Applied Multivariate Analysis Homework 5

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Problem 1.

Total inertia

$$\phi^2 = \sum_i \sum_j \frac{\left(p_{ij} - r_i c_j\right)^2}{r_i c_j} = tr\left(QQ'\right) = \text{sum of the eigen values} = 0.1287176$$

```
sum(res.ca$eig[,1])
```

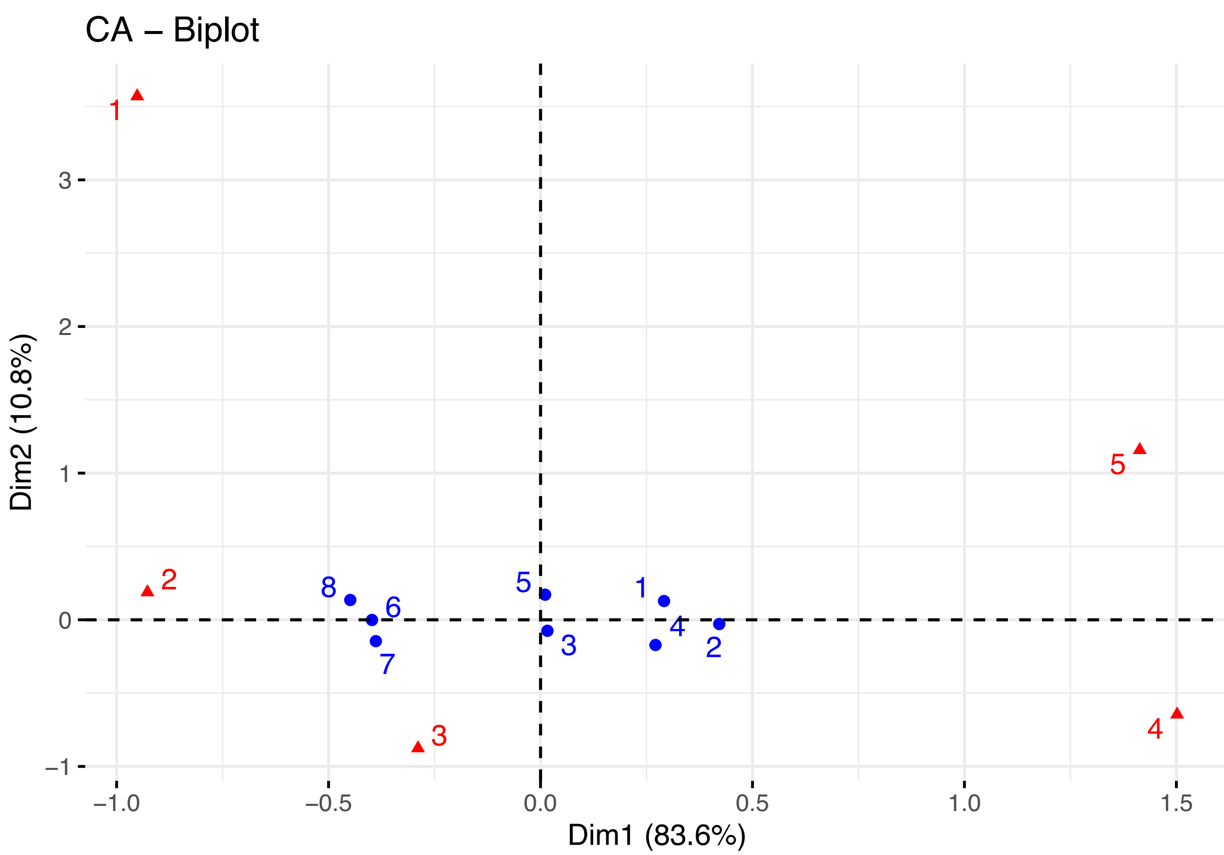
```
## [1] 0.1287176
```

And the respective inertia for the first two coordinates are

```
res.ca$eig[1:2,1]
```

```
##      dim 1      dim 2
## 0.10765619 0.01386851
```

