$$\int \sin^3 x \cos x \, dx$$

Solution 1

$$\int \sin^3 x \cos x \, dx = \int \sin x \sin^2 x \cos x \, dx = \int \sin x (1 - \cos^2 x) \cos x \, dx.$$

Let $u = \cos x$, so $du = -\sin x \, dx$ and the integral above is equal to

$$-\int (1 - u^2) u \, du = -\int u - u^3 \, du = -\frac{u^2}{2} + \frac{u^4}{4} + C = -\frac{\cos^2 x}{2} + \frac{\cos^4 x}{4} + C.$$

Solution 2

From

$$\int \sin^3 x \cos x \, dx$$

because the power of sine and cosine are BOTH odd, it may be easier to do the following: let $u = \sin x$. Then $du = \cos x \, dx$, so the integral above is equal to

$$\int u^3 \, du = \frac{u^4}{4} + C = \frac{1}{4} \sin^4 x + C.$$