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STRATEGIC INVENTORY UNDER SUPPLIERS COMPETITION

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Abstract. This paper investigates the impact of competition and the strategic inventories on the performance of a supply chain comprising two competing suppliers and one retailer. Existing literature has shown that the retailer's optimal strategy in equilibrium is to carry inventories, and the suppliers are unable to prevent this. In contrast, our results show that the suppliers will prevent the retailer from carrying strategic inventories when the degree of competition between suppliers is high, and the retailer's carrying strategic inventory is not necessary to force suppliers to lower the future wholesale price. We also find the substitutable relationship between the effect of strategic inventories and the effect of competition. When the degree of competition increases, the suppliers are worse off but the retailer and the total supply chain are both better off when carrying strategic inventories. The retailer could introduce profit sharing contracts so as to encourage suppliers to support strategic inventories which enhance the entire performance of the supply chain.

1. **Introduction.** Non-cooperative game in the supply chain is known as the major source of inefficiencies. It is the behavior of self-interested firms (modeled using non-cooperative game theory) that results in these inefficiencies, leading to individually optimal decisions that harm the performance of the overall supply chain. The last decade has witnessed active research in supply chain coordination with incentive contracts to achieve first-best profits. A recently emerging strand of research on the effects of non-cooperative optimization in the supply chain is concerned with the effect of multi-period interaction. One of the surprising findings in this literature is that the strategic interaction across periods can be advantageous to the performance of the whole supply chain. To elaborate, Anand et al. [1] reveal that inefficiency is reduced in a multi-period supply chain where retailers carry strategic inventories. Under the multi-period setting, the retailer adopts strategic inventories, which decreases the supplier's monopoly power and weakens the effect of double marginalization, so as to increase its profit. At the same time, the retailer's carrying strategic inventory also benefits the supplier since strategic inventories relax the effective space of vertical contracts and contract-expansion effect works in the supplier's favor.

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Although the above literature provides valuable insights into the strategic inventory problem, they neglect the competition effect between suppliers. It is common for a retailer to sell products from competing suppliers, and the competition effect also reduces the supplier's wholesale price like the effect of strategic inventories. But how these benefits interact with competitive dynamics in a two-period supply chain is not clear. Whether the retailer still holds strategic inventories in a competitive market? What is the relationship between competition effects and strategic inventories effects and under which conditions do suppliers benefit from strategic inventories? How is the performance of the supply chain in the presence of strategic inventories under competitive market? Furthermore, how does the degree of competition among suppliers to strategic inventory levels? These questions motivate this paper.

To address the above questions, we develop a model of a two-period supply chain consisting of two competing suppliers and a retailer. More specifically, we complement the strategic inventory model of Anand et al. [1] by considering competition among suppliers. In the first period, the retailer initially purchases products from suppliers for sale in the retail market, and unsold units of the initially purchased products are carried as inventory to the next period. To weaken the suppliers' monopoly power in the second period, the retailer may strategically purchase excess units in the initial period to lower the wholesale price in the second period. Anticipating the retailer's strategic behavior, the suppliers would set a high wholesale price in the first period to prevent carrying inventories.

We find that the retailer still holds inventories when the unit holding cost is small; the suppliers do not prevent the retailer from carrying strategic inventories when the degree of competition among suppliers is low, and both suppliers and the retailer benefit from that. However, it is not the suppliers who prevent the retailer from using strategic inventories. It is the competition between suppliers that drives this result when the degree of competition is high. Furthermore, the retailer carrying strategic inventories is not necessary to force the suppliers to lower the wholesale price in the second period. These results are in sharp contrast to the results of Anand et al. [1] The key difference in our model that leads to this contrast is that we consider a competitive environment.

2. **Literature review.** Our study is related to the literature on strategic inventories in a multi-period supply chain. Anand et al. [1] are the first to identify the strategic role of inventories in a multi-period supply chain consisting of a single supplier and a single buyer with deterministic demand. Their outcome shows that carrying strategic inventory leads to lowering the wholesale price in the second period and is the optimal strategy for both the buyer and the supplier. Based on the model of Anand et al. [1], Hartwing et al. [8] present the first empirical study to investigate the effect of strategic inventories on supply chain performance. Their surprising insight is that strategic inventories empower buyers, that is, transfer the fair profit share in their favor. The strategic inventory effect leads to better supply chain performance that is even more enhanced than theoretically predicted.

Following Anand et al. [1], Arya et al. [3] demonstrate a manufacturer can employ direct-to-consumers rebates to stave off such exploitation when the retailer exploits excess inventory as a strategic tool to secure lower wholesale prices in a two-period model. They show that with rebates, the retailer is less aggressive in carrying inventories, and the manufacturer is less exploitative in setting wholesale prices. Furthermore, the manufacturer, retailer and consumers all benefit from the use of rebates. Then, Arya et al. [2] use a two-period strategic inventory model to examine the choice of centralization vs. decentralization in procurement. They find that ceding procurement choices to its divisions can help to undercut inventory levels. Mantin and Jiang. [9] take account of quality deterioration in the strategic inventory model. They find that the supplier is always better off and the presence of product deterioration leads to a lower wholesale price in the first period and a higher wholesale price in the second period. All of these papers do not consider the competition of among suppliers, though the competitive environment is universal in reality.

Some researchers have demonstrated that inventories may play a strategic role under horizontal competition. Saloner [11] considers a two-period duopoly model where firms produce and sell in the first period but may salvage inventory to the second period. Although inventories are carried in equilibrium, the firm with the first-mover advantage at the production stage is unable to achieve the Stackelberg result. Rotemberg and Saloner [10] investigate a duopoly model in which duopolists hold inventories to sustain collusive profits by the threat of reverting to competitive behavior. Unlike these works, our focus is on the combined effect of competition effects and strategic inventory effects in a supply chain with two competing suppliers and a retailer. In our model, we demonstrate that the role of competition effects has the same function as strategic inventories effects which lower the monopoly power of suppliers, and the relationship is substitutable between the competition effects and the strategic inventories effects.