The asymmetric plot with principal coordinates for A3 :




```
##                 1     2     3     4     5
## 1           0.037 0.266 0.252 0.193 0.252
## 2           0.016 0.211 0.283 0.247 0.243
## 3           0.036 0.281 0.391 0.141 0.151
## 4           0.013 0.253 0.318 0.260 0.156
## 5           0.060 0.355 0.259 0.145 0.181
## 6           0.046 0.487 0.333 0.067 0.067
## 7           0.021 0.471 0.385 0.064 0.059
## 8           0.084 0.469 0.336 0.049 0.063
## mean_profile 0.037 0.341 0.318 0.150 0.154
```

(1) 高中職的學生 ($A3 = 7, 8, 9$) 明顯分成一群，和其他年級較低的學生相差較遠

(2) 高中職的學生 ($A3 = 7, 8, 9$) 在運動時間較短 ($B7 = 2, 3$) 的 levels 所占比例高於 mean profile，可藉由觀察 $A3 = 7, 8, 9$ 和原點相連所形成的向量跟 $B7 = 2, 3$ 和原點相連所形成的向量的夾角 $<$ 90 度得知。而年級較低的學生 ($A3 = 1, 2, 4$) 在運動時間較長 ($B7 = 4, 5$) 的 levels 所占比例高於 mean profile，一樣可以藉由觀察向量夾角得知

(3) $A3 = 3, 5$ 的學生在圖形上很接近原點，故可以推測他們在各種運動時間的 level 上所佔比例和 mean profile 並沒有太大差距

Problem 2.

(1) PC method with varimax rotation

```
##
## Loadings:
##        RC1    RC2    RC3
## B10_1                 0.115
## B10_2                 0.147
## B10_3                 0.543
## B10_4                 0.530
## B10_5                 0.503
## B10_6
## B10_7                 0.133
## B10_8                 0.174
## B10_9
## B10_10
## C9_1                  0.725
## C9_2                  0.765
## C9_3                  0.725
## C9_4                  0.749
## C9_5          -0.138  0.604
## C9_6                  0.627
## C9_7          -0.120  0.572
## C9_8                  0.518
## C9_9                  0.423
## C9_10          0.103  0.283
## C9_11
## C9_12                 0.367
## C9_13                 0.268
## C9_14                 0.280
## C9_15                 0.300
## C9_16          0.138
## C9_17
## C9_18          0.106
## C9_19
## C10_1   0.719
## C10_2   0.611        -0.101
## C10_3   0.860
## C10_4   0.947
## C10_5   0.649
## C10_6   0.946
## C10_7   0.933
## C10_8   0.960
## C10_9   0.902
```

```
## C10_10  0.873
## C10_11         0.807
## C10_12         0.710
## C10_13         0.703
## C10_14         0.763
## C10_15         0.886
## C10_16         0.851
## C10_17         0.646
## C10_18         0.881
## C10_19         0.742
## C10_20         0.759
##
##                  RC1   RC2   RC3
## SS loadings    7.285 6.251 5.162
## Proportion Var 0.149 0.128 0.105
## Cumulative Var 0.149 0.276 0.382
```

(2) PC method with quartimax rotation

```
##
## Loadings:
##        RC1    RC2    RC3
## B10_1                 0.115
## B10_2                 0.147
## B10_3                 0.545
## B10_4                 0.532
## B10_5                 0.499
## B10_6
## B10_7                 0.127
## B10_8                 0.170
## B10_9
## B10_10
## C9_1                  0.726
## C9_2                  0.769
## C9_3                  0.727
## C9_4                  0.750
## C9_5          -0.101  0.611
## C9_6                  0.632
## C9_7                  0.579
## C9_8                  0.522
## C9_9                  0.423
## C9_10          0.122  0.276
## C9_11
## C9_12                 0.367
## C9_13                 0.263
```

```
## C9_14                0.276
## C9_15                0.299
## C9_16         0.134
## C9_17
## C9_18         0.102
## C9_19
## C10_1   0.720
## C10_2   0.613        -0.107
## C10_3   0.859
## C10_4   0.947
## C10_5   0.650
## C10_6   0.946
## C10_7   0.934
## C10_8   0.959
## C10_9   0.903
## C10_10  0.875
## C10_11         0.808
## C10_12         0.714
## C10_13  0.103  0.699
## C10_14         0.766
## C10_15         0.890
## C10_16         0.850
## C10_17         0.639
## C10_18         0.884
## C10_19         0.741
## C10_20         0.758
##
##                  RC1   RC2   RC3
## SS loadings    7.297 6.215 5.187
## Proportion Var 0.149 0.127 0.106
## Cumulative Var 0.149 0.276 0.382
```

