Physics 421 / PCSE 503
Lecture 18

Basic Fourier Analysis

① Brush vs. Wikipedia vs
scipy vs. .....

→ minor differences ...

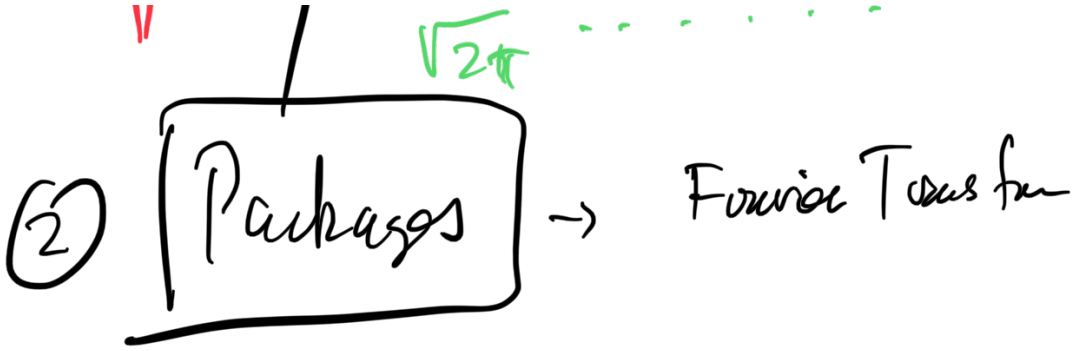
$$\int_{-P}^{P} \text{ vs. } \int_{0}^{2P} \text{ vs. } \cos\left(\frac{\pi n t}{T}\right)$$

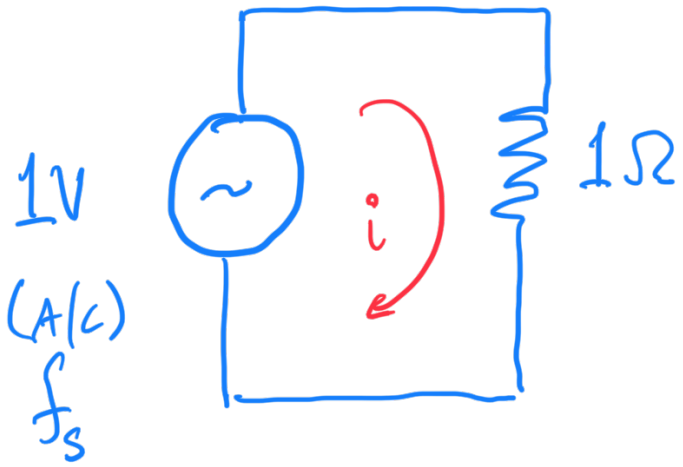
$$\text{vs. } \cos\left(\frac{2\pi n t}{T}\right)$$

Weekend

$$2, \pi, \frac{1}{2}, \frac{1}{\pi}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{\pi}}$$

