

Does  $\sum_{n=1}^{\infty} \frac{n}{n!}$  diverge, converge absolutely, or converge conditionally?

### Solution

$$\begin{aligned} L &= \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| \\ &= \lim_{n \rightarrow \infty} \left| \frac{n+1}{(n+1)!} \cdot \frac{n!}{n} \right| \\ &= \lim_{n \rightarrow \infty} \left| \frac{n+1}{(n+1) \cdot n!} \cdot \frac{n!}{n} \right| \\ &= \lim_{n \rightarrow \infty} \left| \frac{1}{n} \right| \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \\ &= 0 \end{aligned}$$

Since  $L < 1$ , by the Ratio Test, the series  $\sum_{n=1}^{\infty} \frac{n}{n!}$  converges absolutely.