

Does $\sum_{n=1}^{\infty} \frac{n^e}{2^n}$ diverge, converge absolutely, or converge conditionally?

Solution

$$\begin{aligned} L &= \lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \lim_{n \rightarrow \infty} \sqrt[n]{\left| \frac{n^e}{2^n} \right|} \\ &= \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^e}{2^n}} \\ &= \lim_{n \rightarrow \infty} \frac{n^{e/n}}{2} \\ &= \frac{\lim_{n \rightarrow \infty} n^{e/n}}{\lim_{n \rightarrow \infty} 2} \\ &= \frac{1}{2} \end{aligned}$$

Since $L < 1$, the series $\sum_{n=1}^{\infty} \frac{n^e}{2^n}$ converges absolutely by the Root Test.