Physics 421 / PCSE 503
Lecture 7

Kater's Pendulum →

A case study in data analysis

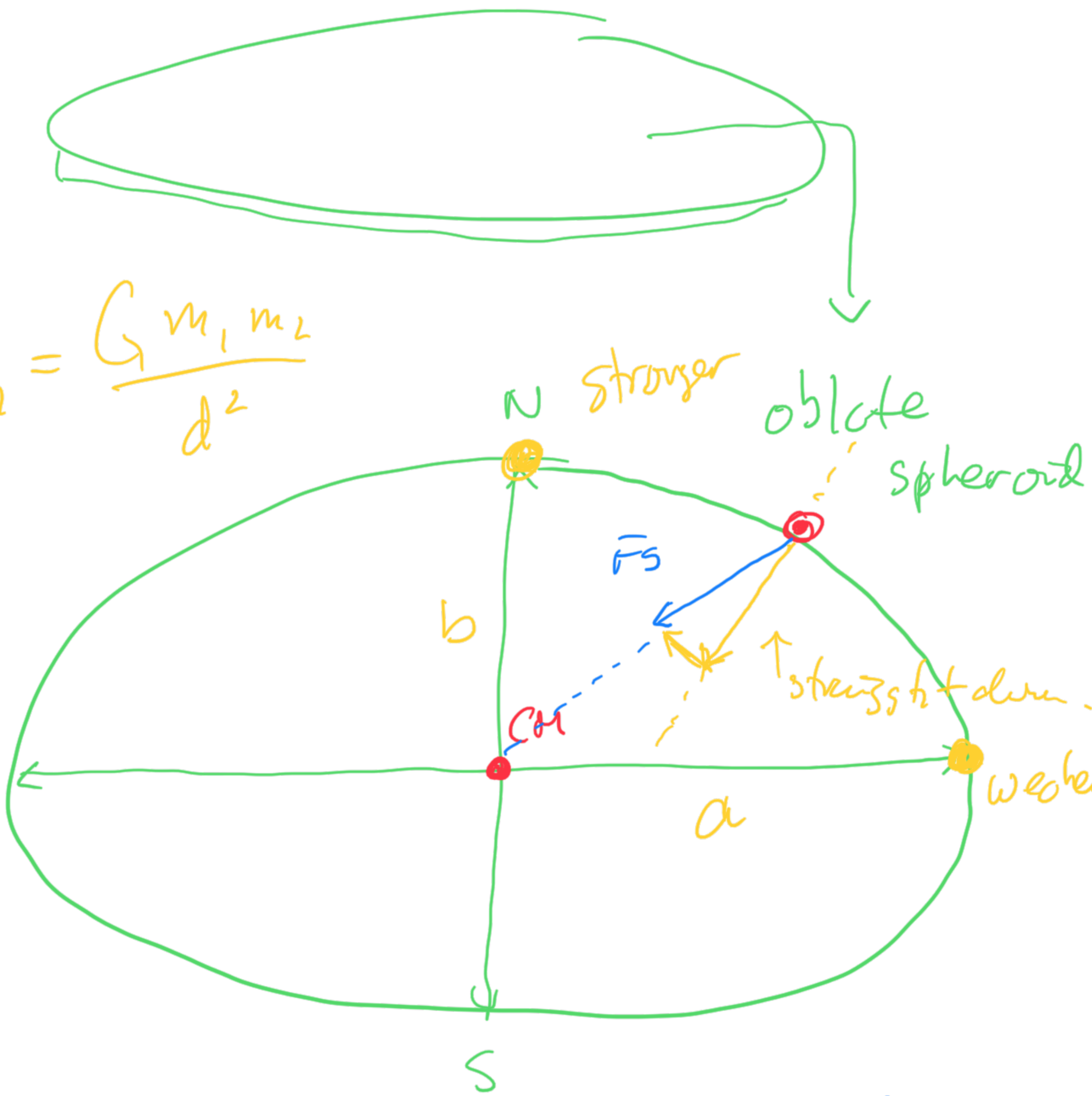
Henry Kater

→ measurement of g

$g = 9.8\ m/s^2$

Earth is not a sphere.

