Functional Regression Models in Human Movement Biomechanics

Methodology and Applications

Edward Gunning

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Background

Winter (1979) defines **human movement biomechanics** as

> *"...the inter-discipline that describes, analyses, and assesses human movement"*

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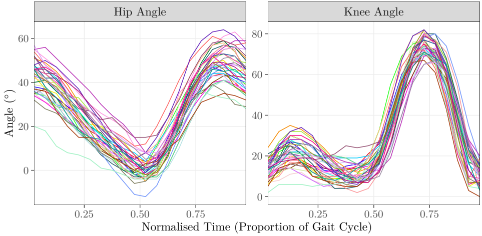

Figure: The childrens' gait dataset (Rice and Silverman, 1991).

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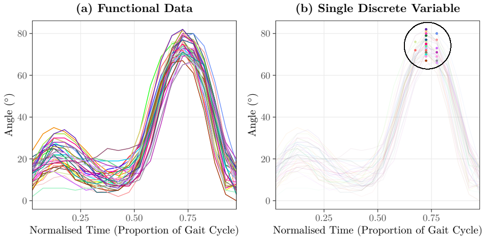
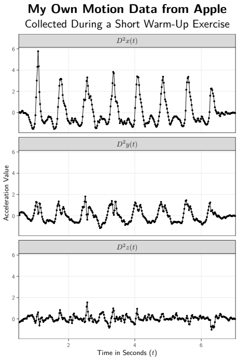


Figure: Data reduction of the knee angle curve to a single dicrete variable (peak knee flexion).

Second-Generation Functional Data in Biomechanics

Volume



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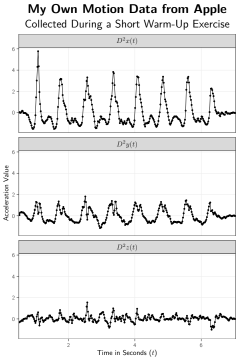

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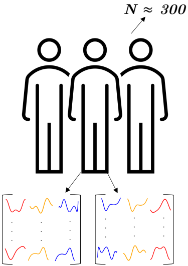

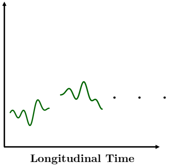

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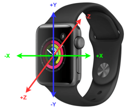

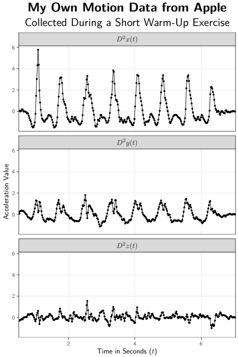

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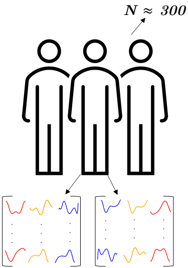
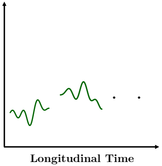
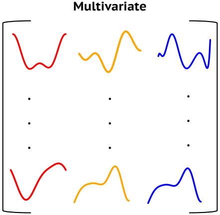




Variety



Human movement as a system of multiple related parts $\rightarrow$ **multivariate** and **dynamical systems** approaches to analysis.

Compare and Contrast

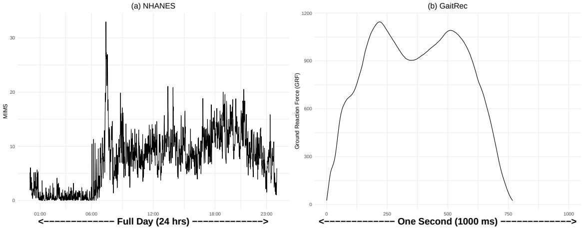


Figure: **(a)** NHANES (Crainiceanu, Goldsmith, et al., 2024); **(b)** GaitRec (Horsak et al., 2020).

