



Functional Regression Models in Human Movement Biomechanics

Methodology and Applications

Edward Gunning

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Background

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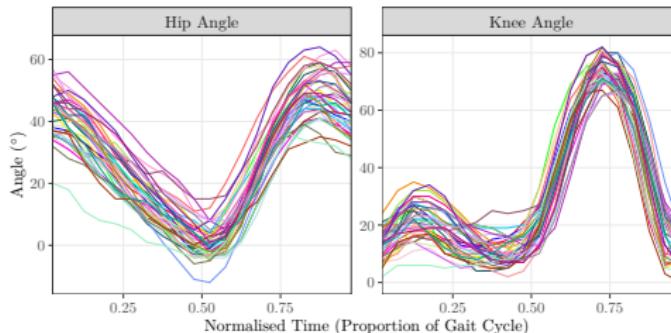


Figure: The childrens' gait dataset (Rice and Silverman, 1991).

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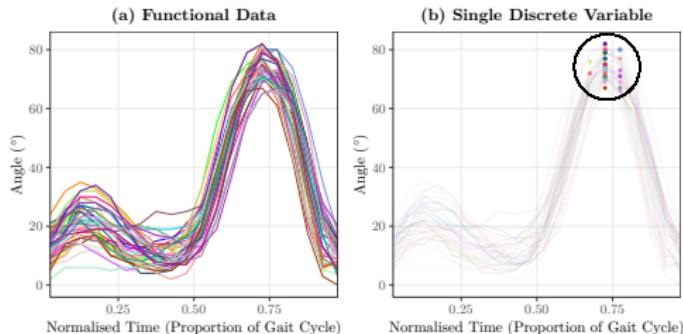


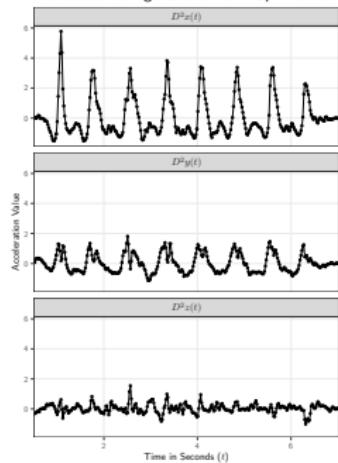
Figure: Data reduction of the knee angle curve to a single discrete variable (peak knee flexion).

Second-Generation Functional Data in Biomechanics

Volume



My Own Motion Data from Apple
Collected During a Short Warm-Up Exercise

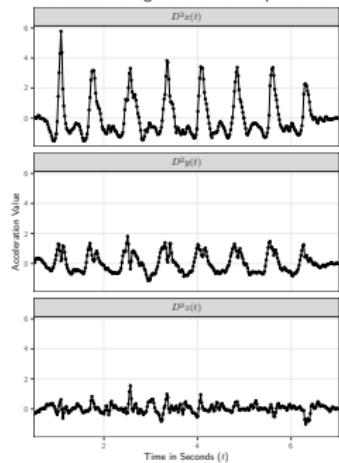


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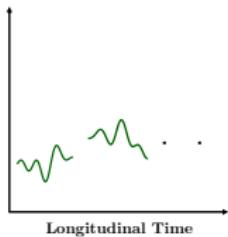
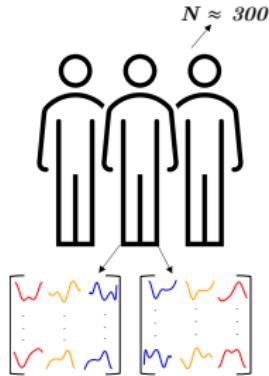
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Complexity

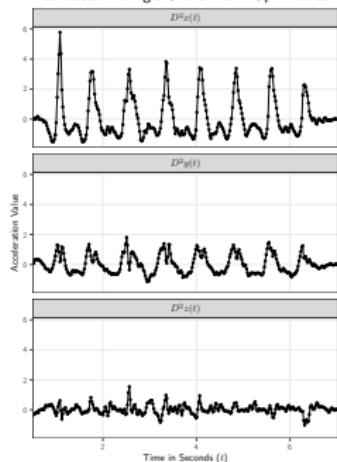


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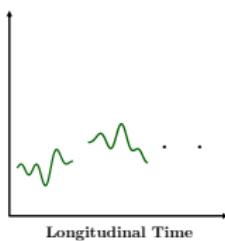
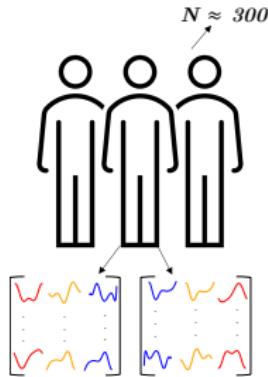
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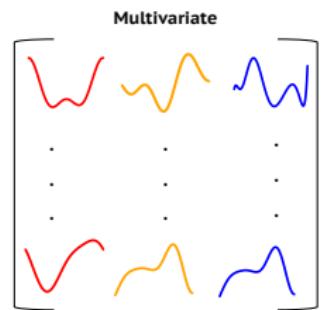
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Complexity



Variety



Human movement as a system of multiple related parts → **multivariate** and **dynamical systems** approaches to analysis.

