Does $\sum_{n=1}^{\infty} \frac{\cos \pi n}{n^2}$ diverge, converge absolutely, or converge conditionally?

Solution 1

The series $\sum_{n=1}^{\infty} \frac{\cos \pi n}{n^2}$ is an alternating series. Note $b_n = |a_n| = \frac{1}{n^2}$. Since the sequence b_n is decreasing and $b_n \to 0$, by the Alternating Series Test, $\sum_{n=1}^{\infty} \frac{\cos \pi n}{n^2}$ converges.

To figure out whether $\sum_{n=1}^{\infty} \frac{\cos \pi n}{n^2}$ converges absolutely or conditionally, we consider the series

$$\sum_{n=1}^{\infty} \left| \frac{\cos \pi n}{n^2} \right| = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

which converges by the p-test, so the series $\sum_{n=1}^{\infty} \frac{\cos \pi n}{n^2}$ converges absolutely.

Solution 2

We consider

$$\sum_{n=1}^{\infty} \left| \frac{\cos \pi n}{n^2} \right| = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

which converges by the p-test, so the series $\sum_{n=1}^{\infty} \frac{\cos \pi n}{n^2}$ converges by the Absolute Convergence Test. In fact, the series $\sum_{n=1}^{\infty} \frac{\cos \pi n}{n^2}$ converges absolutely.