



iCyPhy



# Software Design for Cyber-Physical Systems

*Edward A. Lee*

## Module 9: Consistency and Availability Tradeoffs

Technical University of Vienna  
*Vienna, Austria, May 2022*

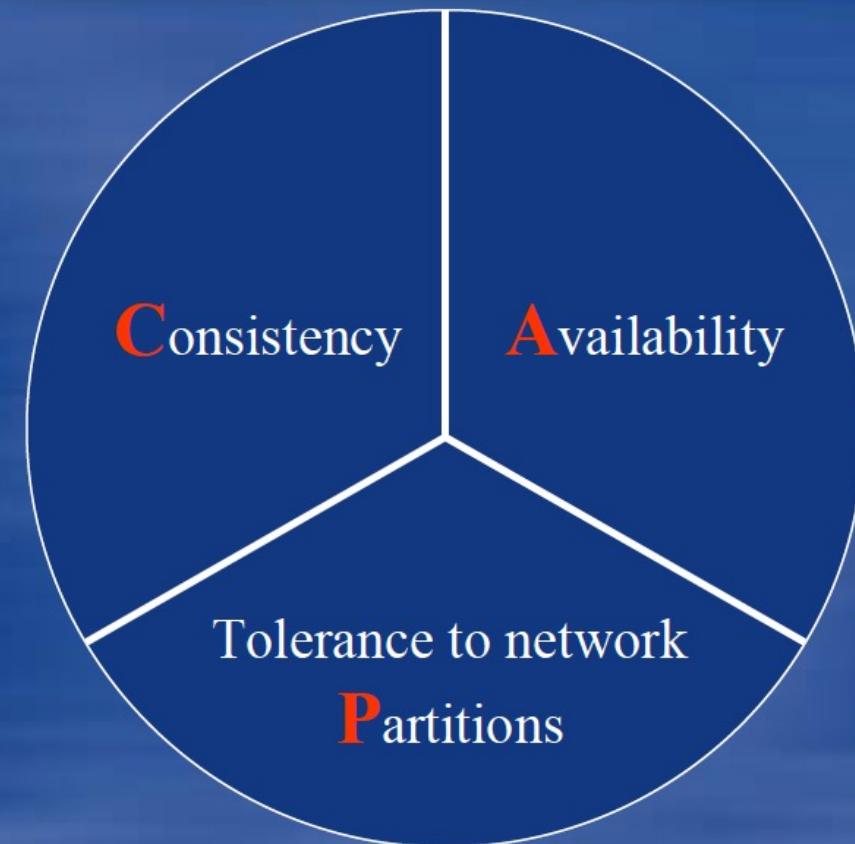


University of California, Berkeley



# The CAP Theorem

## The CAP Theorem



Eric Brewer  
Berkeley & Google

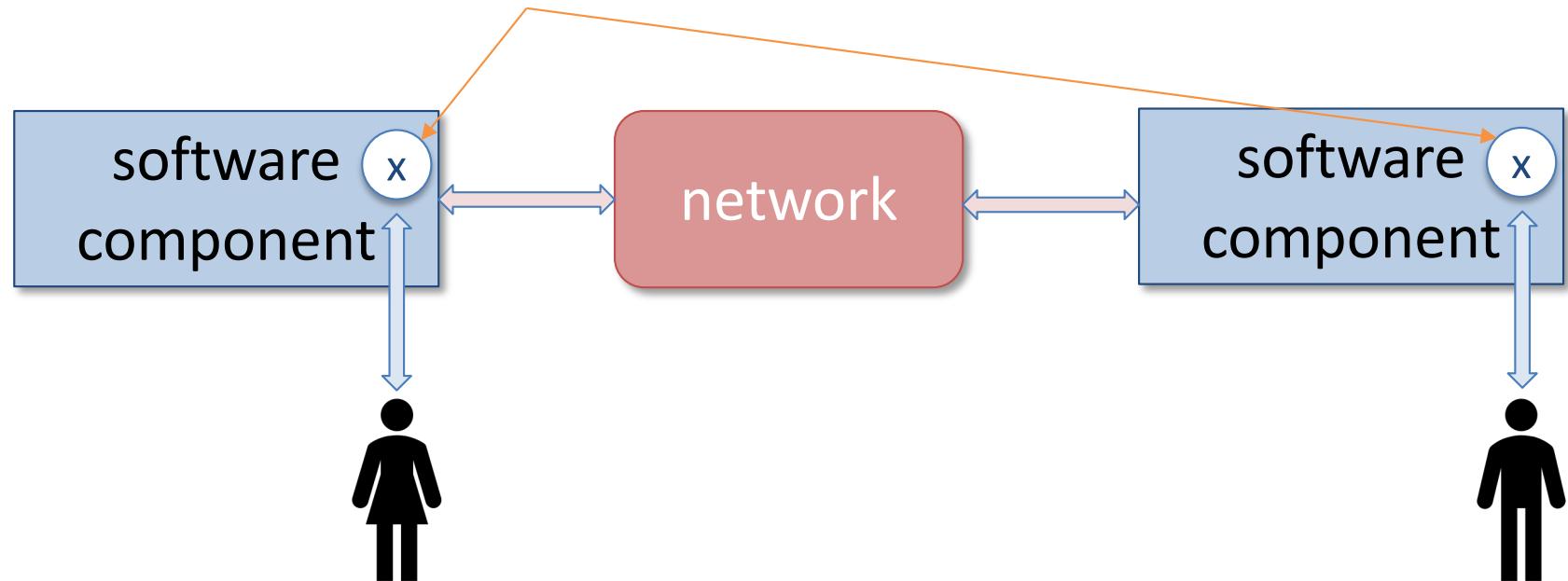


Theorem: You can have **at most two** of these properties for any shared-data system



# Consistency and Availability in Distributed Software

- **Consistency:** agreement on the values of shared variables

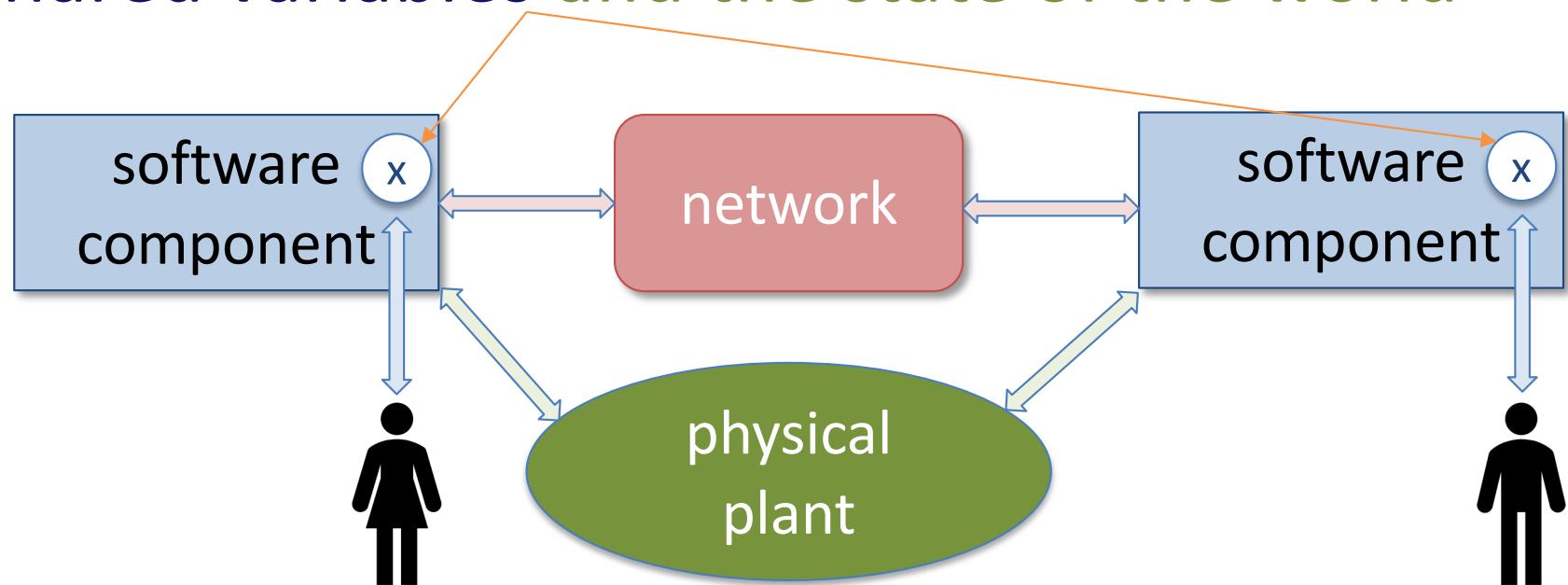


- **Availability:** ability to respond to reads and writes accessing those shared variables



# Consistency and Availability in Cyber-Physical Systems

- **Consistency:** agreement on the values of shared variables and the state of the world



