Intermediate Data Analysis & Econometrics

Review Session 1

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APPLICATION:
ASSET DIVERSIFICATION

Application – Asset Diversification

- Let r denote the rate of return on a security, defined as:

$$r = \frac{p_1 + d - p_0}{p_0}$$

where

$$\begin{aligned} p_1 &= \text{price of security at end of time period,} \\ d &= \text{dividends (if any) paid during time period,} \\ p_0 &= \text{price of security at beginning of time period.} \end{aligned}$$

- Investors like high expected returns, but dislike risk.
  - Risk typically quantified by the variance of the return.
  - Risk-Return tradeoff.

Asset Diversification – Expected Returns and Risks

- Consider two Securities, Ford and Tesla stock, with returns r_F and r_T:

$$r_F = \begin{cases} .20 & \text{with probability } .5 \\ -.10 & \text{with probability } .5. \end{cases}$$

$$r_T = \begin{cases} .60 & \text{with probability } .5 \\ -.40 & \text{with probability } .5. \end{cases}$$

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How did we compute those expected values? Variances?

Asset Diversification – Expected Returns and Risks

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$$r_F = \begin{cases} .20 & \text{with probability } .5 \\ -.10 & \text{with probability } .5. \end{cases} \Rightarrow \begin{aligned} E[r_F] &= .05, \\ Var[r_F] &= .0225 \end{aligned}$$

$$r_T = \begin{cases} .60 & \text{with probability } .5 \\ -.40 & \text{with probability } .5. \end{cases} \Rightarrow \begin{aligned} E[r_T] &= .10, \\ Var[r_T] &= .25 \end{aligned}$$

Which has higher expected return?

Which has higher risk?

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Which asset is the better investment?

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Which has higher expected return?

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Which asset is the better investment?

What if you could hold both securities in a portfolio?

Asset Diversification – Risk and Return Tradeoff

```r
library(ggplot2)

Company <- c("Tesla", "Ford")
Er      <- c(0.1,0.05)
Var     <- c(0.25,0.0225)

df <- data.frame(Company,Er,Var)

ggplot(df, aes(x = Var, y = Er, color = Company)) +
    geom_point(size = 5) +
    theme_bw() + ggtitle("Risk-Return Tradeoff") +
    xlab("Variance") + ylab("Expected Returns")
```

Asset Diversification – Risk and Return Tradeoff

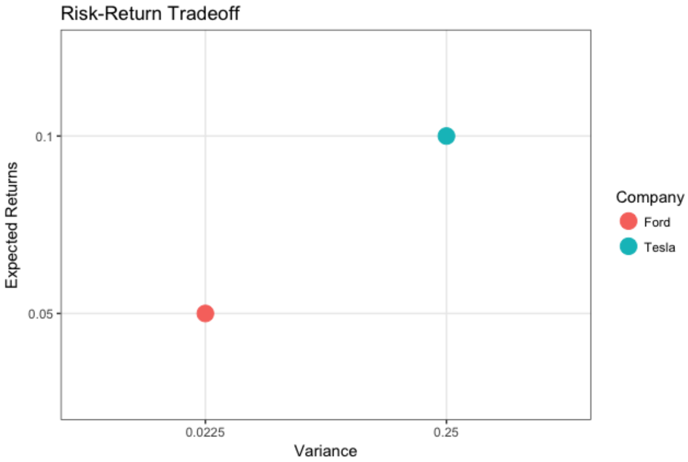

Figure 0.1: Two Risky Assets in the Variance-Expected Return Space

Asset Diversification – Expected Return on Portfolio

- Let w_T and w_F denote the fraction of funds invested in Tesla and Ford.
- Return on portfolio:
$$r_p = w_F \cdot r_F + w_T \cdot r_T.$$
- Expected return on portfolio:
$$E[r_p] = w_F \cdot E[r_F] + w_T \cdot E[r_T].$$

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- Expected return on portfolio:

$$E[r_p] = w_F \cdot E[r_F] + w_T \cdot E[r_T].$$

How does the expected return of holding the portfolio compare to holding just Ford or Tesla?

