110024516 邱繼賢 HW6

1. (a) $\bar{X} \sim N_4(\mu, \frac{1}{60}\Sigma)$

(b) $(X_1 - \mu) \sim N_4(0, \Sigma) \Rightarrow \Sigma^{-\frac{1}{2}}(X_1 - \mu) \sim N_4(0, I)$

$\therefore (X_1 - \mu)' \Sigma^{-1} (X_1 - \mu) \sim \chi^2_4$

(c) $(\bar{X} - \mu) \sim N_4(0, \frac{1}{60}\Sigma) \Rightarrow \sqrt{60}\, \Sigma^{-\frac{1}{2}} (\bar{X} - \mu) \sim N_4(0, I)$

$\therefore 60 (\bar{X} - \mu)' \Sigma^{-1} (\bar{X} - \mu) \sim \chi^2_4$

(d) $59S \sim W_{59}(\cdot \mid \Sigma)$

(e) $59BSB' \sim W_{59}(\cdot \mid B\Sigma B')$

2. (a) $|\Sigma| = (4\times5\times2) - (5\times1\times1) = 35$

(b) $Var\left(\begin{bmatrix} X_3 \\ \hline X_1 \\ X_2 \end{bmatrix}\right) = \left[\begin{array}{c|cc} 2 & -1 & 0 \\ \hline -1 & 4 & 0 \\ 0 & 0 & 5 \end{array}\right]$

$Var\left(X_3 \middle| (X_1, X_2) = (3,4)\right) = \begin{bmatrix} 2 \end{bmatrix} - \begin{bmatrix} -1 & 0 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$= 2 - \frac{1}{4} = \frac{7}{4}$

3. Take $A_{p\times p} = \begin{array}{c} \\ \\ \\ \end{array}\begin{array}{cc} \begin{bmatrix} I & -\Sigma_{12}\Sigma_{22}^{-1} \\ 0 & I \end{bmatrix} & \begin{array}{l} \}\, q \\ \}\, p-q \end{array} \\ \begin{array}{cc} \quad q \quad & \quad p-q \quad \end{array} & \end{array}$

$\therefore A(X-\mu) = A\begin{bmatrix} X_1-\mu_1 \\ X_2-\mu_2 \end{bmatrix} = \begin{bmatrix} X_1-\mu_1-\Sigma_{12}\Sigma_{22}^{-1}(X_2-\mu_2) \\ X_2-\mu_2 \end{bmatrix}$

$\sim N_p(0, A\Sigma A')$ where $A\Sigma A' = \begin{bmatrix} \Sigma_{11}-\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} & 0 \\ 0 & \Sigma_{22} \end{bmatrix}$

$\because Cov(A(X_1-\mu_1), A(X_2-\mu_2)) = 0 \Rightarrow$ They're independent

$\therefore A(X_1-\mu_1) \mid A(X_2-\mu_2) = (X_1-\mu_1-\Sigma_{12}\Sigma_{22}^{-1}(X_2-\mu_2)) \mid (X_2-\mu_2)$

$\sim (X_1-\mu_1-\Sigma_{12}\Sigma_{22}^{-1}(X_2-\mu_2)) \sim N_p(0, \Sigma_{11}-\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21})$

Given $X_2 = x_2$:

$(X_1-\mu_1-\Sigma_{12}\Sigma_{22}^{-1}(x_2-\mu_2)) \sim N_p(0, \Sigma_{11}-\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21})$

$\Rightarrow X_1 \mid X_2 = x_2 \sim N_p(\mu_1+\Sigma_{12}\Sigma_{22}^{-1}(x_2-\mu_2), \Sigma_{11}-\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21})$ $\square$

Problem 4.

Compute the sample covariance matrix for first five variables

$$S \;=\; \frac{1}{130-1}\sum_{j=1}^{130}(x_j-\bar{x})(x_j-\bar{x})'$$

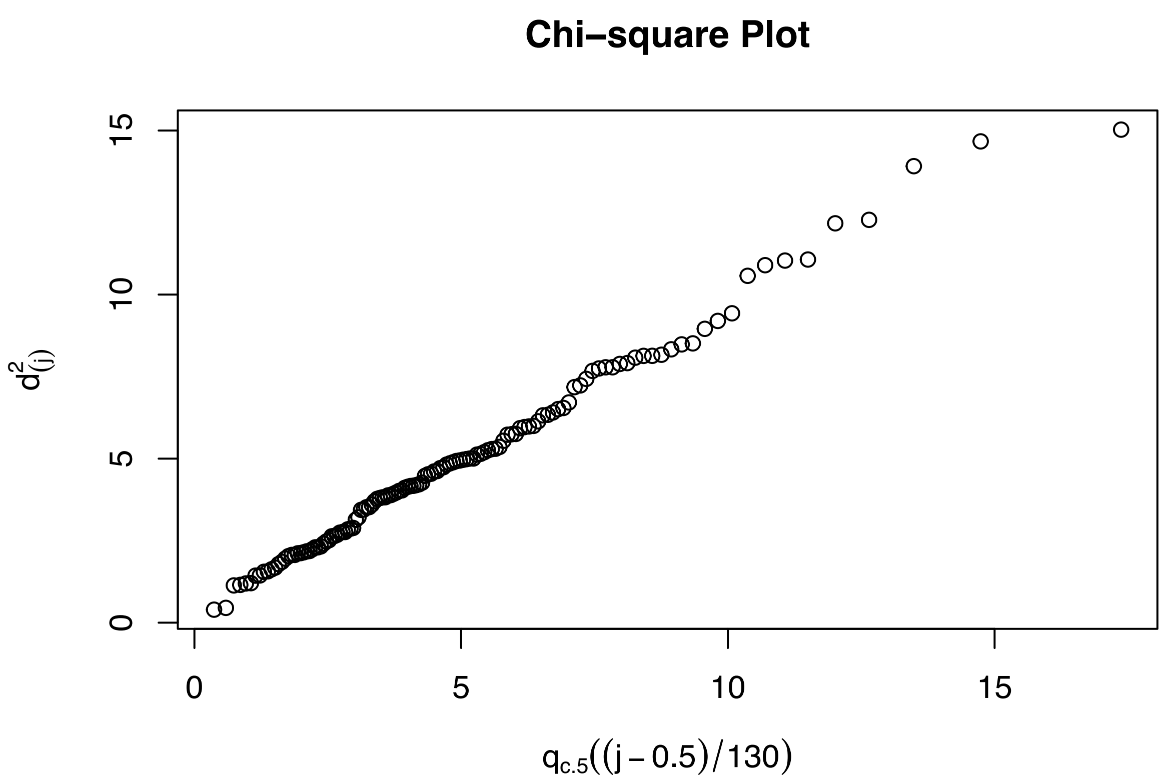
Then compute the squared generalized distances

$$d_j^2 \;=\; (x_j-\bar{x})'S^{-1}(x_j-\bar{x}) \quad , \quad j=1,2,...,130$$

and order them $d_{(1)}^2 \;\leq\; d_{(2)}^2 \;\leq\; ... \;\leq\; d_{(130)}^2$

Graph the Chi-square plot with pairs

$$\left(q_{c.5}\left(\frac{j-0.5}{130}\right)\;,\; d_{(j)}^2\right) \quad , \quad j=1,2,...,130$$



It looks like a straight line in the Chi-square plot, means that the first five columns (variables) are closed to multivariate normal distribution.