Probability and Random Variables (AI1110) Assignment-1

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Q- 12.13.6.6 In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $\frac{5}{6}$. What is the probability that he will knock down fewer than 2 hurdles?

Solution : We can assign the following probabilities for whether the player clears the hurdle or not.

$$\Pr(\text{Cleared}) = \frac{5}{6} \tag{1}$$

$$\Pr(\text{Not Cleared}) = 1 - \frac{5}{6} \tag{2}$$
$$= \frac{1}{6}$$

Let X be the random variable that denotes the number of hurdles not cleared by the player. This is a binomial distribution where not clearing a hurdle is considered a success.

The probability of getting r successes in a binomial distribution having n independent Bernoulli trials and probability of success in each trial being p is

$$\Pr(X = r) = {}^{n}C_{r}p^{r}(1 - p)^{n-r} \tag{3}$$

Here, $p = \Pr(\text{Not Cleared}) = \frac{1}{6}$ and $n = 10$. So, we can write

$$X \sim Bin\left(10, \frac{1}{6}\right) \tag{4}$$

$$\Pr(X = r) = {}^{10}C_{r}\left(\frac{1}{6}\right)^{r}\left(\frac{5}{6}\right)^{10-r} \tag{5}$$

The probability that the player knocks down less than two hurdles can be written as the following cumulative distribution function

$$\Pr(X < 2) = \Pr(X <= 1) \tag{6}$$
$$= F_X(1) \tag{7}$$
$$= \sum_{i=0}^{1} \Pr(X = i) \tag{8}$$
$$= \sum_{i=0}^{1} {}^{10}C_i\left(\frac{1}{6}\right)^{i}\left(\frac{5}{6}\right)^{10-i} \tag{9}$$
$$= {}^{10}C_0\left(\frac{1}{6}\right)^{0}\left(\frac{5}{6}\right)^{10} + {}^{10}C_1\left(\frac{1}{6}\right)^{1}\left(\frac{5}{6}\right)^{9} \tag{10}$$
$$= 1 \cdot 1 \cdot \left(\frac{5}{6}\right)^{10} + 10 \cdot \left(\frac{1}{6}\right)^{1} \cdot \left(\frac{5}{6}\right)^{9} \tag{11}$$
$$= \left(\frac{5}{6}\right)^{10} + 10 \cdot \left(\frac{1}{6}\right)^{1} \cdot \left(\frac{5}{6}\right)^{9} \tag{12}$$
$$= \left(\frac{5}{6}\right)^{9} \cdot \left[\frac{5}{6} + \frac{10}{6}\right] \tag{13}$$
$$= \left(\frac{5}{6}\right)^{9} \cdot \left[\frac{5}{2}\right] \approx 0.48452 \tag{14}$$

Parameter	Description	Value
p	Probability of clearing hurdle	$\frac{5}{6}$
X	R.V. denoting no. of knocked hurdles	$\{0, 1, ...10\}$
n	Number of hurdles	10

TABLE 0
PARAMETERS AND DESCRIPTIONS