$$\int \sin^{11} x \cos x \, dx$$

Solution 1

$$\int \sin^{11} x \cos x \, dx = \int \sin x (\sin^2 x)^5 \cos x \, dx = \int \sin x (1 - \cos^2 x)^5 \cos x \, dx.$$

Let $u = \cos x$, so $du = -\sin x \, dx$ and the integral above is equal to

$$-\int (1-u^2)^5 u \, du$$

Now the integral above can be completed, but requires a five-fold FOILing of $1 - u^2$ first. We'll quit this method to try a second solution below, which is much shorter.

Solution 2

From

$$\int \sin^{11} x \cos x \, dx$$

because the power of sine and cosine are BOTH odd, it may be easier to do the following: let $u = \sin x$. Then $du = \cos x \, dx$, so the integral above is equal to

$$\int u^{11} \, du = \frac{u^{12}}{12} + C = \frac{1}{12} \sin^{12} x + C.$$