$$\int \sec x \, dx$$

Solution

$$\begin{aligned}\int \sec x \, dx &= \int \sec x \frac{\sec x + \tan x}{\sec x + \tan x} \, dx \\ &= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \, dx\end{aligned}$$

Let $u = \sec x + \tan x$, so $du = \sec x \tan x + \sec^2 x \, dx$. So the integral above is equal to

$$\begin{aligned}\int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \, dx &= \int \frac{1}{u} \, du \\ &= \ln |u| + C \\ &= \ln |\sec x + \tan x| + C.\end{aligned}$$