- **Availability:** ability to respond to reads and writes accessing those state variables



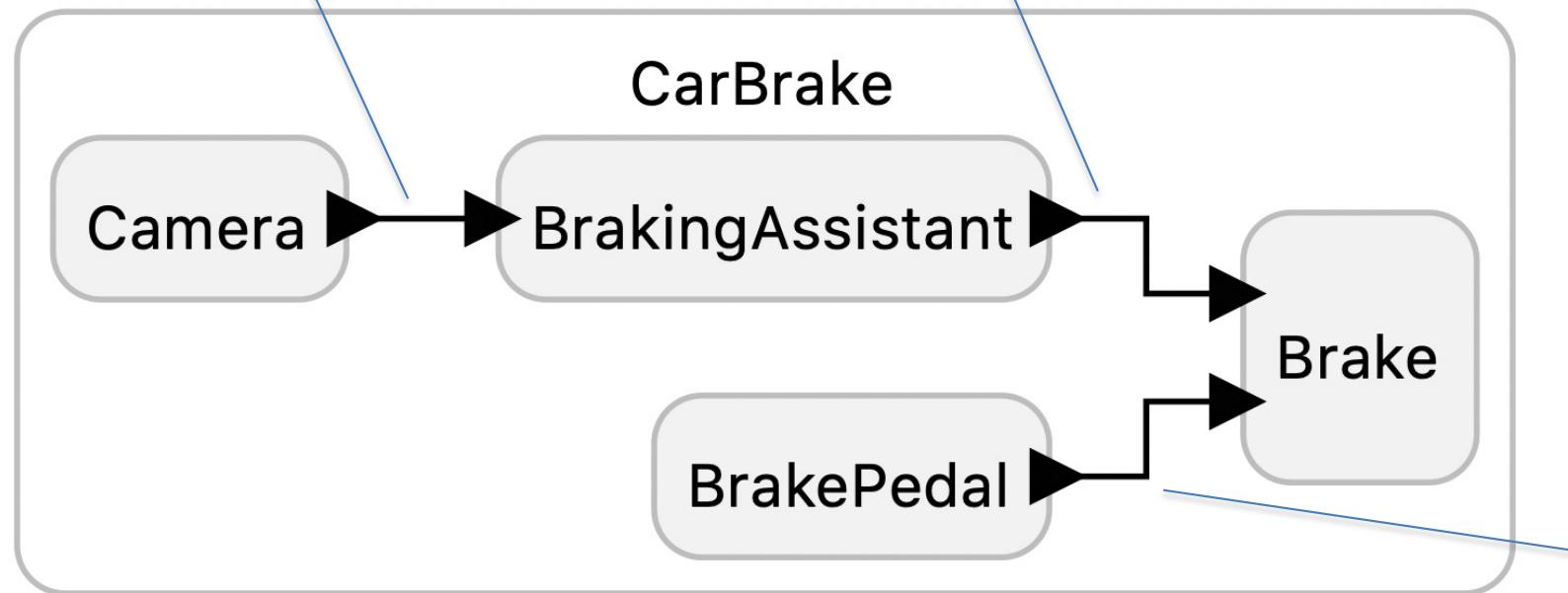
# Availability or Consistency?



Snapshot  
at time  $T$

State of  
the world  
at time  $T$

A software architecture:



Denso autonomous braking demonstrating Advanced Driver-Assistance System (ADAS) in Oct. 2018 [Reported in The Daily Times]

State of the  
world at  
time  $T + \varepsilon$

Thanks to Christian Menard (TU Dresden) for this example.



# Availability or Consistency?

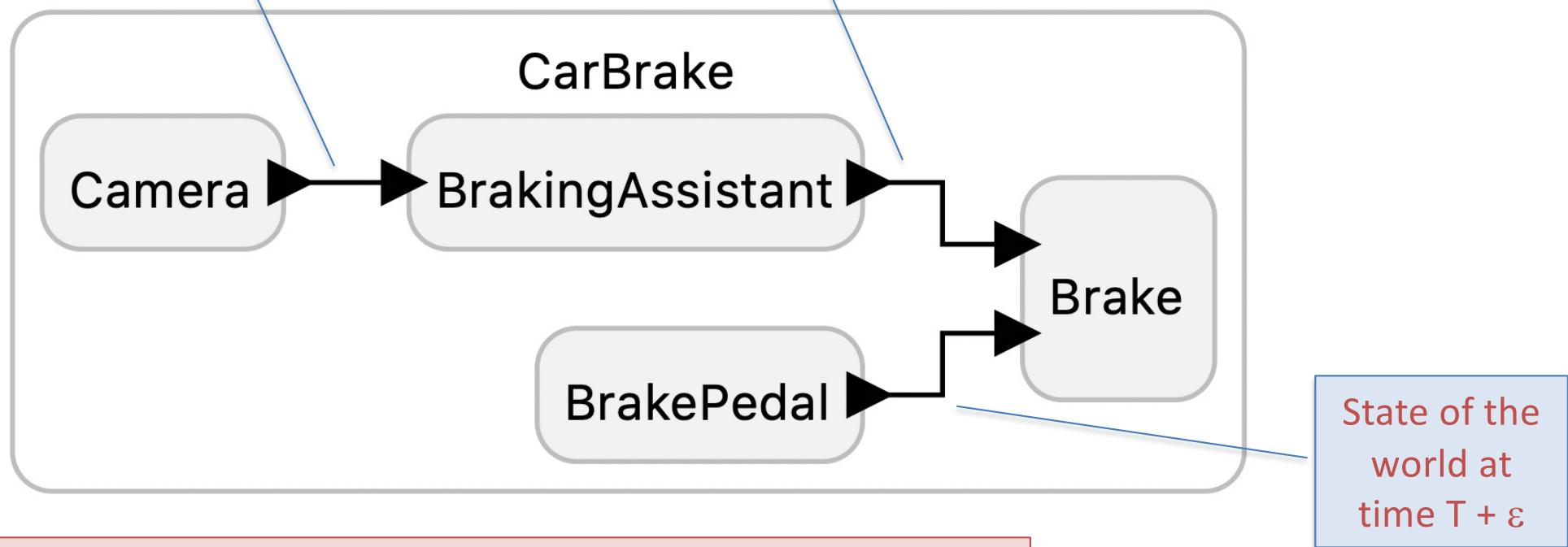
Snapshot  
at time T

State of  
the world  
at time T

Should the Brake component wait for the analysis of the BrakingAssistant before responding to BrakePedal?

- Yes: emphasizing consistency
- No: emphasizing availability

A software architecture:



Thanks to Christian Menard (TU Dresden) for this example.

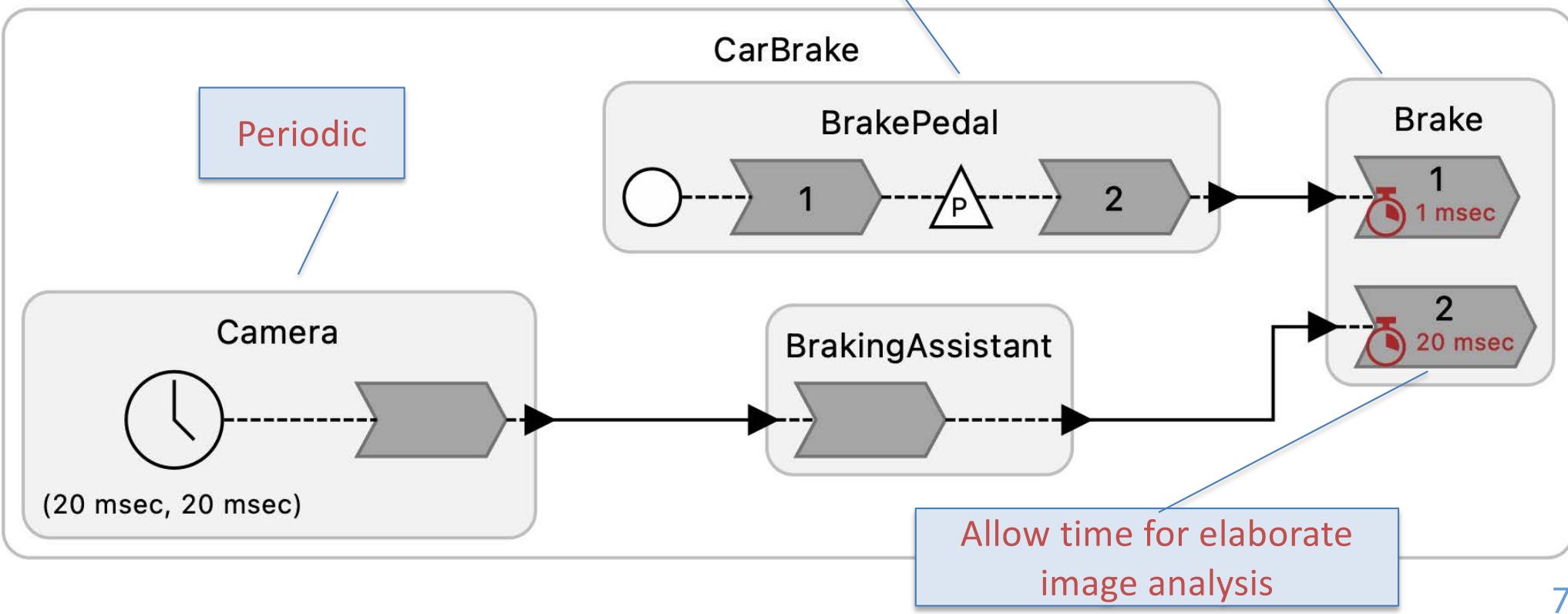


# Availability or Consistency?

**Discussion:** Does this design emphasize availability over consistency?

When are deadline violations likely to occur?

