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1.0 Abstract

This project optimizes WARP Shoe Company's production plan to meet rising demand after a competitor's bankruptcy. Using historical data and operational constraints, a mathematical model was developed to maximize profitability while considering budget, capacity, and storage limits. Solved with AMPL and Gurobi solvers, the study employed linear programming relaxation for efficiency. Results include optimal production levels, binding constraints, and sensitivity analyses to assess resource or operational changes. This framework provides strategic resource allocation, ensuring efficiency and profit maximization, showcasing the use of optimization in addressing real-world production challenges.

2.0 Introduction

The WARP Shoe Company, a well-established footwear manufacturer in Canada, is facing a critical opportunity to optimize its production strategy following the bankruptcy of a major competitor. This development has doubled the predicted demand for February 2006, presenting a unique challenge to efficiently meet increased customer needs while maximizing profitability. To address this, the company has sought a robust production plan leveraging mathematical optimization techniques.

This report explores the development of an optimized production plan for WARP Shoe Company using Operations Research methodologies, specifically Linear Programming (LP) and Integer Programming (IP). By utilizing AMPL and the Gurobi solver, the project integrates demand forecasting, resource allocation, and constraint management to construct a decision-making model that aligns production capacity, budget limitations, and storage requirements with market demand.

The report begins by formulating a mathematical model incorporating decision variables, an objective function to maximize profit, and constraints derived from the company's operational capabilities. The methodology section outlines the assumptions, data analysis, and model construction, followed by detailed implementation and solution analysis. Key results, including optimal production quantities, binding constraints, and sensitivity to resource changes, are discussed. The report concludes with actionable insights and recommendations for WARP Shoe Company's management to implement an efficient and profitable production strategy.

This project not only addresses the immediate production planning needs but also highlights the application of optimization tools in real-world industrial settings, underscoring their value in strategic decision-making.

3.0 Methodology

3.1 Mathematical Model

Maximize Profit (z):

$$\text{maximize } z = \sum_{n}^{\text{Product}_{\text{Num}}} \Bigg[(\text{sales price}_n)(x_n) - \sum_{r}^{\text{RM}_{\text{Num}}} (\text{cost of raw materials}_r)(\text{quantity of raw matrial}_{nr})(x_n)$$

$$- 10\left((2)\left(\sum_{s}^{\text{Store}} \sum_{y}^{\text{Year}} \sum_{m}^{\text{Month}} \frac{(\text{demand}_{sym})}{7 \times 12} \right) - x_n \right)$$

$$- \sum_{k}^{\text{Machine}_{\text{Num}}} \left((\text{machine operation cost}_k) - \frac{25}{3600} \right) (\text{average duration to produce part}_{nk})(x_n) \Bigg]$$

$$- \sum_{w}^{\text{Warehouse Num}} (\text{warehouse operation cost}_w)(\text{warehouse used}_w)$$

Constraints:

subject to

$$\sum_{r}^{\text{RM}_{\text{Num}}} (\text{cost of raw materials}_r)(\text{quantity of raw matrial}_{nr})(x_n) \leq 10000000 \tag{1}$$

$$\sum_{n}^{\text{Product}_{\text{Num}}} (\text{average duration to produce part}_{nk})(x_n) \leq 1209600 \quad \forall\ k \in \text{Machine Number} \tag{2}$$

$$\sum_{n}^{\text{Product Number}} (\text{cost of raw materials}_r)(\text{quantity of raw matrial}_{nr})(x_n) \leq (\text{max supply}_r) \quad \forall\ r \in \text{Raw Material} \tag{4}$$

$$x_n \leq S \sum_{w}^{\text{Warehouse}_{\text{Num}}} (\text{warehouse operation cost}_w)(\text{warehouse used}_w) \quad \forall\ n \in \text{Product Number} \tag{5}$$

$$x_n \leq 0 \quad \forall\ n \in \text{Product Num} \tag{6}$$

$$\text{warehouse used}_w \leq 0 \quad \forall\ w \in \text{Warehouse Num} \tag{7}$$

3.2 Assumptions

- The closing inventory of January 2006 is zero for all types of shoes.
- All sales happen at the end of the month.
- The demand for all types of shoes will double for the month of February 2006.
- The demand for each type of shoe in February 2006 is double the average monthly demand from the years 1997 to 2003.
- The demand estimations from historical data are representative of future demand.
- The productivity of the machines and workers remains consistent with no decrease in efficiency.
- The set up times and cost of the machines are negligible.
- Transportation costs of the products are ignored.
- Manufacturing sequences are ignored.

