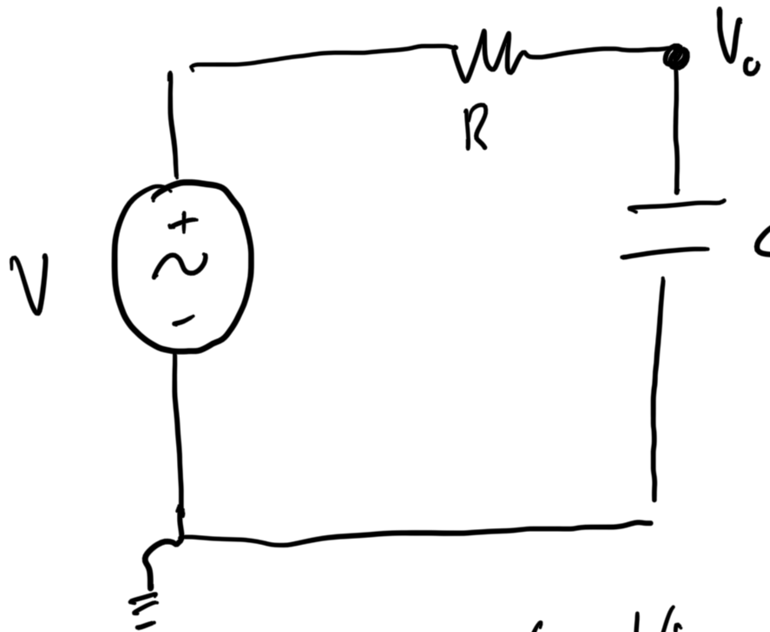
RC Circuit



$Z_C = \frac{1}{j\omega C}$

$$V_c = V\left(\frac{1/j\omega c}{\frac{1}{j\omega c} + R}\right)$$

$$V_c = V\left(\frac{\frac{1}{j\omega c}}{\frac{1 + j\omega RC}{j\omega c}}\right) = V\left(\frac{1}{1 + j\omega RC}\right)$$

$$|V_c| = |V| \cdot \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}}$$

$R = 1000$

$C = 1 \times 10^{-6}$

$\omega = 2\pi f = 2\pi(100) = 200\pi$

$\omega RC = (1000)(10^{-6})(10^{3}.)(.2\pi) = 0.2\pi$

$\sqrt{1 + (.2\pi)^2} = 1.1810 \qquad \dfrac{1}{\sqrt{1+(.2\pi)^2}} = 0.846$

$\therefore \boxed{|V_c|_{max} = 0.8467 \text{ V}}$

<u>phase</u>:

$$V_c = V\left(\frac{1}{1+j\omega RC}\right)\left(\frac{1-j\omega RC}{1-j\omega RC}\right)$$

$$V_c = \frac{V}{1+\omega^2R^2C^2}\left(1 - j\omega RC\right)$$

$$e^{i\phi} = \cos\phi + i\sin\phi$$

$$\therefore \quad \cos\phi = \frac{1}{1+\omega^2R^2C^2}$$

$$\sin\phi = \frac{-\omega RC}{1+\omega^2R^2C^2}$$

$$\tan\phi = -\omega RC \qquad \boxed{\phi = -32.1^\circ}$$

$$T = \frac{1}{f} = 0.01\ \text{s} = 10\ \text{ms}$$

$$\phi_T = \frac{-32.1^\circ}{360^\circ} \times 10\ \text{ms} = -0.89\ \text{ms}$$

Interesting Cases:

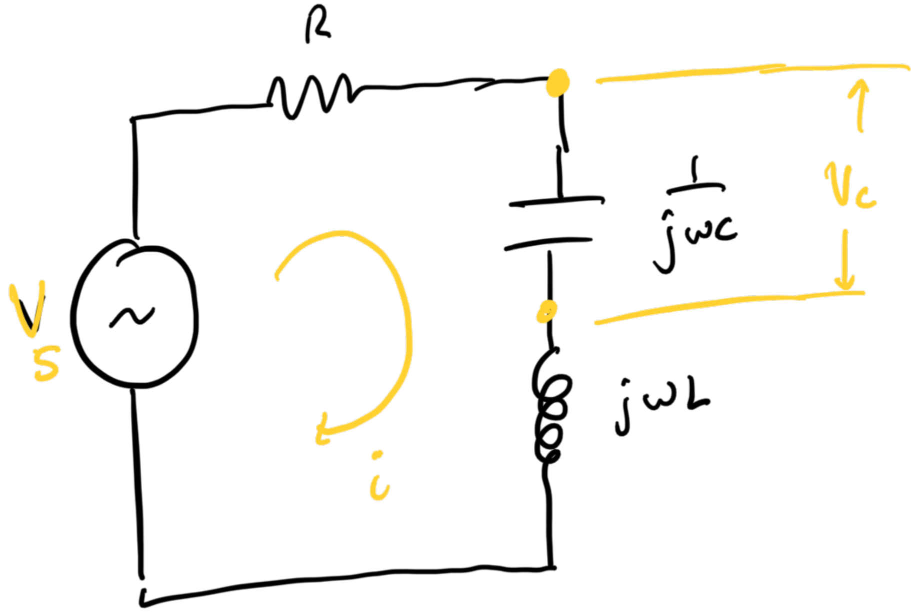
(i) $\omega \ll RC \qquad \left|\frac{V_C}{V}\right| \approx 1, \ \phi \approx 0$

