

Does $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$ diverge, converge absolutely, or converge conditionally?

Solution

$$\begin{aligned}\lim_{n \rightarrow \infty} a_n &= \lim_{n \rightarrow \infty} \frac{2^n}{n^2} \\ &= \lim_{n \rightarrow \infty} \frac{2^n \ln 2}{2n} \quad \text{by L'Hopital's} \\ &= \lim_{n \rightarrow \infty} \frac{2^n \ln 2 \ln 2}{2} \quad \text{by L'Hopital's again} \\ &= \infty\end{aligned}$$

So the series $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$ diverges by the Test for Divergence.

Solution 2 (outline)

The Root Test will work, but it is a lot of work compared to the solution presented.

Solution 3 (outline)

The Ratio Test will work, but it is a lot of work compared to the solution presented.