Probability and Random Variables (AI1110) Assignment-1

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Q- 12.13.4.4 Find the probability distribution of

1) number of heads in two tosses of a coin.
2) number of tails in the simultaneous tosses of three coins.
3) number of heads in four tosses of a coin.

Solution :

1) Let X denote the number of heads. In two coin tosses, the random variable X can take values 0, 1 or 2.

   In one coin toss,

$$\text{Probability of Head} = \Pr(Head) = \frac{1}{2} \tag{1}$$

$$\text{Probability of Tail} = \Pr(Tail) = \frac{1}{2} \tag{2}$$

   Since the multiple coin tosses are independent events, we can obtain the probabilities by multiplication rule, (1) and (2) .

$$\begin{aligned} \Pr(X = 0) &= \Pr(Tail) \times \Pr(Tail) \\ &= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \end{aligned} \tag{3}$$

$$\begin{aligned} \Pr(X = 1) &= 2 \times \Pr(Tail) \times \Pr(Tail) \\ &= 2 \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{2} \end{aligned} \tag{4}$$

$$\begin{aligned} \Pr(X = 2) &= \Pr(Head) \times \Pr(Head) \\ &= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \end{aligned} \tag{5}$$

   Thus, the required probability distribution is:

X	0	1	2
$\Pr(X)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

2) The sample space of this event is

$$S = \{HHH, HHT, HTT, HTH, THH, THT, TTT, TTH\}$$

   X is a random variable that represents the number of tails and can take values 0, 1, 2 and 3.

$$\Pr(X = 0) = \frac{\binom{3}{0}}{2^3} = \frac{1}{8} \tag{1}$$

$$\Pr(X = 1) = \frac{\binom{3}{1}}{2^3} = \frac{3}{8} \tag{2}$$

$$\Pr(X = 2) = \frac{\binom{3}{2}}{2^3} = \frac{3}{8} \tag{3}$$

$$\Pr(X = 3) = \frac{\binom{3}{3}}{2^3} = \frac{1}{8} \tag{4}$$

   Thus, the required probability distribution is:

X	0	1	2	3
$\Pr(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

3) We can calculate the probability distribution of the number of heads in four tosses of a coin using combinations. Let X be the random variable denoting the number of heads in 4 coin tosses. It can take values 0, 1, 2, 3 and 4.

$$\Pr(X = 0) = \frac{\binom{4}{0}}{2^4} = \frac{1}{16} \tag{1}$$

$$\Pr(X = 1) = \frac{\binom{4}{1}}{2^4} = \frac{1}{4} \tag{2}$$

$$\Pr(X = 2) = \frac{\binom{4}{2}}{2^4} = \frac{3}{8} \tag{3}$$

$$\Pr(X = 3) = \frac{\binom{4}{3}}{2^4} = \frac{1}{4} \tag{4}$$

$$\Pr(X = 4) = \frac{\binom{4}{4}}{2^4} = \frac{1}{16} \qquad (5)$$

Thus, the required probability distribution is:

X	0	1	2	3	4
$\Pr(X)$	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$