Experimental Design and Analysis Homework 5

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Problem 1. (3-26)

此實驗總共有：

(1) 四個 two-level treatment factors，分別為 *shield, knob a, knob b, knob c*

(2) 一個 four-level block factor *day* with block size = 16

因此我們可以將全部 2^4 level combinations 安排進每一天 (block) 的實驗之中，但是本實驗主要的目的是探討哪種 *knobs (a,b,c)* 的設定組合在兩種不同的 *shield* 可以造成最大的 difference，考慮到日照強度會隨著時間有顯著變化，在相同的 *knobs* level combination 下，搭配兩種 *shield* 所執行的兩次實驗應該安排為連續進行，故實驗的設計可以將 30 分鐘為一單位，將一天分為 8 等分，把 2^3 種 *knobs (a,b,c)* 的 level combinations 隨機安排進去，然後再將 *shield* I & II 以各 15 分鐘分配給八個 *knobs* 所形成的 level combinations，在相同的 *knobs* 設定值之下，*shield* I & II 要以 balanced randomize 的方式安排進入實驗的四天 (隨機的兩天先 I 後 II，剩餘兩天先 II 後 I)

此實驗設計為一個 Split-plot design：

(1) 3 whole-plot factors : *knob a, knob b, knob c*

(2) 1 sub-plot factor : *shield*

(3) 1 block factor : *day*

(4) whole-plot experimental unit : 30 min

(5) sub-plot experimental unit : 15 min

level combinations for whole-plot factors :

knob a	**knob b**	**knob c**	**level combination**
+	+	+	1
+	+	-	2
+	-	+	3
+	-	-	4
-	+	+	5
-	+	-	6
-	-	+	7
-	-	-	8

Design plan of four successive days :

Run	**Day 1**	**Day 2**	**Day 3**	**Day 4**
1	(1,I) (1,II)	(1,I) (1,II)	(1,II) (1,I)	(1,II) (1,I)
2	(2,I) (2,II)	(2,II) (2,I)	(2,II) (2,I)	(2,I) (2,II)
3	(3,I) (3,II)	(3,II) (3,I)	(3,I) (3,II)	(3,II) (3,I)
4	(4,II) (4,I)	(4,II) (4,I)	(4,I) (4,II)	(4,I) (4,II)
5	(5,II) (5,I)	(5,I) (5,II)	(5,I) (5,II)	(5,II) (5,I)
6	(6,I) (6,II)	(6,II) (6,I)	(6,II) (6,I)	(6,I) (6,II)
7	(7,II) (7,I)	(7,I) (7,II)	(7,II) (7,I)	(7,I) (7,II)
8	(8,I) (8,II)	(8,I) (8,II)	(8,II) (8,I)	(8,II) (8,I)

每天實驗進行的順序應再進行 randomize，此處僅為展示方便

建構模型以供分析：

$$y = \eta + \tau + \alpha + \epsilon^W + \beta + (\alpha\beta) + \epsilon^S$$

Terms related to the whole-plot :

(1) τ : block (*day*) effect

(2) α : all factorial effects of whole-plot factors (*knob a,b,c*)

(3) ϵ^W : whole-plot error

Terms related to the sub-plot :

(1) β : all factorial effects of sub-plot factor (*shield*)

(2) $(\alpha\beta)$: all interaction effects of whole-plot and sub-plot factors (*knob a,b,c* $\times$ *shield*)

(3) ϵ^S : sub-plot error

Problem 3. (4-11)

```
library(dplyr)
library(knitr)
task_data = read.table("TaskEfficiency.txt")
colnames(task_data) = c("setup", "flasher", "inertia", "task", "time", "order")
kable(task_data)
```

setup	flasher	inertia	task	time	order
1	A	low	Y	11	1
2	B	low	Y	12	4
3	A	high	Y	10	5
4	B	high	Y	11	3
5	A	low	Z	16	2
6	B	low	Z	14	6
7	A	high	Z	15	7
8	B	high	Z	19	8

(a)

