Reliability Analysis Homework 3

110024516 邱繼賢

Problem 1.

(a)

(1) simple binomial method

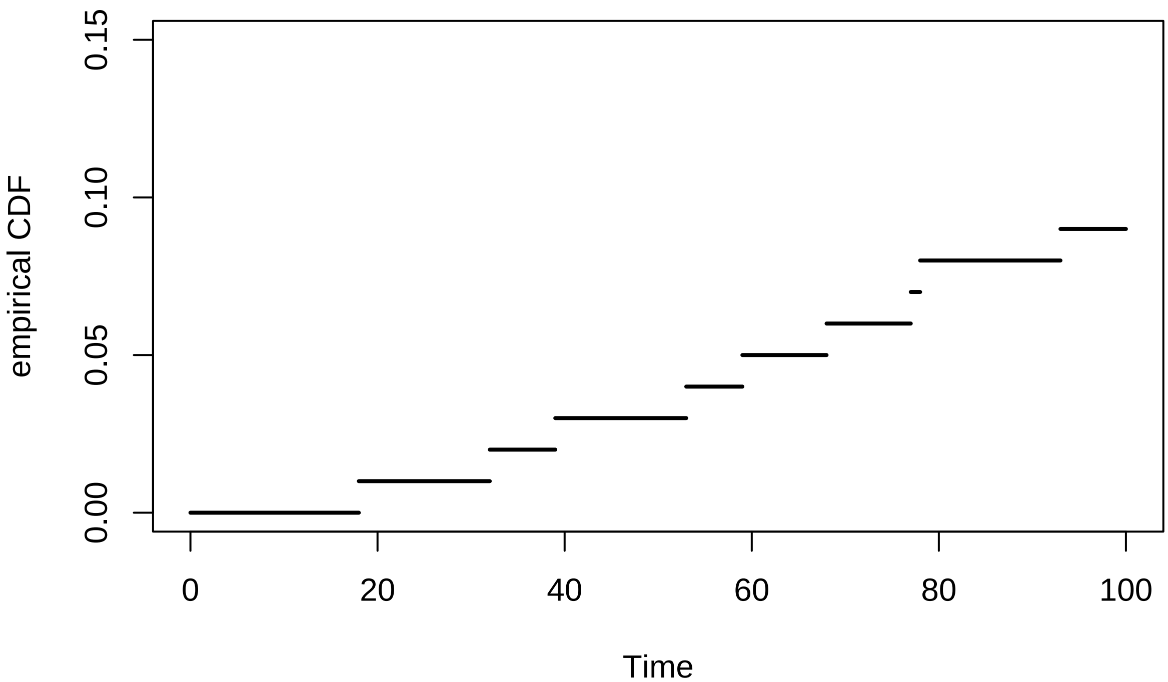
$$\hat{F}_1(t) \;=\; \frac{1}{n}\sum_{i=1}^{n} I\left(t_i \;\leq\; t\right)$$

(2) Kaplan-Meier method

$$\hat{S}(t) \;=\; \prod_{t_i \leq t}\left(1-\frac{d_i}{n_i}\right) \;\;\Rightarrow\;\; \hat{F}_2(t) \;=\; 1-\hat{S}(t)$$

t	$\hat{F}_1(t)$	$\hat{F}_2(t)$
18	0.01	0.01
32	0.02	0.02
39	0.03	0.03
53	0.04	0.04
59	0.05	0.05
68	0.06	0.06
77	0.07	0.07
78	0.08	0.08
93	0.09	0.09
100	0.09	0.09

(b)



(c)

