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Math 136

2 Spans, Sections, Lines

2.1 Linear Combinations and Span

Definition: Recall

Recall definition: Let $\overrightarrow{v_1}, \overrightarrow{v_2}, \cdots, \overrightarrow{v_n} \in \mathbb{F}^n$. A linear combination of $\overrightarrow{v_1} \cdots, \overrightarrow{v_n}$ is a vector of the form:

$$c_1\overrightarrow{v_1} + c_2\overrightarrow{v_2} + \cdots + c_n\overrightarrow{v_n}$$

Problem 1

1. In $\mathbb{R}^2$:
$$2\begin{bmatrix}1\\1\end{bmatrix} + (-1)\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$$
is a linear combination of $\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}1\\2\end{bmatrix}$
2. In $\mathbb{F}^n$
$\overrightarrow{v} = 1 \cdot \overrightarrow{v}$
$\overrightarrow{v} = 1 \cdot \overrightarrow{v} + 0\overrightarrow{u_1} + 0\overrightarrow{u_2} + \cdots + 0\overrightarrow{u_n}$
3. In $\mathbb{F}^n$ $\overrightarrow{0} = 0\overrightarrow{v_1} + \cdots + 0\overrightarrow{v_n}$

Problem 2

Let $\overrightarrow{v_1}, \cdots, \overrightarrow{v_n} \in \mathbb{F}^n$. The span of $\overrightarrow{v_1}, \cdots, \overrightarrow{v_n} \in \mathbb{F}^n$ is the set span $\{\overrightarrow{v_1}, \cdots, \overrightarrow{v_n}\} = \{c_1\overrightarrow{v_1} + c_2\overrightarrow{v_2} + \cdots + c_n\overrightarrow{v_n} : c_1, \cdots, c_n \in \mathbb{F}\}$

Warning:

$$span\{\overrightarrow{v_1}, \cdots, \overrightarrow{v_n}\} = c_1\overrightarrow{v_1} + \cdots + c_n\overrightarrow{v_n}$$

The left side is a set, while the right side is a vector.

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Problem 4

1. T/F: $\mathbb{R}^2 = span\{\overrightarrow{e_1}, \overrightarrow{e_2}\} = span\{\begin{bmatrix}1\\0\end{bmatrix}\}, \begin{bmatrix}1\\0\end{bmatrix}\}$

   Proof.

$$\begin{aligned}
&span\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\}\\
=&\{a\begin{bmatrix}1\\0\end{bmatrix} + b\begin{bmatrix}0\\1\end{bmatrix} : a, b \in \mathbb{R}\}\\
=&\{\begin{bmatrix}a\\b\end{bmatrix} : a, b \in \mathbb{R}\}\\
=&\mathbb{R}^2
\end{aligned}$$

$\square$

2. T/F: $\mathbb{R}^2 = span\{\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}3\\4\end{bmatrix}\}$

Problem 5

Evaluate $\int_0^2 |x^2 - 3x + 2| dx$

$$\begin{aligned}
\int_0^2 |x^2 - 3x + 2| dx &= \int_0^1 (x^2 - 3x + 2) dx + \int_1^2 (-x^2 + 3x - 2) dx\\
&= (\frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x)\Big|_0^1 + (-\frac{1}{3}x^3 + \frac{3}{2}x^2 - 2x)\Big|_1^2\\
&= (\frac{1}{3} - \frac{3}{2} + 2) - 0 + (-\frac{8}{3} + 6 - 4) - (-\frac{1}{3} + \frac{3}{2} - 2)\\
&= 1
\end{aligned}$$