Compare and Contrast

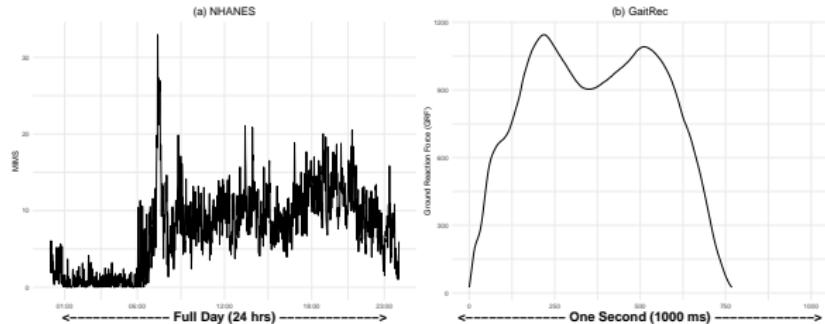


Figure: (a) NHANES (Crainiceanu, Goldsmith, et al., 2024); (b) GaitRec (Horsak et al., 2020).

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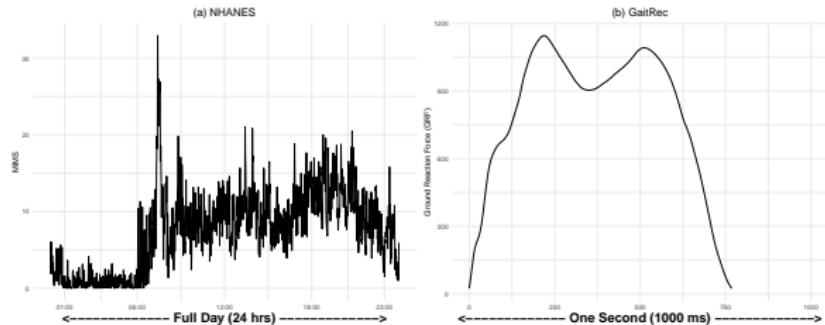


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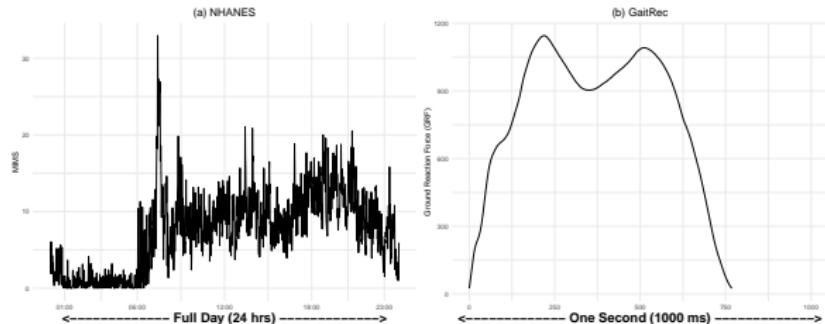


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- Movement on macro vs. micro scales.

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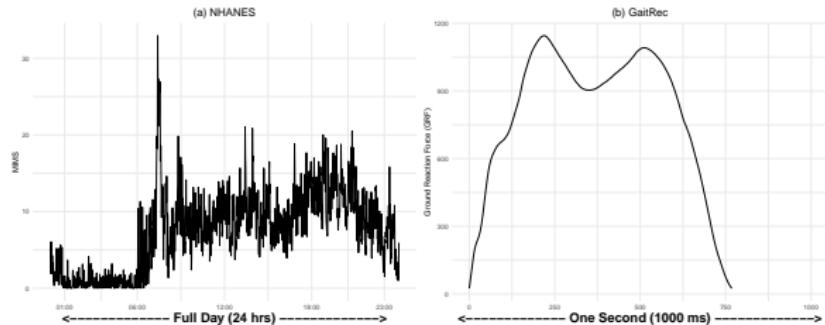


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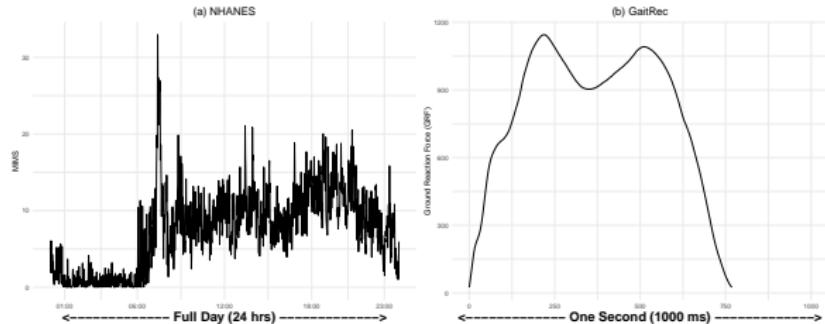


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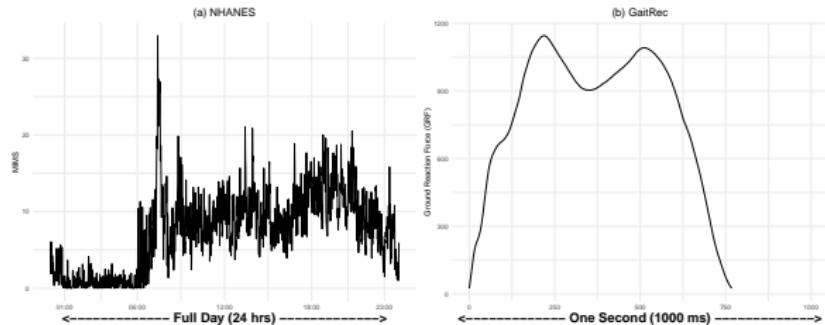


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- Complex temporal features (scalar summaries insufficient).

