$$\int \cot^5 x \csc^3 x \, dx$$

Solution

$$\begin{aligned}
\int \cot^5 x \csc^3 x \, dx &= \int \cot^4 x \csc^2 x \csc x \cot x \, dx \\
&= \int (\cot^2 x)^2 \csc^2 x \csc x \cot x \, dx \\
&= \int (\csc^2 x - 1)^2 \csc^2 x \csc x \cot x \, dx \\
&= \int (u^2 - 1)^2 u^2 \, du \qquad \text{using } u = \csc x \qquad du = -\csc x \cot x \, dx \\
&= -\int (u^4 - 2u^2 + 1) u^2 \, du \\
&= -\int (u^6 - 2u^4 + u^2) \, du \\
&= -\left(\frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3} \right) + C \\
&= -\left(\frac{\csc^7 x}{7} - \frac{2 \csc^5 x}{5} + \frac{\csc^3 x}{3} \right) + C.
\end{aligned}$$