

Introducing Signals and Systems

The Berkeley Approach



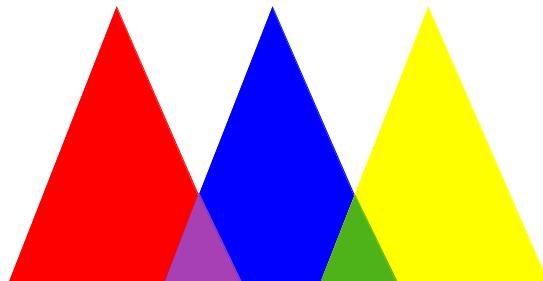
Edward A. Lee
Pravin Varaiya
UC Berkeley

A computer without
networking, audio,
video, or real-time
services.

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Starting Point

EE CE CS



Electronics
Optics
E&M
Plasmas
...

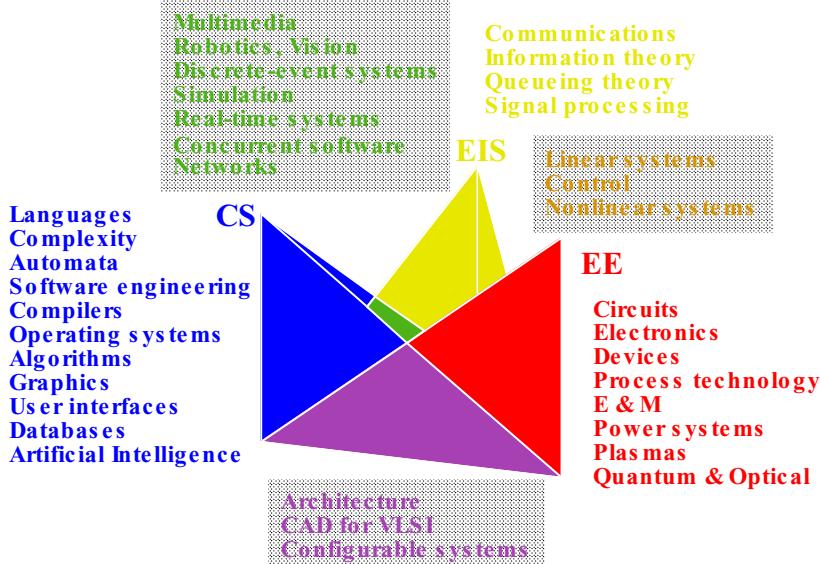
Architecture
CAD
Systems
...

Languages
AI
Theory of Computation
...

But the juncture of EE and CS is not just hardware.
It is also mathematical modeling and system design.

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Intellectual Grouping of EE, CE, CS



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Six Intellectual Groupings

- Blue: Computer Science
- Green: Computer Information Systems
- Yellow: Electronic Information Systems
- Orange: Electronic Systems
- Red: Electronics
- Purple: Computer hardware

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New Introductory Course Needed

CS
(CS 61A)

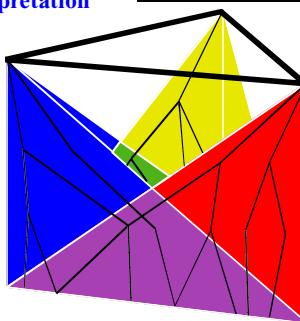
The Structure and Interpretation
of Computer Programs

EIS
(EECS 20)

Structure & Interpretation of
Signals & Systems

EE
(EE 40)

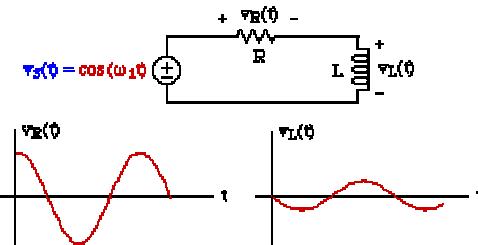
Introduction to
Electronic Circuits



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The Roots of Signals and Systems

- Circuit theory
- Continuous-time
- Calculus-based



Major models

- Frequency domain
- Linear time-invariant systems
- Feedback

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Changes in Content

- **Signal**
 - used to be: voltage over time
 - now may be: discrete messages
- **State**
 - used to be: the variables of a differential equation
 - now may be: a process continuation in a transition system
- **System**
 - used to be: linear time invariant transfer function
 - now may be: Turing-complete computation engine



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Changes in Intellectual Scaffolding

