

1 Vectors in \mathbb{R}^n

1.1 Introduction

Definition

A vector is an element of the set $\vec{x} = \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ x_n \end{bmatrix}$

1.1.1 Properties of Vectors

I. **Equality:** $\vec{x} = \vec{u}$. If both belong to \mathbb{R}^n , then $x_1 = u_1, x_n = u_n$.

II. **Addition:** $\vec{x} + \vec{u} = \begin{bmatrix} v_1 + u_1 \\ \cdot \\ \cdot \\ v_n + u_n \end{bmatrix}$.

III. **Additive Inverse:** $\vec{x} + (-\vec{x}) = \vec{0}$

IV. **Commutativity:** $\vec{x} + \vec{u} = \vec{u} + \vec{x}$.

V. **Associativity:** $\vec{x} + \vec{u} + \vec{w} = \vec{u} + (\vec{x} + \vec{w})$.

VI. **Zero vector:** $\vec{0} = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$

VII. **Scalar Multiplication:** Let $c \in \mathbb{R}$: $c \cdot \vec{v} = \begin{bmatrix} cv_1 \\ \cdot \\ \cdot \\ cv_n \end{bmatrix}$

VIII. **Associativity of Multiplication:** Let $c, d \in \mathbb{R}$: $(cd) \vec{v} = c(d \vec{v})$

IX. **Distributive Property:** $c(\vec{x} + \vec{v}) = c\vec{x} + c\vec{v}$

X. **Inverse Property:** $0 \cdot \vec{v} = \vec{0}$

XI. If $c\vec{v} = \vec{0}$, then either $c = 0$ or $\vec{v} = \vec{0}$

Example

Problem: Prove XI

Solution:

Left side:

$$c\vec{v} = \vec{0} \iff \begin{bmatrix} cv_1 \\ . \\ cv_n \end{bmatrix} = \begin{bmatrix} 0 \\ . \\ 0 \end{bmatrix}$$

Right side:

$$\begin{aligned} c\vec{v} &= \vec{0} \\ \frac{1}{c}c\vec{v} &= \frac{1}{c}\vec{0} \\ \vec{v} &= \vec{0} \end{aligned}$$