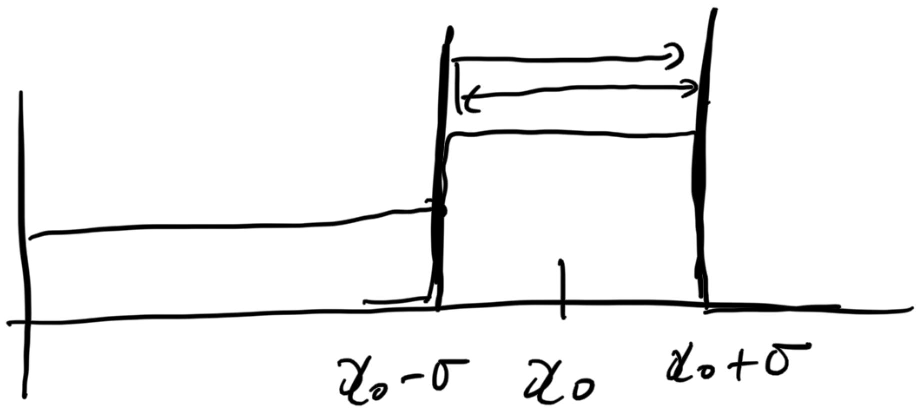
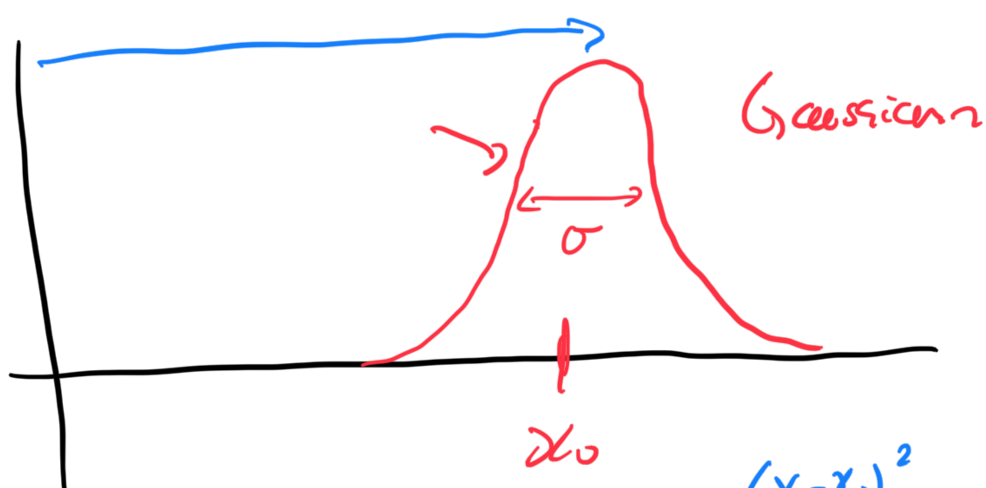
Simulations

→ random # generation.



$$2\sigma\boxed{[0,1]} + (x_0 - \sigma)$$

Real Life.



$$-\frac{(x-x_0)^2}{2\sigma^2}$$

1

$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e$$

If: $x_0 = 0$ $\quad P(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{x^2}{2\sigma^2}}$

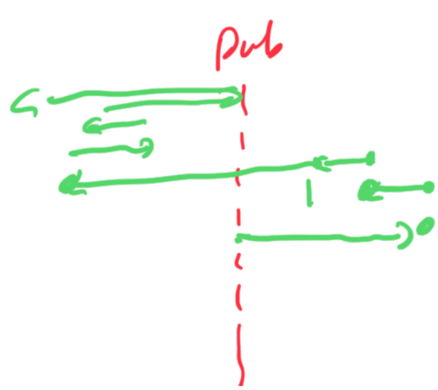
$$\sigma^2 = \int_{-\infty}^{\infty} P(x)\, x^2\, dx$$

$= \langle x^2 \rangle$

$\boxed{\text{Drunken}}$ Sailor Problem.

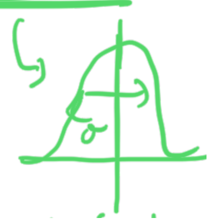
→ Drunken Sailor leaves the pub at closing time.

→ He starts walking.



→ random direction.

→ random length.



N_{steps}

step length

$x = 0$

distribution of possible ending positions.

