Does $\sum_{n=1}^{\infty} \frac{n^e}{n^2}$ diverge, converge absolutely, or converge conditionally?

Solution 1

$$\begin{aligned}
\lim_{n\to\infty} a_n &= \lim_{n\to\infty} \frac{n^e}{n^2} \\
&= \lim_{n\to\infty} \frac{en^{e-1}}{2n} \quad \text{by l'Hopital's rule} \\
&= \lim_{n\to\infty} \frac{e(e-1)n^(e-2)}{2} \quad \text{by l'Hopital's rule} \\
&= \infty
\end{aligned}$$

so the series $\sum_{n=1}^{\infty} \frac{n^e}{n^2}$ diverges by the Test for Divergence.

Solution 2

$\sum_{n=1}^{\infty} \frac{n^e}{n^2} = \sum_{n=1}^{\infty} n^{e-2} = \sum_{n=1}^{\infty} \frac{1}{n^{2-e}}$ is a p-series with $p = 2 - e$. Since $p \leq 1$, the series $\sum_{n=1}^{\infty} \frac{n^e}{n^2}$ diverges by the p-series test.