COMP-2310 Formula Sheet
Midterm Test 2

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Chapter 3

Definition (Principle of Extension). *Two sets A and B are **equal**, denoted by $A = B$, iff* $(\forall x)(x \in A \Leftrightarrow x \in B)$

Lemma (3.2.1). *Let A, B, C be sets.*

(i) $A = A$

(ii) If $(A = B)$, *then* $B = A$

(iii) If $(A = B)$ *and* $(B = C)$, *then* $A = C$

Lemma (3.2.2). $A \neq B$ *iff* $(\exists x)(x \in A \wedge x \notin B) \vee (\exists x)(x \notin A \wedge x \in B)$

Corollary (3.2.2.1). *Let A and B be sets.*
$(\exists x)(x \in A \wedge x \notin B) \Rightarrow A \neq B$

Definition (Subset). *Let A, B be two sets. A is a **subset** of B or B **includes** A, denoted by $A \subseteq B$, iff* $(\forall x)(x \in A \Rightarrow x \in B)$.
*A is a **proper subset** of B, denoted by $A \subset B$, iff $A \subseteq B$ and $A \neq B$.*

Lemma (3.2.3). *Let A, B, C be sets.*

(i) $A \subseteq A$

(ii) If $A \subseteq B$ and $B \subseteq A$, then $A = B$

(iii) If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$

Lemma (3.2.4). *Let A, B be sets. If $A = B$, then* $(A \subseteq B) \wedge (B \subseteq A)$

Lemma (3.2.5). $A \nsubseteq B \Leftrightarrow (\exists x)(x \in A \wedge x \notin B)$

Lemma (3.2.6). *If $A \subset B$, then* $(\exists x)(x \in B \wedge x \notin A)$

Definition (Principle of Specification). *For every set A and every formula $S(x)$, there exists a set B whose elements are exactly those elements of A for which $S(x)$ is true. The set B is denoted by:* $\{x | x \in A \wedge S(x)\}$ *or* $\{x \in A | S(x)\}$

Definition (Power Set). *Let A be a set. The **power set** of A, denoted by $\mathcal{P}(A)$, is the set* $\{X | X \subseteq A\}$

Chapter 4

Definition (Union)**.** *Let A, B be two sets. The **union** of A and B is the set $A \cup B = \{x | x \in A \vee x \in B\}$*

Theorem (4.1.1)**.** *Let A, B be sets.*

(i) $A \cup \emptyset = A$

(ii) $A \cup A = A$

(iii) $A \cup B = B \cup A$

(iv) $(A \cup B) \cup C = A \cup (B \cup C)$

(v) $A \subseteq B$ *iff* $A \cup B = B$

Definition (Intersection)**.** *Let A, B be two sets. The **intersection** of A and B is the set $A \cap B = \{x | x \in A \wedge x \in B\}$*

Theorem (4.2.2)**.** *Let A, B be sets.*

(i) $A \cap \emptyset = \emptyset$

(ii) $A \cap A = A$

(iii) $A \cap B = B \cap A$

(iv) $(A \cap B) \cap C = A \cap (B \cap C)$

(v) $A \subseteq B$ *iff* $A \cap B = A$

Corollary (4.2.2.1)**.** *Let A, B be any two sets. Then, $A \cap B \subseteq A \subseteq A \cup B$*

Theorem (4.2.3)**.** *Let A, B, C be sets.*

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Definition (Disjoint)**.** *Two sets A and B are **disjoint** if $A \cap B = \emptyset$*

Definition (Relative Complement)**.** *Let A, B be two sets. The **relative complement** of B in A is the set $A - B = \{x | x \in A \wedge x \notin B\}$*

Theorem (4.3.4)**.** *Let A, B, C be sets.*

(i) $A \subseteq B$ *iff* $A - B = \emptyset$

(ii) $A - (A - B) = A \cap B$

(iii) $A \cap (B - C) = (A \cap B) - (A \cap C)$

Definition (Absolute Complement)**.** *The* ***(relative) Universal set****, denoted by* $\mathbf{U}$*, is a set which contains every object (thereby includes every set) in our discussion. Let* A *be any set such that* $A \subseteq \mathbf{U}$*. The* ***absolute complement*** *of* A *is the set* $\overline{A} = \mathbf{U} - A$

[4.3.5]

Lemma. *Let* $A \subseteq \mathbf{U}$*. Then,* $x \in \overline{A} \Leftrightarrow x \notin A$

Theorem (4.3.6 DeMorgan's Theorem)**.** *Let A, B be any two subsets of* $\mathbf{U}$

(i) $\overline{A \cup B} = \overline{A} \cap \overline{B}$

(ii) $\overline{A \cap B} = \overline{A} \cup \overline{B}$

Theorem (4.3.7)**.** *Let A, B be any two subsets of* $\mathbf{U}$*.*

(i) $\overline{\overline{A}} = A$

(ii) $\overline{\emptyset} = \mathbf{U}$ *and* $\overline{\mathbf{U}} = \emptyset$