PF method with varimax rotation

```
##
## Loadings:
##        PA1    PA2    PA3
## B10_1
## B10_2                0.119
## B10_3                0.497
## B10_4                0.483
## B10_5                0.457
## B10_6
## B10_7                0.107
## B10_8                0.141
```

```
## B10_9
## B10_10
## C9_1                   0.710
## C9_2                   0.761
## C9_3                   0.711
## C9_4                   0.741
## C9_5          -0.127  0.559
## C9_6                   0.579
## C9_7          -0.110  0.523
## C9_8                   0.463
## C9_9                   0.368
## C9_10                  0.237
## C9_11
## C9_12                  0.313
## C9_13                  0.221
## C9_14                  0.230
## C9_15                  0.250
## C9_16          0.113
## C9_17
## C9_18
## C9_19
## C10_1   0.676
## C10_2   0.563
## C10_3   0.840
## C10_4   0.952
## C10_5   0.601
## C10_6   0.950
## C10_7   0.932
## C10_8   0.969
## C10_9   0.893
## C10_10  0.857
## C10_11         0.785
## C10_12         0.673
## C10_13         0.662
## C10_14         0.733
## C10_15         0.888
## C10_16         0.839
## C10_17         0.598
## C10_18         0.881
## C10_19         0.706
## C10_20         0.726
##
##                  PA1   PA2   PA3
## SS loadings    7.052 5.843 4.537
## Proportion Var 0.144 0.119 0.093
```

```
## Cumulative Var 0.144 0.263 0.356
```

PF method with quartimax rotation

```
##
## Loadings:
##        PA1    PA2    PA3
## B10_1
## B10_2                 0.119
## B10_3                 0.499
## B10_4                 0.485
## B10_5                 0.454
## B10_6
## B10_7                 0.102
## B10_8                 0.138
## B10_9
## B10_10
## C9_1                  0.711
## C9_2                  0.764
## C9_3                  0.713
## C9_4                  0.742
## C9_5                  0.565
## C9_6                  0.584
## C9_7                  0.528
## C9_8                  0.467
## C9_9                  0.368
## C9_10          0.101  0.231
## C9_11
## C9_12                 0.313
## C9_13                 0.218
## C9_14                 0.227
## C9_15                 0.249
## C9_16          0.109
## C9_17
## C9_18
## C9_19
## C10_1   0.677
## C10_2   0.565        -0.104
## C10_3   0.840
## C10_4   0.952
## C10_5   0.602
## C10_6   0.950
## C10_7   0.932
## C10_8   0.969
## C10_9   0.894
```

```
## C10_10  0.858
## C10_11         0.786
## C10_12         0.677
## C10_13         0.658
## C10_14         0.736
## C10_15         0.892
## C10_16         0.838
## C10_17         0.593
## C10_18         0.884
## C10_19         0.705
## C10_20         0.724
##
##                  PA1   PA2   PA3
## SS loadings    7.060 5.811 4.560
## Proportion Var 0.144 0.119 0.093
## Cumulative Var 0.144 0.263 0.356
```

