Does $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ diverge, converge absolutely, or converge conditionally?

- **Solution.** The series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ is alternating.

  Let $a_n = \frac{(-1)^n}{n}$. Then $b_n = |a_n| = \frac{1}{n}$. The sequence b_n is decreasing and

  $$\lim_{n \to \infty} b_n = 0.$$

  By the Alternating Series Test, $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges.

  Does $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converge absolutely or conditionally? We study $\sum |a_n|$,
  namely $\displaystyle\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{n} \right|$, which is the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$, which diverges by the p-
  series test, so $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges conditionally.