$$\int \csc x \, dx$$

Solution

$$\int \csc x \, dx = \int \csc x \frac{\csc x + \cot x}{\csc x + \cot x} \, dx$$
$$= \int \frac{\csc^2 x + \csc x \cot x}{\csc x + \cot x} \, dx$$

Let $u = \csc x + \cot x$, so $du = -\csc x \cot x - \csc^2 x \, dx$. So the integral above is equal to

$$\int \frac{\csc^2 x + \csc x \cot x}{\csc x + \cot x} \, dx = -\int \frac{1}{u} \, du$$
$$= -\ln|u| + C$$
$$= -\ln|\csc x + \cot x| + C.$$

1