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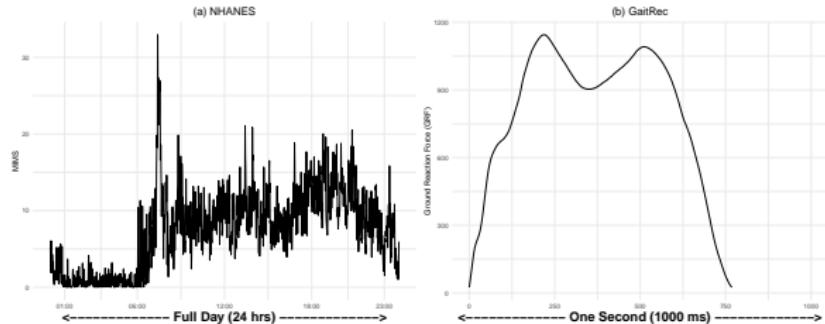


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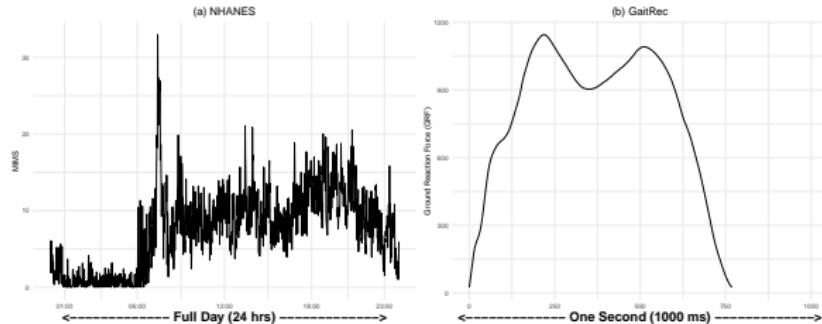


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→ We need principled and efficient statistical modelling approaches to extract information from rich, complex and structured functional data.

GaitRec (Horsak et al., 2020)

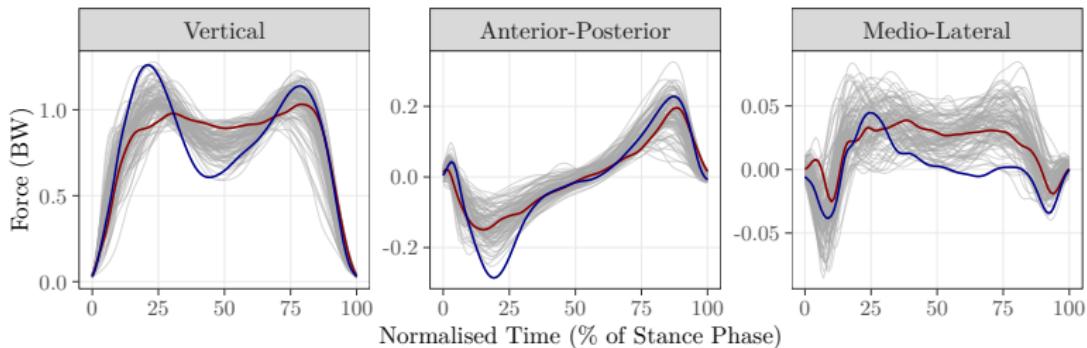


Figure: A sample of observations from the GaitRec dataset (Horsak et al., 2020).

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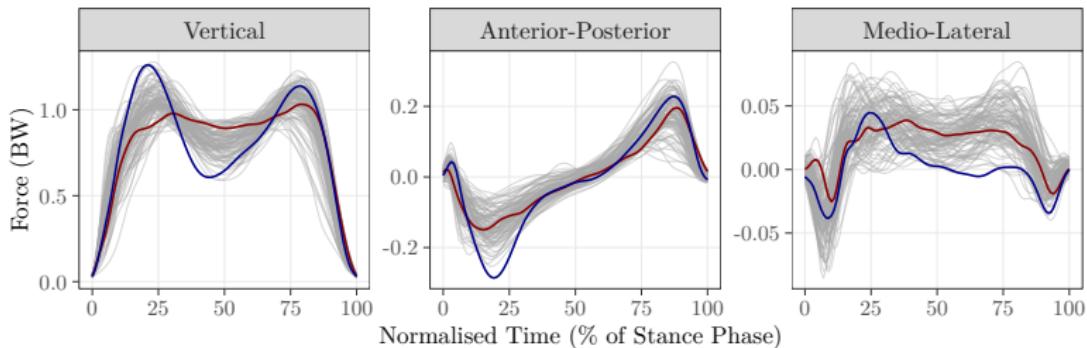


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- Ground reaction force and centre of pressure during walking (kinetics).

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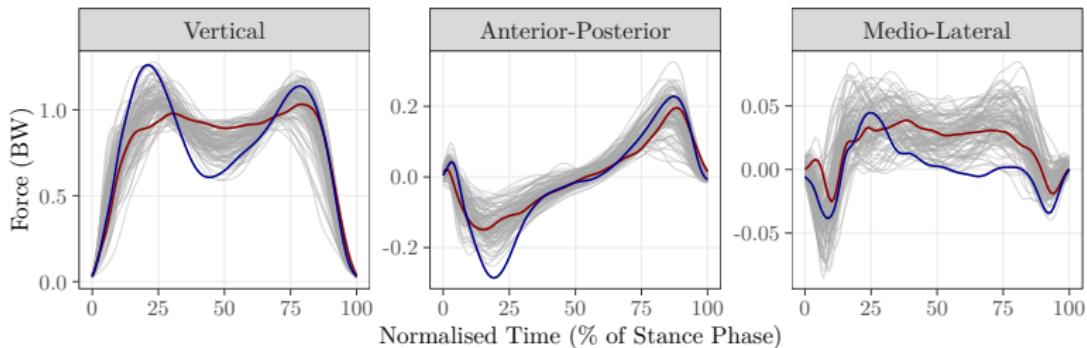


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- 2,295 individuals measured under different conditions (75,732 trials in total).

