$$\int \frac{x+3}{x^2-1}\,dx$$

## Solution 1

To integrate the rational function $\frac{x+3}{x^2-1}$, since the numerator has power 1 and the denominator has power 2, we skip long division. Set up partial fractions

$$\frac{x+3}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}$$

So

$$x+3 = A(x-1) + B(x+1)$$

By substituting,

- Using $x = 1$ gives us $B = 2$
- Using $x = -1$ gives us $A = -1$.

So

$$\begin{aligned}\int \frac{x+3}{x^2-1}\,dx &= \int \frac{-1}{x+1} + \frac{2}{x-1}\,dx \\ &= -\ln|x+1| + 2\ln|x-1| + C\end{aligned}$$

## Solution 2

Split the original integral in two:

$$\int \frac{x+3}{x^2-1}\,dx = \int \frac{x}{x^2-1}\,dx + \int \frac{3}{x^2-1}\,dx$$

The first integral becomes $\frac{1}{2}\ln|x^2-1| + C$ using the substitution $u = x^2 - 1$. For the second integral: to integrate the rational function $\frac{3}{x^2-1}$, since the numerator has power 0 and the denominator has power 2, we skip long division. Set up partial fractions

$$\frac{3}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}$$

So

$$3 = A(x-1) + B(x+1)$$

By substituting,

- Using $x = 1$ gives us $B = \frac{3}{2}$
- Using $x = -1$ gives us $A = -\frac{3}{2}$.

So

$$\begin{aligned}\int \frac{3}{x^2-1}\,dx &= \int \frac{-3/2}{x+1} + \frac{3/2}{x-1}\,dx \\ &= -\frac{3}{2}\ln|x+1| + \frac{3}{2}\ln|x-1| + C\end{aligned}$$

Putting the work together, the original integral is equal to

$$\frac{1}{2}\ln|x^2-1| - \frac{3}{2}\ln|x+1| + \frac{3}{2}\ln|x-1| + C$$