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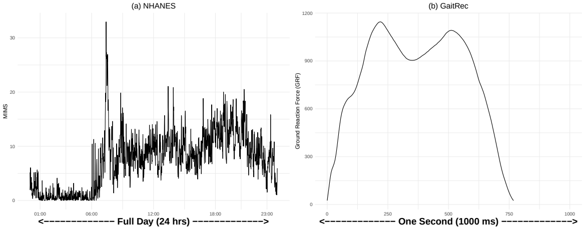

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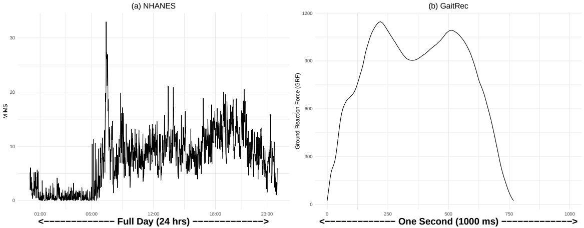

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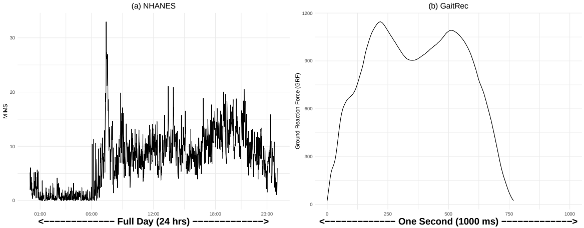

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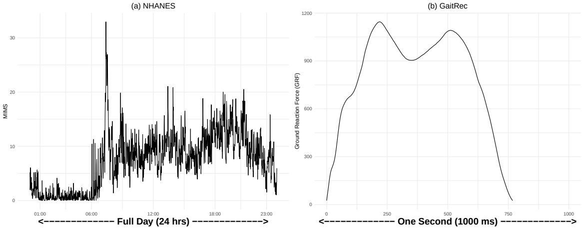

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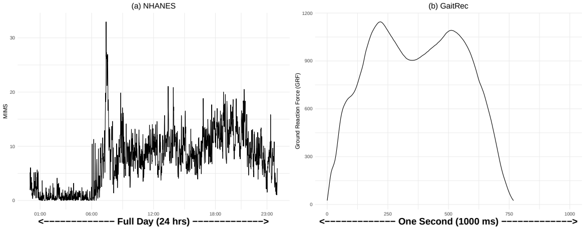

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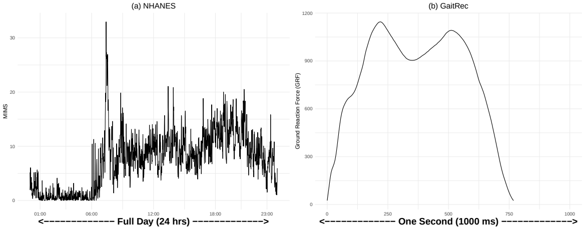

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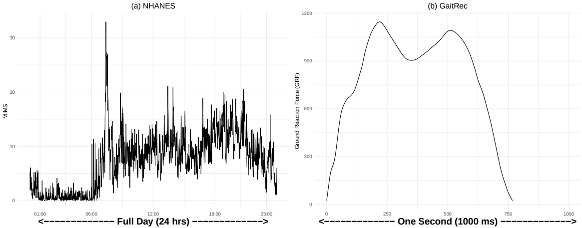

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→ *We need principled and efficient statistical modelling approaches to extract information from rich, complex and structured functional data.*

GaitRec (Horsak et al., 2020)

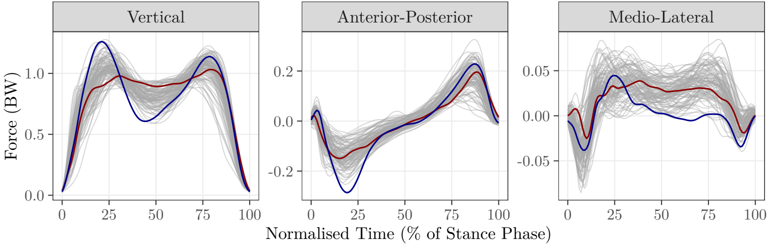

Figure: A sample of observations from the GaitRec dataset (Horsak et al., 2020).

