110024516 統研碩一 邱繼賢

3.5 (iv)

$$ARL_0 = \frac{1}{\alpha} = \frac{1}{0.0027} = 370.37$$

$$\beta = P\left(\mu_0 - Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{m}} \le \bar{X} \le \mu_0 + Z_{1+\frac{\alpha}{2}} \frac{\sigma}{\sqrt{m}} \middle| \mu = \mu_1\right)$$

$$= P\left(\frac{-1 \cdot \sqrt{5}}{0.8650} - 3 \le \frac{\bar{X} - \mu_1}{\sigma/\sqrt{m}} \le \frac{-1 \cdot \sqrt{5}}{0.8650} + 3\right)$$

$$\approx \Phi(0.415) - \Phi(-5.585) \approx 0.6609$$

$$ARL_1 = \frac{1}{1-\beta} \approx 2.9490$$

3.6 (i)

$\bar{\bar{X}} = 35.304$, $\bar{S} = \frac{1}{10}\sum_{i=1}^{10} S_i = 0.821$, $\alpha = 0.0027$

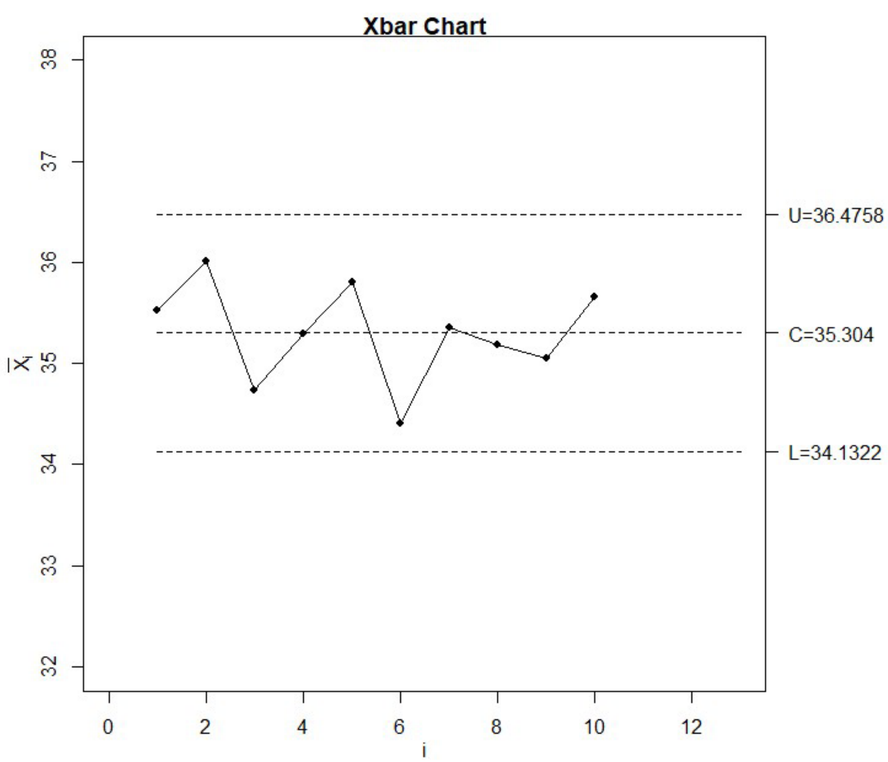
$d_3(5) = \frac{3}{4}\sqrt{\frac{\pi}{2}} \approx 0.94$

Control limits of the $\bar{X}$ chart:

$$U = \bar{\bar{X}} + \frac{Z_{1-\frac{\alpha}{2}}\ \bar{S}}{d_3(5)\ \sqrt{5}} \approx 36.4758$$

$$C = \bar{\bar{X}} = 35.304$$

$$L = \bar{\bar{X}} - \frac{Z_{1-\frac{\alpha}{2}}\ \bar{S}}{d_3(5)\ \sqrt{5}} \approx 34.1322$$



