Experimental Design and Analysis Homework 3

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Problem 1. (2.1)

建構模型

$$y_{ij} \;=\; \eta \;+\; \tau_i \;+\; \epsilon_{ij} \quad , \quad \epsilon_{ij} \;\sim\; N\left(0, \sigma^2\right)$$

τ_i 為 treatment (operator) effect, $i \;=\; 1, 2, 3, 4 \quad , \quad j \;=\; 1, 2, 3, 4, 5$

```
pulp_data = c(59.8,60,60.8,60.8,59.8,
              59.8,60.2,60.4,59.9,60,
              60.7,60.7,60.5,60.9,60.3,
              61,60.8,60.6,60.5,60.5)
operator = gl(4,5,labels = c("A","B","C","D"))
pulp_table = aov(pulp_data ~ operator)
summary(pulp_table)
```

```
##             Df Sum Sq Mean Sq F value Pr(>F)
## operator     3   1.34  0.4467   4.204 0.0226 *
## Residuals   16   1.70  0.1062
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$\Rightarrow \;\; N = 20 \quad , \quad k = 4 \quad , \quad n_1 = n_2 = n_3 = n_4 = 5 \quad , \quad \hat{\sigma}^2 = 0.1062$

(1) Bonferroni Method :

the lower and upper bounds are

$$\bar{y}_{i\cdot} - \bar{y}_{j\cdot} \;\pm\; t_{N-k, \frac{\alpha}{2k'}} \;\hat{\sigma}\sqrt{\frac{1}{n_i} + \frac{1}{n_j}} \quad , \quad k' \;=\; C^4_2 \;=\; 6$$

```r
library(asbio)
bonfCI(pulp_data, operator)
```

```
##
## 95% Bonferroni confidence intervals
##
##          Diff    Lower   Upper Decision Adj. p-value
## muA-muB  0.18 -0.44018 0.80018   FTR H0           1
## muA-muC -0.38 -1.00018 0.24018   FTR H0    0.503359
## muB-muC -0.56 -1.18018 0.06018   FTR H0    0.091504
## muA-muD -0.44 -1.06018 0.18018   FTR H0    0.291823
## muB-muD -0.62 -1.24018 0.00018   FTR H0    0.050093
## muC-muD -0.06 -0.68018 0.56018   FTR H0           1
```

the interval length is

$$2 \times t_{N-k, \frac{\alpha}{2k'}} \; \hat{\sigma} \sqrt{\frac{1}{n_i} + \frac{1}{n_j}}$$

```r
2*qt(1-0.05/(2*6), 20-4)*sqrt(0.1062*(1/5+1/5))
```

```
## [1] 1.240076
```

(2) Tukey Method :

the lower and upper bounds

$$\bar{y}_{i\cdot} - \bar{y}_{j\cdot} \; \pm \; \frac{1}{\sqrt{2}} \, q_{k,N-k,\alpha} \; \hat{\sigma} \sqrt{\frac{1}{n_i} + \frac{1}{n_j}}$$

```r
tukeyCI(pulp_data, operator)
```

```
##
## 95% Tukey-Kramer confidence intervals
##
##          Diff    Lower    Upper  Decision Adj. p-value
## muA-muB  0.18 -0.40981  0.76981    FTR H0     0.818543
## muA-muC -0.38 -0.96981  0.20981    FTR H0     0.290304
## muB-muC -0.56 -1.14981  0.02981    FTR H0     0.065794
## muA-muD -0.44 -1.02981  0.14981    FTR H0     0.184479
## muB-muD -0.62 -1.20981 -0.03019 Reject H0     0.037669
## muC-muD -0.06 -0.64981  0.52981    FTR H0     0.991078
```

the interval length is

$$\sqrt{2} \times q_{k,N-k,\alpha} \ \hat{\sigma} \sqrt{\frac{1}{n_i} + \frac{1}{n_j}}$$

```
sqrt(2)*qtukey(0.95,4,20-4)*sqrt(0.1062*(1/5+1/5))
```

```
## [1] 1.179351
```

可以發現 Tukey Method 的信賴區間長度較短，代表此種方式更為 powerful

Problem 2. (2.2)

(a)

給定顯著水準 $\alpha = 0.01$，建構每一對 treatments 的 99% confidence interval