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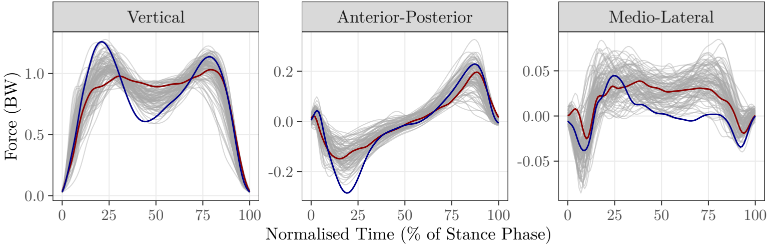

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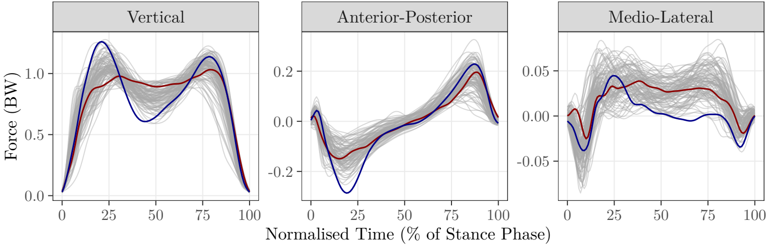

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- Ground reaction force and centre of pressure during walking (kinetics).
- 2,295 individuals measured under different conditions (75,732 trials in total).

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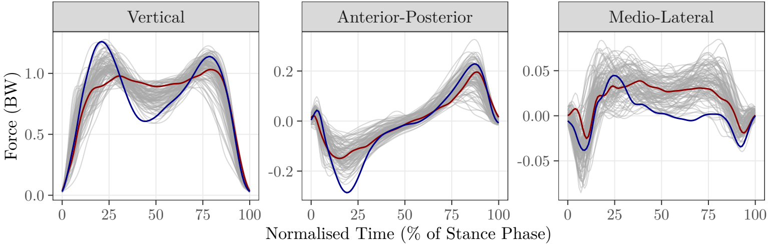

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- Ground reaction force and centre of pressure during walking (kinetics).
- 2,295 individuals measured under different conditions (75,732 trials in total).
- Healthy controls and individuals with impairments in the hip, knee, ankle and calcaneus.

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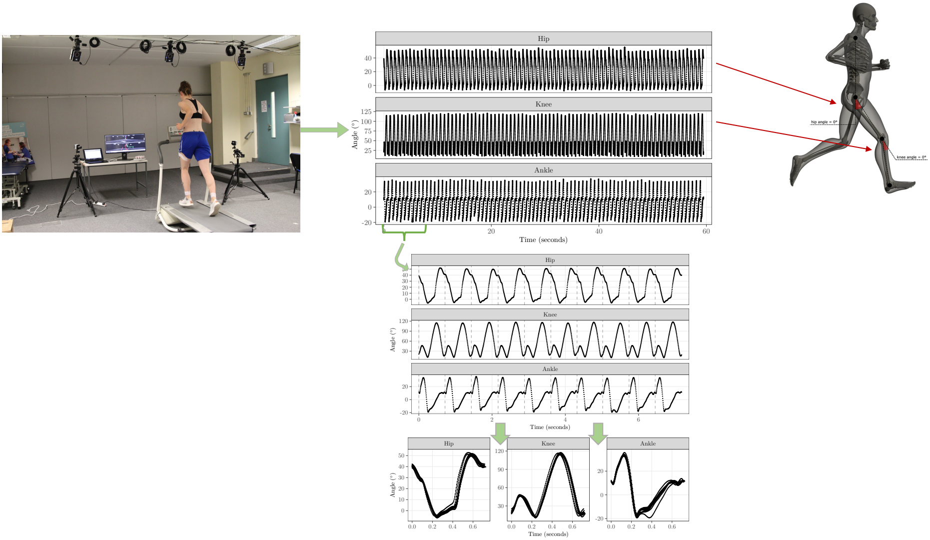