$\forall \bar{X}_i$, $i = 1, 2, \dots, 10$

$L < \bar{X}_i < U$

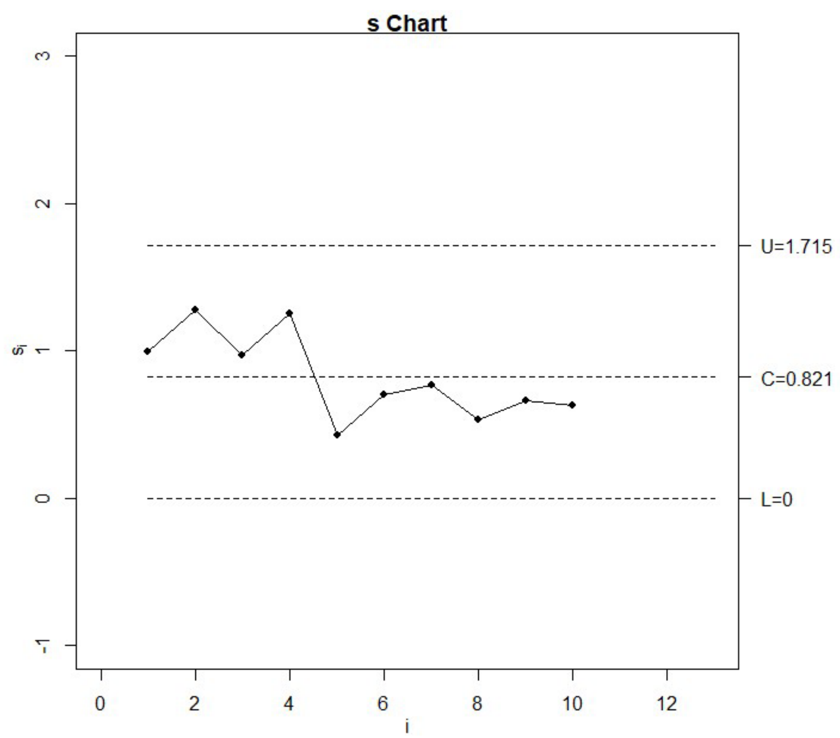
Control limits of the s Chart:

$$U = \left[1 + \frac{Z_{1-\frac{\alpha}{2}} \sqrt{1 - d_3^2(5)}}{d_3(5)} \right] \bar{s} \approx 1.715$$

$$C = \bar{s} = 0.821$$

$$L = \left[1 - \frac{Z_{1-\frac{\alpha}{2}} \sqrt{1 - d_3^2(5)}}{d_3(5)} \right] \bar{s} \approx -0.0729$$