$$F_G = \frac{G m_1 m_2}{d^2}$$

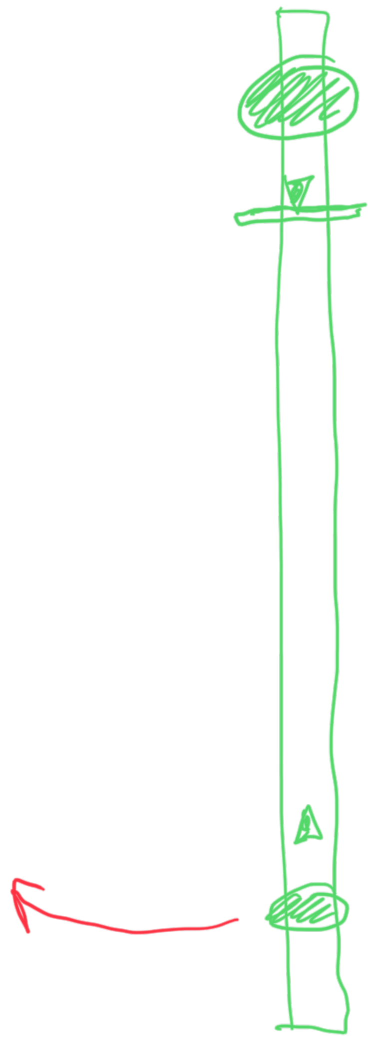


NN $\quad 9.79893 \ m/s^2$

North Pole $\quad 9.84555 \ m/s^2$

Equator $\quad 9.77939 \ m/s^2$

jlabdaq.pcs.cnu.edu/wiki



SMD

→ small mass dn

$\boxed{T_d \text{ oscillation}}$

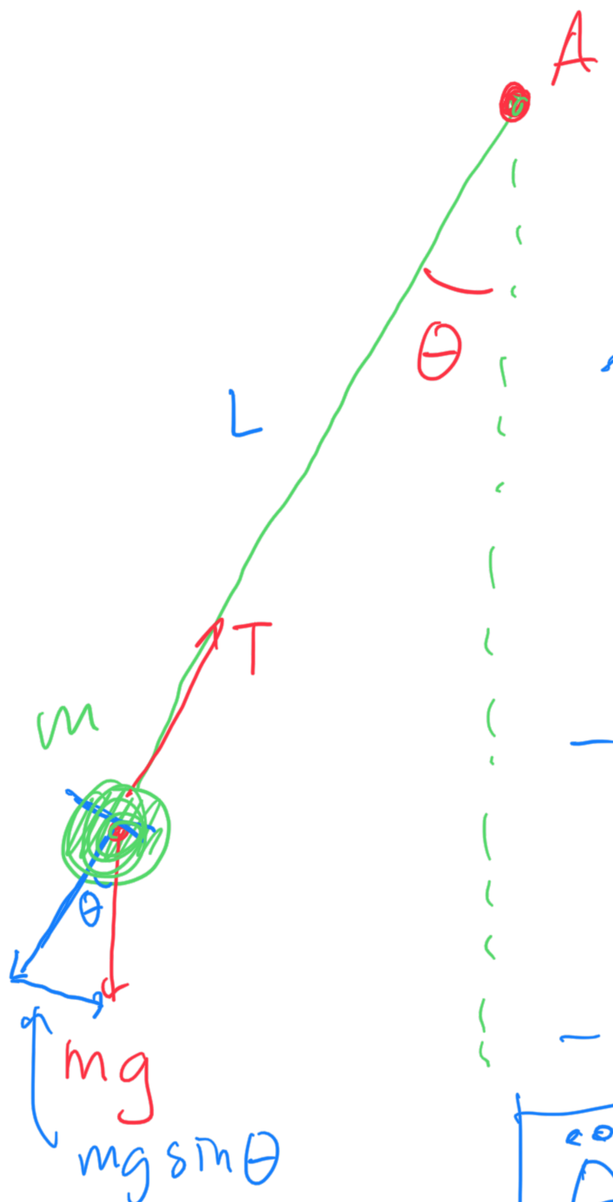
SMU

→ small mass up

$\boxed{T_U \text{ oscillation}}$

— adjust the positions of masses until $T_{up} = T_{down}$

Simple Pendulum



$\boxed{\text{Torque}}$

$$\tau = (mg\sin\theta)L$$

$$= I\alpha$$

$$-mgL\sin\theta = I\alpha$$

$$= I\frac{d^2\theta}{dt^2}$$

$$-mgL\sin\theta = I\ddot{\theta}$$

$$\boxed{\ddot{\theta} = \frac{-mgL\sin\theta}{I}}$$

→ Hard

→ elliptic integrals

If oscillations are <u>small</u>

$$\sin\theta \approx \theta$$

$$\theta = 5^\circ \quad \to \quad 5^\circ * \frac{\pi}{180} = 0.0876$$

$$\sin(5^\circ) = 0.0872$$

$$\ddot{\theta} = \frac{-mgL\,\cancel{\sin\theta}}{I}$$

$$\theta = A\sin(\omega t + \phi)$$

$$\omega = \sqrt{\frac{mgL}{I}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgL}}$$

