# Reliability Analysis
# Homework 1

## Problem 1: Exercise 1.3

In the development and presentation of traditional statistical methods, description and inference are often presented in terms of means and variances (or standard deviations) of distributions.

(a) Use some of the examples in this chapter to explain why, in many applications, reliability or design engineers would be more interested in the time at which 1% (or some smaller percentage) of a particular component will fail instead of the time at which 50% will fail.

(b) Explain why means and variances of time to failure may not be of such high interest in reliability studies.

(c) Give at least one example of a product for which mean time to failure would be of interest. Explain why?

## Problem 2: Exercise 1.6

An important part of quantifying product reliability is specification of an appropriate time scale (or time scales) on which life should be measured (e.g., hours of operation, cycles of operation). For each of the products listed in Exercise 1.5, suggest and give reasons for an appropriate scale (or scales) on which one might measure life for the following products. Also, discuss possible environmental factors that might affect the lifetime of individual units.

<u>Exercise 1.5</u>
For each of the following products, explain your best understanding of the underlying failure mechanism. Also, describe possible ways in which an analyst could define failure.
(a) Painted surface of an automobile.
(b) Automobile lead–acid battery.
(c) Automobile windshield wipers.
(d) Automobile tires.
(e) LED light bulbs.

## Problem 3: Exercise 2.3

The transmission for the Model X automobile has a failure-time cdf

$$F(t) = 1 - \exp\left[-\left(\frac{t}{130}\right)^{2.5}\right],\ t > 0$$

where time is measured in thousands of miles. A Model X automobile with 120 thousand miles of previous service is being offered for sale.

(a) What is the probability that the automobile's transmission will fail before 150 thousand miles?

(b) What is the median of the automobile transmission's remaining-life distribution?

## Problem 4: Exercise 2.6

Consider a random variable with cdf $F(t) = t/2, 0 < t \le 2$. Do the following:

(a) Derive expressions for the corresponding pdf and hf.

(b) Use the results of part (a) to verify the relationship in (2.2)

$$F(t) = 1 - \exp[-H(t)] = 1 - \exp\left[-\int_0^t h(x)dx\right].$$

(c) Make a plot of the cdf and pdf.

(d)Make a plot of the hf. Give a clear intuitive reason for the behavior of $h(t)$ as $t \to 2$ from below. Hint: By the time $t = 2$, all units in the population must have failed.

(e) Derive an expression for $t_p$, the $p$ quantile of $F(t)$, and use this expression to compute $t_{0.4}$. Illustrate this on your plots of the cdf and pdf functions.

(f) Compute $\Pr(0.1 < T \le 0.2)$ and $\Pr(0.8 < T \le 0.9)$. Illustrate or indicate these probabilities on your graphs.

(g) Compute $\Pr(0.1 < T \le 0.2 \mid T > 0.1)$ and $\Pr(0.8 < T \le 0.9 \mid T > 0.8)$. Compare your answers with the approximation in (2.1)

$$h(t) \times \Delta t \approx \Pr(t < T \le t + \Delta t \mid T > t).$$

(h) Explain the results in part (g) and give a general result on the relationship between $\Pr(t < T < t + \Delta t \mid T > t)$ and the approximation in (2.1)

$$h(t) \times \Delta t \approx \Pr(t < T \le t + \Delta t \mid T > t).$$

## Problem 5: Exercise 2.10

An electronic system contains 20 copies of a particular integrated circuit that is at risk of failure during operation. The manufacturer of the component claims that the component's hf over the first 2 years of operation is 75 FITs. For the 1500 systems that are currently in operation, compute a prediction for the total number of these integrated circuits that will fail over the next 2 years of operation.

## Problem 6: Exercise 2.16

Consider the setting in Section 2.2.

(a) Derive the relationship in (2.6)

$$p_i = \Pr\left(t_{i-1} < T \leq t_i \mid T > t_{i-1}\right) = \frac{F\left(t_i\right) - F\left(t_{i-1}\right)}{1 - F\left(t_{i-1}\right)} = \frac{\pi_i}{S\left(t_{i-1}\right)}.$$

(b) Show that

$$\begin{aligned}
\pi_1 &= p_1, \\
\pi_i &= p_i \prod_{j=1}^{i-1}\left(1 - p_j\right), i = 2, \ldots, m, \\
\pi_{m+1} &= \prod_{j=1}^{m}\left(1 - p_j\right).
\end{aligned}$$

(c) Provide an argument to show that if $\pi_1 > 0, \ldots, \pi_{m+1} > 0$, then $0 < p_i < 1$ is the only restriction on the $p_i$ values for $i = 1, \ldots, m$.

(d) Derive the relationship in (2.7)

$$S\left(t_i\right) = \prod_{j=1}^{i}\left(1 - p_j\right), i = 1, \ldots, m+1.$$

## Problem 7: Exercise 2.18

Refer to equation (2.3)

$$L\left(t_0\right) = \mathrm{E}(U) = \frac{1}{1 - F\left(t_0\right)} \int_{t_0}^{\infty} [1 - F(z)] dz.$$

(a) Show that the cdf of the continuous random variable $T$ is related to the function $L(t)$ through the relationship

$$F(t) = 1 - \exp\left[-\int_0^t \frac{1 + L'(z)}{L(z)} dz\right]$$

where $L(t)$ is a differentiable function and $L'(z) = dL(z)/dz$.

(b) Use the result in (a) and the relationships between $F(t)$ and $S(t), f(t), h(t)$ in Section 2.1.1 to obtain expressions for $S(t), f(t), h(t)$ as function of $L(t)$ only. For example,

$$h(t) = \frac{1 + L'(t)}{L(t)}$$

## Problem 8:

Let $X \sim Bin(n,p)$, where $0 < p < 1$ is unknown. Show that the upper end point of the Clopper-Pearson interval is qbeta(1-$\alpha$/2, x+1, n-x).