First go through the book and write down each of the series tests. (Be sure to clearly write the **requirements** for using each test. Some of the tests will immediately require you to study a sequence.) Compare and contrast: Take one row at a time. After completing a row, reflect on what test/strategy was needed for each series. Then make up an example which can be solved using the same test. In each case, determine if the series diverges, converges absolutely, or converges conditionally. **Click on a problem for a link to sample solutions.**

(a) Does the infinite sequence $a_n = \frac{1}{n}$ converge? Does the infinite series $\sum_{n=1}^{\infty} \frac{1}{n}$ converge?

(b) $\sum_{n=1}^{\infty} \frac{1}{n}$ $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ $\sum_{n=1}^{\infty} \frac{1}{n^{0.9}}$ $\sum_{n=1}^{\infty} \frac{1}{0.9^n}$ $\sum_{n=1}^{\infty} \frac{1}{n^2}$ $\sum_{n=1}^{\infty} \frac{1}{2^n}$ $\sum_{n=1}^{\infty} n^2$ $\sum_{n=1}^{\infty} 2^n$

(c) $\sum_{n=1}^{\infty} \frac{3}{2^n}$ $\sum_{n=1}^{\infty} \frac{3^n}{2^n}$ $\sum_{n=1}^{\infty} \frac{2^n}{3^n}$ $\sum_{n=1}^{\infty} \frac{e^e}{2^n}$ $\sum_{n=1}^{\infty} \frac{e^n}{2^n}$ $\sum_{n=1}^{\infty} \frac{n^e}{2^n}$ $\sum_{n=1}^{\infty} \frac{e^n}{n^2}$ $\sum_{n=1}^{\infty} \frac{n^e}{n^2}$

(d) $\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)^n$ $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n$ $\sum_{n=1}^{\infty} \left(\frac{8}{9}+\frac{1}{n}\right)^n$ $\sum_{n=1}^{\infty} \left(\frac{8n+1}{9n-1}+\frac{1}{n}\right)^n$ $\sum_{n=1}^{\infty} \left(\frac{\ln n}{9n-1}\right)^n$ $\sum_{n=1}^{\infty} \left(\frac{\sin n}{9n-1}\right)^n$

(e) $\sum_{n=2}^{\infty} \frac{\ln n}{n}$ $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{100}}$ $\sum_{n=2}^{\infty} \left(\frac{1}{\ln n} - \frac{1}{\ln(n+1)}\right)$ $\sum_{n=1}^{\infty} n^7 e^{-n^8}$

(f) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$

(g) $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$ $\sum_{n=2}^{\infty} \frac{2n}{n^2-1}$ $\sum_{n=2}^{\infty} \frac{n+2}{n^2-1}$ $\sum_{n=2}^{\infty} \frac{n}{n^3-1}$ $\sum_{n=2}^{\infty} \frac{n+2}{n^3-1}$ $\sum_{n=2}^{\infty} \frac{n^2+2}{n^3-1}$

(h) $\sum_{n=2}^{\infty} \frac{1}{n^2+1}$ $\sum_{n=2}^{\infty} \frac{2n}{n^2+1}$ $\sum_{n=2}^{\infty} \frac{n+2}{n^2+1}$ $\sum_{n=2}^{\infty} \frac{n}{n^3+1}$ $\sum_{n=2}^{\infty} \frac{n+2}{n^3+1}$ $\sum_{n=2}^{\infty} \frac{n^2+2}{n^3+1}$ $\sum_{n=2}^{\infty} \frac{\arctan n}{n^2+1}$

(i) $\sum_{n=1}^{\infty} \frac{1}{3n-1}$ $\sum_{n=1}^{\infty} \frac{1}{3n^5+n^4+n^3+n^2+n+1}$ $\sum_{n=1}^{\infty} \frac{15n^4+4n^3+3n^2+2n+1}{3n^5+n^4+n^3+n^2+n+1}$

(j) $\sum_{n=1}^{\infty} \frac{1}{5^n+1}$ $\sum_{n=1}^{\infty} \frac{1}{5^n-1}$ $\sum_{n=1}^{\infty} \frac{5^n \ln 5}{5^n-1}$

(k) $\sum_{n=1}^{\infty} \frac{2+\cos \pi n}{n}$ $\sum_{n=1}^{\infty} \frac{\cos \pi n}{n}$ $\sum_{n=1}^{\infty} \frac{\cos \pi n}{n^2}$ $\sum_{n=1}^{\infty} \frac{\cos \frac{\pi}{3} n}{n^2}$ $\sum_{n=1}^{\infty} \frac{2n+\cos n}{n^2+\sin n}$ $\sum_{n=1}^{\infty} \left(\frac{2\pi+\cos n}{n}\right)^n$

(l) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+1}$ $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+1}$ $\sum_{n=1}^{\infty} \frac{1}{(n+1)^n}$

(m) $\sum_{n=1}^{\infty} \frac{1}{n!}$ $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ $\sum_{n=1}^{\infty} n!$ $\sum_{n=1}^{\infty} \frac{n}{n!}$ $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$ $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ $\sum_{n=1}^{\infty} \frac{n!}{(2n)!}$

(n) $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ $\sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$ $\sum_{n=1}^{\infty} \frac{n^2+1}{n^3+7}$ $\sum_{n=1}^{\infty} \frac{n^2+1}{n^4+7}$ $\sum_{n=1}^{\infty} \frac{n^2-1}{n^3+7}$ $\sum_{n=1}^{\infty} \frac{n^2-1}{n^4+7}$

Techniques for series:

- If you can find the limit of s_n, the sequence of partial sums, do this, because this is the value of the series.
  - Sometimes, finding the limit of s_n is done after cancellations in a telescoping series.
  - Sometimes, finding the limit of s_n is done after partial fractions leads to cancellations in a telescoping series.
- Test for Divergence
- Geometric Series Test
- Integral Test
- p-series Test
- (Direct) Comparison Test
- Limit Comparison Test
- Alternating Series Test
- Absolute Convergence Test
- Ratio Test
- Root Test