There is an extensive literature on incentive contracts and vertical channel coordination. Cachon et al. [4] provide an elaborate review. Especially, profit sharing is widely used in vertical contracts, such as Fitzroy and Keaft [6] indicate that a profit sharing contract could motivate cooperation to increase productivity. Faros et al. [7] find that an upstream firm could prevent destructive competition between downstream firms by using a price-dependent profit sharing contract. Following this literature, we also use profit sharing as an incentive contract to maximize the channel profit, but we focus on the two-period models which take strategic inventories into account.

Our work is also related to the literature on channel structure with multiple suppliers and a single retailer. Such as, Choi [5] considers that a monopoly common retailer who sells two competing products. Their focus is on the effects of the retailer's ability to coordinate retail prices of multiple products for his profit maximization. In addition, the retailer in this scenario can be powerful players that can assume leadership positions against the suppliers. Shang et al. [12] study the problem of information sharing in a supply chain with two competing manufacturers and a common retailer. However, unlike our model, the suppliers are the leader, and we take the strategic inventory into account in a dynamic two-period model.

Our work contributes to the literature in three main aspects. Firstly, our work is the first to show the substitutable relationship between competition effects and strategic inventories effects under competing suppliers. Secondly, we further study the impact of competition on the strategic inventory levels and the retailer's strategies. Thirdly, we further provide conditions under which the suppliers prevent the retailer from carrying strategic inventories and the profit sharing contract could be used to improve the entire supply chain performance.

The remainder of this paper is organized as follows. We present the model in Section 2 and provide the results and analysis in Section 3. We compare the dynamic contract to the commitment contract in Section4. Conclusion and further research are provided in Section 5 and all proofs are relegated to the Appendix.

3. **The model.** We extend the model in Anand et al. [1] by adding a competing supplier and consider a dynamic two-period model with full information and no uncertainty which allows us to focus on the interactions between the retailer and suppliers via strategic inventories. In each of two periods, two competing suppliers produce and sell substitutable products to a single retailer who resells the products to consumers. The retailer may carry inventories in the first period and sells the inventories in the second period. Following Rotemberg and Saloner [10], the inverse demand function for the product in period $t\,(t=1,2)$ is $p_t^i(q)=a-q_t^i-rq_t^{3-i}$, $p_t^i(q)$ and q_t^i are the price and sales quantity of product $i(i=1,2)$ in period t respectively and $r\in(0,1)$ is the degree of substitute among products (i.e. the competition intensity of among suppliers). The marginal cost to produce is normalized to zero.

As the benchmark to analyze the equilibrium outcome in the dynamic contract, this paper firstly analyzes the commitment contract. Under the commitment contract, the suppliers commit to a price sequence at the beginning of the horizon. That is, the supplier $i(i=1,2)$ sets a wholesale price w_1^i and w_2^i at the beginning of the first period, and the retailer sells quantity q^i at a price given by $p^i(q)=a-q^i-rq^{3-i}$. Therefore, the market is cleared and no inventory is carried by the retailer at the end of the second period. With the retailer and two suppliers all maximizing their individual profits, the unique equilibrium result of product i is the wholesale price $w^i=\frac{a(1-r)}{2-r}$, the sales quantity $q^i=\frac{a}{2(2-r)(1+r)}$, and retail price $p^i=\frac{a}{2(2-r)}$. Each supplier's profit is $\pi_m^i=\frac{a^2(1-r)}{(r+1)(2-r)^2}$, the retailer's profit is $\pi_b=\frac{a^2}{(r+1)(2-r)^2}$, and the profit of the supply chain is $\Pi=\frac{a^2(3-2r)}{(r+1)(2-r)^2}$. Since the optimal system profit in the centralized model is $\Pi^f=\frac{a^2}{1+r}$, where the superscript f denotes the first-best solution, the loss of double marginalization is $\frac{a^2(1-r)^2}{(r+1)(2-r)^2}$.

Under the dynamic model, at the start of each period, the supplier $i(i=1,2)$ sets the wholesale price w_t^i, and the retailer determines the order quantity Q_t^i and sells q_t^i units in the market. At the end of period 1, the retailer carries excess purchases as inventory $I^i{=}Q_1^i-q_1^i$ and in period 2, both inventories and purchase quantity will be sold. Thus the sales quantity in period 2 is $q_2^i=Q_2^i+I^i$ and we assume the holding cost of each unit of inventories is $h(h\geq 0)$. Superscript d denotes the situation of the dynamic model.

3.1. **The dynamic (two-period) model.** In period 2, with backward deduction, the retailer determines the purchase quantity considering not only the wholesale price w_2^i but also the inventory which it has carried in period 1 I^i. To be precise, the retailer's period 2 sells q_2^i of the product i, and the profit function of the retailer in period 2 is

$$\pi_{b2}^d=(a-q_2^1-rq_2^2)q_2^1-w_2^1Q_2^1+(a-q_2^2-rq_2^1)q_2^2-w_2^2Q_2^2 \tag{1}$$

The profit function of suppler i in period 2 is

$$\pi_{m2}^{di}=w_2^iQ_2^i \quad i=1,2 \tag{2}$$

According to the first-order condition, we can get that the retailer's optimal purchase quantities, profit and the wholesale price respectively are,

$$Q_2^i = \frac{a - 2I^i - ar + 2I^i r^2 - w_2^i + rw_2^{3-i}}{2(1-r^2)} \tag{3}$$

$$w_2^i = \frac{(1-r)(2a - 4I^i + ar - 4I^i r - 2I^{3-i} r - 2I^{3-i} r^2)}{4 - r^2} \tag{4}$$

$$\begin{aligned} \pi_{b2}^d = & \frac{a(a + 2(I^1 + I^2)(3 - r - 3r^2 + r^3))}{2(2-r)^2(1+r)} \\ & - \frac{(1-r^2)(I^1(12-r^2) + I^2(12-r^2) + 2I^1 I^2 r(8-r^2))}{(4-r^2)^2} \end{aligned} \tag{5}$$

Given the equilibrium outcome in period 2, the retailer chooses Q_1^i and I^i to maximize total profit over the two periods:

$$\pi_b^d = (a - q_1^1 - rq_1^2)q_1^1 - w_1^1 Q_1^1 + (a - q_1^2 - rq_1^1)q_1^2 - w_1^2 Q_1^2 - h(I^1 + I^2) + \pi_{b2}^d \tag{6}$$

