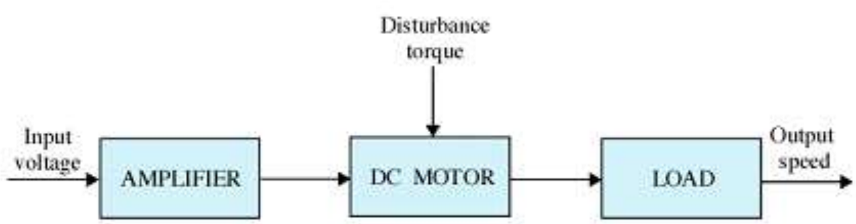
Chapter 3.   Block diagrams and Signal Flow Graphs

3-1 Block diagram

- to describe the composition and interconnection of a system
- to describe the cause-and –effect relationship through out the system

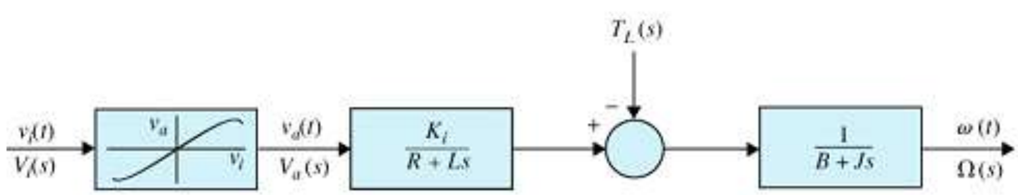
Examples.

(a) a dc-motor control system.



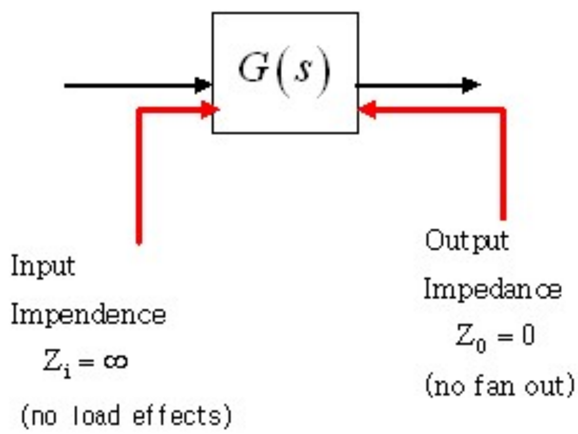
(a)

(b) transfer functions and amplifier characteristics.



(b)

- Assumptions



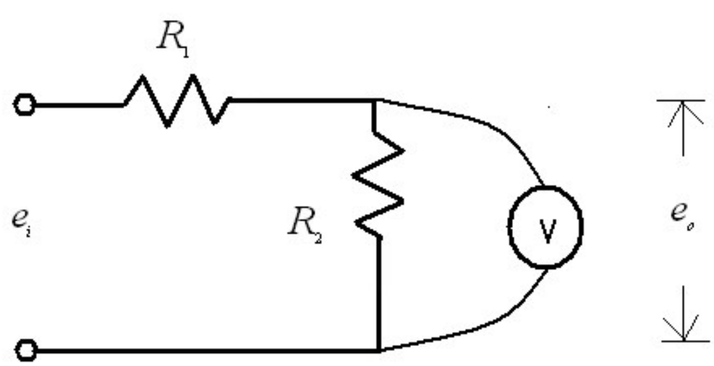
- large input impedance (pre-amp + power-amp)

  -> battery can drive very heavy load

- small output impedance -> assume that system can

  drive many loads

* input impedance of voltmeter

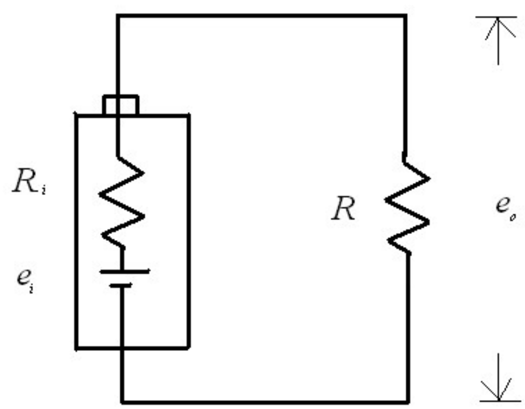


- Voltmeter internal resistance should be very large.
- It takes a small amount of energy from the system

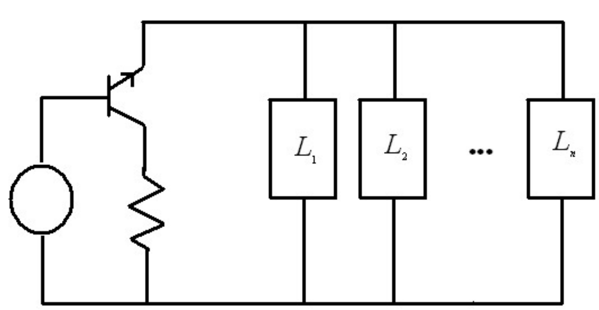
and excite the moving coil and so that the needle of the meter rotates proportional to the measuring voltage.

