$$\int \frac{1}{x^2+2x-24}\,dx$$

Solution

Since the denominator of $\dfrac{1}{x^2+2x-24}$ factors as $x^2+2x-24=(x+6)(x-4)$, we set up a partial fraction decomposition:

$$\frac{1}{x^2+2x-24}=\frac{A}{x+6}+\frac{B}{x-4}.$$

Multiply both sides by $(x+6)(x-4)$ to get

$$1=A(x-4)+B(x+6)$$

- Using the equation above, substituting $x=4$, we have $1=0+B(4+6)$, and see that $B=\frac{1}{10}$.
- Reusing the above equation, substituting $x=-6$, we get $A=-\frac{1}{10}$.

So

$$\frac{1}{x^2+2x-24}=\frac{1}{10}\frac{1}{x-4}-\frac{1}{10}\frac{1}{x+6}.$$

Based on all the algebra above,

$$\begin{aligned}\int \frac{1}{x^2+2x-24}\,dx &= \int\left(\frac{1}{10}\frac{1}{x-4}-\frac{1}{10}\frac{1}{x+6}\right)dx\\ &= \frac{1}{10}\ln|x-4|-\frac{1}{10}\ln|x+6|+C,\end{aligned}$$

where each of the integrals is evaluated using substitution (either $u=x-4$ or $u=x+6$).