And supplier i chooses w_1^i that maximizes this profit over the two periods

$$\pi_m^{di} = w_1^i Q_1^i + \pi_{m2}^{di} \tag{7}$$

Maximizing its profits with respect to purchase quantity q_1^i and inventory level I^i, and substituting the first order conditions of equation (6) into equation (7), we can get that the wholesale price of supplier i is

$$\mathrm{w}_1^i = \frac{(2-r)(18a + 30h - 18ar - 9hr - 2ar^2 - 3hr^2 + 2ar^3 + hr^3)}{102 - 81r + 9r^2 + 8r^3 - 2r^4} \tag{8}$$

Moreover, the equilibrium purchase quantity and strategic inventory level in period 1 are

$$\mathrm{Q}_1^i = \frac{2a(1-r)(39 - r(9+2r)) - h(2-r)(54 - r(33 + 6r - 4r^2))}{2(1-r^2)(102 - 81r + 9r^2 + 8r^3 - 2r^4)}, \tag{9}$$

$$I^i = \frac{a(30 - 39r + 8r^2 + r^3) - h(2-r)^2(30 - 9r - 3r^2 + r^3)}{2(1-r^2)(102 - 81r + 9r^2 + 8r^3 - 2r^4)}. \tag{10}$$

Substituting the equilibrium wholesale price, strategic inventory level and purchase quantity back into equation(6) and equation(7), we can get that the retailer's profit and each supplier's profit are

$$\pi_b^d = \frac{h^2(2-r)^2 A - 2ahB + a^2 C}{2(1-r^2)(102 - 81r + 9r^2 + 8r^3 - 2r^4)^2}, \tag{11}$$

$$\pi_m^{di} = \frac{h^2(2-r)^2 D - 4ahE + 4a^2 K}{2(1-r^2)(102 - 81r + 9r^2 + 8r^3 - 2r^4)^2}. \tag{12}$$

Notes

$$\begin{aligned} A &= 2736 - 2664r + 432r^2 + 372r^3 - 171r^4 + 9r^5 + 10r^6 - 2r^7 \\ B &= 2124 - 3384r + 1071r^2 + 339r^3 - 141r^4 - 9r^5 - 2r^6 + 2r^7 \\ C &= 5580 - 8604r + 2718r^2 + 702r^3 - 438r^4 + 26r^5 + 20r^6 - 4r^7 \\ D &= 1224 - 1224r + 162r^2 + 246r^3 - 78r^4 - 12r^5 + 5r^6 \\ E &= 612 - 1836r + 1791r^2 - 435r^3 - 261r^4 + 137r^5 - r^6 - 8r^7 + r^8 \\ K &= (3-r)^3(1-r)^2(51 + 17r - 3r^2 - r^3) \end{aligned}$$

And the purchase quantity and the wholesale price in period 2 are respectively

$$Q_2^i = \frac{(2-r)(2a(3-r)(1-r)(3+r)+h(30+r(-9+(-3+r)r)))}{2(1-r^2)(102-r(81+r(-9+2(-4+r)r)))} \tag{13}$$

$$w_2^i = \frac{(2-r)(2a(-3+r)(-1+r)(3+r)+h(30+r(-9+(-3+r)r)))}{102-r(81+r(-9+2(-4+r)r))} \tag{14}$$

Proposition 3.1. *The retailer carries strategic inventories when the unit holding cost is less than a threshold value* $h_0 = \frac{a(30-39r+8r^2+r^3)}{(2-r)^2(30-9r-3r^2+r^3)}$ *which is a function of the degree of competition. Moreover, the strategic inventory level is decreasing in r (i.e. a higher competition intensity leads to lowering the strategic inventory level).*

Proof. *of Proposition 3.1.*

Note: $z = h/a$, $I^i = \frac{a(30-39r+8r^2+r^3)-h(2-r)^2(30-9r-3r^2+r^3)}{2(1-r^2)(102-81r+9r^2+8r^3-2r^4)}$,

$$D_1 = 1548 - 4116r + 2199r^2 + 414r^3 - 479r^4 + 60r^5 + 6r^6$$

$$E_1 = 6192 - 33336r + 48780r^2 - 29058r^3 + 3594r^4 + 4266r^5 - 2031r^6 + 142r^7 + 109r^8 - 28r^9 + 2r^{10}$$

Because $0 < r < 1$, then $102 - 81r + 9r^2 + 8r^3 - 2r > 0, 30 - 9r - 3r^2 + r^3 > 0$. Let $h_0 = \frac{a(30-39r+8r^2+r^3)}{(2-r)^2(30-9r-3r^2+r^3)}$. When $h < h_0$, then $I^i > 0$. While, $\frac{\partial I^i}{\partial r} = \frac{-aD_1+hE_1}{2(1-r^2)^2(102-81r+9r^2+8r^3-2r^4)^2}$.

The $h < h_0$ and $0 < r < 1$, So $-aA + hB < 0$, $\frac{\partial I^i}{\partial r} < 0$ □

The proposition reveals that whether the retailer carries strategic inventories depending on the degree of competition among suppliers. In the absence of competition ($r = 0$), the necessary condition for the retailer to carry strategic inventories is $h < a/4$, which is the same as the assumption in Anand et al. [1]. But while there exists competition among suppliers, as illustrated in Fig. 1, a smaller holding cost is required to ensure the retailer carrying strategic inventories. To be specific, if $r > 0$, strategic inventories are not carried by the retailer when $h > h_0$ and h_0 is decreasing in r.

In addition, the strategic inventory also decreases with the degree of competition among suppliers, as illustrated in Fig. 2. In other words, when the degree of competition between suppliers is high, the retailer reduces the strategic inventory level. This conclusion is intuitive since the competition among suppliers lowers the wholesale price, and thus the retailer needs not carry high inventories which indirectly reduce the future wholesale price of suppliers. The strategic inventory effect is worth nothing that if the degree of competition between suppliers is sufficiently high ($r \to 1$), then $h_0 \to 0$ and $I^i \to 0$, which means that the retailer will not carry strategic inventories even though the holding cost is approaching zero. Not like the conclusion that the retailer always carries strategic inventories in Anand et al. [1], this proposition indicates the substitutable relationship between the competition effect and the strategic inventory effect since those two effects have a similar function on decreasing the wholesale price in the second period. Under severe competition among suppliers, the wholesale price is low enough, and the retailer would incur the profit loss from strategic inventories.