- **Fundamental limits**
 - used to be: thermal noise, the speed of light
 - now may be: chaos, computability, complexity
- **Mathematics**
 - used to be: calculus, differential equations
 - now may be: mathematical logic, topology, set theory, partial orders
- **Building blocks**
 - used to be: capacitors, resistors, transistors, gates, op amps
 - now may be: microcontrollers, DSP cores, algorithms, software components

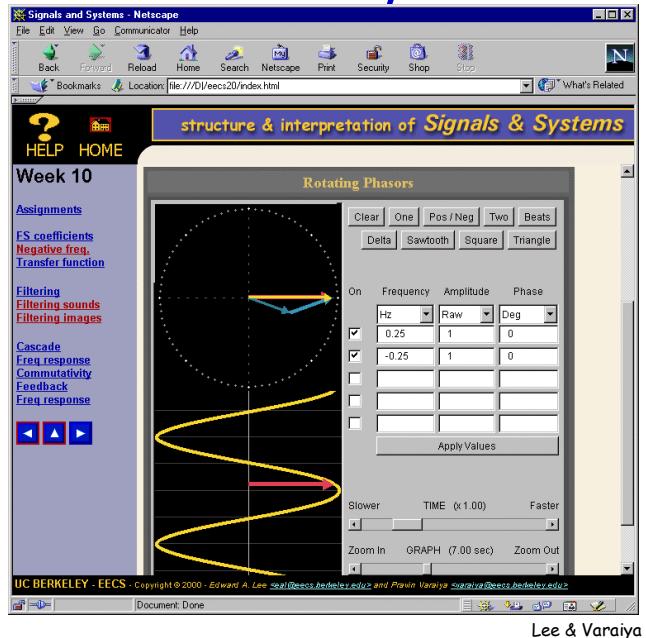


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Action at Berkeley

Berkeley has instituted a new sophomore course that addresses mathematical modeling of signals and systems from a very broad, high-level perspective.

The web page at the right contains an applet that illustrates complex exponentials used in the Fourier series.



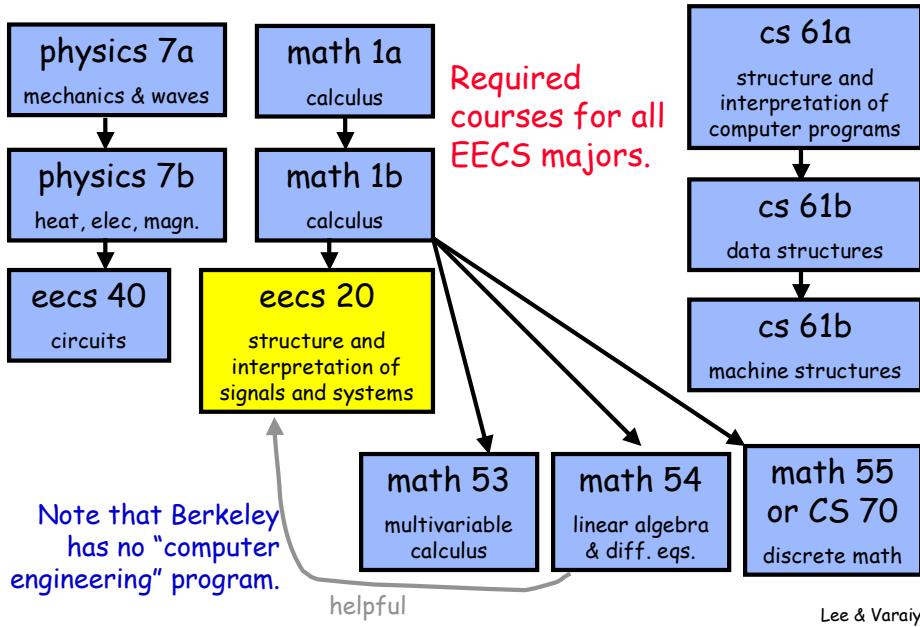
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Themes of the Course

