品質管制 Homework 11

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5.20

(ii)
$ARL_0 = 100$, $\eta(e_n) = \eta_1(e_n)$, $(\delta_1, \delta_2) = (1.0, 4)$, and $v = 0.05$
by Table 5.6 $\Rightarrow$ $h = 0.7874$, $\lambda = 0.1813$, $u = 2.5752$
The charting statistic of the AEWMA chart is defined by

$$A_n \;=\; A_{n-1} \;+\; \eta_1(e_n)\;, \quad \text{where} \;\; A_0 \;=\; \mu_0 \;=\; 0\;,\; e_n \;=\; X_n \;-\; A_{n-1}$$

and

$$\eta_1(e_n) \;=\; \begin{cases} e_n \;+\; (1-\lambda)\; u & ,\ \text{if} \;\; e_n < -u \\ \lambda\; e_n & ,\ \text{if} \;\; |e_n| \le u \\ e_n \;-\; (1-\lambda)\; u & ,\ \text{if} \;\; e_n > u \end{cases}$$

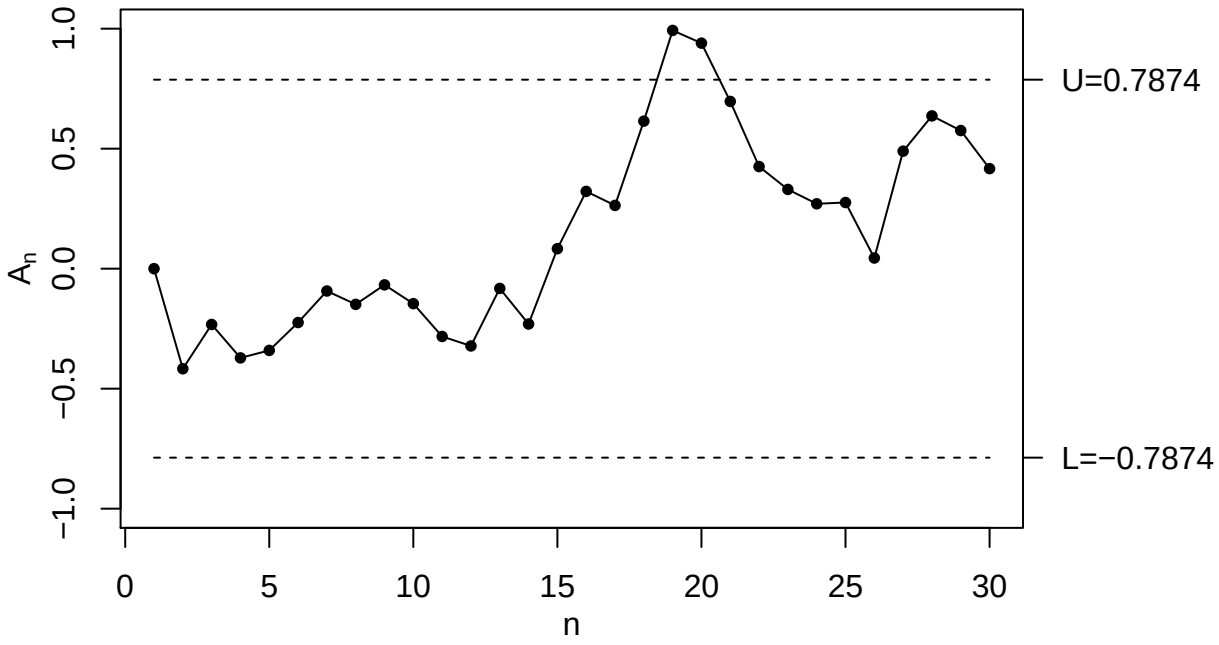
The results of charting statistics are shown as below :

X_n	e_n	A_n
0.0	0.0000	0.0000
-2.3	-2.3000	-0.4170
0.6	1.0170	-0.2326
-1.0	-0.7674	-0.3717
-0.2	0.1717	-0.3406
0.3	0.6406	-0.2245
0.5	0.7245	-0.0931
-0.4	-0.3069	-0.1488
0.3	0.4488	-0.0674
-0.5	-0.4326	-0.1458
-0.9	-0.7542	-0.2826
-0.5	-0.2174	-0.3220
1.0	1.3220	-0.0823
-0.9	-0.8177	-0.2306
1.5	1.7306	0.0832
1.4	1.3168	0.3219
0.0	-0.3219	0.2636
2.2	1.9364	0.6146
2.7	2.0854	0.9927
0.7	-0.2927	0.9396
-0.4	-1.3396	0.6968
-0.8	-1.4968	0.4254
-0.1	-0.5254	0.3301
0.0	-0.3301	0.2703
0.3	0.0297	0.2757
-1.0	-1.2757	0.0444
2.5	2.4556	0.4896

X_n	e_n	A_n
1.3	0.8104	0.6365
0.3	-0.3365	0.5755
-0.3	-0.8755	0.4168

The chart signals if $|A_n - \mu_0| > h$

$$\Rightarrow \begin{cases} U = \mu_0 + h = 0 + 0.7874 = 0.7874 \\ L = \mu_0 - h = 0 - 0.7874 = -0.7874 \end{cases}$$



The AEWMA chart detects mean shift at the 19th time point.

