Reliability Analysis
Homework 3

Problem 1: Exercise 6.7

A sample of 100 specimens of a titanium alloy were subjected to a fatigue test to determine time to crack initiation. The test was run up to a limit of 100,000 cycles. The observed times of crack initiation (in units of 1000 cycles) were $18, 32, 39, 53, 59, 68, 77, 78, 93$. No crack had initiated in any of the other 91 specimens. The data are in file Titanium01.csv.

(a) Compute a nonparametric estimate, $\widehat{F}(t)$, of the cdf $F(t)$ using both the simple binomial method and the Kaplan-Meier method (because the data are singly right censored, these two methods should provide the same answer).

(b) Plot $\widehat{F}(t)$ on linear axes.

(c) Use $\widehat{F}(t)$ to obtain plotting positions and plot the data on Weibull probability plot scales. Use the plot to obtain an estimate of the Weibull distribution shape parameter β.

(d) Comment on the adequacy of the Weibull distribution.

(e) Comment on the adequacy of the available data if the purpose of the experiment was to estimate $t_{0.10}$.

Problem 2: Exercise 6.9

Using the life-test data on silicon photodiode detectors from Exercise 3.12 (PhotoDetector.csv), construct probability plots for the exponential, Weibull, and lognormal distributions. Which distributions look like they might provide an adequate model for photodiode detector life?

Problem 3: Exercise 6.12

The FREC(μ, σ) distribution cdf is

$$F(t) = \Phi_{\text{lev}}\left[\frac{\log(t) - \mu}{\sigma}\right], \; t > 0, \; -\infty < \mu < \infty, \; \sigma > 0.$$

(a) Find the probability scales that will linearize all the cdfs in the Fréchet family.

(b) Use the scales to generate a properly labeled graph, and display the FREC$(1, 1)$ and FREC$(1, 2)$ cdfs.

(c) Which quantile of this distribution corresponds to the scale parameter $\exp(\mu)$?

Problem 4: Exercise 6.13

Suppose that T has cdf

$$F(t) = 1 - \left[1 + \left(\frac{\log(t) - \mu}{\sigma}\right)\right]^{-1}, \ t > \exp(\mu), \ -\infty < \mu < \infty, \ \sigma > 0.$$

(a) Find the p quantile t_p of the distribution.

(b) Find the probability scales that will linearize all the cdfs in this distribution family.

(c) Generate a simple random sample of size $n = 20$ from a population with $\mu = 3$ and $\sigma = 2$ and construct a probability plot using the scales derived in part (b). This is accomplished by ordering the sample points in increasing order $t_{(1)} \leq \cdots \leq t_{(20)}$. Then plot $t_{(i)}$ versus $(i - 0.5)/n$ on the derived axes.

(d) What do μ and σ represent in the probability plot?