Main effect of task

$$ME(\text{task}) \;=\; \bar{z}(\text{task} = Y) \;-\; \bar{z}(\text{task} = Z)$$

$$= \frac{1}{4}(11 + 12 + 10 + 11) \;-\; \frac{1}{4}(16 + 14 + 15 + 19) \;=\; -5$$

```
(11+12+10+11)/4 - (16+14+15+19)/4
```

```
## [1] -5
```

(b)

Construct linear model for three treatment factors : *flasher (A)* , *inertia (B)* , *task (C)*

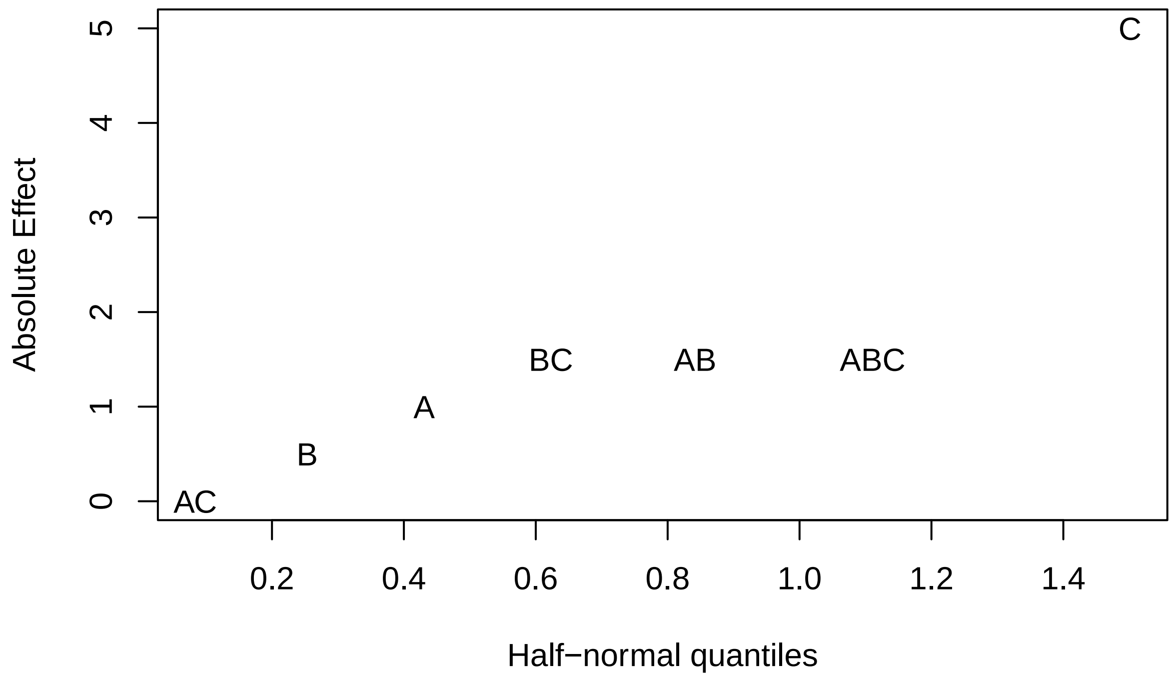
$$\text{time} \;=\; X\beta \;+\; \epsilon$$

where X is the 2^3 full factorial design

Then the factorial effects

$$\hat{\theta} \;=\; 2\hat{\beta}$$

```r
options(contrasts=c("contr.sum","contr.poly"))
mod3.1 = lm(time ~ flasher*inertia*task, task_data)
factorial_effect = coef(mod3.1)[-1]*2
names(factorial_effect) = c("A","B","C","AB","AC","BC","ABC")
"halfnorm" <- function (x, nlab = 2, labs = as.character(1:length(x)), ylab = "Sorted Data") {
    x <- abs(x)
    labord <- order(x)
    x <- sort(x)
    i <- order(x)
    n <- length(x)
    ui <- qnorm((n + 1:n)/(2 * n + 1))
    plot(ui, x[i], xlab = "Half-normal quantiles", ylab = ylab, ylim=c(0,max(x)),
         type = "n")
    if(nlab < n)
        points(ui[1:(n - nlab)], x[i][1:(n - nlab)])
    text(ui[(n - nlab + 1):n], x[i][(n - nlab + 1):n], labs[labord][(n - nlab + 1):n])
}
halfnorm(factorial_effect, nlab = length(factorial_effect),
         labs = names(factorial_effect), ylab = "Absolute Effect")
```



By the half-normal plot above, we can see that only the main effect of factor *task (C)* is relatively significant.

