$$\int_4^5 x^2(x^3+1)^9\,dx$$

indef : $\int x^2(x^3+1)^9\,dx = \int x^2 u^9 \frac{du}{3x^2} = \int \frac{1}{3}u^9\,du = \frac{1}{3}\frac{u^{10}}{10} + C = \frac{1}{3}\frac{(x^3+1)^{10}}{10} + C$

$u = x^3+1$

$\frac{du}{dx} = 3x^2$

$du = 3x^2\,dx$

$\frac{du}{3x^2} = dx$

$$\int_4^5 x^2(x^3+1)^9\,dx = \frac{1}{3}\frac{(5^3+1)^{10}}{10} - \frac{1}{3}\frac{(4^3+1)^{10}}{10}$$

$\boxed{\int_4^5 x^2 u^9 \frac{du}{3x^2}}$ ← ☹

$\boxed{\int_4^5 \frac{1}{3}u^9\,du}$ ← ☹

$$\int_2^7 \frac{x}{\sqrt{x+2}}\,dx$$

indef $\int \frac{x}{\sqrt{x+2}}\,dx = \int \frac{x}{\sqrt{u}}\,du = \int \frac{u-2}{\sqrt{u}}\,du = \int \frac{u}{\sqrt{u}} - \frac{2}{\sqrt{u}}\,du = \int u^{1/2} - 2u^{-1/2}\,du$

$u = x+2$ → $u-2 = x$

$\frac{du}{dx} = 1$

$du = dx$

$= \frac{u^{3/2}}{3/2} - 2\frac{u^{1/2}}{1/2} + C$

$= \frac{(x+2)^{3/2}}{3/2} - 2\frac{(x+2)^{1/2}}{1/2} + C$

$$\int_2^7 \frac{x}{\sqrt{x+2}}\,dx = \left(\frac{(7+2)^{3/2}}{3/2} - 2\cdot\frac{(7+2)^{1/2}}{1/2}\right) - \left(\frac{(2+2)^{3/2}}{3/2} - 2\cdot\frac{(2+2)^{1/2}}{1/2}\right)$$

$\boxed{\int_2^7 \frac{x}{\sqrt{u}}\,du}$ ← ☹

$\boxed{\int_2^7 \frac{u-2}{\sqrt{u}}\,du}$ ← ☹

$$\int_2^4 \frac{3x^2+3}{x^3+3x}\,dx$$

indef $\displaystyle\int \frac{3x^2+3}{x^3+3x}\,dx = \int \frac{3x^2+3}{u}\,\frac{du}{3x^2+3} = \int \frac{1}{u}\,du = \ln|u| + C = \ln|x^3+3x| + C$

$u = x^3+3x$

$\frac{du}{dx} = 3x^2+3$

$du = (3x^2+3)\,dx$

$\frac{du}{3x^2+3} = dx$

$$\int_2^4 \frac{3x^2+3}{x^3+3x}\,dx = \ln|4^3+3(4)| - \ln|2^3+3(2)|$$

$\boxed{\int_2^4 \frac{3x^2+3}{u}\,\frac{du}{3x^2+3}}$:($\boxed{\int_2^4 \frac{1}{u}\,du}$:(

These are not u-values, but this says du

$$\int_e^{e^e} \frac{1}{x\ln x}\,dx$$

$u = \ln x$ so $\frac{du}{dx} = \frac{1}{x}$ so $du = \frac{dx}{x}$ so $x\,du = dx$

indef $\displaystyle\int \frac{1}{x\ln x}\,dx = \int \frac{1}{xu}\,x\,du = \int \frac{1}{u}\,du = \ln|u| + C = \ln|\ln x| + C$

e to the _ is e^e? e to the _ is e?

$$\int_e^{e^e} \frac{1}{x\ln x}\,dx = \ln|\ln e^e| - \ln|\ln e|$$

e to the _ is 1?

$$= \ln|e| - \ln|1| = \ln e - \ln 1$$

$$= 1 - 0$$

$$= 1$$

$\boxed{\int_e^{e^e} \frac{1}{xu}\,x\,du}$:($\boxed{\int_e^{e^e} \frac{1}{u}\,du}$:(