

$$\int x^2 \ln x \, dx$$

Solution

Integrate by parts with $u = \ln x$ and $dv = x^2 \, dx$. So $du = \frac{1}{x} \, dx$ and $v = \frac{x^3}{3}$. Thus,

$$\int x \ln x \, dx = \frac{x^3}{3} \ln x - \int \frac{x^3}{3} \cdot \frac{1}{x} \, dx = \frac{x^3}{3} \ln x - \frac{1}{3} \int x^2 \, dx = \frac{x^3}{3} \ln x - \frac{1}{3} \cdot \frac{x^3}{3} + C.$$