If string is "light"

$$I \approx mL^2$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

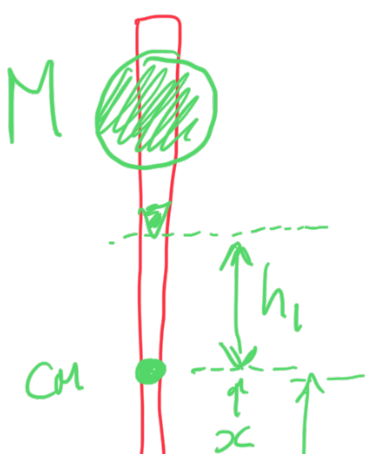
$$T^2 = 4\pi^2 \frac{L}{g}$$

$$\boxed{g = \frac{4\pi^2 L}{T^2}}$$

measure

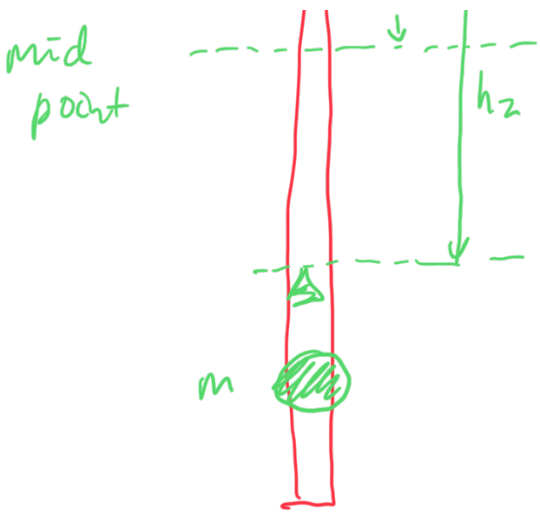
measure

Kater's Pendulum.



$$T = 2\pi\sqrt{\frac{I}{mgL}}$$

distance from pivot pt



to cm.

$$T_{down}^{sm} = 2\pi \sqrt{\frac{I_{smd}}{m g h_1}}$$

$$T_{up}^{sm} = 2\pi \sqrt{\frac{I_{smu}}{m g h_2}}$$

Parallel axis theorem.

