

$$\int \frac{1}{x^2 + 2x + 2} dx$$

Solution

The denominator of this fraction won't factor nicely, so there is no need to bother with partial fractions. Instead, let's complete the square in the denominator:

$$\int \frac{1}{x^2 + 2x + 2} dx = \int \frac{1}{x^2 + 2x + 1 + 1} dx = \int \frac{1}{(x + 1)^2 + 1} dx =$$

Let $u = x + 1$. Then $du = dx$. So, the integral above is equal to

$$\begin{aligned} \int \frac{1}{u^2 + 1} du &= \tan^{-1}(u) + C \\ &= \tan^{-1}(x + 1) + C. \end{aligned}$$