Does $\displaystyle\sum_{n=1}^{\infty} \frac{2 + \cos \pi n}{n}$ diverge, converge absolutely, or converge conditionally?

Solution

The terms of the series $\displaystyle\sum_{n=1}^{\infty} \frac{2 + \cos \pi n}{n}$ are positive. Note, in fact, that

$$\cos(\pi n) \geq -1$$

so by adding 2 to both sides,

$$2 + \cos(\pi n) \geq 1$$

thus

$$\frac{2 + \cos(\pi n)}{n} \geq \frac{1}{n}$$

Since the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by the p-test, the series $\displaystyle\sum_{n=1}^{\infty} \frac{2 + \cos \pi n}{n}$ dinverges by the Direct Comparison Test.