Does $\displaystyle\sum_{n=2}^{\infty} \frac{n+2}{n^2-1}$ diverge, converge absolutely, or converge conditionally?

Solution 1

The series $\sum \frac{1}{n}$ diverges by the p-test. Let $a_n = \frac{n+2}{n^2-1}$ and $b_n = \frac{1}{n}$. Then

$$
\begin{aligned}
\lim_{n\to\infty} \frac{a_n}{b_n} &= \lim_{n\to\infty} \frac{n^2+2n}{n^2-1} \\
&= \lim_{n\to\infty} \frac{2n+2}{2n} \text{ by L'hopital} \\
&= \lim_{n\to\infty} \frac{2}{2} \text{ by L'hopital} \\
&= \lim_{n\to\infty} 2 \\
&= 2
\end{aligned}
$$

Since this limit is a finite, positive number, by the Limit Comparison Test, the series $\displaystyle\sum_{n=2}^{\infty} \frac{n+2}{n^2-1}$ diverges.

Solution 2

This series can also be examined using the Direct Comparison Test, but we skip this method for this series.