品質管制 Homework 10

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5.13

Compute the charting statistics

$$E_n \;=\; \lambda\,(\frac{X_n-\mu_0}{\sigma_0})^2 \;+\; (1-\lambda)E_{n-1}\;,\;\text{where } E_0 \;=\; 1\;,\;\lambda \;=\; 0.1\;,\;(\mu_0,\sigma_0)=(50,5)$$

A signal of upward process variance shift is given at the n-th time point if

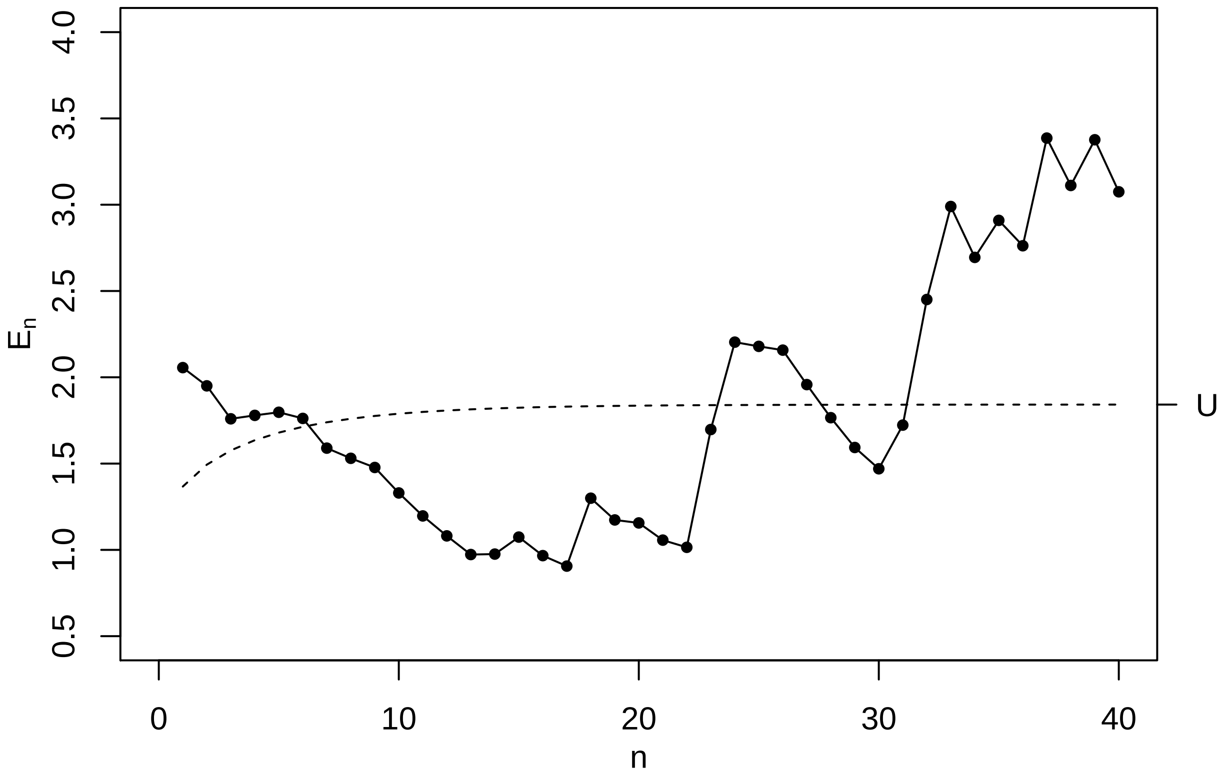
$$E_n \;>\; U \;=\; 1 \;+\; \rho_U\,\sqrt{\frac{2\lambda}{2-\lambda}[1-(1-\lambda)^{2n}]}$$

where $ARL_0 = 200$ and $\lambda = 0.1$ by Table 5.4 $\Rightarrow\;\rho_U = 2.595$

The charting statistics, control limits, and the EWMA chart are shown as below.

