

$$\int \csc x \, dx$$

**Solution**

$$\begin{aligned} \int \csc x \, dx &= \int \csc x \frac{\csc x + \cot x}{\csc x + \cot x} \, dx \\ &= \int \frac{\csc^2 x + \csc x \cot x}{\csc x + \cot x} \, dx \end{aligned}$$

Let  $u = \csc x + \cot x$ , so  $du = -\csc x \cot x - \csc^2 x \, dx$ . So the integral above is equal to

$$\begin{aligned} \int \frac{\csc^2 x + \csc x \cot x}{\csc x + \cot x} \, dx &= - \int \frac{1}{u} \, du \\ &= -\ln |u| + C \\ &= -\ln |\csc x + \cot x| + C. \end{aligned}$$