Does $\sum_{n=1}^{\infty} \frac{1}{n^2}$ diverge, converge absolutely, or converge conditionally?

Solution 1

The function $f(x) = \frac{1}{x^2}$ is continuous, positive, and decreasing on $[1, \infty)$. Now,

$$\int_1^{\infty} \frac{1}{x^2}\, dx = \lim_{t \to \infty} \int_1^t \frac{1}{x^2}\, dx = \lim_{t \to \infty} \left(\frac{-1}{t} - \frac{-1}{1} \right) = 1.$$

Since the integral $\int_1^{\infty} \frac{1}{x^2}\, dx$ converges, the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by the Integral Test.

Since $\sum |a_n| = \sum a_n$, the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges absolutely.

Solution 2

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a p-series with $p = 2$. Since $p > 1$, the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by the p-series Test.

Since $\sum |a_n| = \sum a_n$, the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges absolutely.