Probability and Random Variables (AI1110) Assignment-1

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Q- 12.13.6.6 In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $\frac{5}{6}$. What is the probability that he will knock down fewer than 2 hurdles?

Solution : We can assign the following probabilities for whether the player clears the hurdle or not.

$$\Pr(\text{Cleared}) = \frac{5}{6} \tag{1}$$

$$\Pr(\text{Not Cleared}) = 1 - \frac{5}{6} = \frac{1}{6} \tag{2}$$

Let X be the random variable that denotes the number of hurdles not cleared by the player. This is a binomial distribution where not clearing a hurdle is considered a success.

The probability of getting r successes in a binomial distribution having n independent Bernoulli trials and probability of success in each trial being p is

$$\Pr(X = r) = {}^{n}C_r p^r (1-p)^{n-r} \tag{3}$$

Here, $p = \Pr(\text{Not Cleared}) = \frac{1}{6}$ and $n = 10$. So, we can write

$$X \sim Bin\left(10, \frac{1}{6}\right) \tag{4}$$

$$\Pr(X = r) = {}^{10}C_r \left(\frac{1}{6}\right)^r \left(\frac{5}{6}\right)^{10-r} \tag{5}$$

The probability that the player knocks down less than two hurdles can be written as

$$\Pr(X < 2) = \Pr(X = 0) + \Pr(X = 1) \tag{6}$$

$$= {}^{10}C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{10} + {}^{10}C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^9 \tag{7}$$

$$= 1 \cdot 1 \cdot \left(\frac{5}{6}\right)^{10} + 10 \cdot \left(\frac{1}{6}\right)^1 \cdot \left(\frac{5}{6}\right)^9 \tag{8}$$

$$= \left(\frac{5}{6}\right)^{10} + 10 \cdot \left(\frac{1}{6}\right)^1 \cdot \left(\frac{5}{6}\right)^9 \tag{9}$$

$$= \left(\frac{5}{6}\right)^9 \cdot \left[\frac{5}{6} + \frac{10}{6}\right] \tag{10}$$

$$= \left(\frac{5}{6}\right)^9 \cdot \left[\frac{5}{2}\right] \approx 0.48452 \tag{11}$$