$\int (\sin x + 6)^2 \, dx$

Solution

$$\int (\sin x + 6)^2 \, dx = \int (\sin^2 x + 12 \sin x + 36) \, dx$$

Off to the side, there are two ways to do $\int \sin^2 x \, dx$.

- The first option is to integrate by parts with $u = \sin x$ and $dv = \sin x \, dx$. So $du = \cos x \, dx$ and $v = -\cos x$. So

$$\begin{aligned} \int \sin^2 x \, dx &= -\cos x \sin x + \int \cos^2 x \, dx \\ &= -\cos x \sin x + \int 1 - \sin^2 x \, dx. \end{aligned}$$

  So

$$\int \sin^2 x \, dx = -\cos x \sin x + \int 1 - \sin^2 x \, dx$$

  and using some algebra, we get

$$\int \sin^2 x \, dx = \frac{-\cos x \sin x + x}{2} + C$$

- Using the trigonometric identity

$$\sin^2 \theta = \frac{1 - \cos(2\theta)}{2}$$

  we rewrite:

$$\int \sin^2 x \, dx = \int \frac{1}{2} \, dx - \frac{1}{2} \int \cos 2x \, dx$$

  The second integral is completed using a substitution of $u = 2x$, so the integral becomes equal to

$$\frac{1}{2}x - \frac{1}{4} \sin 2x + C.$$

So,

$$\int (\sin^2 x + 12 \sin x + 36) \, dx = \frac{-\cos x \sin x + x}{2} - 12 \cos x + 36x + C.$$

Alternately, using the second method of integrating the square of the sine function, the final answer is:

$$\int (\sin^2 x + 12 \sin x + 36) \, dx = \frac{1}{2}x - \frac{1}{4} \sin 2x - 12 \cos x + 36x + C.$$