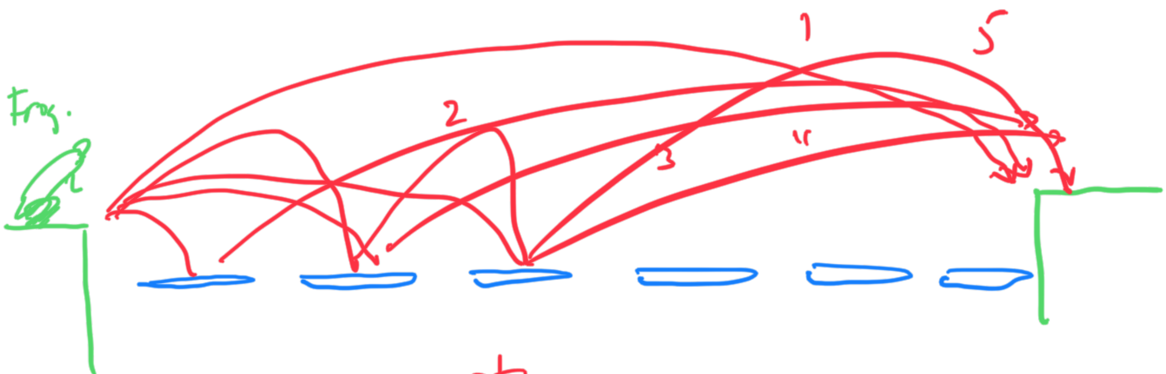
Frog and Lily Pad Problem

Frog & Lilipad Problem.



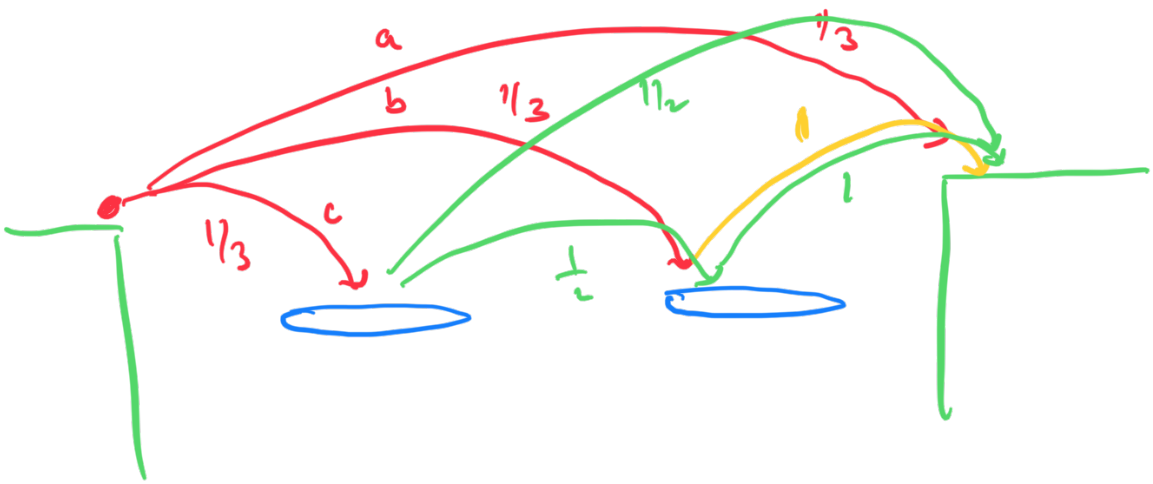
etc.

Question: What is the average # of jumps, if at each opportunity, the frog can jump anywhere between 1 and the remaining distance?

Point 1: An analytical solution seems impossible ... so, let's simulate it!

Let's start with a simple example,

where there are only ≤ 111 paths.