$\Rightarrow$ make $L = 0$

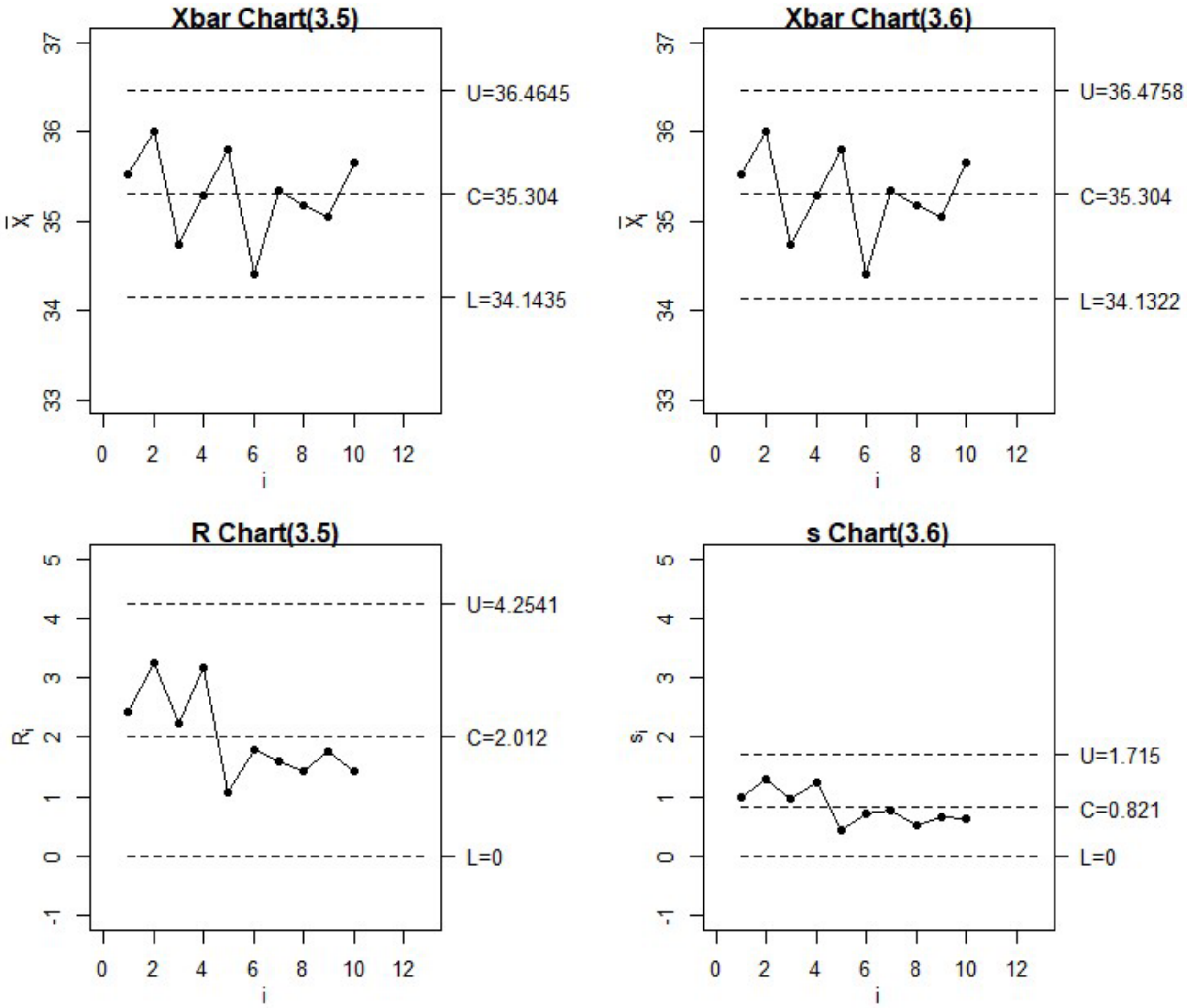


$\forall s_i, \ i = 1, \cdots, 10$

$L < s_i < U$

$\therefore$ The process seems to be in statistical control $\square$

(ii)



（1）兩題各自的 Xbar chart 並沒有太大的差別，中心相同，control limits 寬度也差不多。

（2）R chart 和 s chart 雖然是探討不同的統計量，但都是在呈現資料的離散程度，可以看到兩圖形有類似的上下波動，但 3.5 的 R chart 明顯變化幅度更大，control limits 寬度也更大。

(iii)

$$\hat{\sigma}_1 = \frac{\bar{R}}{d_1(5)} \approx 0.8650$$

$$\hat{\sigma}_2 = \frac{\bar{S}}{d_3(5)} \approx 0.8734$$

$$\hat{\sigma}_3 = \sqrt{\frac{1}{50-1} \sum_{\substack{1 \le i \le 10 \\ 1 \le j \le 5}} \left(X_{ij} - \bar{\bar{X}}\right)^2} \approx 0.9099$$

（1）第一種算法因為只需要知道一組樣本的range，所以方便於計算。

（2）第二種算法會使用到整個樣本的全部資訊，較為精準。

（3）第三種算法需要沒一組樣本都來自同一種分配時才能使用。

3.7 (i)

(a) $n=10$, $\tilde{\alpha}=0.001 \approx 1-(1-\alpha_1)^{10}$

$\Rightarrow \alpha_1 \approx 0.0001$

(b) $n=100$, $\tilde{\alpha}=0.001 \approx 1-(1-\alpha_2)^{100}$

$\Rightarrow \alpha_2 \approx 0.00001$

n 從 10 變成 100，後者為前者的十倍，同時 α_2 則變為 α_1 的十分之一，由此可見，隨著樣本變大時，在 overall FAR $\tilde{\alpha}$ 不變的情況下，individual α FAR 則會由此而變小。

因為 individual FAR $\alpha = P(\text{sample OC} \mid \text{process IC})$，隨著 α 因為樣本數增加而變小時，Xbar chart 的 control limits 範圍就會跟著變大，以此來降低樣本 out of control 的機率。

(ii)

