

$$\int \cos^4 x \, dx$$

Solution

$$\begin{aligned} \int \cos^4 x \, dx &= \int \cos^2 x \cos^2 x \, dx \\ &= \int \frac{1 + \cos 2x}{2} \cdot \frac{1 + \cos 2x}{2} \, dx \\ &= \frac{1}{4} \int (1 + \cos 2x)^2 \, dx \\ &= \frac{1}{4} \int 1 + 2 \cos 2x + \cos^2 2x \, dx \\ &= \frac{1}{4} \int 1 + 2 \cos 2x + \frac{1 + \cos 4x}{2} \, dx \\ &= \frac{1}{4} \int 1 \, dx + \frac{1}{2} \int \cos 2x \, dx + \frac{1}{8} \int 1 \, dx + \frac{1}{8} \int \cos 4x \, dx \\ &= \frac{1}{4}x + \frac{1}{4} \sin 2x + \frac{1}{8}x + \frac{1}{32} \sin 4x + C. \end{aligned}$$

Where the integrals of $\cos 2x$ and of $\cos 4x$ are both done by substitution ($u = 2x$ or $u = 4x$).