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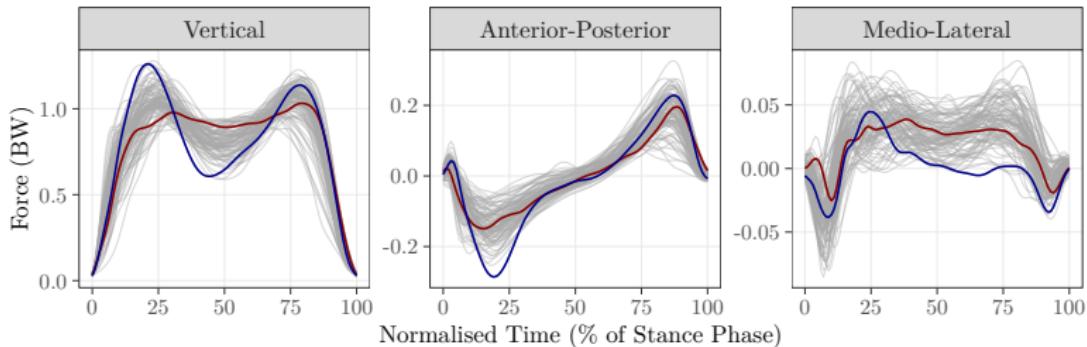
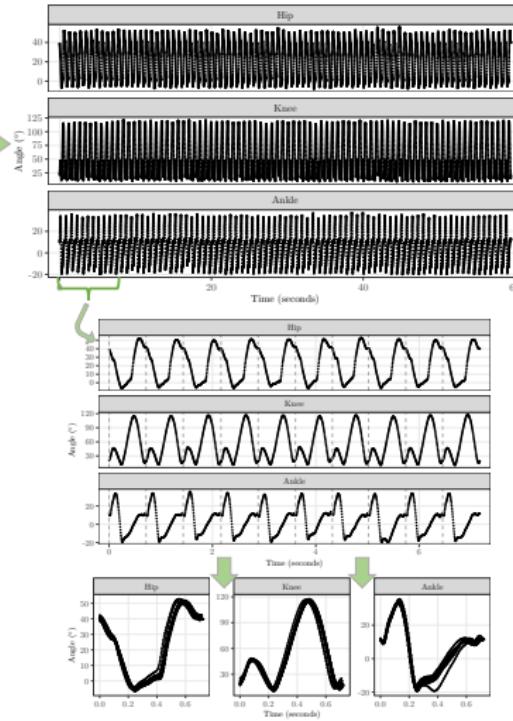


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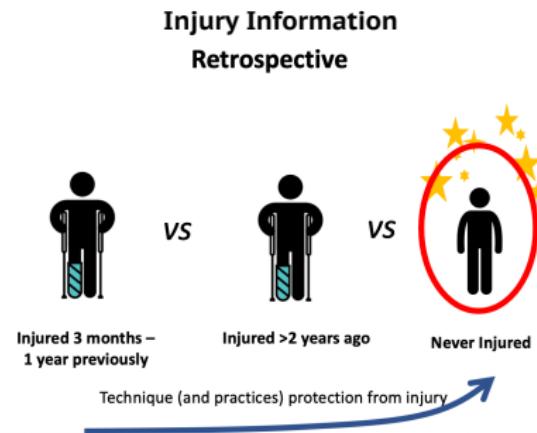
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- 2,295 individuals measured under different conditions (75,732 trials in total).
- Healthy controls and individuals with impairments in the hip, knee, ankle and calcaneus.

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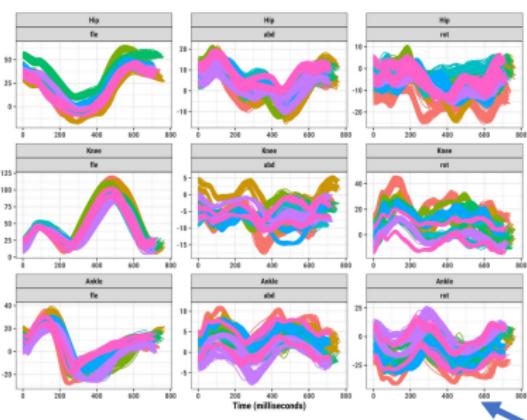


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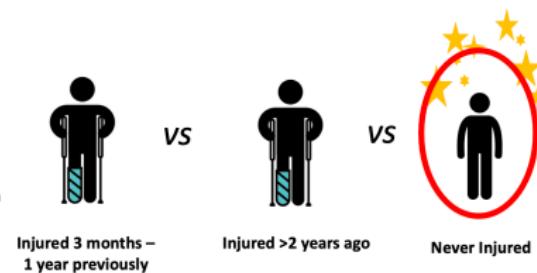


Motivating Dataset

Multivariate Functional Data



Injury Information
Retrospective



Covariates Affecting Running Technique



adjust for

Initial Approach (Gunning et al., 2024b)

