110024516 統研碩一 邱繼賢

3.5 $X_{i1}, X_{i2}, X_{i3}, X_{i4}, X_{i5}$, $1 \le i \le 10$

(i) $\bar{\bar{X}} = \frac{1}{10} \sum_{i=1}^{10} \bar{X}_i = 35.304$, $\bar{R} = \frac{1}{10} \sum_{i=1}^{10} R_i = 2.012$

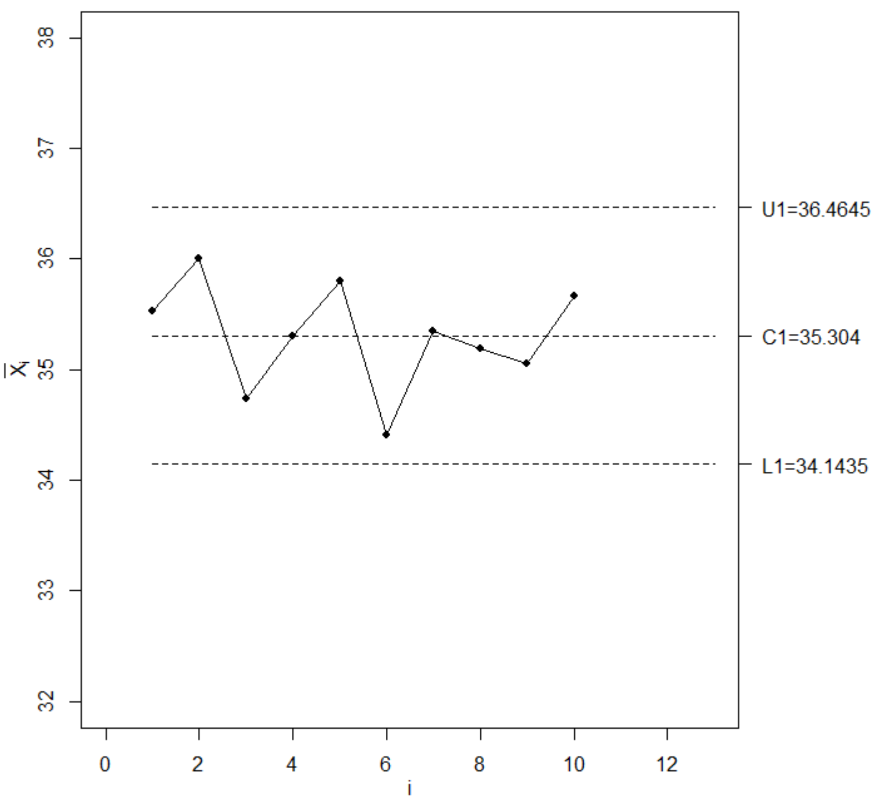
$Z_{1-\frac{\alpha}{2}} \doteq 3$, $d_1(5) = 2.326$, $d_2(5) = 0.864$

Control limit of the $\bar{X}$ chart:

$$U_1 = \bar{\bar{X}} + \frac{Z_{1-\frac{\alpha}{2}}}{d_1(5)\sqrt{5}} \bar{R} \doteq 36.4645$$

$$C_1 = \bar{\bar{X}} = 35.304$$

$$L_1 = \bar{\bar{X}} - \frac{Z_{1-\frac{\alpha}{2}}}{d_1(5)\sqrt{5}} \bar{R} \doteq 34.1435$$