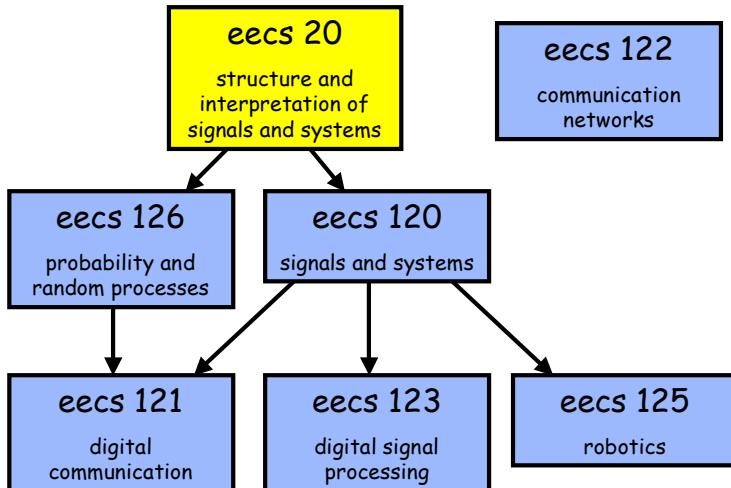
- The connection between *imperative* (Matlab) and *declarative* (Mathematical) descriptions of signals and systems.
- The use of *sets and functions* as a universal language for declarative descriptions of signals and systems.
- State machines and frequency domain analysis as complementary tools for designing and analyzing signals and systems.
- Early and often discussion of applications.

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Role in the EECS Curriculum

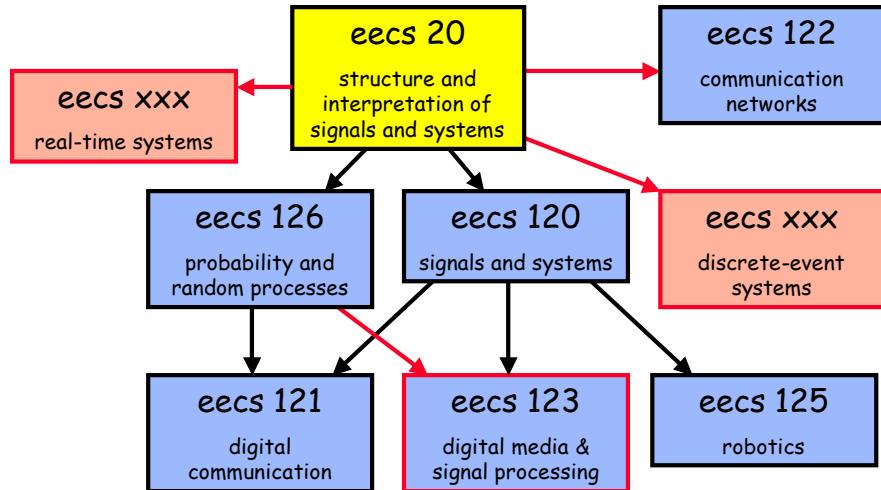


Current Role in EE



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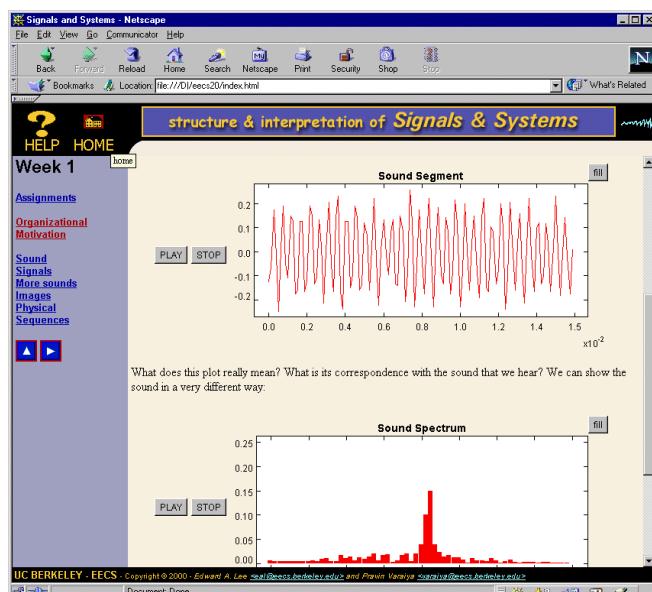
Future Role in EECS (speculative)



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Outline

- 1. Signals ←
- 2. Systems
- 3. State
- 4. Determinism
- 5. Composition
- 6. Linearity
- 7. Freq Domain
- 8. Freq Response
- 9. LTI Systems
- 10. Filtering
- 11. Convolution
- 12. Transforms
- 13. Sampling
- 14. Design
- 15. Examples

