$\int \ln(x^2-4)\,dx$

Solution

$\int \ln(x^2-4)\,dx = \int \ln((x+2)(x-2))\,dx = \int \ln(x+2)+\ln(x-2)\,dx = \int \ln(x+2)\,dx + \int \ln(x-2)\,dx.$

The two separate integrals are essentially integrating $\ln(x)$. While this is not quite true, consider what you'd get if you took the first integral $\int \ln(x+2)\,dx$ and did a substitution of $u = x+2$. Because I'd like to reserve the variable u, consider a t substitution.

- First, recall how to integrate the natural logarithm: Integrate by parts with $u = \ln x$ and $dv = dx$. So $du = \frac{1}{x}\,dx$ and $v = x$. Thus,

$$\int \ln x\,dx = x\ln x - \int x\cdot\frac{1}{x}\,dx = x\ln x - \int 1\,dx = x\ln x - x + C.$$

- Then convert

$$\int \ln(x+2)\,dx$$

  by a t-substitution: if $t = x+2$, then $dt = dx$, so

$$\int \ln(x+2)\,dx = \int t\,dt$$

  Then the integral $\int t\,dt$ can just borrow the work above, and we have

$$\int \ln(x+2)\,dx = \int t\,dt = t\ln t - t + C = (x+2)\ln(x+2) - (x+2) + C.$$

- Similarly convert the second integral using $t = x-2$ and $dt = dx$ to have

$$\int \ln(x-2)\,dx = \int t\,dt = t\ln t - t + C = (x-2)\ln(x-2) - (x-2) + C.$$

Putting everything together,

$$\int \ln(x+2)\,dx + \int \ln(x-2)\,dx = \quad (x+2)\ln(x+2) - (x+2) \quad + \quad (x-2)\ln(x-2) - (x-2) \quad + C.$$