$\Rightarrow$ for all $\bar{X}_i$, $i = 1, 2, \cdots, 10$

$$L_1 < \bar{X}_i < U_1$$

$\therefore$ The process seems to be IC

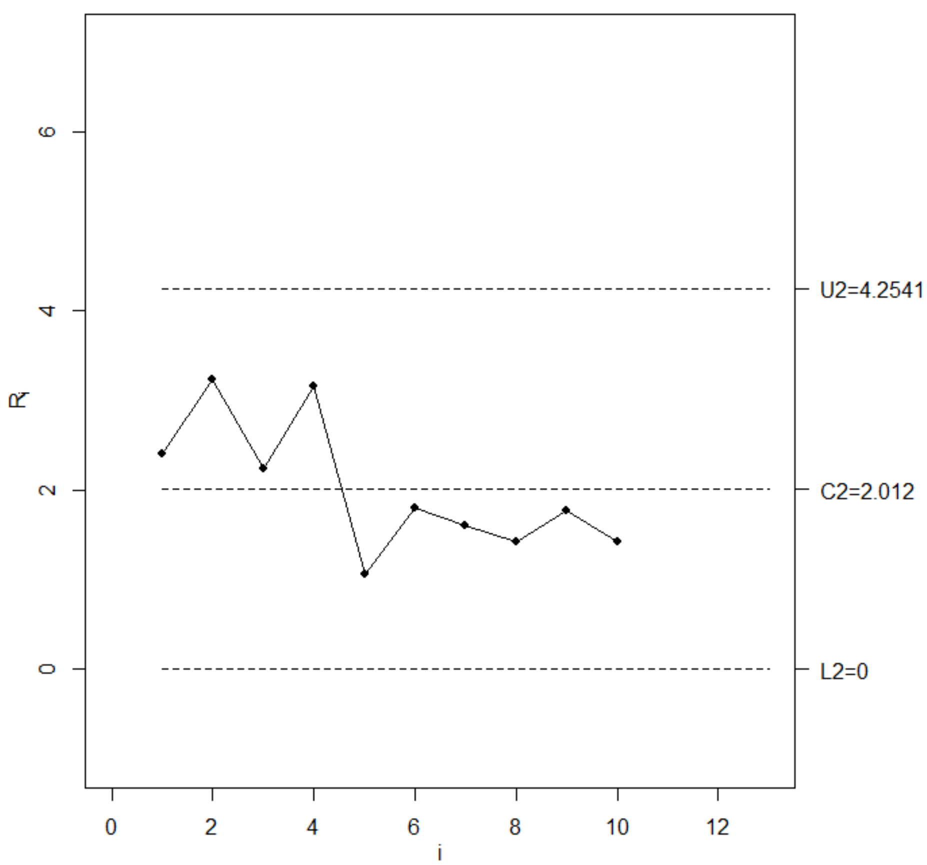
Control limit of the R chart :

$$U_2 = \bar{R} + \frac{Z_{1-\frac{\alpha}{2}} d_2(5)}{d_1(5)} \bar{R} \doteqdot 4.2541$$

$$C_2 = \bar{R} = 2.012$$

$$L_2 = \bar{R} - \frac{Z_{1-\frac{\alpha}{2}} d_2(5)}{d_1(5)} \bar{R} \doteqdot -0.2301$$

$\Rightarrow$ make $L_2 = 0$



$\Rightarrow$ for all R_i, $i = 1, 2, \cdots, 10$

$L_2 < R_i < U_2$

$\therefore$ The process seems to be IC $\square$

(ii) natural to estimate μ_0 by the grand sample mean

$$\hat{\mu}_0 = \bar{\bar{X}} = \frac{1}{10}\sum_{i=1}^{10} \bar{X}_i = 35.304$$

$\because$ the process is IC, then $d_1(5) = E\left(\frac{R_i}{\sigma}\right)$

$\therefore$ a natural estimate of σ

$$\hat{\sigma} = \frac{\bar{R}}{d_1(5)} \doteqdot 0.8650$$

$\square$

(iii)

Let $X_1^*, X_2^*, \ldots, X_5^*$ be the new sample

$$\overline{X}^* = \frac{1}{5}\sum_{i=1}^{5} X_i^*$$

a signal of mean shift $\Rightarrow$ the new sample is OC

$P(\overline{X}^* \text{ is OC} \mid \text{the process is IC})$

$= \alpha = 0.0027$ □

(iv)

$$ARL_0 = \frac{1}{\alpha} = \frac{1}{0.0027} \fallingdotseq 370.3704$$

Note that $\hat{\mu}_1 = \hat{\mu}_0 + 1 = 36.304$

$$\beta = P(L_1 < \bar{X} < U_1 \mid \mu = \mu_1)$$

$$= P\left(-3 - \frac{\sqrt{5}}{\hat{\sigma}} < \frac{\bar{X} - \hat{\mu}_1}{\hat{\sigma}/\sqrt{5}} < 3 - \frac{\sqrt{5}}{\hat{\sigma}}\right)$$

$$\fallingdotseq P(-5.5850 < Z < 0.4150)$$

$$= \Phi(0.4150) - 1 + \Phi(5.5850) \fallingdotseq 0.6609$$

$$ARL_1 = \frac{1}{1-\beta} \fallingdotseq 2.9490$$

□