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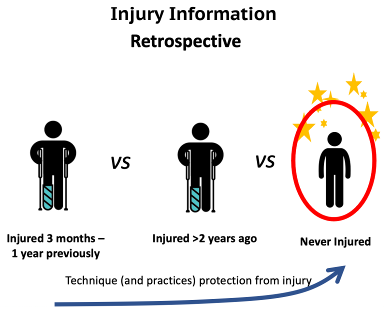
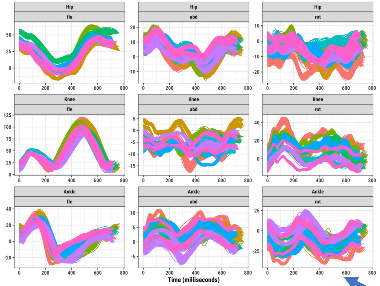


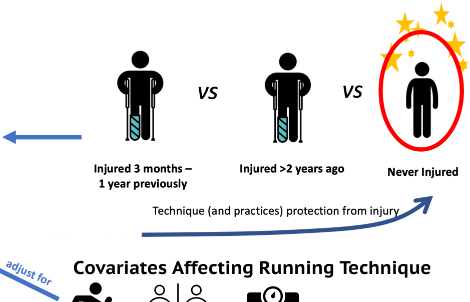
Motivating Dataset

Multivariate Functional Data

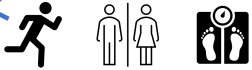


Injury Information

Retrospective



Covariates Affecting Running Technique



Initial Approach (Gunning et al., 2024b)

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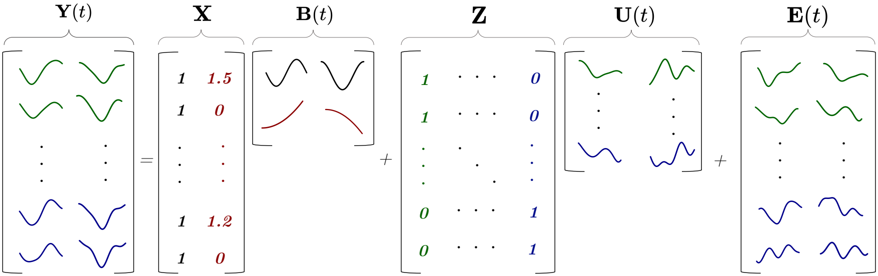

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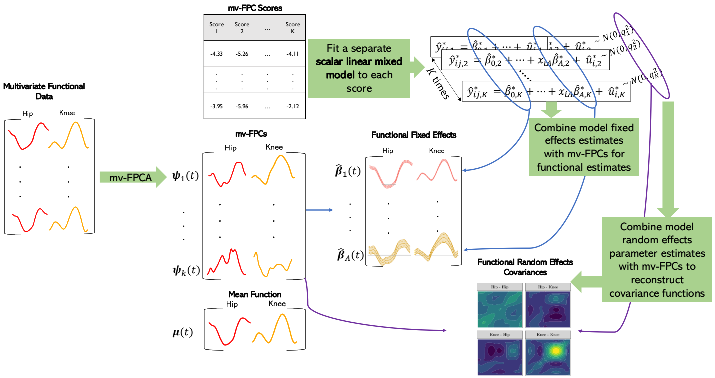
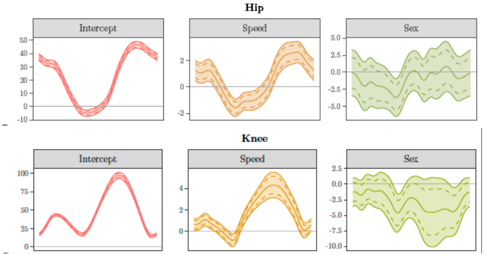


Figure: Flowchart of methodology.

