

- The sequence $a_n = \frac{1}{n}$ converges to zero. We either write

$$\lim_{n \rightarrow \infty} a_n = 0$$

or plug in our specific a_n to write

$$\lim_{n \rightarrow \infty} \frac{1}{n} = 0.$$

We can also write

$$a_n \rightarrow 0$$

or write

$$\frac{1}{n} \rightarrow 0.$$

However, do not write

$$\lim_{n \rightarrow \infty} a_n \rightarrow 0$$

or

$$\lim_{n \rightarrow \infty} \frac{1}{n} \rightarrow 0.$$

- By contrast, the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is the harmonic series, and diverges. There are several ways to show this:

– It is possible to take a look at the sequence of partial sums s_n . One can note that $s_2 \geq 1$ and $s_4 \geq 2$ and $s_8 \geq 3$ and $s_{16} \geq 4$ and so on. It is probably easier to do one of the other things below:

– The function $f(x) = \frac{1}{x}$ is continuous, positive, and decreasing on $[1, \infty)$. Then

$$\int_1^{\infty} \frac{1}{x} dx = \lim_{t \rightarrow \infty} \int_1^t \frac{1}{x} dx = \lim_{t \rightarrow \infty} (\ln |t| - \ln |1|) = \infty$$

since the integral $\int_1^{\infty} \frac{1}{x} dx$ diverges, the series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by the Integral Test

– $\sum_{n=1}^{\infty} \frac{1}{n}$ is a p -series with $p = 1$. Since $p \leq 1$, the series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.