Seminar 1.

META-LANGUAGES

Ex.1: Specify, using BNF, all nonempty sequences of letters

<nonemptyletterseq> ::= <letter> | <letter><nonemptyletterseq>
<letter> ::= a | b | c | … | z | A | B | … | Z

Ex.2: Specify, using BNF, both signed and unsigned integers, with the following constraints:

- 0 does not have a sign
- numbers of at least two digits cannot start with 0

<zero> ::= 0
<nonzero_digit> :: = 1 | 2 | … | 9
<sign> ::= + | -
<digit> ::= <zero> | <nonzero_digit>
<digit_seq> ::= <digit> | <digit><digit_seq>
<abs_val> ::= <nonzero_digit> | <nonzero_digit><digit_seq>
<int> ::= <zero> | <abs_val> | <sign><abs_val>

12, +12, -12, 0
C1: ~~-0~~, ~~+0~~
C2: ~~012~~

Ex.3: Ex. 2 reloaded, in EBNF

int = ["+" | "-"]not_zero{digit} | "0"
not_zero = "1" | … | "9"
digit = not_zero | "0"

Sample mini-language spec

id = letter {letter | digit}
const int a = 4;
?

"abc" 'a' ~~'X'~~ '␣'
'$' """

Seminar 2.

SCANNING

input: source.txt
tokens file

output: PIF + ST +
lex err (if any)

```
VAR     a: integer;
        b: integer;
        c: integer;
BEGIN
        a := 10;
        b := a + 10;
        WRITE("A message: ");
        WRITE(b);
END.
```

Program Internal Form = array of pairs

symbol table

PIF

token	ST_pos
VAR	-1
id	0
:	-1
integer	-1
;	-1
id	1
:	-1
integer	-1
;	-1
id	2
:	-1
string	-1
BEGIN	-1
id	0
:=	-1

ST

ST_pos	symbol
0	a
1	b
2	c
3	10
4	"..."

only identifiers and constants

constant	3
;	-1
id	1
:=	-1
id	0
+	-1
constant	3
;	-1
WRITE	-1
(	-1
constant	4
)	-1
;	-1
WRITE	-1
(	-1
id	1
)	-1
;	-1
END	-1
.	-1

Lexical errors

1) 1a → "wrong id"
2) # → "illegal alphabet char"
3) "A msg
4) " , 'a, 'ab'

} not enough quotes

'ab' → this is a string not a char

b := ~~2~~ ???

Seminar 3.

GRAMMARS

$(ab)^2 = abab$
$a^2 b^2 = aabb$
$(ab)^2 \neq a^2 b^2$

1. Given the grammar $G = \{N, \Sigma, P, S\}$
$N = \{S, C\}$, $\Sigma = \{a, b\}$

$P: S \to ab^{1} \mid aCSb^{2}$
$C \to S^{3} \mid bSb^{4}$
$CS \to b^{5}$

a) prove that $w = ab(ab^2)^2 \in L(G)$

$S \underset{2}{\Rightarrow} aCSb \underset{4}{\Rightarrow} abSbSb \underset{1}{\Rightarrow} ababbSb \underset{1}{\Rightarrow} ababbabb \Rightarrow$

$\Rightarrow ab(ab^2)^2 = w$

b) $w = a^2bab^2$ $\quad w \in L(G)$?

$S \underset{2}{\Rightarrow} aCSb \underset{3}{\Rightarrow} aSSb \underset{1}{\Rightarrow} aabSb \underset{1}{\Rightarrow} aababb \Rightarrow a^2bab^2 = w$

2. $L = \{a^{2n}bc \,;\, n >= 0\}$, $N = \{S\}$, $\Sigma = \{a, b, c\}$
$P: S \to a^2S \mid bc$

$S \Rightarrow a^2S \Rightarrow a^2bc$
$S \Rightarrow a^2S \Rightarrow a^2a^2S \Rightarrow a^2a^2bc$
$S \Rightarrow a^2S \Rightarrow a^2a^2S \Rightarrow a^2a^2S \Rightarrow a^2a^2a^2S \Rightarrow a^2a^2a^2bc$
$S \Rightarrow bc$

? $L = L(G)$

1. ? $L \subseteq L(G)$

$\forall n \in \mathbb{N},\ a^{2n}bc \in L(G)$

$P(n): a^{2n}bc \in L(G)$

? $P(n)$ true, $\forall n \in \mathbb{N}$ math induction

$=$	equal
$\rightarrow$	production
$\Rightarrow$	derivation

I. verification step

$P(0): a^0bc = bc \in L(G)$

$S \Rightarrow bc \Rightarrow P(0)$ is true

II. proof step

Let's suppose for a given $k \in \mathbb{N}^*$ $P(k)$ is true. We will try to prove that $P(k+1)$ is also true.

$P(k) = \text{True} \Rightarrow a^{2k}bc \in L(G) \Rightarrow S \overset{*}{\Longrightarrow} a^{2k}bc$

$P(k+1) = a^{2(k+1)}bc$

$S \underset{1}{\Rightarrow} a^2S \underset{ind}{\Rightarrow} a^2a^{2k}bc = a^{2(k+1)}bc = P(k+1) \in L(G)$

So from I and II $\Rightarrow P(n)$ is true, $\forall n \in \mathbb{N}$

2. $L(G) \subseteq L$

$S \Rightarrow bc$

$\Rightarrow a^2S \Rightarrow a^2bc$

$\Rightarrow a^4S \Rightarrow a^4bc$

$\Rightarrow a^6S \ldots$

$\Rightarrow L(G) \subseteq L$

We can notice that using all productions in all possible combinations we only get as sequences of terminals sequences of shape $a^{2n}bc$, $\forall n \in \mathbb{N} \Rightarrow L(G) \subseteq L$

3. $L = \{0^n 1^n 2^m \mid n, m \in \mathbb{N}^*\}$, $G = (N, \Sigma, P, S)$

$N = \{S, R, P\}$

$\Sigma = \{0, 1, 2\}$

$(0\,1)^n \neq 0^n 1^n$

P: $S \to RP$

$R \to 0R1 \mid 01$

$P \to 2P \mid 2$

S: S

? $L = L(G)$

1. $L \subseteq L(G)$, $\forall\, n, m \in \mathbb{N}^*$

$0^n 1^n 2^m \in L(G)$

Let $n, m \in \mathbb{N}^*$ given

$$S \underset{1}{\Rightarrow} RP \underset{a)}{\overset{n}{\Longrightarrow}} 0^n 1^n P \underset{b)}{\overset{m}{\Longrightarrow}} 0^n 1^n 2^m \in L(G)$$

a) $R \overset{n}{\Longrightarrow} 0^n 1^n$, $\forall\, n \in \mathbb{N}^*$

b) $P \overset{m}{\Longrightarrow} 2^m$, $\forall\, m \in \mathbb{N}^*$

$\}$ needs proof

2. ? $L(G) \subseteq L$

a) $S \to RP$ is the only production of S

b) Tree for $R \Rightarrow R$ can only generate $0^n 1^n$, $n \in \mathbb{N}$

c) Tree for $P \Rightarrow P$ can only generate 2^m, $m \in \mathbb{N}$

$$a) + b) + c) \underset{\text{after proof}}{\Longrightarrow} L(G) \subseteq L$$

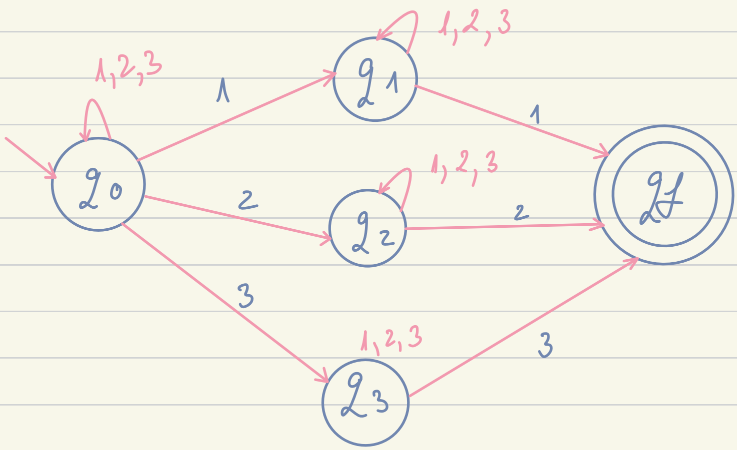
Seminar 4.