Options

a → 1 jump → $P_a = \frac{1}{3}$

b → 2 jumps → $P_b = \frac{1}{3}$

c → 2 jumps → $P_{c2} = \frac{1}{3}\left(\frac{1}{2}\right.$

3 jumps → $P_{c2} = \frac{1}{3}\left(\right.$

$$\bar{n} = \frac{1}{3}(1) + \frac{1}{3}(2) + \frac{1}{6}(2)$$

$$\frac{1}{3}(1) + \frac{1}{2}(2) + \frac{1}{6}(3)$$

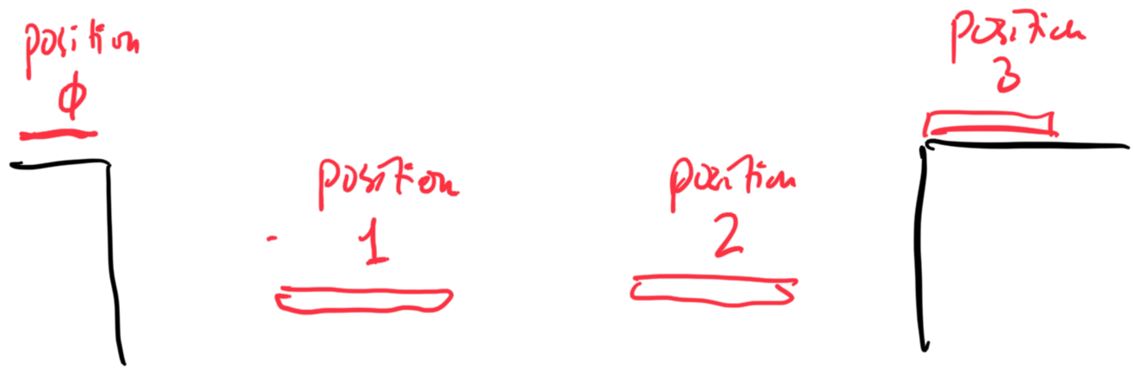
$$= \underbrace{\frac{1}{3} + \frac{2}{3}}_{1} + \underbrace{\frac{1}{3} + \frac{1}{2}}_{5/6}$$

$$\boxed{\bar{n} = \frac{11}{6}}$$

5:31 p
pizza
ordere

Let's simulate this first, and verify that we can reproduce this number!

[EJB Good Simulation Practice Rule:



\# of lilipads = npad (= 2 in this case)

position after first jump = $(1, 2, \ldots, npad+1)$

position after any jump = $(start+1, start+2, \ldots, np$

end = npad + 1

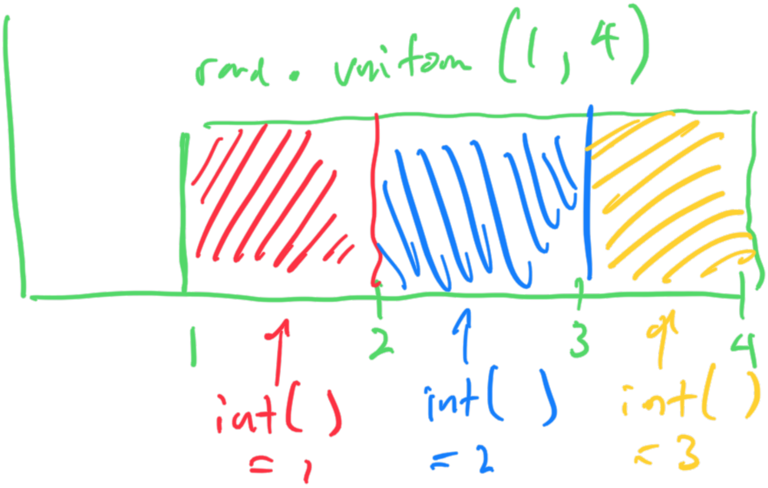
$$\boxed{\text{jump} = \text{int}\big(\text{random.uniform}(1, (\text{end}-\text{start})}$$

let's test this:

$\text{start} = 0$

$\text{end} = 3$

$\text{end} - \text{start} + 1$
$= 3 - 0 + 1$
$= 4$



Theory: As I said, it is not immediately obvious!

