Resolution Exercise Solutions

2. Consider the following axioms:

1. Every child loves Santa.
   $\forall$ *x (CHILD(x) → LOVES(x,Santa))*

2. Everyone who loves Santa loves any reindeer.
   $\forall$ *x (LOVES(x,Santa) → ∀ y (REINDEER(y) → LOVES(x,y)))*

3. Rudolph is a reindeer, and Rudolph has a red nose.
   REINDEER(Rudolph) ∧ REDNOSE(Rudolph)

4. Anything which has a red nose is weird or is a clown.
   ∀ x (REDNOSE(x) → WEIRD(x) ∨ CLOWN(x))

5. No reindeer is a clown.
   ¬ ∃ x (REINDEER(x) ∧ CLOWN(x))

6. Scrooge does not love anything which is weird.
   ∀ x (WEIRD(x) → ¬ LOVES(Scrooge,x))

7. (Conclusion) Scrooge is not a child.
   ¬ CHILD(Scrooge)

3. Consider the following axioms:

1. Anyone who buys carrots by the bushel owns either a rabbit or a grocery store.
   ∀ x (BUY(x) → ∃ y (OWNS(x,y) ∧ (RABBIT(y) ∨ GROCERY(y))))

2. Every dog chases some rabbit.
   ∀ x (DOG(x) → ∃ y (RABBIT(y) ∧ CHASE(x,y)))

3. Mary buys carrots by the bushel.
   BUY(Mary)

4. Anyone who owns a rabbit hates anything that chases any rabbit.
   ∀ x ∀ y (OWNS(x,y) ∧ RABBIT(y) → ∀ z ∀ w (RABBIT(w) ∧ CHASE(z,w) → HATES(x,z)))

5. John owns a dog.
   ∃ x (DOG(x) ∧ OWNS(John,x))

6. Someone who hates something owned by another person will not date that person.
   ∀ x ∀ y ∀ z (OWNS(y,z) ∧ HATES(x,z) → ¬ DATE(x,y))

7. (Conclusion) If Mary does not own a grocery store, she will not date John.
   ((¬ ∃ x (GROCERY(x) ∧ OWN(Mary,x))) → ¬ DATE(Mary,John))

4. Consider the following axioms:

1. Every Austinite who is not conservative loves some armadillo.
   ∀ x (AUSTINITE(x) ∧ ¬ CONSERVATIVE(x) → ∃ y (ARMADILLO(y) ∧ LOVES(x,y)))

2. Anyone who wears maroon-and-white shirts is an Aggie.
   $\forall x\ (WEARS(x) \rightarrow AGGIE(x))$

3. Every Aggie loves every dog.
   $\forall x\ (AGGIE(x) \rightarrow \forall y\ (DOG(y) \rightarrow LOVES(x,y)))$

4. Nobody who loves every dog loves any armadillo.
   $\neg \exists x\ ((\forall y\ (DOG(y) \rightarrow LOVES(x,y))) \wedge \exists z\ (ARMADILLO(z) \wedge LOVES(x,z)))$

5. Clem is an Austinite, and Clem wears maroon-and-white shirts.
   $AUSTINITE(Clem) \wedge WEARS(Clem)$

6. (Conclusion) Is there a conservative Austinite?
   $\exists x\ (AUSTINITE(x) \wedge CONSERVATIVE(x))$

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( ( (not (Austinite x))  (Conservative x)  (Armadillo (f x)) )
  ( (not (Austinite x))  (Conservative x)  (Loves x (f x)) )
  ( (not (Wears x))  (Aggie x) )
  ( (not (Aggie x))  (not (Dog y))  (Loves x y) )
  ( (Dog (g x))  (not (Armadillo z))  (not (Loves x z)) )
  ( (not (Loves x (g x)))  (not (Armadillo z))  (not (Loves x z)) )
  ( (Austinite (Clem)) )
  ( (Wears (Clem)) )
  ( (not (Conservative x))  (not (Austinite x)) ) )
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5. Consider the following axioms:

1. Anyone whom Mary loves is a football star.
   $\forall x\ (LOVES(Mary,x) \rightarrow STAR(x))$

2. Any student who does not pass does not play.
   $\forall x\ (STUDENT(x) \wedge \neg PASS(x) \rightarrow \neg PLAY(x))$

3. John is a student.
   $STUDENT(John)$

4. Any student who does not study does not pass.
   $\forall x\ (STUDENT(x) \wedge \neg STUDY(x) \rightarrow \neg PASS(x))$

5. Anyone who does not play is not a football star.
   $\forall x\ (\neg PLAY(x) \rightarrow \neg STAR(x))$

6. (Conclusion) If John does not study, then Mary does not love John.
   $\neg STUDY(John) \rightarrow \neg LOVES(Mary,John)$

6. Consider the following axioms:

1. Every coyote chases some roadrunner.
   $\forall x\ (COYOTE(x) \rightarrow \exists y\ (RR(y) \wedge CHASE(x,y)))$

2. Every roadrunner who says ``beep-beep'' is smart.
   $\forall x\ (RR(x) \wedge BEEP(x) \rightarrow SMART(x))$

3. No coyote catches any smart roadrunner.
   $\neg \exists x\ \exists y\ (COYOTE(x) \wedge RR(y) \wedge SMART(y) \wedge CATCH(x,y))$

4. Any coyote who chases some roadrunner but does not catch it is frustrated.
   $\forall x\ (COYOTE(x) \land \exists y\ (RR(y) \land CHASE(x,y) \land \neg CATCH(x,y)) \rightarrow FRUSTRATED(x))$

