$$\int \frac{1}{x^2+9}\,dx$$

Solution

The 9 causes a problem because the constant in the denominator for the derivative of arctangent is a 1, not a 9. Using some clever factoring/algebra,

$$\int \frac{1}{x^2+9}\,dx = \int \frac{1}{9(\frac{1}{9}x^2+1)}\,dx = \int \frac{1}{9((\frac{x}{3})^2+1)}\,dx$$

Let $u = \frac{x}{3}$. So $du = \frac{1}{3}\,dx$, and $dx = 3\,du$. Then

$$\int \frac{1}{9((\frac{x}{3})^2+1)}\,dx = \frac{1}{9}\int \frac{1}{u^2+1}3\,du = \frac{1}{3}\int \frac{1}{u^2+1}\,du = \frac{1}{3}\tan^{-1}(u)+C = \frac{1}{3}\tan^{-1}(\tfrac{x}{3})+C.$$