$$\int \sin^2 x \, dx$$

Solution 1

Integrate by parts with $u = \sin x$ and $dv = \sin x \, dx$. So $du = \cos x \, dx$ and $v = -\cos x$. So

$$\begin{aligned} \int \sin^2 x \, dx &= -\cos x \sin x + \int \cos^2 x \, dx \\ &= -\cos x \sin x + \int 1 - \sin^2 x \, dx. \end{aligned}$$

So

$$\int \sin^2 x \, dx = -\cos x \sin x + \int 1 - \sin^2 x \, dx$$

and using some algebra, we get

$$\int \sin^2 x \, dx = \frac{-\cos x \sin x + x}{2} + C$$

Solution 2

Using the trigonometric identity

$$\sin^2 \theta = \frac{1 - \cos(2\theta)}{2}$$

we rewrite:

$$\int \sin^2 x \, dx = \int \frac{1}{2} \, dx - \frac{1}{2} \int \cos 2x \, dx$$

The second integral is completed using a substitution of $u = 2x$, so the integral becomes equal to

$$\frac{1}{2}x - \frac{1}{4} \sin 2x + C.$$