Does $\sum_{n=2}^{\infty} \frac{n^2+2}{n^3+1}$ diverge, converge absolutely, or converge conditionally?

Solution 1

The series $\sum \frac{1}{n}$ diverges by the p-test. Let $a_n = \frac{n^2+2}{n^3+1}$ and $b_n = \frac{1}{n}$.

$$\begin{aligned}
\lim_{n\to\infty} \frac{a_n}{b_n} &= \lim_{n\to\infty} \frac{n^3+2n}{n^3+1} \\
&= \lim_{n\to\infty} \frac{3n^2+2}{3n^2} \text{ by l'hopital} \\
&= \lim_{n\to\infty} \frac{6n}{6n} \text{ by l'hopital} \\
&= \lim_{n\to\infty} \frac{6}{6} \text{ by l'hopital} \\
&= 1
\end{aligned}$$

The Limit Comparison Test applies, since this limit was a positive, finite number. Therefore, the series $\sum \frac{n^2+2}{n^3+1}$ diverges by the Limit Comparison Test.

Note

Another (more involved) solution uses the Direct Comparison Test