MLE method with varimax rotation

```
##
## Loadings:
##        ML1    ML2    ML3
## B10_1
## B10_2
## B10_3                 0.481
## B10_4                 0.463
## B10_5                 0.442
## B10_6
## B10_7
## B10_8
## B10_9
## B10_10               -0.171
## C9_1                  0.779
## C9_2                  0.824
## C9_3                  0.779
## C9_4                  0.819
## C9_5                  0.497
## C9_6                  0.494
## C9_7                  0.446
## C9_8                  0.376
## C9_9                  0.310
## C9_10                 0.178
## C9_11
## C9_12                 0.241
## C9_13                 0.142
```

```
## C9_14                0.151
## C9_15                0.177
## C9_16               -0.128
## C9_17
## C9_18               -0.135
## C9_19               -0.159
## C10_1   0.636
## C10_2   0.512       -0.125
## C10_3   0.862
## C10_4   0.977
## C10_5   0.557
## C10_6   0.947
## C10_7   0.902
## C10_8   0.991
## C10_9   0.897
## C10_10  0.848
## C10_11         0.798
## C10_12         0.701
## C10_13         0.650
## C10_14         0.755
## C10_15         0.898
## C10_16         0.837
## C10_17         0.569 -0.114
## C10_18         0.887
## C10_19         0.690
## C10_20         0.712
##
##                  ML1   ML2   ML3
## SS loadings    6.925 5.790 4.468
## Proportion Var 0.141 0.118 0.091
## Cumulative Var 0.141 0.259 0.351
```

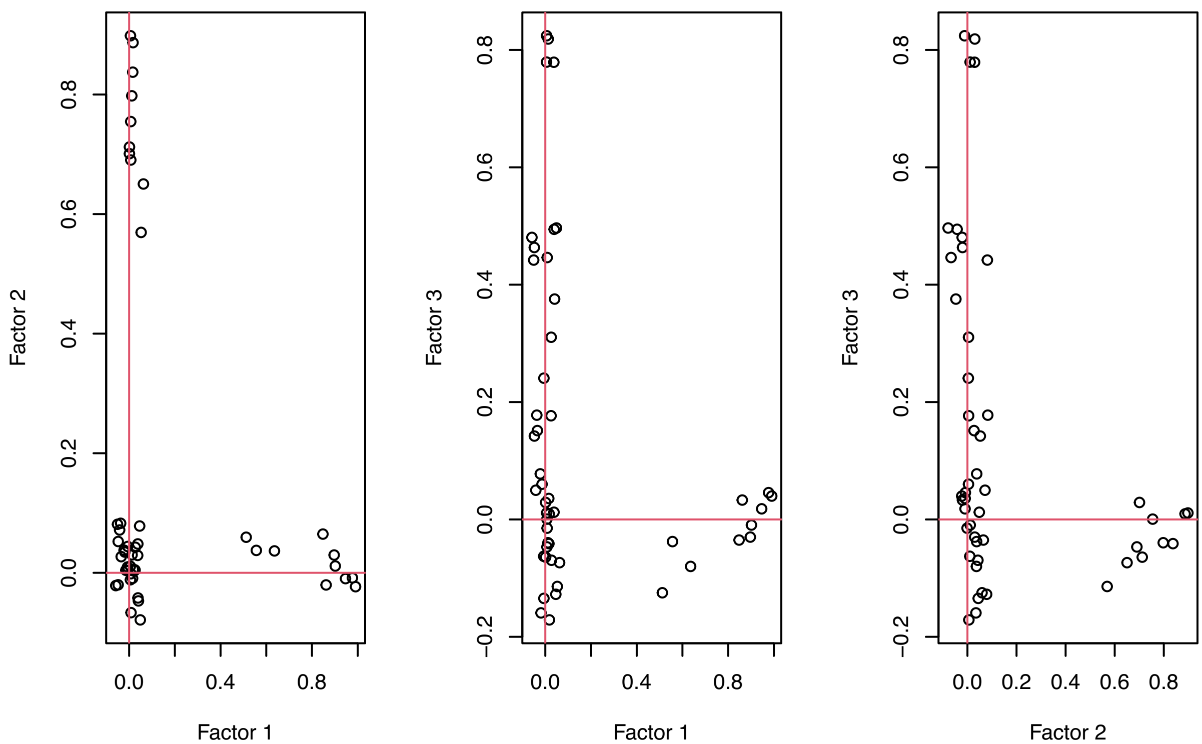
MLE method with quartimax rotation

```
##
## Loadings:
##        ML1    ML2    ML3
## B10_1
## B10_2
## B10_3                0.480
## B10_4                0.463
## B10_5                0.443
## B10_6
## B10_7
## B10_8
```

```
## B10_9
## B10_10               -0.171
## C9_1                  0.779
## C9_2                  0.824
## C9_3                  0.779
## C9_4                  0.819
## C9_5                  0.495
## C9_6                  0.493
## C9_7                  0.445
## C9_8                  0.375
## C9_9                  0.310
## C9_10                 0.179
## C9_11
## C9_12                 0.241
## C9_13                 0.143
## C9_14                 0.152
## C9_15                 0.177
## C9_16                -0.126
## C9_17
## C9_18                -0.134
## C9_19                -0.158
## C10_1   0.636
## C10_2   0.512        -0.124
## C10_3   0.862
## C10_4   0.977
## C10_5   0.556
## C10_6   0.947
## C10_7   0.902
## C10_8   0.991
## C10_9   0.897
## C10_10  0.848
## C10_11         0.798
## C10_12         0.700
## C10_13         0.652
## C10_14         0.755
## C10_15         0.898
## C10_16         0.838
## C10_17         0.572 -0.103
## C10_18         0.886
## C10_19         0.691
## C10_20         0.713
##
##                  ML1   ML2   ML3
## SS loadings    6.924 5.802 4.457
## Proportion Var 0.141 0.118 0.091
```

```
## Cumulative Var 0.141 0.260 0.351
```