- output impedance : Battery :

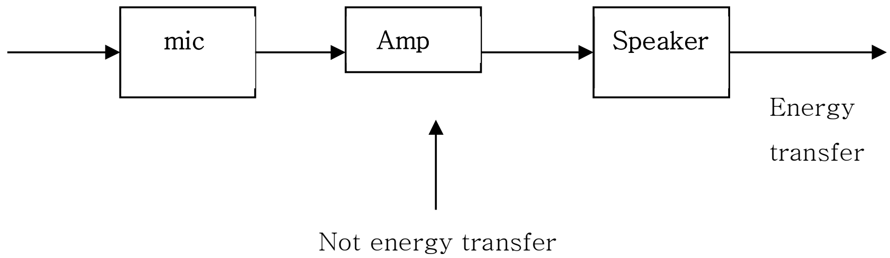
$R_i << R \quad \rightarrow \quad e_o = e_i$



- logic circuit : too many loads cause fan-out

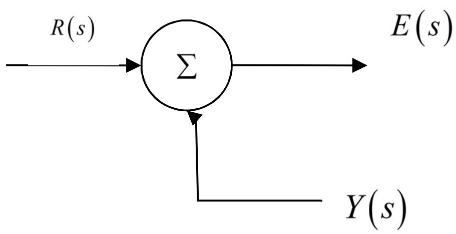


- signal flows only in direction of arrow



* Block diagram elements

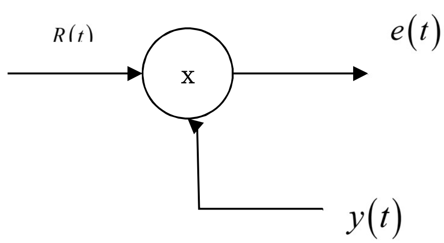
– comparator



$$E(s) = R(s) - Y(s)$$

Addition & subtraction

– multiplier



$$e(t) = r(t) \cdot y(t)$$

Modulation in communication system

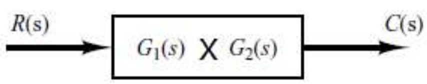
$$\left(E(s) = R(s) * y(s)\right)$$

- Block diagram representation of LTI system is not unique

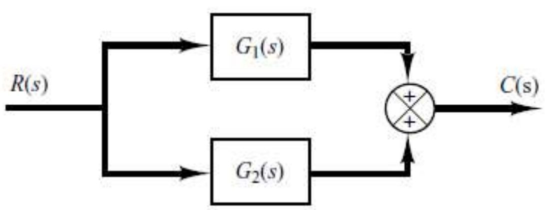
- Examples :

