品質管制 Homework 7

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4.13

(i) $\sigma_0 \ = \ 1, \ \sigma_1 \ = \ 2, \ ARL_0 \ = \ 370$
To detect the upward variance shift, the CUSUM chart is:

$$C_n^+ = max(0 \ , \ C_{n-1}^+ \ + \ (\frac{X_n - \mu_0}{\sigma_0})^2 \ - \ k^+) \ , \text{ where } C_0^+ \ = \ 0$$

and $k^+ \ = \ \dfrac{2 \ log(\sigma_0/\sigma_1)}{(\sigma_0/\sigma_1)^2 - 1} \ = \ 1.8484$

This chart gives a signal of upward variance shift if $C_n^+ > h_U$

Where $h_U \ = \ 8.97$ is computed by $ANYGETH.EXE$

$\therefore$ The optimal values of $(k^+, h_U) \ = \ (1.848, 8.97)$

(ii) $\sigma_0 \ = \ 1, \ \sigma_1 \ = \ 0.5, \ ARL_0 \ = \ 370$
To detect the downward variance shift, the CUSUM chart is:

$$C_n^- = min(0 \ , \ C_{n-1}^- \ + \ (\frac{X_n - \mu_0}{\sigma_0})^2 \ - \ k^-) \ , \text{ where } C_0^- \ = \ 0$$

and $k^- \ = \ \dfrac{2 \ log(\sigma_0/\sigma_1)}{(\sigma_0/\sigma_1)^2 - 1} \ = \ 0.4621$

This chart gives a signal of downward variance shift if $C_n^- < h_L$

Where $h_L \ = \ -2.843$ is computed by $ANYGETH.EXE$

$\therefore$ The optimal values of $(k^-, h_L) \ = \ (0.4621, -2.843)$

(iii) $\sigma_0 \ = \ 0.5, \ \sigma_1 \ = \ 1, \ ARL_0 \ = \ 370$
To detect the upward variance shift, the CUSUM chart is:

$$C_n^+ = max(0 \ , \ C_{n-1}^+ \ + \ (\frac{X_n - \mu_0}{\sigma_0})^2 \ - \ k^+) \ , \text{ where } C_0^+ \ = \ 0$$

and $k^+ \ = \ \dfrac{2 \ log(\sigma_0/\sigma_1)}{(\sigma_0/\sigma_1)^2 - 1} \ = \ 1.8484$

This chart gives a signal of upward variance shift if $C_n^+ > h_U$

Where $h_U \ = \ 8.906135$ is computed by the bisection simulation

and the last $ARL_0 \ = \ 369.67$

$\therefore$ The optimal values of $(k^+, h_U) \ = \ (1.848, 8.906135)$

4.14
計算 $k^- \ = \ \frac{2 \ log(\sigma_1/\sigma_0) \ \sigma_0^2 \ \sigma_1^2}{\sigma_1^2 \ - \ \sigma_0^2} \ = \ \frac{2 \ log(0.5/1) \ 0.25}{0.25 \ - \ 1} \ = \ 0.4621$，再使用 $ANYGETH.EXE$ 軟體套件計算 $h_L \ = \ -0.681$

```
k_minus = (1*0.25*log(0.25/1))/(0.25-1)
h_l = -0.681
```