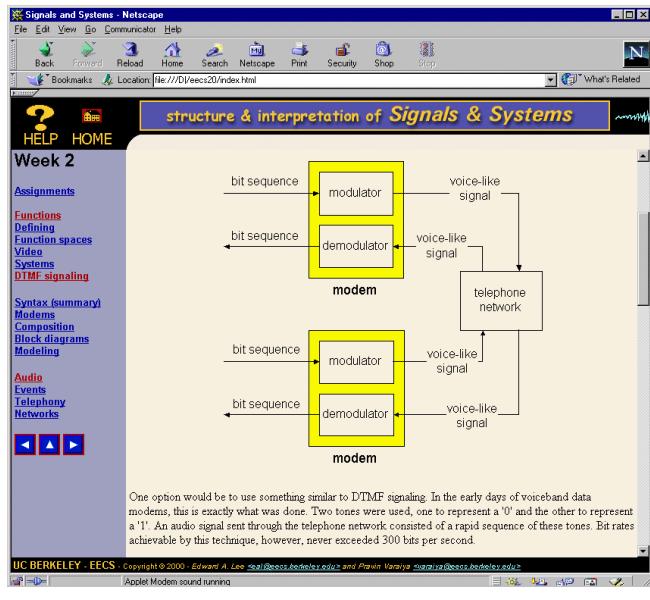


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2. Systems ←



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Notation

- Sets and functions
 - *Sound*: $\text{Reals} \rightarrow \text{Reals}$
 - *Digital Sound*: $\text{Ints} \rightarrow \text{Reals}$
 - *Sampler*: $[\text{Reals} \rightarrow \text{Reals}] \rightarrow [\text{Ints} \rightarrow \text{Reals}]$
- Our notation unifies
 - discrete and continuous time
 - event sequences
 - images and video, digital and analog
 - spatiotemporal models

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Problems with Standard Notation

- The form of the argument defines the domain

- $x(n)$ is discrete-time, $x(t)$ is continuous-time.
- $x(n) = x(nT)?$ Yes, but...

- $X(jw) = X(s)$ when $jw = s$
- $X(e^{jw}) = X(z)$ when $z = e^{jw}$
- $X(e^{jw}) = X(jw)$ when $e^{jw} = jw?$ No.

- $x(n)$ is a function

- $y(n) = x(n) * h(n)$
- $y(n-N) = x(n-N)*h(n-N)?$ No.



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Using the New Notation

- Discrete-time Convolution :

Convolution: $[Ints \rightarrow Reals] \times [Ints \rightarrow Reals]$
 $\rightarrow [Ints \rightarrow Reals]$

- Shorthand:

$$x * y = \text{Convolution}(x, y)$$

- Definition:

$$(x * y)(n) = \sum_{k=-\infty}^{\infty} x(k)y(n-k)$$

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Week 3

[Assignments](#)

[Definition styles](#)

[Defining signals](#)

[Control logic](#)

[State](#)

[Sets & Functions](#)

[Operation](#)

[Stuttering](#)

[Parking meter](#)

[Diagram](#)

[Answering machine](#)

[Table](#)

[Code recognizer](#)

[Equal](#)

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Example - Answering Machine

Verbal description:

- On the third ring, answer the phone, play a greeting, and record a message.
- After recording a message, hang up.
- If a telephone is taken off hook within three rings, do nothing.

State transition diagram:

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Week 4

[Assignments](#)

[Non-determinism](#)

[Example](#)

[Model](#)

[Possible updates](#)

[Relations](#)

[Abstraction](#)

[Equivalence](#)

[Simulation](#)

[Deterministic](#)

[Abstraction](#)

[Non-deterministic](#)

[Behaviors](#)

[Theorem](#)

[Not theorem](#)

[Uses](#)

◀ ▶

Simulation and Bisimulation

Consider deterministic state machine A:

and nondeterministic state machine B:

Consider a game, where each machine starts in its initial state. Then, given an input, A reacts, and B tries to react in such a way as to produce the same output (given the same input). For the above examples, B can always do this, so B is said to **simulate A**. Equivalently, we say that A **simulates B**.

The game can be turned around, where B makes a move and A tries to match it. In the above examples, this is again possible, so A simulates B. A and B are said to be **bisimilar**.