Gaussian $\quad x_0 = 0$

$$\boxed{\sigma_N = \sqrt{N_{steps}}\ \sigma}$$

Diffusion, Heat Transfer,

Collective Motion

Gaussian $\rightarrow$ $\boxed{\begin{array}{l} x_0 = 0 \\ \sigma = 1 \end{array}}$

stats.norm $\rightarrow$ Gaussian.

.rvs (N)

random variate sample

$$r = (\underline{0.0}, \underline{-0.2}, \underline{+0.33}, \underline{\quad}, \underline{\quad}, \ldots)$$

step 1 step 2 step 3

N elements.

Final position $\rightarrow \sum_{i=1}^{n\text{steps}} r$

n part-walk n

$\rightarrow$ X vec

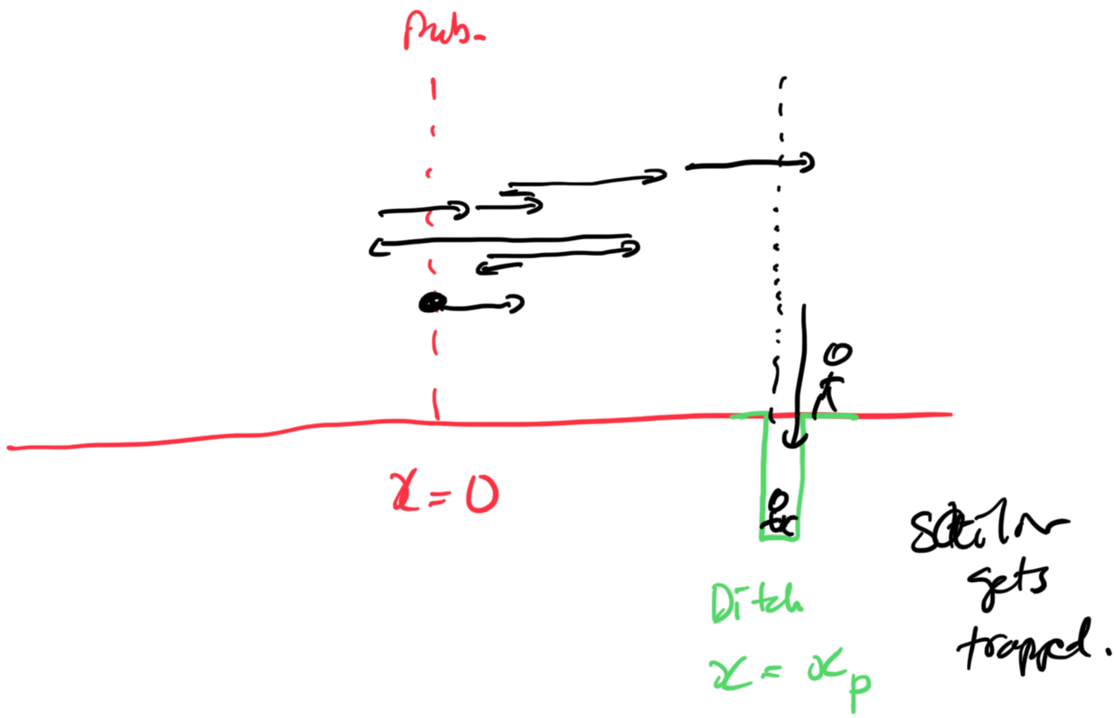
$$(\underline{x_f^{(1)}}, \underline{x_f^{(2)}}, \underline{x_f^{(3)}}, \underline{\quad}, \ldots)$$