FINITE AUTOMATA (FA)

1. Given the FA: $M=(Q,\Sigma,\delta,q_0,F)$, $Q=\{q_0,q_1,q_2,q_3,q_f\}$, $\Sigma=\{1,2,3\}$, $F=\{q_f\}$

δ	1	2	3
q_0	$\{q_0,q_1\}$	$\{q_0,q_2\}$	$\{q_0,q_3\}$
q_1	$\{q_1,q_f\}$	$\{q_1\}$	$\{q_1\}$
q_2	$\{q_2\}$	$\{q_2,q_f\}$	$\{q_2\}$
q_3	$\{q_3\}$	$\{q_3\}$	$\{q_3,q_f\}$
q_f	$\emptyset$	$\emptyset$	$\emptyset$

Prove that $w=12321 \in L(M)$



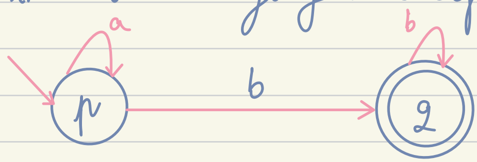
$(q_0, 12321) \vdash (q_1, 2321) \vdash^3 (q_1, 1) \vdash (q_f, \varepsilon) \Rightarrow$

$\Rightarrow \underbrace{(q_0, w)}_{i.c} \vdash^* \underbrace{(q_f, \varepsilon)}_{f.c} \Rightarrow w \in L(M)$

initial

final

2. Find the language accepted by the FA below:



$L = \{ a^n b^m \mid n \in \mathbb{N},\ m \in \mathbb{N}^* \}$

? $L = L(M)$

1) $L \subseteq L(M)$

2) $L(M) \subseteq L$

1) $L \subseteq L(M)$

$\forall\, n \in \mathbb{N},\ m \in \mathbb{N}^*,\ a^n b^n \in L(M)$

Let n, m - fixed, $n \in \mathbb{N}$, $m \in \mathbb{N}^*$

I) $(p, a^n) \vdash^{n} (p, \varepsilon)$, $\forall\, n \in \mathbb{N}$

II) $(q, b^k) \vdash^{k} (q, \varepsilon)$, $\forall\, k \in \mathbb{N}$

$(p, a^n b^m) \underset{(I)}{\vdash^{n}} (p, b^m) \vdash (q, b^{m-1}) \underset{(II)}{\vdash^{m-1}} (q, \varepsilon) \Rightarrow$

$\Rightarrow \quad a^n b^m \in L(M)$

I) $P(n) : (p, a^n) \vdash^{n} (p, \varepsilon)$, $n \in \mathbb{N}$

1) Verification step:

$P(0) : (p, a^0) \vdash^{0} (p, \varepsilon)$ ~~true~~

2) Proof step:

$P(k) : (p, a^k) \vdash^{k} (p, \varepsilon)$, we assume ~~true~~

$$P(k+1): (p, a^{k+1}) \vdash^{k+1} (p, \varepsilon)$$

$$(p, a^{k+1}) \vdash (p, a^{k}) \underset{P(k)}{\vdash^{k}} (p, \varepsilon) \Rightarrow (p, a^{k+1}) \vdash^{k+1} (p, \varepsilon)$$

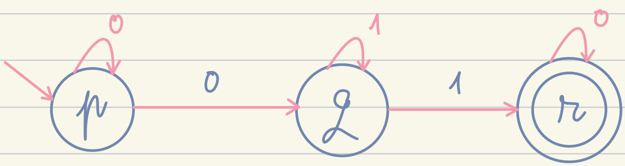
$\Rightarrow P(k+1)$ true

$a^n b b^m$, $n, m \in \mathbb{N}$

$\Updownarrow$

$a^n b^m$, $n \in \mathbb{N}$, $m \in \mathbb{N}^*$

3. a) $L_1 = \{0^n 1^m 0^q \mid n, m \in \mathbb{N}^*, q \in \mathbb{N}\}$

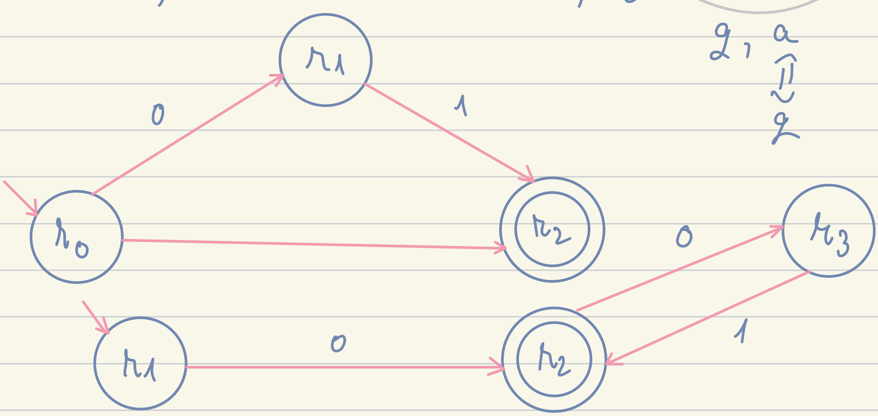


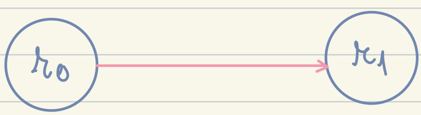
b) $L_2 = \{0(01)^n \mid n \in \mathbb{N}\}$, $\delta : \underbrace{Q \times \Sigma}_{} \longrightarrow \mathcal{P}(Q)$

q, a

$\Updownarrow$

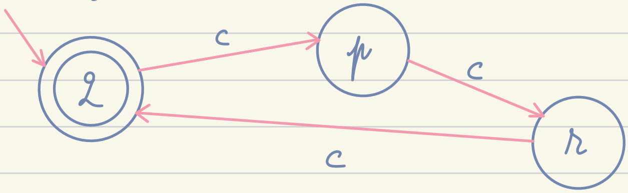
q

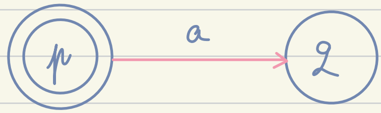




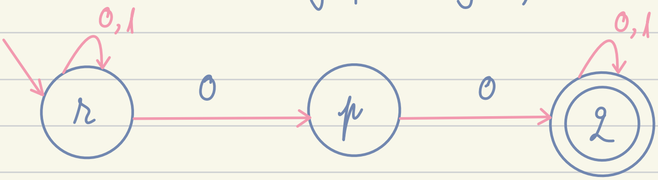
$\varepsilon \notin \Sigma$; $\varepsilon \in \Sigma^*$

c) $L_3 = \{ c^{3n} \mid n \in \mathbb{N} \}$

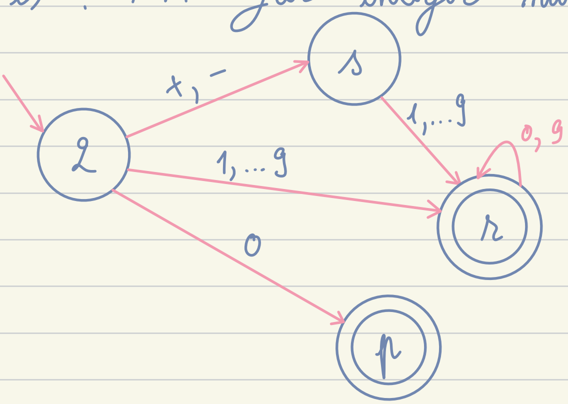


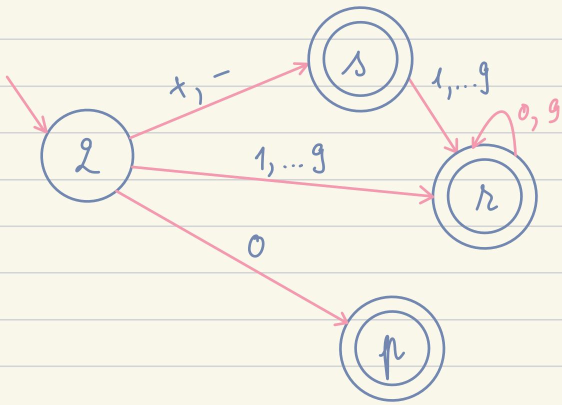


d) $L_4 = \{ x00y \mid x, y \in \{0,1\}^* \}$



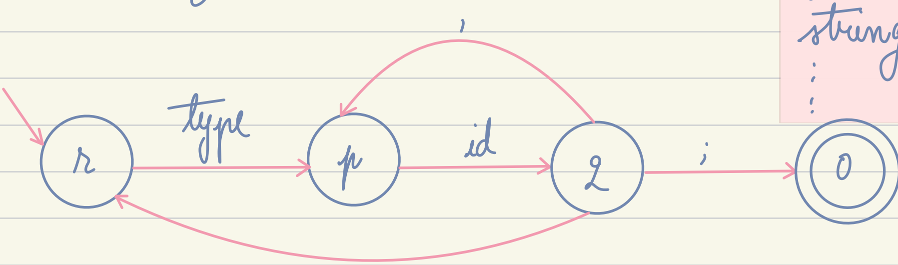
e) ? FA for integer numbers





f) ? FA for variable decl.

int a, b, ... ;
string c, d, ... ;
:
:



Seminar 5.