We can track this game by looking at the state responses of the two machines. They each start in their initial

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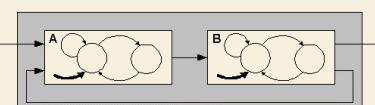
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Feedback Composition

Consider two state machines connected with a feedback loop:



Assumption:

- $Outputs_A \subset Inputs_B$
- $Outputs_{B_0} \subset Inputs_{A_0}$

Definition of the composition:

- $States = States_A \times States_B$
- $Inputs = Inputs_A$
- $Outputs = Outputs_B$

updates function is found by iteration to a fixed point:

- Start with unknowns on the feedback arc

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Infinite State Systems

We study state machine with

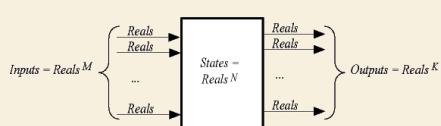
$$States = \mathbb{Reals}^N$$

$$Inputs = \mathbb{Reals}^M$$

$$Outputs = \mathbb{Reals}^K$$

The number of states and the sizes of the input and output alphabets are infinite, which tends to make things more complicated. However, these sets now have arithmetic properties, which opens up a huge new range of modeling and analysis possibilities.

A block diagram of such a system shows the input coming over M input ports and the output delivered over K output ports. (MIMO stands for multi-input, multi-output.)



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Western Musical Scale

Time in seconds

Frequency in Hz

Notes: A, B flat, B, C, C sharp, D, D sharp, E, F, F sharp, G, A flat, A, Sum.

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Discrete Fourier series examples

The following applet illustrates how, unlike the continuous Fourier series, the discrete Fourier series converges to an exact representation of a periodic waveform in a finite number of steps.

Time Domain

Frequency Domain

Square wave include harmonics:

- all
- none
- 0
- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8
- 9
- 10
- 11
- 12
- 13
- 14
- 15
- 16

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Why LTI is desirable for audio signals

The following applets selectively introduce a component to an audio signal that is generated by a nonlinear system. The slider on the left controls a constant a in the expression $x(t) + ax^2(t)$

When a is greater than zero, this introduces what audio engineers call second-harmonic distortion. The slider on the right controls a constant b in the expression $x(t) + bx^3(t)$

Audio engineers call this effect third-harmonic distortion. Note that moving the sliders only has an effect the next time you play the sound.

PLAY STOP

Sound Spectrum

Sound Spectrum

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Filtering Images

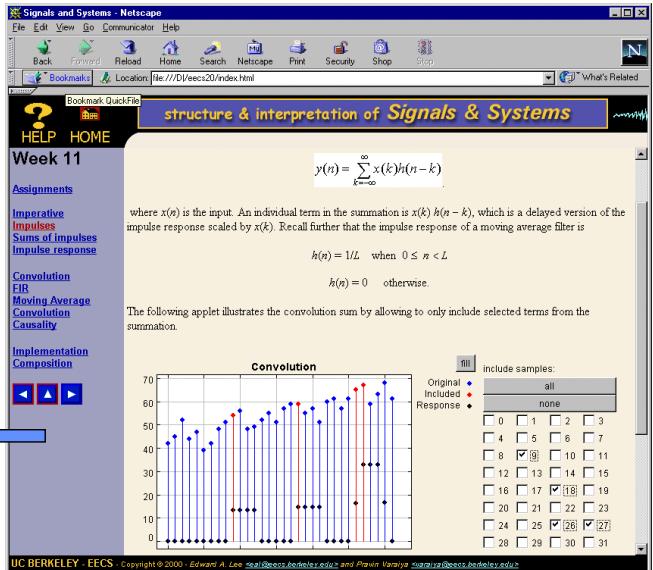
Below is an original image, the same image after lowpass filtering, and the same image after highpass filtering.

Lowpass filtering in images has the effect of blurring the image. In fact, optical lenses that blur an image can be modeled as LTI lowpass filters. Highpass filtering has a rather strange effect on an image. Notice that solid blocks of color are replaced with a middle gray. This is because solid blocks of color correspond to low spatial frequencies, which are removed by highpass filtering. Edges correspond to high spatial frequencies, so the regions near the edges are relatively enhanced. Patterns, such as that in Helen's suit above, are largely preserved, since they consist primarily of high spatial frequencies.