(iii) $A \cap \overline{A} = \emptyset$ *and* $A \cup \overline{A} = \mathbf{U}$

(iv) $A \subseteq B$ *iff* $\overline{B} \subseteq \overline{A}$

(v) $A - B = A \cap \overline{B}$

Definition (Ordered Pair)**.** *The* ***ordered pair*** *of a and b is the set*
$(a, b) = \{\{a\}, \{a, b\}\} = \{x | x = \{a\} \vee x = \{a, b\}\}$

Lemma (4.4.8)**.** $(a, b) = (x, y)$ *implies that* $a = x$ *and* $b = y$

Lemma (4.4.9)**.** $a = c \wedge b = d \Rightarrow (a, b) = (c, d)$

Definition (Cartesian Product)**.** *Let A, B be two sets. The* ***Cartesian Product*** *of A and B is the set* $A \times B = \{x | (\exists a)(\exists b)(a \in A \wedge b \in B \wedge x = (a, b))\} = \{(a, b) | a \in A \wedge b \in B\}$

Lemma (4.5.10)**.** *Let A, B, X and Y be sets.*

(i) $(A \cup B) \times X = (A \times X) \cup (B \times X)$

(ii) $(A \cap B) \times X = (A \times X) \cap (B \times X)$

(iii) $(A - B) \times X = (A \times X) - (B \times X)$

(iv) $(A \cap B) \times (X \cap Y) = (A \times X) \cap (B \times Y)$

(v) $(A = \emptyset \vee B = \emptyset) \Leftrightarrow A \times B = \emptyset$

Definition (Union of a Collection of Sets)**.** *Let $\mathcal{C}$ be a collection of sets. The* ***union of*** *$\mathcal{C}$ is the set:*

$$\bigcup_{X \in \mathcal{C}} X = \{x | (\exists X)(X \in \mathcal{C} \wedge x \in X)\} = \{x | x \in X \text{ for some } X \in \mathcal{C}\}$$

Definition (Intersection of a Collection of Sets)**.** *Let $\mathcal{C}$ be a collection of sets such that $\mathcal{C} \neq \emptyset$. The* ***intersection of*** *$\mathcal{C}$ is the set:*

$$\bigcap_{X \in \mathcal{C}} X = \{x | (\forall X)(X \in \mathcal{C} \Rightarrow x \in X)\} = \{x | x \in X \text{ for all } X \in \mathcal{C}\}$$

Lemma. *A:* $(X \cap Y) \cup X = X$

Chapter 5

Definition (Relation). *A* ***relation*** *is a set of ordered pairs. Specifically, a set R is a relation if $(\forall x)(x \in R \Rightarrow (\exists a)(\exists b)x = (a, b))$*

Definition (Domain & Range of Relations). *Let R be a relation. The* ***domain*** *of R is the set $Dom(R) = \{x|(\exists y)(x, y) \in R\}$ and the* ***range*** *of R is the set $Ran(R) = \{y|(\exists x)(x, y) \in R\}$*

Definition (Cartesian Product of Relations). *Let $R \subseteq X \times Y$, we say that R is a* ***relation from X to Y****. In particular, when $X = Y$, (ie. $R \subseteq X \times X$), we say that R is a* ***relation in X****.*

Lemma (5.1.1). *Let R be a relation from X to Y. Then $Dom(R) \subseteq X$ and $Ran(R) \subseteq Y$*

Definition (Properties of Relation). *Let R be a relation in a set X*

R is ***reflexive*** *if $(\forall x \in X)(x, x) \in R$;*

R is ***irreflexive*** *if $(\forall x \in X)(x, x) \notin R$;*

R is ***symmetric*** *if $(x, y) \in R \Rightarrow (y, x) \in R$;*

R is ***antisymmetric*** *if $(x, y) \in R \wedge (y, x) \in R \Rightarrow x = y$;*

R is ***asymmetric*** *if $\forall x, y \in X,\ ((x, y) \in R \wedge (y, x) \in R)$, or equivalently, $\forall x, y \in X, (x, y) \in R \Rightarrow (y, x) \notin R$;*

R is ***transitive*** *if $(x, y) \in R \wedge (y, z) \in R \Rightarrow (x, z) \in R$.*

Lemma (5.2.1). *Let R be a relation in X. If $(\exists x \in X)(x, x) \in R$, then R is not asymmetric.*

Definition (Equivalence Relation). *A relation R in a set X is an* ***equivalence relation*** *if R is reflexive, symmetric and transitive.*

Definition (Equivalence Class). *Let R be an equivalence relation in a set X. For each $x \in X$, the* ***equivalence class of x w.r.t. R*** *is the set $[x]/R = \{y|y \in X \wedge (x, y) \in R\} = \{y \in X|(x, y) \in R\}$ or equivalently, $[x]/R = \{y|(x, y) \in R\}$*

Definition (Collection of Equivalence Classes). *Let R be an equivalence relation in a set X.* ***The collection of equivalence classes w.r.t. R*** *is the set: $[X]/R = \{S|(\exists x)(x \in X \wedge S = [x]/R)\} = \{[x]/R|x \in X\}$*

Definition (Partition of a Collection of Sets).

continue on page 122 definition at the very top!