Statistical Computing: Homework 1

Due: March 3 (Thursday) 8:30am

Develop a general algorithm to draw random samples from the following distribution families. Your write-up should include the following parts:

- Give the algorithm in detail steps (for a general parameter value).
- Draw a sample of size 1000 for a specific parameter setting, compare the empirical distribution of your data to the target pdf.
- Evaluate the efficiency (theoretically and empirically) of your algorithm for general parameter values (at least for a special case; better for general cases). Comment on your results.
- Make nice plots and summary tables to show your work.
- Submit your summary and code in two separate files.

(1) Weibull distribution:

$$F(x) = 1 - e^{-(x/\theta)^{\beta}}, \quad \theta > 0, \beta > 0, x > 0.$$

(2) pareto distribution:

$$f(x) = \frac{\beta}{\theta(1 + x/\theta)^{\beta+1}}, \quad \theta > 0, \beta > 0, x > 0.$$

(3) skewed distribution I:

$$f(x) = \frac{2}{\gamma + \frac{1}{\gamma}}\, \phi(x\gamma^{-sign(x)}), \quad x \in R, \gamma > 0,$$

where $\phi(x)$ is the pdf of $N(0,1)$. The parameter γ controls the degrees of asymmetry. In particular, $f(x)$ becomes symmetric when $\gamma = 1$.

(4) skewed distribution II (generated by hidden truncation):

$$f(x) = 2\underline{h(x)}G(\alpha x), \quad x \in R, \alpha \in R,$$

where $G(\cdot)$ is a cdf defined on R and symmetric around zero, and $\underline{h(\cdot)}$ is ~~a~~ also symmetric (with respect to zero) pdf defined on R. If $G'(x) = h(x)$, $f(x)$ is called the skew-"G" distribution. For example, when $G(\cdot)$ and $h(\cdot)$ are cdf and pdf of the same normal distribution, $f(x)$ is called skew-normal distribution.

(a) The skew-t distribution with parameter (α, ν): constructed by choosing $G(\cdot)$ and $h(\cdot)$ as the cdf and pdf of $t(\nu)$

(b) (optional) Make your own experiment to construct a family of skewed distributions based on the constructions given in Problems (3)–(4).

(5) a 2-dimension distribution:

$$f(x, y) = 2(1-x)(1-y)(1-xy)^{-3},\ 0 < x < 1,\ 0 < y < 1.$$