品質管制 Homework3

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3.11 (i)
$\alpha = 0.0027, m = 100$
$X_i = 10, 15, 31, 18, 24, 12, 23, 15, 19, 21, 16, 24, 28, 15, 23, 19, 14, 27, 20, 18$
$\bar{p} = \frac{1}{20} \sum_{i=1}^{20} p_i = \frac{1}{20} \sum_{i=1}^{20} \frac{X_i}{m} = 0.196$

Construct the control limits of the p Chart in Large-Sample Cases:

$$U = \bar{p} + Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\bar{p}(1-\bar{p})}{m}} \approx 0.3151$$

$$C = \bar{p} = 0.196$$

$$L = \bar{p} - Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\bar{p}(1-\bar{p})}{m}} \approx 0.0769$$

The process seems to be in statistical control.

(ii)

$$P(p_i > \bar{p} + Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\bar{p}(1-\bar{p})}{m}} \mid \pi = 0.29) = 0.9 \Rightarrow P(Z > \frac{\bar{p} - \pi + Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\bar{p}(1-\bar{p})}{m}}}{\sqrt{\frac{\pi(1-\pi)}{m}}}) = 0.9$$

$$\Rightarrow \frac{\bar{p} - \pi + Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\bar{p}(1-\bar{p})}{m}}}{\sqrt{\frac{\pi(1-\pi)}{m}}} = \frac{(\bar{p}-\pi)\sqrt{m} + Z_{1-\frac{\alpha}{2}} \sqrt{\bar{p}(1-\bar{p})}}{\sqrt{\pi(1-\pi)}} = \frac{(0.196-0.29)\sqrt{m} + 3\sqrt{0.196(1-0.196)}}{\sqrt{0.29(1-0.29)}} = Z_{0.1}$$

$$\Rightarrow m = 355.5331 \Rightarrow take \ m = 356$$

3.13 (i)
$\alpha = 0.0027, m = 50$
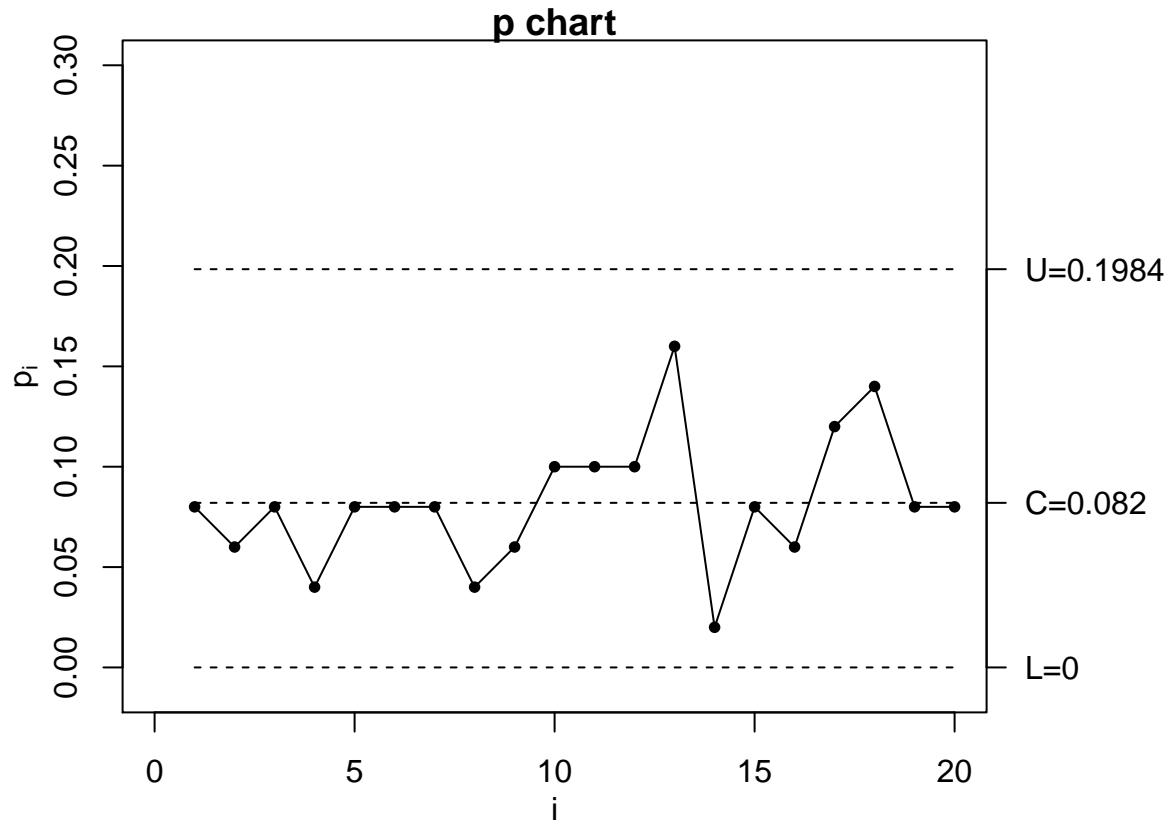$X_i = 4, 3, 4, 2, 4, 4, 4, 2, 3, 5, 5, 5, 8, 1, 4, 3, 6, 7, 4, 4$
$\bar{p} = \frac{1}{20} \sum_{i=1}^{20} p_i = \frac{1}{20} \sum_{i=1}^{20} \frac{X_i}{m} = 0.082$

