$$\int \sin^3 x \cos^{33} x \, dx$$

Solution 1

If we attemp to make $u = \sin x$, we will have to ensure that there is one factor of cosine left in the integral, doing something like this:

$$\int \sin^3 x \cos^{33} x \, dx = \int \sin^3 x \cos^{32} x \cos x \, dx = \int \sin^3 x (\cos^2 x)^{16} \cos x \, dx = \int \sin^3 x (1 - \sin^2 x)^{16} \cos x \, dx$$

Now let $u = \sin x$. Then $du = \cos x \, dx$, so the integral above is equal to

$$\int u^3 (1 - u^2)^{16} \, du.$$

The integral above can technically be done "by hand", but it would take a lot of time to do a sixteen-fold FOILing of $1 - u^2$, so consider instead the solution below:

Solution 2

From

$$\int \sin^3 x \cos^{33} x \, dx$$

because the power of sine and cosine are BOTH odd, it may be easier to do the following:

$$\int \sin^3 x \cos^{33} x \, dx = \int \sin x \sin^2 x \cos^{33} x \, dx = \int \sin x (1 - \cos^2 x) \cos^{33} x \, dx.$$

Let $u = \cos x$, so $du = -\sin x \, dx$ and the integral above is equal to

$$-\int (1 - u^2) u^{33} \, du = -\int u^{33} - u^{35} \, du = -\frac{u^{34}}{34} + \frac{u^{36}}{36} + C = -\frac{\cos^{34} x}{34} + \frac{\cos^{36} x}{36} + C.$$