$$\int \tan^6 x \sec^4 x \, dx$$

Solution

$$\begin{aligned}
\int \tan^6 x \sec^4 x \, dx &= \int \tan^6 x \sec^2 x \sec^2 x \, dx \\
&= \int \tan^6 x (\tan^2 x + 1) \sec^2 x \, dx \\
&= \int u^6 (u^2 + 1) \, du \qquad \text{using } u = \tan x \qquad du = \sec^2 \, dx \\
&= \int u^8 + u^6 \, du \\
&= \frac{u^9}{9} + \frac{u^7}{7} + C \\
&= \frac{\tan^9 x}{9} + \frac{\tan^7 x}{7} + C.
\end{aligned}$$