$$\int \sin x \cos x \, dx$$

Solution 1

Use $u = \sin x$, so $du = \cos x \, dx$. Then

$$\int \sin x \cos x \, dx = \int u \, du = \frac{u^2}{2} + C = \frac{(\sin x)^2}{2} + C.$$

Solution 2

Use $u = \cos x$, so $du = -\sin x \, dx$. Then

$$\int \sin x \cos x \, dx = -\int u \, du = -\frac{u^2}{2} + C = -\frac{(\cos x)^2}{2} + C.$$

Solution 3

Take the trig identity $\sin 2x = 2 \sin x \cos x$ and divide both sides by 2 to obtain $\frac{1}{2} \sin 2x = \sin x \cos x$. So,

$$\int \sin x \cos x \, dx = \frac{1}{2} \int \sin 2x \, dx.$$

Then, by substituting $u = 2x$, we have $du = 2 \, dx$, so the integral above is equal to

$$\frac{1}{4} \int \sin u \, du = -\frac{1}{4} \cos u + C = -\frac{1}{4} \cos 2x + C.$$