$$1$$



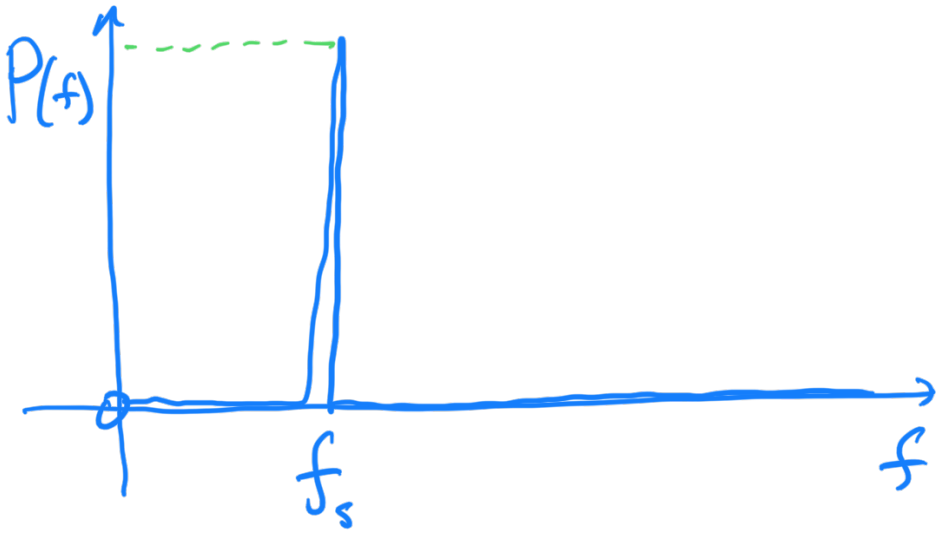


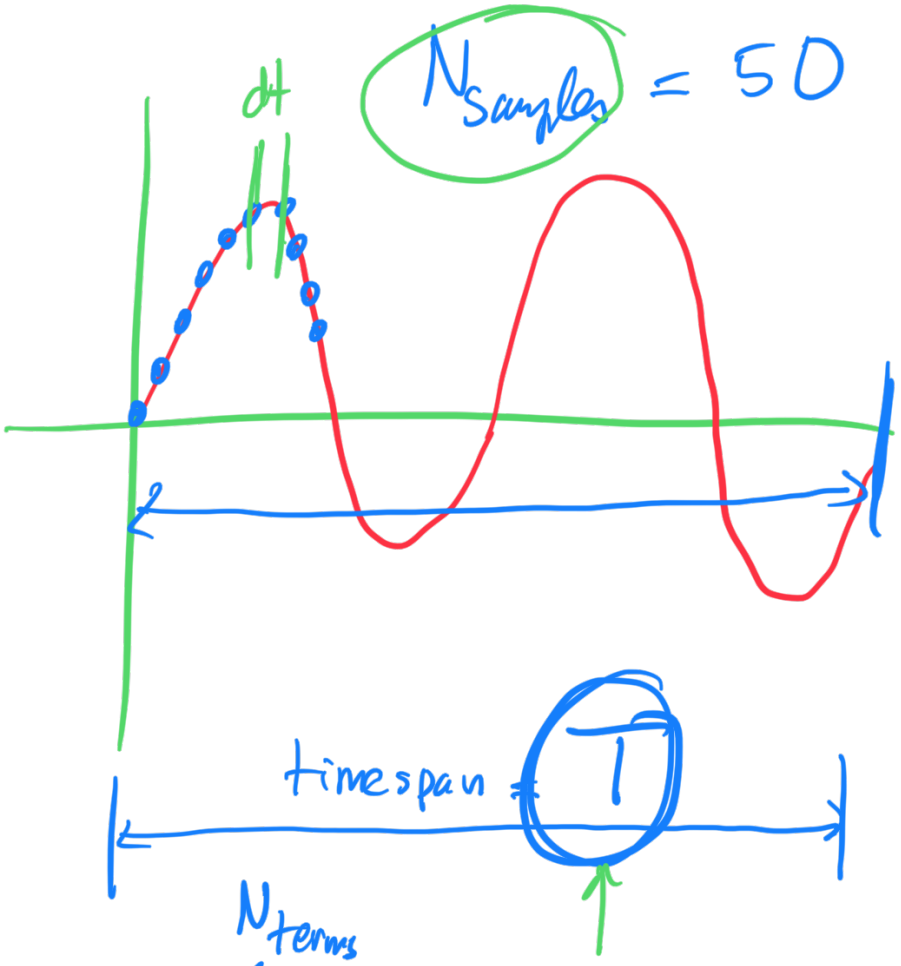
Power

$$V = \sin(\omega t)$$

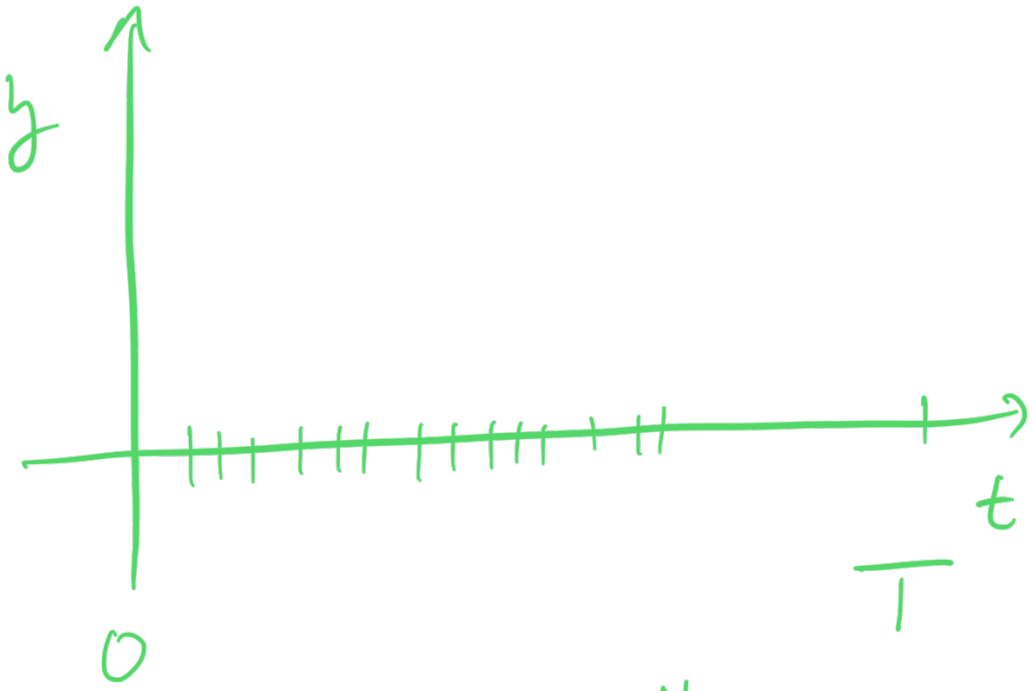
$$P = \frac{V^2}{R} = \sin^2(\omega t)$$

$$\bar{P} = \frac{1}{2} W$$



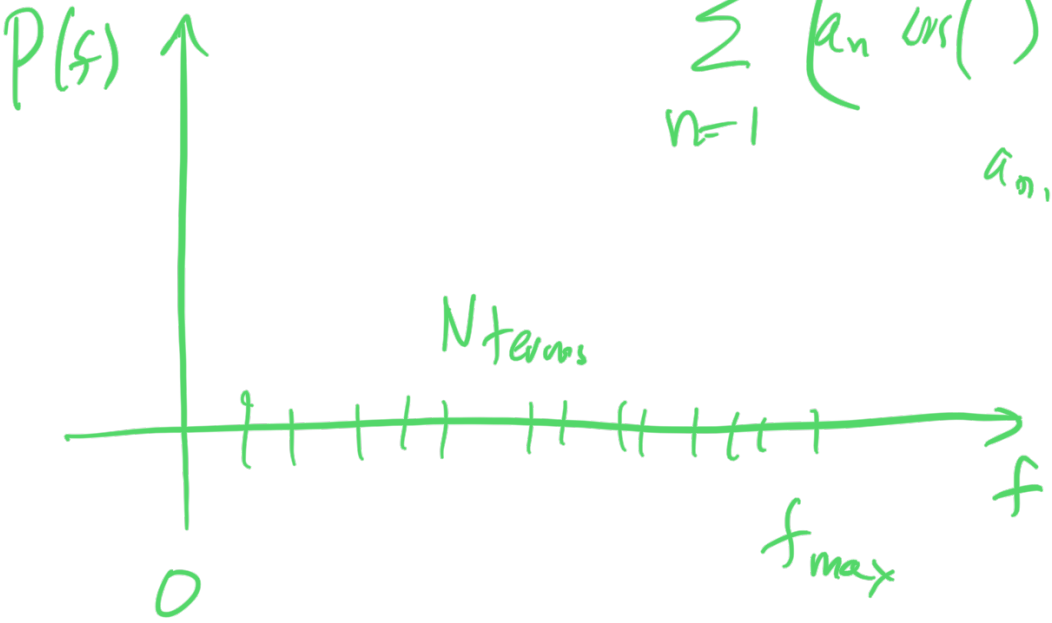


$$\sum_{n=1}$$



$$\sum_{n=1}^{N_{terms}} \left(a_n \cos(\quad) + b_n \sin(\quad \right)$$

a_n, b_n, P_n



$$y(t) = 1 \sin(\omega t)$$

$2\pi f_s$

0 0

$$y(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{2\pi n t}{T}\right) + b_n \sin\left(\frac{2\pi n t}{T}\right) \right]$$

