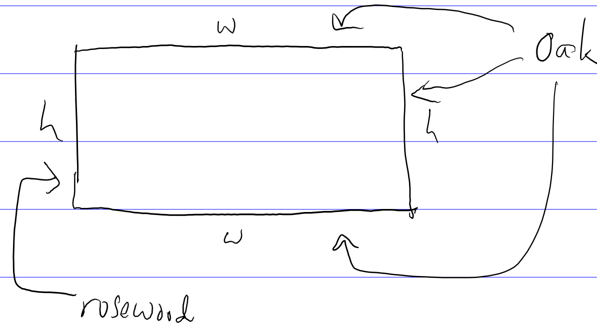
\#1

$h = ht$

$w = \text{width}$

$m = \text{money}$

$A = \text{area} \leftarrow \text{max}$



$$500 = m = 5(\underbrace{w + h + w}_{\text{cost of oak}}) + 10h \quad \leftarrow \text{cost of rosewood}$$

$$A = hw$$

$$500 = 15h + 10w$$

$$\frac{500 - 15h}{10} = w$$

$$A = h\left(\frac{500 - 15h}{10}\right)$$

$$A = 50h - \tfrac{3}{2}h^2$$

$$A' = 50 - 3h$$

$$50 - 3h = 0$$

$$h = \frac{50}{3}$$

$A''(h) = -3$ so $A''(\frac{50}{3}) = -3$, by SDT A has l. max at $h = \frac{50}{3}$

$$w = \frac{500 - 15\left(\frac{50}{3}\right)}{10}$$

Dimensions:

$h = \frac{50}{3}$ ft

$w = \frac{500 - 15\left(\frac{50}{3}\right)}{10}$ ft

#2)

x = width

y = ht

A = area

$$A = xy \qquad\qquad x + y + x = 3000$$

$$2x + y = 3000$$

$$y = 3000 - 2x$$

$$A(x) = x(3000 - 2x) = 3000x - 2x^2$$

maximize $A(x) = 3000x - 2x^2$ with x in $\boxed{[0, 1500]}$

(why 1500?)

$$A'(x) = 3000 - 4x$$

$$3000 - 4x = 0$$

$$x = \frac{3000}{4}$$

$$A\left(\frac{3000}{4}\right) = 3000\left(\frac{3000}{4}\right) - 2\left(\frac{3000}{4}\right)^2$$

endpts
$$\begin{cases} A(0) = 0(3000 - 2\cdot 0) = 0 \\ A(1500) = 1500(3000 - 2\cdot 1500) = 0 \end{cases}$$

Max.

Can skip work in red if using FDT or SDT here

use $x = \frac{3000}{4}$ ft

and $y = 3000 - 2\left(\frac{3000}{4}\right)$ ft.

#3)

$P(x) = 9x - (x^3 - 6x^2 + 15x)$

$P'(x) = 9 - 3x^2 + 12x - 15$

$9 - 3x^2 + 12x - 15 = 0$

$-3x^2 + 12x - 6 = 0$

$-3(x^2 - 4x + 2) = 0$

$x^2 - 4x + 2 = 0$

$$x = \frac{4 \pm \sqrt{16-8}}{2} = \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$

$P'(x) = -3x^2 + 12x - 6$

so $P''(x) = -6x + 12$

$P''(2+\sqrt{2}) = -6(2+\sqrt{2}) + 12 = -12 - 6\sqrt{2} + 12 = -6\sqrt{2}$

so by SDT, P has l. max at $x = 2+\sqrt{2}$

$P''(2-\sqrt{2}) = -6(2-\sqrt{2}) + 12 = -12 + 6\sqrt{2} + 12 = 6\sqrt{2}$

so by SDT, P has l. min at $x = 2-\sqrt{2}$

Produce $(2+\sqrt{2})$ thousand pins

#4)

$P(x) = 6x - (x^3 - 6x^2 + 15x)$

$P(x) = 6x - x^3 + 6x^2 - 15x$

$P'(x) = 6 - 3x^2 + 12x - 15$

$= -3x^2 + 12x - 9$

$-3x^2 + 12x - 9 = 0$

$-3(x^2 - 4x + 3) = 0$

$-3(x-3)(x-1) = 0$

crit: $x = 3, \quad x = 1$

$P''(x) = -6x + 12$

$P''(3) = -6(3) + 12 = -6$ so l. max at 3.