Anand et al. [1] show that the retailer carries strategic inventories to curtail the supplier's wholesale price in the second-period. Interestingly, the result ($w_1^i > w_2^i$) does not always keep in our model. Let $h_1 = \frac{2a(9-9r-r^2+r^3)}{72-72r+12r^2+7r^3-2r^4}$ and $m_t^i = p_t^i - w_t^i$

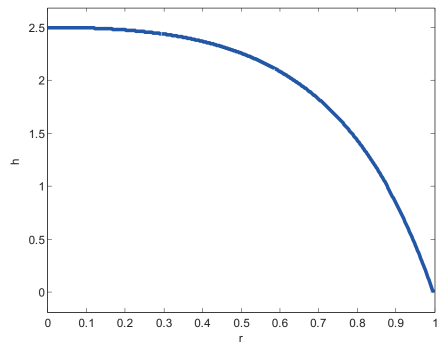

FIGURE 1. The relationship between r and h_0 when $a = 10$

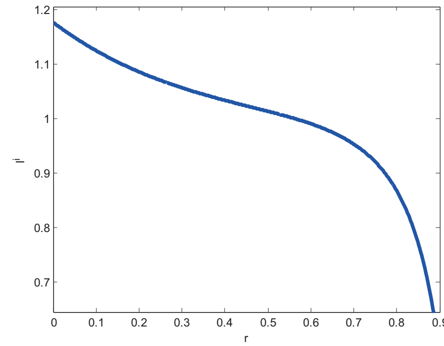

FIGURE 2. The relationship between r and I^i when $a = 10, h = 0.5$

represents the retailer's marginal revenue of product i in period t, so we have the following proposition:

Proposition 3.2. *When strategic inventories are carried,* $w_1^i > w_2^i$*,* $p_1^i > p_2^i$*,* $m_1^i < m_2^i$ *if and only if* $0 < h < h_1$ *and* $w_1^i < w_2^i$*,* $p_1^i < p_2^i$*,* $m_1^i > m_2^i$ *otherwise.*

Proof. of Proposition 3.2. We have following equations

$$p_1^i - p_2^i = \frac{2a(9-9r-r^2+r^3) - h(72-72r+12r^2+7r^3-2r^4)}{2(102-81r+9r^2+8r^3-2r^4)},$$

$$w_2^i - w_1^i = \frac{2a(9-9r-r^2+r^3) + h(-72+72r-12r^2-7r^3+2r^4)}{-102+81r-9r^2-8r^3+2r^4},$$

$$m_1^i - m_2^i = \frac{2a(9-9r-r^2+r^3) - h(72-72r+12r^2+7r^3-2r^4)}{-2(102-81r+9r^2+8r^3-2r^4)}$$

So, We can get

$$h_1 = \frac{2a(9-9r-r^2+r^3)}{72-72r+12r^2+7r^3-2r^4} < h_0 = \frac{a(30-39r+8r^2+r^3)}{30-9r-3r^2+r^3}$$

when $h < h_1$, then $w_1^i > w_2^i$, $p_1^i > p_2^i$ and $m_1^i < m_2^i$, otherwise, the opposition is true. □

When the unit holding cost is relatively low, it is intuitional that the retailer has the incentive to raise strategic inventory level, and in response, the suppliers raise the wholesale price to discourage the retailer from carrying inventories in the first period. However, the high wholesale price reduces the profit from sales, so in the second period, the suppliers set a lower wholesale price to obtain higher profits, and the retailer gains higher marginal revenue too. The above result is the same as that in Anand et al. [1] under the bilateral monopoly channel.

What is new in this paper is that when the unit holding cost is relatively high, although strategic inventories are still carried, the wholesale price in the first period is lower than that in the second period. It is because that in order to maximize the total profits, the suppliers have to balance the profits loss from strategic inventories and the profits gain from the product sales. Anticipating that the retailer would carry less strategic inventories since the high holding cost, the supplier set a lower wholesale price to gain more sales profit in the first period, which exceeds the profit loss from retailer's carrying strategic inventories. In addition, as the competition intensifies, the threshold h_1 becomes smaller. It results in the first period's wholesale price transforming from the high to the low and the second period's wholesale price transforming from the low to the high.

Table.1 The Equilibrium Outcome

	The Dynamic Contract	The Commitment Contract
(w_1^*, w_2^*)	$\left(\begin{array}{c}\frac{2a(3-r)^2(3-2r-r^2)-h(12-24r+9r^2+2r^3-r^4)}{102-81r+9r^2+8r^3-2r^4},\\ \frac{(2-r)(2a(3-r)(1-r)(3+r)+h(30-r(9+(3-r)r)))}{102-81r+9r^2+8r^3-2r^4}\end{array}\right)$	$\left(\frac{a(1-r)}{2-r}, \frac{a(1-r)}{2-r}\right)$
(p_1^*, p_2^*)	$\left(\begin{array}{c}\frac{a(156-153r+21r^2+16r^3-4r^4)-h(2-r)(6-9r+r^3)}{2(102-81r+9r^2+8r^3-2r^4)},\\ \frac{a(138-r(135-r(23+2r(7-2r))))+h(2-r)(30-r(9+(3-r)r))}{2(102-81r+9r^2+8r^3-2r^4)}\end{array}\right)$	$\left(\frac{a}{2(2-r)}, \frac{a}{2(2-r)}\right)$
I^*	$\frac{a(30-39r+8r^2+r^3)-h(2-r)^2(30-9r-3r^2+r^3)}{2(1-r^2)(102-81r+9r^2+8r^3-2r^4)}$	0
(Q_1^*, Q_2^*)	$\left(\begin{array}{c}\frac{2a(1-r)(39-r(9+2r))+h(2-r)(r(33+6r-4r^2)-54)}{2(1-r^2)(102-81r+9r^2+8r^3-2r^4)},\\ \frac{(2-r)(2a(3-r)(1-r)(3+r)+h(30-r(9+(3-r)r)))}{2(1-r^2)(102-81r+9r^2+8r^3-2r^4)}\end{array}\right)$	$\left(\begin{array}{c}\frac{a}{2(2-r)(1+r)},\\ \frac{a}{2(2-r)(1+r)}\end{array}\right)$
π_m^*	$\frac{h^2(2-r)^2D-4ahE+4a^2K}{2(1-r^2)(102-81r+9r^2+8r^3-2r^4)^2}$	$\frac{a^2(1-r)}{(r+1)(2-r)^2}$
π_b^*	$\frac{h^2(2-r)^2A-2ahB+a^2C}{2(1-r^2)(102-81r+9r^2+8r^3-2r^4)^2}$	$\frac{a^2(1-r)}{(r+1)(2-r)^2}$

