Creating Fragility Functions for Performance-Based Earthquake Engineering

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The Applied Technology Council is adapting PEER's performance-based earthquake engineering methodology to professional practice. The methodology's damage-analysis stage uses fragility functions to calculate the probability of damage to facility components given the force, deformation, or other engineering demand parameter (EDP) to which each is subjected. This paper introduces a set of procedures for creating fragility functions from various kinds of data: (A) actual EDP at which each specimen failed; (B) bounding EDP, in which some specimens failed and one knows the EDP to which each specimen was subjected; (C) capable EDP, where specimen EDPs are known but no specimens failed; (D) derived, where fragility functions are produced analytically; (E) expert opinion; and (U) updating, in which one improves an existing fragility function using new observations. Methods C, E, and U are all introduced here for the first time. A companion document offers additional procedures and more examples. [DOI: 10.1193/1.2720892]

INTRODUCTION

BACKGROUND AND OBJECTIVES

A second-generation performance-based earthquake engineering (PBEE-2) procedure has been developed by the Pacific Earthquake Engineering Research (PEER) Center and others that estimates the probabilistic future seismic performance of buildings and bridges in terms of system-level decision variables (DVs), i.e., performance measures that are meaningful to the owner, such as repair cost, casualties, and loss of use (dollars, deaths, and downtime). Under contract to the Federal Emergency Management Agency, the Applied Technology Council has undertaken to transfer the PEER methodology to professional practice (ATC 2005). The methodology involves four stages: hazard analysis, structural analysis, damage analysis, and loss analysis. This paper addresses the damage analysis, whose input is the engineering demand parameters (EDP) calculated in the structural analysis, and whose output is the damage measure (DM) of each

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Table 1. Analysis methods and data employed

Method name	Data used
A. Actual failure EDP	All specimens failed at observed values of EDP
B. Bounding EDP	Some specimens failed; maximum EDP for each is known
C. Capable EDP	No specimens failed; maximum EDP for each is known
D. Derived fragility	Fragility functions produced analytically
E. Expert opinion	Expert judgment is used
U. Updating	Enhance existing fragility functions with new method-B data

damageable structural and nonstructural component in the facility. The analysis uses fragility functions, which in this context give the probability of exceeding a damage state (a value of DM) as a function of EDP. One such fragility function is required for each component type and damage state. Many building-component fragility functions have been created in the past, but no comprehensive set of procedures exists on how to create them. This paper summarizes such a standard developed for ATC-58. See Porter et al. (2006) for more detail, examples, commentary, and alternative approaches.

Damage data come in many forms, but generally comprise knowledge of specimen damage and the EDP imposed. Table 1 lists methods for six situations. Each addresses different data and thus they are not interchangeable. For example, Method A is not applicable when one knows the maximum EDP to which each specimen was subjected, but not the value of EDP at which specimens actually failed. One cannot use Method C if some specimens failed.

The methods proposed here are no substitute for understanding the processes that lead to damage, but are intended to help practitioners and scholars create fragility functions from damage data. No calculus is required, and the only possibly unfamiliar expression is the Gaussian distribution, typically available in spreadsheet software.

DOCUMENTATION REQUIREMENTS

Four requirements are proposed for documenting fragility functions:

1. *Description of specimens*. What is the component type or taxonomic group the fragility function addresses? (See Porter 2005 for an ATC-58 component taxonomy.) Where and how many specimens were tested or observed, how are they counted, and what were their materials, material properties, configuration, and building code (if applicable)? Provide a bibliographic reference of any data source.
2. *Excitation and EDP*. Detail the loading protocol or characteristics of earthquake motion. Identify the EDP(s) examined that might be most closely related to failure probability and define how EDP is calculated or inferred from the loading protocol or observed excitation. Indicate whether EDP is the value at which damage occurred (Method A data) or the maximum to which each specimen was subjected (Methods B, C, and U).

3. *Damage evidence and DM.* What kinds of physical damage or force-deformation were observed? Define damage measures (DMs) quantitatively in terms of repairs required. Damage is assumed to have a repair cost, but note threats to life-safety or potential for loss of use. Explain how DM is inferred from damage or force-deformation evidence.
4. *Observation summary, analysis method, and results.* Present a tabular or graphical listing of specimens, EDP, and DM. Which method was used to derive the fragility function (Table 1)? Present resulting fragility function parameters x_m and β and results of tests to establish fragility function quality (discussed below). Provide sample calculations.

DAMAGE STATE PROBABILITY

$F_{dm}(edp)$ denotes the fragility function for damage state *dm*, defined as the probability that the component reaches or exceeds damage state *dm*, given a particular EDP value (Equation 1), and idealized by a lognormal distribution (Equation 2):

$$F_{dm}(edp) \equiv P[DM \geq dm | EDP = edp] \tag{1}$$

$$F_{dm}(edp) = \Phi\left(\frac{\ln(edp/x_m)}{\beta}\right) \tag{2}$$

where Φ denotes the standard normal (Gaussian) cumulative distribution function (e.g., normsdist in Microsoft Excel), x_m denotes the median value of the distribution, and β denotes the logarithmic standard deviation.