$P''(1) = -6(1) + 12$ so l. min at 1.

Max at $x = 3$.

$P(3) = 6(3) - (3^3 - 6 \cdot 3^2 + 15 \cdot 3) = 0$

Best case scenario is no profit!

#kthxbye!

#5

$$R(x) = x\left(40 - \frac{x}{10}\right)$$

$$R(x) = 40x - \frac{1}{10}x^2$$

$$R'(x) = 40 - \frac{1}{5}x$$

$$40 - \frac{1}{5}x = 0$$

$$40 = \frac{1}{5}x$$

$$200 = x$$

$$R''(x) = -\frac{1}{5} \qquad R''(200) = -\frac{1}{5}$$

so l. max @ 200.

Company should make 200 lamps.

To answer question:

$$p = 40 - \frac{x}{10}$$

$$p = 40 - \frac{200}{10} = 40 - 20 = 20$$

Price per lamp should be \$20

#6

$R(x) = 60x - \frac{1}{5}x^2$

$P(x) = 60x - \frac{1}{5}x^2 - (20x + 200)$
$= 40x - \frac{1}{5}x^2 - 200$

$P'(x) = 40 - \frac{2}{5}x$
$40 - \frac{2}{5}x = 0$
$40 = \frac{2}{5}x$
$\frac{200}{2} = x$
$x = 100$

$P''(x) = -\frac{2}{5}$ $\quad$ $P''(100) = -\frac{2}{5}$ so l. max at $x = 100$

Make 100 toasters.

#7) x, y are the numbers
S is the sum ← minimize

$xy = 20$ $\qquad$ $S = x + y$

$y = \frac{20}{x}$

$S = x + \frac{20}{x}$

$S = x + 20x^{-1}$

$S' = 1 - 20x^{-2}$

$1 - 20x^{-2} = 0$

$1 - \frac{20}{x^2} = 0$

$1 = \frac{20}{x^2}$

$x^2 = 20$

$x = \pm\sqrt{20}$

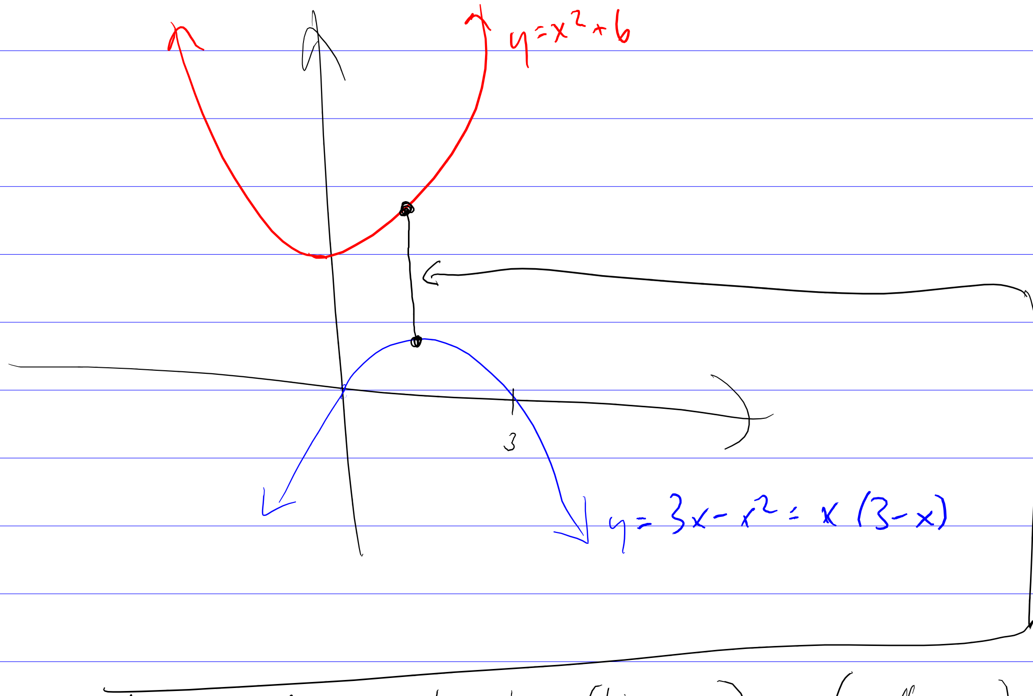
$x = \sqrt{20} \longrightarrow y = \sqrt{20}$