Weibull distribution

$$p \ = \ F(t_p) \ = \ 1 - exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right]$$

$$\Rightarrow t_p \ = \ \eta[-log(1-p)]^{\frac{1}{\beta}}$$

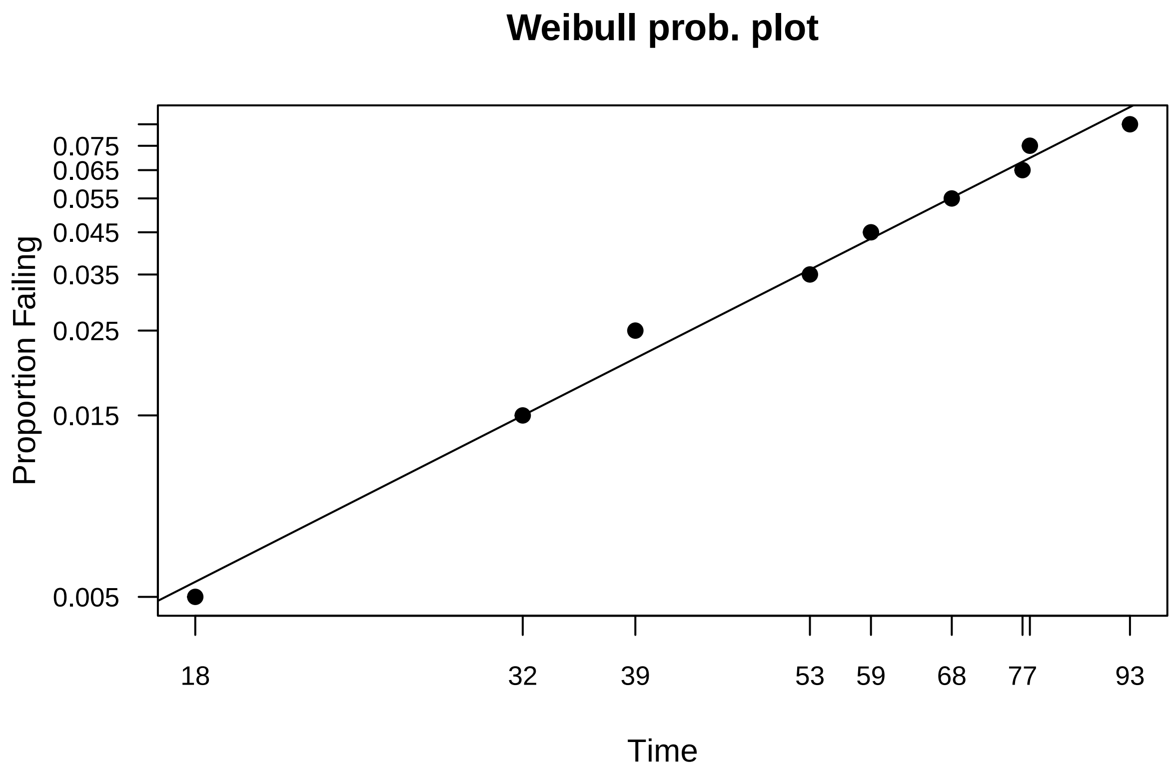
$$\Rightarrow log(t_p) \ = \ log(\eta) \ + \ log(-log(1-p))\frac{1}{\beta}$$

plotting position

$$\left(log\left(T_{(i)}\right) \ , \ \Phi_{sev}^{-1}\left(\frac{i-0.5}{n}\right)\right) \quad , \quad \Phi_{sev}^{-1}(p) \ = \ log\left(-log(1-p)\right)$$

and then relabel at

$$\left(T_{(i)} \ , \ \frac{i-0.5}{n}\right)$$



可以看到資料點分佈在 Weibull probability plot 上大致呈現為一直線，將 $\Phi_{sev}^{-1}\left(\frac{i-0.5}{n}\right)$ 對 $log\left(T_{(i)}\right)$ 做回歸直線，其斜率估計值 1.75869 即為 shape parameter β 的估計值

(d)

```
##
## Call:
## lm(formula = log(-log(1 - Fi)) ~ log(ti), data = plotting_pos)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.104603 -0.051506 -0.005223  0.038570  0.168538
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -10.28786    0.24516  -41.96 1.14e-09 ***
## log(ti)       1.75869    0.06164   28.53 1.67e-08 ***
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.09118 on 7 degrees of freedom
## Multiple R-squared:  0.9915, Adjusted R-squared:  0.9903
## F-statistic: 814.1 on 1 and 7 DF,  p-value: 1.67e-08
```