(a) $n=10$, $\tilde{\alpha}=0.0001 \approx 1-(1-\alpha_1)^{10}$

$\Rightarrow \alpha_1 \approx 0.00001$

(b) $n=100$, $\tilde{\alpha}=0.0001 \approx 1-(1-\alpha_2)^{100}$

$\Rightarrow \alpha_2 \approx 0.000001$

將 overall FAR $\tilde{\alpha}$ 減少為原本的十分之一，則相對應的 individual FAR 也會同樣減少為原本的十分之一，由此可知 individual FAR 跟 individual FAR 之間有著非常明顯的正相關聯動。

綜合以上兩題可知，如果想要降低 individual FAR 可以選擇增加樣本數 n，或是降低 overall FAR $\tilde{\alpha}$，而降低 individual FAR 的同時也會導致 Xbar chart 的 control limits 的範圍變大。

3.8 (i)

$$\bar{\bar{X}} = \frac{1}{10}\sum_{i=1}^{10}\bar{X}_i = 79.609, \quad \bar{S} = \frac{1}{10}\sum_{i=1}^{10} S_i = 4.638$$

$d_3(5) \approx 0.94$, set $\alpha = 0.0027$

For $\bar{X}$ chart:

$$U = \bar{\bar{X}} + \frac{Z_{1-\frac{\alpha}{2}}\,\bar{S}}{d_3(5)\sqrt{5}} \approx 86.2287$$

$$C = \bar{\bar{X}} = 79.609$$

$$L = \bar{\bar{X}} - \frac{Z_{1-\frac{\alpha}{2}}\,\bar{S}}{d_3(5)\sqrt{5}} \approx 72.9893$$

For S chart:

$$U = \left(1 + \frac{Z_{1-\frac{\alpha}{2}}\sqrt{1-d_3^2(5)}}{d_3(5)}\right)\bar{S} \approx 9.6881$$