$S'' = 40x^{-3}$

$S''(\sqrt{20}) = \frac{40}{\sqrt{20}^3}$

so S has l. min @ $x = \sqrt{20}$

8)



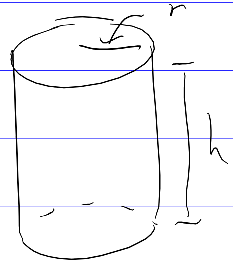
vertical dist for x-value is (bigger y) − (smaller y)

$$d = (x^2+6)-(3x-x^2)$$
$$= x^2+6-3x+x^2$$
$$= 2x^2-3x+6$$
$$d' = 4x-3$$
$$4x-3=0$$
$$x=\frac{3}{4}$$

The x-value $\frac{3}{4}$ will minimize vert. dist.

What is the smallest v. dist? $\left(\left(\frac{3}{4}\right)^2+6\right)-\left(3\left(\frac{3}{4}\right)-\left(\frac{3}{4}\right)^2\right)$

9)



$V = \pi r^2 h = 500$

$A = 2 \cdot \pi r^2 + (2\pi r)(h) \leftarrow$ min.

$h = \frac{500}{\pi r^2}$

$A = 2\pi r^2 + 2\pi r \cdot \frac{500}{\pi r^2}$

$A(r) = 2\pi r^2 + 1000 r^{-1}$

$A'(r) = 4\pi r - 1000 r^{-2}$

$4\pi r - 1000 r^{-2} = 0$

$4\pi r - \frac{1000}{r^2} = 0$

$4\pi r^3 - 1000 = 0$

$r = \sqrt[3]{\frac{1000}{4\pi}}$

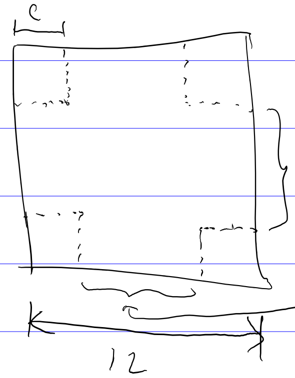
$A''(r) = 4\pi + 1000 r^{-3}$

$A''\left(\sqrt[3]{\frac{1000}{4\pi}}\right) > 0$ so by SDT, A has l. min @ $r = \sqrt[3]{\frac{1000}{4\pi}}$

Dimensions? $r = \sqrt[3]{\frac{1000}{4\pi}}$ cm

$h = \frac{500}{\pi\left(\sqrt[3]{\frac{1000}{4\pi}}\right)^2}$ cm

#10)



V= volume

c = size of cut

both 12-2c
(from picture)

$$\boxed{0 \le c \le 6}$$

$$V = c(12-2c)(12-2c)$$
$$= c(144-48c+4c^2)$$
$$= 144c-48c^2+4c^3$$

$$V' = 144-96c+12c^2$$

$$144-96c+12c^2=0$$
$$12(c^2-8c+12)=0$$
$$12(c-6)(c-2)=0$$
$$c=6 \qquad c=2$$

Abs max of $V(c)$ on $[0,6]$?

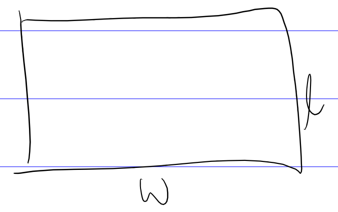
$$V(2) = 128$$
$$V(6) = 0$$
$$V(0) = 0$$

chk endpt

$c=2$ produces volume of $128\ in^3$.

Make cut be 2in.

