$$\int \tan^{-1} x \, dx$$

## Solution

Apply integration by parts with $u = \tan^{-1} x$ and $dv = dx$. So $du = \dfrac{1}{1+x^2} dx$ and $v = x$. So

$$\int \tan^{-1} x \, dx = x \tan^{-1} x - \int \frac{x}{1+x^2} \, dx.$$

To do the integral

$$\int \frac{x}{1+x^2} \, dx$$

do substitution with $u = 1 + x^2$, so $du = 2x \, dx$, thus

$$\int \frac{x}{1+x^2} \, dx = \frac{1}{2} \int \frac{1}{u} \, du = \frac{1}{2} \ln |u| + C = \frac{1}{2} \ln |1 + x^2| + C.$$

Combining the work from earlier,

$$\int \tan^{-1} x \, dx = x \tan^{-1} x - \int \frac{x}{1+x^2} \, dx = x \tan^{-1} x - \frac{1}{2} \ln |1 + x^2| + C.$$