

$$\int_4^{\infty} \frac{6}{\sqrt{x}-1} dx$$

Intuition? $\frac{1}{\sqrt{x}-1} \approx \frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}}$. I know $\int_4^{\infty} \frac{1}{x^{1/2}} dx$ diverges $\left\{ \begin{array}{l} p = \frac{1}{2} \\ p \leq 1 \end{array} \right.$

So $\int_4^{\infty} \frac{6}{\sqrt{x}-1} dx$ probably diverges but to use the Comparison Theorem,

I need to find $g(x)$ such that $\frac{6}{\sqrt{x}-1} \geq g(x)$, and $\int_4^{\infty} g(x) dx$ diverges

$$-1 \leq 0$$

$$\sqrt{x}-1 \leq \sqrt{x}$$

$$\frac{1}{\sqrt{x}-1} \geq \frac{1}{\sqrt{x}}$$

$$\frac{6}{\sqrt{x}-1} \geq \frac{6}{\sqrt{x}}$$

$$\int_4^{\infty} \frac{6}{\sqrt{x}} dx = 6 \int_4^{\infty} \frac{1}{x^{1/2}} dx \text{ diverges}$$

By the Comparison Theorem, $\int_4^{\infty} \frac{6}{\sqrt{x}-1} dx$ diverges