Edward Song
January 17, 2022
Math 136

2 Spans, Sections, Lines

Lines and Planes in $\mathbb{R}^n$

Definition: Recall

A line in $\mathbb{R}^n$ through the origin is a set of the form:

$$\mathbb{L} = \{0 + \overrightarrow{d} : t \in \mathbb{R}\} = span\{\overrightarrow{d}\}$$

where $\overrightarrow{d} \in \mathbb{R}^n$ is non-zero. This is a direction vector.

Example

A line in $\mathbb{R}^n$ is a set of the form:

$$l = \{\overrightarrow{p} + t\overrightarrow{d} : t \in \mathbb{R}\}$$

where $\overrightarrow{p}, \overrightarrow{d} \in \mathbb{R}^n$ and $\overrightarrow{d} \neq \overrightarrow{0}$.

Example

$$l = \{\begin{bmatrix} 2 \\ 1 \end{bmatrix}\} + t\begin{bmatrix} 1 \\ 2 \end{bmatrix} : t \in \mathbb{R}$$

Convention: $\overrightarrow{v} = \begin{bmatrix} a \\ b \end{bmatrix}$ is on l if its terminal part (a, b) is on l. So for example, $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ are on $l \iff (1, 2)$ and $(2, 3)$ are on l.

Problem 4

True or False: The lines $l_1 = \{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} : t \in \mathbb{R}\}$, $l_2 = \{\begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix} + t\begin{bmatrix} 2 \\ 0 \\ -5 \end{bmatrix} : t \in \mathbb{R}\}$ are the same. Graph it and you see it does not work.

Algebraically: $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \in l_1$. Is $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \in l_2 \iff$ can 1 solve for t such that $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ -5 \end{bmatrix}$

Problem 4

True or False: The lines $l_1 = \{\begin{bmatrix}1\\0\end{bmatrix} + t\begin{bmatrix}1\\-1\end{bmatrix} : t \in \mathbb{R}\}$, $l_2 = \{\begin{bmatrix}-1\\2\end{bmatrix} + t\begin{bmatrix}-2\\2\end{bmatrix} : t \in \mathbb{R}\}$ are the same.

Algebraically: Suppose $\overrightarrow{d_1}\begin{bmatrix}1\\-1\end{bmatrix}$ and $\overrightarrow{d_2} = \begin{bmatrix}-2\\2\end{bmatrix}$

$\overrightarrow{d_2} =) -2\overrightarrow{d_1}$ are parallel. Now we need to check if they intersect.

$$\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}-1\\2\end{bmatrix} + t\begin{bmatrix}-2\\2\end{bmatrix}$$
$$\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}-1-2t\\2+2t\end{bmatrix}$$
$$1 = -1 - 2t$$
$$0 = 2 + 2t$$
$$t = -1$$

$\therefore \begin{bmatrix}1\\0\end{bmatrix}$ is on l_2 and they intersect.

Definition

A plane in $\mathbb{R}^n$ through the origin is a set of the form:

$$p = \{s\overrightarrow{u} + t\overrightarrow{v} : s, t \in \mathbb{R}\} = span\{\overrightarrow{u}, \overrightarrow{v}\}$$

where $\overrightarrow{u}, \overrightarrow{v} \in \mathbb{R}^n$, $\overrightarrow{u} \neq c\overrightarrow{v}$ for only $c \in \mathbb{R}$.

Example

$$span\{\begin{bmatrix}1\\0\\0\end{bmatrix}\}, \begin{bmatrix}1\\0\\0\end{bmatrix} = p$$

Definition

A plane in $\mathbb{R}^n$ is a set of the form:

$$p = \{\overrightarrow{w} + s\overrightarrow{u} + t\overrightarrow{v} : s, t \in \mathbb{R}\}$$

where $\overrightarrow{w}, \overrightarrow{u}, \overrightarrow{v} \in \mathbb{R}^n$ and $\overrightarrow{u} \neq c\overrightarrow{v}$ for any $c \in \mathbb{R}$.

Example

$$p = \{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \} + s \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} : s, t \in \mathbb{R}$$