

Dear colleagues,

In our bridge (discrete mathematics and intro to proofs course), I am introducing “flowcharts” as a form of organizing the rules of inference, as opposed to using the so-called syllogism form. The idea is that the flow of logic for a proof could be thought of as a big puzzle, putting together the right puzzle pieces. Not only can this help the student organize the narrative form of a proof, I believe this can be extremely helpful in helping the student who is new to proofs see if their argument really works.

I just gave students the handout titled “Some of the main flowchart forms”. I am proposing a follow-up handout when we get to set membership, tentatively titled “Flowchart forms for using/proving  $x \in S$ ”.

I think we often think of there being two fundamentally different forms of sets (a comma-separated roster, or set builder notation). I would contend that there are *three* forms:

1. A “roster method”, such as  $\mathbb{Z}_{>0} = \{1, 2, 3, 4, \dots\}$
2. A set-builder form, where the conditions both before *and* after the colon must be satisfied. Generically, I wrote this as  $S = \{z \in T : P(z)\}$ . For a specific example, consider  $Z(G) = \{g \in G : \forall h \in G, gh = hg\}$ . I am using  $P(z)$  to generically represent the condition  $\forall h \in G, gh = hg$ .
3. A “gather using a running set” form of set definition, where the roles of the text before and after the colon are *very different*. Now, for each instance of the condition after the colon being satisfied, the expression before the colon is an element in the set. As an example, consider for a function  $f : A \rightarrow B$ , the set  $\text{range}(f) = \{f(x) : x \in A\}$ .

Note that when we compare the last two sets

$$Z(G) = \{g \in G : \forall h \in G, gh = hg\}$$

and

$$\text{range}(f) = \{f(x) : x \in A\},$$

the  $g \in G$  and  $f(x)$  are both in front of the colon, but play *very* different roles. Similarly, the texts  $\forall h \in G, gh = hg$  and  $x \in A$  which both appear *after* the colons play *very* different functional roles in defining the set. I call the third type of set a “gather using a running set” method because I think of it being “as I run through each  $x \in A$ , where  $A$  is my ‘running set’, I pick up an element called  $f(x)$  and throw it into my set.”