

Does $\sum_{n=1}^{\infty} \frac{1}{0.9^n}$ diverge, converge absolutely, or converge conditionally?

Solution 1

Consider the limit of the sequence a_n .

$$\begin{aligned}\lim_{n \rightarrow \infty} a_n &= \lim_{n \rightarrow \infty} \frac{1}{0.9^n} \\ &= \lim_{n \rightarrow \infty} \left(\frac{10}{9} \right)^n \\ &= \infty.\end{aligned}$$

so the series $\sum_{n=1}^{\infty} \frac{1}{0.9^n}$ diverges by the Test for Divergence.

Solution 2

$\sum_{n=1}^{\infty} \frac{1}{0.9^n}$ is a geometric series with $r = \frac{1}{0.9} = \frac{1}{\frac{9}{10}} = \frac{10}{9}$. Since $|r| \geq 1$, the series $\sum_{n=1}^{\infty} \frac{1}{0.9^n}$ diverges by the Geometric Series Test.