Design details:

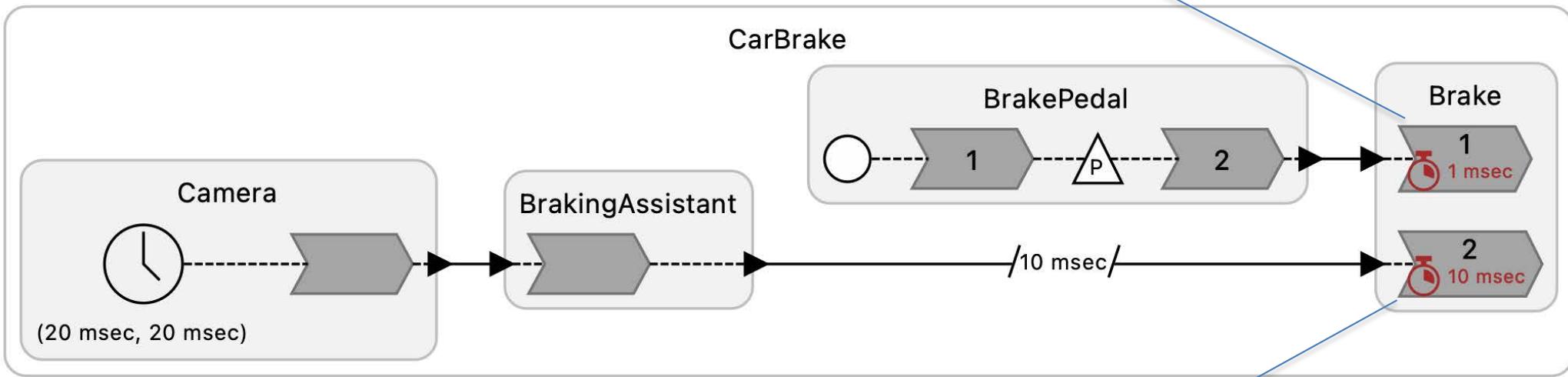




# An Alternative Design

**Discussion:** What should the deadline violation handlers do?

As long as the BrakingAssistant takes less than 10 msec, it will not block reactions to the BrakePedal.



**Discussion:** This design also puts a *minimum* delay on the response to the BrakingAssistant. Is this this a good idea? How could this minimum delay be avoided?

Response time is specified to be between 10 and 20 msec

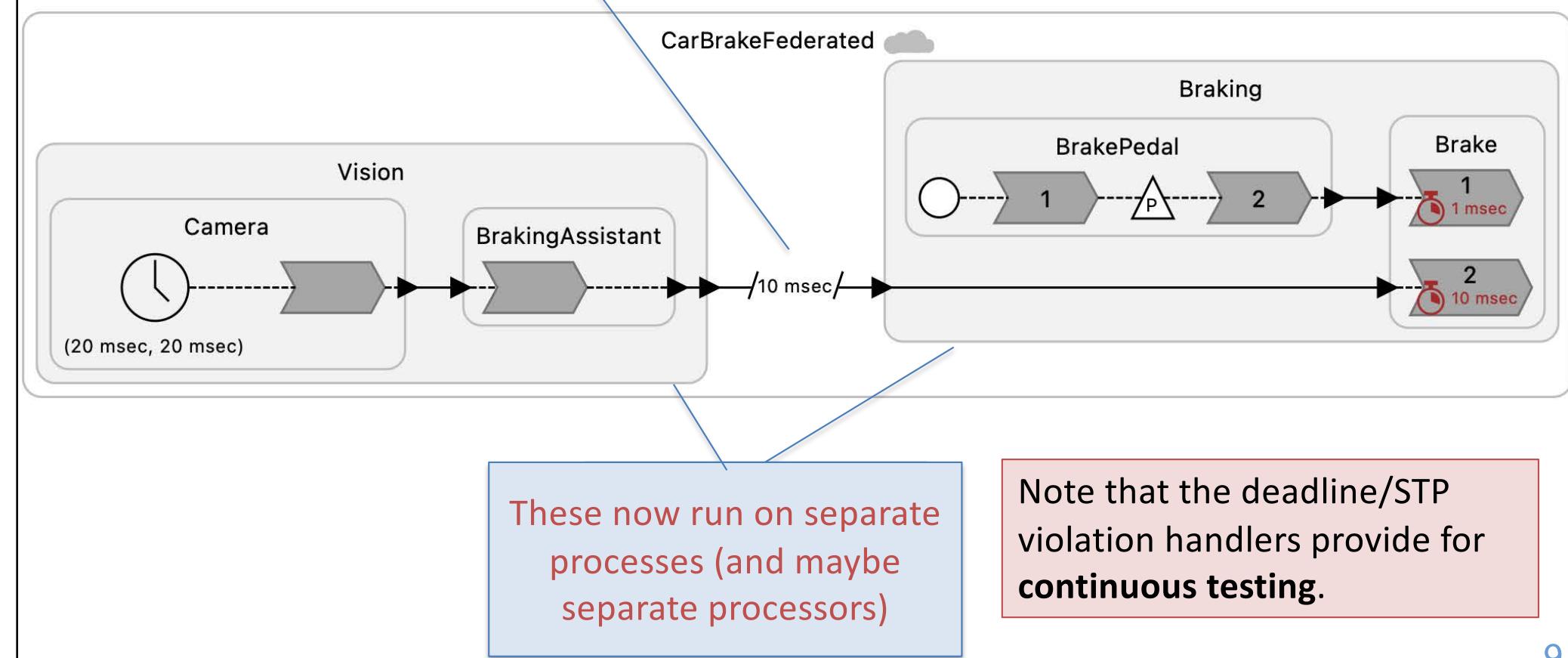


# Federated Design

**Discussion:** Should this use centralized or decentralized coordination?

Explicit inconsistency

**Discussion:** Under decentralized coordination, what should STP violation handlers do?

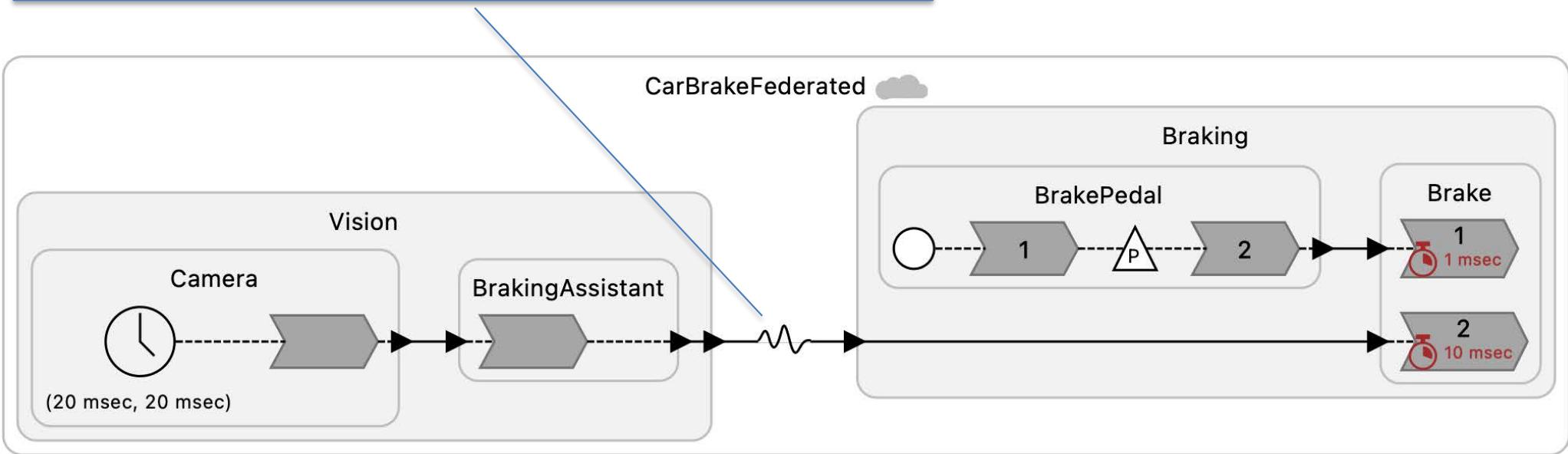




# Asynchronous Design

Physical connection:  
`vision.trigger_brake ~> braking.brake_assistant`

**Discussion:** Advantages  
and disadvantages?



Network failures and failures of the Vision subsystem are undetectable.