11)



$A = wl = 9$ minimize $p = w + l + w + l = 2w + 2l$

$l = \frac{9}{w}$

$p = 2w + 2\left(\frac{9}{w}\right)$

$p(w) = 2w + 18w^{-1}$

$p'(w) = 2 - 18w^{-2}$

$0 = 2 - 18w^{-2}$

$0 = 2 - \frac{18}{w^2}$

$0 = 2w^2 - 18$

$18 = 2w^2$

$9 = w^2$

$\pm 3 = w$

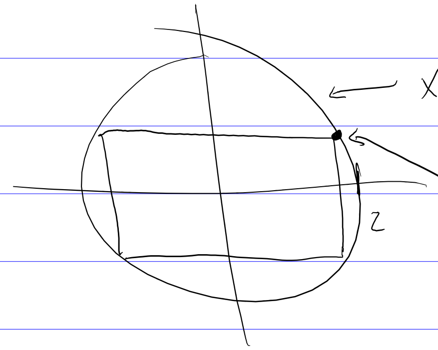
$w = 3$

$p''(w) = 36w^{-3}$ $p''(3) > 0$

$l = \frac{9}{w} = \frac{9}{3} = 3$

Both length and width should be 3 in.

#12)



$x^2+y^2=4$

$(x,y) \leftarrow$ pt in 1st quadrant

Width of rect is $2x$.
Ht of rect. is $2y$.

Area of rect is $(2x)(2y)=4xy$

maximize $A=4xy$

$$x^2+y^2=4$$
$$y=\pm\sqrt{4-x^2}$$
$$y=\sqrt{4-x^2}$$

$$A=4x\sqrt{4-x^2}$$
$$A'(x)=4x\cdot\frac{1}{2\sqrt{4-x^2}}\cdot(-2x)+\sqrt{4-x^2}\cdot(4)$$

$$4x\cdot\frac{1}{2\sqrt{4-x^2}}\cdot(-2x)+\sqrt{4-x^2}\cdot(4)=0$$

$$\frac{-4x^2}{\sqrt{4-x^2}}+4\sqrt{4-x^2}=0$$

mult by $\sqrt{4-x^2}$

$$-4x^2+4(4-x^2)=0$$
$$-4x^2+16-4x^2=0$$
$$16-8x^2=0$$
$$16=8x^2$$
$$2=x^2$$
$$x=\pm\sqrt{2}$$
$$x=\sqrt{2}$$

$$\text{Area}=4(\sqrt{2})\sqrt{4-(\sqrt{2})^2}$$

#13) d = dist between pt on parabola and $(1,4)$

(x,y)

x = x coord of pt on parabola

y = y coord of pt on parabola

$s = d^2$, the square of the distance (easier to deal with)

min d.

$$d = \sqrt{(x-1)^2 + (y-4)^2}$$

parabola: $y^2 = 2x$

$\frac{y^2}{2} = x$

$$d = \sqrt{\left(\frac{y^2}{2} - 1\right)^2 + (y-4)^2}$$

$$\begin{aligned} s &= \left(\frac{y^2}{2} - 1\right)^2 + (y-4)^2 \\ &= \frac{y^4}{4} - y^2 + 1 + y^2 - 8y + 16 \\ &= \frac{1}{4}y^4 - 8y + 17 \\ s' &= y^3 - 8 \\ 0 &= y^3 - 8 \\ 8 &= y^3 \\ 2 &= y \end{aligned}$$

$s''(y) = 3y^2$

$s''(2) = 3\cdot(2)^2 > 0$

By SDT, s has l. min at $y=2$.

$x = \frac{2^2}{2} = \frac{4}{2} = 2$

Closest pt on $y^2 = 2x$ is $(2,2)$.

The distance is $\sqrt{\left(\frac{(2)^2}{2} - 1\right)^2 + (2-4)^2}$.

14

$$\text{dist} = \sqrt{(x-0)^2 + (y-0)^2}$$
$$= \sqrt{x^2 + (17x+17)^2}$$

$$S = \text{dist}^2 = x^2 + (17x+17)^2$$

$$S' = 2x + 2(17x+17)\cdot 17$$
$$2x + 2(17x+17)\cdot 17 = 0$$
$$2x + 17^2\cdot 2x + 17^2\cdot 2 = 0$$
$$(2 + 17^2\cdot 2)x = -17^2\cdot 2$$

$$x = \frac{-17^2\cdot 2}{2+17^2\cdot 2}$$

$$\left(\frac{-17^2\cdot 2}{2+17^2\cdot 2},\ \underbrace{17\left(\frac{-17^2\cdot 2}{2+17^2\cdot 2}\right)+17}_{\text{y coord}}\right)$$