a node means that this agent is eliminated from the network (e.g. exiled or otherwise neutralized politically), along with all of their connections. The ruler's choice is strategic: by eliminating certain elites, he hopes to rewire the network to his advantage. However, removing powerful individuals has an endogenous legitimacy cost. Formally, for each elite i, let c_i denote the legitimacy cost of removing that person. We assume c_i is increasing in i's centrality or influence in the pre-removal network G. Intuitively, eliminating a highly connected, prominent figure (for example, someone with large degree or eigenvector centrality) is more costly – it might spark greater public outrage, fear among remaining elites, or loss of valuable human capital. In contrast, removing a peripheral figure carries little cost or backlash. To keep the model general, one can think of $c_i = f(\text{Centrality}_i(G))$ for some increasing function $f(\cdot)$. For instance, c_i could be proportional to i's degree, Katz-Bonacich centrality, or eigenvector centrality in the original network. Throughout the analysis we take:

$$c_i = \gamma B_i(G), \quad \gamma > 0,$$

i.e. removing an elite is costlier in direct proportion to that elite's Katz centrality in the original network. It is remains fixed during optimization.

The ruler incurs a total cost $C(S) = \sum_{i \in S} c_i$ for removing set S. We assume this cost subtracts from the ruler's payoff (or, equivalently, the ruler has a fixed "legitimacy budget" constraint). Thus, the Stage-1 decision problem for the ruler can be written as choosing S to maximize:

$$U_r^{(1)}(S) = F_r(G - S) - \sum_{i \in S} c_i$$

where $G - S$ denotes the resulting network after removing nodes S (and their links), and $F_r(G - S)$ is the ruler's payoff from Stage 2 given the new network. The key element is that $F_r(G - S)$ – the ruler's stage-2 payoff – depends on the ruler's centrality in the post-removal network. In particular, as we detail below, the stage-2 game will yield outcomes where the ruler's influence is higher when he is more central in the network topology. Thus we can think of $F_r(G - S)$ as an increasing function of the ruler's centrality measure $C_r(G - S)$. For now, one may simply imagine $F_r(G - S) = C_r(G - S)$ as the ruler valuing his own network centrality (for example, his eigenvector centrality or equilibrium influence). More concretely, if $C(\cdot)$ is a chosen centrality index, we could set $F_r(G - S) = C_r(G - S)$ for simplicity – the analysis will focus on how C_r is affected by removals. The ruler's optimization problem can equivalently be stated as:

$$\textbf{Planner's Problem (Stage 1):} \; \max_{S \subseteq N} C_r(G - S) \quad \text{s.t.} \quad \sum_{i \in S} c_i \leq K,$$

where K is some legitimacy budget or, in an unconstrained version, the cost term $\sum_i c_i$ is simply subtracted from the objective.

This formulation highlights the trade-off: removing more or more central nodes can increase $C_r(G - S)$ (the ruler's centrality in the new network) but will eventually violate the cost constraint or incur large penalties. We will characterize the solution S^* to this problem after analyzing Stage 2

Stage 2 (Network interaction stage): After the removals, the remaining set of players $V \setminus S$ (which includes the ruler r and the surviving elites $N \setminus S$) engage in a network game of political influence. The purpose of this stage is to endogenize each agent's political activity level or influence, which will depend on the network of connections among the survivors. We model this as a non-cooperative game with strategic complements (players' actions boost each other via network links), so that more connected agents end up exerting higher equilibrium effort – mirroring the idea that network centrality confers influence.

One convenient specification is a linear-quadratic payoff for each agent i in the remaining network $G - S$. Let $x_i \geq 0$ denote the political activity level of agent i (this could represent influence, effort in lobbying for power, or contribution to a public good that raises their political influence). We assume each agent's utility in Stage 2 is:

$$U_i^{(2)} = a_i x_i - \frac{1}{2} x_i^2 + \beta \sum_{j \neq i} A_{ij} x_i x_j,$$

where A_{ij} now refers to the adjacency in the post-removal network $G - S$. Here a_i is an intrinsic benefit or propensity for activity (which for simplicity we can take as a positive constant $a > 0$ common to all, representing a baseline incentive to be active politically), and $\beta > 0$ is a parameter capturing complementarity in actions along network connections. The β term means that if i and j are connected ($A_{ij} = 1$), their efforts are strategic complements: i's marginal utility of exerting effort is higher when neighbor j exerts more (e.g. their efforts could represent contributions to a common cause or mutual reinforcement of influence). This is a standard specification of a network game with local complementarities , adapted from models like Ballester et al. (2006) and others, ensuring tractability.