Construct the control limits of the p Chart in Large-Sample Cases:

$$U = \bar{p} + Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\bar{p}(1-\bar{p})}{m}} \approx 0.1984$$

$$C = \bar{p} = 0.082$$

$$L = \bar{p} - Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\bar{p}(1-\bar{p})}{m}} \approx -0.0344 \ \rightarrow \ 0$$
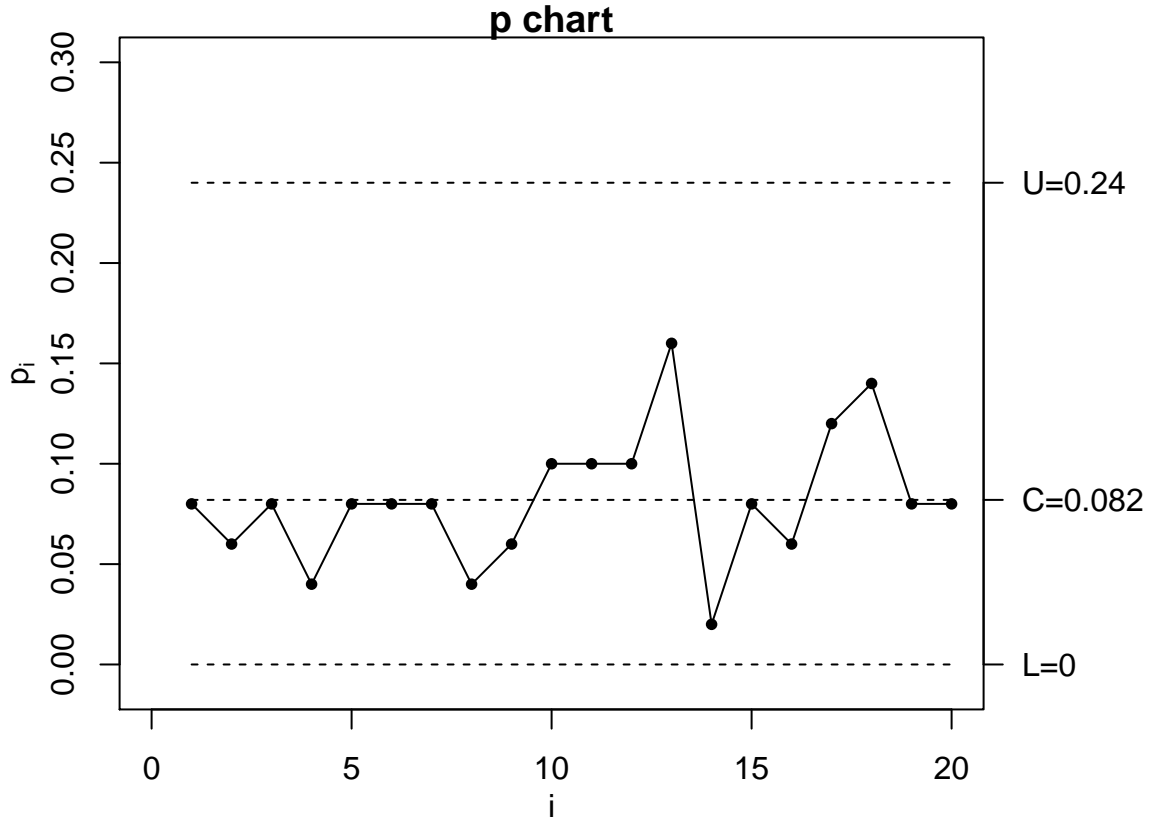
p chart

U=0.1984

C=0.082

L=0

The process seems to be in statistical control.