Notes:

$$
\begin{aligned}
A &= 2736-2664r+432r^2+372r^3-171r^4+9r^5+10r^6-2r^7\\
B &= 2124-3384r+1071r^2+339r^3-141r^4-9r^5-2r^6+2r^7\\
C &= 5580-8604r+2718r^2+702r^3-438r^4+26r^5+20r^6-4r^7\\
D &= 1224-1224r+162r^2+246r^3-78r^4-12r^5+5r^6\\
E &= 612-1836r+1791r^2-435r^3-261r^4+137r^5-r^6-8r^7+r^8\\
K &= (3-r)^3(1-r)^2(51+17r-3r^2-r^3)
\end{aligned}
$$

4. **Comparison.** In this section, we carry out a comparison between the dynamic contract and the commitment contract so as to examine the impact of strategic inventories effects and competition effects on the retailer's strategy. The comparison is summarized in the following propositions. Let $h_2 = \frac{a(6-15r+6r^2+5r^3-2r^4)}{(2-r)^2(6-9r+r^3)}$.

Proposition 4.1. *When strategic inventories are carried, the average wholesale price and average retail price across two periods in the dynamic contract are less than those in the commitment contract. Moreover,* $w_2^i < w^i$, $w_1^i < w^i$ *if* $0 < r \leq 0.6$ *and* $h_2 < h < h_0$ *or* $0.6 < r < 1$ *and* $h < h_0$.

Proof. of Proposition 4.1. We have following equations

$$
w_{avrg}^i = \frac{w_1^i + w_2^i}{2} = \frac{-3h(16-8r-2r^2+r^3)+2a(-45+63r-13r^2-7r^3+2r^4)}{2(-102+81r-9r^2-8r^3+2r^4)},
$$

$$w^i_{avrg} - w^i = -\frac{2a(12-12r+r^2-2r^3+r^4)-3h(2-r)^2(8-r^2)}{2(2-r)(102-81r+9r^2+8r^3-2r^4)}$$

When $h < h_0$, then $2a(12-12r+r^2-2r^3+r^4) > 3h(2-r)^2(8-r^2)$.

Due to $0 < r < 1$, so $102-81r+9r^2+8r^3-2r^4 > 0$, then $w^i_{avrg} < w^i$. Similar the reason, we can prove $p^i_{avrg} < p^i$.

While

$$w^i_1 - w^i = \frac{a(6-15r+6r^2+5r^3-2r^4)-h(2-r)^2(6-9r+r^3)}{(2-r)(102-81r+9r^2+8r^3-2r^4)},$$
$$w^i_2 - w^i = \frac{h(2-r)^2(30-9r-3r^2+r^3)-a(30-39r+8r^2+r^3)}{(2-r)(102-81r+9r^2+8r^3-2r^4)}.$$

When $h < h_0$, then $w^i_2 < w^i$ is always true.

Let $6-15r+6r^2+5r^3-2r^4 = 0$, we get $r = 0.6$.

When $r < 0.6$, then $6-15r+6r^2+5r^3-2r^4 > 0$ and $6-9r+r^3 > 0$. However, if $w^i_1 = w^i$, then $h = h_2 = \frac{a(6-15r+6r^2+5r^3-2r^4)}{(2-r)^2(6-9r+r^3)} < h_0$. So, $w^i_1 < w^i$if $r < 0.6$ and $h_2 < h < h_0$.

When $0.6 < r < 1$, the $6-15r+6r^2+5r^3-2r^4 < 0$, If $0.6 < r \leq 0.7$, then $6-9r+r^3 \geq 0$. Therefore, we can prove $w^i_1 < w^i$. However, $6-9r+r^3 < 0$ when $0.7 < r < 1$. Under this conditions, We can get $h_2 = \frac{a(6-15r+6r^2+5r^3-2r^4)}{(2-r)^2(6-9r+r^3)} > h_0$, and $w^i_1 < w^i$ is true when $h < h_0$ and $0.7 < r < 1$. Therefore, when $0 < r < 0.6$ and $h_2 < h < h_0$ or $0.6 < r < 1$ and $h < h_0$, then $w^i_1 < w^i$. □

No matter the second period's wholesale price and retail price are less than the first period ($w^i_2 < w^i_1$, $p^i_2 < p^i_1$) when the unit holding cost is relatively low or the second period's wholesale price and retail price are greater than the first period ($w^i_2 > w^i_1$, $p^i_2 > p^i_1$) when the unit holding cost is relatively high under dynamic contracts, the average wholesale and average retail prices under the dynamic contract are less than those in the commitment contract. When the degree of competition is relatively high, the wholesale prices of two periods in the dynamic contract are always lower than in the commitment contract. However, when the degree of competition is relatively low, it depends on the unit holding cost.

This proposition implicates that carrying strategic inventory lowers the wholesale price and the retail price regardless of suppliers' competition. Even if the wholesale price and the retail price in the second period are higher than those in the first period, the proposition is still true. This is because the wholesale price and the retail price in the first period under dynamic contracts are lower than those in commitment contracts.

