$$\int \tan^5 x \sec^4 x \, dx$$

Solution

$$\begin{aligned}
\int \tan^5 x \sec^4 x \, dx &= \int \tan^5 x \sec^2 x \sec^2 x \, dx \\
&= \int \tan^5 x (\tan^2 x + 1) \sec^2 x \, dx \\
&= \int u^5 (u^2 + 1) \, du \qquad \text{using } u = \tan x \qquad du = \sec^2 \, dx \\
&= \int u^7 + u^5 \, du \\
&= \frac{u^8}{8} + \frac{u^6}{6} + C \\
&= \frac{\tan^8 x}{8} + \frac{\tan^6 x}{6} + C.
\end{aligned}$$