- 由 **(c)** 的圖形可以看出資料大致落在一直線上

- $\Phi_{sev}^{-1}\left(\frac{i-0.5}{n}\right)$ 對 $log\left(T_{(i)}\right)$ 的線性模型 $R^2 = 99.15\%$，由此可知此直線模型可以很好的解釋資料點的分佈

故可以推論出資料符合 Weibull distribution

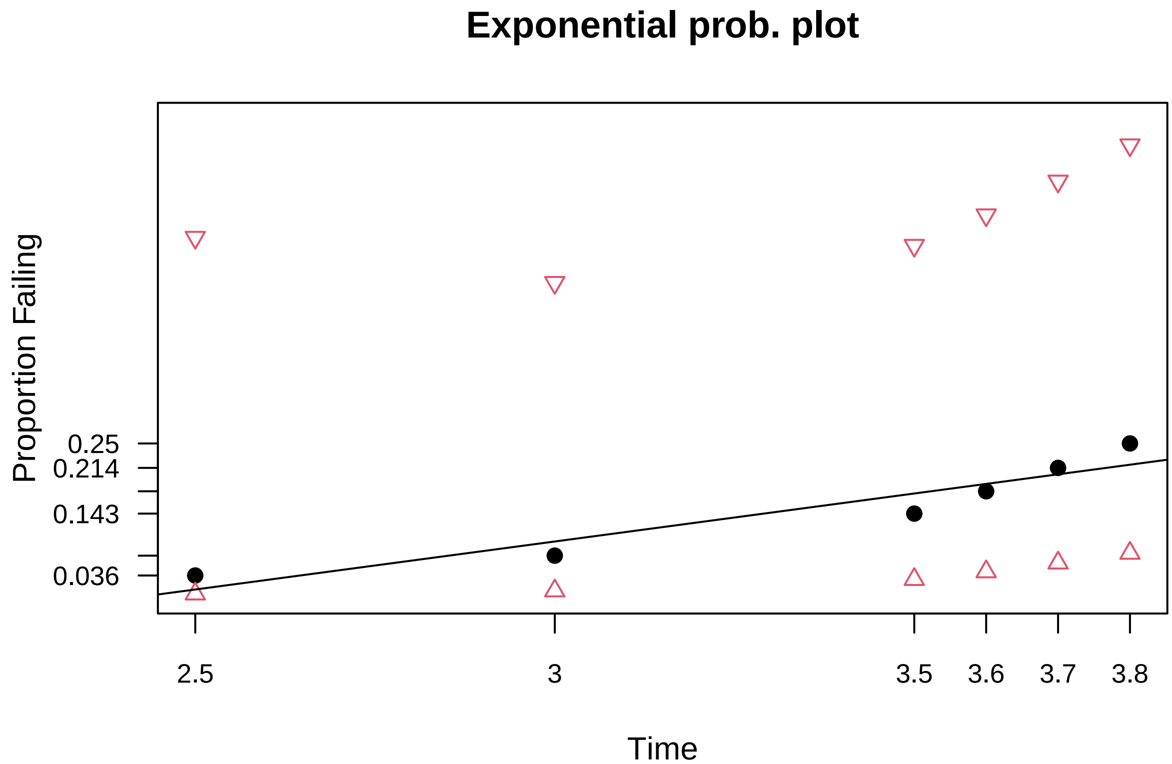
(e)

藉由我們上面配飾的 linear model，可以推得 $\left(\hat{\eta}\ ,\ \hat{\beta}\right) = \left(exp\left(\frac{10.28786}{1.75869}\right),\ 1.75869\right) = (347.1403\ ,\ 1.75869)$，因此 $t_{0.10}$ 可以估計為