FA <=> RG <=> RE

I. 1. **FA <=> RG**

$$G = (\{S, A\}, \{a, b\}, P, S)$$

P: $S \to aA$

$\qquad A \to aA \mid bA \mid a \mid b$

? <=> FA

RG = RLG: $A \to aB$

$\qquad\qquad A \to b$

$\nexists\ A \to \varepsilon \in P$, $A \neq S$

if $S\ \varepsilon \in P$, then S does not appear in rhs

$\cancel{S \to \varepsilon}$

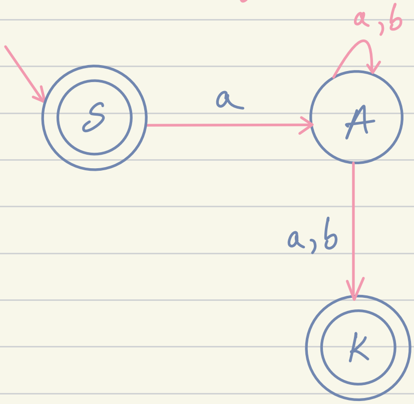
$\cancel{A \to aS}$

$$M = (\{S, A, K\}, \{a, b\}, \dots, \{S\}, \{K\})$$

δ: $\delta(S, a) = A$ $\qquad \delta(A, b) = A$

$\qquad \delta(A, a) = A$ $\qquad \delta(A, a) = K$

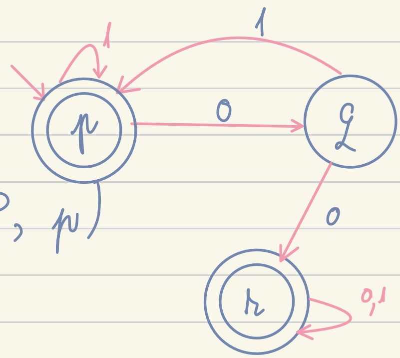
throw a we get A $\qquad \delta(A, b) = K$



Type	Regular Expression	RLG	LLG
Single terminal	e	$S \rightarrow e$	$S \rightarrow e$
Union operation	$(e+f)$	$S \rightarrow e \mid f$	$S \rightarrow e \mid f$
Concatenation	ef	$S \rightarrow eA,\ \ A \rightarrow f$	$S \rightarrow Af,\ \ A \rightarrow e$
Star closure	e^*	$S \rightarrow eS \mid \epsilon$	$S \rightarrow Se \mid \epsilon$
Plus closure	e^+	$S \rightarrow eS \mid e$	$S \rightarrow Se \mid e$
Star closure on union	$(e+f)^*$	$S \rightarrow eS \mid fS \mid \epsilon$	$S \rightarrow Se \mid Sf \mid \epsilon$
Plus closure on union	$(e+f)^+$	$S \rightarrow eS \mid fS \mid e \mid f$	$S \rightarrow Se \mid Sf \mid e \mid f$
Star closure on concatenation	$(ef)^*$	$S \rightarrow eA \mid \epsilon;$ $A \rightarrow fS$	$S \rightarrow Af \mid \epsilon;$ $A \rightarrow Se$
Plus closure on concatenation	$(ef)^+$	$S \rightarrow eA\,;$ $A \rightarrow fS \mid f$	$S \rightarrow Af;$ $A \rightarrow Se \mid e$

2. FA $\Rightarrow$ RLG

? $\Leftrightarrow$ RLG



$G = (\{p, q, r\}, \{0, 1\}, P, p)$

$$
\begin{aligned}
P:\ & p \rightarrow 1p \mid 0q \mid \varepsilon \mid 1 \\
& q \rightarrow 1p \mid 0r \mid 0 \mid 1 \\
& r \rightarrow 0r \mid 1r \mid 0 \mid 1
\end{aligned}
$$

II. 3. RG <=> RE

RE: $0(0+1)^* 1$

? $\Leftrightarrow$ RG

~~$S_1 \rightarrow 0$~~

~~$S_2 \rightarrow 1$~~

$$
\begin{aligned}
0:\ & S_1 \rightarrow 0 \mid 0S_1 \\
1:\ & S_2 \rightarrow 1 \\
0+1:\ & S_3 \rightarrow 0 \mid 1 \mid 0S_1
\end{aligned}
$$

Kleene star = have any number in any order or none of them

$0:\ G_1 = (\{S_1\}, \{0\}, \{S_1 \rightarrow 0\}, S_1)$

$1:\ G_2 = (\{S_2\}, \{1\}, \{S_2 \rightarrow 1\}, S_2)$

$0+1:\ G_3 = (\{S_1, S_2, S_3\}, \{0, 1\}, \{S_1 \rightarrow 0, S_2 \rightarrow 1, S_3 \rightarrow 0 \mid 1\}, S_3)$

$G_3' = (\{S_3\}, \{0, 1\}, \{S_3 \rightarrow 0 \mid 1\}, S_3)$

simplified version

~~$S_3 \rightarrow 0S_3$~~

~~$S_3 \rightarrow \varepsilon$~~

$0+1 \rightarrow \{0, 1\}$

$0^+ \rightarrow \{0, 0^2, 0^3, \dots\}$

$(0+1)^*$:

- $G_4 = (\{S_3\}, \{0, 1\}, \{S_3 \rightarrow \varepsilon \mid 0 \mid 1 \mid 0S_3 \mid 1S_3\}, S_3)$
- $G_4' = (\{S_3\}, \{0, 1\}, \{S_3 \rightarrow \varepsilon \mid 0S_3 \mid 1S_3\}, S_3)$! not regular

RG: $G_1 \to G_4$

$G_2 \to L_2$

$G_3 \to L_1 L_2$ $\quad xy \in L_2$

$S_3 \Rightarrow (0+1)^+$

$S \Rightarrow aA \Rightarrow abB \Rightarrow abcc \Leftrightarrow abcd \quad c \to d$

$G_1 \to L_1$

? $G_2 \to L_1^*$

$0(0+1)^*$:

- $G_5 = (\{S_1, S_3\}, \{0,1\}, \{S_3 \to 0S_3, S_3 \to \varepsilon \mid 0S_3 \mid 1S_3\}, S_1)$! not regular

$0(0+1)^*1$:

- $G_6 = (\{S_1, S_2, S_3\}, \{0,1\}, \{S_1 \to 0S_3, S_3 \to S_2 \mid 0S_3 \mid 1S_3, S_2 \to 1\}, S_1)$! not regular (not RLG or LLG)
- $G_6' = (\{S_1, S_3\}, \{0,1\}, \{S_1 \to 0S_3, S_3 \to 1 \mid 0S_3 \mid 1S_3\}, S_1)$

2. RG $\Rightarrow$ RE

$bb^+ \neq b^+$

$G = (\{S, A, B\}, \{a, b\}, P, S)$

P: $S \to aA$

$A \to aA \mid bB \mid b$

$B \to bB \mid b$

? $\Leftrightarrow$ RE $\quad a^+b^+$

$x = aX + b \Rightarrow X = a^*b$

$b^+ = bb^*$

$a + ab = a(\varepsilon + b)$

$$\begin{cases} S = aA \\ A = aA + bB + b \\ B = bB + b \end{cases}$$

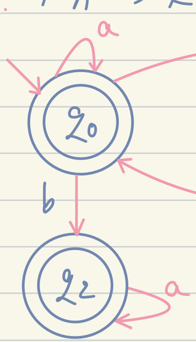
$B = b^*b = b^+$

$A = aA + bb^+ + b \Rightarrow A = a^*b^+$

$S = aA = aa^*b^+ = a^+b^+$

III. FA <=> RE

1. FA => RE



$x = xa + b \qquad x = ba^*$

	a	b
q_0	$\{q_0\}$	$\{q_1, q_2\}$
q_1	$\emptyset$	$\{q_0\}$
q_2	$\{q_2\}$	$\emptyset$

$bb^+ \neq b^+$

$$\begin{cases} q_0 = \varepsilon + q_0 a + q_1 b \\ q_1 = q_0 b \\ q_2 = q_0 b + q_2 a \end{cases}$$

$q_0 + q_2 = (a+bb)^* + (a+bb)^* ba^*$

$q_0 = \varepsilon + q_0 a + q_0 bb$

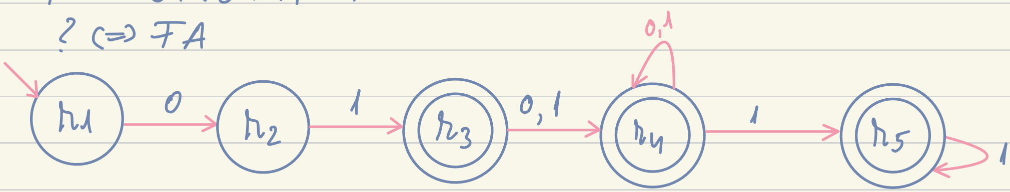
$q_0 = q_0 (a+bb) + \varepsilon$

$$\left.\begin{array}{l} q_0 = (a+bb)^* \\ q_1 = (a+bb)^* b \\ q_2 = (a+bb)^* b + q_2 a \\ q_2 = (a+bb)^* ba^* \end{array}\right| \Rightarrow RE = (a+bb)^* + (a+bb)^* ba^* = (a+bb)^* (\varepsilon + ba^*)$$

2. RE => FA

RE: $01(0+1)^* 1^*$

? <=> FA



Seminar 7.

