

A. L-list of numbers, the predicate has the flow model $(i, 0)$:

$f([], -)$.

$f([H|T], S) :- f(T, S1), S1 < 1, S \text{ is } S1 - H, !$.

$f([-1|T], S) :- f(T, S)$.

In order to avoid the recursive call $f(T, S)$ in both clauses we will create an ~~additional~~ auxiliary predicate $f_aux(H, S, S1)$. Its parameters are the first element of the list, the final result and the result of the recursive call. This predicate has as flow model the model $(i, 0, i)$.

$f_aux(H, S, S1) :-$

$S1 < 1,$

$S \text{ is } S1 - H, !$.

$f_aux(-, S1, S1)$.

$f([], -)$.

$f([H|T], S) :- f(T, S1), f_aux(H, S, S1)$.

In the f_aux predicate we move all the conditions and computations done in the second and third clause from the initial predicate that are not in common.

B. $\text{insertOnEveryPos}(l_1, l_2, \dots, l_m, e) = \begin{cases} e, & \text{if } m=0 \\ e \cup l_1 \cup l_2 \cup \dots \cup l_m \\ l_1 \cup \text{insertOnEveryPos}(l_2, \dots, l_m, e), & \text{otherwise} \end{cases}$

% $\text{insertOnEveryPos}(\text{LST: list}, E: \text{atom}, R: \text{list})$

% Flow model: $(i, i, 0), (0, i, i), (i, 0, i), (0, 0, i)$

$\text{insertOnEveryPos}([], E, [E]) :- !.$

$\text{insertOnEveryPos}(\text{LST}, E, [E | \text{LST}]).$

$\text{insertOnEveryPos}([H | T], E, [H | R]) :-$

$\text{insertOnEveryPos}(T, E, R).$

$\text{arrangements}(l_1, l_2, \dots, l_m, k) = \begin{cases} l_1, & \text{if } k=1 \\ \text{arrangements}(l_2, \dots, l_m, k) \\ \text{insertOnEveryPos}(\text{arrangements}(l_2, \dots, l_m, k+1), l_1), & \text{otherwise} \end{cases}$

% $\text{arrangements}(\text{LST: list}, K: \text{int}, R: \text{list})$

Flow model: $(i, i, 0), (i, i, i)$

$\text{arrangements}([H | _], 1, [H]).$

$\text{arrangements}([_ | T], K, R) :-$

$\text{arrangements}(T, K, R).$

$\text{arrangements}([H | T], K, R) :-$

$K > 1,$

$K1 \text{ is } K-1,$

$\text{arrangements}(T, K1, R),$

$\text{insertOnEveryPos}(R, H, R).$

$\text{sum}(l_1, l_2, \dots, l_m) = \begin{cases} 0, & \text{if } m=0 \\ l_1 + \text{sum}(l_2, \dots, l_m), & \text{otherwise} \end{cases}$

% $\text{sum}(\text{LST: list}, S: \text{int})$

Flow model: $(i, 0), (i, i)$

$\text{sum}([], 0).$

$\text{sum}([H | T], S) :-$

$\text{sum}(T, S1),$

$S \text{ is } S1 + H.$

Cont. B.

oneSol(L, k, S) = arrangements(L, k) if ~~product~~ sum(arrangements(L, k)) = S
L: list, k: int, S: ~~list~~ int, R: list, flow model: (i, i, i, 0), (i, i, i, i), (i, i, 0, i)

oneSol(L, k, S, R):-
 arrangements(L, k, R),
 sum(R, S).

allSol(L, k, S, RL):-
 findall(R, oneSol(L, k, S, R), RL).

The last function is a wrapper function. Flow model: (i, i, i, 0), (i, i, i, i),
(i, i, 0, i).

C. tree (node subtree1 subtree2...)
Mathematical model:

$$\text{nodesOnLevel}(\text{tree}, \text{level}, k) = \begin{cases} \text{nil, if tree is an atom} \\ \text{tree, if tree is an atom and level} = k \\ n \text{ nil, if tree is an atom and level} \neq k \\ \bigcup_{i=1}^n \text{nodesOnLevel}(\text{subtree}_i, \text{level}+1, k), \\ \text{where } n = \text{number of subtrees} \end{cases}$$

```
(defun NodesOnLevel (l level k)
  (cond
    ((and (atom l) (= level k) (list l)))
    ((atom l) nil)
    (t (mapcan #'(lambda (a) (NodesOnLevel a (+ level 1) k)) l))
  )
)
```

```
(defun wrapperNodes (l k)
  (NodesOnLevel (l -1 k))
```

```
+
(defun wrapNodes (l k)
  (NodesOnLevel l -1 k)
)
```

We start with ~~0~~ -1 because when we first call mapcan we will give it this way the level 0 to the root.