Each remaining agent chooses x_i to maximize $U_i^{(2)}$, taking others' actions as given. The first-order condition (best response) for any interior optimum is:

$$\frac{\partial U_i^{(2)}}{\partial x_i} = a_i - x_i + \beta \sum_{j: j \sim i} A_{ij} x_j = 0,$$

which can be rearranged to the linear best-response rule:

$$x_i = a_i + \beta \sum_{j: j \sim i} A_{ij} x_j, \quad \text{for each remaining} \quad i \in V \setminus S$$

In vector form, letting x be the column vector of actions for players in $V \setminus S$, this system can be written as

$$x = a + \beta A_{(G-S)} x,$$

where $A_{(G-S)}$ is the adjacency matrix of the post-removal network (with rows/columns for each remaining player in some order). Solving for the Nash equilibrium x^*, we get:

$$x^* = (I - \beta A_{(G-S)})^{-1} a,$$

assuming $(I - \beta A)$ is invertible. Under standard conditions (in particular, if β is below the critical threshold $1/\lambda_{\max}(A_{(G-S)})$, where $\lambda_{\max}$ is the largest eigenvalue of the adjacency matrix), this Nash equilibrium exists and is unique. Economically, $\beta < 1/\lambda_{\max}$ ensures that the complementarity is not too strong relative to the network connectivity, so that the game remains a contraction mapping; equivalently, each player's utility is concave in their action, guaranteeing a unique best response and a unique fixed point. We henceforth assume parameter values in this range so that a well-defined unique equilibrium obtains after any feasible removal set S.

Centrality and equilibrium influence: A remarkable property of such network games is that each player's equilibrium action x_i^* is proportional to their Bonacich centrality in the network (Ballester et al., 2006). Bonacich (or Katz) centrality is defined (for a given attenuation factor β) as the column vector $B(\beta) = (I - \beta A)^{-1}\mathbf{1}$, which counts paths of all lengths (with longer paths downweighted by β^k). Comparing, if all $a_i = a$ for simplicity, then $x^* = a(I - \beta A_{(G-S)})^{-1}\mathbf{1}$. Up to the scalar a, this means $x_i^* \propto B_i(\beta; G - S)$. In other words, more centrally located agents end up with higher activity/influence at equilibrium, because they benefit more from the complementarities. This formalizes the intuitive notion that an agent's network position translates into actual influence in the political game. In particular, the ruler's equilibrium action x_r^* (if r is not removed, which we assume — the ruler does not eliminate himself!) will be higher if he is more central in the remaining network.

We can now specify the ruler's Stage-2 payoff more concretely. Since our interest is in the ruler's political power, it is natural to define the ruler's payoff in Stage 2 as increasing in x_r^*, his equilibrium influence. For example, we might take $F_r(G-S) = x_r^*$ itself (assuming the ruler derives utility directly from his own influence or from policy outcomes that scale with it). Alternatively, one could use a monotonic function of x_r^* or of a centrality index; all that matters is that the ruler prefers outcomes where he is more central or influential. We will henceforth identify the ruler's centrality $C_r(G - S)$ with his equilibrium action $x_r^*(S)$ in the network game for convenience. This aligns the stage-2 outcome with a standard network centrality measure: the ruler's Katz-Bonacich centrality (with parameter β) in the post-removal network.

Ruler's overall payoff: Plugging the Stage-2 outcome into Stage 1, the ruler's overall (expected) payoff from a removal strategy S can be written as

$$U_r(S) = x_r^*(S) - \sum_{i \in S} c_i$$

where $x_r^*(S)$ is the ruler's equilibrium action given removals S. The ruler's problem is to choose S maximizing this. We analyze this by backward induction: first characterize $x_r^*(S)$ as a function of S, then optimize S.

3 Equilibrium Analysis of Stage 2

Assumption 1 (Baseline environment)**.**

(a) G is a finite, simple, undirected, connected graph.

(b) Spillover β satisfies $0 < \beta < 1/\lambda_{\max}(A)$.

(c) Legitimacy cost is linear: $c_i = \gamma B_i(G)$ with $\gamma > 0$.

