

Does $\sum_{n=1}^{\infty} \frac{1}{n!}$ diverge, converge absolutely, or converge conditionally?

Solution

$$\begin{aligned} L &= \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| \\ &= \lim_{n \rightarrow \infty} \left| \frac{1}{(n+1)!} \cdot \frac{n!}{1} \right| \\ &= \lim_{n \rightarrow \infty} \left| \frac{1}{(n+1) \cdot n!} \cdot \frac{n!}{1} \right| \\ &= \lim_{n \rightarrow \infty} \left| \frac{1}{n+1} \right| \\ &= \lim_{n \rightarrow \infty} \frac{1}{n+1} \\ &= 0. \end{aligned}$$

Since $L < 1$, by the Ratio Test, the series $\sum_{n=1}^{\infty} \frac{1}{n!}$ converges absolutely.