$$\int \sec^4 x \, dx$$

Solution

$$\begin{aligned}\int \sec^4 x \, dx &= \int \sec^2 x \sec^2 x \, dx \\ &= \int (\tan^2 x + 1) \sec^2 x \, dx.\end{aligned}$$

Use $u = \tan x$, so $du = \sec^2 x \, dx$ and the integral above is equal to

$$\int u^2 + 1 \, du = \frac{1}{3}u^3 + u + C = \frac{1}{3}\tan^3 x + \tan x + C.$$