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where $x(n)$ is the input. An individual term in the summation is $x(k) h(n - k)$, which is a delayed version of the impulse response scaled by $x(k)$. Recall further that the impulse response of a moving average filter is

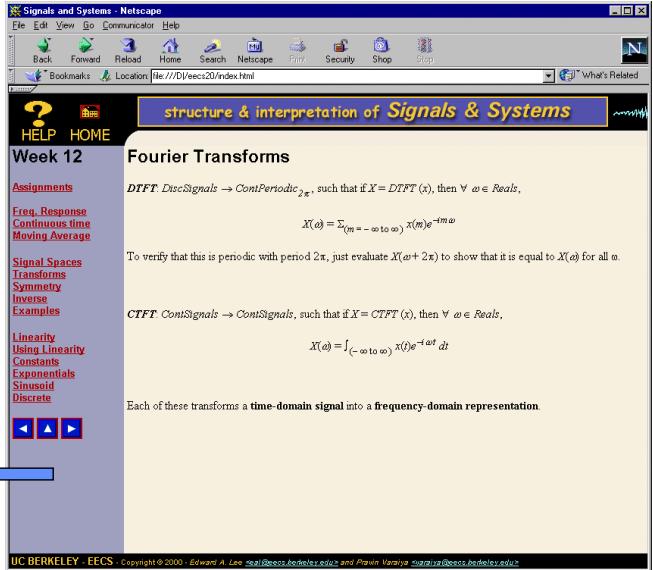
$$h(n) = \begin{cases} 1/L & \text{when } 0 \leq n < L \\ 0 & \text{otherwise.} \end{cases}$$

The following applet illustrates the convolution sum by allowing to only include selected terms from the summation.

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Fourier Transforms

DTFT: $\text{DiscSignals} \rightarrow \text{ContPeriodic}_{2\pi}$, such that if $X = \text{DTFT}(x)$, then $\forall \omega \in \text{Reals}$,

$$X(\omega) = \sum_{m=-\infty}^{\infty} x(m)e^{-im\omega}$$

To verify that this is periodic with period 2π , just evaluate $X(\omega + 2\pi)$ to show that it is equal to $X(\omega)$ for all ω .

CTFT: $\text{ContSignals} \rightarrow \text{ContSignals}$, such that if $X = \text{CTFT}(x)$, then $\forall \omega \in \text{Reals}$,

$$X(\omega) = \int_{(-\infty, \infty)} x(t)e^{-i\omega t} dt$$

Each of these transforms a time-domain signal into a frequency-domain representation.

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Outline

The screenshot shows a Netscape browser window with the title "Signals and Systems - Netscape". The main content area displays an "Outline" with the following items:

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A blue arrow points from the word "Examples" to the left margin of the list.

The right side of the browser window contains a diagram titled "Parametric Speech Synthesis". The diagram illustrates the process of speech synthesis within a human head. Labels include "IDEA", "SPEECH MUSCLES", "ARTICULATORS", "VOCAL TRACT", "LARYNX", "AMPLIFICATION", "SELECTIVE", "TIME-VARIANT", and "NOISE". A legend at the bottom defines these terms: "IDEA" (represented by a speech bubble), "SPEECH MUSCLES" (represented by a hand), "ARTICULATORS" (represented by a mouth), "VOCAL TRACT" (represented by a head profile), "LARYNX" (represented by a larynx icon), "AMPLIFICATION" (represented by a speaker icon), "SELECTIVE" (represented by a filter icon), "TIME-VARIANT" (represented by a waveform icon), and "NOISE" (represented by a cloud icon).

At the bottom of the browser window, it says "UC BERKELEY - EECS Copyright © 2000 Edward A. Lee eelee@cs.berkeley.edu and Pravin Varaiya varaiya@cs.berkeley.edu".