```
bonfCI(pulp_data, operator, conf.level = 0.99)
```

```
##
## 99% Bonferroni confidence intervals
##
##          Diff    Lower   Upper Decision Adj. p-value
## muA-muB  0.18 -0.59772 0.95772   FTR H0          1
## muA-muC -0.38 -1.15772 0.39772   FTR H0   0.503359
## muB-muC -0.56 -1.33772 0.21772   FTR H0   0.091504
## muA-muD -0.44 -1.21772 0.33772   FTR H0   0.291823
## muB-muD -0.62 -1.39772 0.15772   FTR H0   0.050093
## muC-muD -0.06 -0.83772 0.71772   FTR H0          1
```

```
tukeyCI(pulp_data, operator, conf.level = 0.99)
```

```
##
## 99% Tukey-Kramer confidence intervals
##
##          Diff    Lower   Upper Decision Adj. p-value
## muA-muB  0.18 -0.57684 0.93684   FTR H0   0.818543
## muA-muC -0.38 -1.13684 0.37684   FTR H0   0.290304
## muB-muC -0.56 -1.31684 0.19684   FTR H0   0.065794
```

```
## muA-muD -0.44 -1.19684 0.31684   FTR H0     0.184479
## muB-muD -0.62 -1.37684 0.13684   FTR H0     0.037669
## muC-muD -0.06 -0.81684 0.69684   FTR H0     0.991078
```

可以發現每一組 multiple comparison 在兩種方法下的 99% confidence interval 都包含 0，代表我們無法宣稱有任何一對 treatments (operators) 之間有顯著差距。

(b)

在進行 one-way ANOVA 和 multiple comparison test 的時候選擇的顯著水準應該要一致，才不會出現結果矛盾的情況，以下對兩種不同的 α 值所做出的結果進行比較

(1) $\alpha \ = \ 0.01$:

2.1 題中的 ANOVA 表格所顯示的 p-value = $0.0226 \ > \ \alpha$，結果為不拒絕 H_0，代表四位 operators 之間並沒有顯著差異，與 **2.2 (a)** 做 multiple comparison test 的結果一致。

(2) $\alpha \ = \ 0.05$:

ANOVA 表格中的 p-value = $0.0226 \ < \ \alpha$，結果為拒絕 H_0，代表四位 operators 中至少有一對之間有顯著差異，再觀察 **2.1** 題的 95% Tukey Method confidence interval 中的 operator B 和 D，可以發現其對應的 CI 並沒有包含 0，故兩者之間存在顯著差異，與 one-way ANOVA 所做出的結果一致。

(c)

p-value = $0.0226 \ > \ \alpha \ = \ 0.01 \ \Rightarrow$ 我們並沒有足夠的證據來拒絕 H_0，已經可以判定四位 operators 之間並沒有顯著差異了，所以不需要再進行 multiple comparison test 即可得到與 **(a)** 相同的結論。

Problem 3. (2.14)

(a)

建構模型

$$y_{ij} \ = \ \eta \ + \ \tau_i \ + \ \epsilon_{ij} \ \ , \ \ \epsilon_{ij} \ \sim \ N\left(0, \sigma^2\right)$$

where τ_i is the treatment (area) effect, $i = A, B, C, D \ \ , \ \ j = 1, 2, ..., 16$

(1) Over all F test

$$H_0 \ : \ \tau_A = \tau_B = \tau_C = \tau_D$$

$$H_1 \ : \ \text{at least one pair of } \tau_i's \text{ are not the same}$$

```r
mv_data = read.table("mv.txt", header = T)
colnames(mv_data) = c("A","B","C","D")
mv_data2 = stack(mv_data)
mv_data2 = cbind(mv_data2, rep(c(0.016,0.030,0.044,0.058), each = 16))
colnames(mv_data2) = c("value", "treatment", "area")
mv_table = aov(value ~ treatment, data = mv_data2)
summary(mv_table)
```

```
##             Df Sum Sq Mean Sq F value   Pr(>F)
## treatment    3  13515    4505   7.067 0.000381 ***
## Residuals   60  38250     638
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$\Rightarrow$ p-value $= 0.000381 < \alpha = 0.05$，故拒絕 H_0，所以至少有一組 treatmeants 的效應之間具有顯著差異。

