Does $\sum_{n=1}^{\infty} \frac{1}{3n^5 + n^4 + n^3 + n^2 + n + 1}$ diverge, converge absolutely, or converge conditionally?

Solution

Note that $3n^5 \geq n^5$. In fact, it is even further the case that

$$3n^5 + n^4 + n^3 + n^2 + n + 1 \geq n^5$$

and dividing both sides by $(n^5)(3n^5 + n^4 + n^3 + n^2 + n + 1)$, we get

$$\frac{1}{n^5} \geq \frac{1}{3n^5 + n^4 + n^3 + n^2 + n + 1}.$$

Since the series $\sum_{n=1}^{\infty} \frac{1}{n^5}$ converges by the p-series test, the series $\sum_{n=1}^{\infty} \frac{1}{3n^5 + n^4 + n^3 + n^2 + n + 1}$ converges by the Direct Comparison Test.

Since $\sum |a_n| = \sum a_n$, the series $\sum_{n=1}^{\infty} \frac{1}{3n^5 + n^4 + n^3 + n^2 + n + 1}$ converges absolutely.

Comment

This series can be studied using the Limit Comparison Test as well, but this is much more convenient using the Direct Comparison Test.