$$\int \frac{x^3+1}{x^2-1}\,dx$$

Solution 1

To integrate the rational function $\frac{x^3+1}{x^2-1}$, since the numerator has power 3 and the denominator has power 2, we must do long division. The result of long division is

$$\frac{x^3+1}{x^2-1} = x + \frac{x+1}{x^2-1}$$

We could apply partial fraction decomposition to $\frac{x+1}{x^2-1}$, but we can actually cancel common factors instead. so

$$\frac{x^3+1}{x^2-1} = x + \frac{x+1}{x^2-1} = x + \frac{1}{x-1}$$

Therefore

$$\begin{aligned}\int \frac{x^3+1}{x^2-1}\,dx &= \int x + \frac{1}{x-1}\,dx \\ &= \frac{x^2}{2} + \ln|x-1| + C\end{aligned}$$

Solution 2

You might notice that x^3+1 factors as $(x+1)(x^2-x+1)$. So

$$\frac{x^3+1}{x^2-1} = \frac{(x+1)(x^2-x+1)}{(x+1)(x-1)} = \frac{x^2-x+1}{x-1}$$

Long division of x^2-x+1 over $x-1$ gives us

$$\frac{x^2-x+1}{x-1} = x + \frac{1}{x-1}$$

Therefore

$$\begin{aligned}\int \frac{x^3+1}{x^2-1}\,dx &= \int \frac{x^2-x+1}{x-1}\,dx \\ &= \int x + \frac{1}{x-1}\,dx \\ &= \frac{x^2}{2} + \ln|x-1| + C\end{aligned}$$