Let's try Lenth's method

$$PSE \;=\; 1.5 \times \text{median}_{\{|\hat{\theta}_i|<2.5s_0\}}|\hat{\theta}_i| \;=\; 2.5$$

where

$$s_0 \;=\; 1.5\times \;\text{median}|\hat{\theta}_i| \;=\; 2.5$$

```
s0 = 1.5*median(abs(factorial_effect))
pse = 1.5*median(abs(factorial_effect[abs(factorial_effect)<2.5*s0]))
```

Compute test statistic

$$t_{PSE,i} \;=\; \frac{\hat{\theta}_i}{PSE}$$

```
round(factorial_effect/pse, 4)
```

```
##       A       B       C      AB      AC      BC     ABC
## -0.4444  0.2222 -2.2222 -0.6667  0.0000 -0.6667  0.6667
```

If $|t_{PSE,i}|$ exceeds the critical value $IER_{0.1} = 1.71$, we conclude that factorial effect is significant.
The main effect of factor *task* is significant under Lenth's method. The result is the same with the half-normal plot.

(c)

$$\hat{\theta}_C = \frac{1}{4}(z_1 + z_2 + z_3 + z_4) - \frac{1}{4}(z_5 + z_6 + z_7 + z_8)$$
$$\Rightarrow Var\left(\hat{\theta}_C\right) = \frac{1}{16}\sum_{i=1}^{8} Var(z_i) = \frac{1}{16}(4 \times 1^2 + 4 \times 4^2) = 4.25$$

Problem 4. (4-14)

(a)

No, they can't. Because the effects of width and filler are totally aliasing.

(b)

They can do the 2^2 factorial design as below design matrix

Run	Part Width	Filler
1	36	40
2	50	20
3	36	20
4	50	40

which is an orthogonal array with strength 2

(c)

They have to add more than 2 additional runs if they want to estimate all main and interaction effects. Namely construct a 2^3 full factorial design whose design matrix is an orthogonal array with strength 3.

Run	Part Width	Filler	Temperature
1	36	40	20
2	50	20	20
3	36	20	40

Run	Part Width	Filler	Temperature
4	50	40	40
5	36	40	20
6	50	20	20
7	36	20	40
8	50	40	40

However, if we assume that the interaction among the three factors can be neglected and estimating only all main effects is their purpose, they can just add 2 additional runs in the design plan.

Run	Part Width	Filler	Temperature
1	36	40	20
2	50	20	20
3	36	20	40
4	50	40	40

which is still an orthogonal array with strength 2, and it is enough to estimate the main effects.

Problem 5. (4-16)

(a)

Construct location model and dispersion model

$$\bar{y} \;=\; X\beta \;+\; \epsilon \quad , \quad \ln s^2 \;=\; X\gamma \;+\; \delta$$

where X is the 2^3 full factorial design.

Then the factorial effects for location and dispersion effects are

$$\hat{\theta} \;=\; -2\hat{\beta} \quad , \quad \hat{\psi} \;=\; -2\hat{\gamma}$$

the negative sign is because the signs of sum coding in model matrix and $(-,+)$ levels for the factors are just the opposite.

```
drive_data = read.table("DriveShaft.txt")
colnames(drive_data) = c("run", "A", "B", "C", "y1", "y2", "y3", "y4", "y5")
y_bar = apply(drive_data[,5:9],1,mean)
```

```
s_square = apply(drive_data[,5:9],1,var)
drive_data = drive_data %>% mutate(y_bar, s_square)
options(contrasts=c("contr.sum","contr.poly"))
mod_loc = lm(y_bar ~ A*B*C, drive_data)
mod_dis = lm(log(s_square) ~ A*B*C, drive_data)
loc_effect = -coef(mod_loc)[-1]*2 ; names(loc_effect) = c("A","B","C","AB","AC","BC","ABC")
loc_effect
```

```
##      A      B      C     AB     AC     BC    ABC
##  -6.02 -13.60  14.60  -4.54  -3.14  -3.04  -3.34
```

