

Probability and Random Variables (AI1110)

Assignment-1

Edward Nathan Varghese
CS22BTECH11020

Q- 12.13.4.4 Find the probability distribution of

X	0	1	2
Pr(X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

- 1) number of heads in two tosses of a coin.
- 2) number of tails in the simultaneous tosses of three coins.
- 3) number of heads in four tosses of a coin.

Solution :

- 1) Let X denote the number of heads. In two coin tosses, the random variable X can take values 0, 1 or 2.

In one coin toss,

$$\text{Probability of Head} = \Pr(\text{Head}) = \frac{1}{2} \quad (1)$$

$$\text{Probability of Tail} = \Pr(\text{Tail}) = \frac{1}{2} \quad (2)$$

Since the multiple coin tosses are independent events, we can obtain the probabilities by multiplication rule, (??) and (??) .

$$\begin{aligned} \Pr(X = 0) &= \Pr(\text{Tail}) \times \Pr(\text{Tail}) \\ &= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \end{aligned} \quad (3)$$

$$\begin{aligned} \Pr(X = 1) &= 2 \times \Pr(\text{Tail}) \times \Pr(\text{Head}) \\ &= 2 \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{2} \end{aligned} \quad (4)$$

$$\begin{aligned} \Pr(X = 2) &= \Pr(\text{Head}) \times \Pr(\text{Head}) \\ &= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \end{aligned} \quad (5)$$

Thus, the required probability distribution is:

- 2) The sample space of this event is

$$S = \{HHH, HHT, HTT, HTH, THH, THT, TTT, TTH\}$$

X is a random variable that represents the number of tails and can take values 0, 1, 2 and 3.

$$\Pr(X = 0) = \frac{\binom{3}{0}}{2^3} = \frac{1}{8} \quad (1)$$

$$\Pr(X = 1) = \frac{\binom{3}{1}}{2^3} = \frac{3}{8} \quad (2)$$

$$\Pr(X = 2) = \frac{\binom{3}{2}}{2^3} = \frac{3}{8} \quad (3)$$

$$\Pr(X = 3) = \frac{\binom{3}{3}}{2^3} = \frac{1}{8} \quad (4)$$

Thus, the required probability distribution is:

X	0	1	2	3
Pr(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

- 3) We can calculate the probability distribution of the number of heads in four tosses of a coin using combinations. Let X be the random variable denoting the number of heads in 4 coin tosses. It can take values 0, 1, 2, 3 and 4.

$$\Pr(X = 0) = \frac{\binom{4}{0}}{2^4} = \frac{1}{16} \quad (1)$$

$$\Pr(X = 1) = \frac{\binom{4}{1}}{2^4} = \frac{4}{16} = \frac{1}{4} \quad (2)$$

$$\Pr(X = 2) = \frac{\binom{4}{2}}{2^4} = \frac{6}{16} = \frac{3}{8} \quad (3)$$

$$\Pr(X = 3) = \frac{\binom{4}{3}}{2^4} = \frac{4}{16} = \frac{1}{4} \quad (4)$$

$$\Pr(X = 4) = \frac{\binom{4}{4}}{2^4} = \frac{1}{16} \quad (5)$$

Thus, the required probability distribution is:

X	0	1	2	3	4
Pr (X)	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$