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$$\begin{array}{c} \text{Y}(t) \\ \left[\begin{array}{cc} \text{green wavy line} & \text{green wavy line} \\ \text{green wavy line} & \text{green wavy line} \\ \vdots & \vdots \\ \vdots & \vdots \\ \text{blue wavy line} & \text{blue wavy line} \\ \text{blue wavy line} & \text{blue wavy line} \end{array} \right] \end{array} = \begin{array}{c} \textbf{X} \\ \left[\begin{array}{cc} 1 & \textcolor{red}{1.5} \\ 1 & \textcolor{red}{0} \\ \vdots & \vdots \\ \vdots & \vdots \\ 1 & \textcolor{red}{1.2} \\ 1 & \textcolor{red}{0} \end{array} \right] \end{array} + \begin{array}{c} \text{B}(t) \\ \left[\begin{array}{cc} \text{green wavy line} & \text{green wavy line} \\ \text{red wavy line} & \text{red wavy line} \end{array} \right] \end{array} + \begin{array}{c} \textbf{Z} \\ \left[\begin{array}{ccc} \textcolor{green}{1} & \cdots & 0 \\ \textcolor{green}{1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \\ 0 & \cdots & 1 \end{array} \right] \end{array} + \begin{array}{c} \textbf{U}(t) \\ \left[\begin{array}{cc} \text{green wavy line} & \text{green wavy line} \\ \vdots & \vdots \\ \text{blue wavy line} & \text{blue wavy line} \end{array} \right] \end{array} + \begin{array}{c} \textbf{E}(t) \\ \left[\begin{array}{cc} \text{green wavy line} & \text{green wavy line} \\ \text{blue wavy line} & \text{blue wavy line} \\ \vdots & \vdots \\ \vdots & \vdots \\ \text{blue wavy line} & \text{blue wavy line} \\ \text{blue wavy line} & \text{blue wavy line} \end{array} \right] \end{array}$$

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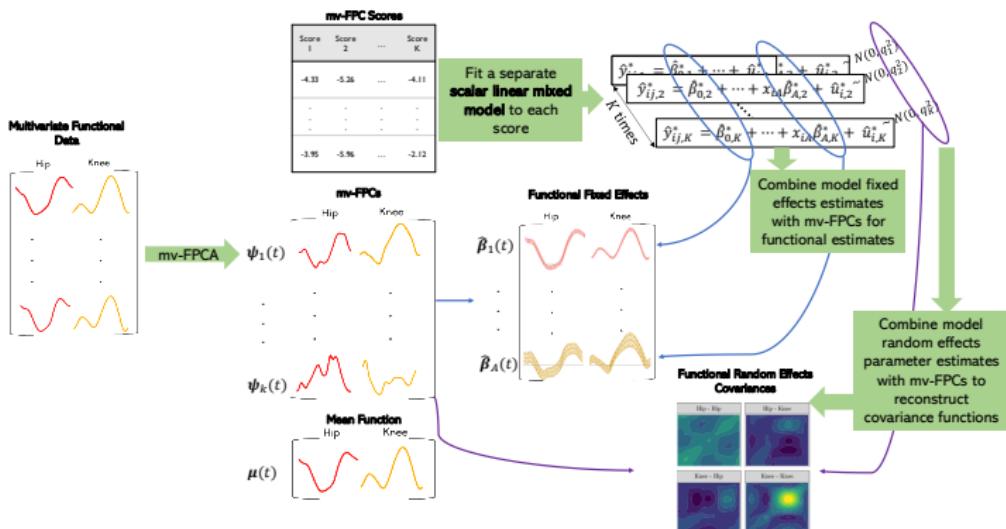
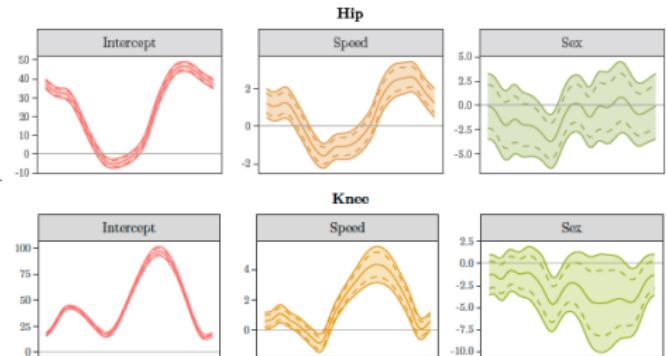


Figure: Flowchart of methodology.

Initial Approach (Gunning et al., 2024b)

Estimation and Inference

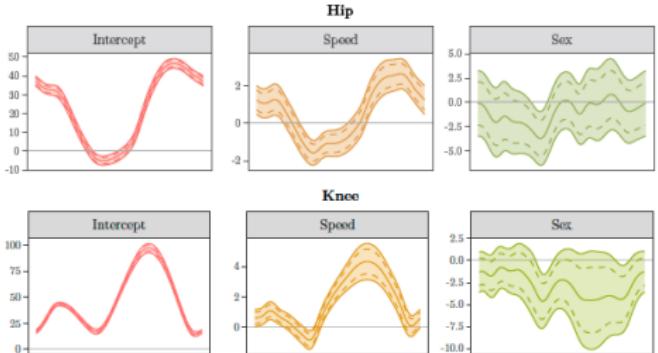
- Bayesian approach of Morris and Carroll (2006) uses MCMC for estimation and inference of functional parameters.
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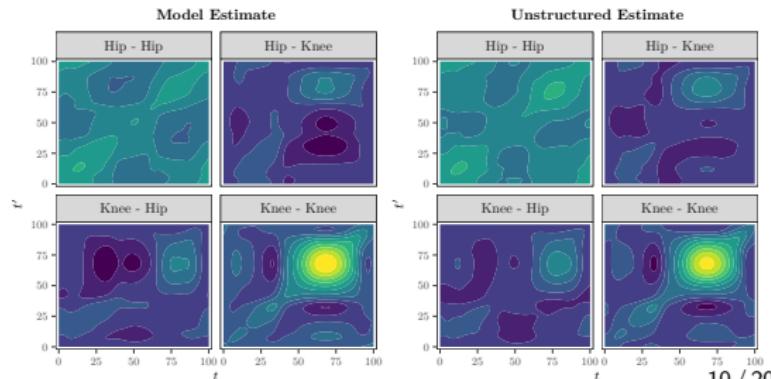
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Covariance Analysis

- Modelling each basis coefficient separately makes potentially limiting assumptions on the form of the functional random effects and error covariances.
- We propose to check this by using an extension of multilevel FPCA (Di et al., 2009) to calculate unstructured estimates to compare with.