(2) Multiple comparison：Tukey method

建構 95% 的 confidence interval (方法同 **2.1**)

```r
tukeyCI(mv_data2$value, mv_data2$treatment)
```

```
##
## 95% Tukey-Kramer confidence intervals
##
##             Diff     Lower    Upper  Decision Adj. p-value
## muA-muB     7.95 -15.6393  31.5393    FTR H0     0.809747
## muA-muC 30.56875   6.97945 54.15805 Reject H0     0.005991
## muB-muC 22.61875  -0.97055 46.20805    FTR H0     0.064808
## muA-muD 34.18125  10.59195 57.77055 Reject H0      0.00172
## muB-muD 26.23125   2.64195 49.82055 Reject H0     0.023551
## muC-muD   3.6125  -19.9768  27.2018    FTR H0     0.977394
```

由上表可知，AC、AD、BD 三組的 confidence intervals 皆不包含 0，拒絕 H_0，故此三組中兩兩之間的效應具有顯著差異。

(b)

Define the first and second degree polynomials

$$
\begin{aligned}
P_1(x) &= 2\left(\frac{x-m}{\Delta}\right) \\
P_2(x) &= \left(\frac{x-m}{\Delta}\right)^2 - \left(\frac{k^2-1}{12}\right)
\end{aligned}
$$

where $m = 0.037$, $\Delta = 0.014$, $k = 4$, then

$$
\begin{aligned}
(P_1(0.016)\ ,\ P_1(0.030)\ ,\ P_1(0.044)\ ,\ P_1(0.058)) &= (-3,-1,1,3) \\
(P_2(0.016)\ ,\ P_2(0.030)\ ,\ P_2(0.044)\ ,\ P_2(0.058)) &= (1,-1,-1,1)
\end{aligned}
$$

建構模型

$$
y = \beta_0 + \beta_1 \frac{P_1(x)}{\sqrt{20}} + \beta_2 \frac{P_2(x)}{2} + \epsilon
$$

```
P1 = function(x) {
    2*(x-0.037)/0.014
}
P2 = function(x) {
    ((x-0.037)/0.014)^2-5/4
}
fit = lm(value ~ I(P1(area)/sqrt(20)) + I(P2(area)/2), data = mv_data2)
summary(fit)
```

