$$\int_3^8 \frac{1}{(x-3)(8-x)}\,dx = \int_3^6 \frac{1}{(x-3)(8-x)}\,dx + \int_6^8 \frac{1}{(x-3)(8-x)}\,dx$$

These could be 7s instead or 4.61108s.

$$= \underbrace{\lim_{t\to3^+}\int_t^6 \frac{1}{(x-3)(8-x)}\,dx}_{} + \underbrace{\lim_{t\to8^-}\int_6^t \frac{1}{(x-3)(8-x)}\,dx}_{}$$

indef $\int \frac{1}{(x-3)(8-x)}\,dx$

$$\frac{1}{(x-3)(8-x)} = \frac{A}{x-3} + \frac{B}{8-x}$$

$$1 = A(8-x) + B(x-3)$$

If $x=3$, $1 = 5A$ so $A = \frac{1}{5}$

If $x=8$, $1 = 5B$ so $B = \frac{1}{5}$

minus? $u = 8-x$ so $du = -dx$

$$= \int \frac{\frac{1}{5}}{x-3} + \frac{\frac{1}{5}}{8-x}\,dx = \frac{1}{5}\ln|x-3| - \frac{1}{5}\ln|8-x| + C$$

$$= \underbrace{\lim_{t\to3^+}\int_t^6 \frac{1}{(x-3)(8-x)}\,dx}_{} = \lim_{t\to3^+}\left(\left(\tfrac{1}{5}\ln|6-3| - \tfrac{1}{5}\ln|8-6|\right) - \left(\tfrac{1}{5}\ln\underbrace{|t-3|}_{\text{small positive}} - \tfrac{1}{5}\ln|8-t|\right)\right)$$

$\frac{1}{5}\ln|t-3| \to -\infty$

Integral diverges. We don't even have to look at this

If we did, we'd get an additional way of seeing the original integral diverges.