We use the lognormal because it fits a variety of structural component failure data well (e.g., Beck et al. 2002, Aslani 2005, Pagni and Lowes 2006), as well as nonstructural failure data (Reed et al. 1991 [Appendix J], Porter and Kiremidjian 2001, Badillo-Almaraz et al. 2006), and building collapse by IDA (e.g., Cornell et al. 2005). It has strong precedent in seismic risk analysis (e.g., Kennedy and Short 1994, Kircher et al. 1997). Finally, there is a strong theoretical reason to use the lognormal: it has zero probability density at and below zero EDP, is fully defined by measures of the first and second moments—$\ln(x_m)$ and β—and imposes the minimum information given these constraints, in the information-theory sense (Goodman 1985).

Both x_m and β are established for each component type and damage state using methods presented later. The probability that the component is *in* damage state *dm*, given EDP=*edp*, is given by

$$\begin{aligned} P[DM = dm | EDP = edp] &= 1 - F_1(edp) \quad dm = 0 \\ &= F_{dm}(edp) - F_{dm+1}(edp) \quad 1 \leq dm < N \\ &= F_{dm}(edp) \quad dm = N \end{aligned} \tag{3}$$

where N denotes the number of possible damage states for the component, in addition to the undamaged state, and $dm=0$ denotes the undamaged state. Where $N \geq 2$ and $\beta_i \neq \beta_j$ for two damage states $i \neq j$, Equation 3 can produce a meaningless negative probability at some levels of EDP. This situation is addressed later.

CREATING FRAGILITY FUNCTIONS

This section provides mathematical procedures for developing fragility functions.

METHOD A, ACTUAL EDP: ALL SPECIMENS FAILED AT OBSERVED EDP

These are the most informative data for creating fragility functions. They are most common where DM can be associated with a point on the observed force-deformation behavior of a component, such as a yield point. Alternatively, specimens are subjected to increasing levels of EDP. The test is interrupted after each level of EDP is imposed, and the specimen examined for damage. Let

M=number of specimens tested to failure
i=index of specimens, $i \in \{1, 2, \ldots M\}$
r_i=EDP at which damage was observed to occur in specimen i.

From the basic definitions of x_m and β (e.g., Ang and Tang 1975),

$$x_m = \exp\left(\frac{1}{M}\sum_{i=1}^{M} \ln r_i\right) \quad \beta = \sqrt{\frac{1}{M-1}\sum_{i=1}^{M} (\ln(r_i/x_m))^2} \tag{4}$$

One tests the resulting fragility function using the Lilliefors goodness-of-fit test (presented below). If it passes at the 5% significance level, the fragility function is acceptable.

Example 1. Aslani (2005) provides a table of peak transient drift ratios at which 43 specimens of pre-1976 reinforced concrete slab-column connections experienced cracking of no more than 0.3 mm width, repaired by applying a surface coating. The data are repeated in Table 2 with original specimen numbers. Calculate the fragility function and test goodness of fit.

Solution. The data are sorted in order of increasing r, an index i is added, the statistics $\ln(r_i)$ and $\ln(r_i/x_m)^2$ calculated and summed. Using Equation 4, $x_m=0.38$, and $\beta=0.39$. The lognormal distribution with these parameters passes the Lilliefors goodness-of-fit test at the 5% significance level. The math is omitted here, but the test is illustrated in Figure 1.

METHOD B, BOUNDING EDP: SOME SPECIMENS FAILED, PEAK EDP KNOWN

Here, the data include the maximum EDP to which each of M specimens was subjected, and knowledge of whether the specimen exceeded the damage state of interest.

Table 2. Example 1 slab-column connection damage data; *s* = specimen, *r* = peak transient drift ratio, %

s	*r*	*s*	*r*	*s*	*r*	*s*	*r*	*s*	*r*	*s*	*r*
3	0.43	11	0.28	22	0.40	60	0.54	72	0.19	80	0.43
4	0.30	12	0.35	23	0.36	61	0.40	73	0.28	81	0.50
5	0.28	16	0.31	24	0.28	62	0.80	74	0.28	82	0.70
6	0.65	17	0.31	25	0.20	66	0.50	75	0.40	$\Sigma \ln r$	−41.6
7	0.22	18	0.28	26	0.50	67	0.50	76	0.74	x_m	0.38
8	0.32	19	0.22	27	0.25	68	0.50	77	0.54	$\Sigma \ln(r/x_m)^2$	6.40
9	0.43	20	0.22	28	0.50	69	0.50	78	0.43	β	0.39
10	0.42	21	0.31	59	0.64	71	0.19	79	0.71		

(Specimens without PTD data are omitted)

Some specimens must be damaged. The method works best where $M \geq 25$. Data must not be biased by damage state, i.e., specimens must not be selected because they experienced damage. The data are grouped into bins by ranges of EDP, where each bin has approximately the same number of specimens in it. For each bin, one calculates the fraction of specimens that failed and the bin-average EDP. These serve as independent data points of failure probability and EDP. The following approach converts Equation 2 to a linear regression problem by taking the inverse Gaussian cumulative distribution function of each side and fitting a line $\hat{y}=sx+c$ to the data (e.g., see "probability paper" in Ang and Tang 1975). Let (*cont.*)

