$$\int \frac{x+1}{x^2-1}\,dx$$

Solution 1

To integrate the rational function $\frac{x+1}{x^2-1}$, since the numerator has power 1 and the denominator has power 2, we skip long division. Set up partial fractions

$$\frac{x+1}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}$$

So

$$x+1 = A(x-1) + B(x+1)$$

By substituting,

- Using $x = 1$ gives us $B = 1$

- Using $x = -1$ gives us $A = 0$.

So

$$\begin{aligned}
\int \frac{x+1}{x^2-1}\,dx &= \int \frac{0}{x+1} + \frac{1}{x-1}\,dx \\
&= \int 0 + \frac{1}{x-1}\,dx \\
&= \int \frac{1}{x-1}\,dx \\
&= \ln|x-1| + C
\end{aligned}$$

Solution 2

We can rewrite the integrand $\frac{x+1}{x^2-1} = \frac{x+1}{(x+1)(x-1)} = \frac{1}{x-1}$. Thus,

$$\int \frac{x+1}{x^2-1}\,dx = \int \frac{1}{x-1}\,dx = \ln|x-1| + C$$