品質管制 Homework 9

110024516 統研碩一邱繼賢

5.2
Note that the IC distribution is $\{X_i\}_{i=1}^n \overset{iid}{\sim} N(\mu_0\ ,\ \sigma^2)$, and the OC distribution is $\{X_i\}_{i=1}^n \overset{iid}{\sim} N(\mu_1\ ,\ \sigma^2)$.
By (5.1)

$$E_n \ = \ \lambda X_n \ + \ (1-\lambda)E_{n-1} \ = \ \lambda \sum_{i=1}^{n}(1-\lambda)^{n-i}\ X_i \ + \ (1-\lambda)^n\ \mu_0$$

When the process is IC up to the time point n, then

$$\mu_{E_n} \ = \ E(E_n) \ = \ \lambda \sum_{i=1}^{n}(1-\lambda)^{n-i}\ E(X_i) \ + \ (1-\lambda)^n\ \mu_0 \ = \ [\ \lambda \sum_{i=1}^{n}(1-\lambda)^{n-i} \ + \ (1-\lambda)^n\ ]\ \mu_0 \ = \ \mu_0$$

$$\sigma^2_{E_n} \ = \ Var(E_n) \ = \ \lambda^2 \sum_{i=1}^{n}(1-\lambda)^{2n-2i}\ Var(X_i) \ = \ \lambda^2 \times \frac{1-(1-\lambda)^{2n}}{1-(1-\lambda)^2}\ \sigma^2 \ = \ \frac{\lambda}{2-\lambda}\ [1-(1-\lambda)^{2n}]\ \sigma^2 \quad (5.2)$$

$\because E_n$ is a linear combination of X_i which are all from normal distributions

$\therefore E_n$ is also from a normal distribution $\sim N(\mu_{E_n}\ ,\ \sigma^2_{E_n}) \ = \ N(\mu_0\ ,\ \frac{\lambda}{2-\lambda}\ [1-(1-\lambda)^{2n}]\ \sigma^2) \quad (5.3)$

In cases when the process has a mean shift from μ_0 to μ_1 at the time point $1 \leq \tau \leq n$

$$E_{n,\tau} \ = \ \lambda\ [\ \sum_{i=1}^{\tau-1}(1-\lambda)^{n-i}X_i \ + \ \sum_{i=\tau}^{n}(1-\lambda)^{n-i}X_i\ ] \ + \ (1-\lambda)^n\mu_0$$

where $\{X_i\}_{i=1}^{\tau-1} \overset{iid}{\sim} N(\mu_0, \sigma^2)$ and $\{X_i\}_{i=\tau}^{n} \overset{iid}{\sim} N(\mu_1, \sigma^2)$

$$\Rightarrow\ \mu_{E_{n,\tau}} \ = \ E(E_{n,\tau}) \ = \ \lambda \sum_{i=1}^{n}(1-\lambda)^{n-i}\mu_0 \ + \ (1-\lambda)^n\mu_0 \ + \ \lambda \sum_{i=\tau}^{n}(1-\lambda)^{n-i}(\mu_1-\mu_0)$$

$$= \ \mu_0 \ + \ \lambda\ \frac{1-(1-\lambda)^{n-\tau+1}}{1-(1-\lambda)}(\mu_1-\mu_0) \ = \ \mu_0 \ + \ [\ 1-(1-\lambda)^{n-\tau+1}\ ]\ (\mu_1-\mu_0) \quad (5.4)$$

As $n \to \infty$, the variance of E_n converges to

$$\tilde{\sigma}^2_{0,\lambda} \ = \ \lim_{n\to\infty} \sigma^2_{E_n} \ = \ \lim_{n\to\infty} \frac{\lambda}{2-\lambda}\ [1-(1-\lambda)^{2n}]\ \sigma^2 \ = \ \frac{\lambda}{2-\lambda}\ \sigma^2\ (\because 0 < 1-\lambda < 1) \quad (5.5)$$