Initial Approach (Gunning et al., 2024b)

Estimation and Inference

- Bayesian approach of Morris and Carroll (2006) uses MCMC for estimation and inference of functional parameters.
- We adapt resampling and simulation ideas from Crainiceanu, Staicu, et al. (2012) to implement in a frequentist setting using `lme4`.



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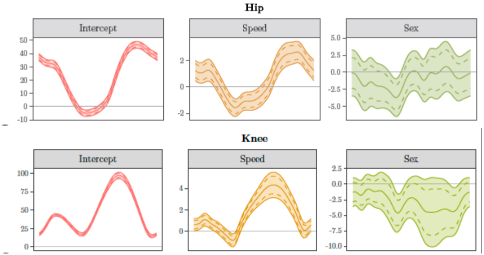
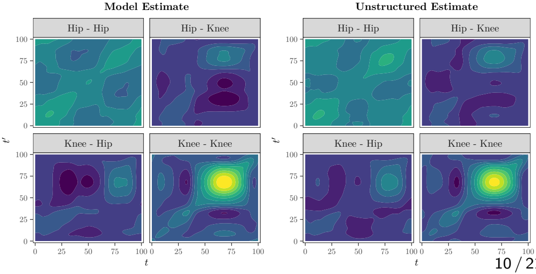


Covariance Analysis

- Modelling each basis coefficient separately makes potentially limiting assumptions on the form of the functional random effects and error covariances.
- We propose to check this by using an extension of multilevel FPCA (Di et al., 2009) to calculate unstructured estimates to compare with.



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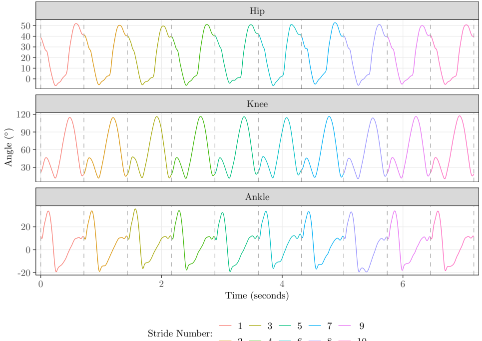
- Scientific results consistent with and expand upon existing biomechanical literature (e.g., based on scalar values).
- **However, working with an average stride is potentially wasteful.** For just a one-minute run, this is a data reduction of almost 100 : 1.



Multivariate Multilevel Longitudinal Functional Model (Gunning et al., 2024a)

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- Hip, knee and ankle $\rightarrow$ **multivariate**.
- Multiple strides measured bilaterally for each subject $\rightarrow$ **multilevel**.
- Strides have a natural time ordering $\rightarrow$ **longitudinal**.

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Our multivariate multilevel longitudinal functional model is

$$\mathbf{y}_{ijl}(t) = \boldsymbol{\beta}_0(t, T_{ijl}) + \sum_{a=1}^{A} x_{ija}\boldsymbol{\beta}_a(t) + \mathbf{u}_i(t, T_{ijl}) + \mathbf{v}_{ij}(t, T_{ijl}) + \boldsymbol{\varepsilon}_{ijl}(t),$$

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Again, take a basis modelling approach[1] by representing

$$\mathbf{y}_{ijl}(t) = \sum_{k=1}^{K} y^*_{ijl,k} \boldsymbol{\psi}_k(t),$$

where $y^*_{ijl,k}$ are scalar basis coefficients and $\boldsymbol{\psi}_k(t)$ are multivariate basis functions in the *functional direction* (we choose mv-FPCs).

[1] Similar idea proposed in univariate two-level case by Park and Staicu (2015).