(ii)

$X_i \sim Bin(50, 0.082)$, $\alpha = 0.0027$, $m = 50$

$\because P(X \leq -1) = 0 < \frac{\alpha}{2}$ $\quad and \quad$ $P(X \geq 12) \approx 0.0006 < \frac{\alpha}{2}$

$\Rightarrow L^* = \frac{max\{a : P(X \leq a) \leq \frac{\alpha}{2}\}}{m} = -1 \rightarrow 0$, $U^* = \frac{min\{a : P(X \geq a) \leq \frac{\alpha}{2}\}}{m} = 0.24$

p chart

跟 (i) 題中的 p chart 相比，下界 L 一樣維持是 0，但上界 U 的數值變大了，因此會導致 α 的數值隨之變小，下一小題將會對此進行計算。

(iii)

$\tilde{\alpha} = P(X < mL^*) + P(X > mU^*) = P(X < 0) + P(X > 12) \approx 0.000147$
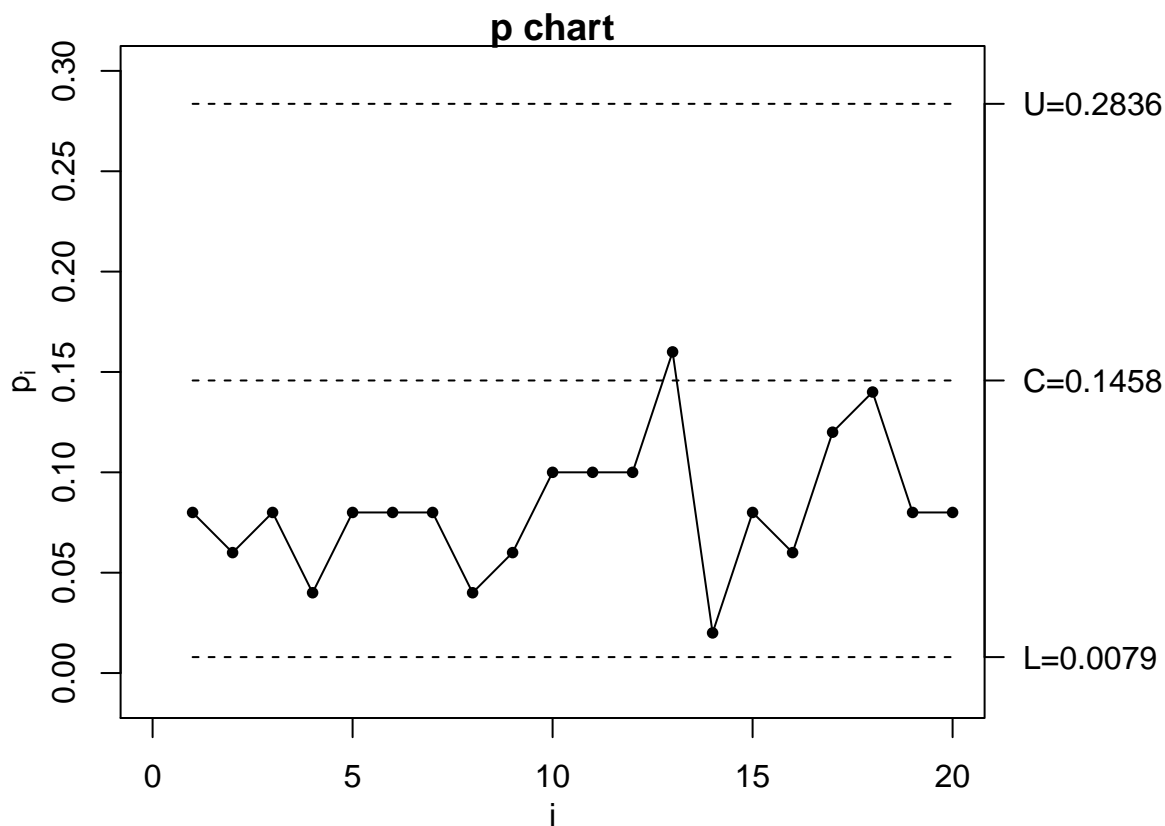
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(iv)