5. (Conclusion) If all roadrunners say ``beep-beep'', then all coyotes are frustrated.
   $(\forall x\ (RR(x) \rightarrow BEEP(x)) \rightarrow (\forall y\ (COYOTE(y) \rightarrow FRUSTRATED(y)))$

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( ( (not (Coyote x))  (RR (f x)) )
  ( (not (Coyote x))  (Chase x (f x)) )
  ( (not (RR x))  (not (Beep x))  (Smart x) )
  ( (not (Coyote x))  (not (RR y))  (not (Smart y))  (not (Catch x y)) )
  ( (not (Coyote x))  (not (RR y))  (not (Chase x y)) (Catch x y)
    (Frustrated x) )
  ( (not (RR x))  (Beep x) )
  ( (Coyote (a)) )
  ( (not (Frustrated (a))) ) )
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7. Consider the following axioms:

1. Anyone who rides any Harley is a rough character.
   $\forall x\ ((\exists y\ (HARLEY(y) \land RIDES(x,y))) \rightarrow ROUGH(x))$

2. Every biker rides [something that is] either a Harley or a BMW.
   $\forall x\ (BIKER(x) \rightarrow \exists y\ ((HARLEY(y) \lor BMW(y)) \land RIDES(x,y)))$

3. Anyone who rides any BMW is a yuppie.
   $\forall x\ \forall y\ (RIDES(x,y) \land BMW(y) \rightarrow YUPPIE(x))$

4. Every yuppie is a lawyer.
   $\forall x\ (YUPPIE(x) \rightarrow LAWYER(x))$

5. Any nice girl does not date anyone who is a rough character.
   $\forall x\ \forall y\ (NICE(x) \land ROUGH(y) \rightarrow \neg DATE(x,y))$

6. Mary is a nice girl, and John is a biker.
   $NICE(Mary) \land BIKER(John)$

7. (Conclusion) If John is not a lawyer, then Mary does not date John.
   $\neg LAWYER(John) \rightarrow \neg DATE(Mary,John)$

8. Consider the following axioms:

1. Every child loves anyone who gives the child any present.
   $\forall x\ \forall y\ \forall z\ (CHILD(x) \land PRESENT(y) \land GIVE(z,y,x) \rightarrow LOVES(x,z)$

2. Every child will be given some present by Santa if Santa can travel on Christmas eve.
   $TRAVEL(Santa,Christmas) \rightarrow \forall x\ (CHILD(x) \rightarrow \exists y\ (PRESENT(y) \land GIVE(Santa,y,x)))$

3. It is foggy on Christmas eve.
   $FOGGY(Christmas)$

4. Anytime it is foggy, anyone can travel if he has some source of light.
   $\forall x\ \forall t\ (FOGGY(t) \rightarrow (\exists y\ (LIGHT(y) \land HAS(x,y)) \rightarrow TRAVEL(x,t)))$

5. Any reindeer with a red nose is a source of light.
   $\forall x\ (RNR(x) \rightarrow LIGHT(x))$

6. (Conclusion) If Santa has some reindeer with a red nose, then every child loves Santa.
   $(\exists x\ (RNR(x) \wedge HAS(Santa,x))) \rightarrow \forall y\ (CHILD(y) \rightarrow LOVES(y,Santa))$

9. Consider the following axioms:

1. Every investor bought [something that is] stocks or bonds.
   $\forall x\ (INVESTOR(x) \rightarrow \exists y\ ((STOCK(y) \vee BOND(y)) \wedge BUY(x,y)))$

2. If the Dow-Jones Average crashes, then all stocks that are not gold stocks fall.
   $DJCRASH \rightarrow \forall x\ ((STOCK(x) \wedge \neg\ GOLD(x)) \rightarrow FALL(x))$

3. If the T-Bill interest rate rises, then all bonds fall.
   $TBRISE \rightarrow \forall x\ (BOND(x) \rightarrow FALL(x))$

4. Every investor who bought something that falls is not happy.
   $\forall x\ \forall y\ (INVESTOR(x) \wedge BUY(x,y) \wedge FALL(y)\ \text{\&rarrm;}\ \neg\ HAPPY(x))$

5. (Conclusion) If the Dow-Jones Average crashes and the T-Bill interest rate rises, then any investor who is happy bought some gold stock.
   $(DJCRASH \wedge TBRISE) \rightarrow \forall x\ (INVESTOR(x) \wedge HAPPY(x) \rightarrow \exists y\ (GOLD(y) \wedge BUY(x,y)))$

10. Consider the following axioms:

1. Every child loves every candy.
   $\forall x\ \forall y\ (CHILD(x) \wedge CANDY(y) \rightarrow LOVES(x,y))$

2. Anyone who loves some candy is not a nutrition fanatic.
   $\forall x\ ((\exists y\ (CANDY(y) \wedge LOVES(x,y))) \rightarrow \neg\ FANATIC(x))$

3. Anyone who eats any pumpkin is a nutrition fanatic.
   $\forall x\ ((\exists y\ (PUMPKIN(y) \wedge EAT(x,y))) \rightarrow FANATIC(x))$

4. Anyone who buys any pumpkin either carves it or eats it.
   $\forall x\ \forall y\ (PUMPKIN(y) \wedge BUY(x,y) \rightarrow CARVE(x,y) \vee EAT(x,y))$

5. John buys a pumpkin.
   $\exists x\ (PUMPKIN(x) \wedge BUY(John,x))$

6. Lifesavers is a candy.
   $CANDY(Lifesavers)$

7. (Conclusion) If John is a child, then John carves some pumpkin.
   $CHILD(John) \rightarrow \exists x\ (PUMPKIN(x) \wedge CARVE(John,x))$

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