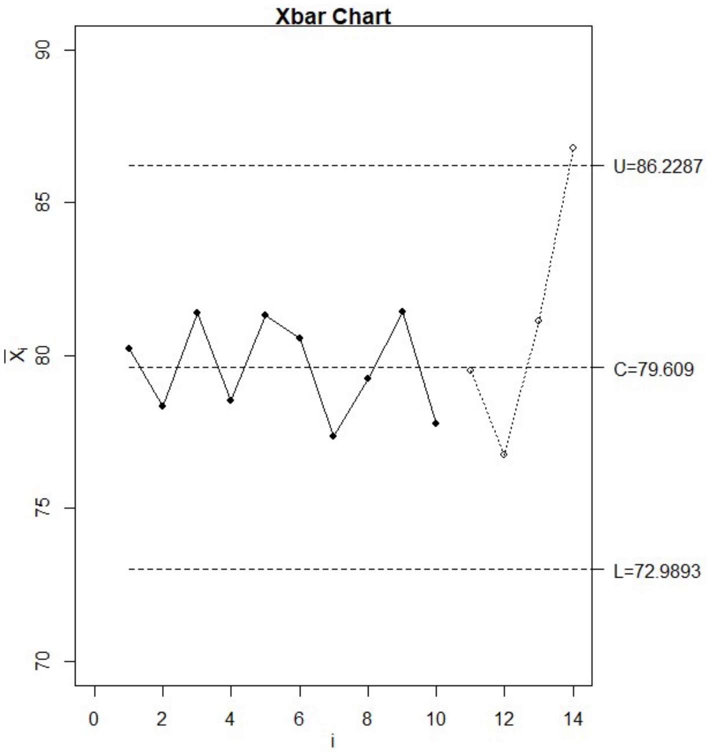
$$C = \bar{S} = 4.638$$

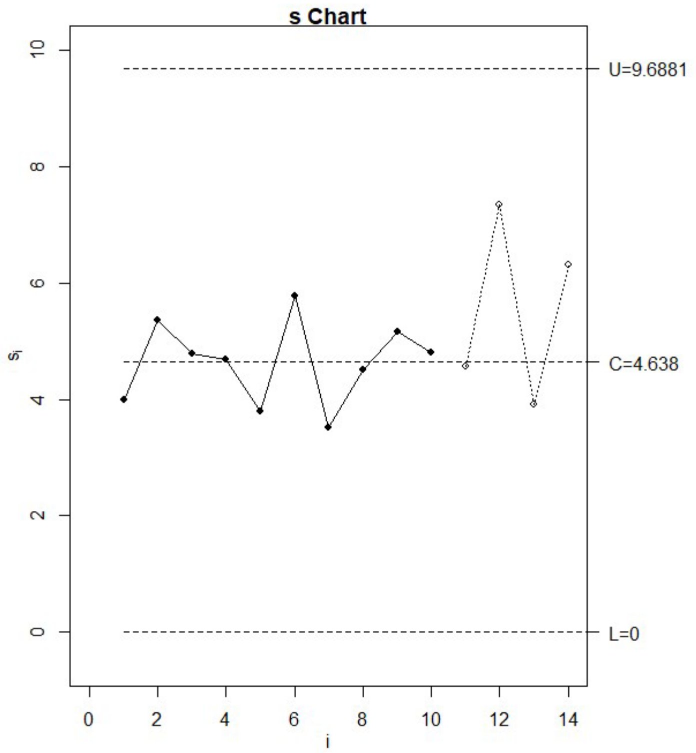
$$L = \left(1 - \frac{Z_{1-\frac{\alpha}{2}}\sqrt{1-d_3^2(5)}}{d_3(5)}\right)\bar{S} \approx -0.4121$$

$\Rightarrow$ let $L = 0$

$\because \forall i = 1, \ldots, 10$, $\bar{X}_i$ and S_i are both within their control limits

$\therefore$ The process seems to be in statistical control.





Since the process is IC at the first 10 time points, the charts are used for phase II process monitoring. We get a signal of process mean shift at the 14th time point, and the process should be stopped at that time point for investigation of possible special causes of the signal.

(ii) $\overline{S^2} = \frac{1}{10}\sum_{i=1}^{10} S_i^2 \approx 21.9654$, $\alpha = 0.0027$

$\chi^2_{1-\frac{\alpha}{2}}(4) = 17.8004$, $\chi^2_{\frac{\alpha}{2}}(4) = 0.1058$

Control limits of S^2 chart :

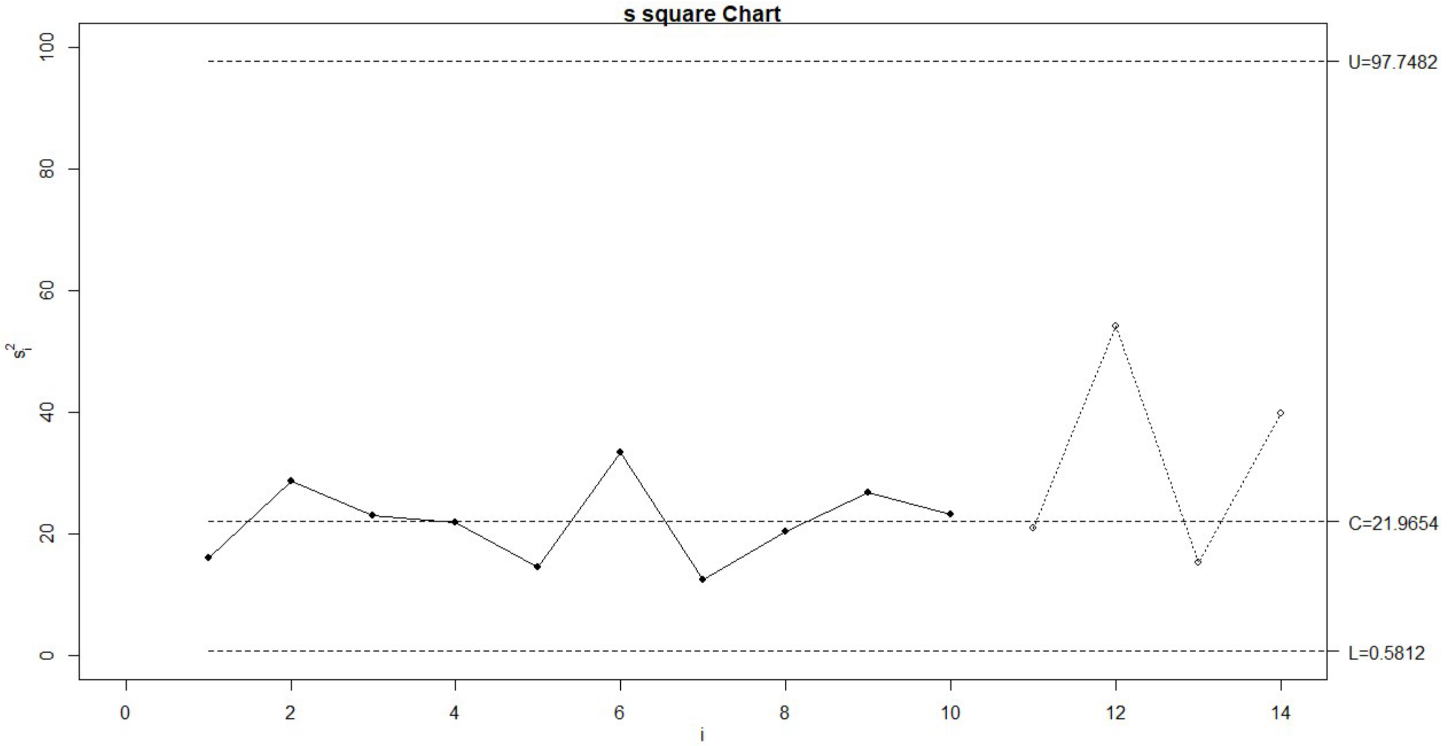
$$U = \frac{\overline{S^2}\,\chi^2_{1-\frac{\alpha}{2}}(4)}{4} \approx 97.7482$$

