Assignment 1 - Example:

0: (~X1 X2 X3)

1: (X1 X3 X4)

2: (X1 X3 ~X4)

3: (X1 ~X3 X4)

4: (X1 ~X3 ~X4)

5: (~X2 ~X3 X4)

6: (~X1 X2 ~X3)

7: (~X1 ~X2 X3)

c DIMACS representation:

p cnf 4 8

-1 2 3 0

-1 3 4 0

-1 3 -4 0

-1 -3 4 0

-1 -3 -4 0

-2 -3 4 0

-1 2 -3 0

-1 -2 3 0

Paper implementation of the DP algorithm:

Initial Formula:

0: (~X1 X2 X3)

1: (X1 X3 X4)

2: (X1 X3 ~X4)

3: (X1 ~X3 X4)

4: (X1 ~X3 ~X4)

5: (~X2 ~X3 X4)

6: (~X1 X2 ~X3)

7: (~X1 ~X2 X3)

Initial Assignment of truth values: X1: 0 X2: 0 X3: 0 X4: 0   
 (0 means unassigned. 1 means assigned true. -1 means assigned false)

**Step1:** set X1: 1

Assignment: X1: 1 X2: 0 X3: 0 X4: 0

Formula:

0: (~X1 X2 X3) changed to: ( X2 X3)

~~1: (X1 X3 X4)~~ clause satisfied

~~2: (X1 X3 ~X4)~~  clause satisfied

~~3: (X1 ~X3 X4)~~ clause satisfied

~~4: (X1 ~X3 ~X4)~~ clause satisfied

5: (~X2 ~X3 X4) remains unchanged

6: (~X1 X2 ~X3) changed to ( X2 ~X3)

7: (~X1 ~X2 X3) changed to ( ~X2 X3)

**Step 2:** set X2: 1

Assignment: X1: 1 X2: X3: 0 X4: 0

Formula (truncated):

0: ( X2 X3) clause satisfied

5: (~X2 ~X3 X4) changed to ( ~X3 X4)

6: ( X2 ~X3) clause satisfied

7: ( ~X2 X3) changed to ( X3)

**Step 3:** set X3: 1

Assignment: X1: 1 X2: 1 X3: 1 X4: 0

Formula:

5: ( ~X3 X4) changed to ( X4)

7: ( X3) clause satisfied

**Step 4:** set X4: 1

Assignment: X1: 1 X2: 1 X3: 1 X4: 1

Formula:

5: ( ~X3 X4) clause satisfied.

The formula is satisfiable and an assignment is X1: 1 X2: 1 X3: 1 X4: 1

**The following trace of the above example with different assignment of truth values illustrates backtracking situations and the data structure that facilitates backtracking**.

Internal representation of the formula (as a linked list) using clause labels:

Initial Formula : 0 1 2 3 4 5 6 7 -1  
initial Assignment of Boolean values to variables: 1: 0 2: 0 3: 0 4: 0   
 (0 means unassigned. 1 means assigned true. -1 means assigned false)

formula is not empty and it has no empty clauses  
**step 1:** Set var 1 to F. (Note: I have chosen to set the var1 to F and not to T as in the previous trace. This choice illustrates the backtracking process and the appropriate data structure.)   
Assignment of Boolean values to variables: 1: -1 2: 0 3: 0 4: 0  
Formula : 1 2 3 4 5 -1 7 6 0 -1

formula is not empty and it has no empty clauses  
**Step 2:** Set var 2 to F.  
Assignment of Boolean values to variables: 1: -1 2: -1 3: 0 4: 0  
Formula : 1 2 3 4 -1 5 -1 7 6 0 -1

formula is not empty and it has no empty clauses  
**Step 3:** Set var 3 to F  
Assignment of Boolean values to variables: 1: -1 2: -1 3: -1 4: 0  
Formula :   
1 2 -1 4 3 -1 5 -1 7 6 0 -1

formula is not empty and it has no empty clauses  
**Step 4:** Set var 4 to F  
Assignment of Boolean values to variables: 1: -1 2: -1 3: -1 4: -1  
Formula :   
1 -1 2 -1 4 3 -1 5 -1 7 6 0 -1

formula is not empty; but it has (at least) one empty clause. Time to backtrack. i.e. go back to the previous step. Note: a clause is empty if all its variables are assigned but the clause is not satisfied. Even if one clause is not satisfied then the formula is not satisfied.  
 **Step 5:** Set var 4 to T (unset 4 and reset it to T).  
Assignment of Boolean values to variables: 1: -1 2: -1 3: -1 4: 1  
Formula :   
2 -1 1 -1 4 3 -1 5 -1 7 6 0 -1

formula is not empty; but it has (at least) one empty clause. Time to backtrack.  
**Step 6:** Set var 3 to T (unset 4, unset 3 and reset 3 to T).  
Assignment of Boolean values to variables: 1: -1 2: -1 3: 1 4: 0  
Formula :   
 3 4 -1 1 2 -1 5 -1 7 6 0 -1

Note: step 5 and step 6 illustrate two types of backtracking. Be careful while implementing these.

And, So on ….

A final satisfying assignment for this formula is: 1: 1 2: 1 3: 1 4: 1