Proposition 4.2. *The retailer is strictly better off under the dynamic contract under the following two conditions:* (1) *when the degree of competition among suppliers is low* ($0 < r < 0.2845$) *and the unit holding cost is small* ($0 < h < h_3 = \frac{a(A-L\sqrt{36-108r-99r^2+126r^3-21r^4-8r^5+2r^6})}{(2-r)^2D}$)*;* (2) *the degree of supplier competition is relatively high* ($0.2845 < r < 1$)

Notes:

$$L = 102-183r+90r^2-r^3-10r^4+2r^5.$$

Proof. of Proposition 4.2. Notes $G_1 = 756+72r-99r^2-381r^3+213r^4-5r^5-14r^6+2r^7$.

We have

$$\pi_b^d - \pi_b = \frac{h^2A - 2ah(2-r)^2B + 2a^2(1-r)^2G_1}{2(2-r)^2(1-r^2)(102-81r+9r^2+8r^3-2r^4)^2},\ A>0.$$

Let $z^2A - 2z(2-r)^2B + 2(1-r)^2G_1 = 0$. When $0 < r < 0.2845$, then $\Delta > 0$, so

we get $z = \dfrac{(2124-3384r+1071r^2+339r^3-141r^4-9r^5-2r^6+2r^7) - (102-183r+90r^2-r^3-10r^4+2r^5)\sqrt{36-108r-99r^2+126r^3-21r^4-8r^5+2r^6}}{(2-r)^2(2736-2664r+432r^2+372r^3-171r^4+9r^5+10r^6-2r^7)}$

Let $h_1 = az$. When $0 < h < h_1$, then $\pi_b^d > \pi_b$. While $0.2845 < r < 1$, then $\Delta < 0$. Moreover $A > 0$, so $\pi_b^d > \pi_b$. □

The proposition shows that the competition effect dominates the strategic inventories effect when the degree of competition is relatively high. This is because when the competition among suppliers increases, the double marginalization effect is dampened and the wholesale price decreases, and thus the retailer has less incentive to carry strategic inventories. But when the degree of competition is low, the decreasing wholesale price could not promise that the retailer is better off; in response, the retailer would rather carry more strategic inventories under low unit holding cost. In this case, the retailer uses strategic inventories to undercut the future wholesale price when the effect of competition is weak.

The above proposition shows the conditions under which the retailer always adopts the strategic inventories. But whether suppliers benefit from the retailer's carrying strategic inventories? We answer this question in the following proposition.

Proposition 4.3. *Under the dynamic contract, the suppliers are better off if and only if* $0 < r < 0.44$ *and* $0 < h < h_4$ *where* $h_4 = \frac{a(2(2-r)G - rL\sqrt{144+60r-64r^2-8r^3+6r^4})}{(2-r)^2D}$.

Notes:

$$G = (306 - 765r + 513r^2 + 39r^3 - 111r^4 + 13r^5 + 6r^6 - r^7)$$

Proof. of Proposition 4.3. We have the following equation

$$\pi_m^{di} - \pi_m^i = \frac{h^2(2-r)^4D - 4ah(2-r)^3G - 2a^2(1-r)^2k_3}{2(2-r)^2(1-r^2)(102-81r+9r^2+8r^3-2r^4)^2}$$

Let $z^2(2-r)^4D - 4z(2-r)^3G - 2(1-r)^2k_3 = 0$. We can get

$$z = \frac{2(-2+r)(-306+765r-513r^2-39r^3+111r^4-13r^5-6r^6+r^7) - \sqrt{2}r(102-183r+90r^2-r^3-10r^4+2r^5)\sqrt{72+30r-32r^2-4r^3+3r^4}}{(2-r)^2(1224-1224r+162r^2+246r^3-78r^4-12r^5+5r^6)},$$

Denote $h_2 = za$. When $r < 0.44$, if $0 < h < h_2$, then $\pi_m^{di} > \pi_m^i$; otherwise, if $h_2 < h < h_0$, then $\pi_m^{di} < \pi_m^i$. When $r \geq 0.44$, then $h_2 < 0$, therefore, if $0 < h < h_0$, then $\pi_m^{di} < \pi_m$. □

Differing from the results in Anand et al.[1], which shows that the supplier is always better off under the dynamic contract when strategic inventories are carried, this proposition reveals that the suppliers would be better off if and only if the degree of competition among suppliers is weak and the unit holding cost is sufficiently small. This is because that under the dynamic contract, carrying strategic inventories not only increases the retailer's profit but also benefits the suppliers. However, when the competition of suppliers intensifies or the unit holding cost is high, the suppliers are worse off and prefer to implement the commitment contract. The intensified competition between suppliers leads to a lower wholesale price, reducing the suppliers' profit directly. According to proposition 4.2 and 4.3, we can get the following proposition.

Proposition 4.4. *When strategic inventories exist, both suppliers and the retailer are better off if (1)* $0 < r \leq 0.152$ *and* $0 < h < h_3$ *or (2)* $0.152 < r < 0.44$ *and* $0 < h < h_4$

Proof. of Proposition 4.4. According to the proposition 4.2 and 4.3, we can get: When $0 < r < 0.152$, then $h_1 < h_2$, and when $0.152 < r < 0.44$, then $h_1 > h_2$. Therefore, we get when $0 < r \leq 0.152$ and $h_1 > h$, or $0.152 < r < 0.44$ and $h_2 > h$, then $\pi_m^{di} > \pi_m^i$ and $\pi_b^d > \pi_b$; otherwise, when $r > 0.44$, then $\pi_{b1}^1 > \pi_b$ but $\pi_m^{di} < \pi_m^i$. $\square$

This proposition implication: both the suppliers and the retailer benefit from carrying strategic inventories when the degree of competition among suppliers is low and the unit holding cost is small. In this case, carrying strategic inventories can increase players' profit and channel performance. However, if the competition intensity is fierce, although the retailer can gain a lower wholesale price and higher profit, the suppliers will prevent the retailer from carrying strategic inventories because the suppliers are worse off since the surplus is transferred from the suppliers to the retailer. When would the suppliers support the retailer to carry strategic inventories? This is illustrated in Fig. 3. Whether the suppliers support the retailer to carry strategic inventories that strictly depends on the unit holding cost and the degree of competition among suppliers. Moreover, the upper bound of holding cost becomes smaller when the degree of competition among suppliers increases. When the degree of competition among suppliers is strong, the retailer and the whole supply chain could achieve Pareto achievement at the cost of suppliers' profit loss.

