Reliability Analysis
Homework 4

Problem 1: Exercise 7.15

Consider n observations with inspection data from an EXP(θ) distribution with $0 < \theta < \infty$. Show the following:

(a) The ML estimate of θ does not exist (i.e., there is no unique maximum of the likelihood function) when $d_1 = n$ or $d_{m+1} = n$.

(b) When $d_i = n$ $(i \neq 1, m+1)$, the ML estimator is

$$\widehat{\theta} = \frac{t_i - t_{i-1}}{\log(t_i) - \log(t_{i-1})}$$

Problem 2: Exercise 8.22

Consider the diesel generator fan data in file Fan.csv.

(b) Fit the exponential distribution to the data. Make a Weibull plot with the ML estimate of $F(t)$ on it.

(c) Report the ML estimate and a 95% confidence interval for the fraction failing before 1250 hours of service, based on the exponential distribution.

(d) Report the ML estimate and a 95% confidence interval for the 0.10 quantile of the life distribution, based on the exponential distribution.

(e) Repeat parts (b) and (c) using the Weibull distribution. How do the results compare?

(f) Report the ML estimate and 95% confidence interval for the Weibull shape parameter. What do these results say about the use of an exponential distribution to describe the diesel generator fan data? How would the conclusion affect fan replacement policy?

(g) Assuming a Weibull distribution, compute approximate 95% confidence intervals for $t_{0.10}$, the time at which 10% of the fan population will fail, based on $Z_{\widehat{t}_{0.10}} \dot{\sim} \text{NORM}(0,1)$ and $Z_{\log(\widehat{t}_{0.10})} \dot{\sim} \text{NORM}(0,1)$, respectively.

(h) Comment on the fact that the smallest observation falls outside of the confidence intervals for $t_{0.10}$.

Problem 3: Exercise 8.27

Use the bearing-cage fracture data in file Bearingcage.csv to do a Weibull analysis of the data.

(a) Obtain a Weibull plot that also shows the ML estimate and a set of 95% confidence intervals for the cdf ranging between 100 and 10,000 hours of operation.

(b) Obtain a contour plot of the relative likelihood of μ and σ. What does the shape of the contour plot suggest?

(c) Compute a Wald 95% confidence interval for $t_{0.10}$.

(d) Compute an LR-based 95% confidence interval for $t_{0.10}$.

(e) Which interval is more trustworthy? Why?

(f) Plot the estimated hazard function with pointwise CIs between 100 and 10000 hours.

(g) Obtain a 95% joint confidence region of μ and the 0.1 quantile $t_{0.1}$.