Length of $y = \frac{1}{2}(e^x + e^{-x})$ for $0 \le x \le 2$

$f(x) = \frac{1}{2}(e^x + e^{-x})$ $\quad f'(x) = \frac{1}{2}(e^x - e^{-x})$ ← from Chain Rule

$$(f'(x))^2 = \frac{1}{4}(e^{2x} - 2 + e^{-2x})$$

$$1 + (f'(x))^2 = \frac{1}{4}\cdot 4 + \frac{1}{4}(e^{2x} - 2 + e^{-2x})$$
$$= \frac{1}{4}(4 + e^{2x} - 2 + e^{-2x})$$
$$= \frac{1}{4}(e^{2x} + 2 + e^{-2x})$$
$$= \frac{1}{4}(e^x + e^{-x})^2$$

so $\sqrt{1 + (f'(x))^2} = \frac{1}{2}(e^x + e^{-x})$

$$L = \int_0^2 \frac{1}{2}(e^x + e^{-x})\,dx$$

$u = -x$ so $du = -dx$

$$= \frac{1}{2}(e^x - e^{-x})\Big|_0^2$$
$$= \frac{1}{2}(e^2 - e^{-2}) - \frac{1}{2}(e^0 - e^{-0})$$