Does $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n^2}$ diverge, converge absolutely, or converge conditionally?

Solution

$$\begin{aligned}
\lim_{n\to\infty} a_n &= \lim_{n\to\infty} \frac{e^n}{n^2} \\
&= \lim_{n\to\infty} \frac{e^n}{2n} \quad \text{by l'Hopital's rule} \\
&= \lim_{n\to\infty} \frac{e^n}{2} \quad \text{by l'Hopital's rule} \\
&= \infty
\end{aligned}$$

so the series $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n^2}$ diverges by the Test for Divergence.