n	X_n	E_n	U
1	33	2.0560	1.3670
2	55	1.9504	1.4937
3	51	1.7594	1.5763
4	57	1.7794	1.6354
5	57	1.7975	1.6795
6	56	1.7617	1.7132
7	49	1.5896	1.7394
8	45	1.5306	1.7599
9	55	1.4775	1.7762
10	50	1.3298	1.7891
11	50	1.1968	1.7994
12	49	1.0811	1.8077
13	50	0.9730	1.8143
14	55	0.9757	1.8196
15	57	1.0741	1.8239
16	50	0.9667	1.8273
17	47	0.9061	1.8301
18	39	1.2995	1.8324
19	51	1.1735	1.8342
20	55	1.1562	1.8357
21	52	1.0565	1.8369
22	54	1.0149	1.8378
23	36	1.6974	1.8386
24	37	2.2037	1.8392
25	43	2.1793	1.8398
26	57	2.1574	1.8402
27	52	1.9576	1.8405
28	49	1.7659	1.8408
29	51	1.5933	1.8410

n	X_n	E_n	U
30	47	1.4699	1.8412
31	40	1.7230	1.8413
32	35	2.4507	1.8414
33	36	2.9896	1.8415
34	51	2.6946	1.8416
35	39	2.9092	1.8417
36	44	2.7623	1.8417
37	35	3.3860	1.8418
38	54	3.1114	1.8418
39	38	3.3763	1.8418
40	47	3.0747	1.8418



We detect the upward variance shifts at the 1st to 6th, 24th to 27th, and time points after 32nd.

5.14
Compute the charting statistics

$$E_n \; = \; \lambda \, (\frac{X_n - \mu_0}{\sigma_0})^2 \; + \; (1-\lambda)E_{n-1} \; , \; \text{where } E_0 \; = \; 1 \; , \; \lambda \; = \; 0.05 \; , \; (\mu_0, \sigma_0) = (0, 1)$$

A signal of upward(downward) process variance shift is given at the n-th time point if

$$E_n \; > \; U \; = \; 1 \; + \; \rho_U \sqrt{\frac{2\lambda}{2-\lambda}[1-(1-\lambda)^{2n}]}$$

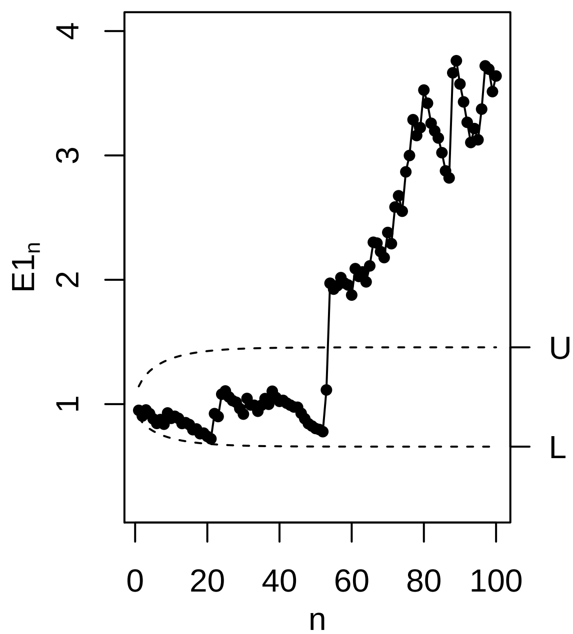
$$\text{or } E_n \; < \; L \; = \; 1 \; - \; \rho_L \sqrt{\frac{2\lambda}{2-\lambda}[1-(1-\lambda)^{2n}]}$$