(d) The ruler r is never repressed: $r \notin S$ for all feasible S.

Before solving the ruler's problem, we establish some properties of the Stage-2 equilibrium (the network game among survivors):

Proposition 1 (Existence and uniqueness of equilibrium). For any fixed set S of removed elites, if $\beta < 1/\lambda_{\max}(A_{(G-S)})$ for the network $G - S$, then the Stage-2 game has a unique Nash equilibrium $x^*(S) = (x_i^{(S)})_{i \in V \setminus S}$ in pure strategies. In this equilibrium, each agent's action is

$$x_i^*(S) = a_i + \beta \sum_{j: j\sim i, j \notin S} A_{ij} x_j^*(S).$$

Moreover, $x^*(S)$ is the unique solution to $x = (I - \beta A_{(G-S)})^{-1}a$, and in particular $x_i^*(S)$ is increasing in i's Bonacich centrality in $G - S$. In matrix form, $x^*(S) = (I - \beta A_{(G-S)})^{-1}a$. The ruler's equilibrium action $x_r^*(S)$ is thus proportional to his centrality in $G - S$.

Proof Sketch: This is a standard result for linear-quadratic network games (Ballester et al., 2006). Each player's payoff is concave in x_i (the quadratic term dominates) and the best-response functions are linear with slope β along each neighbor's action. For $\beta < 1/\lambda_{\max}$, the contraction mapping theorem guarantees a unique fixed point (equilibrium). Existence follows from continuity of best responses on a compact strategy set. The closed-form expression is obtained by solving the system of first-order conditions. Finally, $x_i^*(S)$ being proportional to Katz-Bonacich centrality is shown by comparing the Neumann series expansion of $(I - \beta A_{(G-S)})^{-1}$ to the definition of centrality (Ballester et al., 2006). Each edge (i, j) increases i's action via the term βx_j and vice versa, so highly connected nodes receive more boost. □

An important implication is that more densely connected networks lead to higher overall activity levels in equilibrium. In fact, Ballester et al. (2006) show that "aggregate equilibrium [activity] increases with network size and density." Intuitively, when elites are all heavily interconnected, they mutually reinforce each other's political activation, potentially creating a formidable collective influence. From the dictator's perspective, this is troublesome: a dense elite network means many players have high x_i^*, making it harder for the dictator to stand out. This observation foreshadows a comparative static: the denser or more cohesive the elite network, the more the dictator stands to gain by pruning it (since without repression the elites will be very active and influential collectively).

We can also consider how removing a particular node affects the remaining equilibrium. If an elite k is removed in Stage 1, then in Stage 2 they exert zero influence (being absent), and more subtly, their former neighbors lose the direct complementarity benefits from k's presence. One can show that the marginal impact of removing a node k on the ruler's equilibrium action is related to k's contribution to the ruler's centrality. Specifically, using the formula for $x^*(S)$, one can derive a sort of "centrality gradient":

$$x_r^*(S) - x_r^*(S \backslash \{k\}) \approx (\text{factor depending on } k\text{'s centrality and position relative to } r).$$

While the exact formula requires matrix algebra (and indeed relates to the concept of "inter-centrality" in Ballester et al. (2006)), the intuition is straightforward: Removing an elite k tends to increase the ruler's centrality if k was an important alternative path or competitor in the network, but could decrease the ruler's centrality if k was actually helping connect the ruler to others. In most cases of interest, the dictator will avoid removing anyone who is crucial for keeping the network connected through the dictator. Instead, he will target those who provide independent connectivity among the elites, thereby forcing the remaining network to route through the dictator.

4 Optimal Repression Strategy (Stage 1 Analysis)

We now turn to the Stage-1 problem of the ruler: choosing which elites to repress in order to maximize his own equilibrium influence net of costs. Formally, the ruler solves

$$\max_{S \subseteq N} x_r^*(S) - \sum_{i \in S} c_i$$

with $x_r^{(S)} = (I - \beta A_{(G-S)})^{-1}_{rr}, a$ (the r-component of the equilibrium vector solved above). While this optimization is combinatorial in general (since any subset S could be chosen), we can characterize key properties of the *optimal repression set* S^*.