$$\rightarrow I_{smd} = I_{cm} + M_{total} h_1^2$$

$$\rightarrow I_{smu} = I_{cm} + M_{total} h_2^2$$

$$T_{smd} = 2\pi \sqrt{\frac{I_{cm} + M_T h_1^2}{M_T g h_1}}$$

$$T_{smu} = 2\pi \sqrt{\frac{I_{cm} + M_T h_2^2}{}}$$

$V \quad M_T g h_2$

If $\boxed{T_{smd} = T_{smv}}$

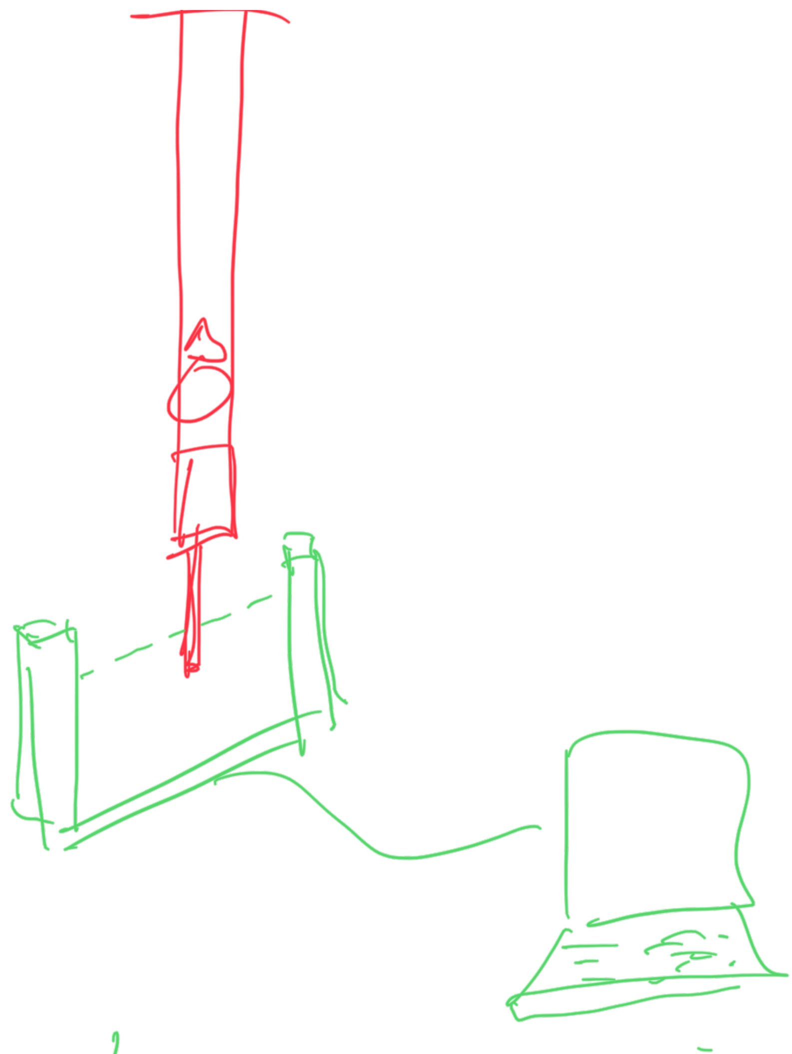
$$\boxed{I_{cm} = M_T h_1 h_2}$$

$$\boxed{T_{equil}} = 2\pi\sqrt{\frac{h_1 + h_2}{g}}$$

$1/10{,}000$

$h_1 + h_2 \rightarrow$ measured very accurately

super accurately!

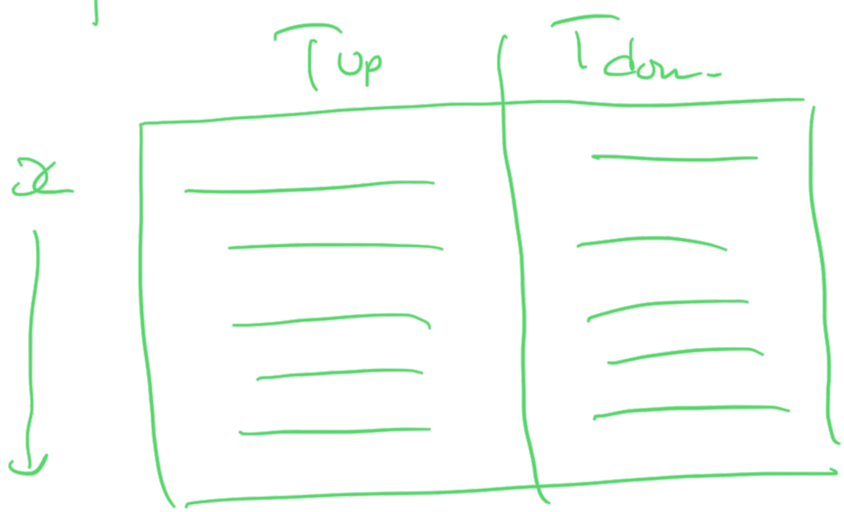


t
1.00 s
3.28 s
5.17 s
7.32 s
⋮

⋮

$\boxed{\text{14 data files.}}$

7 positions of small mass



→ 1.02

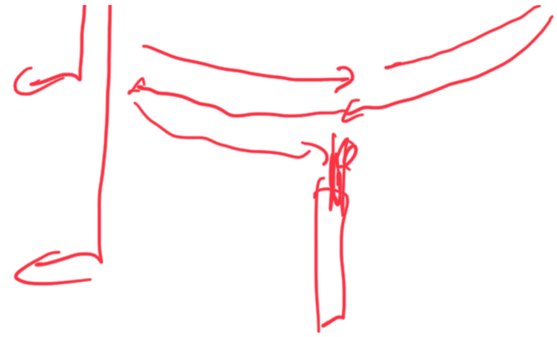
→ 2.03

→ 3.04

→ 4.05

$t_3 - t_1 = T$

→ 5.01



$\rho = .6$

0.13, .19, .35, .33

$$g = \frac{2h}{t^2}$$ ←

b) $$\delta g = \left|\frac{\partial g}{\partial h}\right| \delta h + \left|\frac{\partial g}{\partial t}\right| \delta t$$

$$\frac{\partial g}{\partial h} = \frac{2}{t^2} \qquad \frac{\partial g}{\partial t} = \frac{-4h}{t^3}$$

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dgdt = 2/t**2
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dgdh = -4*h/t**3

dg = dgdh*dh + dgdt
*dt

print (dg)

c) $(\delta g) = \sqrt{\left(\frac{\partial g}{\partial h}\right)^2 \delta h^2 + \left(\frac{\partial g}{\partial t}\right)^2 \delta t}$

dg = np.sqrt(dgdh**2 *dh**2
+ dgdt**2 *dt**

$h_1^2 h_2 + h_2 R^2 = h_1 h_2^2 + h_1 R$

$\cancel{(h_2 - h_1)} R^2 = h_1 h_2 \cancel{(h_2 - h_1}$

$R^2 = h_1 h_2$

1.9881

$1.96967 \quad \cancel{1.97884}$

$$\left[1.97970 \ldots \cancel{1.98606}\right]$$

1.99278

$9.814 \rightarrow 0.0085$

$9.799 \rightarrow 0.0042$