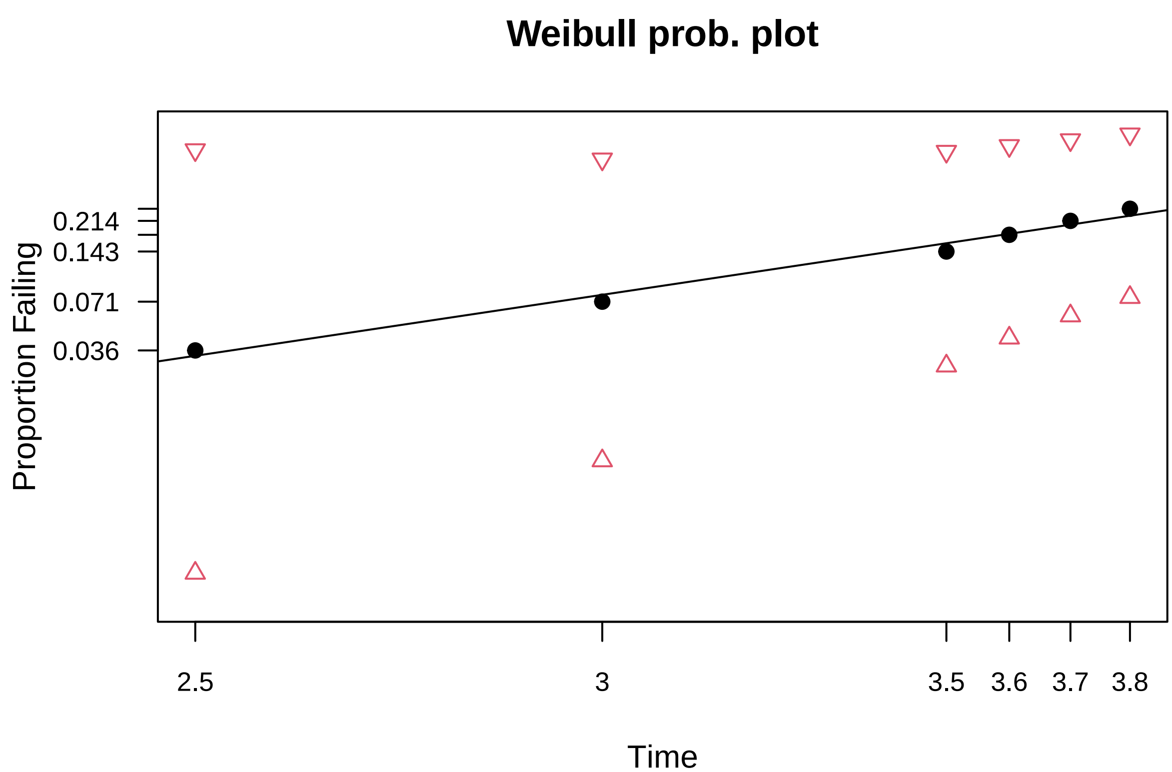
$$\hat{t}_{0.10} = \hat{\eta}\left[-log(1-0.10)\right]^{1/\hat{\beta}} = 96.55944$$

但是目前的資料所估計出的 propotion failing 只介於 0 到 0.085 之間，我們只能說機率值落在此範圍的數據大致服從 Weibull distribution，並不能保證在此範圍之外也依舊如此，而 $\hat{t}_{0.10}$ 就是一個外差估計值，其所估計出的結果可能會有較為明顯的誤差。