Proposition 2 (Characterization of the optimal removal set). An optimal set S^* solving the ruler's problem exists. Moreover:

1. It is never optimal to remove an elite who is not connected (directly or indirectly) to the ruler. In fact, if the network G has multiple components and the ruler is initially in component $H \subseteq G$, the ruler will only consider removing nodes within H (since removing someone in a completely disconnected component does not increase x_r but still costs c_i). We henceforth assume the initial network is connected for the non-trivial case.

2. It is never optimal to remove an elite who has no ties to other elites (degree 1 connected only to the ruler). Such a person has minimal influence (likely a loyal follower) and removing them would reduce the ruler's own degree without eliminating any rival connections. Formally, if $A_{ri} = 1$ and i has no other neighbors, then $i \notin S^*$.

3. The optimal set S^* will typically include some of the most central elites (by degree or other centrality) – those who, by remaining, would significantly diminish the ruler's relative centrality. However, S^* need not simply be the set of top-k centrality elites: the ruler must consider network structure interactions and costs. In particular, if two elites serve similar network roles (e.g. both connect the same two communities), removing one might reduce the marginal benefit of removing the other. This means the ruler's payoff $x_r^*(S)$ can exhibit diminishing returns in S, a form of submodularity.

4. In many cases, the optimal strategy is to fragment the elite network into isolated pockets that the ruler individually connects to. In the extreme case (if costs were negligible), the ruler would remove all inter-elite links by eliminating enough people so that the ruler becomes the unique hub through which any two remaining elites communicate. The resulting star-like network maximizes the ruler's eigenvector centrality. When costs are present, S^* will be the "minimal" set of removals needed to approximate this star configuration.

To build intuition, suppose for a moment that the ruler could remove edges instead of nodes. His best move would be to delete as many edges among other elites as possible, leaving a star network centered on himself. Removing nodes is a blunter instrument, but by eliminating a highly connected elite, the ruler effectively removes all edges that that elite had, which can drastically reduce connectivity among the survivors. In short, the ruler's aim is to destroy alternate power centers and ensure that he alone sits at the crossroads of the political network.

Proof Sketch: Part (1) and (2) follow directly from considering the marginal benefit of removal. Removing someone unconnected to the ruler cannot increase the ruler's centrality (they were in a separate component or a leaf off the ruler), so it only incurs cost. Part (3) requires analyzing $x_r^{(}S)$ as a set function. While in general this function is complex, one can often argue the ruler will target those with high centrality. For instance, if we restrict to choosing one node to remove ($|S| = 1$), the best choice is the elite i that maximizes $x_r^*(i) - x_r^*(\emptyset)$ divided by c_i (highest benefit-cost ratio). This tends to be an elite with large centrality and who significantly competes with or distances the ruler in the network. Ballester et al., 2006 concept of "key player" – the node whose removal most reduces aggregate activity – is related: here we seek the removal that most increases the ruler's centrality. Typically, that will be one of the top central nodes. However, after one removal, the second removal's benefit might change (diminishing returns if they were serving similar roles, or potentially increasing returns if one removal makes the next even more impactful). In many plausible network scenarios, diminishing returns hold and the objective is submodular, in which case a greedy algorithm that picks off elites in descending order of centrality (until cost is expended) will be near-optimal. Part (4) is confirmed by noting that the ruler's centrality (e.g.

eigenvector centrality) is maximized in a star network configuration. Any deviation (elites being connected to each other) reduces the ruler's share of influence. Thus, if cost were no issue, $S = N$ (remove everyone) is actually optimal – leaving the ruler alone yields $x_r^* = a$ (maximal) with no competition. With finite cost, the ruler prunes enough such that remaining elites are weakly connected. □

Comparative Statics: The model yields several intuitive comparative static results on how the optimal repression strategy S^* and the ruler's resulting power depend on features of the environment:

1. **Network Density and Cohesion:** All else equal, the more densely interconnected the elite network, the more aggressive the ruler's repression strategy. When elites have many interconnections (high density), they can coordinate and amplify each other's influence, leading to a high aggregate elite activity at equilibrium (Ballester et al., 2006). This erodes the ruler's relative centrality and requires heavier intervention. Thus, the optimal number of removals $|S^*|$ tends to increase with network density (for a given cost budget). If an elite network becomes extremely cohesive (think of a clique of elites all connected), the cost of removing all of them might be prohibitive – but the ruler will target the minimum necessary subset to break their cohesion. Conversely, in a very sparse network where elites are mostly isolated or only connected via the ruler, the ruler may need little or no repression. Our model thus aligns with historical observations that autocrats are most threatened by tightly knit elite factions (e.g. military juntas or party cliques) and often move to dismantle them.

