品質管制 Homework 8

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4.20

$$(\hat{\mu}_0\ ,\ \hat{\sigma}_0)\ =\ (11.5\ ,\ 1.5)\ ,\ (k^+\ ,\ h^+)\ =\ (0.5\ ,\ 3.502)\ ,\ \text{nomial}\ ARL_0\ =\ 200$$

The chart (4.46)-(4.47) in this case is equivalent to the chart (4.7)-(4.8) for detecting a distributional shift from $N(0,1)$ to $N(\frac{\mu_0-\hat{\mu}_0}{\hat{\sigma}_0},(\frac{\sigma_0}{\hat{\sigma}_0})^2)$

(i)

$$X_n\ \sim\ N(10\ ,\ 1^2)\ \Rightarrow\ \frac{X_n-\hat{\mu}_0}{\hat{\sigma}_0}\ \sim\ N(\frac{10-11.5}{1.5}\ ,\ (\frac{1}{1.5})^2)\ =\ N(-1\ ,\ (\frac{1}{1.5})^2)\ \Rightarrow\ \delta\ =\ -1\ ,\ \lambda\ =\ \frac{1}{1.5}$$

$$\therefore k^*\ =\ \frac{k^+-\delta}{\lambda\ \sigma}\ =\ (0.5+1)\times 1.5\ =\ 2.25\ ,\ h^*\ =\ \frac{h^+}{\lambda\ \sigma}\ =\ 3.502\times 1.5\ =\ 5.253$$

take $(k^*\ ,\ h^*)$ into Siegmund formula $\Rightarrow\ ARL_0\ =\ 346272677538$

(ii)

$$X_n\ \sim\ N(10\ ,\ 1.5^2)\ \Rightarrow\ \frac{X_n-\hat{\mu}_0}{\hat{\sigma}_0}\ \sim\ N(\frac{10-11.5}{1.5}\ ,\ 1^2)\ =\ N(-1\ ,\ 1^2)\ \Rightarrow\ \delta\ =\ -1\ ,\ \lambda\ =\ 1$$

$$\therefore k^*\ =\ \frac{k^+-\delta}{\lambda\ \sigma}\ =\ 0.5+1\ =\ 1.5\ ,\ h^*\ =\ \frac{h^+}{\lambda\ \sigma}\ =\ 3.502$$

take $(k^*\ ,\ h^*)$ into Siegmund formula $\Rightarrow\ ARL_0\ =\ 268313.2$

(iii)

$$X_n\ \sim\ N(11.5\ ,\ 1^2)\ \Rightarrow\ \frac{X_n-\hat{\mu}_0}{\hat{\sigma}_0}\ \sim\ N(\frac{11.5-11.5}{1.5}\ ,\ (\frac{1}{1.5})^2)\ =\ N(0\ ,\ (\frac{1}{1.5})^2)\ \Rightarrow\ \delta\ =\ 0\ ,\ \lambda\ =\ \frac{1}{1.5}$$

$$\therefore k^*\ =\ \frac{k^+-\delta}{\lambda\ \sigma}\ =\ 0.5\times 1.5\ =\ 0.75\ ,\ h^*\ =\ \frac{h^+}{\lambda\ \sigma}\ =\ 3.502\times 1.5\ =\ 5.253$$

take $(k^*\ ,\ h^*)$ into Siegmund formula $\Rightarrow\ ARL_0\ =\ 13494.22$

(iv)

$$X_n\ \sim\ N(13\ ,\ 2^2)\ \Rightarrow\ \frac{X_n-\hat{\mu}_0}{\hat{\sigma}_0}\ \sim\ N(\frac{13-11.5}{1.5}\ ,\ (\frac{2}{1.5})^2)\ =\ N(1\ ,\ (\frac{2}{1.5})^2)\ \Rightarrow\ \delta\ =\ 1\ ,\ \lambda\ =\ \frac{2}{1.5}$$

$$\therefore k^*\ =\ \frac{k^+-\delta}{\lambda\ \sigma}\ =\ (0.5-1)\times 0.75\ =\ -0.375\ ,\ h^*\ =\ \frac{h^+}{\lambda\ \sigma}\ =\ 3.502\times 0.75\ =\ 2.6265$$

take $(k^*\ ,\ h^*)$ into Siegmund formula $\Rightarrow\ ARL_0\ =\ 6.764607$

結論：

(1) (i)(ii)(iii) 所計算出的 $actual\ ARL_0$ 皆遠大於 $nominal\ ARL_0\ =\ 200$，(iv) 算出的則遠小於 200。

(2) 由 (i) 和 (ii) 可以得知，在 μ_0 相同的情況下，σ_0 的數值越接近 $\hat{\sigma}_0 = 1.5$，最後所算出的 ARL_0 數值越小。

(3) 由 (i) 和 (iii) 可以得知，在 σ_0 相同的情況下，μ_0 的數值越接近 $\hat{\mu}_0 = 11.5$，最後所算出的 ARL_0 數值越小。

(4) 由 (iv) 可以得知，(μ_0, σ_0) 皆分別大於 $(\hat{\mu}_0, \hat{\sigma}_0)$ 時，所計算出的 ARL_0 會遠小於 200，因為 Siegmund formula 是設計來偵測正向的平均偏移。