a. Cascade system

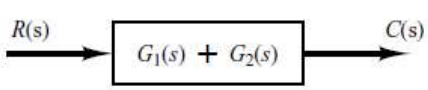
equivalent block diagram



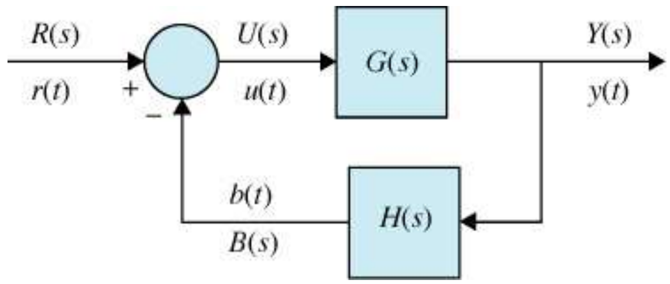
b. Parallel system



equivalent block diagram

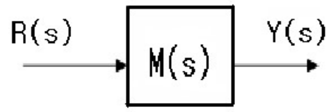


c. feedback control system

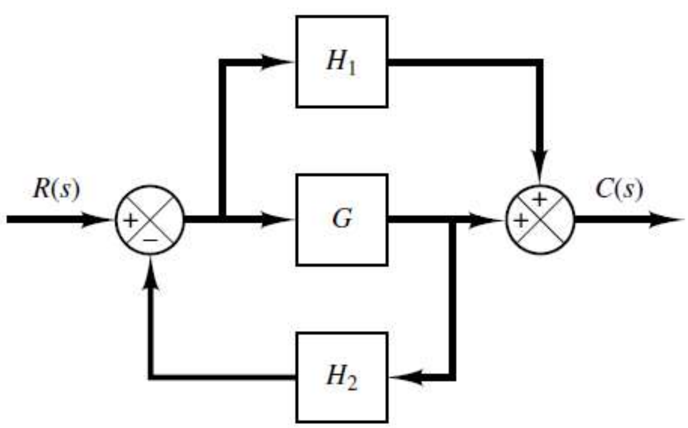


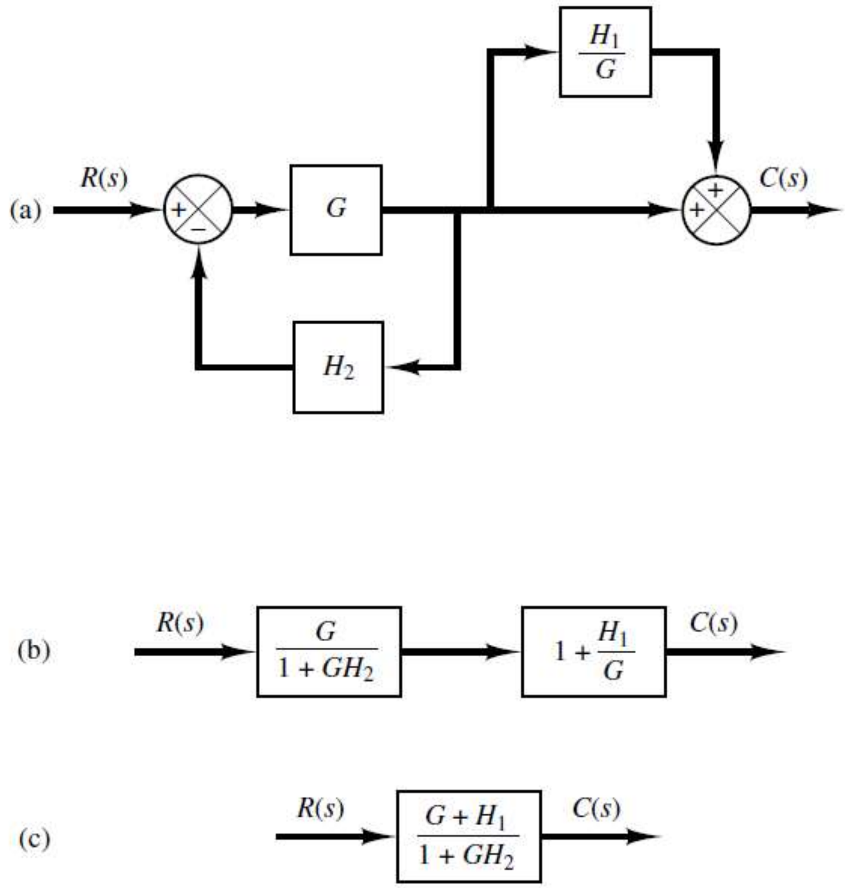
$$Y(s) = G(s)U(s)$$
$$U(s) = R(s) - H(s)Y(s)$$
$$\therefore Y(s) = G(s)\{R(s) - H(s)Y(s)\}$$
$$\therefore M(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

.equivalent block diagram

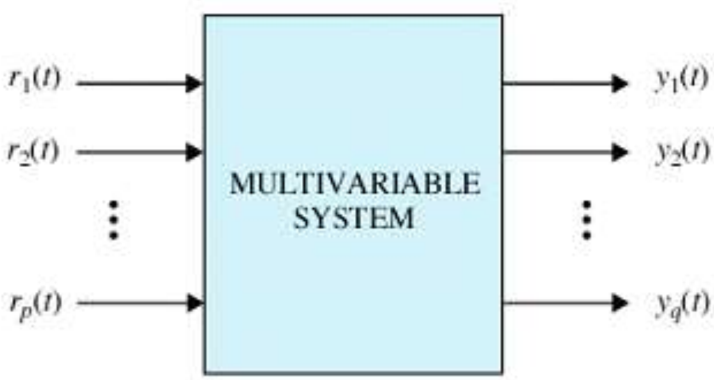


d. Other example.





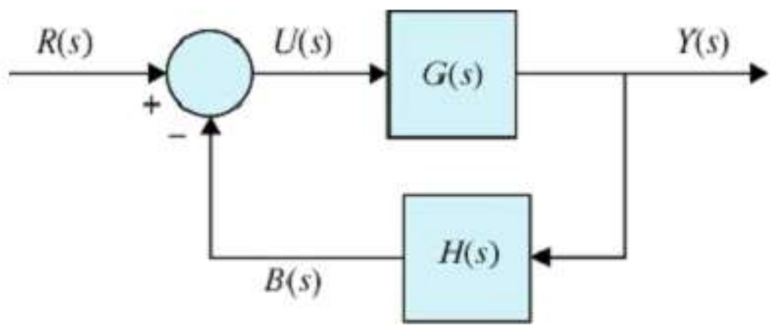
3-1-2 Multi-variable system： Vector Matrix form.