5.4

First, compute the charting statistic E_n from (5.1)

$$E_n \ = \ \lambda \, X_n \ + \ (1-\lambda) \, E_{n-1} \quad \text{, where} \quad E_0 \ = \ \mu_0 \ = \ 10$$

with respective λ values in (i), (ii), (iii) and (iv). The results are shown as below :

n	X_n	(i) E_n	(ii) E_n	(iii) E_n	(iv) E_n
1	13	10.300000	11.500000	10.300000	11.500000
2	12	10.470000	11.750000	10.470000	11.750000
3	10	10.423000	10.875000	10.423000	10.875000
4	10	10.380700	10.437500	10.380700	10.437500
5	9	10.242630	9.718750	10.242630	9.718750
6	7	9.918367	8.359375	9.918367	8.359375
7	11	10.026530	9.679688	10.026530	9.679688
8	12	10.223877	10.839844	10.223877	10.839844
9	7	9.901490	8.919922	9.901490	8.919922
10	10	9.911341	9.459961	9.911341	9.459961
11	11	10.020207	10.229980	10.020207	10.229980
12	10	10.018186	10.114990	10.018186	10.114990
13	11	10.116367	10.557495	10.116367	10.557495
14	10	10.104731	10.278748	10.104731	10.278748
15	9	9.994257	9.639374	9.994257	9.639374
16	18	10.794832	13.819687	10.794832	13.819687
17	14	11.115349	13.909843	11.115349	13.909843
18	16	11.603814	14.954922	11.603814	14.954922
19	18	12.243432	16.477461	12.243432	16.477461
20	12	12.219089	14.238730	12.219089	14.238730
21	13	12.297180	13.619365	12.297180	13.619365
22	17	12.767462	15.309683	12.767462	15.309683
23	18	13.290716	16.654841	13.290716	16.654841
24	15	13.461644	15.827421	13.461644	15.827421
25	18	13.915480	16.913710	13.915480	16.913710
26	15	14.023932	15.956855	14.023932	15.956855
27	14	14.021539	14.978428	14.021539	14.978428
28	18	14.419385	16.489214	14.419385	16.489214
29	14	14.377446	15.244607	14.377446	15.244607
30	16	14.539702	15.622303	14.539702	15.622303

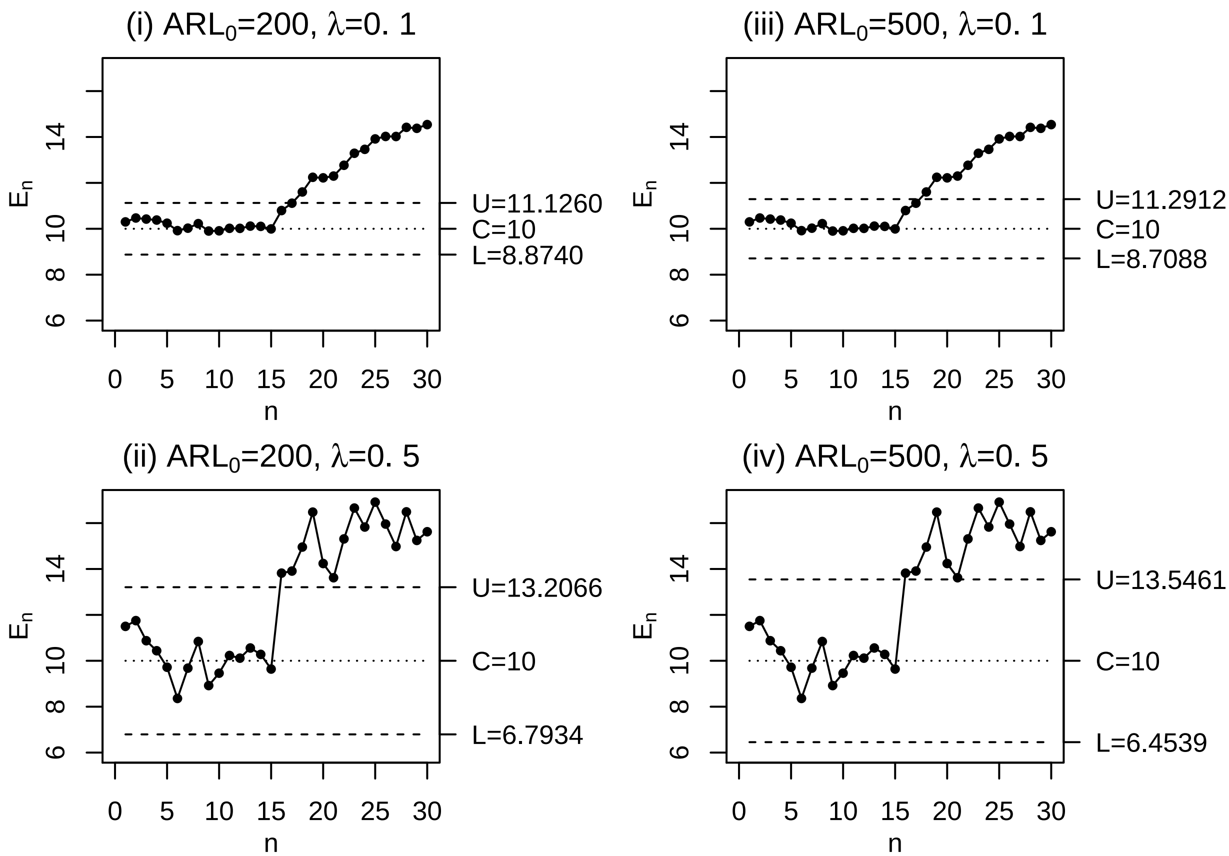
Second, find the values of ρ from Table 5.1 in the textbook with respective ARL_0 and λ values, and then compute the control limits with such ρ and λ values by the definition from (5.7)

