品質管制 Homework 4

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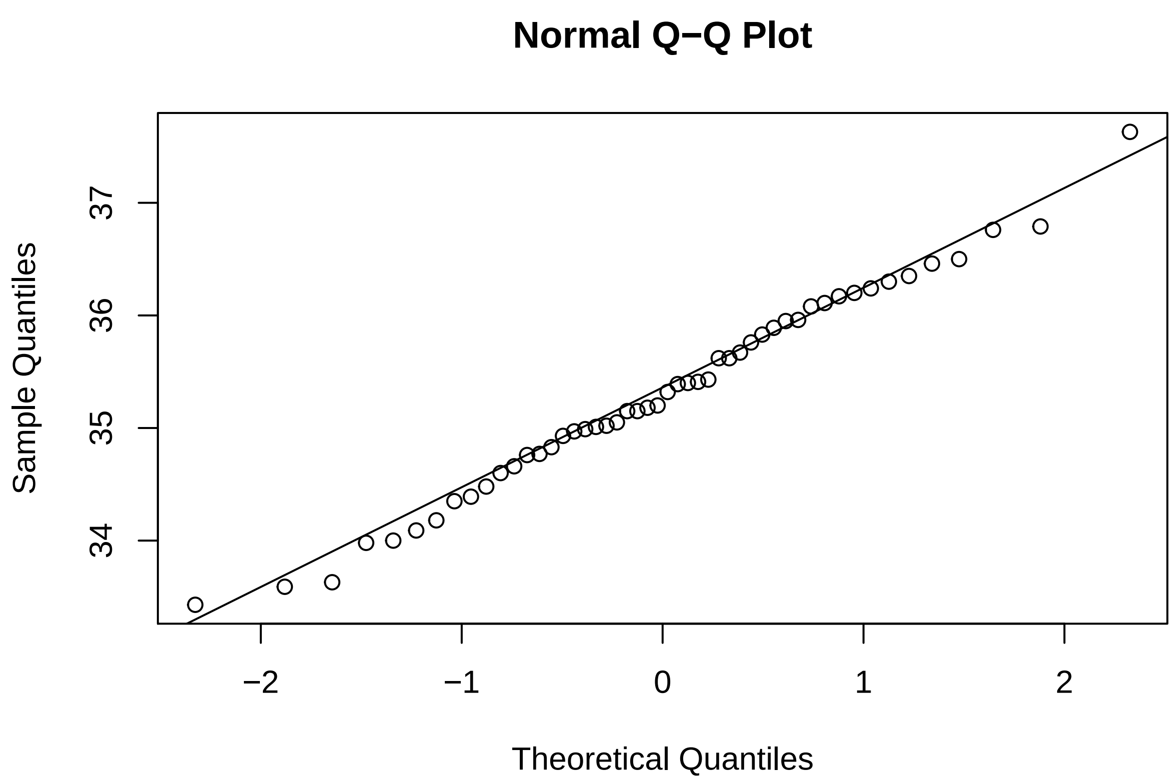
3.19 (i)



The data are plotted against a theoretical normal distribution in such a way that the points should form an approximate straight line. Departures from this straight line indicates from normality. Thus, the combined sample has been checked for the normality.

(ii)

$\overline{X} = \frac{1}{50}\sum_{i=1}^{50} X_i = 35.3046$, and $S = \sqrt{\frac{1}{50-1}\sum_{i=1}^{50}(X_i - \bar{X})^2} \approx 0.9099$

$P(LSL \leq X \leq USL) = P(\frac{LSL-\overline{X}}{S} \leq Z \leq \frac{USL-\overline{X}}{S})$
$= \Phi(\frac{USL-\overline{X}}{S}) - \Phi(\frac{LSL-\overline{X}}{S}) \approx \Phi(2.9625) - \Phi(-2.5329) \approx 0.9928$

(iii)

In Exercise 3.5 (i), $\overline{X} = 35.304, \ and \ \hat{\sigma} = \frac{\overline{R}}{d_1(5)} \approx 0.8650$

$P(LSL \leq X \leq USL) = P(\frac{LSL-\overline{X}}{\hat{\sigma}} \leq Z \leq \frac{USL-\overline{X}}{\hat{\sigma}}) \approx \Phi(3.1167) - \Phi(-2.6636) \approx 0.9952$