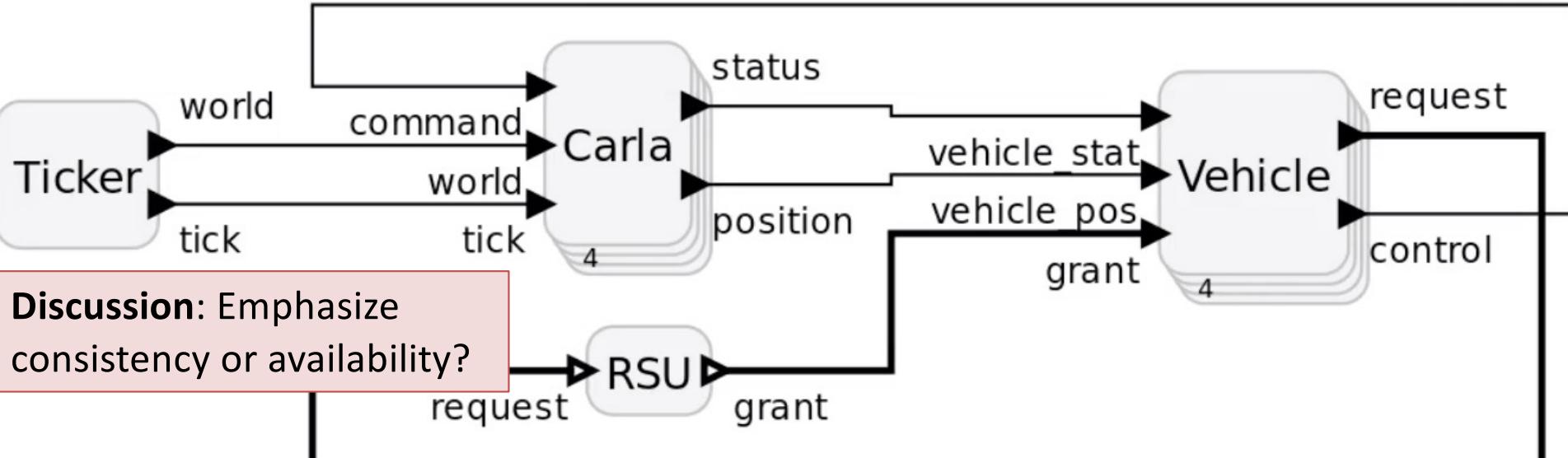


# Intersection

- **Consistency:** agreement on the state of the intersection.
- **Availability:** ability to enter the intersection.



CarlaIntersection

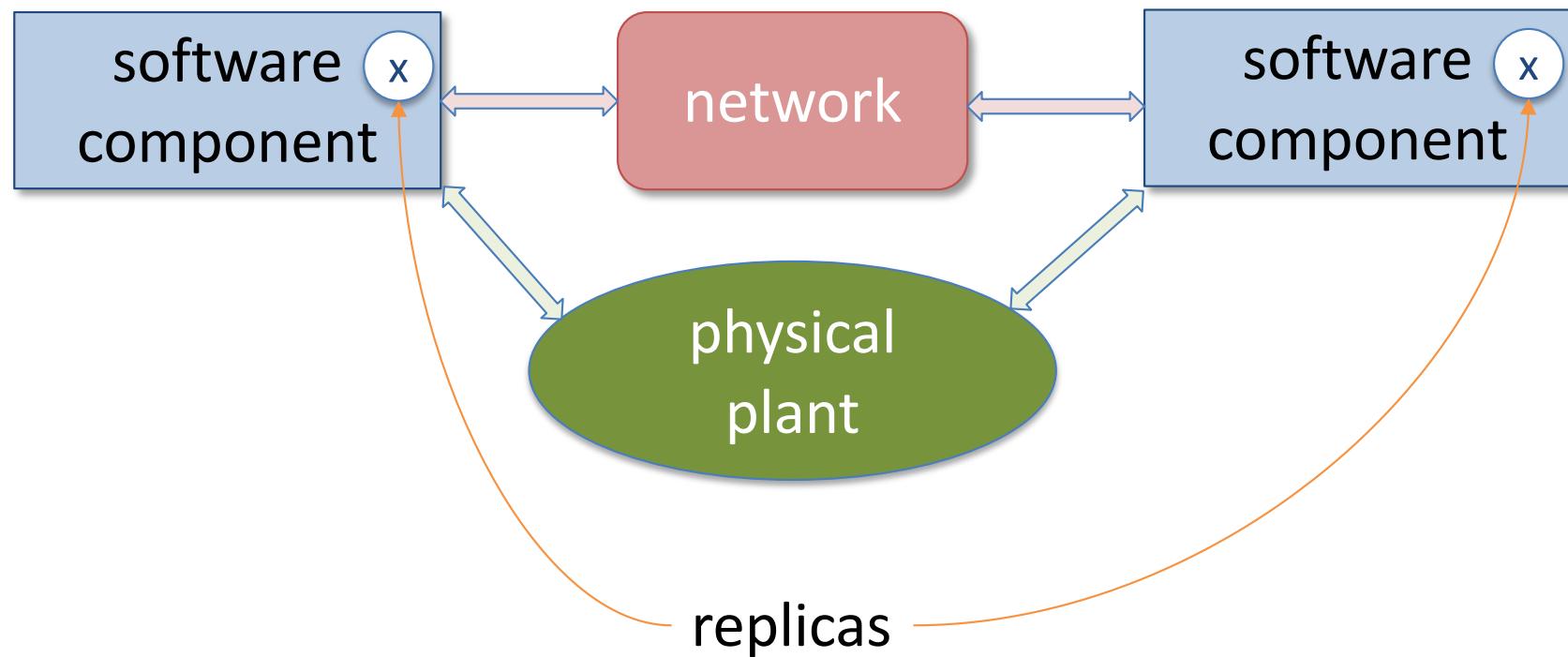


Thanks to **Soroush Bateni** (UT Dallas) and **Ravi Akella** (Denso) for this example.



# The Need For Replicas

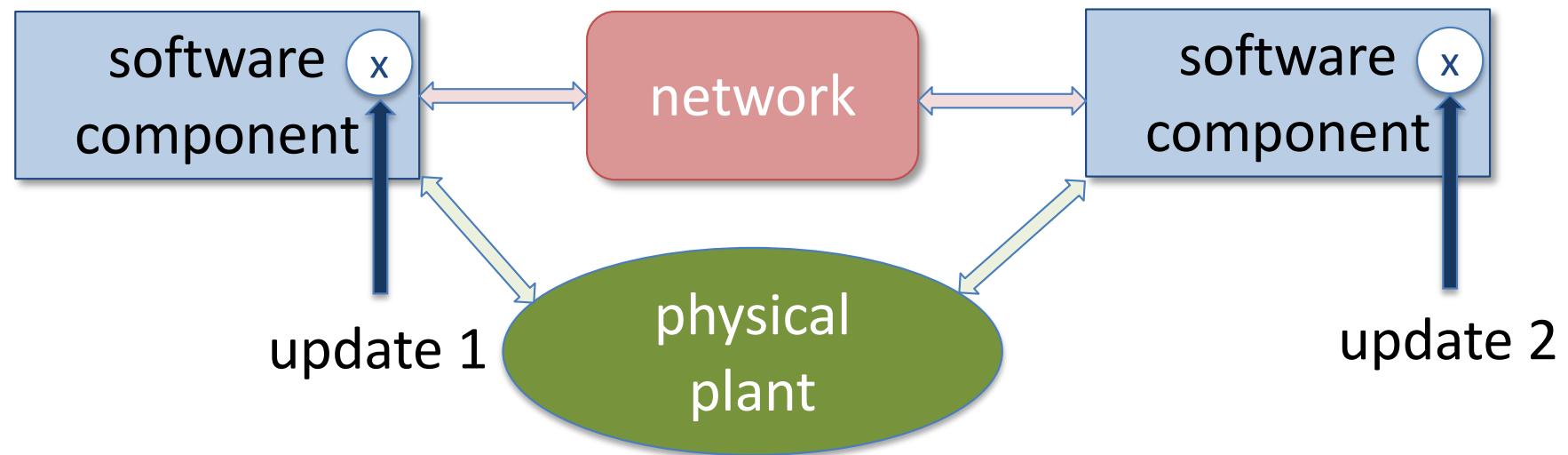
If there are shared variables, then, to tolerate network latency, replication is necessary.





