$$\int \sin^2 x \cos^5 x \, dx$$

Solution

Since the power of sine is even yet the power of cosine is odd, we have no choice here. Let us reserve one factor of cosine. The remaining factors of cosines should become sines.

$$\begin{aligned}
\int \sin^2 x \cos^5 x \, dx &= \int \sin^2 x \cos^4 x \cos x \, dx \\
&= \int \sin^2 x (\cos^2 x)^2 \cos x \, dx \\
&= \int \sin^2 x (1 - \sin^2 x)^2 \cos x \, dx.
\end{aligned}$$

Now let $u = \sin x$ so $du = \cos x \, dx$ and the integral above is equal to

$$\begin{aligned}
&\int u^2 (1 - u^2)^2 \, du \\
&= \int u^2 (1 - 2u^2 + u^4) \, du \\
&= \int (u^2 - 2u^4 + u^6) \, du \\
&= \frac{1}{3} u^3 - \frac{2}{5} u^5 + \frac{1}{7} u^7 + C \\
&= \frac{1}{3} \sin^3 x - \frac{2}{5} \sin^5 x + \frac{1}{7} \sin^7 x + C.
\end{aligned}$$