匯入資料並計算

$$C_n^- = min(0, C_{n-1}^- + S_n^2 - k^-),\ where\ C_0^- = 0$$

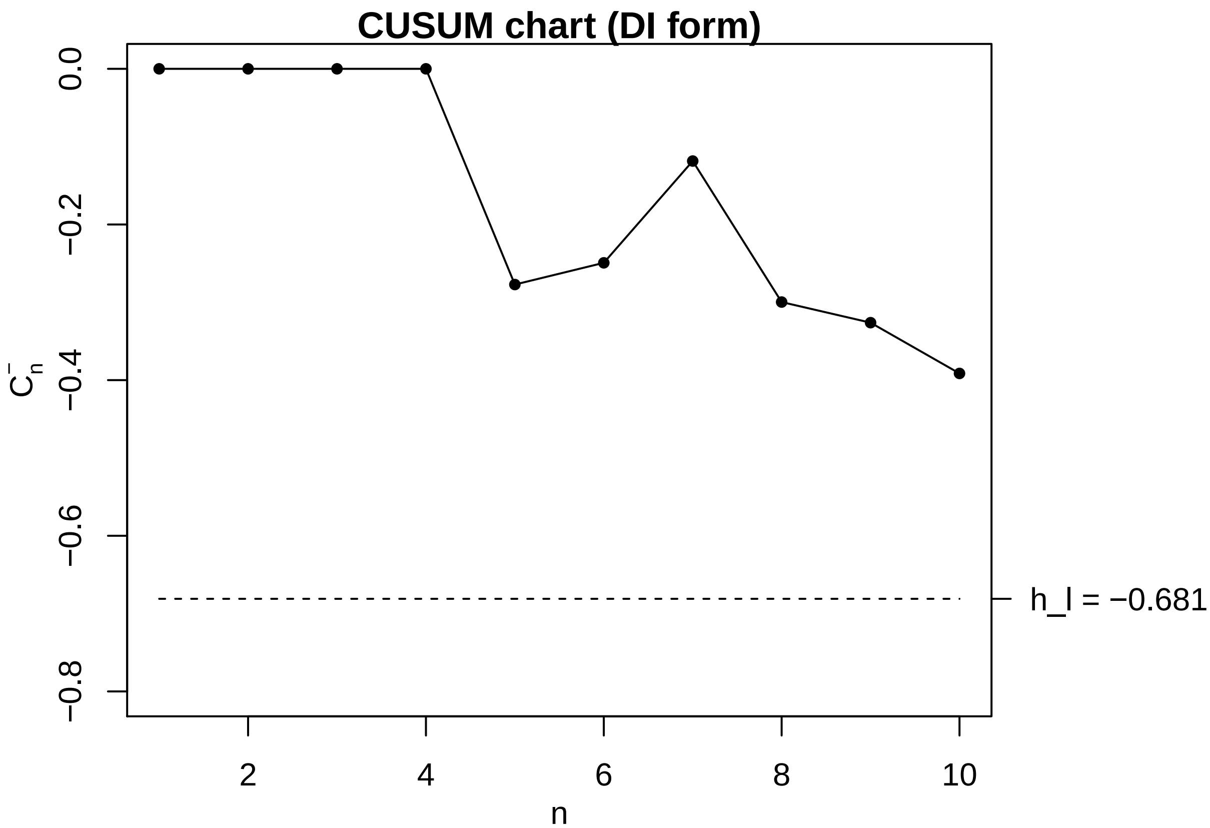
其結果呈現如下

```
library(knitr)
data = read.table("ex35.dat.txt", header = T)
data$C_n[1] = min(0, 0+data$s[1]^2-k_minus)
for (i in 2:10) {
    data$C_n[i] = min(0, data$C_n[i-1]+data$s[i]^2-k_minus)
}

kable(data.frame(C_n = round(data$C_n, 4)), row.names = 1:10)
```

	C_n
1	0.0000
2	0.0000
3	0.0000
4	0.0000
5	-0.2772
6	-0.2493
7	-0.1185
8	-0.2997
9	-0.3262
10	-0.3914

繪製 CUSUM chart

```
ii = seq(1,10)
par(mfrow = c(1,1), mar=c(4,4,1,6))
plot(ii,data$C_n,type="o",lty=1,pch=16,xlab="n",
     ylab=expression(C[n]^{"-"}),mgp=c(2,1,0),xlim=c(1,10),
     ylim=c(-0.8,0),cex=0.8)
lines(ii,rep(h_l,10),lty=2,cex=0.8)
axis(4, at = h_l, labels = "h_l = -0.681", cex=0.8,las=1)
title("CUSUM chart (DI form)")
```



In the 10 samples from a process producing bearings, the CUSUM chart does not give any signals. Therefore, we can not detect downward variance shifts from the observed data.

4.15

$$
\begin{aligned}
&\because \frac{1}{ARL_{0,J}} \approx \frac{1}{ARL_{0,M}} + \frac{1}{ARL_{0,V}}, \ and \ ARL_{0,J} = 200 \\
&\Rightarrow \ Make \ that \ ARL_{0,M} = ARL_{0,V} = 400 \\
&And \ (k,h) = (0.5, 0.128), \ (k^+, h_U) = (0.4621, 0.633) \\
&(Note \ that \ k = \frac{\mu_1 - \mu_2}{2} = 0.5 \ , \\
&and \ h = 0.128 \ by \ the \ software \ package \ ANYGETH.EXE \\
&k^+ = \frac{2\sigma_0\sigma_1 log(\sigma_1/\sigma_0)}{\sigma_1^2 - \sigma_0^2} = 0.4621 \ , \\
&and \ h_U = 0.633 \ by \ the \ software \ package \ ANYGETH.EXE)
\end{aligned}
$$

```
k_mean = 0.5
h_mean = 0.128

k_vplus = (2*0.25*1*log(1/0.5))/(1-0.25)
h_vplus = 0.633
```

匯入資料並計算

$$
\begin{cases}
CUSUM-M : C_n^+ = max(0 \ , \ C_{n-1}^+ + (\overline{X}_n - \mu_0) - k) \ , \ where \ C_0^+ = 0 \\
CUSUM-V : C_n^+ = max(0 \ , \ C_{n-1}^+ + S_n^2 - k^+) \ , \ where \ C_0^+ = 0
\end{cases}
$$