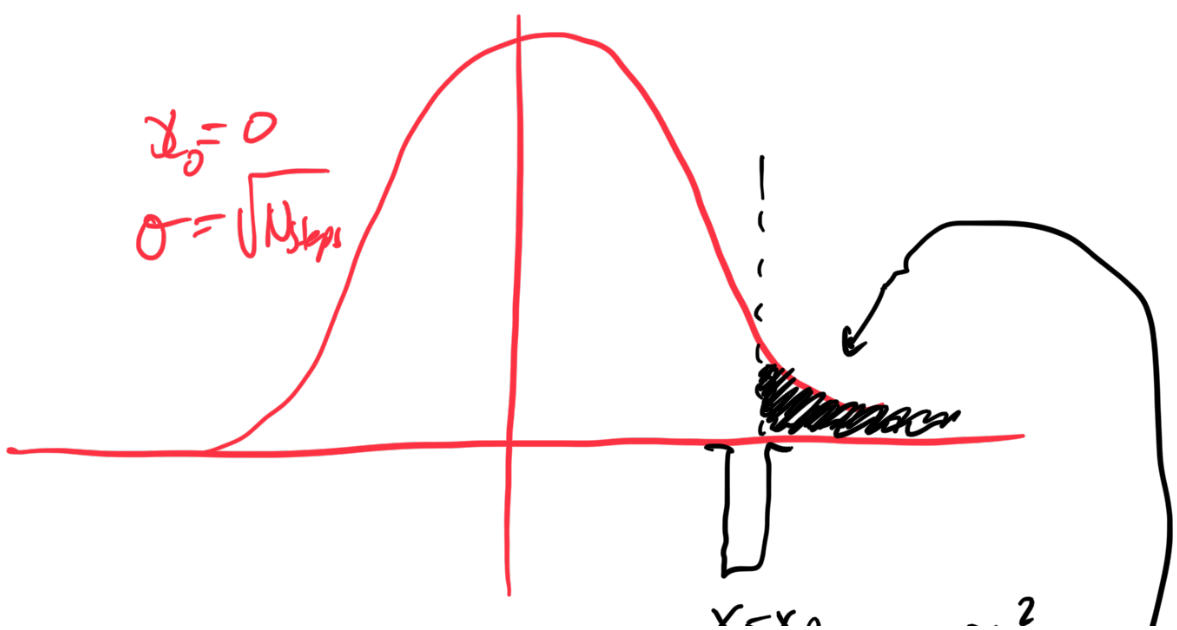
N part

Gaussian $\quad x_0 = 0$

$$\sigma_N = \sqrt{N_{steps}}\ \sigma$$

1



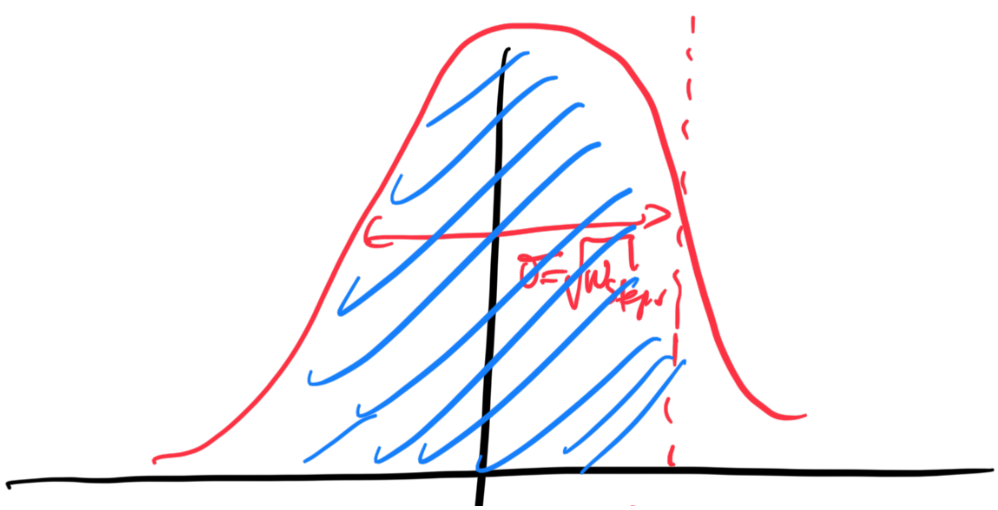


$$\text{fraction} = \int_{x_p}^{\infty} \frac{1}{\sqrt{2\pi\sigma_N^2}}\, e^{-\frac{x^2}{2\sigma_N^2}}\, dx$$

$$= \frac{1}{2}\,\mathrm{erfc}\left(\frac{x_p}{\sqrt{2}\,\sigma_N}\right)$$

$\boxed{\text{walk\_ditch}}$

$$x_p = \frac{\sigma}{\sqrt{N_{steps}}}$$



x_p

Electron.

