Homework 03

Submission Notices:

- *Conduct your homework by filling answers into the placeholders in this file (in Microsoft Word format). Questions are shown in black color, instructions/hints are shown in italics and blue color, and your content should use any color that is different from those.*
- *After completing your homework, prepare the file for submission by exporting the Word file (filled with answers) to a PDF file, whose filename follows the following format,*
  <StudentID-1>_<StudentID-2>_HW01.pdf (Student IDs are sorted in ascending order)
  E.g., ***2112001_2112002_HW02.pdf***
  and then submit the file to Moodle directly WITHOUT any kinds of compression (.zip, .rar, .tar, etc.).
- *Note that you will get zero credit for any careless mistake, including, but not limited to, the following things.*
  1. *Wrong file/filename format, e.g., not a pdf file, use "-" instead of "_" for separators, etc.*
  2. *Disorder format of problems and answers*
  3. *Conducted not in English*
  4. *Cheating, i.e., copying other students' works or letting other students copy your work.*

Problem 1. (3pts) Consider a first-order knowledge base that describes worlds containing people, songs, albums (e.g., "Meet the Beatles") and disks (i.e., particular physical instances of CDs). The vocabulary contains the following symbols:

- CopyOf(d, a): Disk d is a copy of album a
- Owns(p, d):Person p owns disk d
- Sings(p, s, a): Album a includes a recording of song s sung by person p
- Wrote(p, s): Person p wrote song s

Express the following statements in first-order logic:

1. Gershwin did not write "Eleanor Rigby."
2. Either Gershwin or McCartney wrote "The Man I Love."
3. Joe owns a copy of *Revolver*.
4. Every song that McCartney sings on *Revolver* was written by McCartney.
5. Gershwin did not write any of the songs on *Revolver*.
6. Every song that Gershwin wrote has been recorded on some album. (Possibly different songs are recorded on different albums.)
7. There is a single album that contains every song that Joe has written
8. Joe owns a copy of an album that has Billie Holiday singing "The Man I Love."
9. Joe owns a copy of every album that has a song sung by McCartney. (Of course, each different album is instantiated in a different physical CD.)

Please fill your answers in the table below

No.	Original statements and FOL sentences
1.	Gershwin did not write "Eleanor Rigby."
	¬Wrote(Gershwin, EleanorRigby)

2.	Either Gershwin or McCartney wrote "The Man I Love."
	Wrote(Gershwin, TheManILove) ∨ Wrote(McCartney, TheManILove)
3.	Joe owns a copy of *Revolver*.
	∃d Owns(Joe, d) ∧ CopyOf(d, Revolver)
4.	Every song that McCartney sings on *Revolver* was written by McCartney.
	∀s Sings(McCartney, s, Revolver) ⟹ Wrote(McCartney, s)
5.	Gershwin did not write any of the songs on *Revolver*.
	¬∃s ∀p Sings(p, s, Revolver) ∧ Wrote(Gershwin, s)
6.	Every song that Gershwin wrote has been recorded on some album. (Possibly different songs are recorded on different albums.)
	∀s Wrote(Gershwin, s) ⟹ ∃p,a Sings(p, s, a)
7.	There is a single album that contains every song that Joe has written
	∃a (¬∃s Wrote(Joe, s) ∧ ∀p ¬Sings(p, s, a))
8.	Joe owns a copy of an album that has Billie Holiday singing "The Man I Love."
	∃d,a Owns(Joe, d) ∧ CopyOf(d, a) ∧ Sings(BillieHoliday, TheManILove, a)
9.	Joe owns a copy of every album that has a song sung by McCartney. (Of course, each different album is instantiated in a different physical CD.)
	∀a ∃s,d Sings(McCartney, s, a) ∧ Owns(Joe, d) ∧ CopyOf(d, a)

Problem 2. (3pts) The Horned and Magical Unicorn. If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned. Prove by resolution that "the unicorn is both horned and magical."

You are allowed to use these following propositional variables only

Y = unicorn is mYthical R = unicorn is moRtal M = unicorn is a maMmal

H = unicorn is Horned G = unicorn is magical

It is worth nothing that "immortal" = "not mortal"

Please fill your answer in the following table.