Asset Diversification – Risk of Portfolio

- Return on portfolio:

$$r_p = w_F \cdot r_F + w_T \cdot r_T.$$

- Variance of return on portfolio:

$$Var(r_p) = w_F^2 \cdot Var(r_F) + w_T^2 \cdot Var(r_T) + 2 \cdot w_F \cdot w_T \cdot Cov(r_F, r_T)$$

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Need joint distribution of (r_F, r_T) *to calculate* $Cov(r_F, r_T)$.

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Need joint distribution of (r_F, r_T) to calculate $Cov(r_F, r_T)$.

- We will calculate $Cov(r_F, r_T)$ under two scenarios:

  1. Main uncertainty is total demand for cars, implying strong positive correlation in returns for Ford and Tesla.
  2. Main uncertainty is nature of demand for cars, whether electric cars largely replace conventional cars, implying strong negative correlation in returns for Ford and Tesla.

First Scenario: Positive Return Covariance

- Suppose Ford's and Tesla's performance largely driven by overall demand for cars, with the probabilities of the four possible outcomes given by:

Returns	Ford .2	Ford -.1
Tesla .6	0.4	0.1
Tesla -.4	0.1	0.4

$$\begin{aligned} Cov(r_F, r_T) &= E[(r_F - E[r_F])(r_T - E[r_T])] \\ &= 0.4 \cdot (.2 - .05) \cdot (.6 - .1) + 0.1 \cdot (-.1 - .05) \cdot (.6 - .1) \\ &+ 0.1 \cdot (.2 - .05) \cdot (-.4 - .1) + .4 \cdot (-.1 - .05) \cdot (-.4 - .1) \\ &= 0.045 \end{aligned}$$

$$Corr(r_F, r_T) = .6$$

First Scenario: Positive Return Covariance (cont'd)

- Consider portfolio with equal investment in Ford and Tesla, $w_F = w_T = 1/2$.

$$E[r_p] = .5 \cdot E[r_F] + .5 \cdot E[r_T] = .5 \cdot .05 + .5 \cdot .1 = .075$$

$$\begin{aligned} Var[r_p] &= .5^2 \cdot Var[r_F] + .5^2 \cdot Var[r_T] + 2 \cdot .5 \cdot .5 \cdot Cov(r_F, r_T) \\ &= .5^2 \cdot .0225 + .5^2 \cdot .25 + 2 \cdot .5^2 \cdot .045 \\ &= .09 \end{aligned}$$

- Expected return on portfolio half way between expected return on Tesla and expected return on Ford.
- Variance of portfolio return in-between that on Ford and that on Tesla but not the average.
- How to evaluate risk-return tradeoff for portfolio vs. only Ford or Tesla?

First Scenario: Positive Return Covariance (cont'd)

```r
library(ggplot2)
er_F   <- 0.05   # Expected Returns
er_T   <- 0.1
var_F  <- 0.0225 # Variances (i.e. Risk)
var_T  <- 0.25
cov_FT <- 0.045  # Covariance

weights <- seq(from = 0, to = 1, length.out = 1000)
tab <- data.frame(wF = weights, wT = 1 - weights)
tab$er_p  <-  tab$wF * er_F + tab$wT * er_T
tab$var_p <-  tab$wF^2 * var_F +
              tab$wT^2 * var_T +
              2 * tab$wF * (1 - tab$wF) * cov_FT

ggplot() + geom_point(data = tab,
                      aes(x = var_p, y = er_p, color = wF)) +
    theme_bw() +
    ggtitle("Possible....") +
    xlab("Volatility") +
    ylab("Expected Returns")
```

First Scenario: Positive Return Covariance (cont'd)

