$$\int \frac{1}{x^2+2x-3}\,dx$$

Solution

The denominator factors: $x^2+2x-3=(x+3)(x-1)$. So, we should apply partial fractions. We set up

$$\frac{1}{x^2+2x-3}=\frac{A}{x+3}+\frac{B}{x-1}$$

Multiplying both sides by $(x+3)(x-1)$, we get

$$1=A(x-1)+B(x+3)$$

Use the above equation twice:

- Plug $x=1$ in to the equation $1=A(x-1)+B(x+3)$ to get $B=\frac{1}{4}$.
- Plug $x=-3$ in to the equation $1=A(x-1)+B(x+3)$ to get $A=-\frac{1}{4}$.

So,

$$\frac{1}{x^2+2x-3}=\frac{-1/4}{x+3}+\frac{1/4}{x-1},$$

which I'll prefer to write (so the minus sign is part of the second term)

$$\frac{1}{x^2+2x-3}=\frac{1}{4}\cdot\frac{1}{x-1}-\frac{1}{4}\cdot\frac{1}{x+3}.$$

Back to the integral:

$$\begin{aligned}\int \frac{1}{x^2+2x-3}\,dx &= \int \frac{1}{4}\cdot\frac{1}{x-1}-\frac{1}{4}\cdot\frac{1}{x+3}\,dx\\ &= \frac{1}{4}\int \frac{1}{x-1}\,dx-\frac{1}{4}\int\frac{1}{x+3}\,dx\\ &= \frac{1}{4}\ln|x-1|-\frac{1}{4}\ln|x+3|+C.\end{aligned}$$

where the last two integrals are both done by substitution ($u=x-1$ for the first integral, and using $u=x+3$ for the second integral).