$$C = \overline{S^2} = 21.9654$$

$$L = \frac{\overline{S^2}\,\chi^2_{\frac{\alpha}{2}}(4)}{4} \approx 0.5812$$

$\because \forall\, i = 1, \ldots, 10$, $L < S_i^2 < U$

$\therefore$ The process variability seems to be in statistical control.



Since the process variability is IC at the first 10 time points, the chart is used for phase II process monitoring. We do not get any signals of process variability shift at the 11th~14th time points, so the process should continue doing phase II monitoring until we get a signal of process variability shift.

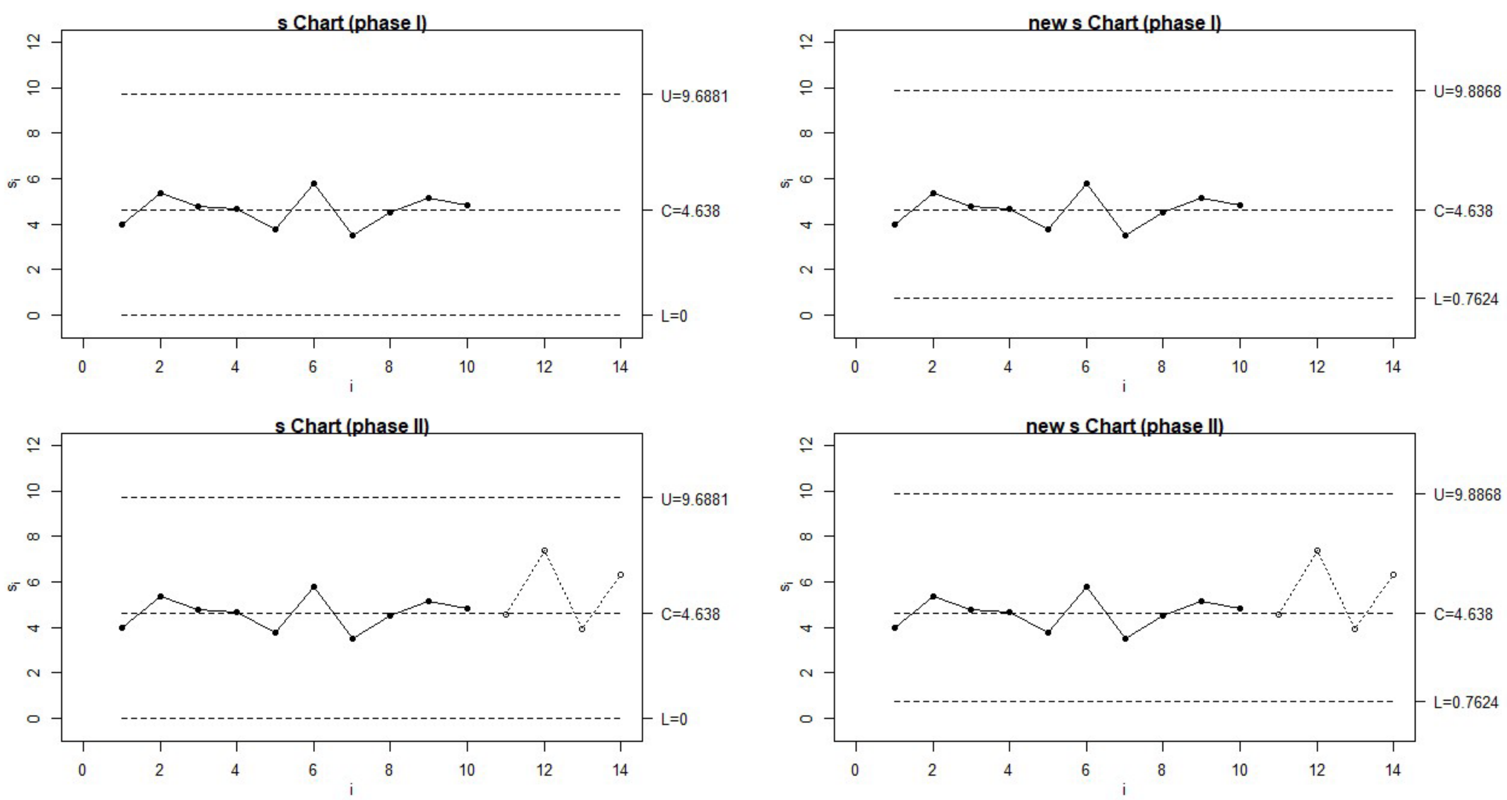
(iii)