Summary and Limitations of Initial Approach

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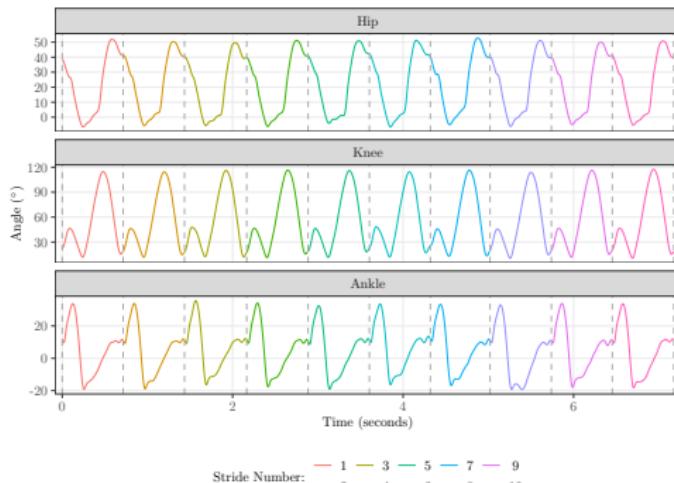
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- Scientific results consistent with and expand upon existing biomechanical literature (e.g., based on scalar values).
- **However, working with an average stride is potentially wasteful.** For just a one-minute run, this is a data reduction of almost 100 : 1.



Multivariate Multilevel Longitudinal Functional Model (Gunning et al., 2024a)

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- Hip, knee and ankle → **multivariate**.
- Multiple strides measured bilaterally for each subject → **multilevel**.
- Strides have a natural time ordering → **longitudinal**.

Multivariate Multilevel Longitudinal Functional Model (Gunning et al., 2024a)

Our multivariate multilevel longitudinal functional model is

$$\mathbf{y}_{ijl}(t) = \boldsymbol{\beta}_0(t, T_{ijl}) + \sum_{a=1}^A x_{ija} \boldsymbol{\beta}_a(t) + \mathbf{u}_i(t, T_{ijl}) + \mathbf{v}_{ij}(t, T_{ijl}) + \boldsymbol{\varepsilon}_{ijl}(t),$$

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$$\mathbf{y}_{ijl}(t) = \underbrace{\beta_0(t, T_{ijl})}_{\text{Intercept}} + \sum_{a=1}^A x_{ija}\beta_a(t) + \mathbf{u}_i(t, T_{ijl}) + \mathbf{v}_{ij}(t, T_{ijl}) + \varepsilon_{ijl}(t),$$

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Again, take a basis modelling approach¹ by representing

$$\mathbf{y}_{ijl}(t) = \sum_{k=1}^K y_{ijl,k}^* \psi_k(t),$$

where $y_{ijl,k}^*$ are scalar basis coefficients and $\psi_k(t)$ are multivariate basis functions in the *functional direction* (we choose mv-FPCs).

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Reduces the problem to fitting a series of K mixed models of the form:

$$y_{ijl,k}^* = \beta_{0,k}^*(T_{ijl}) + \sum_{a=1}^A x_{ia} \beta_{a,k}^* + u_{i,k}^*(T_{ijl}) + v_{ij,k}^*(T_{ijl}) + \varepsilon_{ijl,k}^*,$$

¹Similar idea proposed in univariate two-level case by Park and Staicu (2015).

Multivariate Multilevel Longitudinal Functional Model (Gunning et al., 2024a)

Again, take a basis modelling approach¹ by representing

$$\mathbf{y}_{ijl}(t) = \sum_{k=1}^K y_{ijl,k}^* \psi_k(t),$$

where $y_{ijl,k}^*$ are scalar basis coefficients and $\psi_k(t)$ are multivariate basis functions in the *functional direction* (we choose mv-FPCs).

Reduces the problem to fitting a series of K mixed models of the form:

$$y_{ijl,k}^* = \beta_{0,k}^*(T_{ijl}) + \sum_{a=1}^A x_{ia} \beta_{a,k}^* + u_{i,k}^*(T_{ijl}) + v_{ij,k}^*(T_{ijl}) + \varepsilon_{ijl,k}^*,$$

which is a standard multilevel functional model (Di et al., 2009) in the *longitudinal direction*.

¹ Similar idea proposed in univariate two-level case by Park and Staicu (2015).

Multivariate Multilevel Longitudinal Functional Model (Gunning et al., 2024a)

Again, we use basis expansions, this time in the longitudinal direction:

$$\beta_{0,k}^*(T) = \sum_{d=1}^D \beta_{0,k,d}^* \xi_d(T), \quad u_{i,k}^*(T) = \sum_{d=1}^D u_{i,k,d}^* \xi_d(T) \quad \text{and} \quad v_{ij,k}^*(T) = \sum_{d=1}^D v_{ij,k,d}^* \xi_d(T).$$

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Question: Which basis functions should we use for $u_{i,k}^*(T)$ and $v_{ij,k}^*(T)$?