$\widetilde{m} = m + Z_{1-\frac{\alpha}{2}}^2 = 59$, $\tilde{p} = \frac{\bar{X} + Z_{1-\frac{\alpha}{2}}^2/2}{\widetilde{m}} \approx 0.1458$

The control limits of the p chart:

$$U = \tilde{p} + Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\tilde{p}(1-\tilde{p})}{\widetilde{m}}} \approx 0.2836$$

$$C = \tilde{p} \approx 0.1458$$

$$L = \tilde{p} - Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\tilde{p}(1-\tilde{p})}{\widetilde{m}}} \approx 0.0079$$

從此 p chart 可看出，U、L、C 三個數值跟前兩種 p chart 相比之下皆變大，特別是 C 的數值上升幅度最大，因此幾乎所有的資料都落在 C 值的下方，由此可以推得，此種繪製 p chart 的方法並不適合本題的資料數據。
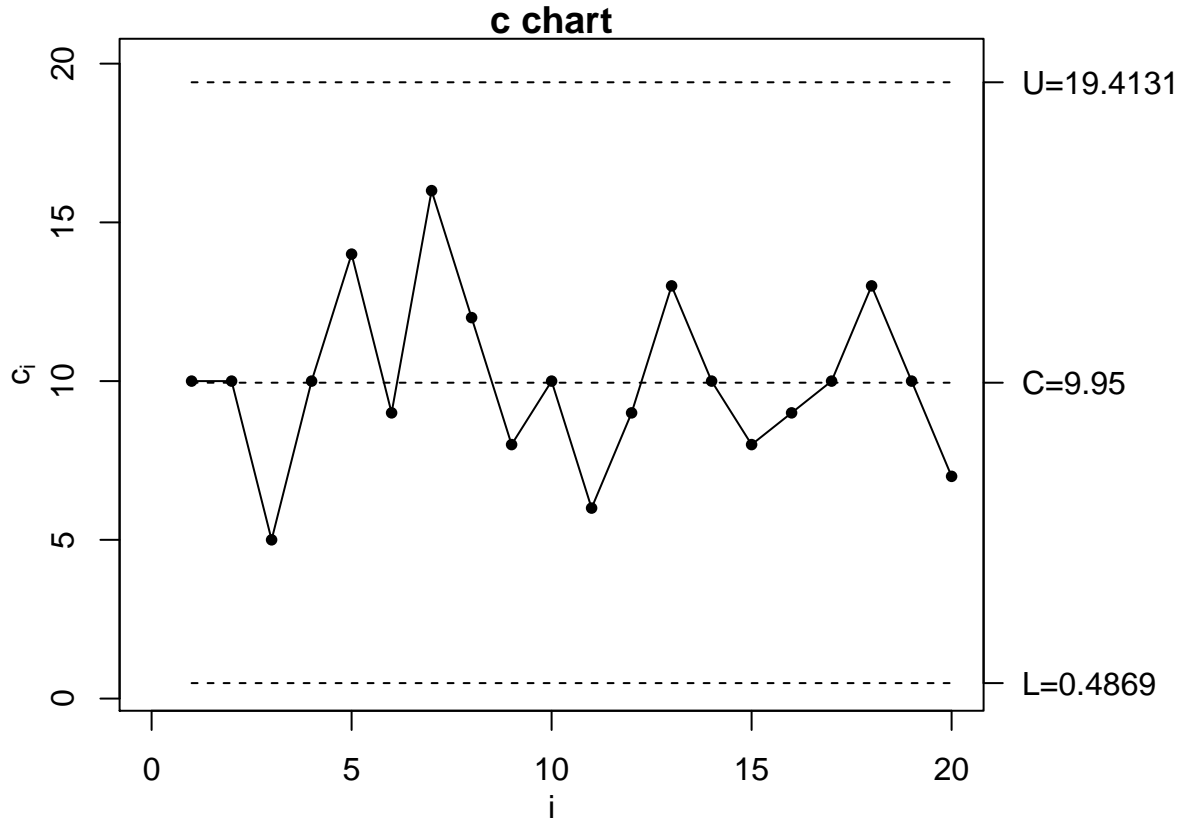
3.16