CFG

1. $G = (\{S, A, B\}, \{0, 1\}, P, S)$

$P:\quad S \to 0B \mid 1A$

$\quad\quad A \to 0 \mid 0S \mid 1AA$

$\quad\quad B \to 1 \mid 1S \mid 0BB$

$w = 0001101110$

? build left / right most deriv. for w + parse trees

<u>left most</u>

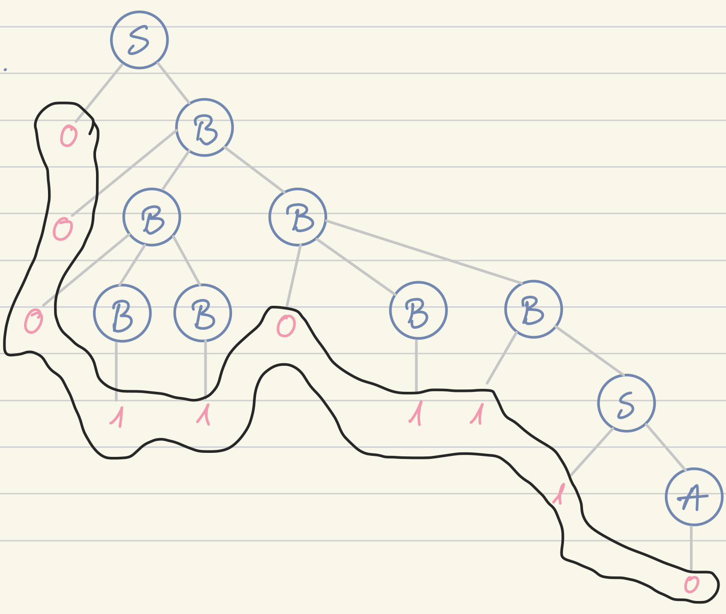
I. 1886686723

II. $S \Rightarrow 0B \xRightarrow[8]{} 00BB \xRightarrow[8]{} 000BBB \xRightarrow[6]{} 0001BB \xRightarrow[6]{} 00011B \xRightarrow[8]{}$

$\xRightarrow[8]{} 000110BB \xRightarrow[6]{} 0001101B \xRightarrow[7]{} 00011011S \xRightarrow[2]{} 000110111A \xRightarrow[3]{}$

$\xRightarrow[3]{} 0001101110$

III.



rightmost

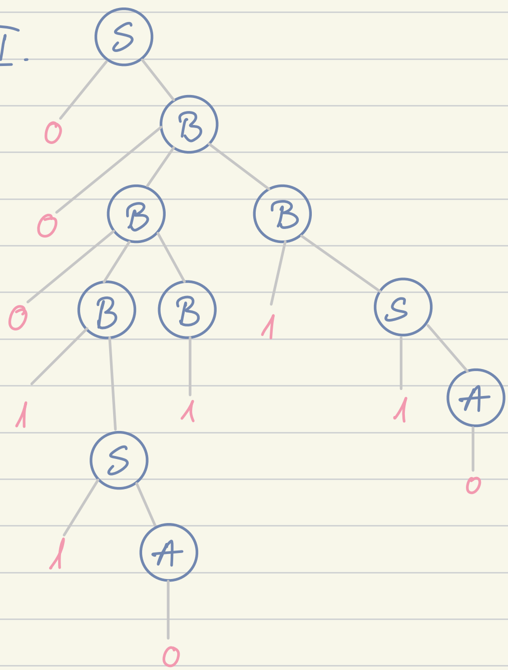
I. 1872386723

II. $S \underset{1}{\Rightarrow} 0B \underset{8}{\Rightarrow} 00BB \underset{7}{\Rightarrow} 00B1S \underset{2}{\Rightarrow} 00B11A \underset{3}{\Rightarrow} 00B110 \underset{8}{\Rightarrow}$

$\underset{8}{\Rightarrow} 000BB110 \underset{6}{\Rightarrow} 000B110 \underset{7}{\Rightarrow} 0001S1110 \underset{2}{\Rightarrow} 00011A1110 \underset{3}{\Rightarrow}$

$\underset{3}{\Rightarrow} 0001101110$

III.



2. ? G is ambigaus or not

a. $G_1 = (\{S, B, C\}, \{a, b, c\}, P, S)$

P: $S \to \overset{1}{abc} \mid \overset{2}{aB}$

$B \to \overset{3}{bC}$

$C \to \overset{4}{c}$

b. $G_2 = (\{E\}, \{a, +, *, (,)\}, P, E)$

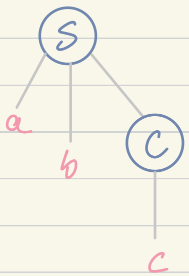
$P: E \rightarrow E + E \mid E * E \mid (E) \mid a$

- $w = abc$

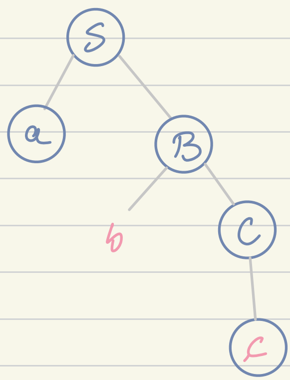
$$S \underset{1}{\Rightarrow} abc \underset{2}{\Rightarrow} abc$$

$$S \underset{2}{\Rightarrow} aB \underset{3}{\Rightarrow} abc \underset{4}{\Rightarrow} abc$$

tree 1 :



tree 2 :

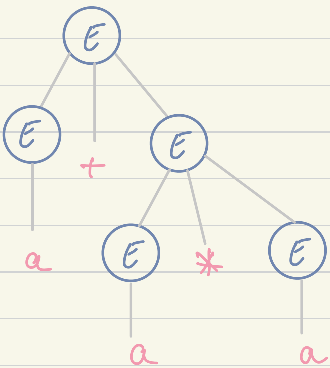


- $w = a + a * a$

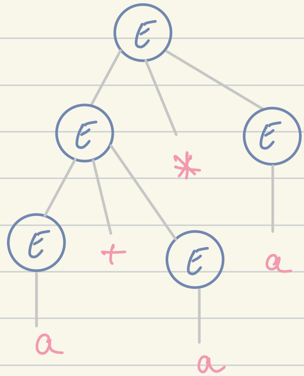
$$E \underset{1}{\Rightarrow} E + E \underset{4}{\Rightarrow} a + E \underset{2}{\Rightarrow} a + E * E \underset{2}{\Rightarrow} a + a * E \underset{2}{\Rightarrow} a + a * a$$

$$E \underset{2}{\Rightarrow} E * E \underset{1}{\Rightarrow} E + E * E \underset{4}{\Rightarrow} a + E * E \underset{4}{\Rightarrow} a + a * E \underset{4}{\Rightarrow} a + a * a$$

tree 1 :



tree 2 :



Recursive descendent parser

$$G = (\{S\}, \{a, b, c\}, \{S \to \overset{S_1}{aSbS} \mid \overset{S_2}{aS} \mid \overset{S_3}{c}\}, S)$$

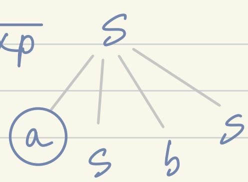
$w = aacbc$

? $w \in L(G)$

$(s, i, \overrightarrow{\alpha}, \overleftarrow{\beta})$

$s \in \{q, b, f, e\}$

Ⓢ $\vdash_{exp}$



$\boxed{aacbc}$

$(q, 1, \varepsilon, S) \vdash_{exp} (q, 1, S_1, aSbS) \vdash_{adv} (q, 2, S_1 a, SbS) \vdash_{exp}$

$(q, 2, S_1 a S_1, aSbSbS) \vdash_{adv} (q, 3, S_1 a S_1 a, SbSbS) \vdash_{exp}$

$(q, 3, S_1 a S_1 a S_1, aSbSbSbS) \vdash^{2}_{mi+at}$

$(b, 3, S_1 a S_1 a S_1, aSbSbSbS) \vdash_{at} (q, 3, S_1 a S_1 a S_2, aSbSbS)$

$\vdash^{2}_{mi+at} (q, 3, S_1 a S_1 a S_3, cbSbS) \vdash_{adv} (q, 4, S_1 a S_1 a S_3 c, bSbS) \vdash_{adv}$

$(q, 5, S_1 a S_1 a S_3 cb, SbS) \vdash^{5}_{exp, mi, at, mi, at}$

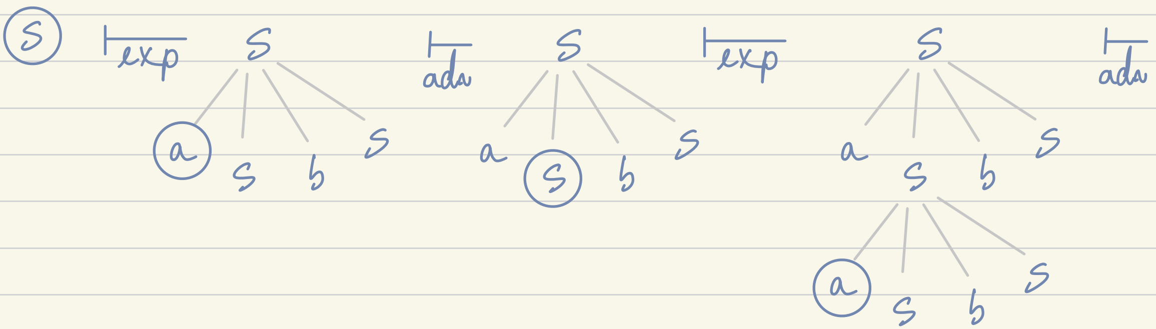
$(q, 5, S_1 a S_1 a S_3 cb S_3, cbS) \vdash_{adv} (q, 6, S_1 a S_1 a S_3 cb S_3 c, bS)$