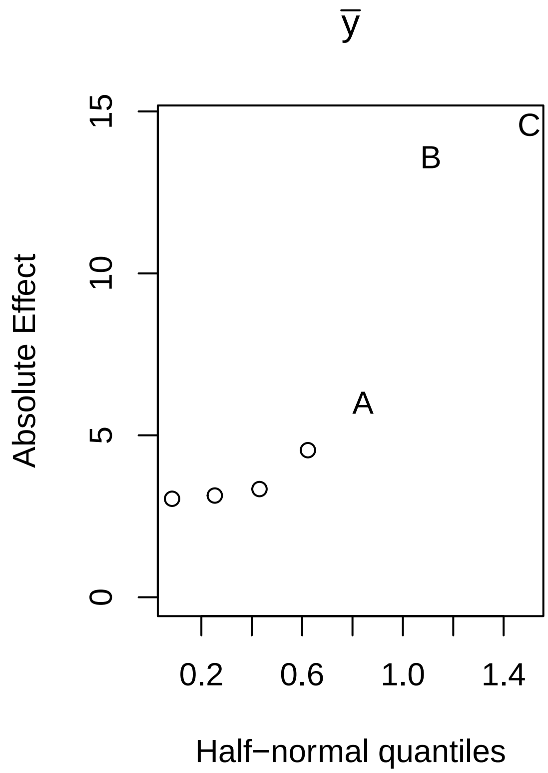
```
dis_effect = -coef(mod_dis)[-1]*2 ; names(dis_effect) = c("A","B","C","AB","AC","BC","ABC")
round(dis_effect,4)
```

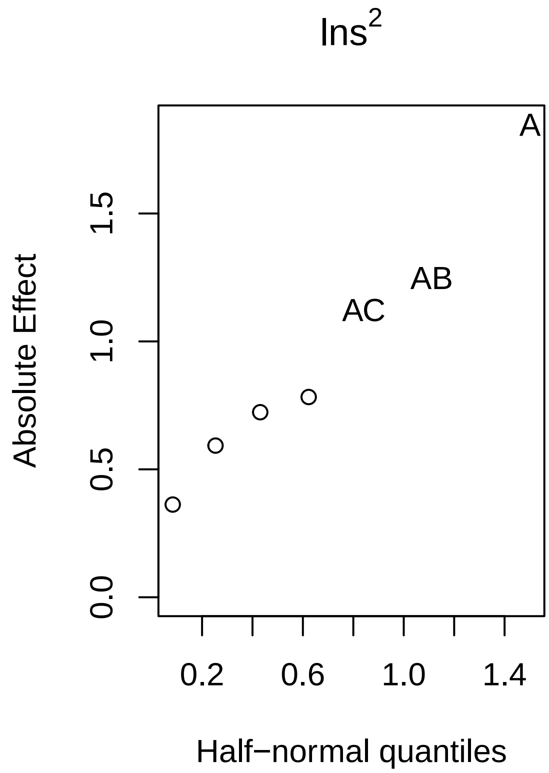
```
##       A       B       C      AB      AC      BC     ABC
## -1.8484 -0.5927 -0.7232 -1.2496 -1.1243 -0.7826 -0.3623
```

Let's check the significance of location and dispersion effects :

(1) Half-normal plot method

```
library(latex2exp)
par(mfrow = c(1,2))
halfnorm(loc_effect, nlab = 3, labs = c("A","B","C","AB","AC","BC","ABC"),
         ylab = "Absolute Effect")
title(TeX("$\\bar{y}$"))
halfnorm(dis_effect, nlab = 3, labs = c("A","B","C","AB","AC","BC","ABC"),
         ylab = "Absolute Effect")
title(TeX("$ln\ s^2$"))
```





For $\bar{y}$, main effects of B and C are significant. For $\ln s^2$, only the main effect of A looks slightly significant.