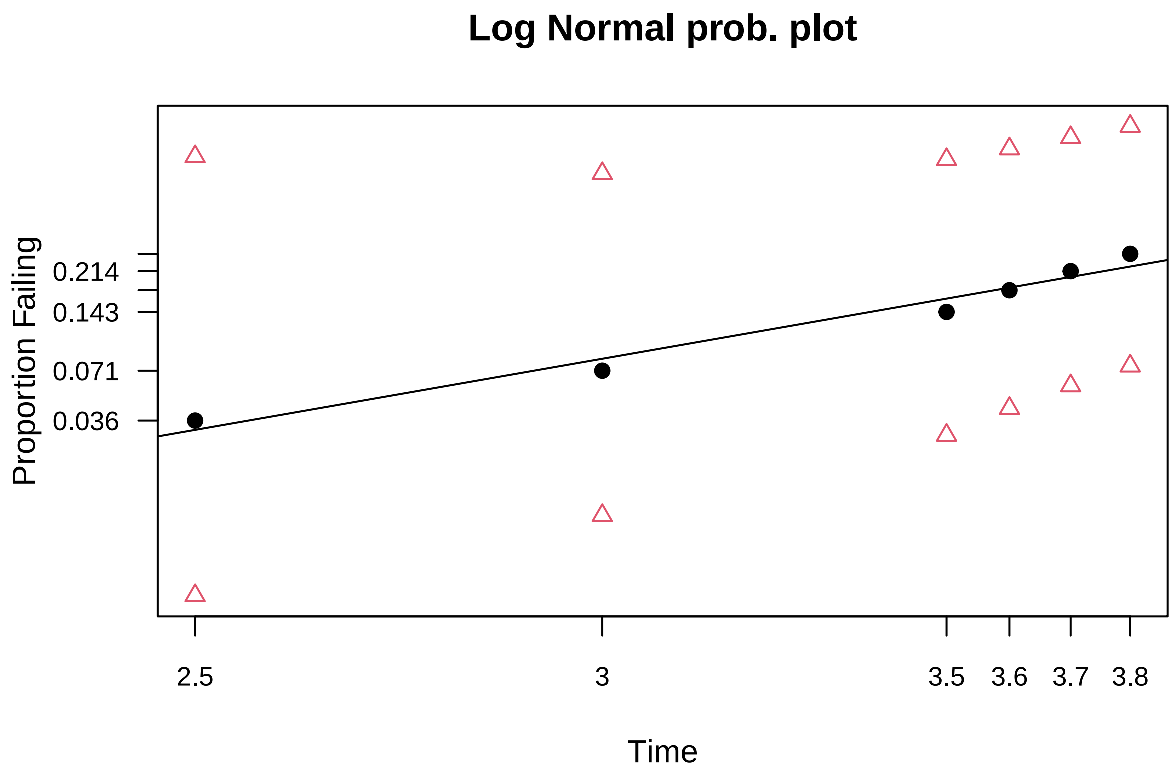
Problem 2.



Exponential prob. plot
Proportion Failing
0.25
0.214
0.143
0.036
2.5
3
3.5
3.6
3.7
3.8
Time



Weibull prob. plot
Proportion Failing
0.214
0.143
0.071
0.036
2.5
3
3.5
3.6
3.7
3.8
Time



- 三種分布的 probability plots 皆可以在其 simultaneous confidence bands 中劃出一條直線
- Exponential probability plot 中資料點分布最不接近一條直線
- Weibull 和 lognormal probability plot 中資料點大致都落在一直線上，其中又以 Weibull 的更為接近一直線

故 Weibull distribution 比較適合用來配適此筆資料的模型。

Problem 3.

(a)