(a)

(b)

- Vector-matrix block diagram



$$\mathbf{y} = \mathbf{G}\mathbf{U}, \quad \mathbf{U} = \mathbf{R} - \mathbf{H}\mathbf{y}$$

$$\therefore \mathbf{y} = \mathbf{G}(\mathbf{R} - \mathbf{H}\mathbf{y})$$

$$\therefore (\mathbf{I} + \mathbf{G}\mathbf{H})\mathbf{y} = \mathbf{G}\mathbf{R}$$

$$\therefore \mathbf{y} = (\mathbf{I} + \mathbf{G}\mathbf{H})^{-1}\mathbf{G}\mathbf{R}$$

$$\therefore \mathbf{M} = (\mathbf{I} + \mathbf{G}\mathbf{H})^{-1}\mathbf{G}$$

3-2 Signal-Flow Graph

- Introduced by S.Mason

- Simplified version of a block diagram

- Constrained by more rigid rule
- System is Laplace transformed

Definition : Graphical means of portraying the input-output relationships between the variables of a set of linear algebraic equations.

Output variable = ∑ gain x input variables

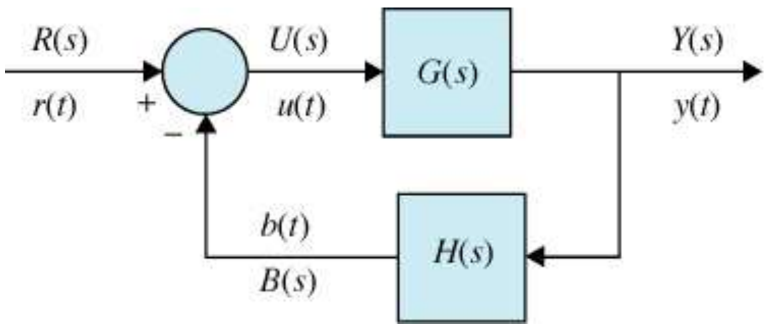
3-2-1 Basic elements

Node: represents variables (input, state, output)

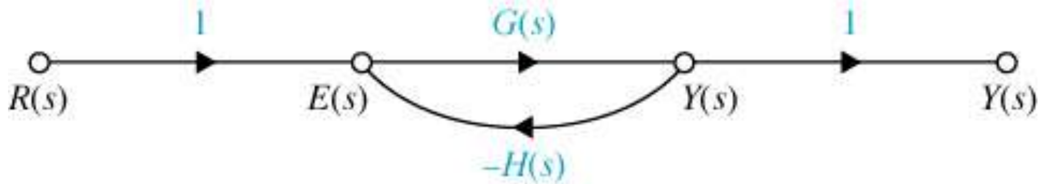
Branch: represents gain and direction

- Ex. Signal flow graph of $y_2 = a_{12} y_1$.

- Block diagram of feedback control system.



- Signal Flow Graph of feedback control system.



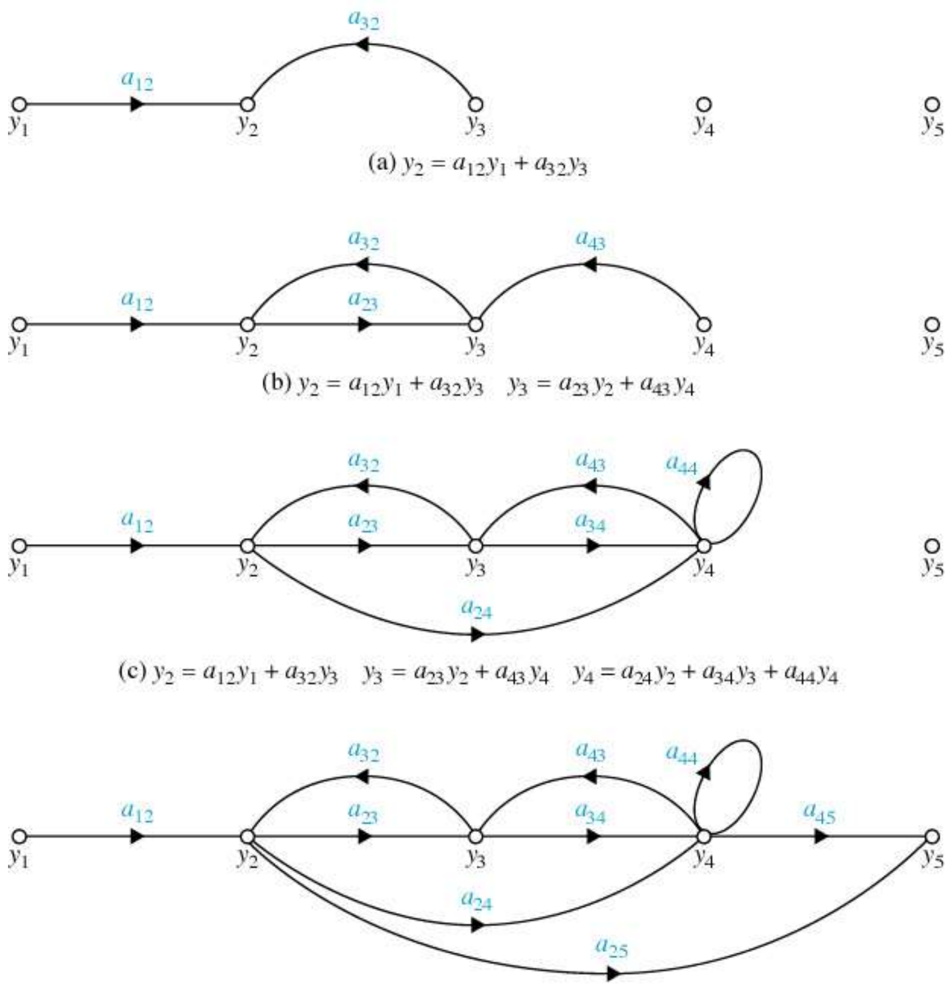
Ex. Construction of SFG for following system:

$$y_2 = a_{12}y_1 + a_{32}y_3$$

$$y_3 = a_{23}y_2 + a_{43}y_4$$

$$y_4 = a_{24}y_2 + a_{34}y_3 + a_{44}y_4$$

$$y_5 = a_{25}y_2 + a_{45}y_4$$



(a) $y_2 = a_{12}y_1 + a_{32}y_3$

(b) $y_2 = a_{12}y_1 + a_{32}y_3 \quad y_3 = a_{23}y_2 + a_{43}y_4$

(c) $y_2 = a_{12}y_1 + a_{32}y_3 \quad y_3 = a_{23}y_2 + a_{43}y_4 \quad y_4 = a_{24}y_2 + a_{34}y_3 + a_{44}y_4$

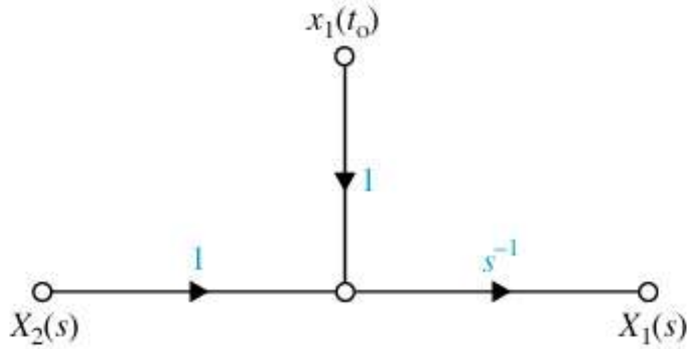
(d) Complete signal-flow graph

3-2-2 Basic properties

- only for linear systems (need to be Laplace transformed like T.F.)
- takes care of initial conditions unlike T.F.

$$\dot{X}_1(t) = X_2(t)$$

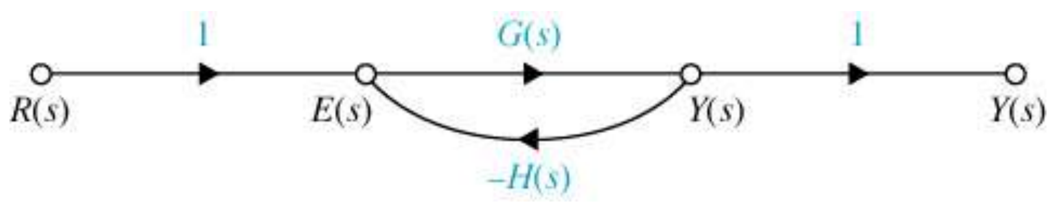
$$SX_1(S) - X_1(t_0) = X_2(S)$$
$$\therefore X_1(S) = \frac{1}{S}X_2(S) + \frac{1}{S}X_1(t_0)$$

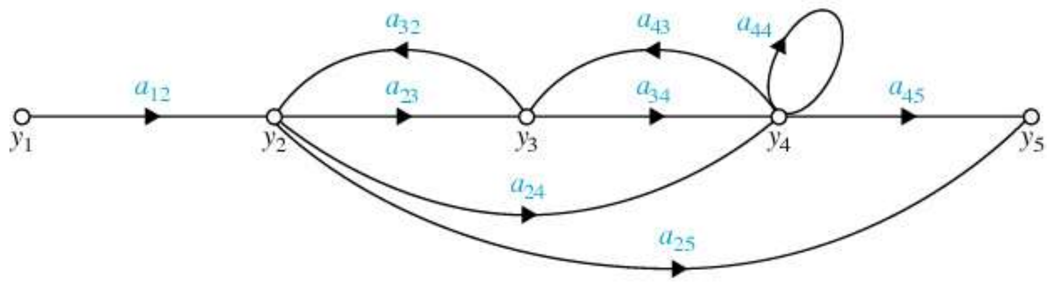


- The branch directing from node y_k to y_j represents the dependence of y_j on y_k, but not the reverse

