$$\int \sin^{-1} x \, dx$$

Solution

Apply integration by parts with $u = \sin^{-1} x$ and $dv = dx$. So $du = \dfrac{1}{\sqrt{1-x^2}} dx$ and $v = x$. So

$$\int \sin^{-1} x \, dx = x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} \, dx.$$

To do the integral

$$\int \frac{x}{\sqrt{1-x^2}} \, dx$$

do substitution with $u = 1 - x^2$, so $du = -2x \, dx$, thus

$$\int \frac{x}{\sqrt{1-x^2}} \, dx = -\frac{1}{2} \int u^{-1/2} \, du = -\frac{1}{2} \frac{u^{1/2}}{\frac{1}{2}} + C = -\sqrt{u} + C = -\sqrt{1-x^2} + C.$$

Combining the work from earlier,

$$\int \sin^{-1} x \, dx = x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} \, dx = x \sin^{-1} x + \sqrt{1-x^2} + C.$$