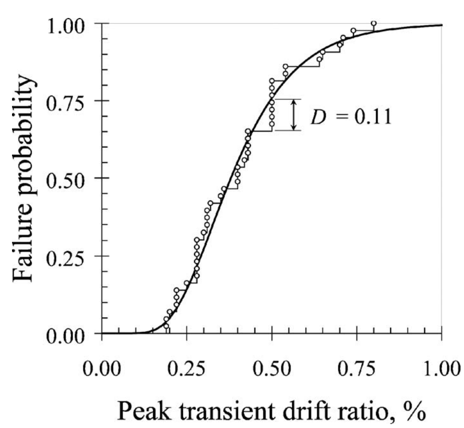

Figure 1. Example 1 fragility function (smooth curve) and sample cumulative distribution (stepped curve).

M =number of specimens observed
i =index of specimens, $i \in \{1, 2, \ldots M\}$
r_i =maximum EDP to which specimen i was subjected
f_i =failure indicator for specimen i, 1 if the specimen failed, 0 otherwise
N =number of EDP bins

$$N = \lfloor \sqrt{M} \rfloor \tag{5}$$

where $\lfloor\rfloor$ means the largest integer less than or equal to the term inside

j =index of data bins, $j \in \{1, 2, \ldots N\}$
a_j =lower EDP bound of bin j

$$a_j = r_{N(j-1)+1} \tag{6}$$

M_j = number of specimens with $a_j \leq r < a_{j+1}$

$$\begin{aligned} M_j &= \sum_{i=1}^{M} H(r_i - a_j) - H(r_i - a_{j+1}) \quad & j < N \\ &= \sum_{i=1}^{M} H(r_i - a_j) & j = N \end{aligned} \tag{7}$$

x_j =natural logarithm of the average r within bin j

$$\begin{aligned} x_j &= \ln\left(\frac{1}{M_j}\sum_{i=1}^{M} r_i(H(r_i - a_j) - H(r_i - a_{j+1}))\right) \quad & j < N \\ &= \ln\left(\frac{1}{M_j}\sum_{i=1}^{M} r_i H(r_i - a_j)\right) & j = N \end{aligned} \tag{8}$$

m_j = number of failed specimens in bin j, i.e.,

$$\begin{aligned} m_j &= \sum_{i=1}^{M} f_i(H(r_i - a_j) - H(r_i - a_{j+1})) \quad & j < N \\ &= \sum_{i=1}^{M} f_i H(r_i - a_j) & j = N \end{aligned} \tag{9}$$

y_j =inverse standard normal distribution of the failed fraction specimens in bin j,

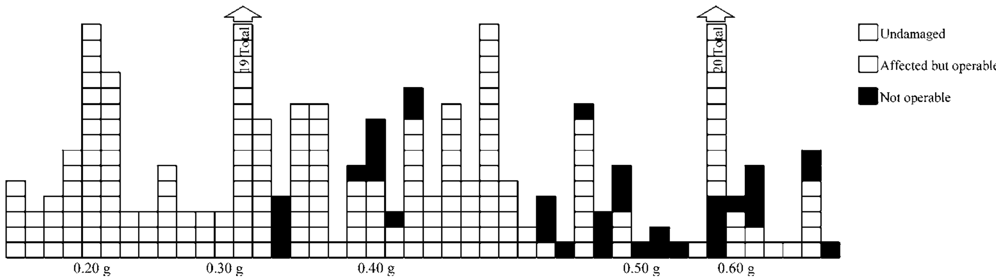

Figure 2. Example 2 damage data.

$$y_j = \Phi^{-1}\left(\frac{m_j + 1}{M_j + 1}\right) \tag{10}$$

where Φ^{-1} denotes the inverse standard normal distribution (e.g., normsinv in MS Excel) and

$$H(s) = 1 \text{ if } s > 0, \quad 1/2 \text{ if } s = 0, \quad 0 \text{ if } s < 0 \tag{11}$$

The parameters x_m and β are determined by fitting a line $\hat{y}=sx+c$ to the data:

$$\beta = \frac{1}{s} = \left(\sum_{j=1}^{N}(x_j - \bar{x})^2\right) \Bigg/ \left(\sum_{j=1}^{N}(x_j - \bar{x})(y_j - \bar{y})\right)$$

$$x_m = \exp(-c\beta) = \exp(\bar{x} - \bar{y}\beta) \tag{12}$$

where

$$\bar{x} = \frac{1}{N}\sum_{j=1}^{N} x_j \quad \bar{y} = \frac{1}{N}\sum_{j=1}^{N} y_j \tag{13}$$

In Equation 10, 1 is added to numerator and denominator to deal with cases with zero failures in the bin. Porter et al. (2006) presents an alternative approach using a least-squares fit to the binary failure data, i.e., to the pairs of EDP and a binary (0,1) failure indicator. The alternative approach avoids errors associated with bin-average EDPs.