The control limits of new version s chart:

$$U = \sqrt{97.7482} \approx 9.8868$$

$$C = \bar{S} = 4.638$$

$$L = \sqrt{0.5812} \approx 0.7624$$



兩種 s chart 的 upper control limit 並沒有太大的差距，而 lower control limit 比較有明顯的不同，原本的 s chart 在計算下界時會遇到負值的情況，在此情況下，要將其值取為 0，而新的 s chart 下界值因為是 s square chart 下界開根號，則不會有此狀況。

新的這種繪製 s chart 的方法，可以避免 lower control limit 計算出負值的情況，而且因為是將 s square chart 的 control limits 開根號，所以呈現出的數據和原始資料數據的單位是相同的，會比起 s square chart 更方便解讀。

3.9

The table for X_i, $MR_i(\tilde{m}=2)$, $MR_i(\tilde{m}=5)$:

	x	MR_2	MR_5
1	25	1	15
2	24	15	17
3	39	13	17
4	26	1	5
5	25	3	7
6	22	2	7
7	24	3	7
8	21	7	7
9	28	4	12
10	24	0	10
11	24	2	10
12	22	6	10
13	16	10	10
14	26	1	5
15	25	1	5
16	26	5	5
17	21	4	NA
18	25	2	NA
19	23	1	NA
20	24	NA	NA

(i) $\tilde{m}=2$, $\bar{X} = 24.5$, $\overline{MR} \approx 4.2632$

$d_1(2) = 1.128$, $d_2(2) = 0.853$, $\alpha = 0.0027$

For $\bar{X}$ chart:

$$U = \bar{X} + \frac{Z_{1-\frac{\alpha}{2}}}{d_1(\tilde{m})}\overline{MR} \approx 35.8383$$

$$C = \bar{X} = 24.5$$

$$L = \bar{X} - \frac{Z_{1-\frac{\alpha}{2}}}{d_1(\tilde{m})}\overline{MR} \approx 13.1617$$

For R chart:

$$U = \left(1 + \frac{z_{1-\frac{\alpha}{2}} d_2(\tilde{m})}{d_1(\tilde{m})}\right)\overline{MR} \approx 13.9348$$

$$C = \overline{MR} = 4.2632$$

$$L = \left(1 - \frac{z_{1-\frac{\alpha}{2}} d_2(\tilde{m})}{d_1(\tilde{m})}\right)\overline{MR} \approx -5.4084$$