The FREC distribution cdf

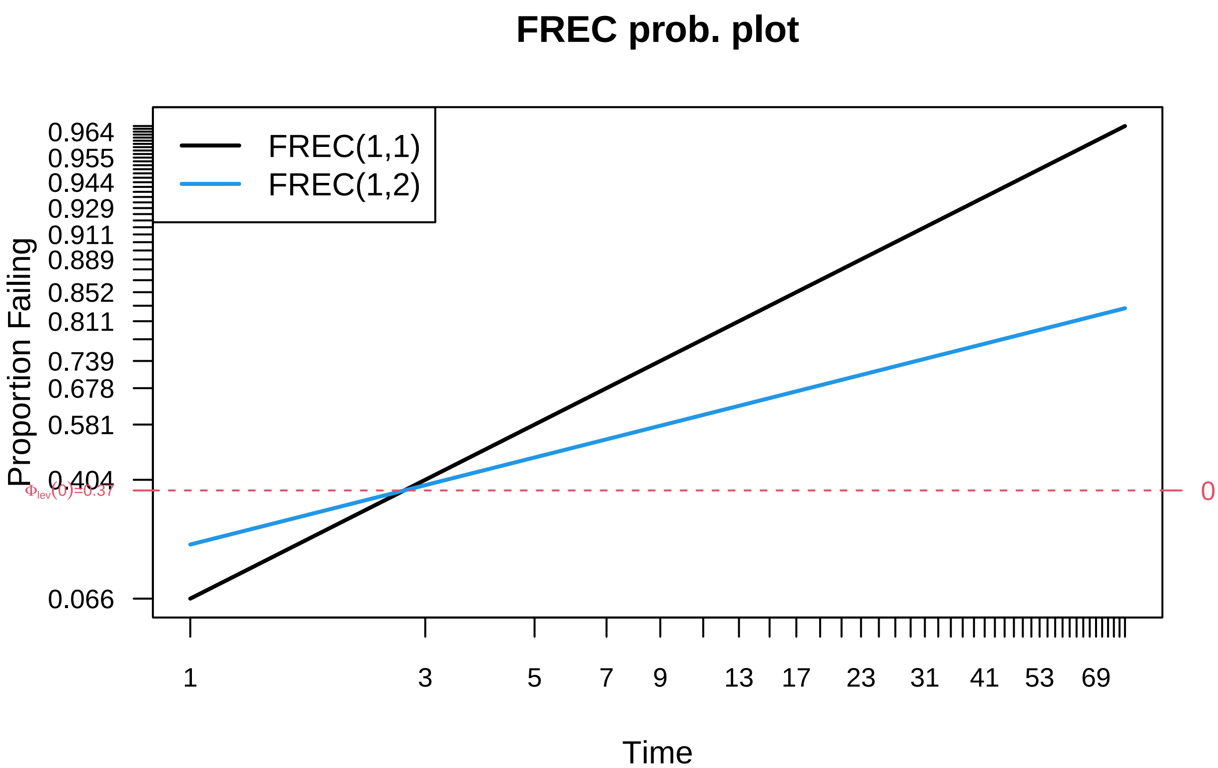
$$p \;=\; F\left(t_p\right) \;=\; \Phi_{lev}\left[\frac{log(t_p)-\mu}{\sigma}\right] \;\Rightarrow\; log(t_p) \;=\; \mu \;+\; \sigma\, \Phi_{lev}^{-1}(p)$$

plotting position

$$\left(log\left(t_p\right)\;,\; \Phi_{lev}^{-1}(p)\right) \quad,\quad \Phi_{lev}^{-1}(p) \;=\; -log(-log(p))$$

(b)

relabel at $(t_p\ ,\ p)$



(c)

$$log\left(t_{p^*}\right) \ = \ log\left(e^{\mu}\right) \ = \ \mu \ + \ \sigma\ \Phi^{-1}_{lev}(p^*) \ \Rightarrow \ \Phi^{-1}_{lev}(p^*) \ = \ 0 \ \Rightarrow \ p^* \ = \ \Phi_{lev}(0) \ = \ 0.37$$

the 0.37 quantile of the distribution corresponds to the scale paremeter e^{μ}

Problem 4.