Example. Consider the (imaginary) damage statistics in Figure 2, which depicts motor control centers (MCCs) observed after various earthquakes in 45 facilities. Each box represents one specimen. Crosshatched boxes represent MCCs that experienced noticeable earthquake effect such as shifting but that remained operable. Black boxes represent MCCs that were inoperable after the earthquake and required service or replacement (potentially causing downtime). Each stack represents one facility. Calculate the fragility function in terms of PGA, binning 0.15−0.24 g, 0.25−0.34 g, etc.

Table 3. Example 2 solution data

j	a_j(g)	$\bar{r}_j$(g)	M_j	m_j	x_j	y_j	$x_j-\bar{x}$	$y_j-\bar{y}$	$(x_j-\bar{x})^2$	$(x_j-\bar{x})(y_j-\bar{y})$
1	0.15	0.2	52	0	−1.61	−2.08	−0.623	−1.031	0.388	0.642
2	0.25	0.3	48	4	−1.20	−1.27	−0.217	−0.223	0.047	0.049
3	0.35	0.4	84	8	−0.92	−1.25	0.070	−0.202	0.005	−0.014
4	0.45	0.5	35	15	−0.69	−0.14	0.294	0.907	0.086	0.266
5	0.55	0.6	41	12	−0.51	−0.50	0.476	0.549	0.226	0.261
Σ=			260		−4.93	−5.23			0.753	1.204
Avg=					−0.99	−1.05				

Solution. The number of bins, N, and the lower EDP bounds, a_j, are dictated by the available data: $N=5$ bins with lower bounds of 0.15 g, 0.25 g, etc. The values of M_j and m_j are found by counting all boxes and black boxes, respectively, in Figure 2, in each bin, and are shown in Table 3. The value of M is found by summing: $M=\Sigma M_j=260$. Values x_j and y_j are calculated as $x_j=\ln(\bar{r}_j)$, and $y_j=\Phi^{-1}((m_j+1)/(M_j+1))$. Average values are calculated as shown: $\bar{x}=-0.99$, $\bar{y}=-1.05$, according to Equation 13. For each bin, the values of $x_j-\bar{x}$ and $y_j-\bar{y}$ are calculated as shown. Then, β and x_m are calculated as shown in Equation 12:

$$\beta=\frac{0.753}{1.204}=0.63 \quad x_m=\exp(-0.99+1.05\cdot 0.63)=0.72g$$

The results can be checked by plotting y versus x and fitting a line, as shown in Figure 3: β is the inverse of the slope of the trendline, $1/1.60=0.62$, and x_m is the value of r at which the line has a y-value of 0, i.e., $x_m=\exp(-0.53/1.60)=0.72$.

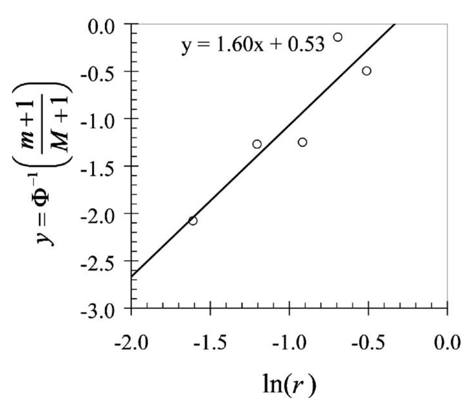

Figure 3. Checking Example 2 results.

METHOD C, CAPABLE EDP: NO SPECIMENS FAILED, EDPS ARE KNOWN

Method C is introduced here to deal with cases with no observations of DM$\geq dm$ and M observations of no damage occurrences of DM$\geq dm$. It addresses the best case for this type of data, i.e., many specimens, none of which had apparent distress, and several of which were subjected to EDP near the maximum value. It also addresses the more general case, including situations where few specimens experienced EDP near the maximum, or where some specimens experienced distress short of the damage state of interest, or both.

The procedure creates a bin-average subjective failure probability S for a bin of specimens at the high end of the tested range of EDP, and assigns a response value to this bin of specimens. The bin includes all specimens with some distress, the lowest of which has EDP$=r_d$, and all specimens without distress that were subjected to EDP of at least r_d or 0.7 times the largest level of EDP to which any specimen was subjected. The specimens in this bin without apparent distress are assigned 0% subjective failure probability, 10% for specimens with distress not suggestive of imminent failure, and 50% for specimens with distress suggestive of imminent failure. It assigns to this bin the median EDP of all the specimens in the bin, denoted by r_m. Combining the point on the fragility function (r_m, S) with an assumed $\beta=0.4$ produces a fragility function consistent with the assigned subjective failure probabilities. The precise interpretation of "distress suggestive of imminent failure" is left to the analyst. To create a fragility function from Method-C data, let

r_i=EDP experienced by specimen i $(i=1,2,\ldots M)$
$r_{max}=max_i\{r_i\}$
r_d=minimum EDP experienced by any specimen with distress
r_a=the smaller of r_d and $0.7r_{max}$
M_A=number of specimens without apparent distress and with $r_i\geq r_a$
M_B=number of specimens at any r_i with distress not suggestive of imminent failure
M_C=number of specimens at any r_i with distress suggestive of imminent failure
$r_m=r_{max}$ if $M_B+M_C = 0$
$\quad =0.5\cdot(r_{max}+r_a)$ otherwise
S=subjective failure probability at r_m

$$S = (0.5M_C + 0.1M_B)/(M_A + M_B + M_C) \tag{14}$$

Use Table 4 to determine $F_{dm}(r_m)$ and Equation 15 to determine β and x_m.

