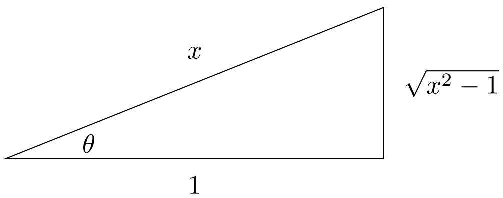
$$\int \frac{\sqrt{25x^2-25}}{x}\,dx$$

Solution

$$\int \frac{\sqrt{25x^2-25}}{x}\,dx = \int \frac{\sqrt{25(x^2-1)}}{x}\,dx = 5\int \frac{\sqrt{x^2-1}}{x}\,dx$$

Let $x = 1\sec\theta$ with θ in quadrants I or III. So, $dx = 1\sec\theta\tan\theta\,d\theta$ and we use $\sec\theta = \frac{x}{1}$ to draw a right triangle with x as the hypotenuse side side and 1 as the adjacent side. The side opposite to θ is $\sqrt{x^2-1}$, which is found using the Pythagorean Theorem, and we have



Then $\tan\theta = \dfrac{\sqrt{x^2-1}}{1}$. Based on this equation, we will use $1\tan\theta = \sqrt{x^2-1}$ if it is helpful. Since $\cos\theta = \dfrac{1}{x}$, we will use $x = \dfrac{1}{\cos\theta} = 1\sec\theta$ if it is helpful.

So

$$\begin{aligned}
5\int \frac{\sqrt{x^2-1}}{x}\,dx &= 5\int \frac{\tan\theta}{\sec\theta}\sec\theta\tan\theta\,d\theta \\
&= 5\int \tan^2\theta\,d\theta \\
&= 5\int (\sec^2\theta - 1)\,d\theta \\
&= 5\left[\tan\theta - \theta\right] + C \\
&= 5\left[\sqrt{x^2-1} - \sec^{-1}x\right] + C
\end{aligned}$$