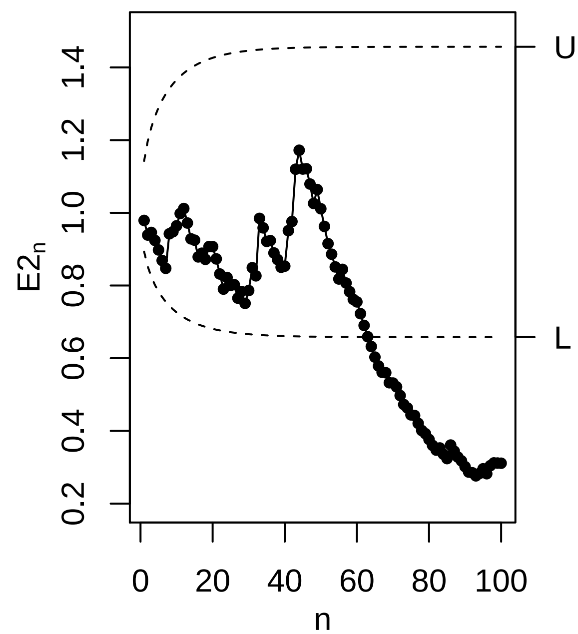
where $ARL_0 = 200$ and $\lambda = 0.05$ by Table 5.4 $\Rightarrow$ $(\rho_U, \rho_L) = (2.017, 1.510)$
The charting statistics, control limits, and the EWMA charts are shown as below.

n	$X_{n,1}$	$E_{n,1}$	$X_{n,2}$	$E_{n,2}$	U	L
1	0.0187	0.9500	-0.7618	0.9790	1.1426	0.8932
2	-0.1843	0.9042	0.4194	0.9389	1.1967	0.8527
3	-1.3713	0.9530	-1.0399	0.9460	1.2351	0.8240
4	-0.5992	0.9233	0.7116	0.9240	1.2650	0.8016
5	0.2945	0.8815	-0.6332	0.8979	1.2893	0.7834
6	0.3898	0.8450	0.5632	0.8688	1.3097	0.7682
7	-1.2081	0.8757	0.6610	0.8472	1.3269	0.7552
8	-0.3637	0.8386	-1.6581	0.9423	1.3418	0.7441
9	-1.6267	0.9289	1.0282	0.9481	1.3546	0.7345
10	-0.2565	0.8858	1.1280	0.9643	1.3658	0.7261
11	1.1018	0.9022	-1.2802	0.9980	1.3757	0.7188
12	0.7558	0.8856	1.1289	1.0118	1.3843	0.7123
13	-0.2382	0.8442	-0.4641	0.9720	1.3920	0.7065
14	0.9874	0.8507	-0.3158	0.9284	1.3988	0.7015
15	0.7414	0.8357	0.9243	0.9247	1.4048	0.6970
16	0.0893	0.7943	0.0771	0.8787	1.4101	0.6930
17	-0.9549	0.8002	1.0399	0.8889	1.4149	0.6894
18	-0.1952	0.7621	0.7419	0.8720	1.4192	0.6862
19	0.9255	0.7668	1.2555	0.9072	1.4230	0.6833
20	0.4830	0.7401	0.9509	0.9070	1.4264	0.6808
21	-0.5963	0.7209	-0.4814	0.8733	1.4295	0.6785
22	-2.1853	0.9236	0.2029	0.8317	1.4322	0.6764
23	-0.6749	0.9002	-0.0317	0.7901	1.4347	0.6746
24	-2.1191	1.0797	-1.1956	0.8221	1.4369	0.6730
25	-1.2652	1.1058	0.6237	0.8004	1.4388	0.6715
26	-0.3737	1.0575	-0.9148	0.8023	1.4406	0.6701
27	-0.6876	1.0282	0.2488	0.7652	1.4422	0.6689
28	-0.8722	1.0149	-1.0626	0.7834	1.4437	0.6679
29	-0.1018	0.9646	-0.3640	0.7509	1.4449	0.6669
30	-0.2538	0.9196	-1.2070	0.7862	1.4461	0.6660
31	-1.8537	1.0455	1.4292	0.8490	1.4472	0.6652
32	-0.0779	0.9935	0.6334	0.8266	1.4481	0.6645
33	0.9686	0.9907	-1.9968	0.9847	1.4490	0.6639
34	0.1849	0.9429	-0.6818	0.9587	1.4497	0.6633
35	-1.3799	0.9910	-0.4601	0.9213	1.4504	0.6628
36	-1.4355	1.0444	-0.9831	0.9236	1.4510	0.6623
37	0.3621	0.9988	0.4953	0.8897	1.4516	0.6619

