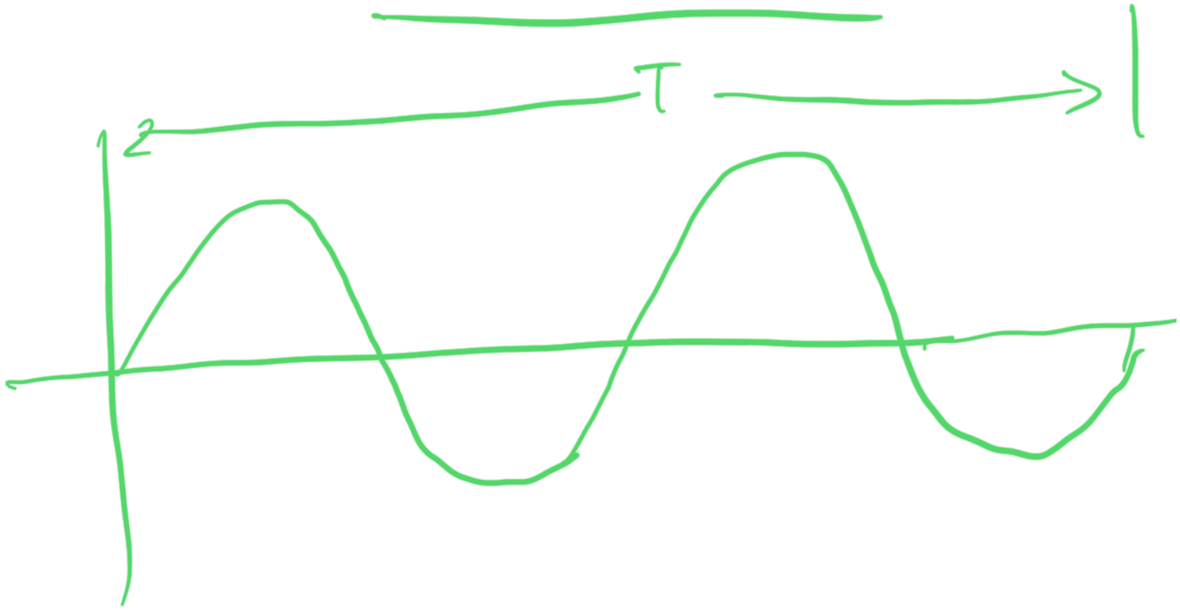
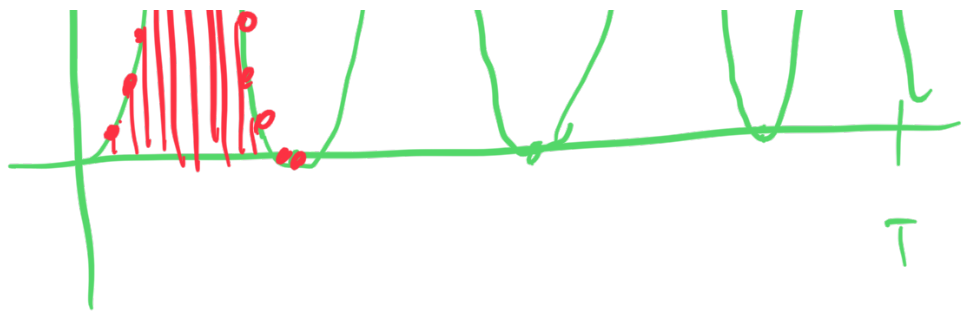
Fourier Integrals



$$y = \sin(2\pi f_s t)$$

$$P = \frac{1}{T}\int_0^T y^2\, dt$$

$$= \frac{1}{T}\int_0^T \sin^2(2\pi f_s t)\, dt \quad = \frac{1}{2}T$$



$$\boxed{\bar{P} = \frac{1}{2}}$$ ← this is what we expect!

$$y = \sum_{n=1}^{N} b_n \sin\left(\frac{2\pi n}{T} t\right)$$

$$\bar{P} = \frac{1}{T}\int_0^T y^2 \, dt$$

$$\bar{P} = \frac{1}{T}\int_0^T \left(\sum_{n=1}^{N_{terms}} b_n \sin\left(\frac{2\pi n}{T} t\right)^2\right) dt$$

$$P = \frac{1}{T}\sum_{i=1}^{N_{samples}} \left(\sum_{n=1}^{N_{terms}} b_n \sin\left(\frac{2\pi n}{T} t_i\right)^2\right)(dt)$$

i-dependence

N_{terms} $N_{samples}$

$$= \frac{1}{T} \sum_{n=1} b_n^2 \underbrace{\sum_{i=1} \sin^2\left(\frac{2\pi n}{T} t_i\right) dt}_{\frac{1}{2} T}$$

$$\boxed{\bar{P} = \frac{1}{2} \sum_{n=1}^{N_{\text{terms}}} b_n^2}$$

Wikipedia

$$y = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{2\pi n}{T} t\right) + b_n \sin\left(\frac{2\pi n}{T} t\right) \right]$$

Bash

vs.

$$y = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{\pi n}{T} t\right) + b_n \sin\left(\frac{\pi n}{T} t\right) \right]$$

What is the difference?

