Does $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ diverge, converge absolutely, or converge conditionally?

- **Solution.** The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ is alternating.

  Let $a_n = \frac{(-1)^n}{n^2}$. Then $b_n = |a_n| = \frac{1}{n^2}$. The sequence b_n is decreasing and

  $$\lim_{n\to\infty} b_n = 0.$$

  By the Alternating Series Test, the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges.

  Does $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converge absolutely or conditionally? We study $\sum |a_n|$, namely $\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{n^2}\right|$, which is the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$, which converges by the p-seriee test, so $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges absolutely.

- **Solution.** We study $\sum |a_n|$, namely $\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{n^2}\right|$, which is the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$, which converges by the p-series test, so by the Absolute Convergence Test, $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges.

  Since $\sum |a_n|$ converges, $\sum a_n$ converges absolutely.