n	$X_{n,1}$	$E_{n,1}$	$X_{n,2}$	$E_{n,2}$	U	L
38	-1.7591	1.1036	0.7258	0.8715	1.4521	0.6615
39	-0.3245	1.0536	0.6673	0.8502	1.4526	0.6612
40	-0.6516	1.0222	0.9548	0.8533	1.4530	0.6609
41	1.0866	1.0301	-1.6753	0.9509	1.4533	0.6606
42	-0.7625	1.0077	-1.2052	0.9760	1.4537	0.6604
43	-0.8287	0.9916	-1.9633	1.1199	1.4540	0.6601
44	0.8345	0.9769	1.4708	1.1721	1.4543	0.6599
45	-0.9677	0.9748	0.3725	1.1204	1.4545	0.6597
46	-0.0288	0.9261	1.0659	1.1212	1.4547	0.6596
47	0.2325	0.8825	0.5306	1.0792	1.4549	0.6594
48	-0.3012	0.8429	0.1020	1.0258	1.4551	0.6593
49	-0.6776	0.8238	1.3378	1.0640	1.4553	0.6592
50	0.6552	0.8040	0.0872	1.0112	1.4554	0.6591
51	-0.8013	0.7959	-0.1956	0.9625	1.4555	0.6590
52	-0.6691	0.7785	-0.1249	0.9152	1.4557	0.6589
53	2.7359	1.1139	0.5776	0.8861	1.4558	0.6588
54	4.2755	1.9722	-0.4324	0.8511	1.4559	0.6587
55	1.0116	1.9247	-0.4333	0.8180	1.4560	0.6587
56	1.5727	1.9522	-1.1605	0.8444	1.4560	0.6586
57	-1.8044	2.0174	0.3044	0.8068	1.4561	0.6585
58	1.0658	1.9733	0.5750	0.7830	1.4562	0.6585
59	-1.2918	1.9581	-0.5998	0.7618	1.4562	0.6585
60	0.5820	1.8771	-0.7900	0.7550	1.4563	0.6584
61	-2.4752	2.0896	0.3266	0.7225	1.4563	0.6584
62	-0.9124	2.0267	-0.2747	0.6902	1.4564	0.6583
63	-1.6606	2.0633	0.2605	0.6591	1.4564	0.6583
64	0.6802	1.9832	-0.3497	0.6322	1.4564	0.6583
65	2.1328	2.1115	-0.2195	0.6030	1.4565	0.6583
66	2.4323	2.3017	-0.3387	0.5786	1.4565	0.6582
67	1.4714	2.2949	0.4796	0.5612	1.4565	0.6582
68	-0.9624	2.2264	-0.7341	0.5601	1.4565	0.6582
69	1.1255	2.1785	0.0919	0.5325	1.4566	0.6582
70	-2.4926	2.3802	-0.7176	0.5316	1.4566	0.6582
71	0.7618	2.2902	-0.5687	0.5212	1.4566	0.6582
72	-2.8609	2.5849	-0.2073	0.4973	1.4566	0.6582
73	-2.0969	2.6755	0.0720	0.4727	1.4566	0.6581
74	-0.4370	2.5513	0.5310	0.4631	1.4566	0.6581
75	-2.9799	2.8677	-0.2854	0.4441	1.4567	0.6581
76	2.3454	2.9994	0.6386	0.4423	1.4567	0.6581
77	-2.9597	3.2874	0.1141	0.4208	1.4567	0.6581
78	-0.8608	3.1601	-0.1544	0.4009	1.4567	0.6581
79	-2.1033	3.2232	0.4799	0.3924	1.4567	0.6581
80	3.0452	3.5257	0.2744	0.3766	1.4567	0.6581
81	1.1857	3.4197	0.2128	0.3600	1.4567	0.6581
82	-0.4453	3.2587	0.3218	0.3472	1.4567	0.6581
83	1.4258	3.1974	-0.6802	0.3529	1.4567	0.6581
84	1.4332	3.1402	-0.0993	0.3358	1.4567	0.6581
85	0.8805	3.0220	0.3097	0.3238	1.4567	0.6581
86	0.3177	2.8759	1.0341	0.3611	1.4567	0.6581
87	1.3195	2.8192	-0.1526	0.3442	1.4567	0.6581
88	4.4410	3.6644	0.1406	0.3280	1.4567	0.6581
89	-2.3679	3.7615	0.3457	0.3175	1.4567	0.6581