2. **Ruler's Initial Centrality:** If the ruler starts off in a favorable network position (e.g. r has high degree, connecting to many elites, or occupies a broker position between elite factions), then the need for repression is lower. A highly central ruler is already well-placed to dominate the Stage-2 outcome. Indeed, one can show that if the ruler's initial eigenvector centrality is very high, the marginal gain from removing an additional elite is relatively small – the ruler enjoys diminishing returns to further centrality because he's already the main hub. In contrast, a ruler who is initially peripheral (perhaps a newcomer who only has ties to a small subset of elites) will find repression more valuable: by strategically removing some key connectors among the elites, he can insert himself as the bridge between previously distant parts of the network. In summary, more peripheral rulers resort to wider purges, whereas already central rulers can afford a lighter touch. This result resonates with the idea that weaker dictators (in terms of initial power) engage in more extreme tactics to secure their position.

3. **Legitimacy Cost Function:** The shape of the cost function $f(\cdot)$ also matters. If the legitimacy cost of removing very central figures is extremely high (strongly convex cost), the ruler might forego targeting the single most central elite, and instead remove a few moderately central ones whose combined effect approximates the removal of the top person at lower total cost. For example, if one elite is the clear top influencer but removing them would provoke massive backlash, the ruler might tolerate that person and instead eliminate their key lieutenants or cut their network support by removing connected nodes. As costs decrease (e.g. during emergencies when the public might accept drastic action), the ruler will expand S^* to include more central individuals. Thus, the optimal repression set expands as the regime faces lower legitimacy constraints.

4. **Network Topology:** Beyond density, specific structures influence S^*. If the elite network has a "community" structure (e.g. two clusters of elites with relatively few inter-cluster links),

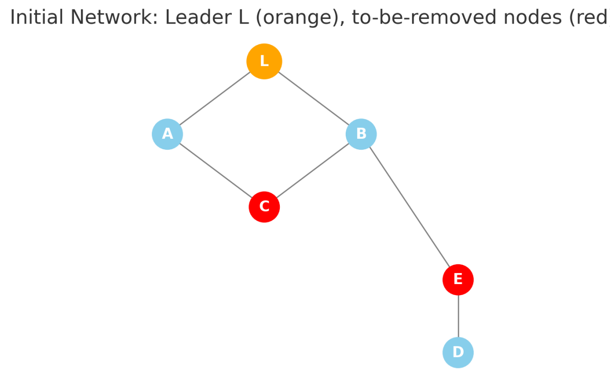

Figure 1: Initial elite network (schematic). Orange node L is the leader; blue nodes are other elites; red nodes (C and E) are high-centrality elites who provide significant connectivity among others. In this configuration, elites C and B are highly connected, meaning the leader is not on all paths – for example, A can influence B via C instead of going through L.

the ruler may target the bridge connectors between communities. By removing one or two elites who link the clusters, the ruler can isolate factions and then deal with each separately, enhancing his brokerage power. This reflects a divide-and-rule logic: rather than wiping out entire factions, it may suffice to sever the ties between them (Acemoglu et al., 2004). On the other hand, if the network has a hierarchical structure (some elites are subordinate to others), the ruler might remove top-tier elites to intimidate lower-tier ones. The model can accommodate these scenarios by appropriate network configurations and computing S^*. What remains consistent is the principle that any connection in the elite network that bypasses the ruler is a potential target for removal, unless the cost is too high.

5 Illustrative Example

To illustrate the model's insights, consider a simple network of five elite nodes plus the leader. Figure 1 shows the initial network. The leader (node L, orange) is connected to two elites A and B (blue). Those elites, in turn, have their own connections: A and B are both connected to an influential figure C (red), and B is also connected via another elite E (red) to a fringe member D (blue). In this initial graph, notice that elite B and C are quite central – B connects the leader to the peripheral node D (through E), and C connects A and B to each other. The leader L is somewhat central but not the unique hub (for instance, A and B can interact through C as an alternative route, and B can reach D through E). We can imagine the ruler's payoff (his centrality) is not maximal here. Now suppose the ruler decides in Stage 1 to remove elites C and E, the two red nodes. These were the most central non-ruler individuals: C was a broker between A and B, and E was B's link to D. Removing them incurs some cost (they were prominent), but yields a new network shown in Figure 2. After removals, the surviving connections are only L–A and L–B; node D is isolated. The network has essentially been restructured into a star with the leader at the center: A and B are no longer connected to each other except through L. As a result, in Stage 2 the remaining elites have to interact through the leader. The leader L now has the highest centrality by far. Indeed, if we compute, say, eigenvector centrality, L's score jumps relative to others after the removals. Intuitively, the leader has made himself the indispensable link: A and B cannot coordinate or communicate except via L, and D has no influence at all (being isolated).