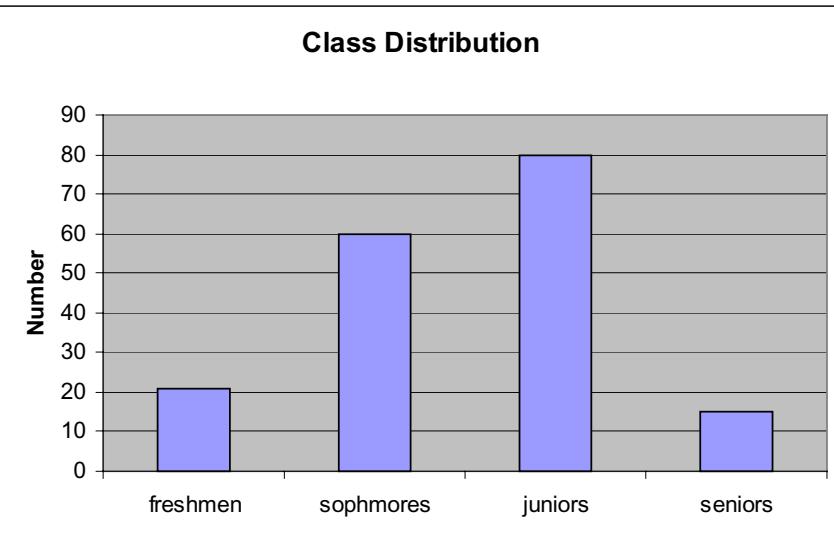
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Analysis of Spring 2000 Offering

- Class standing had little effect on performance.
- On average, the GPA of students was neither lowered nor raised by this class.
- Students who attend lecture do better than those that don't.
- Taking at least one of Math 53, 54, or 55 helps by about $\frac{1}{2}$ grade level.
- Taking Math 54 (linear algebra & differential eqs.) helps by about 1 grade level (e.g. B to B+).
- Computing classes have little effect on performance.

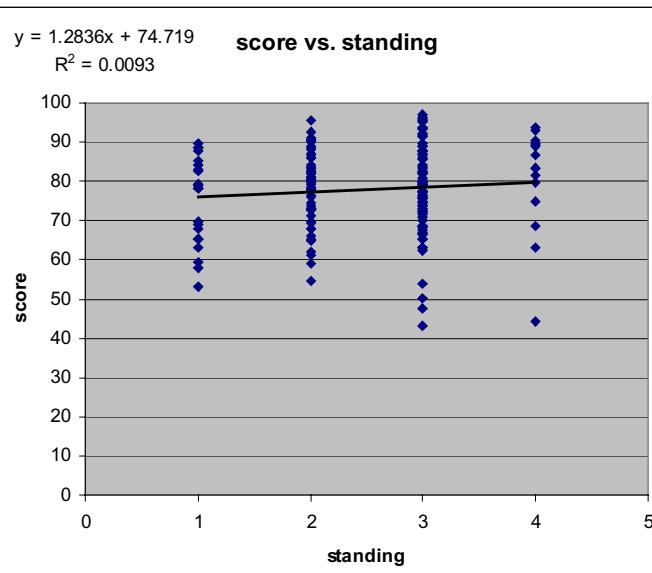
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Distribution by Class Standing



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Effect of Class Standing



80 and above:
A's
63 and above: B's
62 and below: C's

176 of the 227
students
responded (the
better ones).

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Effect of Showing Up

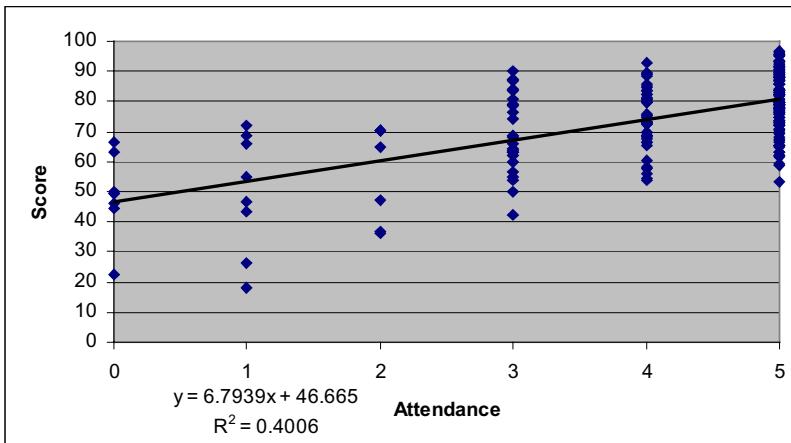
Students who answered the survey were those that showed up for the second to last lab.

- The mean for those who responded was 78, vs. 65 for those who did not respond (two grades, e.g. B to A-).
- The standard deviation is much higher for those who did not respond.
- A t-test on the means shows the data are statistically very significant.

We conclude that the respondents to the survey do not represent a random sample from the class, but rather represent the diligent subset.

