Optimization Problems – A step-by-step process
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1. Identify the quantities
   - Example: Volume, radius of can, height of can, amount of aluminum

2. Assign a variable to each quantity.
   - Example: V, r, h, A

3. How are the quantities related to each other? A picture usually helps. Write equation(s) to relate the quantities together.
   - Example: $V = \pi r^2 h$ and $A = \pi r^2 + \pi r^2 + 2\pi rh$

4. Identify **which** quantity should be maximized/minimized.
   - Example: To minimize the amount of aluminum material used, minimize the quantity A

5. Are there any restrictions/constraints on the quantities? Is there a quantity which is always fixed?
   - Example: $r \geq 0$ and $h \geq 0$, which forces $V \geq 0$ and $A \geq 0$. Also, V is fixed to be 500.

6. Take the equation in Step 3 which mentions the quantity to be maximized/minimized identified in Step 4. Solve for that variable.
   - Example: A is already solved for, but simplify to $A = 2\pi r^2 + 2\pi rh$.

7. Use the other equations to ged rid of all but one variable. (Incorporate any fixed values from Step 5.)
   - Example: In $A = 2\pi r^2 + 2\pi rh$ there are two variables, namely r and h. Use $500 = \pi r^2 h$ to obtain $h = \frac{500}{\pi r^2}$, so $A = 2\pi r^2 + 2\pi r \frac{500}{\pi r^2} = 2\pi r^2 + \frac{1000}{r}$.
     Now, $A = 2\pi r^2 + \frac{1000}{r}$ only has one variable, namely r.

8. Take the new equation as the defintion of a function and find the absolute minimum/maximum (keeping in mind the constraints from Step 5, usually on a closed interval).
   - Example: Find the absolute minimum of $A = 2\pi r^2 + \frac{1000}{r}$ with the constraint that r is in $[0, \infty)$. Think of A is the output (instead of y) and r as the input (instead of x).

9. There are several things to be careful about in the last step:
   (a) Don't forget to check the endpoints of a closed interval.
   (b) If the constraint isn't a closed and bounded interval of the form $[a, b]$, then check whether the point you have is a local maximum or local minimum using FDT or SDT. (It's always good to check this anyway!)
   (c) Be sure to include units.