S

n paid Σ 1 step 2 step 3 step 4 step

0	$\boxed{1}$	1				
1	$\boxed{2}$	1	1			
2	$\boxed{4}$	1	2	1		
3	$\boxed{8}$	1	3	3	1	
4	$\boxed{16}$	1	4	6	4	

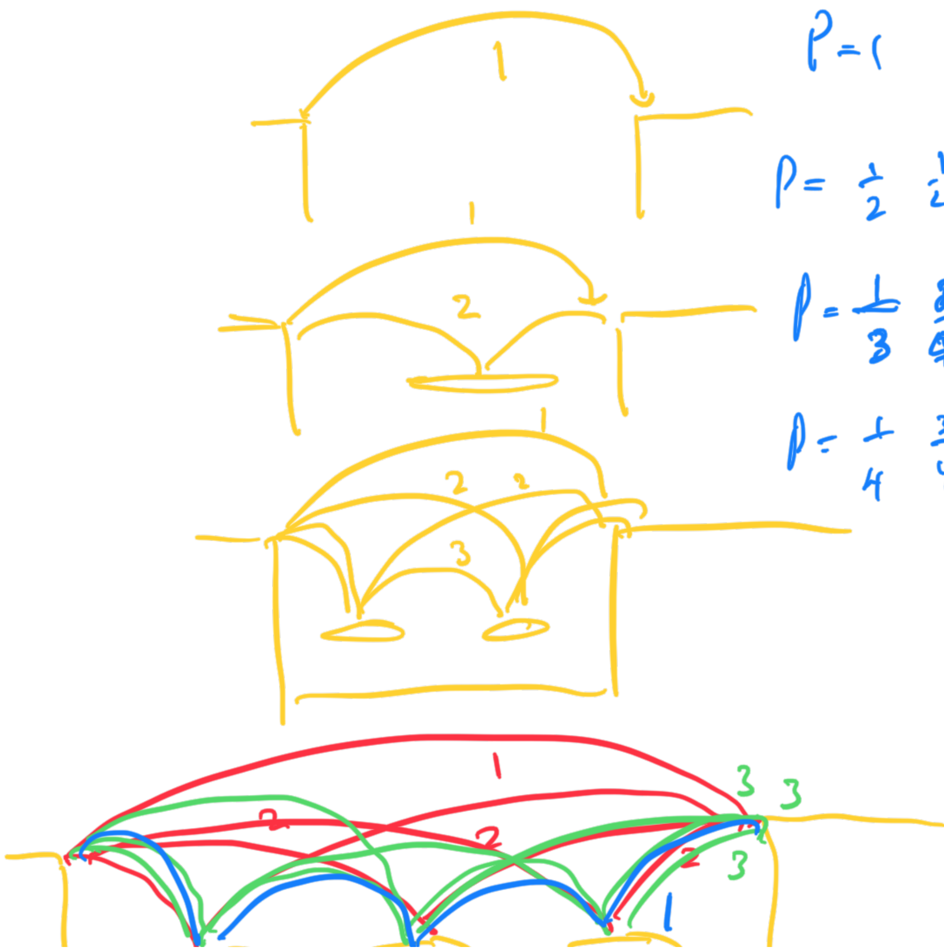
⋮ !

0

1

2

3

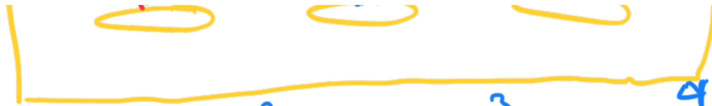


$p = 1$

$p = \frac{1}{2}\ \frac{1}{2}$

$p = \frac{1}{3}\ \frac{2}{4}\ \frac{1}{6}$

$p = \frac{1}{4}\ \frac{3}{4}\ \frac{3}{4}\ \frac{1}{4}$



	1	2	3	4
x	1	3	3	1
	always 1	always n	always n	always 1
$P(x)$	$\frac{1}{n}$	$\frac{3}{n}$	$\frac{3}{n}$	$\frac{1}{n}$

These are the coefficients of $(a+b)^n$

$$= a^n + n a^{n-1} b + \dots + n a b^{n-1}$$

(Pascal's triangle.