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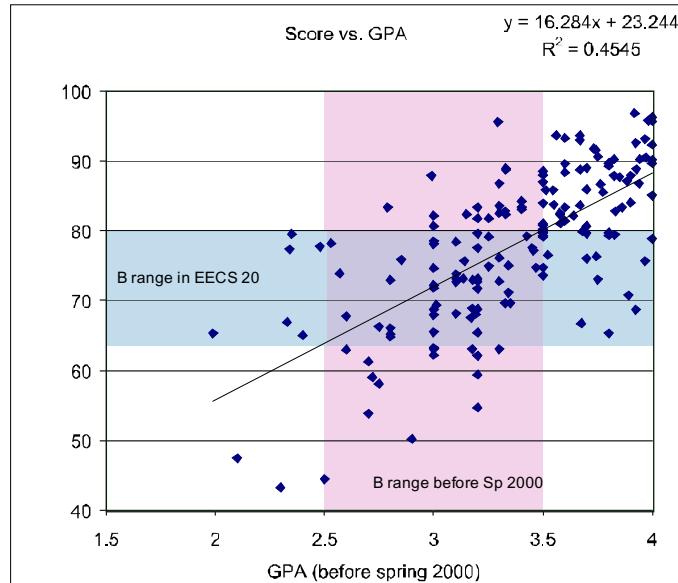
Attendance in Class vs. Score



Attendance is measured by presence for pop quizzes, of which there were five.

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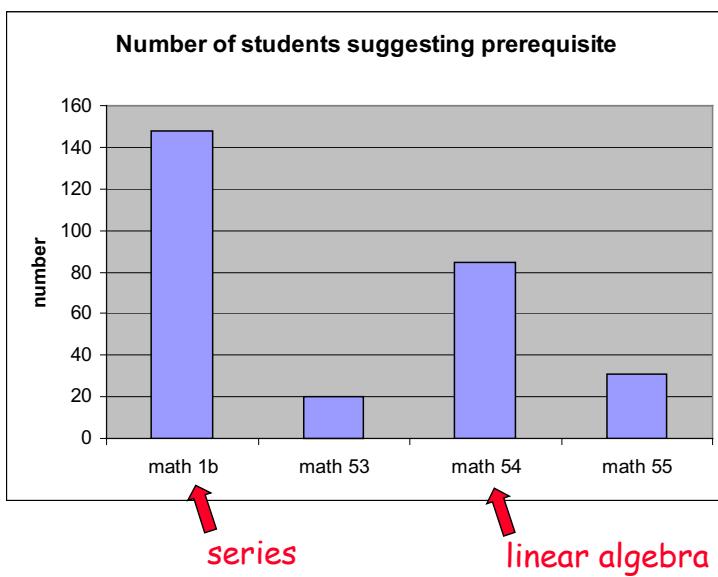
Effect on GPA



On average, students' GPA was not affected by this class.

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Student Opinion on Prerequisites



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Differences from Tradition

- No circuits
- More discrete-time, some continuous-time
- Broader than LTI systems
- Unifying sets-and-functions framework
- Emphasis on applications
- Many applets and demos
- Tightly integrated software lab

Text draft (Lee & Varaiya) and website available.

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Bottom-Up or Top-Down?



Bottom-up:

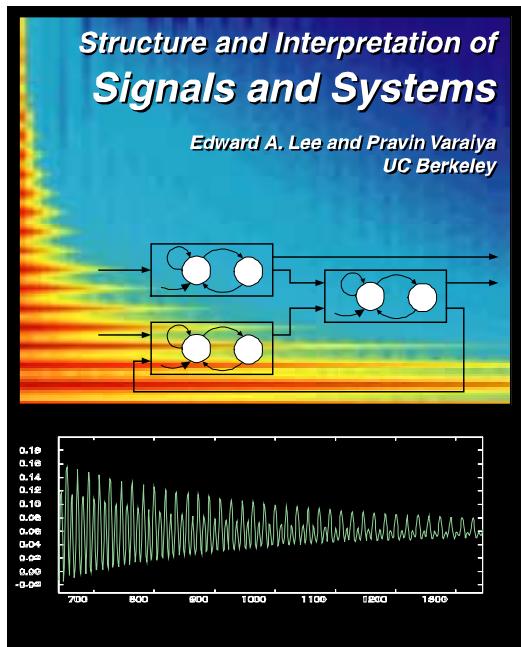
- foundations first
- derive the applications



Top-down:

- applications first
- derive the foundations

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Textbook

Draft available on
the web.

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