3-2-3 Def. of SFG terms

- SFG examples





(d) Complete signal-flow graph

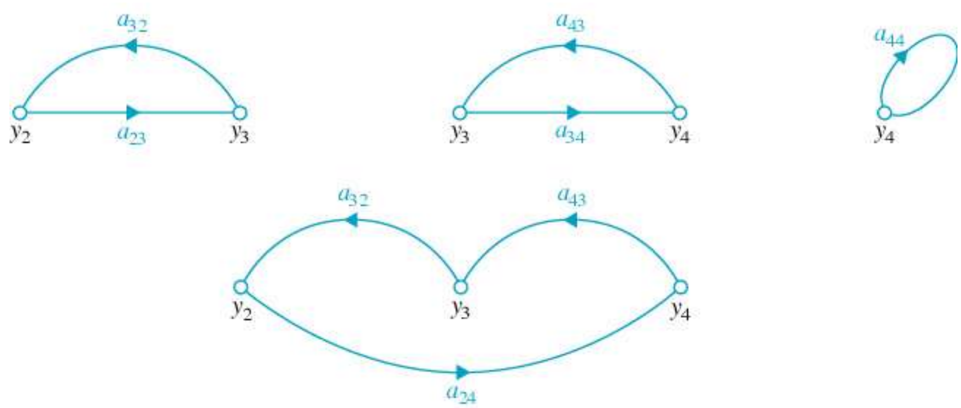
- Input node (Source): has only outgoing branches. Any other nodes can't be input node

- Output node (Sink): has only incoming branches. Any node can be output node by connecting branch with unit gain

- path: any collection of continuous succession of branches traversed in the same direction.

- Forward path: a path that starts at an input node and ends at an output node. (No node transferred more than once in one Forward path)

- loop: originate and terminate on the same node



- non-touching loops: do not share a node

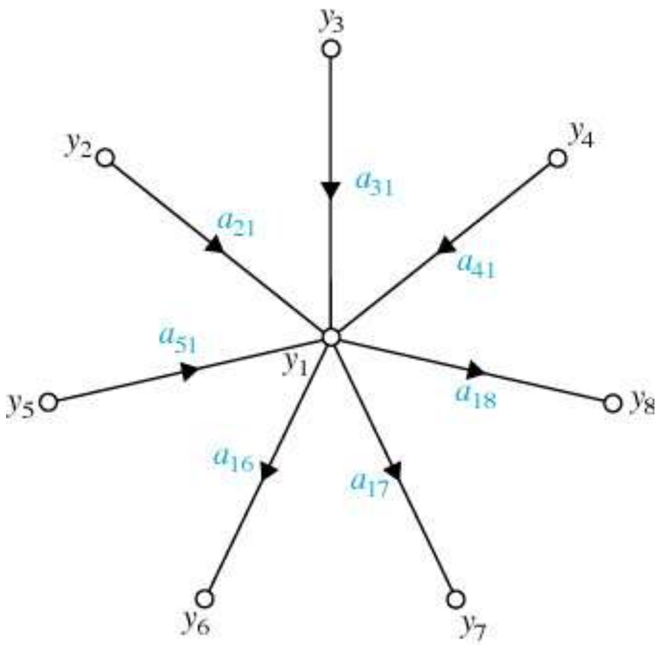
  2 touching loops

  3 non-touching loops: none

- path gain: product of the branch gains

3-2-4 SFG algebra

1. value of node : sum of all entering signals



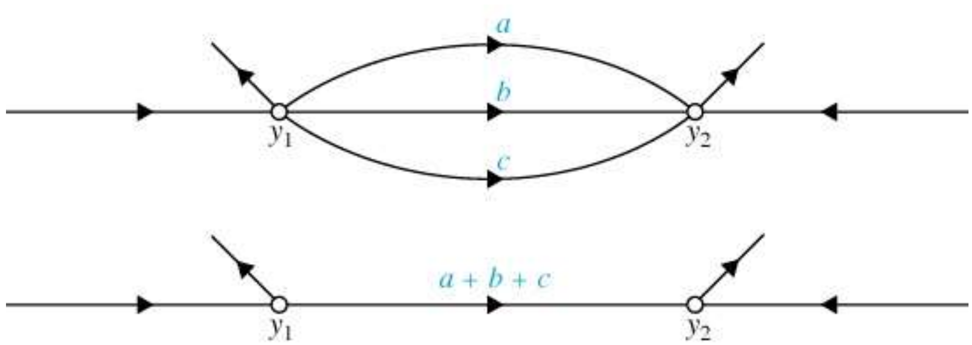
$$y_1 = a_{21}y_2 + a_{31}y_3 + a_{41}y_4 + a_{51}y_5$$

$$y_6 = a_{16}y_1$$

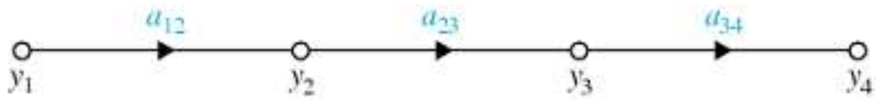
$$y_7 = a_{17}y_1$$

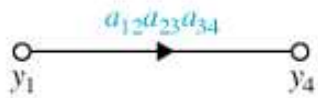
$$y_8 = a_{18}y_1$$

2. parallel branches : can be replaced by a single branch with equivalent gain.



4. serial branches : can be replaced by a single branch with equivalent gain.