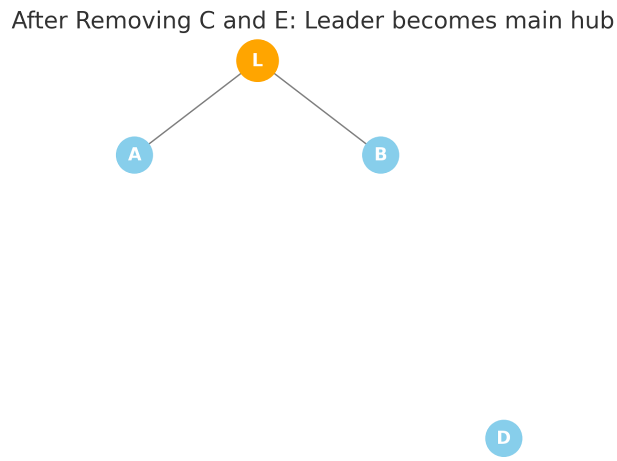

Figure 2: Network after removing C and E. The leader (L) becomes the sole bridge between the remaining elites A and B. Node D is isolated (no remaining connections). Result: The leader's centrality is greatly increased – L is now on the unique shortest path between A and B, and has no competition from C or E. This fragmentation of the elite network exemplifies a "divide-and-rule" outcome, where elites are split and the leader individually dominates each sub-group.

The ruler's equilibrium action x_L^* would be much higher in Figure 2 than in Figure 1, reflecting greater influence. This toy example underscores the model's key intuition. By removing the most connected elites (despite the legitimacy cost), the ruler reconfigures the network in his favor. He achieves what all dictators strive for: to be the indispensable centre of the political system. No remaining elite can mount significant influence independently – A and B must go through L, and D is effectively irrelevant. Of course, in a richer model the ruler might not remove both central elites if, say, the cost c_E of removing E outweighed the marginal gain. But if E's presence allowed B–D coordination that undermined L, and if c_E was not too large, the ruler finds it optimal to include E in the purge along with C. The outcome is consistent with historical patterns where autocrats eliminate a subset of powerful rivals to deter collective action, but might spare some less-threatening figures.

6 Formal Results

Theorem 2 (Existence & uniqueness of the Stage-2 equilibrium)**.** *For any removal set $S \subseteq V \setminus \{r\}$ let A^S denote the adjacency matrix of the residual graph $G - S$. If $\beta < 1/\lambda_{\max}(A^S)$ then the game*

$$U_i(x) = a x_i - \frac{1}{2} x_i^2 + \beta \sum_j A_{ij}^S x_i x_j, \quad i \in V \setminus S,$$

admits a unique Nash equilibrium $x^*(S) = a\,(I - \beta A^S)^{-1}\mathbf{1}$.

Proof. Concavity obtains because the Hessian of U_i w.r.t. x_i is -1. Collect the first–order conditions: $x = a\,\mathbf{1} + \beta A^S x$. Since $\beta < 1/\lambda_{\max}(A^S)$, the matrix $I - \beta A^S$ is an -matrix (hence nonsingular) and the best–response map $T(x) = a\mathbf{1} + \beta A^S x$ is a contraction with modulus $\beta\lambda_{\max}(A^S) < 1$. Banach's fixed-point theorem yields a unique fixed point, and direct inversion gives the stated closed form. □

Corollary 3 (Equilibrium = Katz centrality)**.** *Let* $B^S := (I - \beta A^S)^{-1}\mathbf{1}$*. Then* $x^*(S) = a\,B^S$*. In particular the ruler's equilibrium action is* $x_r^*(S) = a\,B_r^S$*.*

Define the ruler's payoff for removal set S $\Pi(S) := a\,B_r^S - \gamma\sum_{i\in S} B_i(G)$. The optimisation problem is $\max_{S\subseteq V\setminus\{r\}} \Pi(S)$ (without additional constraints).