$$\beta = 0.4$$
$$z = \Phi^{-1}(F_{dm}(r_m))$$
$$x_m = r_m \exp(-z\beta) \tag{15}$$

Example. ANCO Engineers, Inc. (1983) performed shake-table tests on ceiling systems with various lateral restraints. Ten tests simulated conditions with the ceiling attached to

Table 4. Values of $\exp(-z\beta)$

Conditions	$F_{dm}(r_m)$	Z	$\exp(-z\boldsymbol{\beta})$, $\boldsymbol{\beta}=0.4$
$M_A \geq 3$ and $S=0$	0.01	−2.326	2.54
$M_A < 3$ and $S \leq 0.075$	0.05	−1.645	1.93
$0.075 < S \leq 0.15$	0.10	−1.282	1.67
$0.15 < S \leq 0.3$	0.20	−0.842	1.40
$S > 0.3$	0.40	−0.253	1.11

a perimeter wall that provided a boundary. Peak diaphragm acceleration (PDA) from nine of these tests is recorded in Table 5. Failure required replacement of damaged grid and tiles. Calculate the fragility function.

Solution. (a) Here, r_i=PDA for specimen i, $r_{max}=1.03$ g, $r_d=0.76$ g, $0.7r_{max}=0.72$ g, $r_a=\min(0.76\text{ g}, 0.72\text{ g})=0.72$ g, $M_A=1$, $M_B=0$, and $M_C=2$. By Equation 14, $S=(0.5\cdot 2+0.1\cdot 0)/(1+0+2)=0.33$. Since $M_B+M_C>0$, $r_m=0.5\cdot(r_{max}+r_a)=0.88$ g. From Table 4, $S>0.3$, so $F_{dm}(r_m)=0.4$. From Equation 15, $\beta=0.4$ and $x_m=1.11r_m=0.97$ g peak diaphragm acceleration.

METHOD D, DERIVED FRAGILITY FUNCTIONS

The capacity of some components can be calculated by modeling the component as a structural system, and determining the EDP (e.g., acceleration or shear deformation) that would cause the system to reach dm. Other components may be amenable to fault tree analysis; e.g., see Vesely et al. (1981). Let r denote the calculated capacity of the component to resist damage state dm, including consideration of any anchorage or bracing. Then

$$x_m = 0.92r$$

$$\beta = 0.4 \tag{16}$$

Equation 16 assumes that $\beta=0.4$ and calculates the median of a lognormal distribution from the mean value and β.

Table 5. Example 3 ceiling test data

ID	Test, run	PDA (g)	Failure	ID	Test, run	PDA (g)	Failure
5	7-2	0.39	FALSE	12	6-3	0.69	FALSE
7	6-1	0.48	FALSE	13	7-4	0.76	INCIPIENT
8	4-1	0.49	FALSE	14	5-5	0.79	FALSE
10	5-1	0.51	FALSE	16	6-4	1.03	INCIPIENT
11	7-3	0.52	FALSE				

METHOD E, EXPERT OPINION

There are several methods for eliciting expert opinion, from ad hoc to structured processes involving multiple experts, self-judgment of expertise, and iteration to examine major discrepancies between experts. To properly elicit expert opinion on uncertain quantities requires attention to clear definitions, biases, assumptions, and expert qualifications. The method (introduced for the first time here) employs Spetzler and von Holstein (1972) for probability encoding and Dalkey et al. (1970) for expert qualification, with some useful simplifications. See Porter et al. (2006) for more discussion of this method.

To use Method E, select experts with professional experience in the design or post-earthquake observation of the component. Solicit their advice using Figure 4. Representative images should be offered to the experts and recorded. If an expert refuses to provide estimates or limits them to certain conditions, either narrow the component definition accordingly and iterate, or ignore that expert's response and analyze the remaining ones. Let

N	= number of experts providing judgment about a value
i	= index of experts, $i \in \{1, 2, \ldots N\}$
x_{mi}	= estimated median EDP of expert i
x_{li}	= estimated lower-bound EDP of expert i
w_i	= level of expertise of expert i
α	= 1.5

$$x_m = \frac{\sum_{i=1}^{N} w_i^\alpha x_{mi}}{\sum_{i=1}^{N} w_i^\alpha} \qquad x_l = \frac{\sum_{i=1}^{N} w_i^\alpha x_{li}}{\sum_{i=1}^{N} w_i^\alpha} \qquad \beta = \frac{\ln(x_m/x_l)}{1.28} \tag{17}$$

If Equation 17 produces $\beta < 0.4$, either justify β, or replace β and x_m using

$$\beta = 0.4$$

$$x_m = 1.67 x_l \tag{18}$$

Regarding Equation 18, it is common for experts to express overconfidence in an uncertain variable, such as the EDP at which damage will occur. If the results of the survey produce $\beta < 0.4$, and this low value of β cannot be justified, use the judged x_l to anchor the fragility function, apply β=0.4, and calculate the resulting value of x_m. Kennedy and Short (1994) show that by establishing the EDP at which the component has 10% failure probability, the overall reliability of the component is insensitive to β, hence the value of directly encoding experts' judgment of this value in particular.