(2) Lenth's method

Following the same step in **(4-11)** to compute the test statistics $\frac{\hat{\theta}}{PSE_{\hat{\theta}}}$ and $\frac{\hat{\psi}}{PSE_{\hat{\psi}}}$

```
s0_loc = 1.5*median(abs(loc_effect))
pse_loc = 1.5*median(abs(loc_effect[abs(loc_effect)<2.5*s0_loc]))
round(loc_effect/pse_loc,4)
```

```
##       A       B       C      AB      AC      BC     ABC
## -0.8840 -1.9971  2.1439 -0.6667 -0.4611 -0.4464 -0.4905
```

```
s0_dis = 1.5*median(abs(dis_effect))
pse_dis = 1.5*median(abs(dis_effect[abs(dis_effect)<2.5*s0_dis]))
round(dis_effect/pse_dis,4)
```

```
##       A       B       C      AB      AC      BC     ABC
## -1.5745 -0.5049 -0.6161 -1.0644 -0.9577 -0.6667 -0.3086
```

If $|t_{PSE,i}|$ exceeds the critical value $IER_{0.1} = 1.71$, we conclude that factorial effect is significant.
For $\bar{y}$, the main effects of B and C are significant. For $\ln s^2$, no effects look significant, only the test statistic of the main effect A slightly closed to the critical value.

(b)

Fit the location model and dispersion model

$$\hat{\bar{y}} = \hat{\beta}_0 + \frac{1}{2}\hat{\theta}_B x_B + \frac{1}{2}\hat{\theta}_C x_C = 65.42 - 6.8x_B + 7.3x_C$$
$$\ln \hat{s}^2 = \hat{\gamma}_0 + \frac{1}{2}\hat{\psi}_A x_A = 3.8714225 - 0.9241892x_A$$

where

$$\begin{cases} x_A = -1 \ , \ \text{if} \ A = \#5023 \\ x_A = \ \ 1 \ , \ \text{if} \ A = \#5074 \end{cases} , \begin{cases} x_B = -1 \ , \ \text{if} \ B = 800 \\ x_B = \ \ 1 \ , \ \text{if} \ B = 1000 \end{cases} , \begin{cases} x_C = -1 \ , \ \text{if} \ C = 50 \\ x_C = \ \ 1 \ , \ \text{if} \ C = 80 \end{cases}$$

It is appropriate to use the two-step procedure because there exist two adjustment factors B and C.

(1) Choose $A = \#5074$ to minimize $Var(y_x)$, then the predicted variance

$$\hat{\sigma}^2 = \exp[3.8714225 - 0.9241892(1)] = 19.05317$$

(2) Choose x_B and x_C to satisfy $75 = 65.42 - 6.8x_B + 7.3x_C$. For example

$$(x_B \ , \ x_C) = (-1 \ , \ 0.3808219) \Leftrightarrow (B \ , \ C) = (800 \ , \ 70.71233)$$

Problem 6. (4-20)

The first blocking scheme B_I

$B_1 = 126$, $B_2 = 136$, $B_3 = 346$, $B_4 = 456$, $B_{12} = 23$, $B_{13} = 1234$, $B_{14} = 1245$,

$B_{23} = 14$, $B_{24} = 1345$, $B_{34} = 35$, $B_{123} = 246$, $B_{124} = 23456$, $B_{134} = 12356$, $B_{234} = 156$, $B_{1234} = 25$

$\Rightarrow g(B_I) = (0, 4, 6, 3, 2, 0)$

The second blocking scheme B_{II}

$B_1 = 136$, $B_2 = 1234$, $B_3 = 3456$, $B_4 = 123456$, $B_{12} = 246$, $B_{13} = 145$, $B_{14} = 245$,

$B_{23} = 1256$, $B_{24} = 56$, $B_{34} = 12$, $B_{123} = 235$, $B_{124} = 135$, $B_{134} = 236$, $B_{234} = 34$, $B_{1234} = 146$

$\Rightarrow g(B_{II}) = (0, 3, 8, 3, 0, 1)$

Notice that $g_2(B_I) > g_2(B_{II})$, so scheme B_{II} is said to have less aberration than scheme B_I. It is clear that scheme B_{II} sacrifies less number of two-factor interaction than scheme B_I. By the effect hierarchy principle, the second scheme B_{II} is better.