我們可以看到各種方式所做出的結果差異不大，基本上每個變數都只有三個 factors 中的一個的 loading 比較大，這是因為我們有進行 rotation 所造成的結果，接下來選取 MLE method with varimax rotation 來進一步觀察。



可以看出大部份的變數都落在三個 factors 各自的軸上，符合前面對於 loading 數值的描述，以下列出三個 Factors 對應到 loading 較大的變數各為哪些，並試著解釋出三個 Factor 各自可能代表的意義

(i) Factor 1：C10_1 ∼ C10_10，這些變數都是一些不良且可能成癮的嗜好，Factor 1 可能代表著「不良嗜好傾向」

(ii) Factor 2：C10_11 ∼ C10_20，這些變數都是一些跟家庭或同儕有關所造成的反社會行為，Factor 2 可能代表著「社會環境造成的不良影響」

(iii) Factor 3：B10_3 ∼ B10_5 & C9，這些變數都是一些家庭或同儕所給予的正面回饋，Factor 3 可能代表著「社會環境所給予的正面影響」

3.

$\because \operatorname{diag}(S - \tilde{L}\tilde{L}' - \tilde{\Psi}) = \vec{0}$

$\therefore$ SS of entries $(S - \tilde{L}\tilde{L}' - \tilde{\Psi}) \le$ SS of entries $(S - \tilde{L}\tilde{L}')$

and $S - \tilde{L}\tilde{L}' = \hat{P}\hat{\Lambda}\hat{P}'$

where $\hat{P} = [\hat{e}_{m+1} \,\vdots\, \hat{e}_{m+2} \,\vdots\, \cdots \,\vdots\, \hat{e}_p]$, $\hat{\Lambda} = \begin{bmatrix} \hat{\lambda}_{m+1} & & 0 \\ & \ddots & \\ 0 & & \hat{\lambda}_p \end{bmatrix}$

$\because$ SS of entries $(S - \tilde{L}\tilde{L}') =$ SS of entries $(\hat{P}\hat{\Lambda}\hat{P}')$

$= tr[\hat{P}\hat{\Lambda}\hat{P}'\hat{P}\hat{\Lambda}\hat{P}'] = tr[\hat{P}\hat{\Lambda}^2\hat{P}'] = tr[\hat{\Lambda}^2]$

$= \hat{\lambda}_{m+1}^2 + \cdots + \hat{\lambda}_p^2$

$\therefore$ SS of entries $(S - \tilde{L}\tilde{L}' - \tilde{\Psi}) \le \hat{\lambda}_{m+1}^2 + \cdots + \hat{\lambda}_p^2$ $\square$