Does $\sum_{n=1}^{\infty} n!$ diverge, converge absolutely, or converge conditionally?

Solution 1

$$\begin{aligned}
\lim_{n\to\infty} a_n &= \lim_{n\to\infty} n! \\
&= \infty
\end{aligned}$$

Since $\lim_{n\to\infty} a_n$ does not exist (infinity is not a number), by the Test for Divergence, the series $\sum_{n=1}^{\infty} n!$ diverges.

Solution 2

$$\begin{aligned}
L &= \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| \\
&= \lim_{n\to\infty} \left| \frac{(n+1)!}{n!} \right| \\
&= \lim_{n\to\infty} \left| \frac{(n+1)\cdot n!}{n!} \right| \\
&= \lim_{n\to\infty} |n+1| \\
&= \lim_{n\to\infty} (n+1) \\
&= \infty.
\end{aligned}$$

Since $L = \infty$, by the Ratio Test, the series $\sum_{n=1}^{\infty} n!$ diverges.