Objective. This form solicits your judgment about the values of an engineering demand parameter (EDP) at which a particular damage state occurs to a particular building component. Judgment is needed because the component may contribute significantly to the future seismic performance of a building, but relevant empirical and analytical data are currently impractical to acquire. Your judgment is solicited because you have relevant experience with the component of interest.

Definitions. Please provide judgment on the damageability of the following component. Images of a representative sample of the component and damage state may be attached. It is recognized that other EDPs may correlate better with damage, but please consider only the one specified here.

Component name: ______________________________

Component definition: ______________________________

____

Damage state name: ______________________________

Damage state definition: ______________________________

____

Relevant EDP: ______________________________

Definition of EDP: ______________________________

____

Uncertainty; no personal stake. Please provide judgment about this class of components, not a particular instance, and not one that you designed or otherwise have any stake in. There is probably no precise threshold level of EDP that causes damage, because of variability in design, construction, installation, etc., and even if there were, nobody would know it with certainty. To reflect uncertainty, **please provide two values of EDP at which damage occurs: median and lower bound.**

Estimated median EDP: ______________ *Definition*: damage will occur at this level of EDP 5 times in 10. In a single case, you judge an equal chance that failure will occur at lower or higher EDP.

Estimated lower-bound EDP: ______________ *Definition*: damage will occur at this level of EDP 1 time in 10. In a single case, you judge a 10% chance that your estimate is too high. *Judge the lower bound carefully*. Make an initial guess, then imagine conditions that might make the actual failure EDP lower (errors in design or installation, deterioration, poor maintenance, interaction, etc.) and revise accordingly. Without careful thought, expert judgment of the lower bound tends to be too close to the median estimate, so think twice and do not be afraid of showing uncertainty.

Your level of expertise (1-5): ______________ *Definition*: 1 means you have no experience or expertise with this component and damage state, 5 means you are very familiar or highly experienced.

Your name: ______________________________ ***Date***: ________________

Figure 4. Form for soliciting expert judgment on component fragility.

Example. Stone cladding on the exterior of retail buildings may fall in earthquakes. Consider 2-in. x 6-in. x 1-3/16-in. stone veneer adhered to a concrete masonry unit substrate with thin-bed mortar (liquid latex mixed with Portland cement, 100% coverage). Create a fragility function for the probability that any given stone would fall from the building (posing a life-safety threat) and require replacement, as a function of the peak transient drift ratio of the story on which the stone is applied.

Solution. Figure 4 was used to solicit judgment from three (imaginary) engineers on the fragility of the component, using the following definitions.

Component: Stone cladding 1, defined as 2-in. x 6-in. x 1-3/16-in. stone veneer adhered to a concrete masonry unit substrate with thin-bed mortar (liquid latex mixed with Portland cement, 100% coverage)

Damage state: Falling, defined as a given panel becoming delaminated from CMU and falling

EDP: PTD, defined as the peak transient drift ratio of the story and column line of stone veneer

Responses are shown in columns 2, 3, and 4 of the following:

(1) Response i	(2) Expertise w_i	(3) Median x_{mi}	(4) Lower bound x_{li}	(5) $w_i^{1.5}$	(6) $w_i^{1.5} \cdot x_{mi}$	(7) $w_i^{1.5} \cdot x_{li}$
1	2	0.003	0.0015	2.83	0.0085	0.0042
2	1	0.005	0.001	1.00	0.0050	0.0010
3	2	0.010	0.005	2.83	0.0283	0.0141
			$\Sigma=$	6.66	0.0418	0.0194

By Equations 17 and 18,

$$x_m = \frac{0.0418}{6.66} = 0.63\,\% \qquad x_l = \frac{0.0194}{6.66} = 0.29\,\% \qquad \beta = \frac{\ln\left(\dfrac{0.0063}{0.0029}\right)}{1.28} = 0.60$$

METHOD U, UPDATING A FRAGILITY FUNCTION WITH NEW DATA

Here, the data are a pre-existing fragility function and M specimens with known damage state and maximum EDP. It is not necessary that any of the specimens failed. Let

M = number of specimens observed
i = index of specimens, $i \in \{1, 2, \ldots M\}$
r_i = maximum EDP to which specimen i was subjected
f_i = 1 if specimen i failed (reached or exceeded damage state dm), 0 otherwise
x_m = median from pre-existing fragility function
β = logarithmic standard deviation from pre-existing fragility function
x'_m = revised median of the fragility function
β' = revised logarithmic standard deviation of the fragility function

