

Probability and Random Variables (AI1110)

Assignment-1

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Q- 12.13.6.6 In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $\frac{5}{6}$. What is the probability that he will knock down fewer than 2 hurdles?

Solution : We can assign the following probabilities for whether the player clears the hurdle or not.

$$\Pr(\text{Cleared}) = \frac{5}{6} \quad (1)$$

$$\begin{aligned} \Pr(\text{Not Cleared}) &= 1 - \frac{5}{6} \\ &= \frac{1}{6} \end{aligned} \quad (2)$$

Let X be the random variable that denotes the number of hurdles not cleared by the player. This is a binomial distribution where not clearing a hurdle is considered a success.

The probability of getting r successes in a binomial distribution having n independent Bernoulli trials and probability of success in each trial being p is

$$\Pr(X = r) = {}^nC_r p^r (1 - p)^{n-r} \quad (3)$$

Here, $p = \Pr(\text{Not Cleared}) = \frac{1}{6}$ and $n = 10$. So, we can write

$$X \sim \text{Bin}\left(10, \frac{1}{6}\right) \quad (4)$$

$$\Pr(X = r) = {}^{10}C_r \left(\frac{1}{6}\right)^r \left(\frac{5}{6}\right)^{10-r} \quad (5)$$

cumulative distribution function

$$\Pr(X < 2) = \Pr(X \leq 1) \quad (6)$$

$$= F_X(1) \quad (7)$$

$$= \sum_{i=0}^1 \Pr(X = i) \quad (8)$$

$$= \sum_{i=0}^1 {}^{10}C_i \left(\frac{1}{6}\right)^i \left(\frac{5}{6}\right)^{10-i} \quad (9)$$

$$= {}^{10}C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{10} + {}^{10}C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^9 \quad (10)$$

$$= 1 \cdot 1 \cdot \left(\frac{5}{6}\right)^{10} + 10 \cdot \left(\frac{1}{6}\right)^1 \cdot \left(\frac{5}{6}\right)^9 \quad (11)$$

$$= \left(\frac{5}{6}\right)^{10} + 10 \cdot \left(\frac{1}{6}\right)^1 \cdot \left(\frac{5}{6}\right)^9 \quad (12)$$

$$= \left(\frac{5}{6}\right)^9 \cdot \left[\frac{5}{6} + \frac{10}{6}\right] \quad (13)$$

$$= \left(\frac{5}{6}\right)^9 \cdot \left[\frac{5}{2}\right] \approx 0.48452 \quad (14)$$

Table 1: Parameters

Parameter	Description	Value
p	Probability of clearing hurdle	0.833333333333
X	R.V. denoting no. of knocked hurdles	{0, 1, ..., 10}
n	Number of hurdles	10
$\Pr(X = 0)$	Probability that 0 hurdles knocked	0.16150558289
$\Pr(X = 1)$	Probability that 1 hurdle knocked	0.32301116578
$\Pr(X < 2)$	Probability that < 2 hurdles knocked	0.48451674867

The probability that the player knocks down less than two hurdles can be written as the following