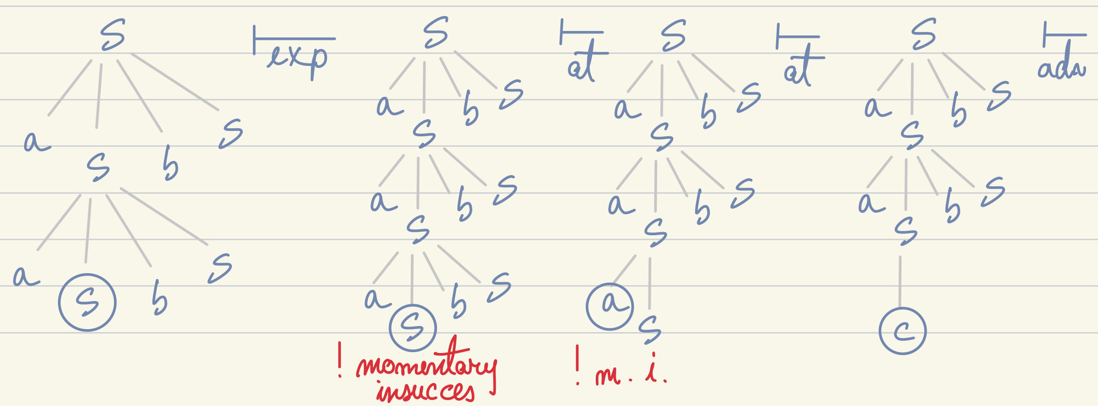
$\vdash_{mi} (b, 6, S_1 a S_1 a S_3 cb S_3 c, bS) \vdash_{back}$

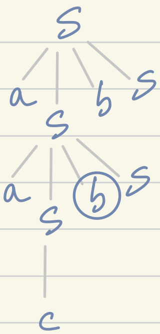
$(b, 5, S_1 a S_1 a S_3 cb \cancel{S_3}, \underbrace{c}_{S} bS) \vdash_{at} (b, 5, S_1 a S_1 a S_3 cb, SbS)$

$\vdash^{3}_{back, back, at} (b, 3, S_1 a S_1 a, SbSbS) \vdash_{back} (b, 2, S_1 a \underbrace{S_1}_{S_2}, aSbSbS)$

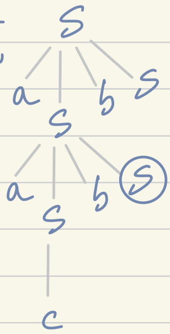
$\vdash_{at} (q, 2, S_1 a S_2, aSbS)$



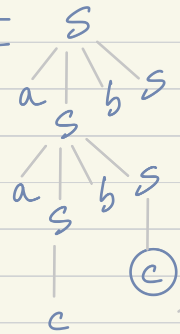




$\vdash$ ada

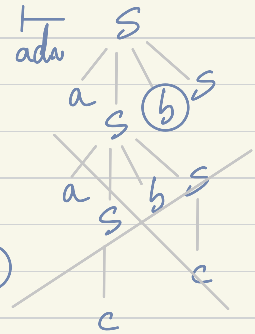


$\vdash$ exp, mi, at, mi, at

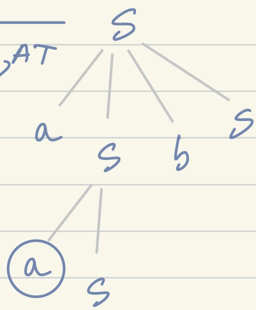


$\vdash$ ada

! m.i



$\vdash$ mi, B, AT, B, B, AT, B, AT



Seminar 8.

LL(1) parser

$G = (\{S, A, B, C, D\}, \{a, +, *, (,)\}, P, S)$

P:
(1) $S \longrightarrow BA$
(2) $A \longrightarrow +BA$
(3) $A \longrightarrow \varepsilon$
(4) $B \longrightarrow DC$
(5) $C \longrightarrow *DC$
(6) $C \longrightarrow \varepsilon$
(7) $D \longrightarrow (S)$
(8) $D \longrightarrow a$

$w = a*(a+a)$

I. FIRST & FOLLOW

$$FIRST(\underbrace{X_1X_2\ldots X_m}_{\alpha}) = FIRST(X_1) \oplus FIRST(X_2) \oplus \ldots \oplus FIRST(X_m)$$

$, \forall i = \overline{1, m}, \; X_i \in N \cup \Sigma$

	F_0	F_1	$= F_2$	$= F_3$	$= \ldots$
S	$\emptyset$	$\emptyset$	$\{(, a\}$	$\{(, a\}$	
A	$\{+, \varepsilon\}$	$\{+, \varepsilon\}$	$\{+, \varepsilon\}$	$\{+, \varepsilon\}$	
B	$\emptyset$	$\{(, a\}$	$\{(, a\}$	$\{(, a\}$	
C	$\{*, \varepsilon\}$	$\{*, \varepsilon\}$	$\{*, \varepsilon\}$	$\{*, \varepsilon\}$	
D	$\{(, a\}$	$\{(, a\}$	$\{(, a\}$	$\{(, a\}$	

FIRST

$S \overset{*}{\Rightarrow} S$

$FIRST(S) = \{(, a\}$
$(A) = \{+, \varepsilon\}$
$(B) = \{(, a\}$
$(C) = \{*, \varepsilon\}$
$(D) = \{(, a\}$

	L_0	L_1	L_2	L_3 =	L_4	= ...
S	$\{\varepsilon\}$	$\{\varepsilon,)\}$	$\{\varepsilon,)\}$	$\{\varepsilon,)\}$	$\{\varepsilon,)\}$	
A	$\emptyset$	$\{\varepsilon\}$	$\{\varepsilon,)\}$	$\{\varepsilon,)\}$	$\{\varepsilon,)\}$	
B	$\emptyset$	$\{+, \varepsilon\}$	$+, \varepsilon$	$\{+, \varepsilon,)\}$	$\{+, \varepsilon,)\}$	
C	$\emptyset$	$\emptyset$	$+, \varepsilon$	$\{+, \varepsilon,)\}$	$\{+, \varepsilon,)\}$	
D	$\emptyset$	$\{*\}$	$*, +, \varepsilon$	$\{*, +, \varepsilon,)\}$	$\{*, +, \varepsilon,)\}$	

FOLLOW

FOLLOW $(S) = \{\varepsilon,)\}$

$(A) = \{\varepsilon,)\}$

$(B) = \{+, \varepsilon,)\}$

$(C) = \{+, \varepsilon,)\}$

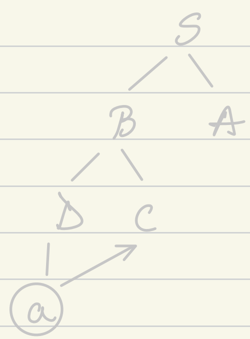
$(D) = \{+, *, \varepsilon,)\}$

II. LL(1) table

	a	+	*	(	)	$
S	BA, 1			BA, 1		
A		+BA, 2			ε, 3	ε, 3
B	DC, 4			DC, 4		
C		ε, 6	*DC, 5		ε, 6	ε, 6
D	a, 8			(S), 7		
a	pop					
+		pop				
*			pop			
(				pop		
)					pop	
$						accept

→ empty is error

→ if there are conflicts => there isn't grammar LL(1)



III. Parse the seq. w

$$
\begin{aligned}
&(a*(a+a)\$,\ S\$,\ \varepsilon) \vdash (a*(a+a)\$,\ BA\$,\ 1) \vdash \\
&(a*(a+a)\$,\ DCA\$,\ 14) \vdash (a*(a+a)\$,\ aCA\$,\ 148) \vdash \\
&(*(a+a)\$,\ CA\$,\ 148) \vdash (*(a+a)\$,\ *DCA\$,\ 1485) \vdash \\
&((a+a)\$,\ DCA\$,\ 1485) \vdash ((a+a)\$,\ (S)CA\$,\ 14857) \vdash \\
&(a+a)\$,\ S)CA\$,\ 14857) \vdash (a+a)\$,\ BA)CA\$,\ 148571) \vdash \\
&(a+a)\$,\ DCA)CA\$,\ 148714) \quad (a+a)\$,\ aCA)CA\$,\ 14857148) \\
&\vdash (+a)\$,\ CA)CA\$,\ 14857148) \vdash (+a)\$,\ A)CA\$,\ 148571486) \\
&\vdash (+a)\$,\ +BA)CA\$,\ 1485714862) \vdash (a)\$,\ BA)CA\$,\ 1485714862) \\
&\vdash (a)\$,\ DCA)CA\$,\ 14857148624) \vdash (a)\$,\ aCA)CA\$,\ 148571486248) \\
&\vdash ()\$,\ CA)A\$,\ 148571486248) \overset{s}{\vdash} \underline{(\$,\ \$,\ 1485714862486363)}
\end{aligned}
$$

Conflict

$$
\left.\begin{aligned} A &\to \alpha\beta \\ A &\to \alpha\gamma \end{aligned}\right. \Longrightarrow \left|\begin{aligned} A &\to \alpha B \\ B &\to \beta / \gamma \end{aligned}\right.
$$

Both options are correct, but because first one has productions starting both with α the LL(1) does not know which one to use.

Seminar 9.