$$2\pi f_s = \frac{2\pi n}{T} \qquad n = T \cdot f_s$$

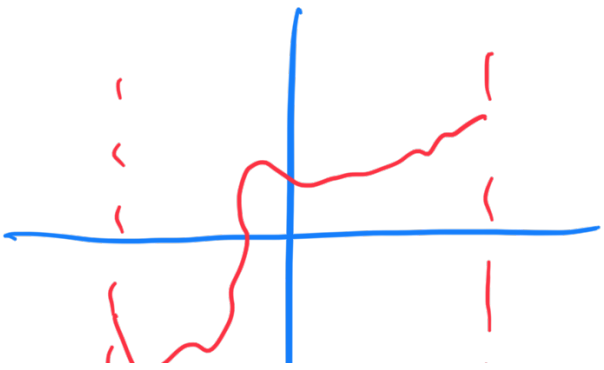
$$b_0 = 0$$

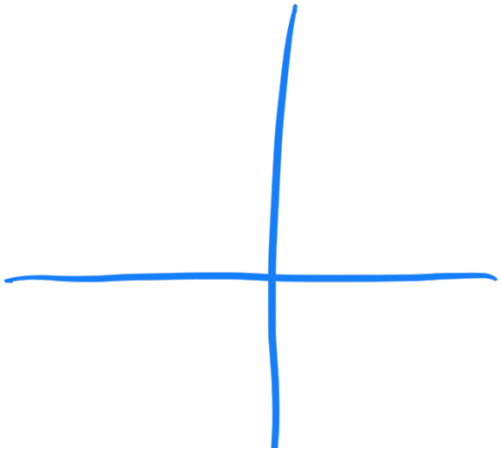
$$b_1 = 0$$

$$\boxed{b_2 = 1}$$

$$b_3 = 0$$

$$\vdots$$





$-p$ $\quad$ p $\quad$ p

LAST CLASS (Brush)

THIS CLASS (With pedin.

$$\frac{1}{p}\int_{-p}^{p}(\;)\,dt \qquad \frac{2}{T}\int_{0}^{T}$$

$\frac{1}{2}$ (intend) $\rightarrow$ $\frac{2}{\text{(intend)}}\int_{-p}^{p}$ $\longleftrightarrow$ same. $\frac{2}{\text{(intend)}}\int_{0}^{T}$

$$a_0 = \frac{2}{T}\int_{0}^{T}\left(f(t)\right)dt$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{2\pi n t}{T}\right) + b_n \sin\left(\frac{2\pi n t}{T}\right)\right.$$

$$a_0 = \frac{2}{T}\int_0^T \left[\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{2\pi n t}{T}\right) + (b_n)\sin\left(\frac{2\pi n t}{T}\right) \right) \right] dt$$

$$= \frac{2}{T}\int_0^T \frac{a_0}{2}\, dt$$

outside loop

inside loop

$$+ \frac{2}{T}\sum_{n=1}^{\infty} a_n \int_0^T \cos\left(\frac{2\pi n t}{T}\right) dt$$

$$+ \frac{2}{T}\sum_{n=1}^{\infty} b_n \int_0^T \sin\left(\frac{2\pi n t}{T}\right) dt$$

y t Real(n) → a_n

y t Imag(n) → b_n

$$a_0 + \sum_{n=1}^{\infty} a_n \cos + b_n \sin$$

$$y(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} c_n e^{i\omega t}$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} \cos(\omega t) + i \sin(\omega t)$$

Real Imag

?

$$P_n = \sqrt{a_n^2 + b_n^2}$$

$a_0 = 0$

$a_n = 0$

$b_2 = 1$

$b_{49} = -1$