No.	Sentences	Notes the sentence's sources
Translate the English sentences into propositional logic sentences		
1.	Y → ¬R	From the statement
2.	¬Y → (R ∧ M)	From the statement
3.	¬R ∨ M → H	From the statement
4.	H → G	From the statement
5.	H ∧ G	(need to be proven)

Convert propositional logic sentences into CNF		
1.	¬Y ∨ ¬R	
2.	Y ∨ M	
3.	Y ∨ R	
4.	R ∨ H	
5.	¬M ∨ H	
6.	¬H ∨ G	
7.	¬H ∨ ¬G	Negated goal
Resolution proof		
8.	¬Y ∨¬R	From 1
9.	Y ∨ ¬R	From 2
10.	Y ∨ M	From 2
11.	¬R ∨ M	From 7, 9
12.	H	From 3, 10
13.	M	From 4, 11

Conclusion: The unicorn is both horned and magical.

Problem 3. (2pts) Consider a vocabulary with the following symbols:

- In(a, b): predicate "a in b"
- Country(a): predicate "a is a country"
- Border(a, b): predicate "a border b"
- MapColor(a): predicate "Color of a in the map"

Translate each FOL sentence into natural language (English)

Please fill your answers in the table below

No.	Original statements and FOL sentences
1.	In(Paris,France) ∧ In(Marseilles,France)
	Paris and Marseilles are both in France.
2.	∃ c Country(c) ∧ Border (c,Iraq) ∧ Border (c,Pakistan)
	There is a country that borders both Iraq and Pakistan.
3.	∀ c Country(c) ∧ Border (c,Ecuador) ⇒ In(c, SouthAmerica)
	All countries that border Ecuador are in South America.
4.	∀ c, d [In(c, SouthAmerica) ∧ In(d,Europe)] ⇒ ¬Borders(c, d)]
	No country in South America borders any country in Europe.
5.	∀ x, y (Country(x) ∧ Country(y) ∧ Borders (x, y) ∧ ¬(x = y)) ⇒ ¬(MapColor (x) = MapColor (y))
	No two adjacent countries have the same map color.

Problem 4. (2pts) Given a 3x3 grid of Sudoku game as below

5	3	a
6	b	c
1	9	8

Suppose

- a1 is the logical variable that it is true then a is 1, else a is not 1
- a2 is the logical variable that it is true then a is 2, else a is not 2
- ….
- a9 is the logical variable that it is true then a is 9, else a is not 9
- Similar for b1, …, b2 and c1, …, c9.

a) Write CNF sentences to restrict available numbers for a, b, c using given logical variables. Each CNF sentence should go with a corresponding sentence in natural language. (1pt)

No.	CNF sentences	Notes
1.	a cannot be 5, 3, 6, 1, 9, 8 $\neg a5 \wedge \neg a3 \wedge \neg a6 \wedge \neg a1 \wedge \neg a9 \wedge \neg a8$	
2.	b cannot be 5, 3, 6, 1, 9, 8 $\neg b5 \wedge \neg b3 \wedge \neg b6 \wedge \neg b1 \wedge \neg b9 \wedge \neg b8$	
3.	c cannot be 5, 3, 6, 1, 9, 8 $\neg c5 \wedge \neg c3 \wedge \neg c6 \wedge \neg c1 \wedge \neg c9 \wedge \neg c8$	
4.	a is 2 if and only if b and c are not 2 $a2 \leftrightarrow \neg b2 \wedge \neg c2$	*Convert ↔ into CNF sentences* $u \leftrightarrow v : (u \rightarrow v) \wedge (v \rightarrow u)$
5.	a is 4 if and only if b and c are not 4 $a4 \leftrightarrow \neg b4 \wedge \neg c4$	*Convert ↔ into CNF sentences* $u \leftrightarrow v : (u \rightarrow v) \wedge (v \rightarrow u)$
6.	a is 7 if and only if b and c are not 7 $a7 \leftrightarrow \neg b7 \wedge \neg c7$	*Convert ↔ into CNF sentences* $u \leftrightarrow v : (u \rightarrow v) \wedge (v \rightarrow u)$
…	(4), (5), (6) are similar in case of b and c	

b) Find all sets of values for (a1, a2, …, a9), (b1, b2, …, b9), and (c1, c2, …, c9) that satisfies all CNF sentences in (a). Fill **T for True** and **leave blank for False** in the given table. (1pt)

a1	a2	a3	a4	a5	a6	a7	a8	a9	b1	b2	b3	b4	b5	b6	b7	b8	b9	c1	c2	c3	c4	c5	c6	c7	c8	c9
	T											T												T		
	T														T						T					
			T							T															T		
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