

Homework 03

Submission Notices:

- Conduct your homework by filling answers into the placeholders in this file (in Microsoft Word format). Questions are shown in black color, *instructions/hints are shown in italics and blue color*, and *your content should use any color that is different from those*.
- After completing your homework, prepare the file for submission by exporting the Word file (filled with answers) to a PDF file, whose filename follows the following format,
 <StudentID-1>_<StudentID-2>_HW01.pdf (Student IDs are sorted in ascending order)
 E.g., **2112001_2112002_HW02.pdf**
and then submit the file to Moodle directly *WITHOUT* any kinds of compression (.zip, .rar, .tar, etc.).
- Note that you will get zero credit for any careless mistake, including, but not limited to, the following things.
 1. Wrong file/filename format, e.g., not a pdf file, use “-” instead of “_” for separators, etc.
 2. Disorder format of problems and answers
 3. **Conducted not in English**
 4. Cheating, i.e., copying other students’ works or letting other students copy your work.

Problem 1. (3pts) For each English sentence below, write the FOL sentence that best expresses its intended meaning, using the following given predicates.

Child(x) for “x is a child”

Cupcake(x) for “x is a cupcake”

Loves(x, y) for “x loves y”

Eats(x, y) for “x eats y”

1. Every child loves every cupcake.
2. For every child, there is a cupcake that the child loves.
3. There is a child who loves every cupcake.
4. Some child loves some cupcake.
5. There is a cupcake that every child loves.
6. For every cupcake, there is a child who loves that cupcake.
7. Every child eats exactly one cupcake.
8. Some child eats at least two cupcakes.
9. Every child eats at most one cupcake.

Please be aware of the precedence of logical connectives if you do not use brackets.

Please fill your answers in the table below

No.	Original statements and FOL sentences
1.	Every child loves every cupcake. $\forall x \text{ Child}(x) \rightarrow [\forall y \text{ Cupcake}(y) \rightarrow \text{Loves}(x, y)]$
2.	For every child, there is a cupcake that the child loves. $\forall x \text{ Child}(x) \rightarrow [\exists y \text{ Cupcake}(y) \wedge \text{Loves}(x, y)]$
3.	There is a child who loves every cupcake $\exists x \text{ Child}(x) \wedge [\forall y \text{ Cupcake}(y) \rightarrow \text{Loves}(x, y)]$
4.	Some child loves some cupcake. $\exists x \text{ Child}(x) \wedge [\exists y \text{ Cupcake}(y) \wedge \text{Loves}(x, y)]$
5.	There is a cupcake that every child loves. $\exists x \text{ Cupcake}(x) \wedge [\forall y \text{ Child}(y) \rightarrow \text{Loves}(y, x)]$
6.	For every cupcake, there is a child who loves that cupcake. $\forall x \text{ Cupcake}(x) \rightarrow [\exists y \text{ Child}(y) \wedge \text{Loves}(y, x)]$
7.	Every child eats exactly one cupcake. $\forall x \text{ Child}(x) \rightarrow [\exists y \text{ Cupcake}(y) \wedge \text{Eats}(x, y) \wedge [\forall z \text{ Cupcake}(z) \wedge \text{Eats}(x, z) \rightarrow (y = z)]]$
8.	Some child eats at least two cupcakes. $\exists x \text{ Child}(x) \wedge [\exists y, z \text{ Cupcake}(y) \wedge \text{Cupcake}(z) \wedge \text{Eats}(x, y) \wedge \text{Eats}(x, z) \wedge \neg(y = z)]$
9.	Every child eats at most one cupcake. $\neg \exists x \text{ Child}(x) \wedge [\exists y, z \text{ Cupcake}(y) \wedge \text{Cupcake}(z) \wedge \text{Eats}(x, y) \wedge \text{Eats}(x, z) \wedge \neg(y = z)]$

Problem 2. (3pts) Consider the following statements

“(1) Either taxes are increased or if expenditures rise then the debt ceiling is raised. (2) If taxes are increased, then the cost of collecting taxes rises. (3) If a rise in expenditures implies that the government borrows more money, then if the debt ceiling is raised, then interest rates increase. (4) If taxes are not increased and the cost of collecting taxes does not increase then if the debt ceiling is raised, then the government borrows more money. (5) The cost of collecting taxes does not increase. (6) Either interest rates do not increase, or the government does not borrow more money. Prove by resolution that (7) Either the debt ceiling is not raised, or expenditures do not rise.”