n	$X_{n,1}$	$E_{n,1}$	$X_{n,2}$	$E_{n,2}$	U	L
90	-0.1479	3.5745	0.0232	0.3017	1.4567	0.6581
91	-0.8327	3.4305	0.0565	0.2868	1.4567	0.6581
92	-0.3830	3.2663	0.4977	0.2848	1.4567	0.6581
93	0.1391	3.1039	-0.3406	0.2764	1.4567	0.6581
94	2.3107	3.2157	-0.6385	0.2829	1.4567	0.6581
95	1.1899	3.1257	-0.7343	0.2958	1.4567	0.6581
96	-2.8393	3.3725	-0.1567	0.2822	1.4567	0.6581
97	-3.2134	3.7201	-0.8518	0.3044	1.4567	0.6581
98	1.7859	3.6936	-0.6753	0.3119	1.4568	0.6581
99	0.2963	3.5133	-0.5510	0.3115	1.4568	0.6581
100	2.4541	3.6388	-0.5498	0.3111	1.4568	0.6581





For the first dataset, we detect the upward variance shift at the 54th time point, and for the second dataset, we detect the downward shift at the 64th time point.

5.17
For detecting the variance upward shifts, compute the charting statistics

$$E_{n,v} \;=\; \lambda\, \frac{S_n^2}{\sigma_0^2} \;+\; (1-\lambda)\; E_{n-1,v} \text{ , where } E_{0,v}=1 \text{ , } \lambda = 0.1 \text{ , } \sigma_0 = 0.5$$

A signal of upward process variance shift is given at the n-th time point if

$$E_{n,v} \;>\; U \;=\; 1+\rho_U \sqrt{\frac{2\lambda}{(2-\lambda)(m-1)}[1-(1-\lambda)^{2n}]} \text{ , where } \rho_U \;=\; 2.836 \text{ by Exercise 5.16}$$

For detecting the mean shift, compute the charting statistics

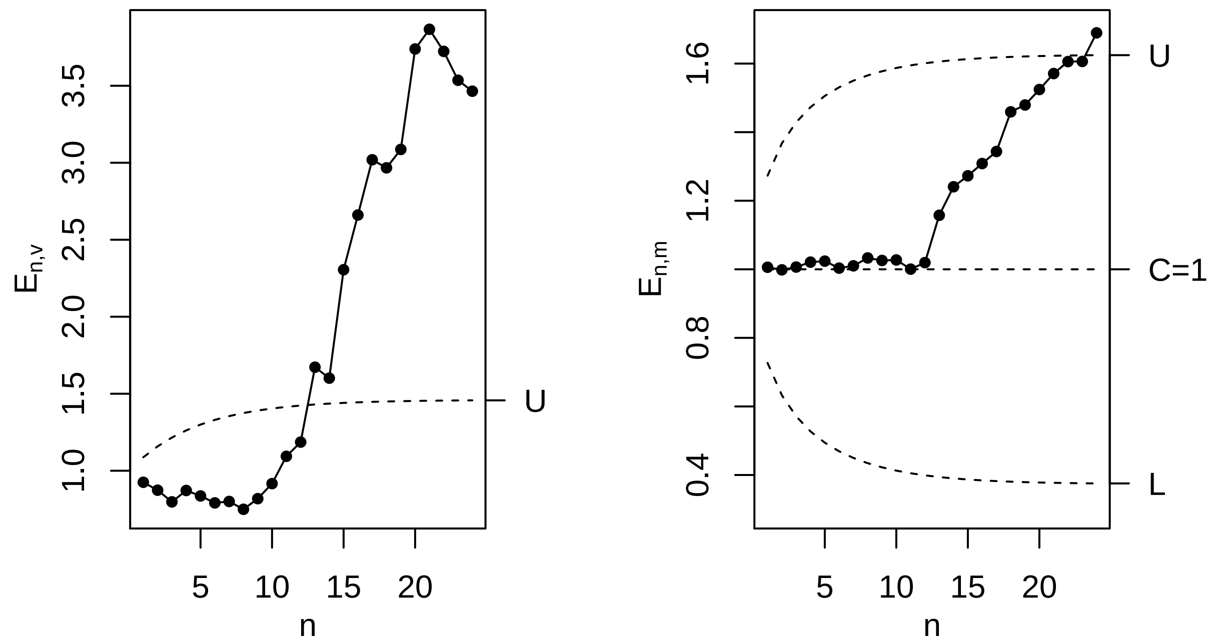
$$E_{n,m} \;=\; \lambda\, \overline{X}_n \;+\; (1-\lambda)\; E_{n-1,m} \text{ , where } E_0 \;=\; \mu_0 \;=\; 1$$