(a)

$$\begin{aligned} p \ &= \ F(t_p) \ = \ 1 - \left[1 + \left(\frac{log(t_p) - \mu}{\sigma}\right)\right]^{-1} \\ \Rightarrow log(t_p) \ &= \ \mu \ + \ \sigma\left(\frac{p}{1-p}\right) \\ \Rightarrow t_p \ &= \ exp\left[\mu \ + \ \sigma\left(\frac{p}{1-p}\right)\right] \end{aligned}$$

(b)

plotting position

$$\left(log(t_p) \;,\; \left(\frac{p}{1-p} \right) \right)$$

which will linearize all the cdfs with slope $= \; \frac{1}{\sigma}$, and x-intercept $= \; \mu$

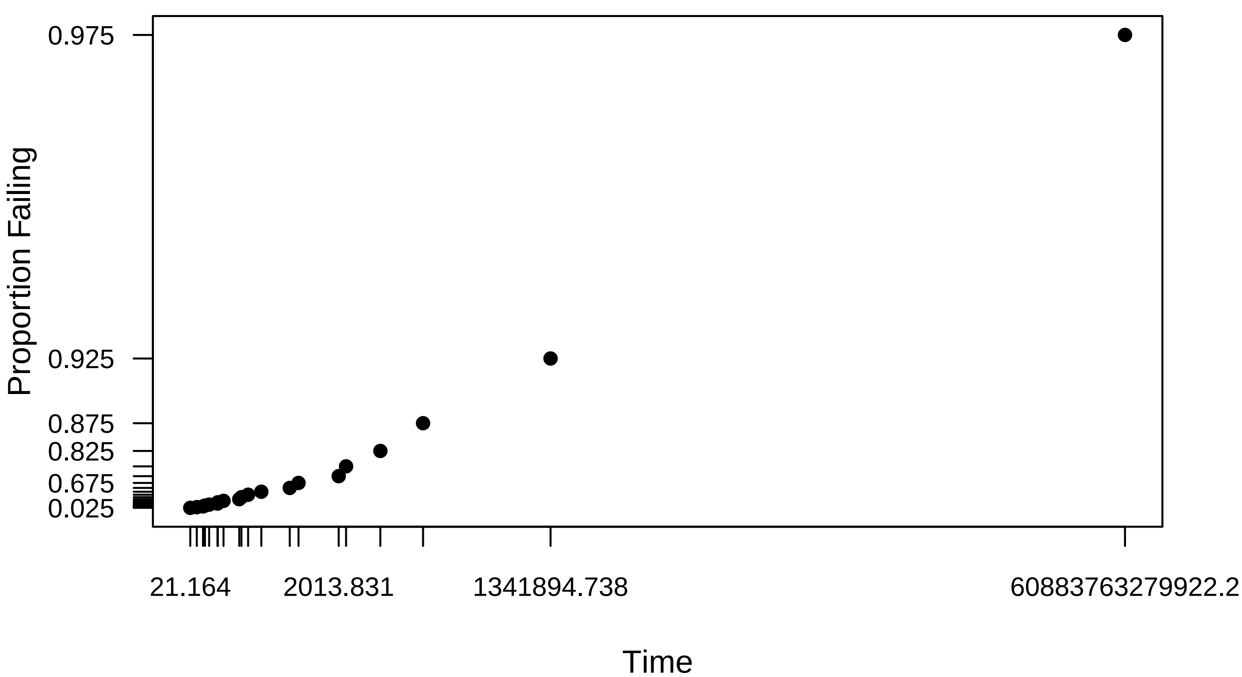
(c)

By inverse CDF method to generate the sample :

(1) draw the sample $\{U_i\}_{i=1}^{20} \overset{iid}{\sim} U(0,1)$

(2) $F^{-1}(U_i) \; = \; T_i \overset{iid}{\sim} F$, where $F^{-1}(p \; ; \; \mu, \sigma) \; = \; exp\left[\mu \; + \; \sigma\left(\frac{p}{1-p}\right)\right]$

$\{T_i\}_{i=1}^{20}$ are the samples what we want.
Taking $\left(log\left(T_{(i)}\right) \;,\; \frac{i-0.5}{20} / \left(1 - \frac{i-0.5}{20}\right)\right)$ as plotting position, and then relabeling at $\left(T_{(i)} \;,\; \frac{i-0.5}{20}\right)$



(d)

μ (or e^{μ} in relabeling) is the x-intercept and $\frac{1}{\sigma}$ is the slope in the probability plot