Homework 6 Due date: June 1

1. Let $X_1, \ldots, X_{60}$ be a random sample of size 60 from a four-variate normal distribution having mean μ and covariance Σ. Specify each of the following completely.
   (a) The distribution of $\bar{X}$.
   (b) The distribution of $(X_1 - \mu)'\Sigma^{-1}(X_1 - \mu)$.
   (c) The distribution of $60(\bar{X} - \mu)'\Sigma^{-1}(\bar{X} - \mu)$.
   (d) The distribution of $59S$.
   (e) The distribution of $59BSB'$ where B is a 2×4 matrix of rank 2.
2. A random vector $(X_1, X_2, X_3)'$ follows a multivariate normal distribution with a covariance matrix $\Sigma = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 5 & 0 \\ -1 & 0 & 2 \end{bmatrix}$.
   (a) What is the generalized variance of this random vector?
   (b) What is the conditional variance of $X_3 | (X_1, X_2) = (3, 4)$?
3. Prove Result 4.6.
4. The data in T4-6.DAT consist of 130 observations generated by scores on a psychological test administered to Peruvian teenagers (ages 15, 16, 17). For each of these teenagers, the gender (the 6th column, male=1, female=2) and socioeconomic status (the 7th column, low=1, medium=2) were also recorded. The scores were accumulated into five subscale scores labeled independence, support, benevolence, conformity, and leadership (column 1-5). Please check for multivariate normality of the first five columns with a chi-square plot and comment on the result.