4.0 Results

4.1 Implementation

The decision variables used to formulate the objective function were derived by interpreting the project instructions and analyzing the provided dataset. This involved identifying relevant values from the dataset rather than relying solely on explicitly stated decision variables. Similarly, the constraints were constructed by carefully interpreting the dataset to ensure all influencing factors were accounted for.

When the objective function was initially formulated as an integer program, Gurobi faced challenges in solving it due to the extensive number of iterations required, resulting in prohibitive computational times. Gurobi employs a branch-and-bound approach to find the optimal solution, which was computationally intensive in this case. To enhance efficiency, the model was relaxed to a linear program (LP), and the resulting variable values were rounded to the nearest integers.

4.2 Results

The calculated optimal objective function value is $11,055,800, suggesting a potential maximum profit for WARP Shoe Company in February 2006. This figure provides a useful approximation for the company's production plan aimed at maximizing profitability. However, after rounding the constraints to the nearest integer, 53 constraints were violated, indicating that the objective function value may be an overestimate. Additionally, the demand figures used were averaged from 1997 to 2003, which may not accurately reflect the demand anticipated for February 2006. These factors introduce certain limitations to the accuracy of the results, yet the approximation remains a practical guide for strategic planning.

4.3 Questions

1. The average monthly demand for each product number was calculated by averaging the demands across all stores for the years 1997 to 2003. This average was doubled to account for the anticipated surge in demand for February 2006.
2. The model has 565 decision variables and 795 constraints, a result of being able to produce 557 products and store shoes in 8 different warehouses. These constraints reflect real-world limitations on raw materials and operational hours, which impact profitability.
3. 53 constraints were violated, specifically the maximum quantity of raw materials available for RM_Num 1, 2, 3, 8, 10, 12, 22, 24, 27, 28, 44, 47, 51, 61, 65, 68, 69, 73, 75, 79, 83, 86, 91, 93, 94, 97, 98, 101, 102, 104, 105, 106, 111, 113, 117, 123, 124, 126, 130, 132, 133, 134, 135, 137, 138, 139, 145, 146, 149, 154, and 162.
4. Binding constraints include the maximum available quantities of raw materials for RM_Num 5, 15, 16, 25, 29, 31, 35, 36, 37, 38, 39, 40, 42, 49, 52, 53, 54, 63, 66, 72, 76, 78, 85, 87, 89, 95, 100, 108, 114, 118, 122, 129, 131, 136, 152, 155, and 157. These constraints show that the optimal value makes full use of all the specified raw materials that are available. In practical terms, this implies that increasing the supply of these specific raw materials will increase profit further.
5. Purchasing additional warehouse space is not economically justified since no warehouse constraint is binding. The current warehouses are not being utilized to their full capacity, making the expenditure on extra space unnecessary.
6. Binding constraints include the maximum available quantities of raw materials for RM_Num 1, 21, 32, 38, 49, 57, 65, 67, 72, 75, 81, 86, 93, 97, 116, 123, 127, 129, 162, 163.When the daily operation time for the machines was reduced from 12 hours to 8 hours over 28 days of the month,

two out of seventy-two constraints related to machine operation time were violated. The violation of these constraints overestimated the objective function, increasing the maximum profit by $500 to a new total of $11,056,300.

7. An additional $7 million budget would not affect the objective function value because the binding constraints are primarily related to raw material availability. The limitation lies in the supply of raw materials rather than the budget constraint which has a slack value of $5,394,110.

5.0 Conclusion

This project demonstrates the application of operations research techniques to optimize the production plan for the WARP Shoe Company under real-world constraints. By formulating a mathematical model in AMPL and utilizing the Gurobi solver, the project addressed key challenges, including resource limitations, demand forecasting, and operational capacity, to maximize profitability.