$|V_0|$

(ii) $\omega = RC$ $\quad \left|\frac{V_c}{V_0}\right| = \frac{1}{\sqrt{2}}$, $\phi = -45^\circ$

(iii) $\omega \gg RC$ $\quad \left|\frac{V_c}{V_0}\right| \approx 0$, $\phi = -90^\circ$

RLC Circuit



$$Z = R + \frac{1}{j\omega C}\cdot\frac{j}{j} + j\omega L$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Phase Angle :

$$\cos\phi = \frac{R}{|Z|}$$

$$\cos\phi = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Resonance :

$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC}$$

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

Current :

$$i = \frac{V_{in}}{Z} = \frac{V_{in}}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

Output Voltage:

$$V_c = i \cdot Z_c$$
$$= \frac{V_{in}}{Z} \cdot \left(\frac{1}{j\omega C}\right)$$

$$V_c = V_{in}\left(\frac{-j}{\omega C\left(R + j\left(\omega L - \frac{1}{\omega C}\right)\right)}\right)$$

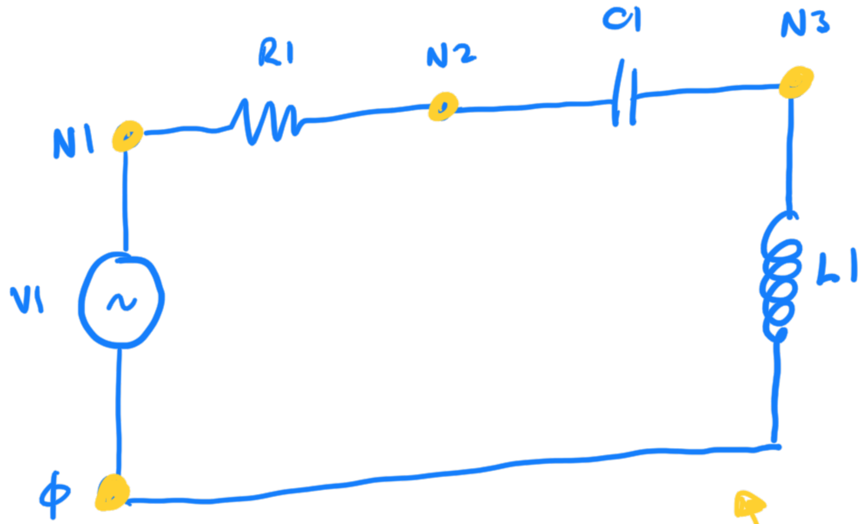
$$\frac{V_c}{V_{in}} = \frac{-j}{\omega RC + j\left(\omega^2 LC - 1\right)} \times \frac{j}{j}$$

$$\boxed{\frac{V_c}{V_{in}} = \frac{+1}{\left(1-\omega^2 LC\right) + j\,\omega RC}}$$

phase: $\boxed{\tan\phi = \frac{-\omega RC}{1-\omega^2 LC}}$

$$\left|\frac{V_c}{V_{in}}\right| = \frac{1}{\sqrt{\quad}}$$

$|V_{in}| \qquad \sqrt{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2}$



From skidl.net file

$f = 50\ \text{Hz}$

$\omega = 2\pi f = 100\pi$

$C = 100 \times 10^{-6}$

Resonance:

$$\omega^2 = \frac{1}{LC}$$

$$L = \frac{1}{\omega^2 C} = \frac{1}{(100\pi)^2 (100) \times 10^{-6}}$$

$$= 0.10132\ \text{H}$$

$$\frac{V_c}{V_{in}} = \frac{1}{(1-\omega^2 LC) + j\omega RC} = Ae^{i\phi}$$

$$A\sin\phi = -\omega RC$$

$$A\cos\phi = (1-\omega^2 LC)$$

$$\tan\phi = \frac{-\omega RC}{1-\omega^2 LC}$$

$$A^2 = \frac{1}{\omega^2 R^2 C^2 + (1-\omega^2 LC)^2}$$

$$A = \frac{1}{\sqrt{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2}}$$

at resonance $\rightarrow$ $A = \dfrac{1}{\omega RC} = \dfrac{1}{(100\pi)(1.2)(1\times 10^{5}}$

$$= 2.654$$

$$\therefore \boxed{|V_c| = 265.4\ V}$$