品質管制 Homework 6

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4.11

給定變數 $(k,\ h)\ =\ (0.5,\ 3.08)$，並定義函數 *compute_arl* 用以計算 ARL 值，如下：

$$ARL\ \approx\ \frac{exp(2k(h+1.166))-2k(h+1.166)-1}{2k^2}$$

再定義函數 *compute_arl_star*，用兩個 one-sided CUSUM chart 計算出 two-sided CUSUM chart 的 ARL 值，如下

$$ARL^{\star}\ =\ (\frac{1}{ARL^{+}}\ +\ \frac{1}{ARL^{-}})^{-1}$$

```
k = 0.5
h = 3.08

compute_arl = function(k, h) {
  return((exp(2*k*(h+1.166))-2*k*(h+1.166)-1)/(2*k^2))
}

compute_arl_star = function(arl_plus, arl_minus) {
  return((1/arl_plus+1/arl_minus)^(-1))
}
```

(i)

```
arl_plus1 = compute_arl(k, h)
arl_minus1 = compute_arl(k, h)

arl_star = compute_arl_star(arl_plus1, arl_minus1)
c(arl_plus1, arl_minus1, arl_star)
```

```
## [1] 129.15910 129.15910  64.57955
```

$$\Rightarrow\ ARL_0^{+}\ =\ ARL_0^{-}\ =\ 129.15910$$
$$\therefore ARL_0^{\star}\ =\ (\frac{1}{ARL_0^{+}}\ +\ \frac{1}{ARL_0^{-}})^{-1}\ =\ 64.57955$$

(ii)

```
delta2 = 1.2
arl_plus2 = compute_arl(k-delta2, 3.08)
arl_minus2 = compute_arl(k+delta2, 3.08)
arl_star2 = compute_arl_star(arl_plus2, arl_minus2)
c(arl_plus2, arl_minus2, arl_star2)
```

```
## [1] 5.047980e+00 3.218974e+05 5.047901e+00
```

$$
\begin{aligned}
&To\ calculate\ ARL_1^+\ ,\ k^* = k-\delta = -0.7\ ,\ h^* = h = 3.08 \\
&and\ then\ take\ (k^*, h^*)\ into\ function\ compute\_arl \\
&\Rightarrow\ ARL_1^+ = 5.04798 \\
&To\ calculate\ ARL_1^-\ ,\ k^* = k-(-\delta) = 1.7\ ,\ h^* = h = 3.08 \\
&and\ then\ take\ (k^*, h^*)\ into\ function\ compute\_arl \\
&\Rightarrow\ ARL_1^- = 3.219 \times 10^5 \\
&\therefore\ ARL_1^* = (\frac{1}{ARL_1^+} + \frac{1}{ARL_1^-})^{-1} = 5.047901
\end{aligned}
$$

(iii)

```
delta3 = -1.2
arl_plus3 = compute_arl(k-delta3, 3.08)
arl_minus3 = compute_arl(k+delta3, 3.08)
arl_star3 = compute_arl_star(arl_plus3, arl_minus3)
c(arl_plus3, arl_minus3, arl_star3)
```

```
## [1] 3.218974e+05 5.047980e+00 5.047901e+00
```

$$
\begin{aligned}
&To\ calculate\ ARL_1^+\ ,\ k^* = k-\delta = 1.7\ ,\ h^* = h = 3.08 \\
&and\ then\ take\ (k^*, h^*)\ into\ function\ compute\_arl \\
&\Rightarrow\ ARL_1^+ = 3.219 \times 10^5 \\
&To\ calculate\ ARL_1^-\ ,\ k^* = k-(-\delta) = -0.7\ ,\ h^* = h = 3.08 \\
&and\ then\ take\ (k^*, h^*)\ into\ function\ compute\_arl \\
&\Rightarrow\ ARL_1^- = 5.04798 \\
&\therefore\ ARL_1^* = (\frac{1}{ARL_1^+} + \frac{1}{ARL_1^-})^{-1} = 5.047901
\end{aligned}
$$

(iv)

(ii) 和 (iii) 兩小題的 ARL_1^+ 及 ARL_1^- 數值為互相交換，因為計算 $mean\ shift\ \delta = 1.2$ 的 ARL_1^+，同等於計算 $mean\ shift\ \delta = -1.2$ 的 ARL_1^-，反之亦然。

因為兩小題的 ARL_1^+ 和 ARL_1^- 僅為數值互換，所以計算出的 ARL_1^* 會完全相同。