The findings highlight the importance of accurate demand estimation and effective constraint management in developing an optimal production plan. The analysis revealed the optimal production quantities, binding constraints, and the potential for improved outcomes through strategic adjustments. Sensitivity analyses further emphasized the impact of resource variations on the solution, providing actionable insights for decision-making.

Despite the utility of the model, it is crucial to acknowledge its limitations. The reliance on historical demand data from 1997 to 2003 to estimate future needs may not accurately reflect the actual demand in February 2006, potentially leading to discrepancies in production plan. Furthermore, the violation of constraints due to the lack of computational efficiency introduces additional inaccuracies, which could overestimate the projected profitability. These factors underscore the necessity for careful interpretation of the model's outputs and suggest that adjustments may be required as more precise and current data becomes available.

In conclusion, this project underscores the value of mathematical optimization in enhancing industrial operations. The results provide WARP Shoe Company with a robust production strategy tailored to meet increased market demand while maintaining cost-efficiency. Future recommendations include exploring dynamic pricing strategies and integrating additional uncertainties, such as fluctuating raw material costs, to further refine the production planning process. This study not only achieves its immediate objectives but also lays a foundation for more advanced operational strategies in the future.

6.0 Appendix

Appendix A. Excel Data Filter

Figure A1. The organization of the .out file to identify binding constraints in excel

RM_Num	val
1	0
21	0
32	0
38	0
49	0
57	0
65	0
67	0
72	0
75	0
81	0
86	0
93	0
97	0
116	0
123	0
127	0
129	0
162	0
163	0

Table A1. Optimal amount of each product to produce

Product Number	Amount Produced
SH011	444
SH013	632
SH019	83
SH024	16
SH025	376
SH027	526
SH029	483

Product Number	Amount Produced
SH031	545
SH032	649
SH034	725
SH038	88
SH039	333
SH040	750
SH041	196
SH044	435
SH047	314
SH050	241
SH057	159
SH058	603
SH061	855
SH062	545
SH068	287
SH072	233
SH076	91
SH080	516
SH081	301
SH082	301
SH088	173
SH090	591
SH094	304
SH095	404
SH096	760
SH105	1064
SH107	39
SH110	689
SH113	422
SH115	72
SH118	113
SH121	1857
SH142	190

Product Number	Amount Produced
SH144	76
SH147	423
SH150	285
SH154	121
SH165	853
SH173	120
SH176	330
SH183	105
SH187	37
SH195	138
SH197	895
SH201	878
SH203	872
SH205	414
SH209	798
SH213	256
SH217	350
SH221	1003
SH223	971
SH224	606
SH225	926
SH226	660
SH227	777
SH229	184
SH230	428
SH233	431
SH234	636
SH235	325
SH236	530
SH238	75
SH245	288
SH248	14
SH249	763

Product Number	Amount Produced
SH253	491
SH254	247
SH256	1166
SH259	25
SH261	717
SH262	1266
SH265	20
SH269	922
SH274	121
SH277	887
SH281	151
SH282	912
SH287	867
SH289	37
SH295	992
SH298	350
SH299	182
SH300	860
SH308	399
SH312	347
SH313	916
SH314	208
SH316	677
SH317	272
SH320	903
SH329	378
SH334	692
SH338	453
SH340	495
SH341	50
SH345	361
SH348	23
SH352	547

Product Number	Amount Produced
SH359	14
SH361	74
SH367	171
SH373	15
SH378	446
SH379	408
SH384	1133
SH395	307
SH399	55
SH407	536
SH411	187
SH417	556
SH419	548
SH421	196
SH423	1469
SH425	724
SH427	1416
SH430	359
SH445	776
SH449	283
SH451	133
SH453	1258
SH456	56
SH462	302
SH465	532
SH466	517
SH467	471
SH470	283
SH471	387
SH473	845
SH474	408
SH476	360
SH477	1052

Product Number	Amount Produced
SH478	589
SH479	451
SH482	262
SH483	859
SH489	276
SH490	1125
SH491	394
SH497	497
SH500	797
SH501	1071
SH508	886
SH511	737
SH512	498
SH513	176
SH514	419
SH516	1111
SH521	469
SH525	405
SH532	616
SH535	727
SH540	538
SH543	186
SH544	29
SH552	107
SH556	788
SH557	394