3-2-6 Gain Formula for SFG

Given a SFG or block diagram, the task of solving for the input-output relations by algebraic manipulation could be quite tedious. Fortunately, there is a general gain formula available that allows the determination of the input-output relations of a SFG by inspection. Given a SFG with N forward paths and L loops, the gain between the input node y_{in} and output node y_{out} is :.

$$M = \frac{y_{out}}{y_{in}} = \sum_{k=1}^{N} \frac{M_k \Delta_k}{\Delta} \qquad \text{(Mason's gain formula)}$$

where

y_{in} = input-node variable

y_{out} = output-node variable

M = gain between y_{in} and y_{out}

N = total number of forward paths between y_{in} and y_{out}

M_k = gain of the kth forward path between y_{in} and y_{out}

$$\Delta = 1 - \sum_i L_{i1} + \sum_j L_{j2} - \sum_k L_{k3} + \dots$$

L_{mr} = gain product of the mth $(m = i, j, k, \dots)$ possible combination of r non-touching loops $(l \le r \le L)$

Or

Δ = 1-(sum of the gains of all individual loops) + (sum of products of gains of all possible combinations of two nontouching loops) – (sum of products of gains of all possible combinations of three nontoucing loops) + ...

Δ_k = the Δ for that part of the SFG that is nontouching with the kth forward path

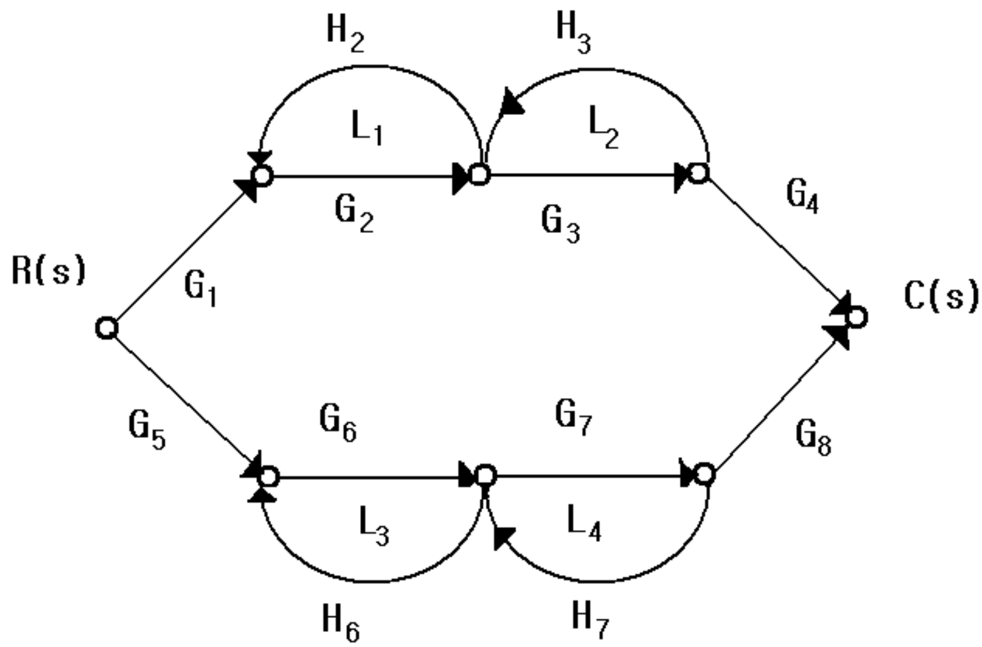
The gain formula in Eq.(3-46) may seem formidable to use at first glance. However, Δ and Δ_k are the only terms in the formula that could be complicated if the SFG has a large number of loops and nontouching loops.

Care must be taken when applying the gain formula to ensure that it is applied between an input node and an output node.

Example.

A two-path signal-flow graph is shown below.

is the multi-legged robot.



Forward paths from input $R(s)$ to output $C(s)$ are:

Path 1: $M_1 = G_1G_2G_3G_4$

Path 2: $M_2 = G_5G_6G_7G_8$

There are four self-loops:

$L_1 = G_2H_2, \quad L_2 = H_3G_3, \quad L_3 = G_6H_6, \quad L_4 = G_7H_7.$

Loops L_1 and L_2 do not touch L_3 and L_4.