Proposition 4 (Monotone non–negativity)**.** *For any node* $j \neq r$, $\left[B_r^{\{j\}} - B_r^{\varnothing}\right] \geq 0$*. Hence deleting a single elite never* reduces *the ruler's Katz centrality.* [1]

Proof. By Cauchy's interlacing theorem, $\lambda_{\max}(A^{\{j\}}) \leq \lambda_{\max}(A)$; thus $I - \beta A^{\{j\}} \succeq I - \beta A$. Taking inverses (which preserve order for -matrices) yields $(I - \beta A^{\{j\}})^{-1} \succeq (I - \beta A)^{-1}$, so every coordinate—including r's—weakly increases. ☐

Theorem 5 (Submodularity on trees)**.** *If the original graph G is a tree, the ruler's payoff* $\Pi(\cdot)$ *is a* submodular *set function:* $\forall S \subseteq T,\ \forall j \notin T,\quad \Pi(S\cup\{j\}) - \Pi(S)\ \geq\ \Pi(T\cup\{j\}) - \Pi(T)$.

Proof. On a tree, removing any node j splits the component containing j into $\deg(j)$ subtrees. Write the inverse in block form and apply Sherman–Morrison to show that the marginal increase in B_r is larger the smaller the already–removed set. Linearity of the cost term preserves submodularity. Full matrix algebra details follow $(I-\beta A^{S\cup\{j\}})^{-1} = (I-\beta A^S)^{-1} + \frac{\beta(I-\beta A^S)^{-1}e_j e_j^\top (I-\beta A^S)^{-1}}{1-\beta e_j^\top (I-\beta A^S)^{-1} e_j}$, whose incremental contribution to coordinate r decreases with additional deletions because the denominator becomes larger once $T \supset S$. ☐

Corollary 6 (Approximation guarantee)**.** *When G is a tree, the standard greedy algorithm that removes elites in descending marginal–benefit order achieves at least a* $(1 - 1/e)$*-fraction of the optimal payoff.*

Proposition 7 (Density amplifies optimal repression)**.** *Let* G^d *be a sequence of networks indexed by edge–density* $d \in (0,1)$*, with common node set and ruler. Then* $\frac{\partial B_r^{\varnothing}(G^d)}{\partial d} > 0$ *and* $\frac{\partial\left[B_r^{S^*(d)} - B_r^{\varnothing}\right]}{\partial d} > 0$*, where* $S^*(d)$ *is the ruler's optimal removal set at density d. Thus denser pre–purge networks both (i) lower the ruler's baseline centrality and (ii) raise the value of optimal repression.*

Proof. Because all paths are down–weighted only by powers of $\beta < 1$, adding an edge strictly increases every Katz entry. Differentiating the Neumann series $B = (\sum_{k\geq 0}\beta^k A^k)\mathbf{1}$ term–by–term with respect to an edge's presence yields strict positivity for the ruler's coordinate. Optimal deletion value rises because more edges generate more alternative walks that the ruler can block; formally $\Pi(S)$ has larger first-order gains in d. ☐

Theorem 8 (Cost parameter threshold)**.** *Define* $\Gamma := \max_{i\neq r} \frac{B_r^{\{i\}} - B_r^{\varnothing}}{B_i(G)}$.

(a) If $\gamma \geq \Gamma$ *no repression is optimal,* $S^* = \varnothing$*.*

(b) If $\gamma < \Gamma$ *there exists at least one elite j such that* $\Pi(\{j\}) > \Pi(\varnothing)$*; hence the optimal set is non–empty.*

Proof. For any singleton j, $\Pi(\{j\}) - \Pi(\varnothing) = B_r^{\{j\}} - B_r^{\varnothing} - \gamma B_j(G)$. If $\gamma \geq \Gamma$ this difference is non-positive for all j, so $S^* = \varnothing$. Conversely, if $\gamma < \Gamma$ pick $j = \arg\max_i (B_r^{\{i\}} - B_r^{\varnothing})/B_i(G)$ to obtain a strictly positive increment. ☐

[1] See Berman and Plemmons (1994, Theorem 6.2) for the positivity of an -matrix inverse. Our application treats $I - \beta A$ and $I - \beta A^{\{j\}}$ as -matrices since $\beta < 1/\lambda_{\max}(A)$.