3.20 (i)
$LSL = 2100,\ USL = 2300,\ T = \frac{USL + LSL}{2},\ m = 50,\ \overline{X} = 2250,\ s = 50$
$\hat{C}_p = \frac{USL - LSL}{6s} \approx 0.6667$
$\hat{C}_{pl} = \frac{\overline{X} - LSL}{3s} = 1,\ \hat{C}_{pu} = \frac{USL - \overline{X}}{3s} \approx 0.3333 \Rightarrow \hat{C}_{pk} = min(\hat{C}_{pl},\ \hat{C}_{pu}) = 0.3333$
$\hat{C}_{pm} = \frac{\hat{C}_p}{\sqrt{1+(\frac{\overline{X} - T}{s})^2}} \approx 0.4714$
Between C_p *and* C_{pk}, C_{pk} is more appropriate to use in this case, because $\overline{X} \neq T$.

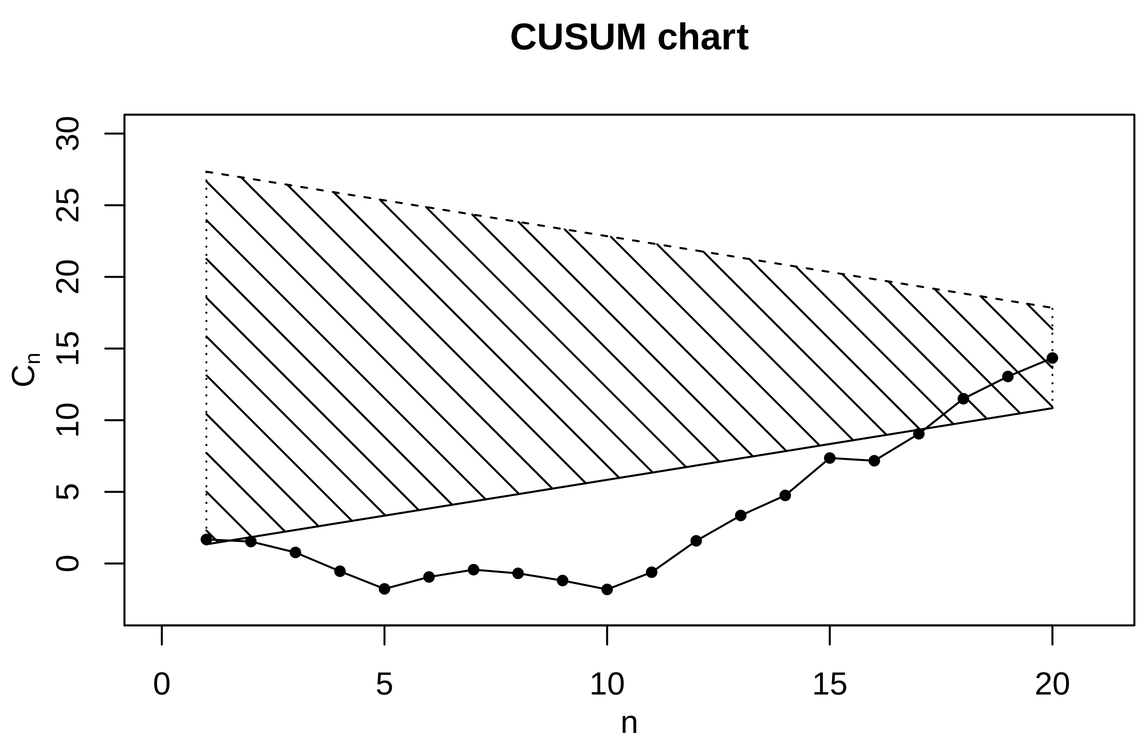
It is natural to observe that $P(LSL < X < USL \mid \overline{X} = T) > P(LSL < X < USL \mid \overline{X} \neq T)$, so we prefer to use C_{pk} in this case.