$$\frac{1}{T} \int^{T} y(t)\, dt$$

ad_2

$$= \frac{1}{T}\int_0 \left(+ \sum_{n=1} a_n \cos\left(\frac{2\pi n}{T} t\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi n t}{T}\right) \right) dt$$

$$= \frac{1}{T}\int_0^T \frac{a_0}{2}\, dt + \sum_{n=1}^{\infty} \left[\frac{1}{T}\int_0^T a_n \cos\left(\frac{2\pi n}{T} t\right) dt + \frac{1}{T}\int_0^T b_n \sin\left(\frac{2\pi n}{T} t\right) dt \right]$$

$$= \frac{1}{T}\int_0^T \frac{a_0}{2}\, dt + \sum_{n=1}^{\infty} \left[+ a_n \left(\frac{1}{T}\int_0^T \cos\left(\frac{2\pi n}{T} t\right) dt \right) + b_n \left(\frac{1}{T}\int_0^T \sin\left(\frac{2\pi n}{T} t\right) dt \right) \right]$$

$$\frac{1}{T}\int_0^T \cos\left(\frac{2\pi n}{T} t\right) dt$$

$$= \frac{1}{T} \times \frac{T}{2\pi n} \sin\left(\frac{2\pi n}{T} t\right)\Big|_0^T$$

$$= \frac{1}{2\pi n}\left[\sin(2\pi n) - 0 \right]$$

$= 0$.

$= 0$!

(Also true for $\int_0^T \sin\left(\frac{2\pi n}{T} t\right.$

$$= \frac{1}{T}\int_0^T \frac{a_0}{2}\,dt$$

!!!

$$= \frac{a_0}{2} \qquad \therefore\ a_0 = \frac{2}{T}\int_0^T y(t)\,dt$$

Note : T is the entire span!!

when one does $\int_{-T/2}^{T/2} y(t)\,dt$, one sees that

$$a_0 = \frac{2}{T}\int_{-T/2}^{T/2} y(t)\,dt$$

$$= \frac{1}{(T/2)}\int_{-T/2}^{T/2} y(t)\,dt$$

$$a_0 = \frac{1}{\ }\int^{P} y(t)\,dt \quad \text{as before}$$

$$a_0 = \frac{1}{p} \int_{-p} g$$

b_n

$$\frac{1}{T}\int_0^T y(t) \sin\left(\frac{2\pi n}{T} t\right) dt$$

$y_m(t)$

$$= \frac{1}{T}\int_0^T \left[\frac{a_0}{2} + \sum_{m=1}^{\infty} a_m \cos\left(\frac{2\pi m}{T} t\right) + \sum_{m=1}^{\infty} b_m \sin\left(\frac{2\pi m}{T} t\right) \right] \left(\sin\left(\frac{2\pi n}{T} t\right)\right) dt$$

$$= \cancel{\frac{1}{T}\int_0^T \frac{a_0}{2} \sin\left(\frac{2\pi n}{T} t\right) dt} \rightarrow 0$$

$$+ \frac{1}{T}\int_0^T \sum_{m=1}^{\infty} a_m \cos\left(\frac{2\pi m}{T} t\right) \sin\left(\frac{2\pi n}{T} t\right) dt$$

$$+ \frac{1}{T}\int_0^T \sum_{m=1}^{\infty} b_m \sin\left(\frac{2\pi m}{T} t\right) \sin\left(\frac{2\pi}{T}\right.$$

$= 0$

$$= \sum_{m=1}^{\infty} b_m \left(\cancel{\frac{1}{T}\int_0^T \cos\left(\frac{2\pi m}{T}t\right)\sin\left(\frac{2\pi n}{T}t\right)dt} \right.$$

$$\left. + \frac{1}{T}\underbrace{\int_0^T \sin\left(\frac{2\pi m}{T}t\right)\sin\left(\frac{2\pi n}{T}t\right)}_{=\frac{1}{2}T\,\delta_{m,n}} \right)$$

$$= \sum_{m=1}^{\infty} b_m \frac{1}{2}\delta_{m,n} = \frac{b_n}{2}$$

$$\therefore \quad \boxed{b_n = \frac{2}{T}\int_0^T y(t)\sin\left(\frac{2\pi n}{T}t\right)dt}$$

$$\boxed{a_n = \frac{2}{T}\int_0^T y(t)\cos\left(\frac{2\pi n}{T}t\right)dt}$$

$$\boxed{a_0 = \frac{2}{T}\int_0^T y(t)\,dt}$$

$$P_n = a_n^2 + b_n^2$$

$$P = \int_0^{f_{max}} P(f)\,df$$

(this is P_n)

$$\boxed{\bar{P} = \frac{1}{f_{max}} \int_0^{f_{max}} P(f)\,df}$$

(Terms look like $\sin(\omega t)$ $\quad \left(\frac{2\pi n}{T} t\right) = 2\pi f_n t$

$\therefore f_n = \frac{n}{T}$

$\therefore f_{max} = \frac{n_{max}}{T}$

$df = ?$

$u +$

$$= f_{n+1} - f_n$$

$$= \left(\frac{n+1}{T}\right) - \left(\frac{n}{T}\right)$$

$$\boxed{df = \frac{1}{T}}$$

$$\bar{P} = \frac{1}{f_{max}} \sum_{n=1}^{N_{terms}} P_n^2 \cdot \frac{1}{T}$$

$$= \frac{T}{n_{max}} \sum_{n=1}^{N_{terms}} P_n^2 \cdot \frac{1}{T}$$

$$\bar{P} = \frac{1}{n_{max}} \sum_{n=1}^{N_{terms}} P_n^2 = \frac{1}{2}\,\text{Wa}$$