$$\int \frac{1}{x^2+2x+1}\,dx$$

Solution 1

The denominator of the integrand factors as $x^2+2x+1=(x+1)^2$. So the integral we want is

$$\frac{1}{x^2+2x+1}=\frac{1}{(x+1)^2}$$

By setting up partial fractions

$$\frac{1}{(x+1)^2}=\frac{A}{x+1}+\frac{B}{(x+1)^2}$$

Multiplying both sides by $x+1$, we have

$$1=A(x+1)+B$$

A convenient number to plug in is $x=-1$. Doing this, we'd get $B=1$. Now that we know B is 1, replace this in the equation above:

$$1=A(x+1)+1$$

Let us now pick a value for x other than -1. You can pick any number you like other than -1. For instance, I'll pick $x=3$. Then we have

$$1=4A+1$$

and by applying algebra, we get $A=0$.

In the end, our partial fraction decomposition becomes

$$\frac{1}{(x+1)^2}=\frac{0}{x+1}+\frac{1}{(x+1)^2}$$

but this simplifies to

$$\frac{1}{(x+1)^2}=\frac{1}{(x+1)^2}$$

which we already knew. Partial fractions didn't get us anywhere for this problem, so follow along the second solution below:

Solution 2

$$\int \frac{1}{x^2+2x+1}\,dx=\int \frac{1}{(x+1)^2}\,dx$$

Let $u=x+1$. Then $du=dx$. So,

$$\begin{aligned}\int \frac{1}{(x+1)^2}\,dx &= \int \frac{1}{u^2}\,du\\ &= \int u^{-2}\,du\\ &= \frac{u^{-1}}{-1}+C\\ &= \frac{-1}{u}+C\\ &= \frac{-1}{x+1}+C.\end{aligned}$$