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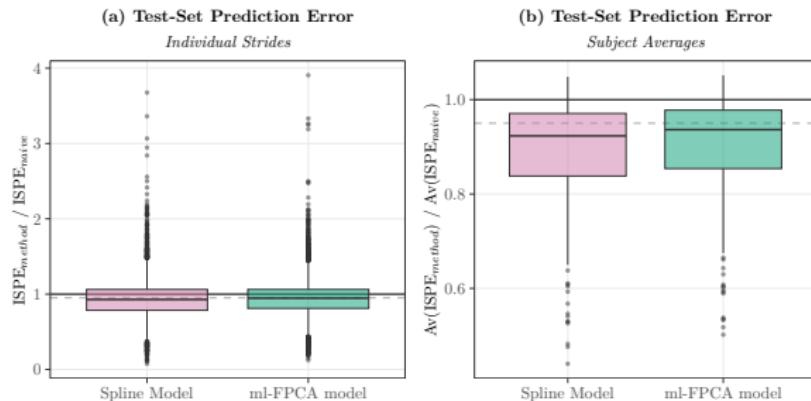


Figure: Representation using pre-specified spline (pink) and empirically-determined ml-FPCA basis (Cui, Li, et al., 2023, green) basis functions gives similar results.

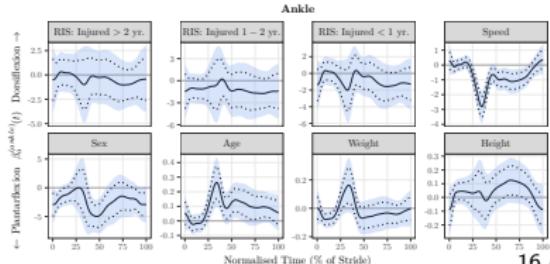
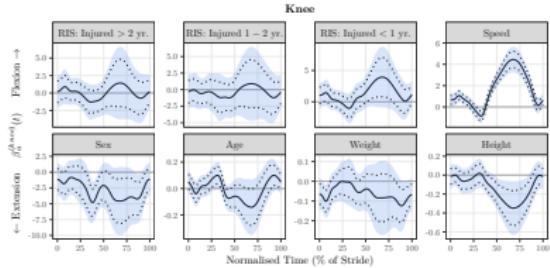
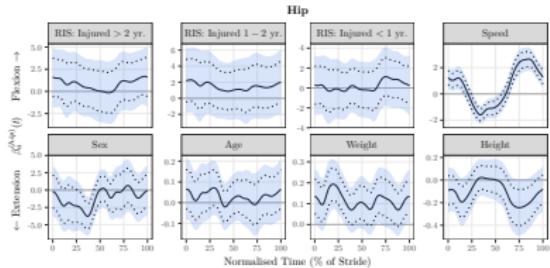
Results: Fixed Effects

- Fixed effects estimates

$$\hat{\beta}_a(t) = \sum_{k=1}^K \hat{\beta}_{a,k}^* \psi_k(t).$$

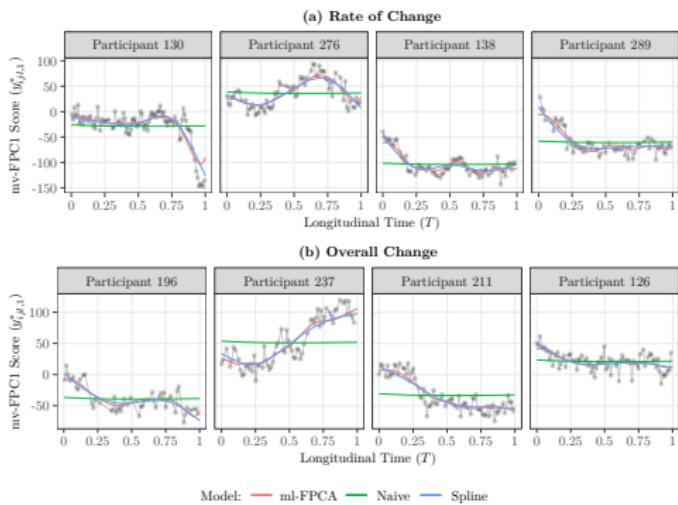
- Simultaneous bands account for multiple comparisons across t and the hip, knee and ankle. Obtained using bootstrap and simulation approaches^a.
- Scientific results consistent with existing biomechanical knowledge.

^aSee Faraway (1997), Ruppert, Wand, and Carroll (2003), Crainiceanu, Staicu, et al. (2012), Park, Staicu, et al. (2018), and Cui, Leroux, et al. (2022).



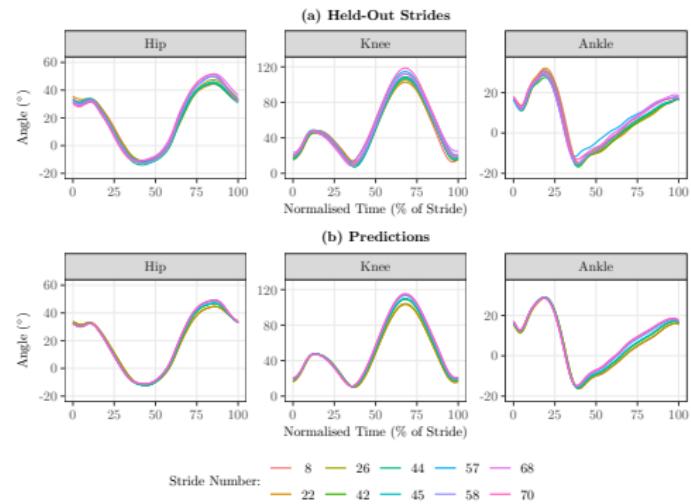
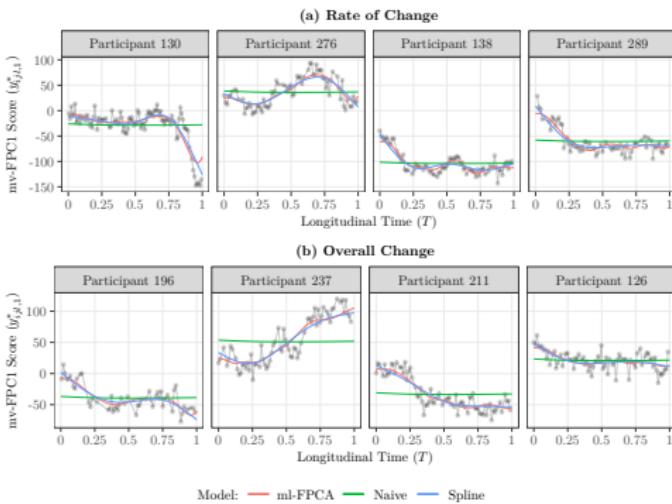
Results: Random Effects

Rates of change of $\hat{\mathbf{u}}_i(t, T)$ and $\hat{\mathbf{v}}_{ij}(t, T)$ w.r.t. T characterise changes over the course of the treadmill run.