$\alpha = 0.0027$, $c_i = 10, 10, 5, 10, 14, 9, 16, 12, 8, 10, 6, 9, 13, 10, 8, 9, 10, 13, 10, 7$, $\bar{c} = \frac{1}{10} \sum_{i=1}^{10} c_i = 9.95$

The control limits of the c chart:

$$U = \bar{c} + Z_{1-\frac{\alpha}{2}} \sqrt{\bar{c}} \approx 19.4131$$

$$C = \bar{c} = 9.95$$

$$L = \bar{c} - Z_{1-\frac{\alpha}{2}} \sqrt{\bar{c}} \approx 0.4869$$

The process producing the plates appears to be in statistical control.
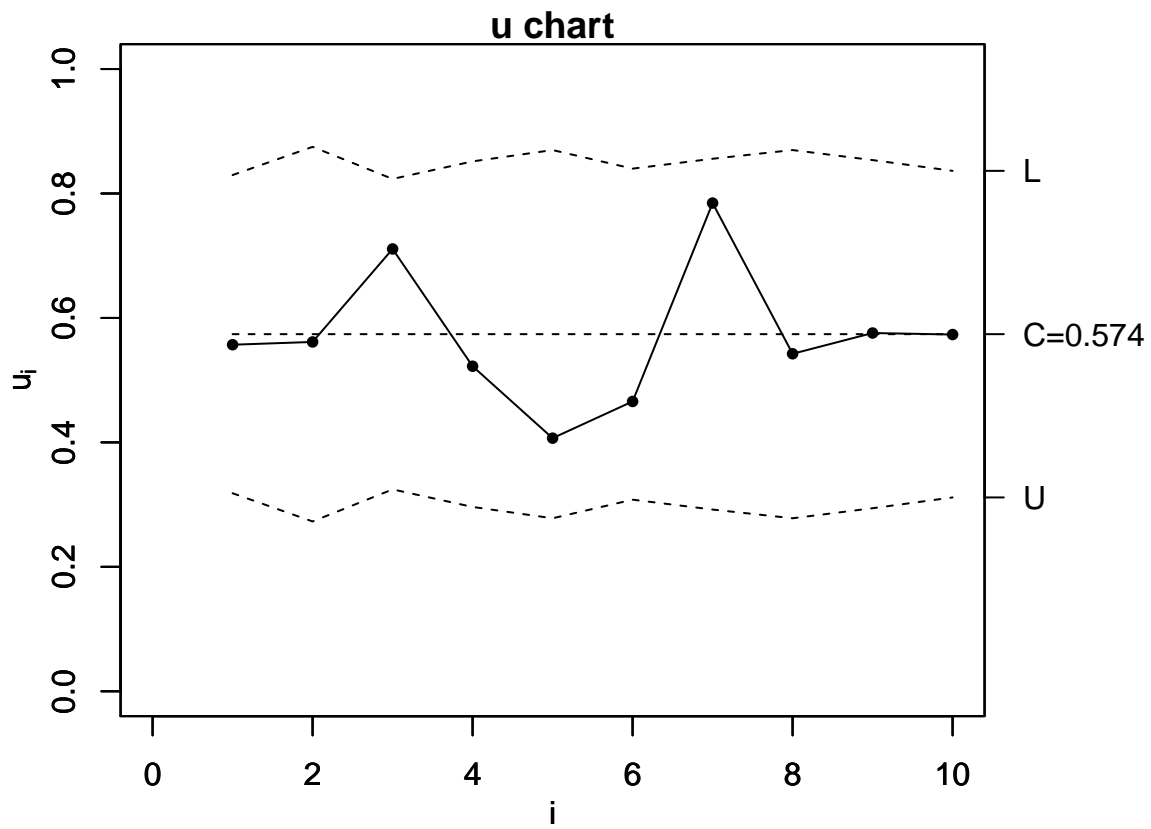
3.17

$\alpha = 0.0027$, $\bar{u} = \frac{\sum_{i=1}^{10} c_i}{\sum_{i=1}^{10} m_i} \approx 0.574$

The control limits of the u chart:

$$U_i = \bar{u} + Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\bar{u}}{m_i}}$$

$$C = \bar{u} \approx 0.574$$

$$L_i = \bar{u} - Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\bar{u}}{m_i}}$$

The production process appears to be in statistical control.