$$\begin{aligned} USL \ &= \ \mu_0 \ + \ \rho \, \sqrt{\frac{\lambda}{2-\lambda}} \, \sigma \\ CL \ &= \ \mu_0 \\ LCL \ &= \ \mu_0 \ - \ \rho \, \sqrt{\frac{\lambda}{2-\lambda}} \, \sigma \end{aligned}$$

where $\mu_0 = 0$, and $\sigma = 2$. The results are shown as below :

	ρ	UCL	CL	LCL
(i)	2.454	11.1260	10	8.8740
(ii)	2.777	13.2066	10	6.7934
(iii)	2.814	11.2912	10	8.7088
(iv)	3.071	13.5461	10	6.4539

Third, construct the EWMA charts with respective charting statistics and control limits :



	ρ	UCL-LCL	Signal time point
(i)	2.454	2.2520	18
(ii)	2.777	6.4132	16
(iii)	2.814	2.5824	18
(iv)	3.071	7.0922	16

結論：

(1) 在 λ 值不變，ARL_0 變大的情況下 ($i \to iii$ or $ii \to iv$)，ρ 數值會變大，進而造成 control limit 的寬度變大，但在此題中並沒有因此造成 signal time point 較晚發生。

(2) 在 ARL_0 值不變，λ 變大的情況下 ($i \to ii$ or $iii \to iv$)，ρ 數值會變大，進而造成 control limit 的寬度變大，但因為 λ 值變大，所以 charting statistics 也跟著改變了，故在此題中發生了，即使 control limit 寬度變大，signal time point 卻提早發生了。

5.6

Compute the values of ARL_0 by the function *xewma.arl()* with respective λ and ρ values. The results are shown as below :

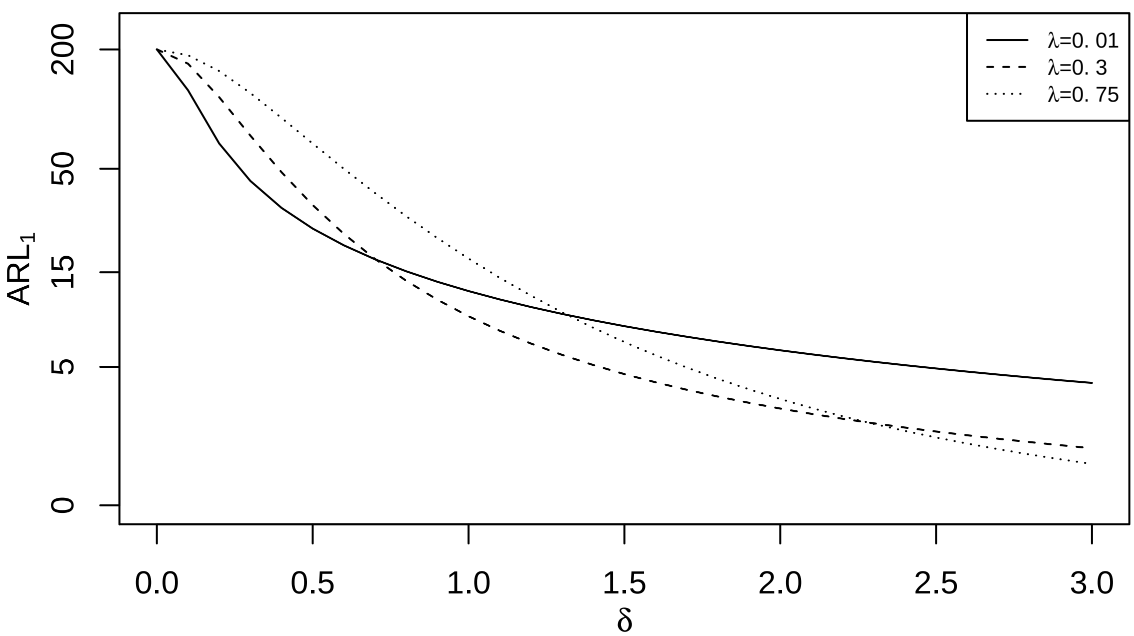
	λ	ρ	ARL_0
(i)	0.1	1	10.4216
(ii)	0.1	2	73.2765
(iii)	0.5	1	3.7861
(iv)	0.5	2	26.4519