# Ordering Updates

Assume updates can occur in multiple places:

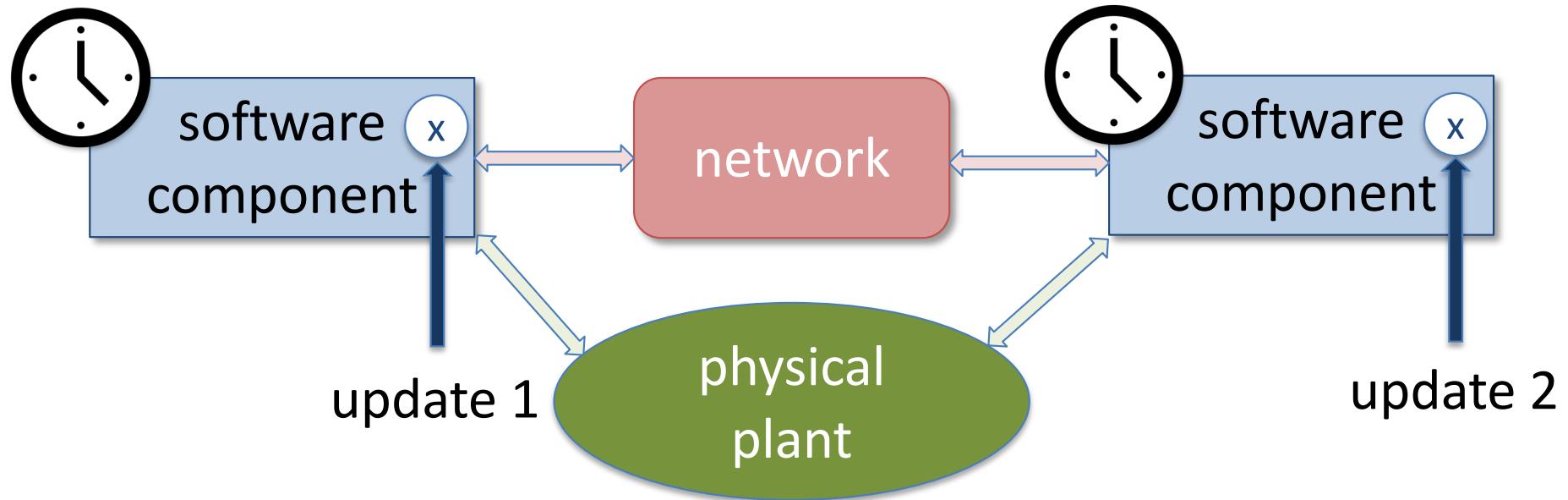


Consistency requires agreement on the order of these updates.



# Physical Time is Imperfect

We have *imperfect* measurements of time.

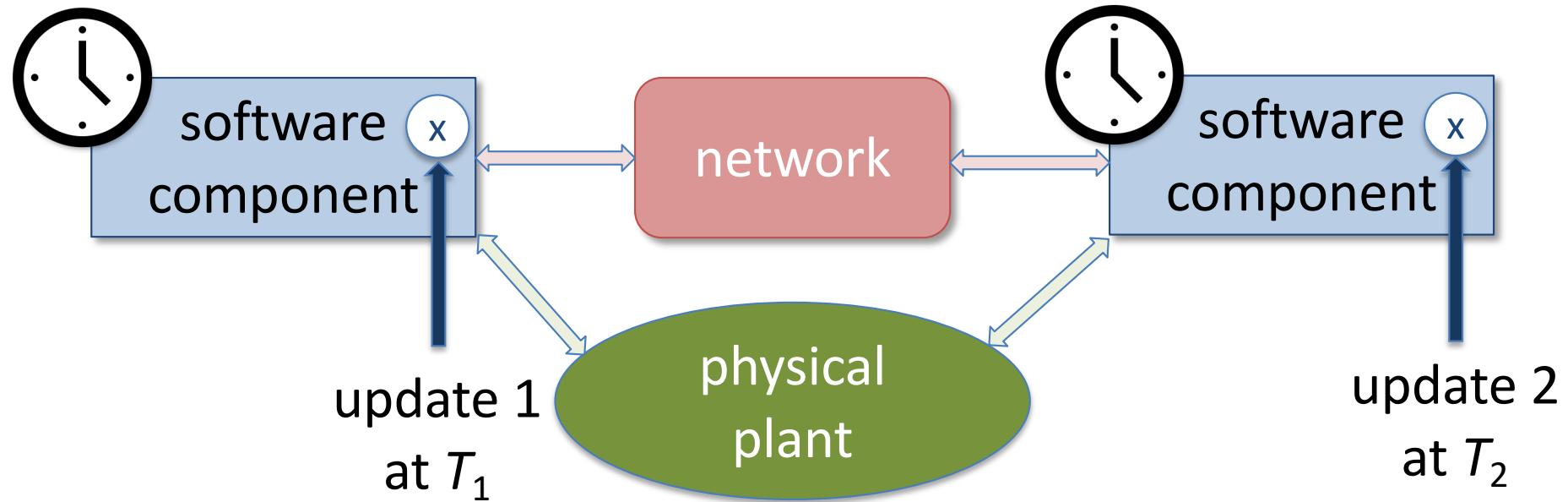


With clock synchronization (albeit imperfect), physical time can be used to assign a logical time.



# Timestamps

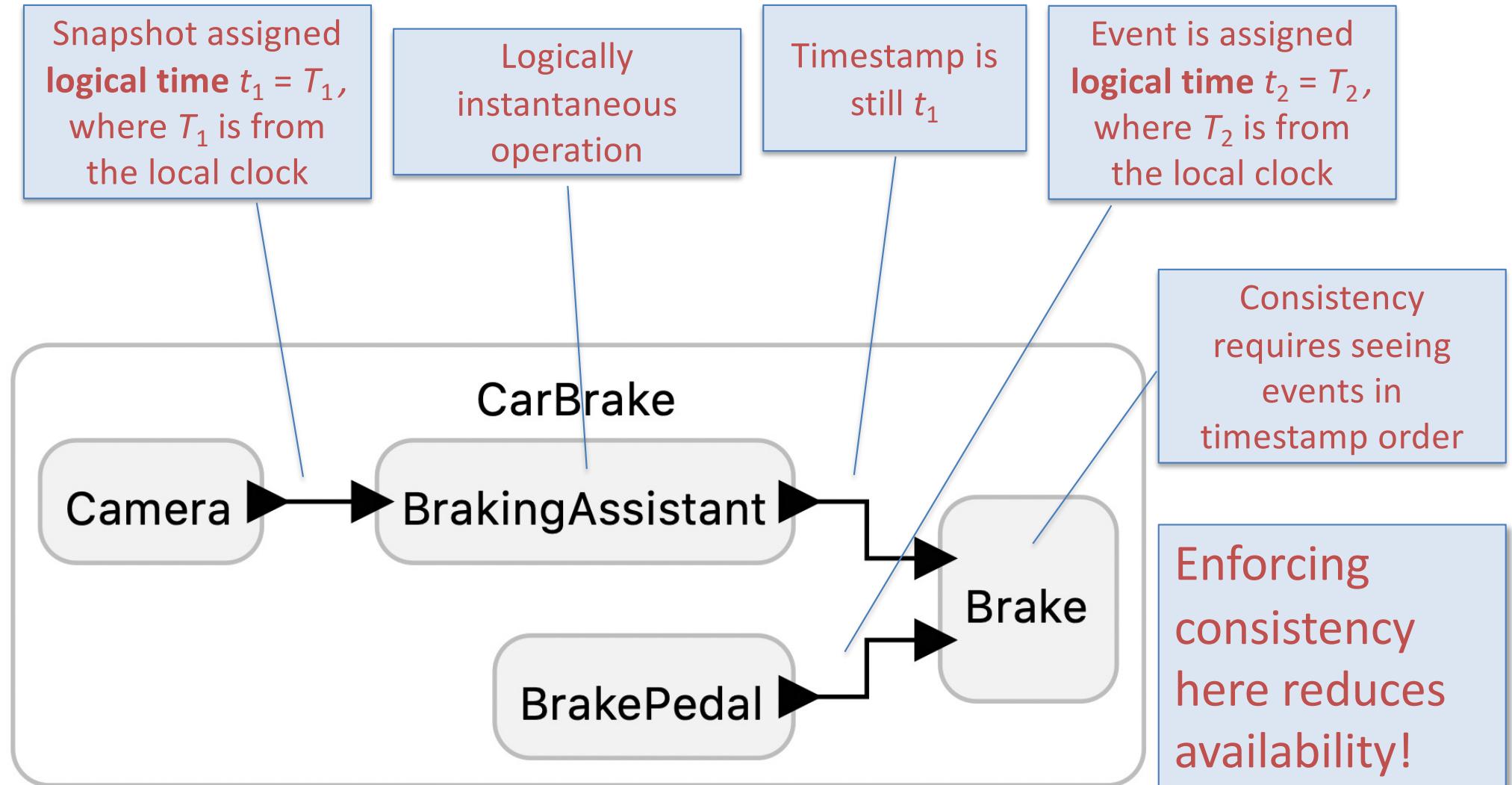
We have *imperfect* measurements of time.