The coefficients are given by the

$\boxed{\text{binomial distribution.}}$ ← physics 341

$$P_B(x) = \binom{N}{x} p^x (1-p)^{N-x}$$

$$E[x] = \int x P(x)\, dx$$

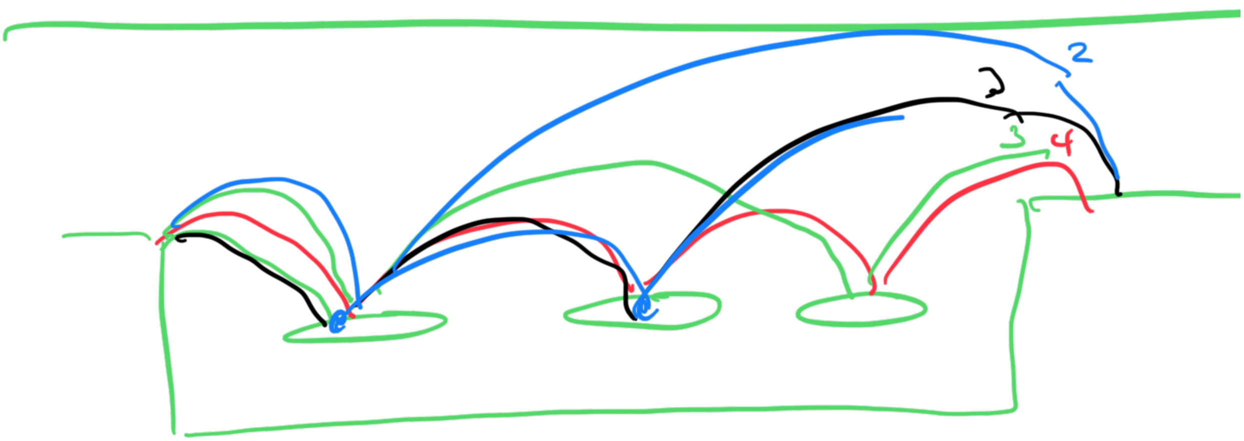
$$\int P(\cancel{x}) \, d\cancel{x} \qquad 1$$

Our coefficients are going to be given by:

$$x = k+1$$

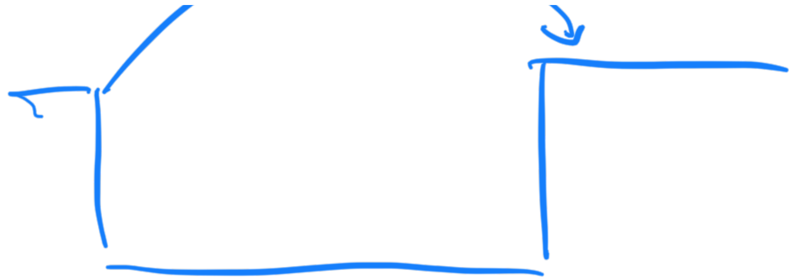
$$e(k) = (k+1)\, P_B(k, N_{pad}, \tfrac{1}{2})\, P_{Binomial}(k, N_{pad}, \tfrac{1}{2})$$