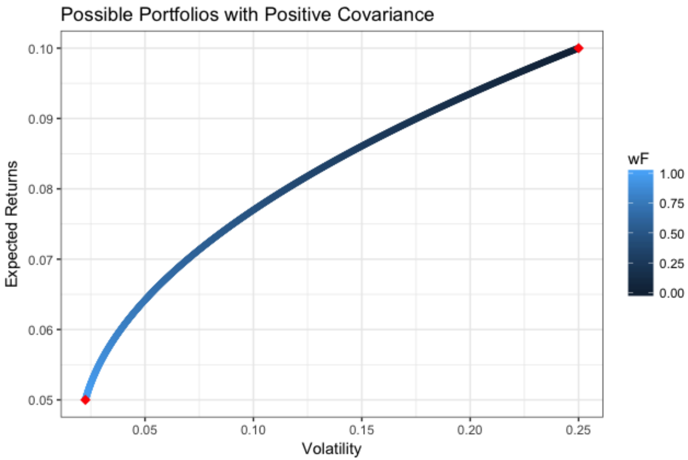


Figure 0.2: Portfolios of Two Positively Correlated Risky Assets

Second Scenario: Negative Return Covariance

- Suppose Ford's and Tesla's largely driven by whether electric cars become dominant in market place or demand for conventional cars continues to dominate, with the probabilities of the four possible outcomes given by:

Returns	Ford .2	Ford -.1
Tesla .6	.1	0.4
Tesla -.4	0.4	0.1

$$\begin{aligned} Cov(r_F, r_T) &= E[(r_F - E[r_F])(r_T - E[r_T])] \\ &= 0.1 \cdot (.2 - .05) \cdot (.6 - .1) + 0.4 \cdot (-.1 - .05) \cdot (.6 - .1) \\ &+ 0.4 \cdot (.2 - .05) \cdot (-.4 - .1) + .1 \cdot (-.1 - .05) \cdot (-.4 - .1) \\ &= -0.045 \end{aligned}$$

$$Corr(r_F, r_T) = -.6$$

Second Scenario: Negative Return Covariance (cont'd)

- Consider portfolio with equal investment in Ford and Tesla, $w_F = w_T = 1/2$.

$$E[r_p] = .5 \cdot E[r_F] + .5 \cdot E[r_T] = .5 \cdot .05 + .5 \cdot .1 = .075$$

$$\begin{aligned} Var[r_p] &= .5^2 \cdot Var[r_F] + .5^2 \cdot Var[r_T] + 2 \cdot .5 \cdot .5 \cdot Cov(r_F, r_T) \\ &= .5^2 \cdot .0225 + .5^2 \cdot .25 + 2 \cdot .5^2 \cdot (-.045) \\ &= .046 \end{aligned}$$

- Expected return on portfolio half way between expected return on Tesla and expected return on Ford.
- Recall variance on portfolio was .09 in first scenario, versus .046 here in second scenario. Explanation?
- How to evaluate risk-return tradeoff for portfolio vs. only Ford or Tesla?

Second Scenario: Negative Return Covariance (cont'd)

- Consider portfolio investing mostly in Ford but partially in Tesla, $w_F = .9, w_T = .1$.

$$E[r_p] = .9 \cdot E[r_F] + .1 \cdot E[r_T] = .9 \cdot .05 + .1 \cdot .1 = .055.$$

$$\begin{aligned} Var[r_p] &= .9^2 \cdot Var[r_F] + .1^2 \cdot Var[r_T] + 2 \cdot .9 \cdot .1 \cdot Cov(r_F, r_T) \\ &= .9^2 \cdot .0225 + .1^2 \cdot .25 + 2 \cdot .9 \cdot .1 \cdot (-.045) \\ &= .013 \end{aligned}$$

- In this scenario, would an investor ever wish to invest 100% in Ford?