(ii)
A 95% confidence interval for C_p is

$$(\hat{C}_p\sqrt{\frac{\chi^2_{\frac{\alpha}{2},m-1}}{m-1}},\ \hat{C}_p\sqrt{\frac{\chi^2_{1-\frac{\alpha}{2},m-1}}{m-1}}) \approx (0.5350,\ 0.7981)$$

(iii)
$P(X < LSL\ or\ X > USL \mid process\ IC)$
$= P(Z < \frac{LSL - \overline{X}}{s}) + P(Z > \frac{USL - \overline{X}}{s}) = \Phi(-3) + (1 - \Phi(1)) \approx 0.1600$

4.2 (i)
$k = 0.5,\ h = 3.502,\ \mu_0 = 0$
The charting statistic $C_n = \sum_{i=1}^{20}(X_i - \mu_0)$



The CUSUM chart detects a positive mean shift occurs before n = 20, because there are some values of the charting statistics falling below that half-line.

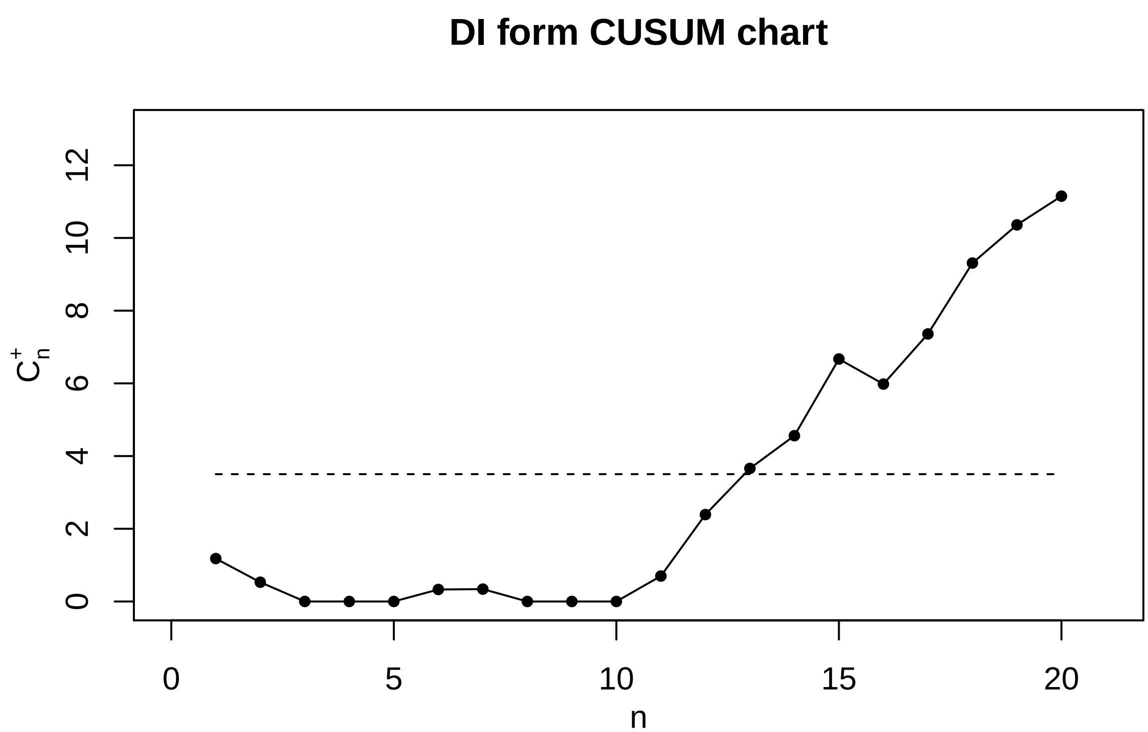
Then, we choose the time point, which is farthest away from the V-mask, is used as the estimate of τ

$$\Rightarrow \ \hat{\tau} \ = \ 10$$

After the estimate of τ, $\hat{\tau}$ is obtained, δ can be estimated by

$$\Rightarrow \ \hat{\delta} \ = \ \frac{C_{20} \ - \ C_{\hat{\tau}}}{20 \ - \ \hat{\tau}} \ = \ \frac{14.34 \ - \ (-1.81)}{20 \ - \ 10} \ = \ 1.615$$

(ii)
$C_n^+ \ = \ max(0, \ C_{n-1}^+ \ + \ (X_n \ - \ \mu_0) \ - \ k) \ , \ \ where \ \ C_0^+ \ = \ 0$



This kind of CUSUM chart also gives a signal of an upward mean shift because

$$C_n^+ \ > \ h \ = \ 3.502, \ for \ n \ \geq \ 13$$

It shows the same results in part(i).