Does $\sum_{n=2}^{\infty} \frac{n+2}{n^2+1}$ diverge, converge absolutely, or converge conditionally?

Solution 1

The series $\sum \frac{1}{n}$ diverges by the p-test. Let $a_n = \frac{n+2}{n^2+1}$ and $b_n = \frac{1}{n}$.

$$\begin{aligned}
\lim_{n\to\infty} \frac{a_n}{b_n} &= \lim_{n\to\infty} \frac{n^2+2n}{n^2+1} \\
&= \lim_{n\to\infty} \frac{2n+2}{2n} \text{ by l'hopital} \\
&= \lim_{n\to\infty} \frac{2}{2} \text{ by l'hopital} \\
&= 1
\end{aligned}$$

The Limit Comparison Test applies, since this limit was a positive, finite number. Therefore, the series $\sum \frac{n+2}{n^2+1}$ diverges by the Limit Comparison Test.

Solution 2

The Direct Comparison Test will work, but will involve finding a $K > 0$ such that

$$\frac{n+2}{n^2+1} \geq K \cdot \frac{1}{n}$$