Does $\sum_{n=1}^{\infty} \frac{1}{n}$ diverge, converge absolutely, or converge conditionally?

- **Solution.** The series $\sum_{n=1}^{\infty} \frac{1}{n}$ is the harmonic series, thus diverges.
- **Solution.** Since $p = 1$, the series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by the p-series test.
- **Solution.** The function $f(x) = \frac{1}{x}$ is continuous, positive, and decreasing on $[1, \infty)$ and $f(n) = a_n$.

  Consider the integral $\int_1^{\infty} \frac{1}{x}\, dx$. This is an improper integral:

  $$\begin{aligned} \int_1^{\infty} \frac{1}{x}\, dx &= \lim_{t \to \infty} \int_1^{t} \frac{1}{x}\, dx \\ &= \lim_{t \to \infty} (\ln|t| - \ln|1|) \\ &= \lim_{t \to \infty} (\ln|t| - 0) \\ &= \infty \end{aligned}$$

  Since the integral $\int_1^{\infty} \frac{1}{x}\, dx$ diverges, the series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.