$$\int \tan^5 x \sec^3 x \, dx$$

Solution

$$\begin{aligned}
\int \tan^5 x \sec^3 x \, dx &= \int \tan^4 x \sec^2 x \sec x \tan x \, dx \\
&= \int (\tan^2 x)^2 \sec^2 x \sec x \tan x \, dx \\
&= \int (\sec^2 x - 1)^2 \sec^2 x \sec x \tan x \, dx \\
&= \int (u^2 - 1)^2 u^2 \, du \qquad \text{using } u = \sec x \qquad du = \sec x \tan x \, dx \\
&= \int (u^4 - 2u^2 + 1) u^2 \, du \\
&= \int (u^6 - 2u^4 + u^2), du \\
&= \frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3} + C \\
&= \frac{\sec^7 x}{7} - \frac{2\sec^5 x}{5} + \frac{\sec^3 x}{3} + C.
\end{aligned}$$