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which is a standard multilevel functional model (Di et al., 2009) in the *longitudinal direction*.

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Multivariate Multilevel Longitudinal Functional Model (Gunning et al., 2024a)

Again, we use basis expansions, this time in the longitudinal direction:

$$\beta^*_{0,k}(T) = \sum_{d=1}^{D} \beta^*_{0,k,d}\ \xi_d(T), \quad u^*_{i,k}(T) = \sum_{d=1}^{D} u^*_{i,k,d}\ \xi_d(T) \quad \text{and} \quad v^*_{ij,k}(T) = \sum_{d=1}^{D} v^*_{ij,k,d}\ \xi_d(T).$$

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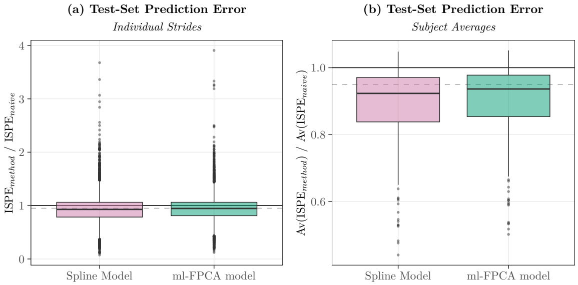


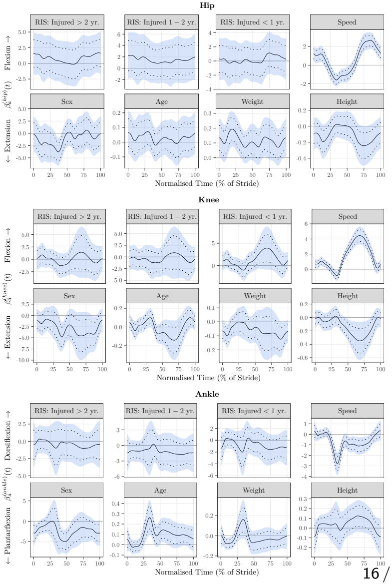
Figure: Representation using pre-specified spline (pink) and empirically-determined ml-FPCA basis (Cui, Li, et al., 2023, green) basis functions gives similar results.

Results: Fixed Effects

- Fixed effects estimates

$$\widehat{\beta}_a(t) = \sum_{k=1}^{K} \widehat{\beta}^*_{a,k}\psi_k(t).$$

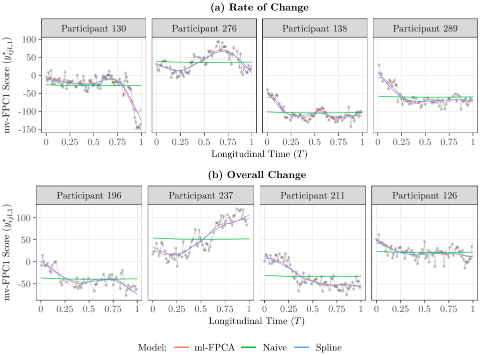
- Simultaneous bands account for multiple comparisons across t and the hip, knee and ankle. Obtained using bootstrap and simulation approaches[a].
- Scientific results consistent with existing biomechanical knowledge.



[a] See Faraway (1997), Ruppert, Wand, and Carroll (2003), Crainiceanu, Staicu, et al. (2012), Park, Staicu, et al. (2018), and Cui, Leroux, et al. (2022).

Results: Random Effects

Rates of change of $\widehat{\mathbf{u}}_i(t, T)$ and $\widehat{\mathbf{v}}_{ij}(t, T)$ w.r.t. T characterise changes over the course of the treadmill run.