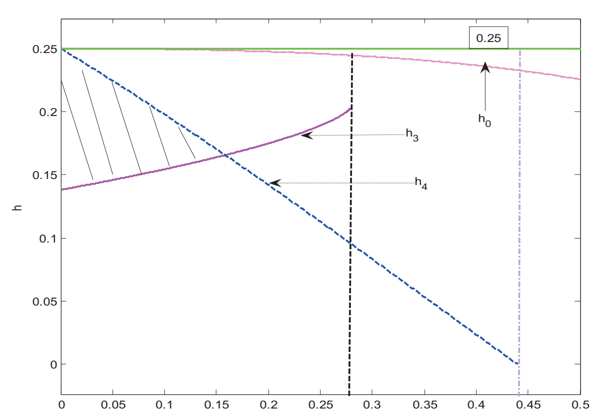

FIGURE 3. the relationship h and r with $a = 1$

According to proposition 4.4, all parties are better off in the presence of strategic inventories when the degree of supplier competition and the unit holding cost are low. So, how many profits of the overall supply chain are increasing? Whether the profits of the supply chain achieve the first-best solution in the presence of strategic inventories and horizontal competition without coordination contracts? The following proposition answers these questions.

Proposition 4.5. *When strategic inventories are carried, the overall performance of the supply chain is better off* ($\Pi^d > \Pi$) *for* $0 < h < h_5 = \frac{a(A_1 - L\sqrt{C_1})}{(2-r)^2 B_1}$ *and regardless of the degree of competition among suppliers, but first-best solution is never achieved. However, the supply chain profit is approaching the first-best profits with the competition intensifies.*

Notes:

$$A_1 = 4572 - 10728r + 8235r^2 - 1401r^3 - 1185r^4 + 539r^5 - 6r^6 - 30r^7 + 4r^8$$

$$B_1 = 5184 - 5112r + 756r^2 + 864r^3 - 327r^4 - 15r^5 + 20r^6 - 2r^7$$

$$C_1 = 36 + 180r - 99r^2 + 606r^3 - 321r^4 - 36r^5 + 38r^6 - 4r^7$$

Proof. of Proposition 4.5. Notes:

$$v_1 = 5184 - 5112r + 756r^2 + 864r^3 - 327r^4 - 15r^5 + 20r^6 - 2r^7,$$

$$v_2 = 1980 - 1620r + 387r^2 - 222r^3 + 145r^4 + 4r^5 - 22r^6 + 4r^7,$$

$$v_3 = 4572 - 10728r + 8235r^2 - 1401r^3 - 1185r^4 + 539r^5 - 6r^6 - 30r^7 + 4r^8.$$

So, we have the following equation

$$\Pi^d - \Pi^c = \frac{h^2(2-r)^4 v_1 - 2ah(2-r)^2 v_3 + 2a^2(1-r)^3 v_2}{2(2-r)^2(1-r^2)(102 - 81r + 9r^2 + 8r^3 - 2r^4)}.$$

Let $z^2(2-r)^4 v_1 - 2z(2-r)^2 v_3 + 2(1-r)^3 v_2 = 0$. Solve this equation, we obtain

$$z_1 = \frac{v_3 - L\sqrt{36 + 180r - 99r^2 + 606r^3 - 321r^4 - 36r^5 + 38r^6 - 4r^7}}{(2-r)^2 v_1}$$

$$z_2 = \frac{v_3 + L\sqrt{36 + 180r - 99r^2 + 606r^3 - 321r^4 - 36r^5 + 38r^6 - 4r^7}}{(2-r)^2 v_1}$$

let $h_3 = az_1$. When $0 < h < h_3$, $\Pi^d > \Pi^c$. Similarly we can get $\Pi^d < \Pi^f$. □

The above proposition is illustrated in Fig. 4. Notes $z = h/a$, $s = (\Pi^d - \Pi^f)/\Pi^f$, as the degree of competition among suppliers increases, the upper bound of unit holding cost is smaller. Therefore, under intensified competition between suppliers, it is difficult to carry strategic inventories for the retailer because the effect of competition gradually substitutes the effect of strategic inventories. The supply chain profits are approaching to first-best profits with the competitive intensity increasing, but it never achieves first-best profits.

5. **Profit sharing contract.**

According to the above proposition, the suppliers will prevent the retailer from carrying strategic inventories when the horizontal competition is fierce. However, the overall supply chain and the retailer benefit from carrying inventories under fierce competition. So, how to encourage the suppliers to agree with retailer's carrying strategic inventories? The introduction of the retailer's profit sharing can serve as an incentive for suppliers. The retailer sets the proportion of profit sharing. We assume that each supplier's sharing ratio is β and the retailer's sharing ratio is $1 - \beta$, the other conditions are keeping. Superscript P denotes the situation of profit sharing. Then the total profits function of the retailer and suppliers respectively are as follows:

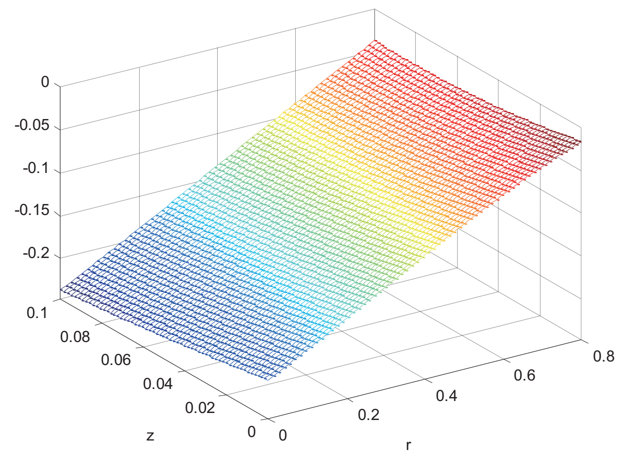

FIGURE 4. Improvement performance

$$\begin{aligned}\pi_b^P &= (1-\beta)[(a-q_1^1-rq_1^2)q_1^1 - w_1^1 Q_1^1 \\ &\quad + (a-q_1^2-rq_1^1)q_1^2 - w_1^2 Q_1^2 - h(I^1+I^2) + \pi_{b2}^P] \quad (15)\\ \pi_m^{Pi} &= w_1^i \mathrm{Q}_1^i + \pi_{m2}^{Pi} + \beta[(a-q_1^1-rq_1^2)q_1^1 - w_1^1 Q_1^1 \\ &\quad + (a-q_1^2-rq_1^1)q_1^2 - w_1^2 Q_1^2 - h(I^1+I^2) + \pi_{b2}^p] \quad (16)\end{aligned}$$