LR(0) parser

$G = (\{S, A\}, \{a, b, c\}, P, S)$

$S' \longrightarrow S$

$P:$ (1) $S \longrightarrow aA$

(2) $A \longrightarrow bA$

(3) $A \longrightarrow c$

$w = abbc$

? $w \in L(G)$

I Canonical collection of states (set)

LR(k) item $[A \longrightarrow \alpha . \beta, u]$

$\hookrightarrow$ kernel $\qquad$ prod $\qquad \Sigma^k$

LR(0) item $[A \longrightarrow \alpha . \beta]$

$$\left(\begin{array}{l} [A \longrightarrow \alpha . \underset{}{\overset{\in N}{B}} \beta] \\ [B \longrightarrow . \gamma] \end{array} \right. \qquad \mathcal{C} = \{s_0, s_1, \ldots\}$$

$s_0 = closure(\{[S' \longrightarrow .S]\}) = \{[S' \longrightarrow .S], [S \longrightarrow .aA]\}$

$s_1 = goto(s_0, S) = closure(\{[S' \longrightarrow S.]\}) = \{[S' \longrightarrow S.]\}$

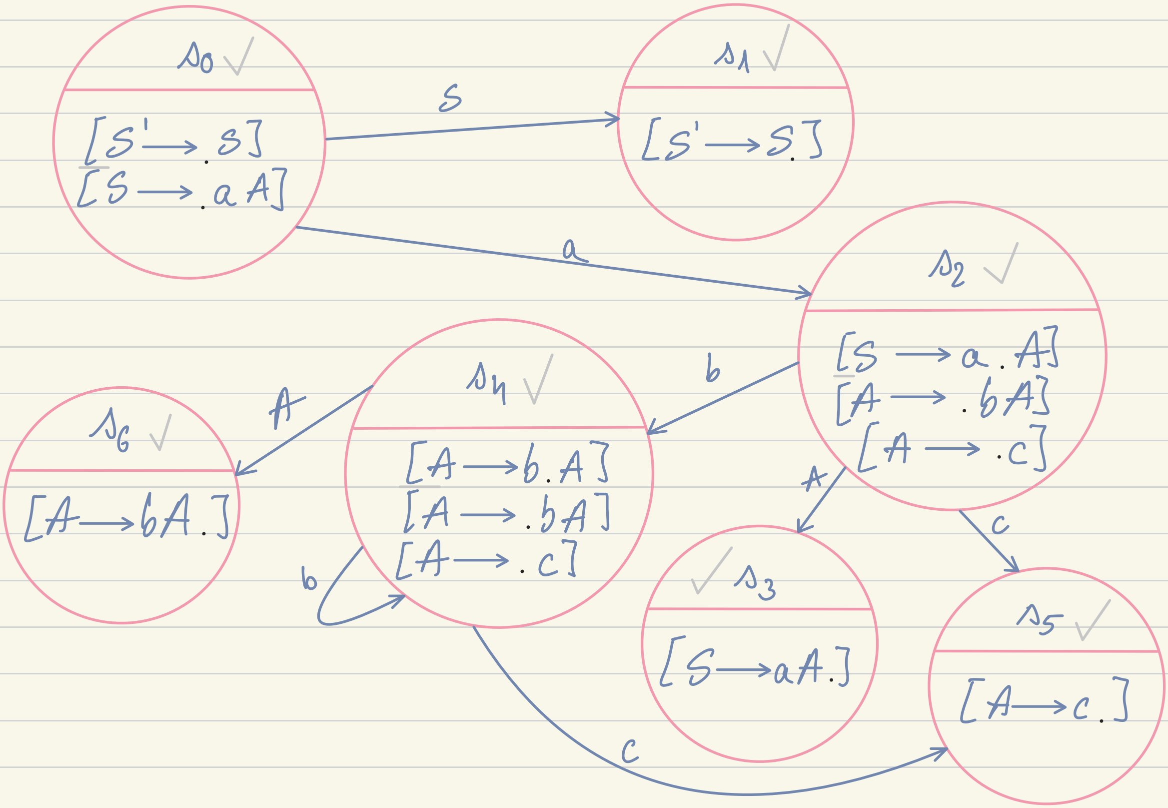
$goto(s_0, A) = \emptyset$

$s_2 = goto(s_0, a) = closure(\{[S \longrightarrow a.A]\}) = \{[S \longrightarrow a.A],$
$[A \longrightarrow .bA], [A \longrightarrow .c]\}$

$goto(s_0, b) = goto(s_0, c) = \emptyset$

$s_3 = goto(s_2, A) \underline{=} closure(\{[S \longrightarrow aA.]\}) = \{[S \longrightarrow aA.]\}$

$s_4 = goto(s_2, b) = closure(\{[A \longrightarrow b.A]\}) = \{[A \longrightarrow b.A],$
$[A \longrightarrow .ba], [A \longrightarrow .c]$



$s_5 = goto(s_2, c) = closure(\{[A \to c.]\} = \{[A \to c.]\}$

$s_6 = goto(s_4, A) = closure(\{[A \to bA.]\}) = \{[A \to bA.]\}$

$goto(s_4, b) = closure(\{[A \to b.A]\}) = s_4$

$goto(s_4, c) = closure(\{[A \to c.]\}) = s_5$

$goto(s_0, A) = \emptyset$

$s_2 = goto(s_0, a) = closure(\{[S \to a.A]\}) = \{[S \to a.A], [A \to .ba], [A \to .c]\}$

$goto(s_0, b) = goto(s_0, c) = \emptyset$

II. LR(0) parsing table

		goto				
	action	S	A	a	b	c
s0	shift	s1		s2		
s1	accept					
s2	shift		s3		s4	s5
s3	reduced 1					
s4	shift		s6		s4	s5
s5	reduced 3					
s6	reduced 2					

III. Parse w

work	input	output
\$s0	abbc\$	ε
\$s0 a s2	bbc\$	ε
\$s0 a s2 b s4	bc\$	ε
\$s0 a s2 b s4 b s4	c\$	ε
\$s0 a s2 b s4 b s4 c s5	\$	ε
\$s0 a s2 b s4 b s4 A s6	\$	3
\$s0 a s2 b s4 A s6	\$	23
\$s0 a s2 A s3	\$	223
\$s0 S s1	\$	1223
\$ accept		

Seminar 11

SLR

1. $G = (\{S', E, T\}, \{+, id, const, (,)\}, P, S)$

$P: S' \longrightarrow S$

(1) $E \longrightarrow T$

(2) $E \longrightarrow E + T$

(3) $T \longrightarrow (E)$

(4) $T \longrightarrow id$

(5) $T \longrightarrow const$

$[A \longrightarrow \alpha . \beta]$

$w = id + const$

I $s_0 = closure(\{[S' \longrightarrow .E]\}) = \{[S' \longrightarrow .E], [E \longrightarrow .T],$
$[E \longrightarrow .E+T], [T \longrightarrow .(E)]\ [T \longrightarrow .id], [T \longrightarrow .const]$

$s_1 = goto(s_0, E) = closure(\{[S' \longrightarrow E.], [E \longrightarrow E.+T]\}) =$
$= \{[S' \longrightarrow E.], [E \longrightarrow E.+T]\}$

$s_2 = goto(s_0, T) = closure(\{[E \longrightarrow T.]\}) = \{[E \longrightarrow T.]\}$

$s_3 = goto(s_0, '(') = closure(\{[T \longrightarrow (.E)]\}) = \{[T \longrightarrow (.E)],$
$[E \longrightarrow .T], [E \longrightarrow .E+T], [T \longrightarrow .(E)],$
$[T \longrightarrow .id], [+ \longrightarrow .const]\}$

$s_4 = goto(s_0, id) = closure(\{[T \longrightarrow id.]\} = \{[T \longrightarrow id.]\}$

$s_5 = goto(s_0, const) = closure(\{[T \longrightarrow const.]\}) = \{[T \longrightarrow const.]\}$

$s_6 = goto(s_1, +) = closure(\{[E \longrightarrow E+.T]\}) = \{[E \longrightarrow E+.T],$
$[T \longrightarrow .(E)], [T \longrightarrow .id], (T \longrightarrow .const]\}$

$s_7 = goto(s_3, E) = closure(\{[T \longrightarrow (E.)], [E \longrightarrow E.+T]\}) =$
$= \{[T \longrightarrow (E.)], [E \longrightarrow E.+T]\}$

$goto(s_3, T) = closure(\{[E \longrightarrow T.]\}) = s_2$ $\quad goto(s_3, '(') = s_3$

$goto(s_3, id) = s_4$ $\quad goto(s_3, const) = s_5$ $\quad goto(s_3, +) = s_6$

$s_8 = goto(s_6, T) = closure(\{[E \longrightarrow E+T.]\} = \{[E \longrightarrow E+T.]\}$

$goto(s_6, '(') = s_3$ $\quad goto(s_6, id) = s_4$ $\quad goto(s_6, const) = s_5$

$s_9 = goto(s_7, ')') = closure(\{[T \longrightarrow (E).]\}) = \{[T \longrightarrow (E).]\}$

$goto(s_7, +) = s_6$

II. $FOLLOW(E) = \{\varepsilon, +,)\} = FOLLOW(T)$

	+	(	)	id	const	\$	E	T
s0		shift s3		shift s4	shift s5		s1	s2
s1	shift s6					accept		
s2	reduce 1		reduce 1			reduce 1		
s3		shift s3		shift s4	shift s5		s7	s2
s4	reduce 4		reduce 4			reduce 4		
s5	reduce 5		reduce 5			reduce 5		
s6		shift s3		shift s4	shift s5			s8
s7	shift s6		shift s9					
s8	reduce 2		reduce 2			reduce 2		
s9	reduce 3		reduce 3			reduce 3		

III.

work	input	output
\$ s0	id + const \$	ε
\$ s0 id s4	+ const \$	ε
\$ s0 T s2	+ const \$	4
\$ s0 E s1	+ const \$	1 4
\$ s0 E s1 + s6	const \$	1 4
\$ s0 E s1 + s6 const s5	\$	1 4
\$ s0 E s1 + s6 T s8	\$	5 1 4
\$ s0 E s1	\$	2 5 1 4
\$ accept		

Seminar 12.