for $k = 0, \dots, N_{pad}$



	eval	eval r
0	1	1
1	$\frac{1}{2}$	$\frac{1}{2}$
2		

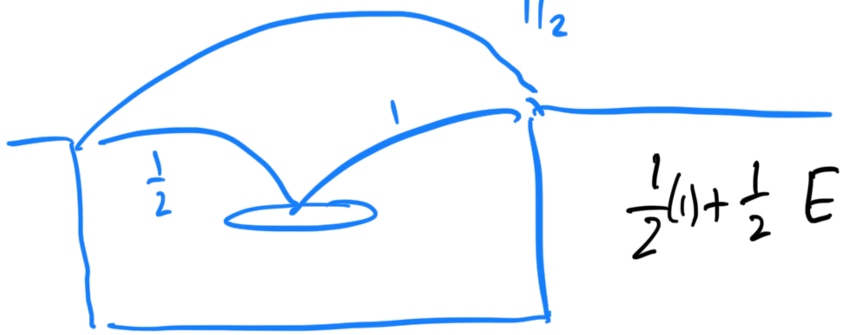
$n = 0$



Jumps	P
1	1

$1(1) = 1$

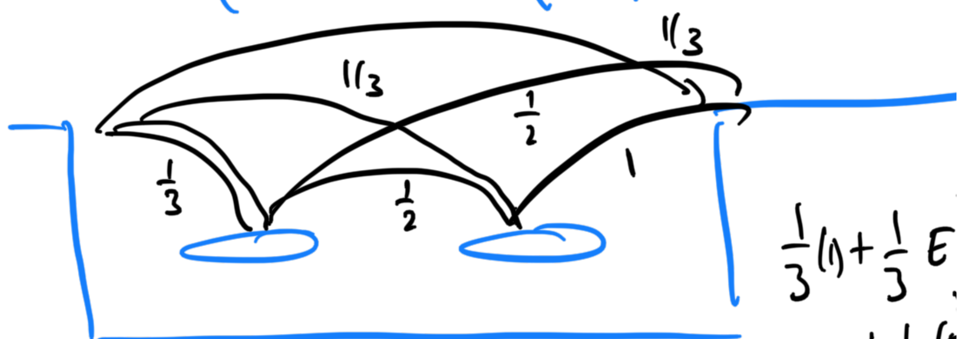
$n = 1$



$\frac{1}{2}(1) + \frac{1}{2} E(0$

$$1\left(\tfrac{1}{2}\right) + 2\left(\tfrac{1}{2}\right) = 1.5$$

$n=2$

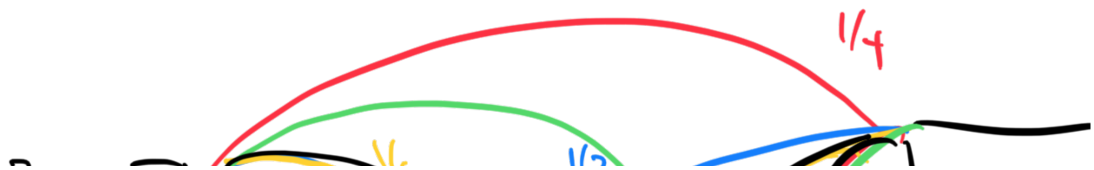


$\frac{1}{3}(1) + \frac{1}{3} E$

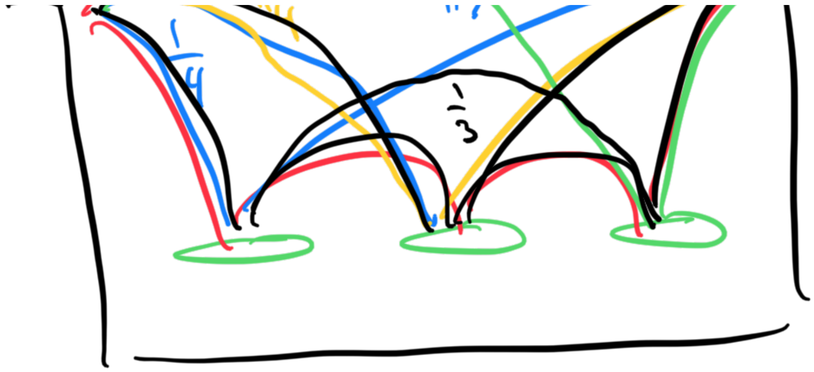
$+ \frac{1}{3}(E$

$$1\left(\tfrac{1}{3}\right) + 2\left(\tfrac{1}{3}\cdot\tfrac{1}{2} + \tfrac{1}{3}\cdot 1\right)$$

$$+ 3\left(\tfrac{1}{3}\right)$$



$n = 3$



$\frac{1}{4} + \frac{1}{4}E($

$+ \frac{1}{4}(E($

$+ \frac{1}{4}($

$$1\left(\frac{1}{4}\right) + 2\left(\frac{1}{4}\cdot\frac{1}{3} + \frac{1}{4}\frac{1}{2} + \frac{1}{4}(1)\right) + 4($$

$$+3\left(\frac{1}{4}\cdot\frac{1}{3}\cdot\frac{1}{2}\right.$$

$$+\frac{1}{4}\cdot\frac{1}{2}\cdot 1 + \frac{1}{4}\cdot\frac{1}{3}\cdot$$

$$\frac{1}{4}(1) + \frac{1}{4}\left(\frac{11}{6}\right) + \frac{1}{4}\left(\frac{6}{6}\right) + \frac{1}{4}(1$$

Recursion

$$E(0) = 1$$

$$E(1) = \frac{1}{2} + \frac{1}{2}E(0)$$

$$E(2) = \frac{1}{3} + \frac{1}{3}\boxed{E(2)}$$

...