One calculates the revised median and logarithmic standard deviation as follows:

$$w'_j = \frac{w_j \prod_{i=1}^{M} L(i,j)}{\sum_{j=1}^{5} w_j \prod_{i=1}^{M} L(i,j)} \qquad x'_m = \exp\left(\sum_{j=1}^{5} w'_j \ln(x_{mj})\right) \qquad \beta' = \sum_{j=1}^{5} w'_j \beta_j \tag{19}$$

where

$$L(i,j) = 1 - \Phi\left(\frac{\ln(r_i/x_{mj})}{0.707\beta_j}\right) \quad if\, f_i = 0 = \Phi\left(\frac{\ln(r_i/x_{mj})}{0.707\beta_j}\right) \quad if\, f_i = 1 \tag{20}$$

$$x_{m1} = x_{m4} = x_{m5} = x_m \quad \beta_1 = \beta_2 = \beta_3 = \beta \quad w_1 = 1/3$$

$$x_{m2} = x_m e^{-1.22\beta} \quad \beta_4 = 0.64\beta \quad w_2 = w_3 = w_4 = w_5 = 1/6$$

$$x_{m3} = x_m e^{1.22\beta} \quad \beta_5 = 1.36\beta \tag{21}$$

Method U is introduced for the first time here. See Porter et al. (2006) for an example. The method uses Bayes' Theorem (e.g., Ang and Tang 1975) to revise x_m and β of an existing fragility function with new observations of M specimens whose EDP and damage state have been observed. Some explanation may be useful to readers unfamiliar with Bayesian updating. It is recognized here that x_m and β are themselves uncertain, and can be assigned probability distributions. The distributions are revised based on how likely it is that the observed damage would have occurred for various possible values of x_m and β.

For those familiar with Bayesian updating, the prior probability distribution of x_m is taken as lognormal with median equal to the x_m value in the pre-existing fragility function, and logarithmic standard deviation taken as $0.707 \times \beta$ of the pre-existing fragility function, consistent with a compound lognormal fragility function and $\beta_r = \beta_u = 0.707\beta$. The prior of β is taken as normal with expected value equal to the β of the pre-existing fragility function, and coefficient of variation (COV) of 0.21. This COV is selected because it provides for 98% probability that β is within the bounds of 0.5 and 1.5 times the prior β, which agrees with the observed range for β of 0.2 to 0.6. The distributions of x_m and β are assumed to be independent. Their joint distribution is approximated by five discrete points (x_{mj}, β_j), each with probability-like weight w_j (where $j = 1, 2, \ldots 5$). Using a method described in Julier (2002), the values of x_{mj}, β_j, and w_j are chosen so that the first five moments of the discrete joint distribution match those of the continuous joint distribution. Figure 5 illustrates the principle, showing a probability density function of two variables x_m and β (the surface) and the discrete points (bars), each with an associated weight (indicated by bar height). The first few moments of the points (the mean, variance, etc.) match those of the surface. In Equation 19, the w values are updated to reflect the observations, and x_m and β are updated using the new w's.

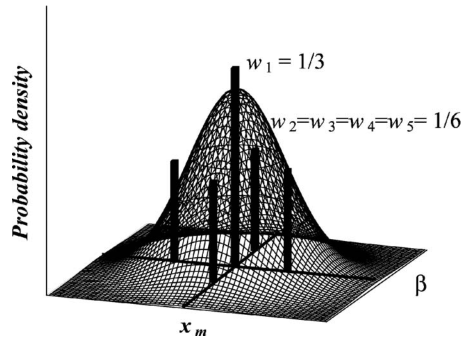

Figure 5. Substituting a sample of five points (bars) for a continuous joint distribution (surface).

ASSESSING FRAGILITY FUNCTION QUALITY

The previous section provided mathematical procedures for developing fragility functions. Issues associated with the quality of those fragility functions are now addressed, particularly the treatment of competing EDPs, goodness-of-fit testing, dealing with fragility functions that cross, and how to assign an overall quality level to a fragility function.

CONSIDERING COMPETING EDPS

One may be uncertain which is the best EDP to use. In such a case, create fragility functions for each alternative and choose the fragility function with the lowest β. See Porter et al. (2006) for choosing between EDPs with differing COV.

GOODNESS OF FIT

A goodness-of-fit test checks that an assumed distribution adequately fits the data. The Lilliefors (1967) test is used here. It is a special case of the Kolmogorov-Smirnov (K-S) test, applicable when the parameters of the distribution are estimated from the same data as are being compared with the distribution, as is the case here. To perform the test, calculate

$$D = \max_X \left| F_{dm}(edp) - S_M(edp) \right| \tag{22}$$

over the range $0 < edp \leq \max\{r_i\}$, where $S_M(edp)$ is given by

$$S_M(edp) = \frac{1}{M}\sum_{i=1}^{M} H(r_i - edp) \tag{23}$$

and H is given by Equation 11. If $D < D_{\mathrm{crit}}$, the fragility function passes the goodness-of-fit test ($\alpha = 0.05$ significance level is used here; the equation approximates Lilliefors' table):

$$D_{crit} = 0.895/(M^{0.5} - 0.01 + 0.85M^{-0.5}) \tag{24}$$

FRAGILITY FUNCTIONS THAT CROSS

Some components have two or more fragility functions. Any two lognormal fragility functions i and j with medians $x_{mj} > x_{mi}$ and logarithmic standard deviations $\beta_i \neq \beta_j$ cross if:

$$edp < \exp\left(\frac{\beta_j \ln x_{mi} - \beta_i \ln x_{mj}}{\beta_j - \beta_i}\right): \quad \beta_i < \beta_j$$

$$edp > \exp\left(\frac{\beta_j \ln x_{mi} - \beta_i \ln x_{mj}}{\beta_j - \beta_i}\right): \quad \beta_i > \beta_j \tag{25}$$

