Homework 2 Due date: March 9

1. Suppose $\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_n$ are *n*-dimension random vectors with mean μ and covariance matrix Σ. If we define $\overline{\boldsymbol{X}} = \frac{1}{n}\sum_{i=1}^{n} \boldsymbol{X}_i$ and $\boldsymbol{S} = \frac{1}{n-1}\sum_{i=1}^{n}(\boldsymbol{X}_i - \overline{\boldsymbol{X}})(\boldsymbol{X}_i - \overline{\boldsymbol{X}})'$. Prove that $\mathrm{E}(\overline{\boldsymbol{X}}) = \mu$, $\mathrm{E}(\boldsymbol{S}) = \Sigma$, and $cov(\overline{\boldsymbol{X}}) = \frac{1}{n}\Sigma$.
2. Prove the following results
   (a) Cauchy Schwarz Inequality
   (b) Extended Cauchy-Schwarz Inquality
   (c) Maximization Lemma
   (d) Maximization of quadratic forms for points on the unit sphere
3. Prove that when n=p in the data matrix, then the determinant of the sample covariance matrix would be zero, that is, |S|=0.
4. Exercise 2.32 in the book of Johnson and Wichern.
5. Exercise 3.1
6. Exercise 3.7. Please provide your codes for sketching the ellipses.