Noted $\beta_1 = \frac{h^2(2-r)^4A-2ah(2-r)^2B+2a^2e_1}{(2-r)^2(h^2(2-r)^2A-2ahB+a^2C)}$ is a threshold when $\pi_b^P = \pi_b$ and $\beta_2 = \frac{2h^2(2-r)^4D+4ah(2-r)^3G-2a^2(1-r)^2k_3}{(2-r)^2(h^2(2-r)^2A-2ahB+a^2C)}$ is a threshold when $\pi_m^{Pi} = \pi_m^i$.
Notes:

$$e_1 = (1-r)^2(756+72r-99r^2-381r^3+213r^4-5r^5-14r^6+2r^7),$$
$$k_3 = -612+1836r-1053r^2+114r^3-77r^4+68r^5+6r^6-12r^7+2r^8.$$

We summarize the role of profit sharing contracts in the following proposition.

Proposition 5.1. *When strategic inventories exist, all parties are better off regardless of competition intensity if and only if* $\beta_2 < \beta < \beta_1$ *and* $0 < h < h_t$.

Proof. of Proposition 5.1. Let $\pi_b^{P1} = \pi_b, \pi_m^{Pi} = .\pi_m^i$, we have

$$\beta_1 = (h^2(2-r)^4A - 2ah(2-r)^2B + 2a^2e_1)/(2(2-r)^2(h^2(2-r)^2A-2ahB+a^2C))$$
$$\begin{aligned}\beta_2 &= (h^2(2-r)^4D + 4ah(2-r)^3G \\ &\quad - 2a^2(1-r)^2k_3)/((2-r)^2(h^2(2-r)^2A-2ahB+a^2C))\end{aligned}$$

When $0 < h < h_t = \frac{a(v_3-L\sqrt{C_1})}{(2-r)^2v_1}$, then $\beta_1 > \beta_2$. When $\beta_1 > \beta > \beta_2$, then $\pi_b^{P1} > \pi_b, \pi_m^{Pi} > .\pi_m^i$. □

The proposition reveals that in the presence of horizontal competition, suppliers and the retailer benefit from carrying strategic inventories regardless of competitive intensity when the unit holding cost is lower and the ratio of profit sharing is in region (β_2, β_1). That is, the suppliers agreed to carry strategic inventories as long as the unit holding cost is low and the ratio of profit sharing locates in certain regions even if the degree of horizontal competition is high. Under fierce horizontal competition setting, the retailer successfully encourages suppliers to support carrying strategic inventories by profit sharing contracts which improves the system's profits and make members of the supply chain achieving a win-win solution. The proposition is illustrated in Fig. 5. As the degree of competition among suppliers increases, the space of performance improvement of the supply chain is decreasing and finally disappears. In region, the degree of competition is low and β_2 is negative. This means the suppliers benefit from carrying strategic inventories without profit sharing contracts when the degree of competition is low. Therefore, the retailer does not need to implement the profit sharing contract for the region I. In region II, the degree of competition is high and β_2 is positive. This is because the retailer must use the profit sharing contract to encourage the suppliers to agree with carrying strategic inventories. Furthermore, both the suppliers and the retailer benefit from it because the overall performance of the supply chain is enhanced when the degree of competition increases, then members of the supply chain obtain more profits by the profit sharing contract.

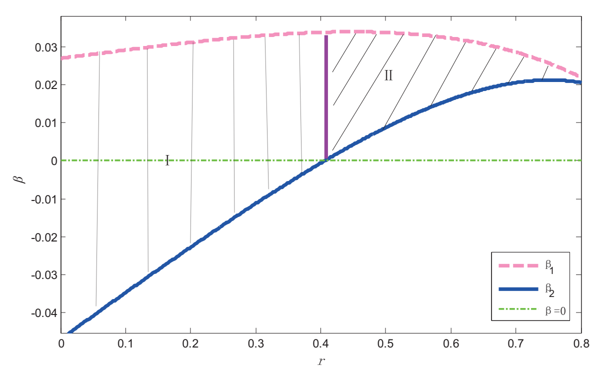

FIGURE 5. the relationship between β and r with $a = 10, h = 0.2$

6. **Conclusion.** This paper demonstrates that the retailer could exploit excess inventory as a strategic tool to secure lower wholesale prices under competing suppliers. Due to supplier competition, we find that the range of parameter values within which the retailer holds strategic inventories under the two-period model is limited. The suppliers support the retailer to hold strategic inventories if only if the degree of supplier competition is low and the unit holding cost is small. The suppliers prevent the retailer from carrying strategic inventories when the competition between suppliers is strong, but the whole supply chain benefits from the competition. In addition, we discuss the feasibility of the profit sharing contract to coordinate the supply chain. The suppliers, the retailer and consumers all benefit from carrying strategic inventories when the value of holding cost is smaller and the ratio of profit sharing is within a certain region. Furthermore, we also find that for the retailer, carrying strategic inventories is not the only way to reduce the suppliers' monopoly power since the substitutable relationship between the competition effects and the strategic inventories effects. The performance of the entire supply

chain is approaching to the first-best profits in the presence of competition effects and strategic inventories effects. These results are different from Anand et al. [1].

The concept of strategic inventories has only recently been defined and has attracted increasing attention to the various emerging implication. Our modeling framework can be extended to include other features. For example, incorporating random demand can facilitate risks. The consumer behavior is also an essential factor. Strategic consumers' purchase behavior affects the retailer's strategy.

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