Let T be taxes are increased

C be cost of collecting taxes rises

E be expenditures rise

G be government borrows more money

D be debt ceiling is raised

I be interest rates increase

Note that you do not need to handle the exclusiveness of the grammar structure “either...or”, just consider them as simple “or”.

(1pt) Translate the English sentences into propositional logic sentences

No.	Sentences	No.	Sentences	No.	Sentences
1	$T \vee (E \rightarrow D)$	4	$(\neg T \wedge \neg C) \rightarrow (D \rightarrow G)$	7	$\neg D \vee \neg E$
2	$T \rightarrow C$	5	$\neg C$		
3	$(E \rightarrow G) \rightarrow (D \rightarrow I)$	6	$\neg I \vee \neg G$		

(1pt) Convert propositional logic sentences into CNF

No.	Sentences	Notes the sentence's sources
1	$T \vee \neg E \vee D$	$T \vee (E \rightarrow D)$
2	$\neg T \vee C$	$T \rightarrow C$
3	$\neg D \vee I \vee E$	$(E \rightarrow G) \rightarrow (D \rightarrow I)$
4	$\neg D \vee I \vee \neg G$	$(E \rightarrow G) \rightarrow (D \rightarrow I)$
5	$\neg D \vee G \vee C \vee T$	$(\neg T \wedge \neg C) \rightarrow (D \rightarrow G)$
6	$\neg C$	$\neg C$
7	$\neg I \vee \neg G$	$\neg I \vee \neg G$
8	D	Negated goal
9	E	Negated goal

(1pt) Prove by resolution that either the debt ceiling is not raised, or expenditures do not rise.

No.	Sentences	Notes the sentence's sources
10	$\neg T$	2, 6
11	G	5, 6, 8, 10
12	$\neg I$	7, 11
13	$\neg D$	4, 11, 12
14	{ }	8, 13
15		
16		

Conclusion: either the debt ceiling is not raised, or expenditures do not rise

Problem 3. (2.5pts) Consider a vocabulary with the following symbols:

Dog(x) predicate "x is a dog"

Cat(x) predicate "x is a cat"

Cute(x) predicate "x is cute"

Scary(x) predicate "x is scary"


Owns(x, y) predicate "x owns y"

Bites(x, y) predicate "x bites y"

Please fill your answers in the table below





No.	Original statements and FOL sentences
1.	$\exists x (\text{Owns}(\text{Joe}, x) \wedge \text{Dog}(x) \wedge \text{Cute}(x))$ Joe owns a cute dog.
2.	$\forall x ((\text{Owns}(\text{Joe}, x) \wedge \text{Dog}(x)) \Rightarrow \text{Cute}(x))$ All dogs that Joe own is cute.
3.	$\neg(\exists x (\text{Owns}(\text{Joe}, x) \wedge \text{Dog}(x))) \Rightarrow \text{Scary}(\text{Joe})$ If Joe does not own any dog, then Joe is scary.
4.	$\forall x (\text{Dog}(x) \wedge (\exists y \text{Dog}(y) \wedge (\exists z \text{Cat}(z)) \wedge (\text{Bites}(x,y) \vee \text{Bites}(x,z))) \Rightarrow \neg \text{Cute}(x).$ For every dog, if dog bites some dogs or bites some cats, that dog is not cute.
5.	$\exists x (\text{Dog}(x) \wedge \neg (\text{Scary}(x) \wedge \text{Cute}(x)))$ There is a dog that is not both scary and cute.

Problem 4. (1.5 pts) Given a chess board with 4 rows and 4 columns (4x4) as below.

a) (0.5pt) Place 4 queens on the board so that they are not able to attack each other.

Please fill your answer in the given table below

b) (1pt) Assign a Boolean variable to each cell of the board as below (*1, 2, 3, etc. are variable names*)

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

If a variable is True, then there is a queen on the corresponding cell and vice versa.

Use CNF clauses to describe constraints to place a queen on cell no. 1 (*use -1 to denote NOT 1*).

Hint: if there is a queen on cell no. 1, then there mustn't any queens on cell no. 2, 3, etc.

Please fill your answer in the table below (add more rows as needed)

1.	$-1 \vee -(5 \vee 9 \vee 13)$	
2.	$-1 \vee -(2 \vee 3 \vee 4)$	
3.	$-1 \vee -(6 \vee 11 \vee 16)$	

s