

$$\int \frac{3}{x^2 - 1} dx$$

Solution

To integrate the rational function $\frac{3}{x^2-1}$, since the numerator has power 0 and the denominator has power 2, we skip long division. Set up partial fractions

$$\frac{3}{x^2 - 1} = \frac{A}{x + 1} + \frac{B}{x - 1}$$

So

$$3 = A(x - 1) + B(x + 1)$$

By substituting,

- Using $x = 1$ gives us $B = \frac{3}{2}$
- Using $x = -1$ gives us $A = -\frac{3}{2}$.

So

$$\begin{aligned} \int \frac{3}{x^2 - 1} dx &= \int \frac{-3/2}{x + 1} + \frac{3/2}{x - 1} dx \\ &= -\frac{3}{2} \ln |x + 1| + \frac{3}{2} \ln |x - 1| + C \end{aligned}$$