結論：

(1) 在 λ 固定的情況下 (i$\rightarrow$ii or iii$\rightarrow$iv)，ρ 和 ARL_0 呈現正相關。

(2) 在 ρ 固定的情況下 (i$\rightarrow$iii or ii$\rightarrow$iv)，λ 和 ARL_0 呈現負相關。

5.8



結論：

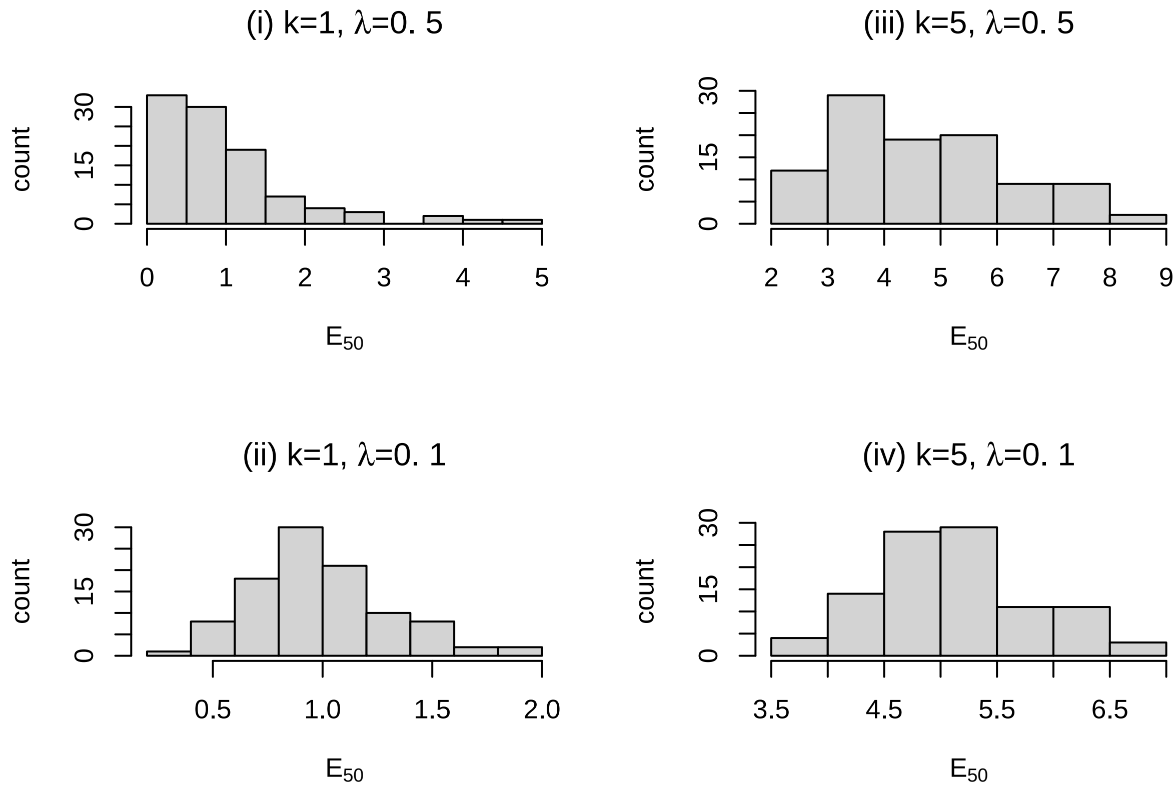
(1) 不論 λ 數值大小，δ 和 ARL_1 皆呈現負相關。

(2) 在 δ 數值較小時 $(0 \leq \delta \leq 0.6)$，較小的 λ 值 $(\lambda = 0.01)$ 表現得較好 (即 ARL_1 值較小)。

(3) 在 δ 數值介於中間時 $(0.6 \leq \delta \leq 2.3)$，介於中間的 λ 值 $(\lambda = 0.3)$ 表現得較好。

(4) 在 δ 數值較大時 $(2.3 \leq \delta \leq 3)$，較大的 λ 值 $(\lambda = 0.75)$ 表現得較好。

5.11



結論：
隨著 χ^2 的自由度 k 數值變大以及 λ 數值變小，E_{50} 的分佈會從類似 χ^2 的右偏分佈，逐漸變成類似常態分佈左右對稱且等尾的形式。