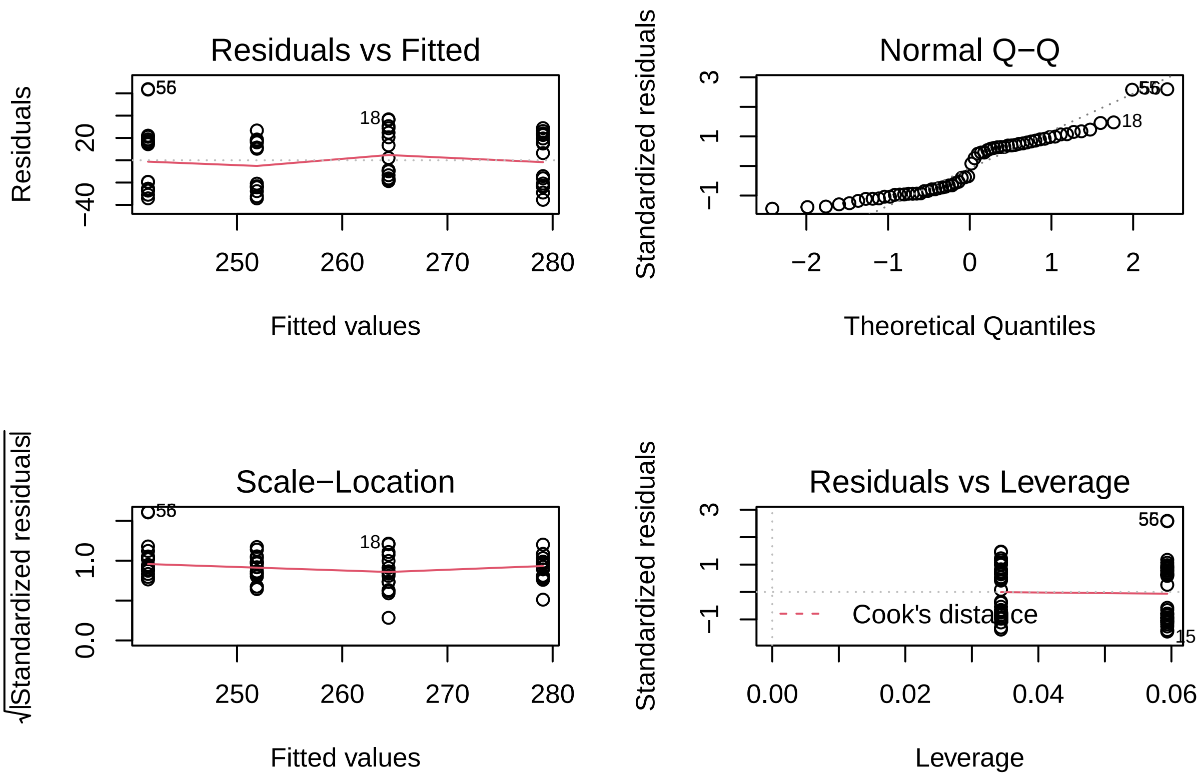
```
##
## Call:
## lm(formula = value ~ I(P1(area)/sqrt(20)) + I(P2(area)/2), data = mv_data2)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -35.484 -23.133  -3.349  18.953  63.765
##
## Coefficients:
##                      Estimate Std. Error t value Pr(>|t|)
## (Intercept)           259.225      3.167  81.851  < 2e-16 ***
## I(P1(area)/sqrt(20))  -27.987      6.334  -4.419 4.15e-05 ***
## I(P2(area)/2)           2.169      6.334   0.342    0.733
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 25.34 on 61 degrees of freedom
## Multiple R-squared:  0.2436, Adjusted R-squared:  0.2188
## F-statistic:  9.82 on 2 and 61 DF,  p-value: 0.0002008
```

線性效應的 p-value 結果顯著對模型有貢獻，而且其係數為負數，代表隨著排氣孔的面積增大，炮擊的速度隨之下降，與題目一開始的假設相符合；而二次效應的 p-value 結果呈現不顯著。

對模型檢查 diagnostic

```r
par(mfrow = c(2,2))
plot(fit)
```



可以看出 residual 基本上沒有出現 non-constant variance 和 mean curvature，但是從 QQ plot 可以看出，residual 明顯不服從常態分配。

Problem 4. (2.17)

(a)

(1) F test for devices

建構模型 (random effect model)

$$y_{ij} = \eta + \tau_i + \epsilon_{ij} \ , \ \tau_i \sim N\left(0, \sigma_\tau^2\right) \ , \ \epsilon_{ij} \sim N\left(0, \sigma^2\right)$$

where τ_i is the random effect of devices, $i = 1, 2, 3$, $j = 1, 2, ..., 15$, and σ^2 and σ_τ^2 are two variance components of the model.

Test

$$H_0 : \sigma_\tau^2 = 0$$
$$H_1 : \sigma_\tau^2 > 0$$

ANOVA table :

```
data = read.table("BloodPressure.txt", skip = 1, header = T)
dev_data = data[,2:4]
dev_data2 = stack(dev_data)
colnames(dev_data2) = c("value", "device")
dev_table = aov(value ~ device, data = dev_data2)
summary(dev_table)
```

```
##             Df Sum Sq Mean Sq F value Pr(>F)
## device       2      0    0.01       0      1
## Residuals   42   3699   88.08
```

estimates of variance components :

$$\hat{\sigma}^2 = MSE = 88.08$$
$$\hat{\sigma}_\tau^2 = \frac{MSTr - MSE}{n} = \frac{0.01 - 88.08}{15} < 0 \Rightarrow \hat{\sigma}_\tau^2 = 0$$

(2) F test for doctors

建構模型 (random effect model)

$$y_{ij} = \eta + \tau_i + \epsilon_{ij} \ , \ \tau_i \sim N\left(0, \sigma_\tau^2\right) \ , \ \epsilon_{ij} \sim N\left(0, \sigma^2\right)$$

where τ_i is the random effect of doctors, $i = 1, 2, 3$, $j = 1, 2, ..., 15$, and σ^2 and σ_τ^2 are two variance components of the model.