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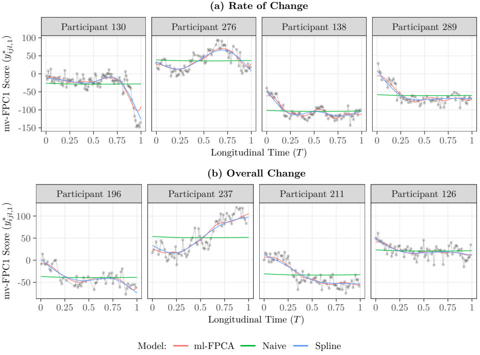

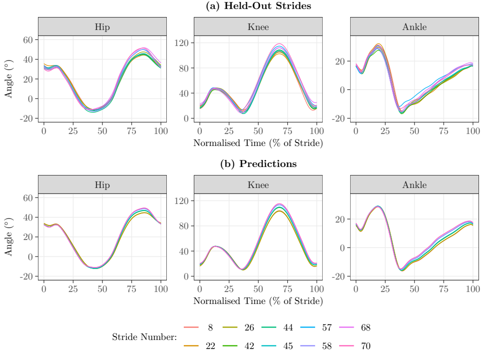

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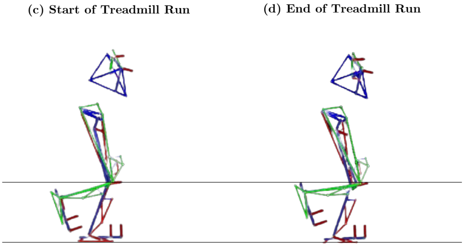

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- Simple and flexible approach for modelling streams of smooth multivariate functional data that arise in biomechanics.
- Characterise population-level fixed effects and intra-individual longitudinal changes during the treadmill run – practically meaningful insights.
- Potential to use the model in different contexts and extend it to more complex settings – we are only "scratching the surface".

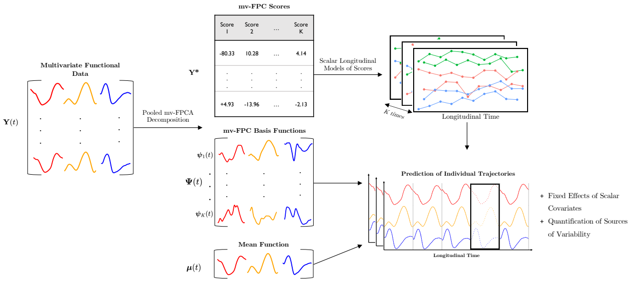

Figure: Summary of the modelling approach.

Thank you!

More Information

Literature:

- E Gunning et al. (2024b). *Analysing Kinematic Data from Recreational Runners using Functional Data Analysis*. arXiv:2408.08200 [stat]
- E Gunning et al. (2024a). "A Multivariate Multilevel Longitudinal Functional Model for Repeatedly Observed Human Movement Data". In: arXiv:2408.08481 [stat]

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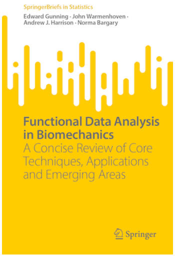
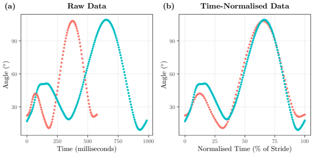
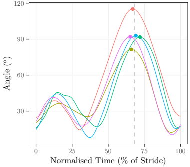
Website: `https://edwardgunning.github.io/`
Upcoming Book:

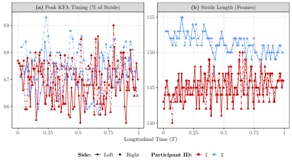


Future Work & Current Interests

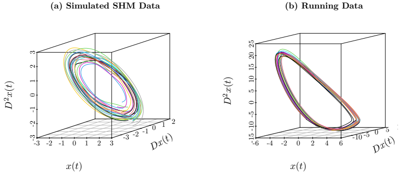
Registration for Structured Models

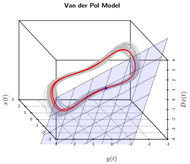






Functional Regression for ODEs





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