$\Rightarrow$ Let $L = 0$

(ii) $\tilde{m} = 5$, $\bar{X} = 24.5$, $\overline{MR} \approx 9.3125$

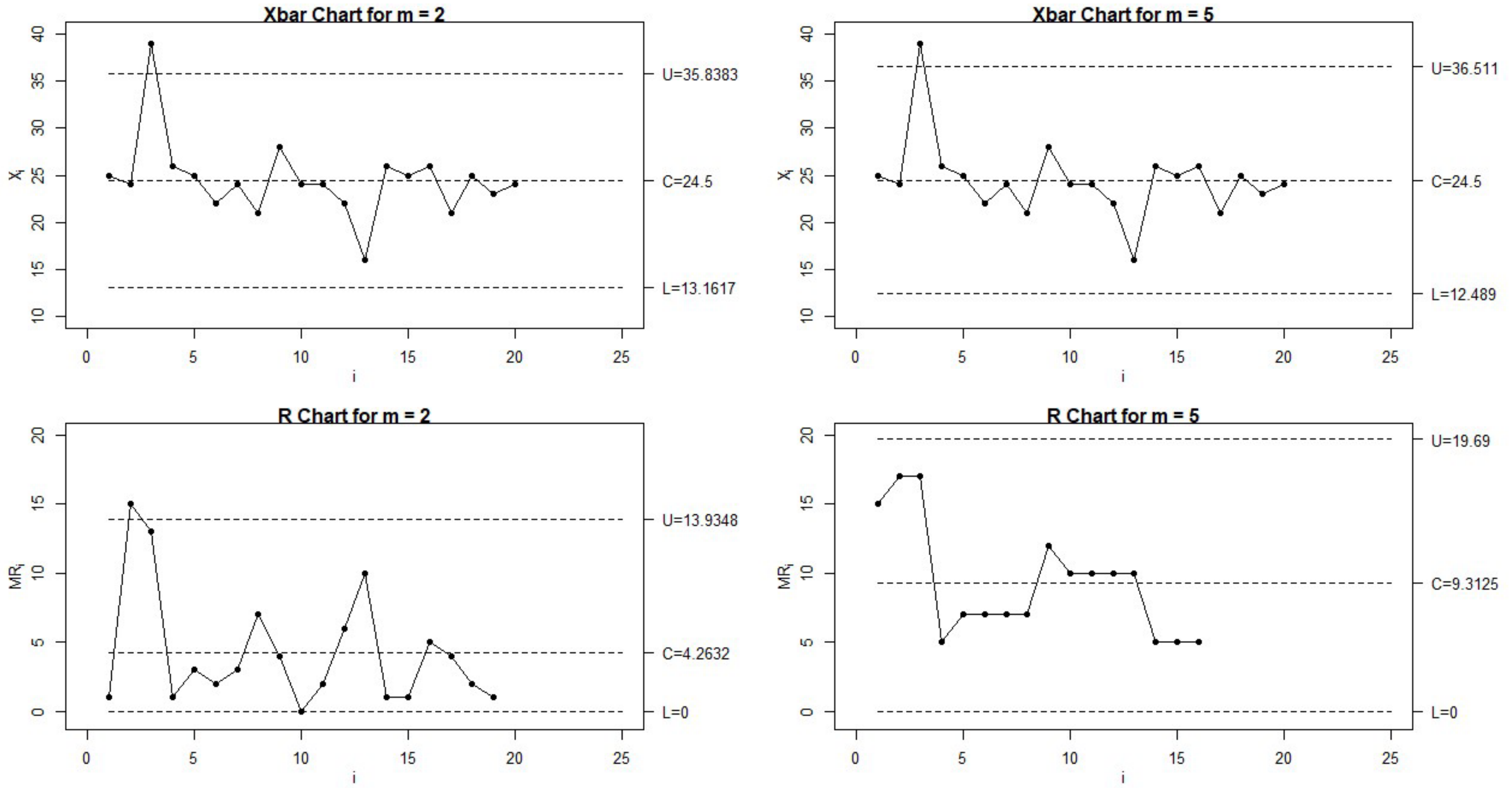
$d_1(5) = 2.326$, $d_2(5) = 0.864$

For $\bar{X}$ chart:

$U = 36.5110$, $C = 24.5$, $L = 12.4890$

For R chart:

$U = 19.690$, $C = 9.3125$, $L = 0$



（1）對於 Xbar chart 來說，將 $\tilde{m}$ 從 2 提升到 5，只有 control limits 的上下界範圍稍微變大一點點，第三筆資料依舊呈現為 out of control 。

（2）對於 R chart 在 $\tilde{m}$ 上升後，因為每一個 MR 會使用到更多筆的資料，所以整體圖形變得較為平滑（出現數組相連的數據點是同一水平高），而且 control limits 的上界也有明顯的提升，不再出現 out of control 的數據點。