3.18

$\alpha = 0.0027, \ w_1 = 0.6, \ w_2 = 0.3, \ w_3 = 0.1$

$\bar{c_1}^* = \frac{1}{20} \sum_{i=1}^{20} c_{i1}^* = 4.5, \ \bar{c_2}^* = \frac{1}{20} \sum_{i=1}^{20} c_{i2}^* = 9.7, \ \bar{c_3}^* = \frac{1}{20} \sum_{i=1}^{20} c_{i3}^* = 22.8$
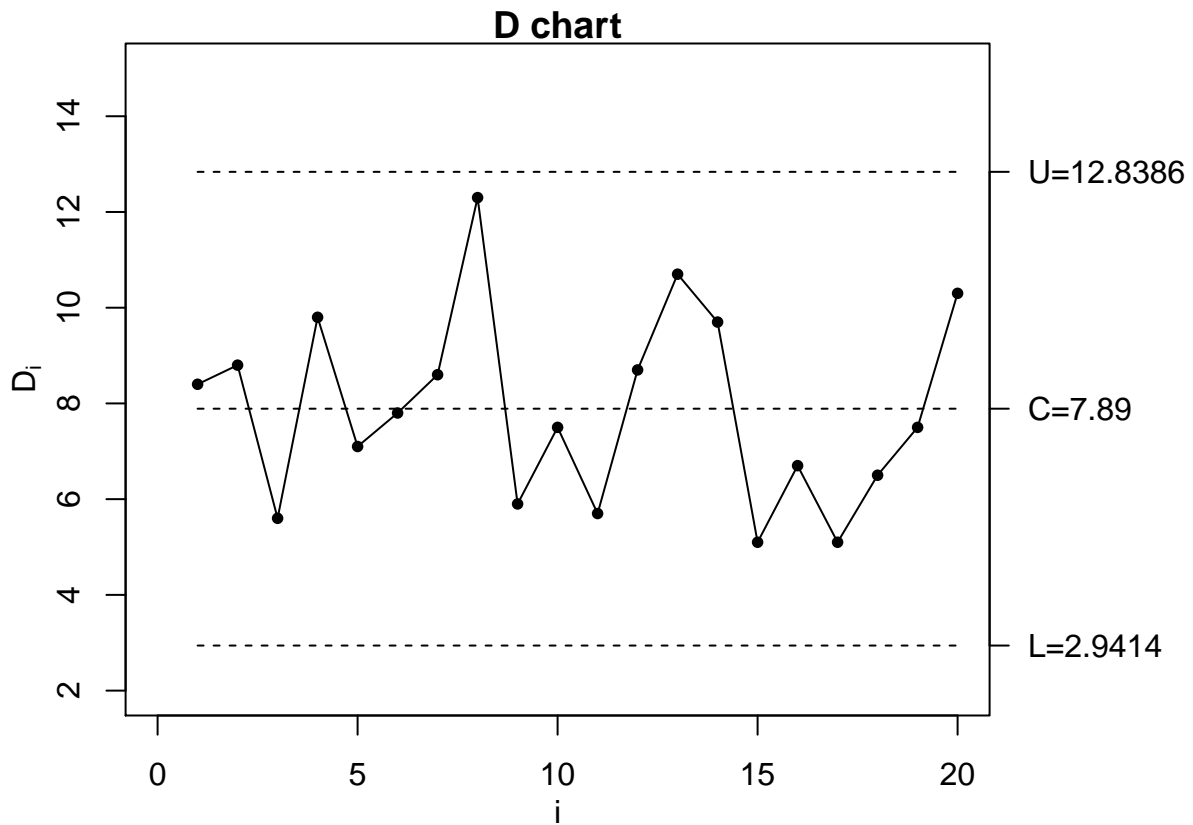
$\bar{D} = \frac{1}{20} \sum_{i=1}^{20} D_i = \frac{1}{20} \sum_{i=1}^{20} \sum_{j=1}^{3} w_j c_{ij}^* = \sum_{j=1}^{3} w_j \bar{c_j}^* = 7.89$

The control limits of the D chart:

$$U = \bar{D} + Z_{1-\frac{\alpha}{2}} \sqrt{\sum_{j=1}^{3} w_j^2 \bar{c_j}^*} \approx 12.8386$$

$$C = \bar{D} = 7.89$$

$$L = \bar{D} - Z_{1-\frac{\alpha}{2}} \sqrt{\sum_{j=1}^{3} w_j^2 \bar{c_j}^*} \approx 2.9414$$

The production process appears to be in statistical control.