$$\int \cos^2 x \, dx$$

Solution 1

Integrate by parts with $u = \cos x$ and $dv = \cos x \, dx$. So $du = -\sin x \, dx$ and $v = \sin x$. So

$$\begin{aligned}\int \cos^2 x \, dx &= \cos x \sin x + \int \sin^2 x \, dx \\ &= \cos x \sin x + \int 1 - \cos^2 x \, dx.\end{aligned}$$

So

$$\int \cos^2 x \, dx = \cos x \sin x + \int 1 - \cos^2 x \, dx$$

and using some algebra, we get

$$\int \cos^2 x \, dx = \frac{\cos x \sin x + x}{2} + C$$

Solution 2

Using the trigonometric identity

$$\cos^2 \theta = \frac{1 + \cos(2\theta)}{2}$$

we rewrite:

$$\int \cos^2 x \, dx = \int \frac{1}{2} \, dx + \frac{1}{2} \int \cos 2x \, dx$$

The second integral is completed using a substitution of $u = 2x$, so the integral becomes equal to

$$\frac{1}{2}x + \frac{1}{4} \sin 2x + C.$$