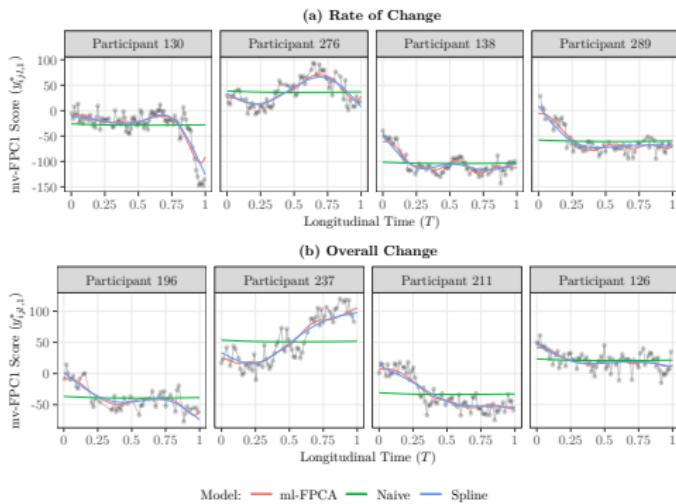
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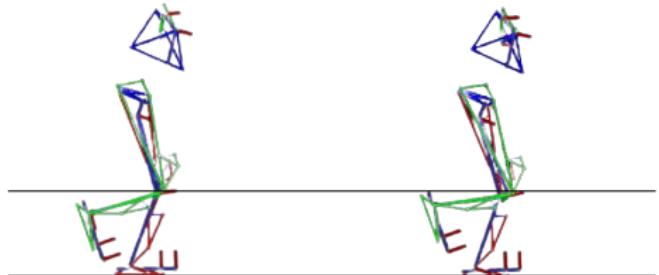


Results: Random Effects

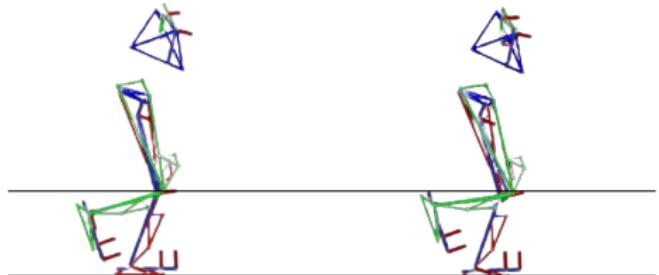
Rates of change of $\hat{\mathbf{u}}_i(t, T)$ and $\hat{\mathbf{v}}_{ij}(t, T)$ w.r.t. T characterise changes over the course of the treadmill run.



(c) Start of Treadmill Run



(d) End of Treadmill Run



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- Simple and flexible approach for modelling streams of smooth multivariate functional data that arise in biomechanics.
- Characterise population-level fixed effects and intra-individual longitudinal changes during the treadmill run – practically meaningful insights.
- Potential to use the model in different contexts and extend it to more complex settings – we are only “scratching the surface”.

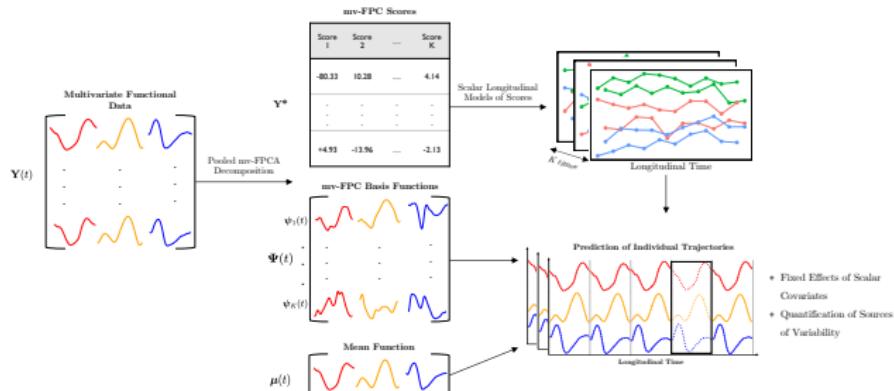


Figure: Summary of the modelling approach.

Thank you!

More Information

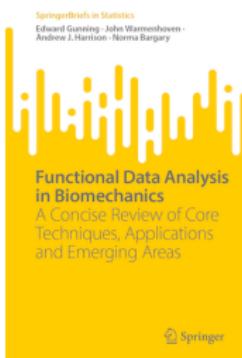
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- E Gunning et al. (2024b). *Analysing Kinematic Data from Recreational Runners using Functional Data Analysis*. arXiv:2408.08200 [stat]
- E Gunning et al. (2024a). “A Multivariate Multilevel Longitudinal Functional Model for Repeatedly Observed Human Movement Data”. In: arXiv:2408.08481 [stat]

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Website: <https://edwardgunning.github.io/>

Upcoming Book:



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