

Section 4.6: Optimization Problems  
E. Kim

1. A homeowner has \$500 to spend on building a fence to surround a garden. Three sides of the fence will be constructed with oak fencing costing \$5 per foot and one side will be constructed with rosewood costing \$10 per foot. What are the dimensions which will form the largest garden area?
2. Dr. K's pet unicorn Charlie has been getting unruly lately. Charlie the Unicorn needs a fenced off playing area made in the shape of a rectangle. One side of the rectangular field will be formed by a flat-walled side of Candy Mountain, which will require no fence. Using 3000 feet of fencing for the other three sides, what are the dimensions of the field which has the largest playing area?
3. Suppose that the cost of producing  $x$  thousand dress shirt pins is

$$C(x) = x^3 - 6x^2 + 15x$$

and the revenue of selling  $x$  thousand dress shirt pins is

$$R(x) = 9x.$$

Is there a production level that maximizes profit? If so, what is it?

4. Your friend from high school wants you to become a partner in a new business where the revenue is  $R(x) = 6x$  and the cost is  $C(x) = x^3 - 6x^2 + 15x$ . Why should you **NOT** join the business venture?
5. The price-demand equation for lamps is given by  $p = 40 - \frac{x}{10}$ . What price should the company charge for each lamp to maximize revenue?
6. The price-demand equation for toasters is given by  $p = 60 - \frac{x}{5}$  and the total cost for producing toasters is given by  $C(x) = 20x + 200$ . How many toasters should be produced to maximize profit?
7. Find two positive numbers whose product is 20 and whose sum is minimum.
8. What is the minimum vertical distance between  $y = x^2 + 6$  and  $y = 3x - x^2$ ?
9. Menard's is starting their own line of soup and wants to design a soup can which will hold 500 cm<sup>3</sup>. What dimensions for an aluminum can will use the least amount of material possible?
10. JCPenney needs to make a new type of gift box. An open-top box is to be made by cutting squares out from the four corners of a 12-inch by 12-inch sheet of cardboard and bending up the sides. How large should the squares be cut to make the box hold as much as possible?
11. The crust of bread is just awful<sup>1</sup>. Awful, awful, awful! Pretending that the bread is flat, and assuming that the bread should be rectangular, what should be the dimensions of the bread if the rectangle's area is 9 in<sup>2</sup> and we'd like to minimize the perimeter (crust)?
12. What is the area of the largest rectangle inscribed in a semicircle of radius 2?
13. Find the point on the parabola  $y^2 = 2x$  that is closest to the point (1, 4).
14. Find the point on the line  $y = 17x + 17$  closest to the origin.

---

<sup>1</sup>... just like the outside crispy ring of a pancake/waffle is awful!