The control limits of the EWMA chart for detecting a mean shift is given by

$$\begin{aligned} U &= \mu_0 \;+\; \rho\sqrt{\frac{\lambda}{2-\lambda}\;[1-(1-\lambda)^{2n}]}\;\frac{\sigma_0}{\sqrt{m}} \\ C &= \mu_0 \\ L &= \mu_0 \;-\; \rho\sqrt{\frac{\lambda}{2-\lambda}\;[1-(1-\lambda)^{2n}]}\;\frac{\sigma_0}{\sqrt{m}} \end{aligned}$$

The charting statistics, control limits, and the EWMA charts are shown as below.

$\bar{X}_n$	S_n	$E_{n,v}$	U_v	$E_{n,m}$	U_m	C_m	L_m
1.06	0.25	0.9250	1.0874	1.0060	1.2731	1	0.7269
0.93	0.32	0.8735	1.1582	0.9984	1.3674	1	0.6326
1.08	0.17	0.7977	1.2156	1.0066	1.4289	1	0.5711
1.15	0.62	0.8717	1.2620	1.0209	1.4728	1	0.5272
1.05	0.36	0.8363	1.2996	1.0238	1.5056	1	0.4944
0.82	0.31	0.7911	1.3301	1.0034	1.5307	1	0.4693
1.07	0.47	0.8004	1.3548	1.0101	1.5502	1	0.4498
1.24	0.27	0.7495	1.3748	1.0331	1.5655	1	0.4345
0.96	0.60	0.8186	1.3910	1.0258	1.5776	1	0.4224
1.04	0.67	0.9163	1.4041	1.0272	1.5872	1	0.4128
0.76	0.82	1.0936	1.4148	1.0005	1.5949	1	0.4051
1.19	0.71	1.1859	1.4234	1.0194	1.6010	1	0.3990
2.40	1.23	1.6725	1.4303	1.1575	1.6060	1	0.3940
1.99	0.49	1.6012	1.4360	1.2407	1.6099	1	0.3901
1.56	1.47	2.3055	1.4406	1.2727	1.6131	1	0.3869
1.63	1.21	2.6606	1.4443	1.3084	1.6157	1	0.3843
1.66	1.25	3.0195	1.4473	1.3436	1.6178	1	0.3822
2.50	0.79	2.9672	1.4497	1.4592	1.6194	1	0.3806
1.66	1.02	3.0866	1.4517	1.4793	1.6208	1	0.3792
1.93	1.55	3.7390	1.4533	1.5244	1.6219	1	0.3781
1.99	1.12	3.8668	1.4546	1.5709	1.6228	1	0.3772
1.92	0.78	3.7235	1.4556	1.6058	1.6235	1	0.3765
1.61	0.68	3.5361	1.4564	1.6062	1.6241	1	0.3759
2.44	0.84	3.4648	1.4571	1.6896	1.6245	1	0.3755



We detect the upward variance shift at the 14th time point, detect the mean shift at the 24th time point.