document Three dimensional coordinates into two dimensional coordinates conversion

Written by Edward Gerhold

Definitions

[Picture of a 3-D coordinate system with ijk-vectors on the axes pointing into three directions]

Let phin be the set of axis angles, one for each axis. The angles start at the same place, at the number zero, you have to arrange the x, y, and z axes like on a piece of paper around the unit circle by giving them the appropriate angles. All three angles start at the default at zero

phin = phix, phiy, phiz

Let en be the set of three two dimensional unit base vectors, namely ex, ey and ez, they point on the two dimensional plane into three directions and represent the axes of the three dimensional coordinate system.

en = ex, ey, ez

To guess no numbers, its easier for us, to go around the unit circle by the angles of the unit vectors, and to use cosine and sine for the correct x-distance and y-distance. For help, you should remember this parametrization of x and y from the unit circle.

x = r cos phi y = r sin phi

Modeling the three two dimensional base vectors after this we get the following

ex = (r*cos(phix), r*sin(phix) )T ey = (r*cos(phiy), r*sin(phiy) )T ez = (r*cos(phiz), r*sin(phiz) )T

The other help we take is from the orthogonal base formula. The sum of the basis multiplied with the coordinates is nothing new. But literature explains only how to multiply square matrices or coordinates and bases with equal dimensions.

x' = sigma en*xn

To make it short, each x,y,z coordinate has to be multiplied for the new x' and the new y' coordinate with its corresponding term of the unit vector. That means, to sum the cos terms up for x' and to sum the sin terms up for y'.

x' = x*r*cos(phix) + y*r*cos(phiy) + z*r*cos(phiz) y' = x*r*sin(phix) + y*r*sin(phiy) + z*r*sin(phiz)

Let A be the matrix containing the three unit vectors in order, one each column. You get a 2x3 matrix, which i call the Gerhold Matrix to distinguish it from other matrices.

A = (ex, ey, ez) = (r*cos(phix), r*cos(phiy), r*cos(phiz); r*sin(phix), r*sin(phiy), r*sin(phiz))

Theorem (Fundamental Theorem of converting 3-D Points into 2-D Points)

If you multiply the matrix containing the three two-dimensional unit vectors with the three coordinate points (x,y,z), the result is a two coordinate point, (x',y'), which is the correct point on the two dimensional plane representing the point in the three dimensional coordinate system we display.

A(x,y,z) = (x',y')

in other words

x' = x*r*cos(phix) + y*r*cos(phiy) + z*r*cos(phiz) y' = x*r*sin(phix) + y*r*sin(phiy) + z*r*sin(phiz)

Proof: [example calculation or higher math variable proof]

Corollary (Converting more Dimensions)

The theorem can be extended to more dimensions, for example can four two-dimensional vectors represent a 4-D space on the 2-D plane. They get converted into the correct 2-D points. For Example, if you use a 2x4 matrix and convert all points at each instance of t you have a moving object into the direction of the fourth vector.

A = (ex, ey, ez) = (r*cos(phix), r*cos(phiy), r*cos(phiz), r*cos(phit); r*sin(phix), r*sin(phiy), r*sin(phiz), r*sin(phit))

A(x,y,z,t)=(x',y')