Edward Song
January 11, 2022
Math 136

1 Vectors in $\mathbb{R}^n$

1.1 Introduction

Definition

A vector is an element of the set $\overrightarrow{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$

1.1.1 Properties of Vectors

I. **Equality:** $\overrightarrow{x} = \overrightarrow{u}$. If both belong to $\mathbb{R}^n$, then $x_1 = u_1, x_n = u_n$.

II. **Addition:** $\overrightarrow{x} + \overrightarrow{u} = \begin{bmatrix} v_1 + u_1 \\ \vdots \\ v_n + u_n \end{bmatrix}$.

III. **Additive Inverse:** $\overrightarrow{x} + (-\overrightarrow{x}) = \overrightarrow{0}$

IV. **Commutativity:** $\overrightarrow{x} + \overrightarrow{u} = \overrightarrow{u} + \overrightarrow{x}$.

V. **Associativity:** $\overrightarrow{x} + \overrightarrow{u} + \overrightarrow{w} = \overrightarrow{u} + (\overrightarrow{x} + \overrightarrow{w})$.

VI. **Zero vector:** $\overrightarrow{0} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$

VII. **Scalar Multiplication:** Let $c \in \mathbb{R}$: $c \cdot \overrightarrow{v} = \begin{bmatrix} cv_1 \\ \vdots \\ cv_n \end{bmatrix}$

VIII. **Associativity of Multiplication:** Let $c, d \in \mathbb{R} : (cd)\overrightarrow{v} = c(d\overrightarrow{v})$

IX. **Distributive Property:** $c(\overrightarrow{x} + \overrightarrow{v}) = c\overrightarrow{x} + c\overrightarrow{v}$

X. **Inverse Property:** $0 \cdot \overrightarrow{v} = \overrightarrow{0}$

XI. If $c\overrightarrow{v} = \overrightarrow{0}$, then either $c = 0$ or $\overrightarrow{v} = \overrightarrow{0}$

Example

Problem: Prove XI

Solution:

Consider two cases: $c \neq 0$ and $c = 0$

Case 1: $c = 0$

$$c\overrightarrow{v} = \overrightarrow{0} \iff \begin{bmatrix} cv_1 \\ \vdots \\ cv_n \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$$

Case 2: $c \neq 0$

$$\begin{aligned} c\overrightarrow{v} &= \overrightarrow{0} \\ \frac{1}{c}c\overrightarrow{v} &= \frac{1}{c}\overrightarrow{0} \\ \overrightarrow{v} &= \overrightarrow{0} \end{aligned}$$

Therefore, claim is true.