With clock synchronization (albeit imperfect), physical time can be used to assign a logical time.



# Timestamps in Use





# Recent Result: CAL Theorem

## Quantifying and Generalizing the CAP Theorem

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Shaokai Lin<sup>1[0000-0001-6885-5572]</sup>, Marten Lohstroh<sup>1[0000-0001-8833-4117]</sup>, and  
Christian Menard<sup>3[0000-0002-7134-8384]</sup>

arXiv:2109.07771v1 [cs.DC] 16 Sep 2021

**Theorem 1.** *Given a trace as defined in Section 3.2, the unavailability in Definition 2 at process  $i$  is, in the worst case,*

$$\bar{A}_i = \max\left(O_i, \max_{j \in N}(\mathcal{L}_{ij} - \bar{C}_{ij})\right), \quad (17)$$

*where  $O_i$  is the processing offset given by Definition 3,  $\mathcal{L}_{ij}$  is apparent latency in Definition 4 (which includes  $O_j$ ), and  $\bar{C}_{ij}$  is the inconsistency of Definition 1 .*



# Causation

Event  $e_1$  **causally effects**  $e_2$  if  $e_2$  cannot behave as if  $e_1$  had not occurred.

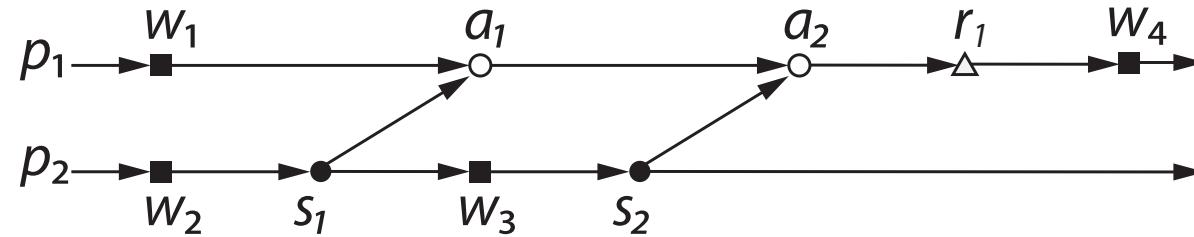
Event  $e_1$  **counterfactually causes**  $e_2$  if  $e_2$  will not occur if  $e_1$  had not occurred.

In both case, we write  $e_1 \rightarrow e_2$

See Lee (2020), *The Coevolution*, Chapter 11, for subtleties around causation.



# A Process Model

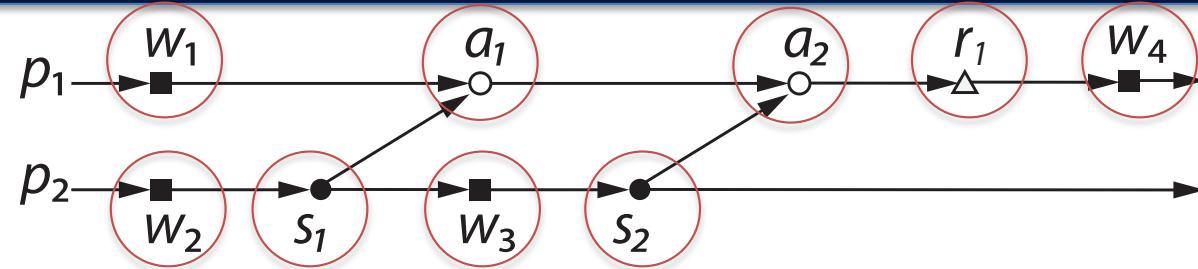


Event types:

- $w_x$ : Merge a value with the local replica of  $x$ .
- $r_x$ : Read the value of the local replica of  $x$ .
- $s_x$ : Send the value of the local replica of  $x$  to some set of other processes.
- $a_x$ : Accept a new value for  $x$  and merge it with the local replica.



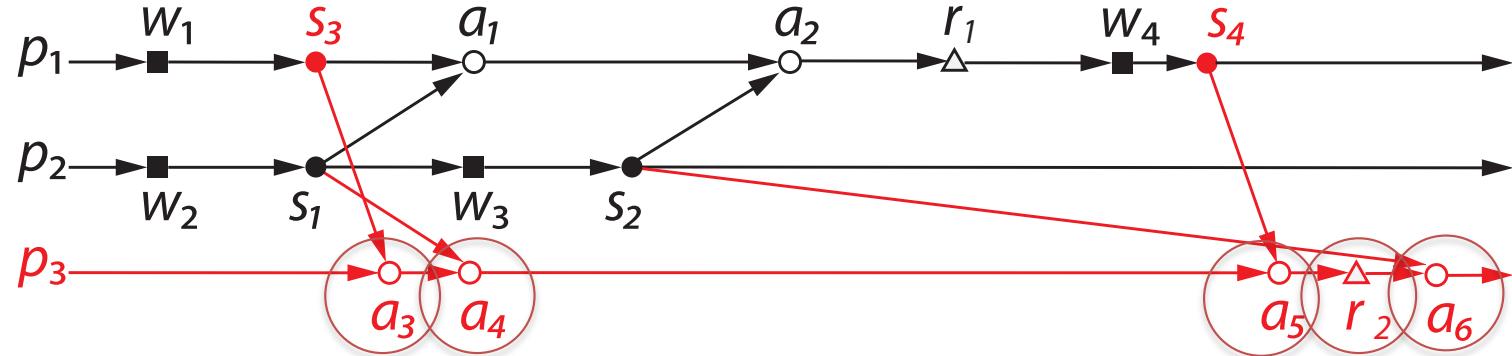
# A Bulletin Board



- $w_1$ : Joe posts a picture of Sally at a recent party by writing to a local copy.
- $w_2$ : Sally posts that her son Billy is missing, writing to a local copy.
- $s_1$ : Sally's message is sent to Joe's process.
- $a_1$ : Joe's machine receives the message and updates his local copy.
- $w_3$ : Sally posts that her son has been found (on the local copy).
- $s_2$ : Sally's message is sent to Joe's process.
- $a_2$ : Joe's machine receives the second message and updates his local copy.
- $r_1$ : Joe reads Sally's messages.
- $w_4$ : Joe posts "That's good news, a relief."



# Bulletin Board Observer

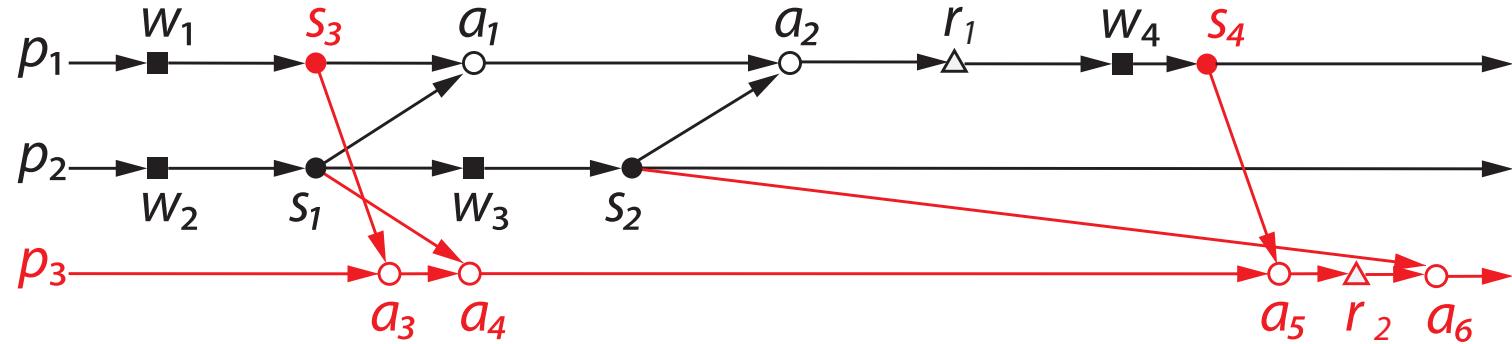


- $a_3$ : Akosh receives Joe's picture of Sally.
- $a_4$ : Akosh receives Sally's post that her son Billy is missing.
- $a_5$ : Akosh receives Joe's post "That's good news, a relief".
- $r_2$ : Akosh reads the posts so far.
- $a_6$ : Akosh receives Sally's post that her son has been found.

This sequence violates **causal consistency**.



