$$\int \frac{x^2+3x+11}{x^2-1}\,dx$$

Solution

To integrate the rational function $\frac{x^2+3x+11}{x^2-1}$, since the numerator has power 2 and the denominator has power 2, we must do long division. The result of long division is

$$\frac{x^2+3x+11}{x^2-1} = 1 + \frac{3x+12}{x^2-1}$$

We apply partial fraction decomposition to

$$\frac{3x+12}{x^2-1} = \frac{3x+12}{(x+1)(x-1)}$$

Set up partial fractions

$$\frac{3x+12}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$$

So

$$3x+12 = A(x-1) + B(x+1)$$

By substituting,

- Using $x = 1$ gives us $B = \frac{15}{2}$
- Using $x = -1$ gives us $A = -\frac{9}{2}$.

So

$$\begin{aligned}\int \frac{x^2+3x+11}{x^2-1}\,dx &= \int 1 + \frac{3x+12}{x^2-1}\,dx \\ &= \int 1 - \frac{9}{2}\cdot\frac{1}{x+1} + \frac{15}{2}\cdot\frac{1}{x-1}\,dx \\ &= x - \frac{9}{2}\ln|x+1| + \frac{15}{2}\ln|x-1| + C.\end{aligned}$$