

Does  $\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)^n$  diverge, converge absolutely, or converge conditionally?

**Solution**

$$\begin{aligned} L &= \lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \lim_{n \rightarrow \infty} \sqrt[n]{\left|\left(\frac{1}{n}\right)^n\right|} \\ &= \lim_{n \rightarrow \infty} \sqrt[n]{\left(\frac{1}{n}\right)^n} \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \\ &= 0 \end{aligned}$$

Since  $L < 1$ , the series  $\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)^n$  converges absolutely by the Root Test.