This produces a (meaningless) negative probability of being in damage state i under Equation 3b. Figure 6a illustrates the point: F_2 has a higher β than F_1, and F_3 has a lower β than F_2. Two methods are proposed to deal with the problem. Either replace Equation 2 with

$$F_i(edp) = \max_j \left\{ \Phi\left(\frac{\ln(edp/x_{mj})}{\beta_j}\right)\right\} \quad \text{for all } j \geq i \tag{26}$$

as shown in Figure 6b, or find x_m and β values for each damage state, and then revise them:

$$\beta_i' = \frac{1}{N}\sum_{i=1}^{N} \beta_i \quad \text{for all } i$$

$$x_{mi}' = \exp(1.28(\beta' - \beta_i) + \ln x_{mi}) \tag{27}$$

as shown in Figure 6c. Equation 27 adjusts the functions so they match the originals at 10% failure probability, with the same justification as discussed in Method E.

ASSIGNING A SINGLE QUALITY LEVEL TO A FRAGILITY FUNCTION

Fragility functions come from data with varying quantity and quality. Table 6 offers a system to assign a high, medium, or low quality to a fragility function. It is based solely on the authors' judgment. The analyst should report the quality of fragility functions used with any loss estimate.

CONCLUSIONS

Six methods for creating fragility functions were presented, including three new ones: one for dealing with cases where no failure has been observed, another for situations where one must rely on expert opinion, and a third for updating an existing fragility function with new damage observations. The procedures are under consideration as a standard for ATC-58, a technology-transfer project by the Applied Technology Council to bring PEER's performance-based earthquake engineering methodology to practice.

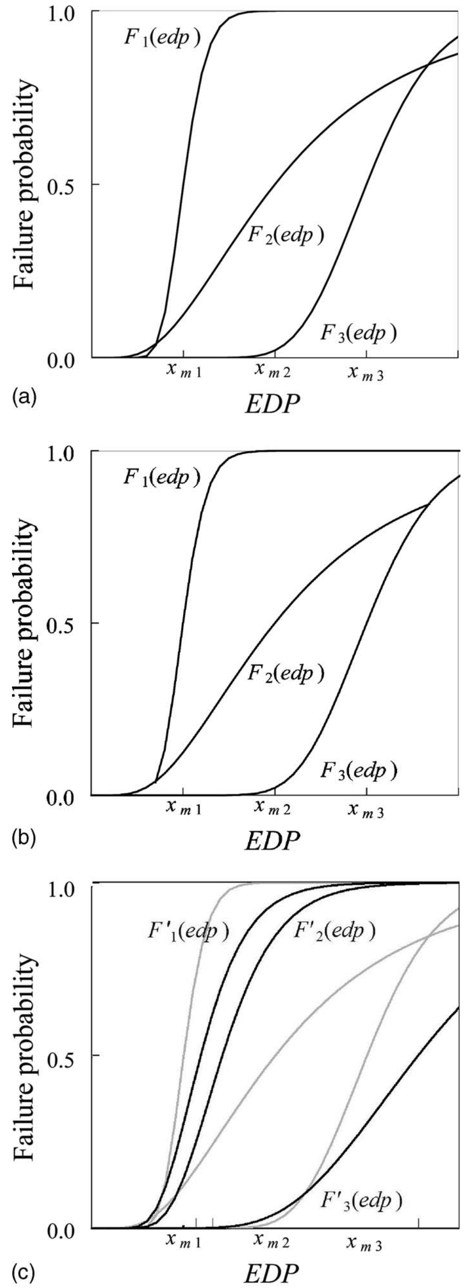

Figure 6. (a) Fragility functions that cross; (b) solution with Equation 26; and (c) Equation 27.

Table 6. Fragility function quality level

Quality	Method	Review*	M	Other
High	A	Yes	≥ 4	Passes Lilliefors test at $\alpha = 5\%$. Justify (a) $\geq 20\%$ difference in x_m or β versus past estimates, and (b) $\beta < 0.2$ or $\beta > 0.6$.
	B	Yes	≥ 20	Justify (a) $\geq 20\%$ difference in x_m or β versus past estimates, and (b) $\beta < 0.2$ or $\beta > 0.6$.
	U	Yes	≥ 6	Prior was at least moderate quality
Medium	A		≥ 3	Examine and justify any case of $\beta < 0.2$ or $\beta > 0.6$.
	B		≥ 16	Examine and justify any case of $\beta < 0.2$ or $\beta > 0.6$.
	C	Yes	≥ 6	
	D	Yes		
	E	Yes		At least 3 experts with $w \geq 3$
	U		≥ 6	or prior was moderate quality
Low				All other cases

* Data and analysis published in peer-reviewed archival journal.

The procedures are intended for engineering professionals who will eventually use PBEE. Little unfamiliar math is involved, and no calculus. A larger document, Porter et al. (2006), presents these procedures with more commentary, some alternative approaches, and more sample problems.

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