LIFE

Differential Equations

$\downarrow$

Every physical process

(DesCartes)

$\downarrow$

$, affection

Ordinary $\boxed{\text{Second-Order}}$ Differential Equations.

$$\rightarrow \vec{F}_{net} = m\vec{a}$$

$$\frac{d^2\vec{x}}{dt^2} \rightarrow \ddot{x}$$

$$\vec{v} = \dot{x}$$

$$\cancel{(\dot{x})^2}, \quad \cancel{\dot{x}\ddot{x}}, \quad \cancel{x^3\dot{x}}$$

$\boxed{\text{Damped}}$ Oscillator.

$$\underbrace{m\ddot{x}}_{ma} + \underbrace{b\dot{x}}_{\vec{F} = -b\vec{v}} + \underbrace{kx}_{\text{restoring force.}} = 0$$

ma

$\vec{F} = -b\vec{v}$

frictional force.

restoring force.

$\vec{F}_{spring} = -k\vec{x}$

$$\boxed{\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0}$$

$\boxed{b=0}$

$$\ddot{x} + \frac{k}{m}x = 0$$

$$x = Ae^{i\omega_0 t}$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$\ddot{x} = -\omega_0^2 Ae^{i\omega_0 t}$$

$$= -\omega_0^2 x$$

$$\ddot{x} + \underbrace{\frac{b}{m}}_{2\varepsilon\omega_0}\dot{x} + \omega_0^2 x = 0$$

$$2\varepsilon\omega_0 = \frac{b}{m}$$

$$\varepsilon = \boxed{\frac{b}{2m\omega_0}} = \frac{b}{2m\sqrt{\frac{k}{m}}}$$

$$\varepsilon = \frac{b}{}$$

$$\boxed{c \ \ \ \ 2\sqrt{mk}}$$

$$x = Ae^{i\omega t}$$

$$r^2 + \frac{b}{m}r + \omega_0^2 = 0$$

$$r = \frac{-\frac{b}{m} \pm \sqrt{\left(\frac{b}{m}\right)^2 - 4\left(\omega_0^2\right)}}{2}$$

(i) $\left(\frac{b}{m}\right)^2 - 4\left(\omega_0^2\right) > 0$ — real, overdamped

(ii) $\left(\frac{b}{m}\right)^2 - 4\omega_0^2 < 0$ — complex, underdamped

(iii) $\boxed{\left(\frac{b}{m}\right)^2 - 4\omega_0^2} = 0$ — double root, critically damped.

$$b = 0.1\sqrt{4mk}$$

$$\left(\frac{b}{m}\right)^2 - 4\omega_0^2 = 0$$

$$\cancel{\frac{b}{m} = 4\omega_0^2}$$

$$\cancel{b = 4\omega_0^2 m}$$

$$\cancel{= 4}\sqrt{\frac{k}{m}}$$

$$\boxed{b = \sqrt{4mk}}$$

$$\ddot{x} + 2\zeta\omega_0\dot{x} + \omega_0^2 x = 0$$

$$\boxed{\ddot{x} = -2\zeta\omega_0\dot{x} - \omega_0^2 x}$$

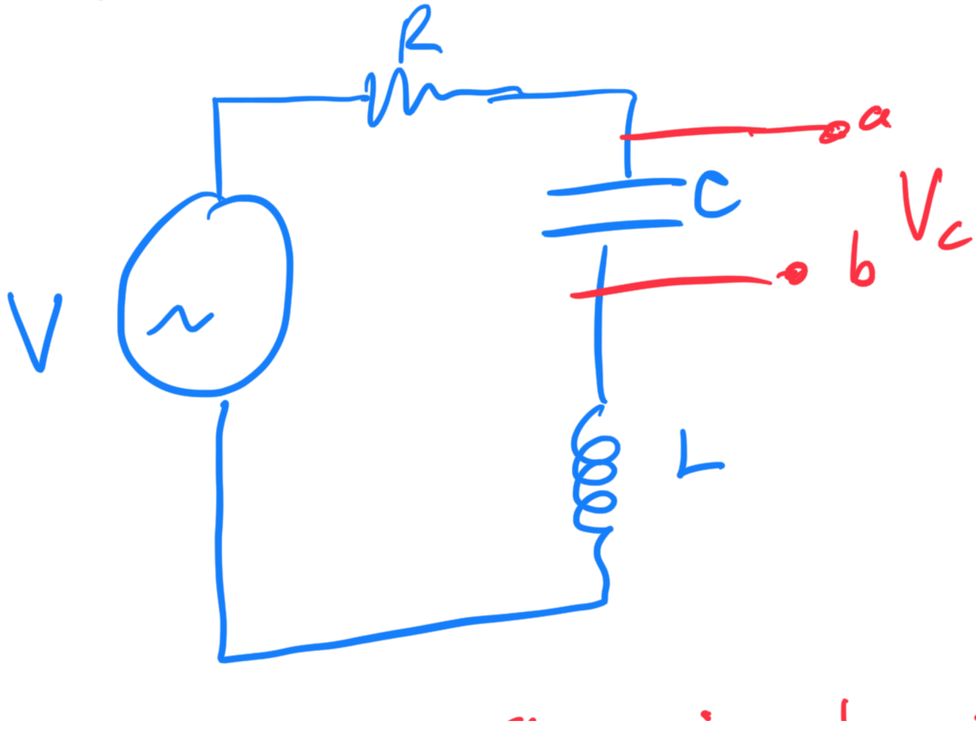
$$y_{vec} = (x, \dot{x})$$

$$y_{init} = (1, 0)$$

Damped **Driven** Oscillator.

$$\cancel{m}\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = \frac{F_0 \sin(\omega t}{m}$$

AC voltage source.



① $$m\ddot{x} + b\dot{x} + kx = 0$$

$$x = c_1 y_1 + c_2 y_2$$

② x_p

- undetermined coefficient
- variation of parameters
- Laplace transforms
- Power Series.

$$\ddot{x} = -\frac{b}{m}\dot{x} - \omega_0^2 x + \underline{\frac{F_0}{m}\sin(\omega}$$

SHM