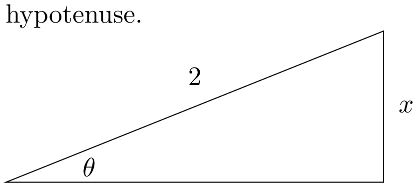
$$\int \sqrt{4-x^2}\,dx$$

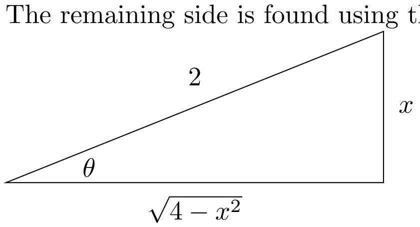
Solution

Because of seeing $\sqrt{a^2-x^2}$ in the integral with $a=2$, we apply trig substitution with $x=2\sin\theta$, for $-\frac{\pi}{2}\leq\theta\leq\frac{\pi}{2}$.

So, $dx=2\cos\theta\,d\theta$ and we use $\sin\theta=\frac{x}{2}$ to draw a right triangle with x as the opposite side at 2 as the hypotenuse.



The remaining side is found using the Pythagorean Theorem, and we have



So our integral becomes

$$\int \sqrt{4-x^2}\,dx=\int \sqrt{4-x^2}\,2\cos\theta\,d\theta$$

We still need to replace $\sqrt{4-x^2}$, so we look to the side of the triangle with that quantity (it is adjacent to θ) and the number side (which is the hypotenuse). Since $\cos\theta=\frac{\sqrt{4-x^2}}{2}$, we have $\sqrt{4-x^2}=2\cos\theta$. So, our integral becomes

$$\begin{aligned}\int 2\cos\theta\,2\cos\theta\,d\theta &= 4\int \cos^2\theta\,d\theta\\ &= 4\cdot\frac{1}{2}\int 1+\cos 2\theta\,d\theta\\ &= 2\int 1+\cos 2\theta\,d\theta\\ &= 2\theta+\sin 2\theta+C\\ &= 2\theta+2\sin\theta\cos\theta+C\\ &= 2\sin^{-1}\frac{x}{2}+2\cdot\frac{x}{2}\cdot\frac{\sqrt{4-x^2}}{2}+C.\end{aligned}$$