Second Scenario: Negative Return Covariance (cont'd)

```
library(ggplot2)
er_F   <- 0.05   # Expected Returns
er_T   <- 0.1
var_F  <- 0.0225 # Variances (i.e. Risk)
var_T  <- 0.25
cov_FT <- -0.045  # Covariance !!---NOW NEGATIVE---!!

weights <- seq(from = 0, to = 1, length.out = 1000)
tab <- data.frame(wF = weights, wT = 1 - weights)
tab$er_p  <-  tab$wF * er_F + tab$wT * er_T
tab$var_p <-  tab$wF^2 *var_F +
              tab$wT^2 * var_T +
              2 * tab$wF * (1 - tab$wF) * cov_FT

ggplot() + geom_point(data = tab,
                      aes(x = var_p, y = er_p, color = wF)) +
    theme_bw() +
    gtitle("Possible....") +
    xlab("Volatility") +
    ylab("Expected Returns")
```

Second Scenario: Negative Return Covariance (cont'd)

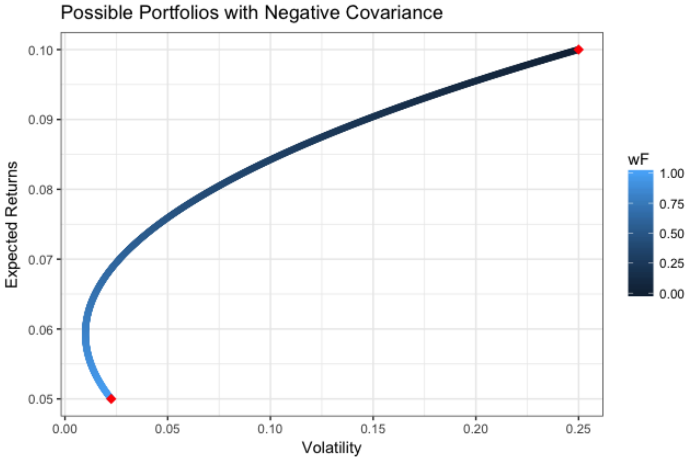
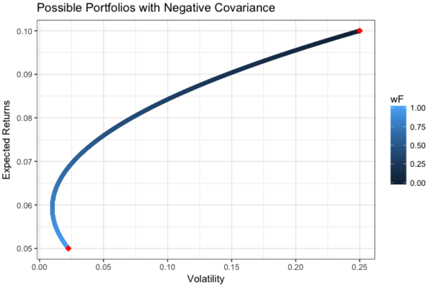
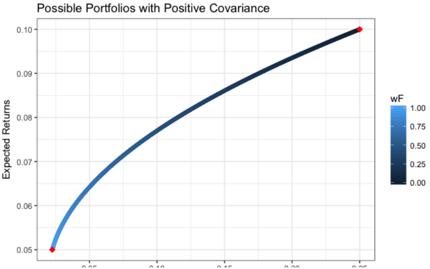


Figure 0.3: Portfolios of Two Negatively Correlated Risky Assets

Comparing the portfolios with pos. vs neg. covariance





Diversification, Insuranace

- For an investor initially only holding Ford, adding Telsa to portfolio provides insurance, with the lower the correlation between their returns the better the insurance.
- For an investor initially only holding Tesla, adding Ford to portfolio provides insurance, with the lower the correlation between their returns the better the insurance.

Diversification, Insuranace

- Diversification central issue in finance, institutional investing.
- Also important for individual investing, e.g.,
  - How much should I invest in stock vs. bonds?
  - How do the equity options my employer gives me affect my investment decision?
- Central issue in private insurance and for public provision of insurance.
- Related: Risk sharing in village economies.

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- Related: Risk sharing in village economies.

(*What about gambling?*)

Pricing of Assets

- Above discussion has implication for pricing of assets.
- Value of an asset to an investor depends not just on expected return and volatility of return, but on how return covaries with returns of other assets (market return).
- Investors willing to have lower expected return for asset that moves less with market returns.
- Investors require higher expected return to invest in asset whose return is strongly correlated with market return.
- One formal model: Capital Asset Pricing Model, will return to later in the course, including formal test of model and tests for excess returns (“alpha”).