其結果呈現如下

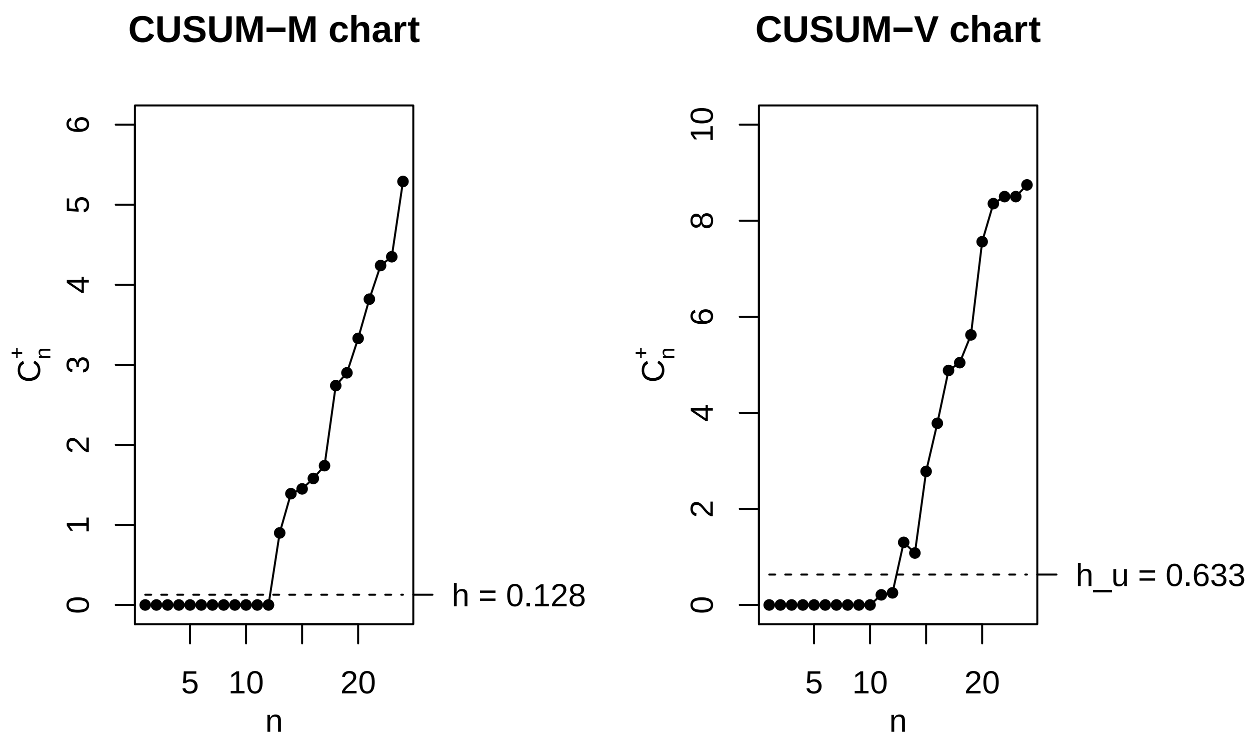
```r
data415 = read.table("ex415.dat.txt", header = T)
data415$C_mean[1] = max(0, 0+(data415$xbar[1]-1)-k_mean)
for (i in 2:24) {
    data415$C_mean[i] = max(0, data415$C_mean[i-1]+data415$xbar[i]-1-k_mean)
}

data415$C_var[1] = max(0,0+data415$xstd[1]^2-k_vplus)
for (i in 2:24) {
    data415$C_var[i] = max(0, data415$C_var[i-1]+data415$xstd[i]^2-k_vplus)
}
C = data.frame(C_n_M = data415$C_mean,
               C_n_V = data415$C_var)
kable(round(C, 4), row.names = 1:24)
```

	C_n_M	C_n_V
1	0.00	0.0000
2	0.00	0.0000
3	0.00	0.0000
4	0.00	0.0000
5	0.00	0.0000
6	0.00	0.0000
7	0.00	0.0000
8	0.00	0.0000
9	0.00	0.0000
10	0.00	0.0000
11	0.00	0.2103
12	0.00	0.2523
13	0.90	1.3031
14	1.39	1.0811
15	1.45	2.7799
16	1.58	3.7819
17	1.74	4.8823
18	2.74	5.0443
19	2.90	5.6226
20	3.33	7.5630
21	3.82	8.3553
22	4.24	8.5016
23	4.35	8.5019
24	5.29	8.7454

繪製 CUSUM-M 和 CUSUM-V chart

```r
par(mfcol = c(1,2), mar=c(5,3.5,4,5.5))
ii = seq(1,24)
plot(ii,data415$C_mean,type="o",lty=1,pch=16,xlab="n",
     ylab=expression(C[n]^{"+"}),mgp=c(2,1,0),xlim=c(1,24),
     ylim=c(0,6),cex=0.8)
lines(ii,rep(h_mean,24),lty=2,cex=0.8)
axis(4, at=h_mean,labels="h = 0.128",cex=0.8,las=1)
title("CUSUM-M chart")
```

```
plot(ii, data415$C_var, type="o",lty=1,pch=16,xlab="n",
     ylab=expression(C[n]^{"+"}),mgp=c(2,1,0),xlim=c(1,24),
     ylim=c(0,10),cex=0.8)
lines(ii, rep(h_vplus, 24), lty=2,cex=0.8)
axis(4, at = h_vplus, labels = "h_u = 0.633", cex=0.8,las=1)
title("CUSUM-V chart")
```



It can be seen that the CUSUM-M chart gives a signal at the 13th time point, and the CUSUM-V chart also gives a signal at the 13th time point.