Does $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ diverge, converge absolutely, or converge conditionally?

Solution

$$\begin{aligned}
L &= \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| \\
&= \lim_{n\to\infty} \left| \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} \right| \\
&= \lim_{n\to\infty} \left| \frac{2^n \cdot 2}{(n+1)\cdot n!} \cdot \frac{n!}{2^n} \right| \\
&= \lim_{n\to\infty} \left| \frac{2}{n+1} \right| \\
&= \lim_{n\to\infty} \frac{2}{n+1} \\
&= 0
\end{aligned}$$

Since $L < 1$, by the Ratio Test, the series $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ converges absolutely.