$$\int x^2 \sin x \, dx$$

Solution

Let $u = x^2$ and $dv = \sin x \, dx$. Then $du = 2x \, dx$ and $v = -\cos x$. So

$$\int x^2 \sin x \, dx = -x^2 \cos x + 2 \int x \cos x \, dx$$

We need to integrate $x \cos x$.

- To evaluate $\int x \cos x \, dx$, we let $u = x$ and $dv = \cos x \, dx$. Then $du = dx$ and $v = \sin x$. So

$$\int x \cos x \, dx = x \sin x - \int \sin x \, dx = x \sin x + \cos x + C.$$

Back to our original integral, by substituting the value of our side integral, we have

$$-x^2 \cos x + 2(x \sin x + \cos x) + C.$$