Does $\displaystyle\sum_{n=1}^{\infty}\left(\frac{\ln n}{9n-1}\right)^n$ diverge, converge absolutely, or converge conditionally?

Solution

$$
\begin{aligned}
L &= \lim_{n\to\infty} \sqrt[n]{|a_n|} \\
&= \lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{\ln n}{9n-1}\right)^n\right|} \\
&= \lim_{n\to\infty} \sqrt[n]{\left(\frac{\ln n}{9n-1}\right)^n} \\
&= \lim_{n\to\infty} \frac{\ln n}{9n-1} \\
&= \lim_{n\to\infty} \frac{\frac{1}{n}}{9} \text{ by L'hopital's rule} \\
&= \lim_{n\to\infty} \frac{1}{9n} \\
&= 0
\end{aligned}
$$

Since $L < 1$, by the Root Test, the series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{\ln n}{9n-1}\right)^n$ converges absolutely.