Test

$$H_0 : \sigma_\tau^2 = 0$$
$$H_1 : \sigma_\tau^2 > 0$$

ANOVA table :

```
doc_data = data[,6:8]
doc_data2 = stack(doc_data)
colnames(doc_data2) = c("value", "doctor")
doc_table = aov(value ~ doctor, data = doc_data2)
summary(doc_table)
```

```
##             Df Sum Sq Mean Sq F value Pr(>F)
## doctor       2  496.3  248.16   139.1 <2e-16 ***
## Residuals   42   74.9    1.78
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

estimates of variance components :

$$\begin{aligned}\hat{\sigma}^2 &= MSE = 1.78 \\ \hat{\sigma}^2_\tau &= \frac{MSTr - MSE}{n} = \frac{248.16 - 1.78}{15} = 16.42533\end{aligned}$$

(b)

(1) p-value $= 1 > 0.05 = \alpha$，不拒絕 H_0，故我們沒有充分的證據顯示所有設備之間有著顯著的變異

(2) p-value $< 2e-16 < 0.05 = \alpha$，拒絕 H_0，故我們有充分的證據顯示所有醫生之間有著顯著的變異

(c)

The 95% confidence intervals limits

$$\hat{\eta} \pm t_{k-1,\frac{\alpha}{2}} \sqrt{\frac{MSTr}{nk}}$$

where $\hat{\eta}_1 = 128.0664$, $\hat{\eta}_2 = 130.0213$, $MSTr_1 = 0.01$, $MSTr_2 = 248.16$, $k = 3$, $n = 15$

```
library(knitr)
eta_hat1 = mean(dev_data2$value)
MSTr1 = 0.01
lower1 = eta_hat1-qt(0.975,3-1)*sqrt(MSTr1/45)
upper1 = eta_hat1+qt(0.975,3-1)*sqrt(MSTr1/45)
eta_hat2 = mean(doc_data2$value)
```

```r
MSTr2 = 248.16
lower2 = eta_hat2-qt(0.975,3-1)*sqrt(MSTr2/45)
upper2 = eta_hat2+qt(0.975,3-1)*sqrt(MSTr2/45)
kable(data.frame(a = c("devices","doctors"), lower = c(lower1,lower2), upper = c(upper1,upper2)),
      col.names = c("","Lower Bound", "Upper Bound"), digits = 4)
```

	Lower Bound	Upper Bound
devices	128.0023	128.1306
doctors	119.9173	140.1254

Problem 5. (2.18)

(a)

In randon effect model

$$y_{ij} = \eta + \tau_i + \epsilon_{ij} \ , \ \tau_i \sim N\left(0, \sigma_\tau^2\right) \ , \ \epsilon_{ij} \sim N\left(0, \sigma^2\right)$$

模型描述的是從整體的 treatment population 中隨機抽出 treatment，此時 τ_i 不再是固定的參數，而是一個稱為 random effect 的隨機變數，所以 $E\left(y_{ij}\right) = E\left(\eta + \tau_i + \epsilon_{ij}\right) = E(\eta) = \eta$，$\eta$ 就是 population mean，同時也代表著整個 treatmeat population 的平均。

(b)

探討不同業務員，對業績的影響：

(1) random effect model

此情況下主要是探討的是整個業務員母體 (公司全體員工) 對業績的影響的變異大小是否顯著，不同的數個業務員只是從整個母體中所抽出的代表，特定業務員之間所帶來的差異並不是我們所關心的，這幾個抽出的業務員是否能代表整個母體，為我們帶來關於母體的資訊才是我們關注的重點，而此時的參數 η 代表的是業務員母體的平均，自然也會是我們所關心的數值。

(2) fixed effect model

此情況下主要探討的是特定幾個業務員之間對業績影響的是否有顯著差異，所選出特定的幾個業務員對母體有沒有代表性，並不是我們關注的重點，所以我們只需要知道 treatment effect τ_i 這些參數之間的關係即可，不用在意參數 η 的數值，因為該數值的大小並不會影響到不同業務員之間的差距，只是一個讓全體數值一起平移的參數。