Does $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+1}$ diverge, converge absolutely, or converge conditionally?

Note

While the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+1}$ is alternating, the sequence $b_n = |a_n| = \frac{n}{n+1}$ satisfies NEITHER property: b_n is not decreasing and the sequence b_n does not have limit zero. So the Alternating Series Test doesn't apply. It is not at this point that we declare the series to diverge. Rather, trying the Alternating Series Test made sense because we had an alternating series, but the problem is that we got NO INFORMATION. We must try a different test.

Solution

$$\lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{(-1)^n n}{n+1}$$

does not exist (as we get every other term of this sequence to be a number close to 1 and every other term of this sequence to be a number close to -1). Since $\displaystyle\lim_{n \to \infty} a_n$ does not exist, the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+1}$ diverges by the Test for Divergence.