

$$\int \sin^4 x \, dx$$

Solution

$$\begin{aligned}
 \int \sin^4 x \, dx &= \int \sin^2 x \sin^2 x \, dx \\
 &= \int \frac{1 - \cos 2x}{2} \cdot \frac{1 - \cos 2x}{2} \, dx \\
 &= \frac{1}{4} \int (1 - \cos 2x)^2 \, dx \\
 &= \frac{1}{4} \int 1 - 2 \cos 2x + \cos^2 2x \, dx \\
 &= \frac{1}{4} \int 1 - 2 \cos 2x + \frac{1 + \cos 4x}{2} \, dx \\
 &= \frac{1}{4} \int 1 \, dx - \frac{1}{2} \int \cos 2x \, dx + \frac{1}{8} \int 1 \, dx + \frac{1}{8} \int \cos 4x \, dx \\
 &= \frac{1}{4}x - \frac{1}{4} \sin 2x + \frac{1}{8}x + \frac{1}{32} \sin 4x + C.
 \end{aligned}$$

Where the integrals of $\cos 2x$ and of $\cos 4x$ are both done by substitution ($u = 2x$ or $u = 4x$).