# Causality Relation

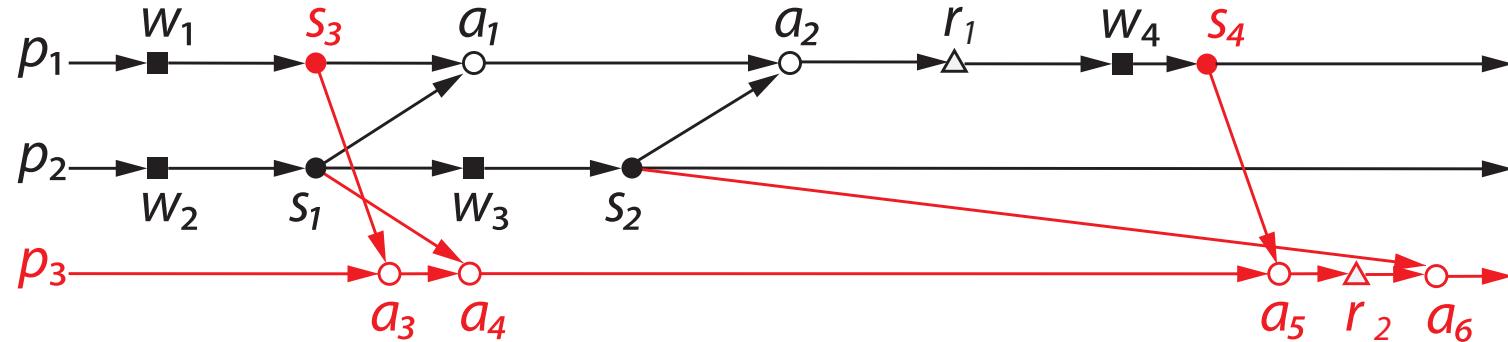


Formally, the **causality relation** is the smallest *transitive* relation such that  $e_1 \rightarrow e_2$  if  $e_1$  precedes  $e_2$  in a process, or  $e_1$  is the sending of a value in one process (event type  $s_x$ ) and  $e_2$  is the acceptance of the value in another process (event type  $a_x$ ).

Schwarz and Mattern (1994) “Detecting causal relationships in distributed computations: in search of the holy grail.” *Distributed Computing*.

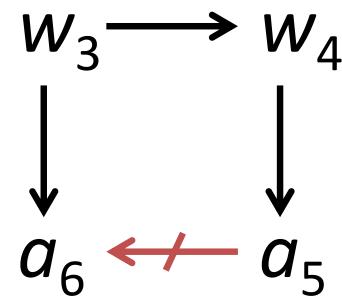


# Causal Consistency



**Causal consistency** requires that if  $w_3 \rightarrow w_4$ ,  $w_3 \rightarrow a_6$ , and  $w_4 \rightarrow a_5$ , then  $a_5 \not\rightarrow a_6$ .

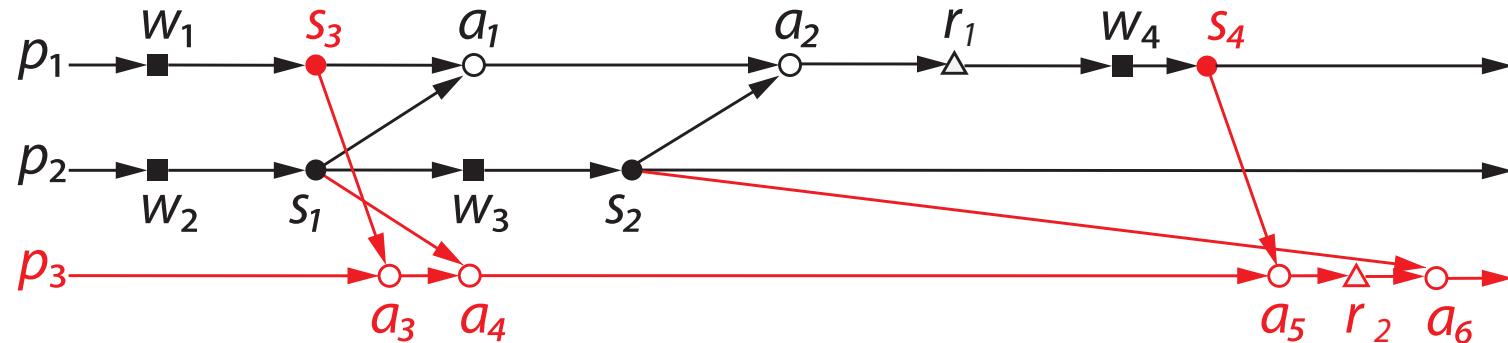
This requirement is violated by the above picture.



Schwarz and Mattern (1994) “Detecting causal relationships in distributed computations: in search of the holy grail.” *Distributed Computing*.



# Lingua Franca is Causally Consistency by Default



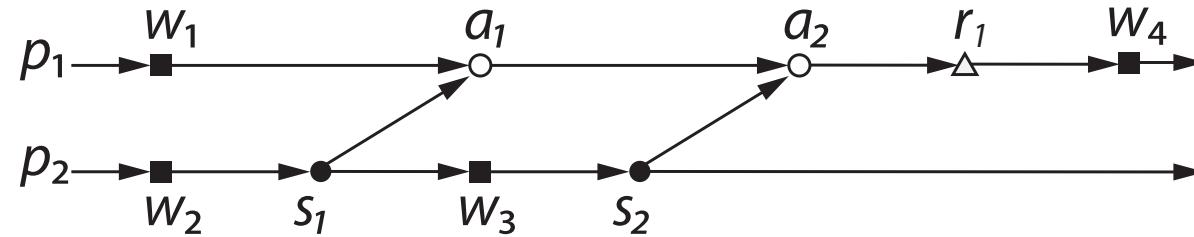
**LF semantics:** If a reaction  $r_1$  can have an effect (state update or output) that influences what reaction  $r_2$  sees in any way, then execution of  $r_1$  must precede execution of  $r_2$ .

Note: This guarantee does not apply to side-effects!

Note: Adding **after** delays can explicitly reverse the ordering.



# Recall Terminology



Event types:

- $w_x$ : **Merge** a value with the local replica of  $x$ .
- $r_x$ : **Read** the value of the local replica of  $x$ .
- $s_x$ : **Send** the value of the local replica.
- $a_x$ : **Accept** a new value for  $x$  and merge.



# Inconsistency

## Definition

For each write event on process  $j$  with tag  $g_j$ , let  $g_i$  be the tag of the corresponding accept event on process  $i$  or  $\infty$  if there is no corresponding accept event. The **inconsistency**  $\bar{C}_{ij} \in \mathbb{I}$  from  $j$  to  $i$  is defined to be

$$\bar{C}_{ij} = \max(\mathcal{T}(g_i) - \mathcal{T}(g_j)), \quad (1)$$

where the maximization is over all write events on process  $j$ . If there are no write events on  $j$ , then we define  $\bar{C}_{ij} = 0$ .

Time stamp of the tag.

Set of intervals.



# Strong Consistency

A strongly consistent system is one where

$$\bar{c}_{ij} = 0$$

for all  $i, j$ .

In Lingua Franca, this means there are no after delays on connections between components.



# Unavailability

## Definition

For each read event on process  $i$ , let  $g_i$  be its tag and  $T_i$  be the physical time at which it is processed. The **unavailability**  $\bar{A}_i \in \mathbb{I}$  at process  $i$  is defined to be

$$\bar{A}_i = \max(T_i - \mathcal{T}(g_i)), \quad (2)$$

where the maximization is over all read events on process  $i$  that are triggered by user requests. If there are no such read events on process  $i$ , then  $\bar{A}_i = 0$ .



# Processing Offset

## Definition

For process  $i$ , the **processing offset**  $O_i \in \mathbb{I}$  is

$$O_i = \max(T_i - \mathcal{T}(g_i)) \quad (3)$$

where  $T_i$  and  $g_i$  are the physical time and tag, respectively, of a write event on process  $i$  that is triggered by a local external input (and hence assigned a timestamp drawn from the local clock). The maximization is over all such write events in process  $i$ . If there are no such write events, then  $O_i = 0$ .



# Apparent Latency

## Definition

Let  $g_j$  be the tag of a write event in process  $j$  that is triggered by an external input at  $j$  (so  $\mathcal{T}(g_j)$  is the physical time of that external input). Let  $T_i$  be the physical time of the corresponding accept event in process  $i$  (or  $\infty$  if there is no such event). (If  $i = j$ , we assume  $T_i$  is the same as the physical time of the write event.) The **apparent latency** or just **latency**  $\mathcal{L}_{ij} \in \mathbb{I}$  for communication from  $j$  to  $i$  is

$$\mathcal{L}_{ij} = \max(T_i - \mathcal{T}(g_j)), \quad (4)$$

where maximization is over all such write events in process  $j$ . If there are no such write events, then  $\mathcal{L}_{ij} = 0$ .