1000 sailor

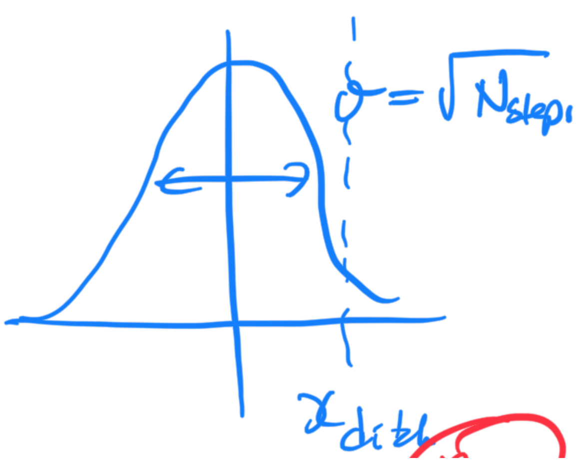
each sailor

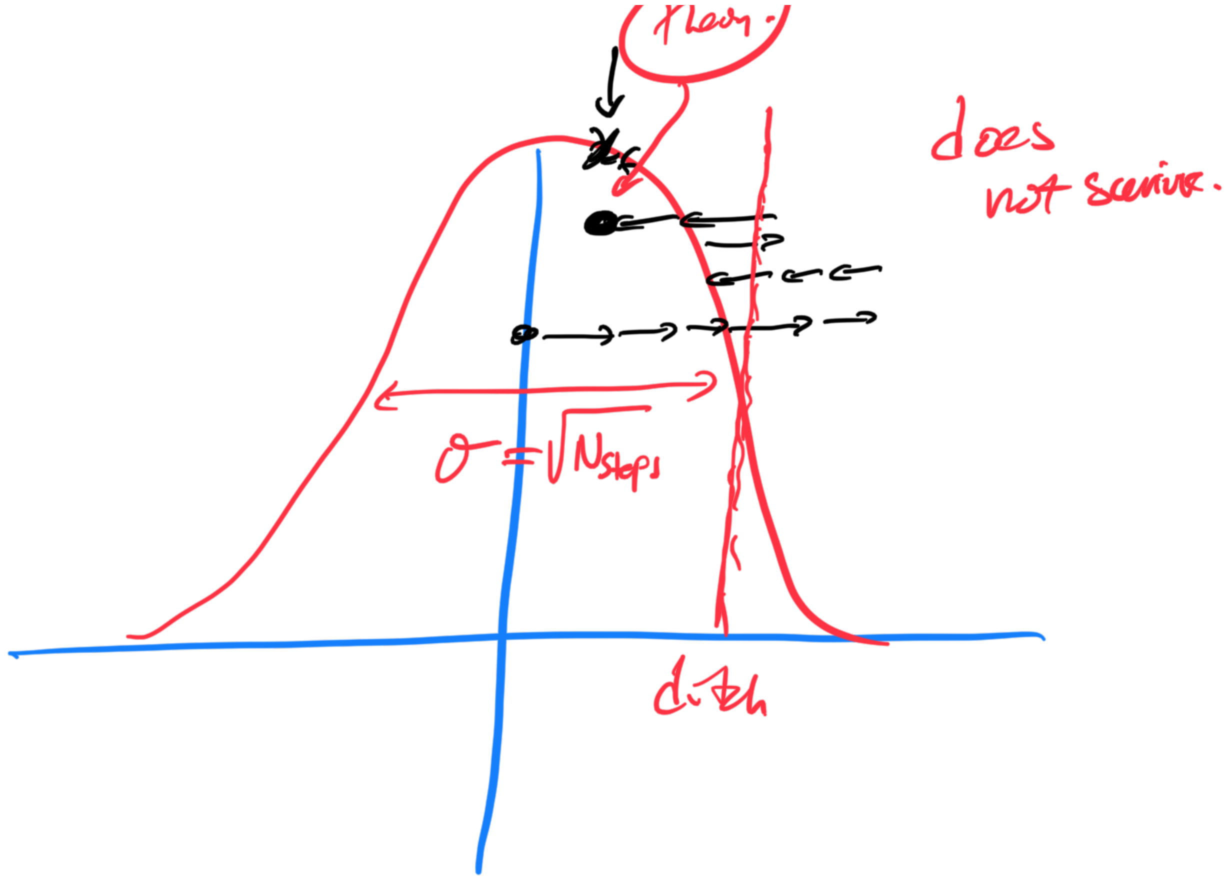
$$\underbrace{1000 \times 100 \text{ steps}}_{P_{survival}}$$

1000 trials

fivethirtyeight.org

$$\bar{P}_{survival} \rightarrow N_{steps}$$




does not survive.
σ = √N_steps
ditch