LR(1)

1. $G = (\{S', S, A\}, \{a, b\}, P, S')$

$P: S' \longrightarrow S$

(1) $S' \longrightarrow AA$

(2) $A \longrightarrow aA$

(3) $A \longrightarrow b$

$w = abab$

LR(1) item $[A \longrightarrow \alpha . \beta, a]$

$a \in \Sigma \cup \{\$\}$

I Canonical collection

$S \longrightarrow AB$

$A \longrightarrow aA \mid b \mid \varepsilon$

$B \longrightarrow c$

$FIRST(A) = \{a, b\} = FIRST(S)$

$[A \longrightarrow \alpha . B\beta, a] \longrightarrow [B \longrightarrow . \gamma, b]$

$[S' \longrightarrow . S, \$]$

$b \in FIRST(\beta a)$

$s_0 = closure(\{[S' \longrightarrow . S, \$]\}) = (\{[S' \longrightarrow . S, \$], [S \longrightarrow . AA, \$]$
$[A \longrightarrow . aA, a], [A \longrightarrow . aA, b], [A \longrightarrow . b, a], [A \longrightarrow . b, b]\})$

$s_1 = goto(s_0, S) = closure(\{[S' \longrightarrow S., \$]\}) = \{[S' \longrightarrow S., \$]\}$

$s_2 = goto(s_0, A) = closure(\{[S \longrightarrow A.A, \$]\}) = \{[S \longrightarrow A.A, \$],$
$[A \longrightarrow . aA, \$], [A \longrightarrow . b, \$]\}$

$s_3 = goto(s_0, a) = closure(\{[A \longrightarrow a.A, a], [A \longrightarrow a.A, b]\} =$
$= \{[A \longrightarrow a.A, a], [A \longrightarrow a.A, b], [A \longrightarrow . aA, a],$
$[A \longrightarrow . aA, b], [A \longrightarrow . b, a], [A \longrightarrow . b, b]\}$

$s_4 = goto(s_0, b) = closure(\{[A \longrightarrow b., a], [A \longrightarrow b., b]\}) =$
$= \{[A \longrightarrow b., a], [A \longrightarrow b., b]\}$

$s_5 = goto(s_2, A) = closure(\{[S \longrightarrow AA., \$]\}) = \{[S \longrightarrow AA., \$]\}$

$s_6 = goto(s_2, a) = closure(\{A \longrightarrow a.A, \$]\}) = \{[A \longrightarrow a.A, \$],$
$[A \longrightarrow . aA, \$], [A \longrightarrow . b, \$]\}$

$s_7 = goto(s_2, b) = closure(\{[A \longrightarrow b., \$]\}) = \{[A \longrightarrow b., \$]\}$

$s_8 = goto(s_3, A) = closure(\{A \longrightarrow aA., a], [A \longrightarrow aA., b]\}) =$

$= \{[A \longrightarrow aA., a], [A \longrightarrow aA., b]\}$

$goto(s_3, a) = \dots = s_3 \qquad goto(s_3, b) = s_4$

$s_9 = goto(s_6, A) = closure(\{A \longrightarrow aA., \$]\} = \{[A \longrightarrow aA., \$]\},$

$goto(s_6, a) = s_6 \qquad goto(s_6, b) = s_7$

II. LR(1) parsing table

	action			goto	
	a	b	$	S	A
s0	SH s3	SH s4		s1	s2
s1			ACC		
s2	SH s6	SH s7			s5
s3	SH s3	SH s4			s8
s4	R(3)	R(3)			
s5			R(1)		
s6	SH s6	SH s7			s9
s7			R(3)		
s8	R(2)	R(2)			
s9			R(2)		

III. Parse w

work	input	output
$s0	abab$	ε
$s0 a s3	bab$	ε
$s0 a s3 b s4	ab$	ε
$s0 a s3 A s8	ab$	3

work	input	output
$ s0 A s2	ab$	23
$ s0 A s2 a s6	b$	23
$ s0 A s2 a s6 b s7	$	23
$ accept		

LALR(1)

I Canonical collection

$C = \{ s_0, s_1, s_2, s_{36}, s_{47}, s_5, s_{89} \}$

$s_{36} = \{ [A \longrightarrow a.A , a/b/\$], [A \longrightarrow .aA, a/b/\$],$
$[A \longrightarrow .b, a/b/\$] \}$

$s_{47} = \{ [A \longrightarrow b. , a/b/\$] \}$

$s_{89} = \{ [A \longrightarrow aA. , a/b/\$] \}$

II. Parsing table

	action			goto	
	a	b	$	S	A
s0	SH s36	SH s47		s1	s2
s1			ACC		
s2	SH s36	SH s47			s5
s36	SH s36	SH s47			s89
s47	R(3)	R(3)	R(3)		
s5			R(1)		
s89	R(2)	R(2)	R(2)		

Seminar 13.

PDA

if and only if

$(p, aw, Z\gamma) \vdash (q, w, \alpha\gamma)$ iff $(q, \alpha) \in \delta(p, a, Z)$

$(p, w, Z\gamma) \vdash (q, w, \alpha\gamma)$ iff $(q, \alpha) \in \delta(p, \varepsilon, Z)$

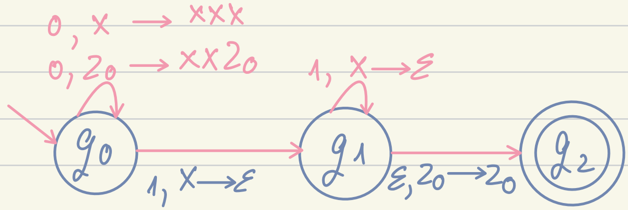
1. $L_1 = \{0^n 1^{2n} \mid n \in \mathbb{N}^*\}$

$w_1 = 0^2 1^4 \in L(M)$

$w_2 = 0^2 1^3 \notin L(M)$

$w_3 = 0^2 1^5 \notin L(M)$

M:

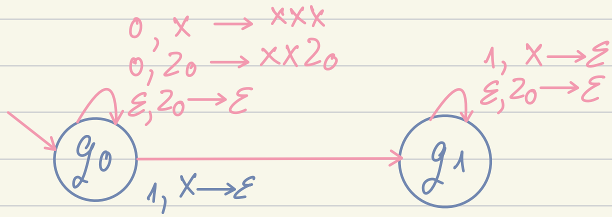


w_1: $(q_0, 0^2 1^4, Z_0) \vdash (q_0, 0^1 1^4, XXZ_0) \vdash (q_0, 1^4, XXXXZ_0)$
$\vdash (q_1, 1^3, XXXZ_0) \vdash^3 (q_1, \varepsilon, Z_0) \vdash (q_2, \varepsilon, Z_0) \Rightarrow$
$\Rightarrow w_1 \in L(M)$

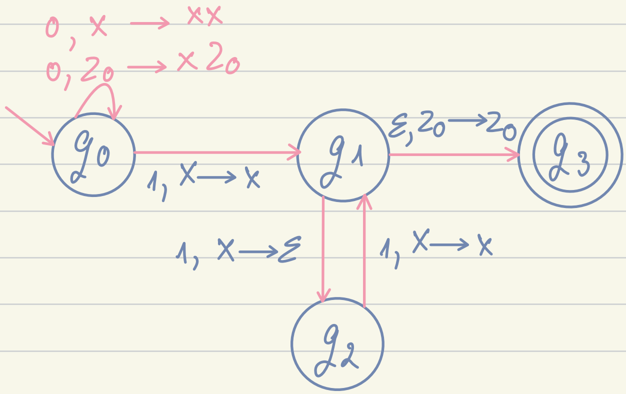
w_2: $(q_0, 0^2 1^3, Z_0) \vdash^2 (q_0, 1^3, X^4 Z_0) \vdash^3 (q_0, \varepsilon, X Z_0) \Rightarrow$
$\Rightarrow w_2 \notin L(M)$

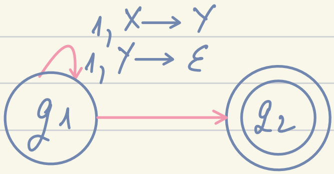
w_3: $(q_0, 0^2 1^5, Z_0) \vdash^2 (q_0, 1^5, X^4 Z_0) \vdash^4 (q_1, 1, Z_0) \vdash$
$\vdash (q_2, 1, Z_0) \Rightarrow w_3 \notin L(M)$

M:

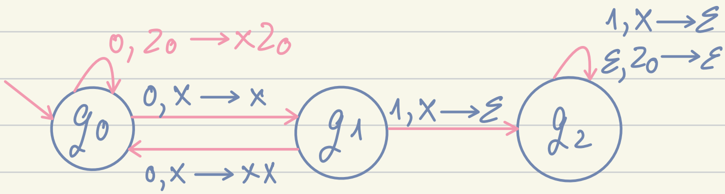


2. I. $L_2 = \{ 0^n (1^2)^n \mid n \in \mathbb{N}^* \}$





II. $L_2 = \{ 0^{2n} 1^n \mid n \in \mathbb{N}^* \}$



$$(q_0, 0^4 1^2, Z_0) \vdash (q_0, 0^3 1^2, xZ_0) \vdash (q_1, 0^2 1^2, xZ_0) \vdash$$
$$(q_0, 01^2, xxZ_0) \vdash (q_1, 1^2, xxZ_0) \vdash^2 (q_2, \varepsilon, Z_0) \vdash (q_2, \varepsilon, \varepsilon) \Rightarrow$$
$$\Rightarrow \in L(M)$$

3. $L_3 = \{ \underbrace{w}_{q_0} \underbrace{w^R}_{q_1} \mid w \in \{a, b\}^* \}$

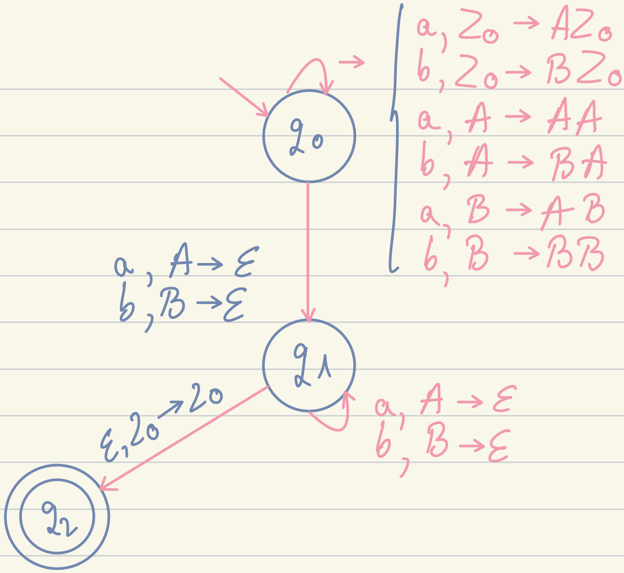
abaaba
abbbba

A aa

B

A

?



q_0
q_1
q_2
$a, Z_0 \to AZ_0$
$b, Z_0 \to BZ_0$
$a, A \to AA$
$b, A \to BA$
$a, B \to AB$
$b, B \to BB$
$a, A \to \varepsilon$
$b, B \to \varepsilon$
$a, A \to \varepsilon$
$b, B \to \varepsilon$
$\varepsilon, Z_0 \to Z_0$

Seminar 14.

Attribute grammars

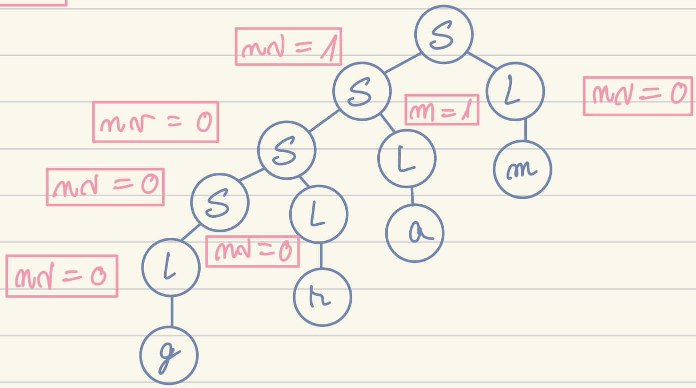
(G, A, R)
cfg

Given AG for
1. computing nr of vowels in a non-empty string
2. computing the value of an attribute expr. with $+, *, /, -, (,)$
3. checking if $n \vdots 3$, $n \in \mathbb{N}$

1. $S \to L$ $\quad S.nr = L.nr$
$S \to SL$ $\quad S_1.nr = L.nr + S_2.nr$
$L \to a \mid e \mid i \mid o \mid u \mid A \mid E \mid I \mid O \mid U$ $\quad L.nr = 1$
$L \to b \mid c \mid \dots \mid z \mid B \mid C \mid \dots \mid Z$ $\quad L.nr = 0$

nr

$\boxed{w = \text{grom}}$



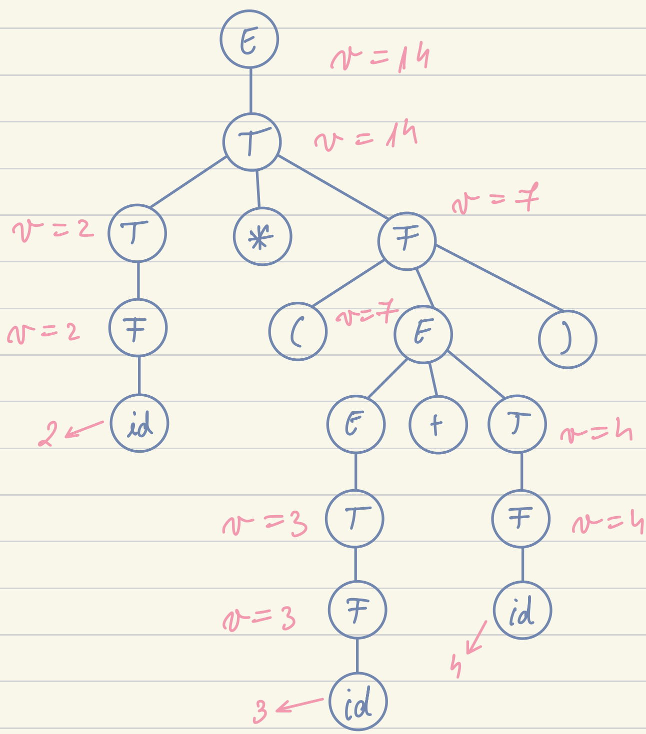
2.

$E \to E + T$	$E_1.v = E_2.v + T.v$
$E \to E - T$	$E_1.v = E_2.v - T.v$
$E \to T$	$E.v = T.v$
$T \to T * F$	$T_1.v = T_2.v * F.v$
$T \to T / F$	$T_1.v = T_2.v / F.v$
$T \to F$	$T.v = F.v$
$F \to (E)$	$F.v = E.v$
$F \to id$	$F.v = id.v$
$F \to const$	$F.v = const.v$

v

$w = a * (b + c)$

$\boxed{id * (id + id)}$

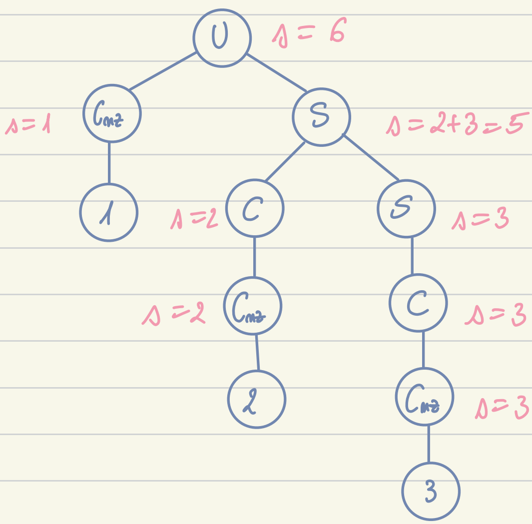


3

$U \to C$	$U.s = C.s$
$U \to C_{nz} S$	$U.s = C_{nz}.s + S.s$
$S \to C$	$S.s = C.s$
$S \to CS$	$S.s = S.s + C.s$
$C_{nz} \to 1$	$C_{nz}.s = 1$
$\dots$	$\dots$
$C_{nz} \to 9$	$C_{nz}.s = 9$
$C \to 0$	$C.s = 0$
$C \to C_{nz}$	$C.s = C_{nz}.s$

$$U.isdiv = (U.s \ \% \ 3 == 0)$$
$$U.isdiv = ((C_{nz}.s + S.s) \ \% \ 3 == 0)$$

for 123



$s=6 \Rightarrow isdiv = true$

3. Address Code

```
if (a < b) OR c AND (e > d)
    then  a := -1;
    else  a := b + c * 3;
endif
```

index	op	arg 1	arg 2	result
1	<	a	b	t_1
2	>	b	d	t_2
3	and	c	t_2	t_3
4	or	t_1	t_3	t_4
5	goto	t_4		(10)
6	*	c	3	t_5
7	+	b	t_5	t_6
8	:=	t_6		a
9	goto			(12)
10	@	1		t_7
11	:=	t_7		a
12	...			

<op> <arg1> <arg2> <result>

```
while (a < b) do
      a := a + 1;
      ...
endwhile
```

index	op	arg 1	arg 2	result
1	<	a	b	
2	>	b	d	
3	and	c	(2)	
4	or	(1)	(3)	
5	goto	(4)	(10)	
6	*	c	3	
7	+	b	(8)	
8	:=	(7)	a	
9	goto		(12)	
10	@	1		
11	:=	(10)	a	
12				

index	op	arg 1	arg 2	result
1	<	a	b	t_1
2	!	t_1		t_2
3	goto	t_2		(7)
4	+	a		t_4
5	:=	t_4	1	a
6	goto			(1)
7				