Does $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ diverge, converge absolutely, or converge conditionally?

Solution

$$\begin{aligned}
L &= \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| \\
&= \lim_{n\to\infty} \left| \frac{(n+1)^2}{2^{n+1}} \cdot \frac{2^n}{n^2} \right| \\
&= \lim_{n\to\infty} \left| \frac{n^2+2n+1}{2^n \cdot 2} \cdot \frac{2^n}{n^2} \right| \\
&= \lim_{n\to\infty} \left| \frac{n^2+2n+1}{2n^2} \right| \\
&= \lim_{n\to\infty} \frac{n^2+2n+1}{2n^2} \\
&= \lim_{n\to\infty} \frac{2n+2}{4n} \quad \text{by L'hopital} \\
&= \lim_{n\to\infty} \frac{2}{4} \quad \text{by another L'hopital} \\
&= \frac{2}{4} = \frac{1}{2}
\end{aligned}$$

Since $L < 1$, the series $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ converges absolutely by the Ratio Test.

Solution 2 (outline)

The Root Test will work, but it is a lot of work compared to the solution presented.