Does $\displaystyle\sum_{n=1}^{\infty} \frac{\cos \pi n}{n}$ diverge, converge absolutely, or converge conditionally?

Solution

The series $\displaystyle\sum_{n=1}^{\infty} \frac{\cos \pi n}{n}$ is an alternating series. Note $b_n = |a_n| = \frac{1}{n}$. Since the sequence b_n is decreasing and $b_n \to 0$, by the Alternating Series Test, $\displaystyle\sum_{n=1}^{\infty} \frac{\cos \pi n}{n}$ converges.

To figure out whether $\displaystyle\sum_{n=1}^{\infty} \frac{\cos \pi n}{n}$ converges absolutely or conditionally, we consider the series

$$\sum_{n=1}^{\infty} \left| \frac{\cos \pi n}{n} \right| = \sum_{n=1}^{\infty} \frac{1}{n}$$

which diverges by the p-test, so the series $\displaystyle\sum_{n=1}^{\infty} \frac{\cos \pi n}{n}$ converges conditionally.