$$\int x^3 \ln x \, dx$$

Solution

Integrate by parts with $u = \ln x$ and $dv = x^3 \, dx$. So $du = \frac{1}{x} \, dx$ and $v = \frac{x^4}{4}$. Thus,

$$\int x \ln x \, dx = \frac{x^4}{4} \ln x - \int \frac{x^4}{4} \cdot \frac{1}{x} \, dx = \frac{x^4}{4} \ln x - \frac{1}{4} \int x^3 \, dx = \frac{x^4}{4} \ln x - \frac{1}{4} \cdot \frac{x^4}{4} + C.$$