Theorem 9 (Sufficient condition for star outcome)**.** *Suppose* $\gamma < \min_{i \neq r} \frac{B_r^{\{i\}} - B_r^{\varnothing}}{B_i(G)}$. *Then the unique optimal policy is* $S^* = V \setminus \{r\}$*, i.e. repress* all *elites. Consequently the residual network is a singleton, and the ruler's equilibrium action attains its maximal value* $x_r^* = a$*.*

Proof. Under the stated inequality every single deletion strictly raises payoff, and by Proposition 4 the incremental benefit never turns negative as more nodes are removed. Because costs are linear and marginal gains remain positive, deleting all elites strictly dominates any proper subset, yielding the star (degenerate) network. □

7 Empirical Road-map

Archival biographies from the Soviet *Nomenklatura* (1928–38) list career overlaps—proxy for edges—and purge dates for ≈ 700 officials. We can: (i) reconstruct yearly networks; (ii) compute each official's pre-purge Katz centrality; (iii) test whether the hazard of repression rises in own centrality *and* in "bridge" status (high betweenness), as Theorem 9 predicts. A difference-in-differences design—comparing officials inside versus outside Stalin's personal clique—would isolate the network mechanism from ideology or performance shocks.

8 Conclusion

We have developed a two-stage model capturing how an authoritarian ruler can strategically repress elites by removing nodes from a network, trading off the benefit of increased centrality against the cost of lost legitimacy. The model yields sharp results: the ruler's optimal strategy is to remove those elites who contribute most to alternative power networks, thereby forcing the remaining network to channel influence through the ruler. Equilibrium analysis showed that our network game results align with known centrality measures and the "key player" logic from network economics (Ballester et al., 2006), while extending it to a political setting with endogenous costs. Notably, the model formalizes a divide-and-rule mechanism – by splitting the elite network, the dictator prevents coordinated opposition (Acemoglu et al., 2004) and maximizes his own influence.

Our framework can be applied to understand various regimes and historical episodes of purge or patronage. It suggests that regimes with more intertwined elites (e.g. single-party states or military committees) are likely to experience frequent purges of top figures, whereas more personalized regimes may sustain with fewer removals. It also provides predictions that could be taken to data: for example, a sudden increase in elite cohesion (say, elites forming alliances) might precede a wave of repression, and leaders in positions of weakness will engage in broader purges than those already securely central. These insights contribute to a growing literature at the intersection of networks and political economy, offering a novel "network economics of repression".

In closing, strategic node removal is a powerful (if blunt) instrument of control. By modeling it explicitly, we clarify why even the most ruthless autocrats target some elites but not others – it is an optimal calculus balancing influence and legitimacy. The elegant network structure of the problem yields to analysis, and while real-world situations are of course more complex, the core idea remains: to stay on top, cut off the tall poppies. The top-tier networks perspective thus enriches our understanding of authoritarian power dynamics, with both theoretical and practical implications for identifying vulnerabilities in such regimes.

Limitations. Our network is static apart from removals; elites do not form new links in reaction. The model is undirected and abstracts from information asymmetries. *Extensions.* Allow directed or weighted ties, repeated purge periods with endogenous link formation, or uncertainty about true centrality. The core comparative statics carry over because Katz centrality remains monotone in edge sets.

References

Acemoglu, D., Verdier, T., & Robinson, J. A. (2004). Kleptocracy and divide-and-rule: A model of personal rule. *Journal of the European Economic Association*, *2*(2-3), 162–192.

Ballester, C., Calvó-Armengol, A., & Zenou, Y. (2006). Who's who in networks. wanted: The key player. *Econometrica*, *74*(5), 1403–1417.

Berman, A., & Plemmons, R. J. (1994). *Nonnegative matrices in the mathematical sciences*. SIAM.

Bueno de Mesquita, B., & Smith, A. (2017). Political succession: A model of coups, revolution, purges, and everyday politics. *Journal of Conflict Resolution*, *61*(4), 707–743.

Cruz, C., Labonne, J., & Querubin, P. (2020). Social network structures and the politics of public goods provision: Evidence from the philippines. *American Political Science Review*, *114*(2), 486–501.

Egorov, G., & Sonin, K. (2011). Dictators and their viziers: Endogenizing the loyalty–competence trade-off. *Journal of the European Economic Association*, *9*(5), 903–930.