# More on Apparent Latency

The apparent latency is a sum of four components,

$$\mathcal{L}_{ij} = O_j + X_{ij} + L_{ij} + E_{ij}, \quad (5)$$

where  $X_{ij}$  is **execution time** overhead at node  $j$  for sending a message to node  $i$ ,  $L_{ij}$  is the **network latency** from  $j$  to  $i$ , and  $E_{ij}$  is the **clock synchronization error**. The three latter quantities are indistinguishable and always appear summed together, so there is no point in breaking apparent latency down in this way. Moreover, these latter three quantities would have to be measured with some physical clock, and it is not clear what clock to use. The apparent latency requires no problematic measurement since it explicitly refers to local clocks and tags.



# CAL Theorem for Strongly Consistent Systems

The unavailability at process  $i$  for a strongly consistent system is

$$\bar{A}_i = \max_{j \in N} \max(\mathcal{L}_{ij}, O_i) = \max(O_i, \max_{j \in N} \mathcal{L}_{ij}). \quad (6)$$

When network latency, clock synchronization error, or execution time increase, the apparent latency increases, and so does the unavailability. “Network partitioning” means that  $\mathcal{L}_{ij}$  diverges and the system becomes unavailable at process  $i$ ,  $\bar{A}_i = \infty$ . When enforcing strong consistency, therefore, network partitioning implies unavailability, as expected from the CAP theorem.



# The CAL Theorem for Arbitrary Consistency

## Theorem

*Given a trace, the unavailability at process  $i$  is, in the worst case,*

$$\bar{A}_i = \max \left( O_i, \max_{j \in N} (\mathcal{L}_{ij} - \bar{C}_{ij}) \right), \quad (7)$$

*where  $O_i$  is the processing offset,  $\mathcal{L}_{ij}$  is apparent latency (which includes  $O_j$ ), and  $\bar{C}_{ij}$  is the inconsistency.*



# Max-Plus Algebra

Operators:

$$a \oplus b = \max(a, b)$$

$$a \otimes b = a + B - \text{side}$$

Algebra properties:

- associativity:

$$(a \oplus b) \oplus c = a \oplus (b \oplus c)$$

$$(a \otimes b) \otimes c = a \otimes (b \otimes c)$$

- commutativity:

$$a \oplus b = b \oplus a$$

(note:  $\otimes$  is not commutative for matrix ops)

- distributivity:

$$(a \oplus b) \otimes c = (a \otimes c) \oplus (b \otimes c)$$



# Connection Matrix

Let  $N$  be the number of nodes, and define an  $N \times N$  matrix  $\Gamma$  such that its elements are given by

$$\Gamma_{ij} = \mathcal{L}_{ij} - \bar{C}_{ij} - O_j. \quad (8)$$

That is, the  $i, j$ -th entry in the matrix is an assumed bound on  $X_{ij} + L_{ij} + E_{ij}$  (execution time, network latency, and clock synchronization error), adjusted downwards by the specified tolerance  $\bar{C}_{ij}$  for inconsistency.



# The CAL Theorem in Max-Plus

Let  $\mathbf{A}$  be a column vector with elements equal to the unavailabilities  $\bar{A}_i$ , and  $\mathbf{o}$  be a column vector with elements equal to the processing offsets  $O_i$ . Then the CAL theorem can be written as

$$\mathbf{A} = \mathbf{o} \oplus \Gamma \mathbf{o}, \quad (9)$$

where the matrix multiplication is in the max-plus algebra. This can be rewritten as

$$\mathbf{A} = (\mathbf{I} \oplus \Gamma) \mathbf{o}, \quad (10)$$

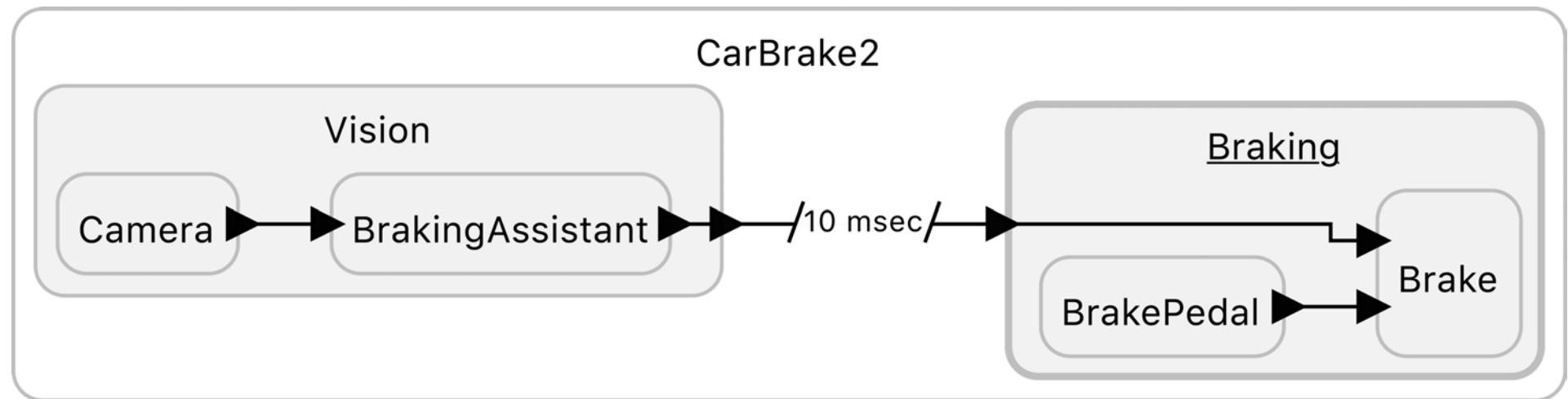
where  $\mathbf{I}$  is the identity matrix in max-plus, which has zeros along the diagonal and  $-\infty$  everywhere else.



# Lingua Franca enables explicit tradeoffs between Consistency and Availability

- Specify your availability requirement (**deadline**)
- State your requirements of the network (**latency**)
- Specify a tolerance for inconsistency (**logical delay**)

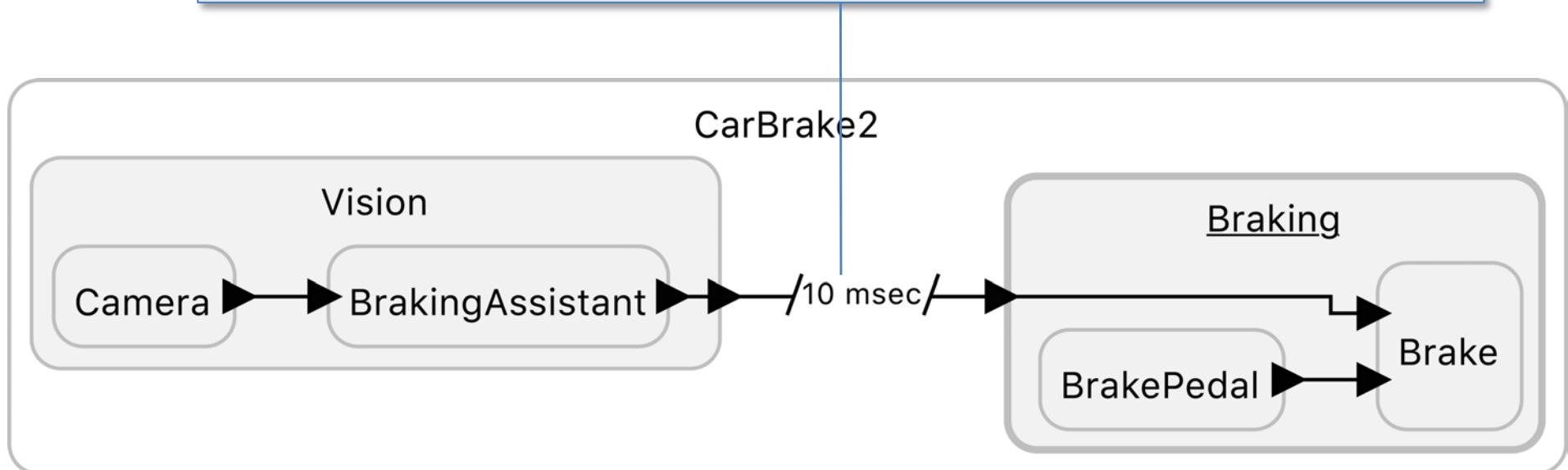
E.g., 10 msec tolerance for logical delay implies that if network latencies are less than 10 msec, availability is instantaneous:





# What if the Execution Time Bound is Violated?

If communication latency exceeds 10 msec...



choice

Sacrifice availability

Sacrifice consistency



# Fault Handling in Lingua Franca

Led by Soroush Bateni (UT Dallas, UC Berkeley)



## Sacrifice availability

### Centralized coordination:

- Use centralized coordination
- Provide deadline miss handlers.

## Sacrifice consistency

### Decentralized coordination:

- Use decentralized coordination.
- Provide safe-to-process violation handlers.