(iv)
$ARL_0 = 100$, $\eta(e_n) = \eta_2(e_n)$, $(\delta_1, \delta_2) = (0.25, 4)$, and $v = 0.05$
by Table 5.6 $\Rightarrow$ $h = 0.3542$, $\lambda = 0.0188$, $u = 12.6145$
The charting statistic of the AEWMA chart is defined by

$$A_n \ = \ A_{n-1} \ + \ \eta_2(e_n) \ , \ \text{ where } \ A_0 \ = \ \mu_0 \ = \ 0 \ , \ e_n \ = \ X_n \ - \ A_{n-1}$$

and

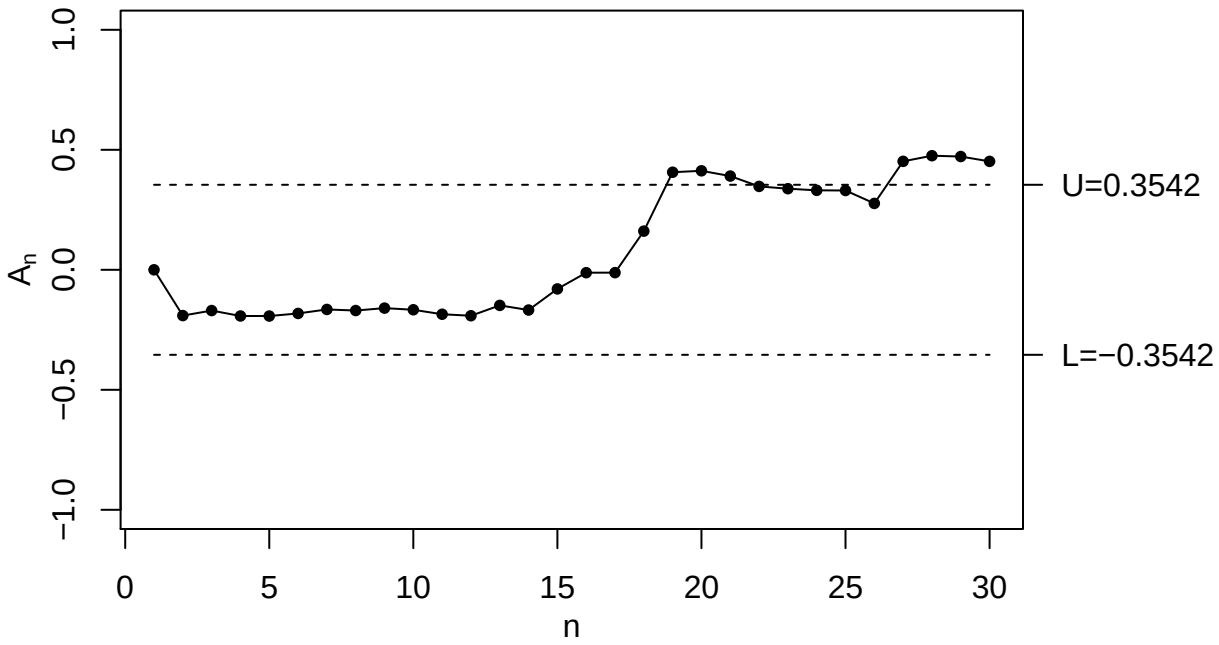
$$\eta_2(e_n) \ = \ \begin{cases} e_n \ [\ 1 - (1-\lambda)(1 - (\frac{e_n}{u})^2)^2 \ ] & , \ \text{if } \ |e_n| \le u \\ e_n & , \ \text{otherwise} \end{cases}$$

The results of charting statistics are shown as below :

X_n	e_n	A_n
0.0	0.0000	0.0000
-2.3	-2.3000	-0.1908
0.6	0.7908	-0.1698
-1.0	-0.8302	-0.1925
-0.2	-0.0075	-0.1926
0.3	0.4926	-0.1819
0.5	0.6819	-0.1652
-0.4	-0.2348	-0.1697
0.3	0.4697	-0.1596
-0.5	-0.3404	-0.1665
-0.9	-0.7335	-0.1852
-0.5	-0.3148	-0.1915
1.0	1.1915	-0.1483
-0.9	-0.7517	-0.1677
1.5	1.6677	-0.0796
1.4	1.4796	-0.0121
0.0	0.0121	-0.0119
2.2	2.2119	0.1611
2.7	2.5389	0.4066
0.7	0.2934	0.4124
-0.4	-0.8124	0.3905
-0.8	-1.1905	0.3474
-0.1	-0.4474	0.3379
0.0	-0.3379	0.3311
0.3	-0.0311	0.3305
-1.0	-1.3305	0.2766
2.5	2.2234	0.4518
1.3	0.8482	0.4753
0.3	-0.1753	0.4719
-0.3	-0.7719	0.4518

The chart signals if $|A_n \ - \ \mu_0| \ > \ h$

$$\Rightarrow \begin{cases} U \ = \ \mu_0 \ + \ h \ = \ 0 \ + \ 0.3542 \ = \ 0.3542 \\ L \ = \ \mu_0 \ - \ h \ = \ 0 \ - \ 0.3542 \ = \ -0.3542 \end{cases}$$



The AEWMA chart detects mean shift at the 19th time point.