$\therefore \Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4).$

The cofactor of the determinant along path 1 is evaluated by removing the loops that touch path 1 from Δ. Therefore we have

$$L_1 = L_2 = 0 \quad \text{and} \quad \Delta_1 = 1 - (L_3 + L_4)$$

Similarly, the cofactor for path 2 is

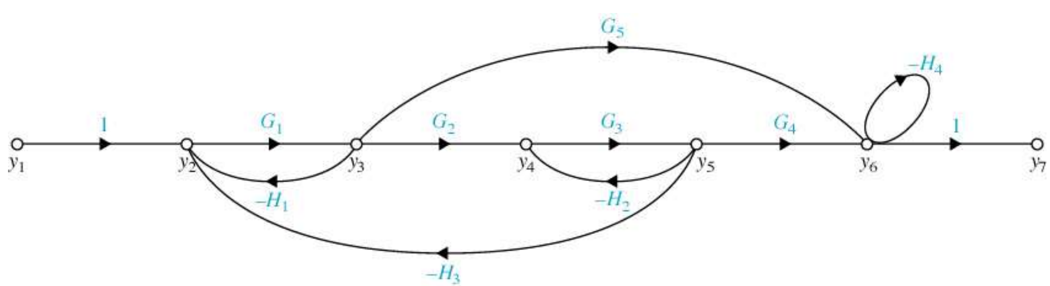
$$\Delta_2 = 1 - (L_1 + L_2)$$

Therefore the transfer function is:

$$\frac{C(s)}{R(s)} = T(s) = \frac{M_1\Delta_1 + M_2\Delta_2}{\Delta}$$

$$= \frac{G_1G_2G_3G_4(1 - L_3 - L_4) + G_5G_6G_7G_8(1 - L_1 - L_2)}{1 - L_1 - L_2 - L_3 - L_4 + L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4}$$

3-2-7 Example



Transfer function is

$$\frac{y_{out}}{y_2} = \frac{\dfrac{y_{out}}{y_{in}}}{\dfrac{y_2}{y_{in}}} = \frac{\dfrac{\sum M_k\Delta_k \big|_{\text{from } y_{in} \text{ to } y_{out}}}{\Delta}}{\dfrac{\sum M_k\Delta_k \big|_{\text{from } y_{in} \text{ to } y_2}}{\Delta}} = \frac{\sum M_k\Delta_k \big| y_{in} - y_{out}}{\sum M_k\Delta_k \big| y_{in} - y_2}$$

$* \ y_{in} -> y_{out}$

$$M_1 = 1 \cdot G_1 \cdot G_2 \cdot G_3 \cdot G_4 \cdot 1$$
$$M_2 = 1 \cdot G_1 \cdot G_5 \cdot 1$$
$$L_1 = -G_1 H_1 \quad L_2 = -G_3 H_2 \quad L_3 = -G_1 G_2 G_3 H_3 \quad L_4 = -H_4$$
$$L_{12} = G_1 H_1 G_3 H_2 \quad L_{14} = G_1 H_1 H_4 \quad L_{24} = G_3 H_2 H_4 \quad L_{34} = G_1 G_2 G_3 H_3 H_4$$
$$L_{124} = -G_1 H_1 G_3 H_2 H_4$$
$$\Delta = 1 - \left(L_1 + L_2 + L_3 + L_4\right) + \left(L_{12} + L_{14} + L_{24} + L_{34}\right) - \left(L_{123}\right)$$
$$\Delta_1 = 1 \quad \left(\text{not include } L_1, L_2, L_3, L_4\right)$$
$$\Delta_2 = 1 - \left(L_2\right) = 1 + G_3 H_2 \quad \left(\text{not include } L_1, L_3, L_4\right)$$
$$\therefore M_1 \Delta_1 + M_2 \Delta_2 = G_1 G_2 G_3 G_4 + G_1 G_5 \left(1 + G_3 H_2\right)$$

$* \ y_{in} -> y_2$

$$M_1 = 1 \quad \Delta_1 = 1 - \left(L_2 + L_4\right) + \left(L_{24}\right) \quad \left(\text{not include } L_1, L_3\right)$$
$$\therefore M_1 \Delta_1 = 1 + G_3 H_2 + H_4 + G_3 H_2 H_4$$
$$\therefore \frac{y_7}{y_2} = \frac{G_1 G_2 G_3 G_4 + G_1 G_5 \left(1 + G_3 H_2\right)}{1 + G_3 H_2 + H_4 + G_3 H_2 H_4}$$

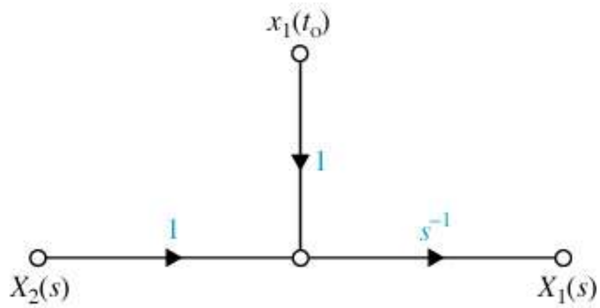
3-3 State diagram

The state diagram is an extension of the SFG to portray state eqn. and differential eqn. using integrator.

$$\frac{dx_1}{dt} = x_2 \quad \dashrightarrow \qquad x_1 = \int_0^t x_2 d\tau + x_1(0)$$

Since SFG handles only integrator in s-domain.

$$x_1(s) = \frac{x_2(s)}{s} + \frac{x_1(0)}{s}$$



Note that Block diagram can only describe the T.F.

- 2^{nd} order system example.

$$\ddot{y} + a_1\dot{y} + a_0 y = r$$