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Week 4 Problem Set:

Section 1.7: 1, 3, 4, 6, 7, 9, 10, 16, 17, 31, 39

#1
$1 + 3 = 4$ □

#3
$2^2 = 4$ □

#4
$m = 2k$ where m is even
$m = -2k = -(2k)$ is even □

#6
$1(3) = 3$

#7
$1^2 = 1$
$2^2 = 4$
$4 - 1 = 3$ □

#9
Suppose that r is rational and i is irrational and $s = r + i$ is rational
$s + (-r) = i$ is rational, which is a contradiction □

#10

$2(1) = 2$ □

#16
$m = 1$
$n = 2$
$mn = 2 = 2k$ □

#17
a. Assume that n is odd, so $n = 2k + 1$ for some integer k
   Then $n^3 + 5 = 2(4k^3 + 6k^2 + 3k + 3)$
   Because $n^3 + 5$ is two times some integer, it is even □
b. Suppose that $n^3 + 5$ is odd and n is odd
   Because n is odd and the product of two odd numbers is odd, it follows that n^2 is odd and then that n^3 is odd
   But then $5 = (n^3 + 5) - n^3$ would have to be even because it is the difference of two odd numbers
   Therefore, the supposition that $n^3 + 5$ and n were both odd is wrong □

#31
$x = 2$
i. $3(2) + 2 = 8$ is even
ii. $(2) + 5 = 7$ is odd
iii. $(2)^2 = 4$ is even

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#39
Proof by Contradiction
Suppose that $a_1, a_2, \ldots, a_n$ are all less than A, where A is the average of these numbers
Then $a_1 + a_2 + \cdots + a_n < nA$
Dividing both sides by n shows that $A = (a_1 + a_2 + \cdots + a_n)/n < A$, which is a contradiction □

Section 1.8: 3, 7, 14, 24, 30, 38

#3
$x \geq y$ and $x < y$
$\max(x, y) + \min(x, y) = x + y$
$x = 3$ and $y = 1$
$\max(3, 1) = 3$
$\min(3, 1) = 1$
$\max(3, 1) + \min(3, 1) = 3 + 1$
$x = 1$ and $y = 3$
$\max(1, 3) = 3$
$\min(1, 3) = 1$
$\max(1, 3) + \min(1, 3) = 3 + 1$ □

#7
There are four cases.
Case 1:
$x \geq 0$ and $y \geq 0$
Then $|x| + |y| = x + y = |x + y|$
Case 2:
$x < 0$ and $y < 0$
Then $|x| + |y| = -x + (-y) = -(x + y) = |x + y|$ because $x + y < 0$
Case 3:
$x \geq 0$ and $y < 0$
Then $|x| + |y| = x + (-y)$
If $x \geq -y$, then $|x + y| = x + y$
But because $y < 0$, $-y > y$, so $|x| + |y| = x + (-y) > x + y = |x + y|$
If $x < -y$, then $|x + y| = -(x + y) = -x + (-y)$
But because $x \geq 0$, $x \geq -x$, so $|x| + |y| = x + (-y) \geq -x + (-y) = |x + y|$
Case 4:
$x < 0$ and $y \geq 0$
Identical to Case 3 with the roles of x and y reversed □

#14
$a = 2$ and $b = 3$
$2^3 = 8$ is also rational □

#24
$\sqrt{(x^2 + y^2)/2}$ Conjecture

#30
There's no solutions in integers x and y for $2x^2 + 5y^2 = 14$

#38
8-Gallon Jug: 8 gallon(s)
5-Gallon Jug: 0 gallon(s)
3-Gallon Jug: 0 gallon(s)

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Pour 5 gallons of the 8-gallon jug into the 5-gallon jug.
8-Gallon Jug: 3 gallon(s)
5-Gallon Jug: 5 gallon(s)
3-Gallon Jug: 0 gallon(s)
Pour 3 gallons of the 5-gallon jug into the 3-gallon jug.
8-Gallon Jug: 3 gallon(s)
5-Gallon Jug: 2 gallon(s)
3-Gallon Jug: 3 gallon(s)
Pour 3 gallons of the 3-gallon jug into the 8-gallon jug.
8-Gallon Jug: 6 gallon(s)
5-Gallon Jug: 2 gallon(s)
3-Gallon Jug: 0 gallon(s)
Pour 2 gallons of the 5-gallon jug into the 3-gallon jug.
8-Gallon Jug: 6 gallon(s)
5-Gallon Jug: 0 gallon(s)
3-Gallon Jug: 2 gallon(s)
Pour 5 gallons of the 8-gallon jug into the 5-gallon jug.
8-Gallon Jug: 1 gallon(s)
5-Gallon Jug: 5 gallon(s)
3-Gallon Jug: 2 gallon(s)
Pour 1 gallon of the 5-gallon jug into the 3-gallon jug.
8-Gallon Jug: 1 gallon(s)
5-Gallon Jug: **4 gallon(s)**
3-Gallon Jug: 3 gallon(s) □

Section 5.1: 3, 5, 10, 14, 20, 25, 33, 35, 37, 50, 51

#3

a. $P(1) = 1(1+1)(2(1)+1)/6 = 1$
b. Both sides of P(1) shown in part (a) equal 1.
c. $k^2 = k(k + 1)(2k + 1)/6$
d. $(k + 1)^2 = (k + 1)((k + 1) +1)(2(k + 1) + 1)/6$
e. $(12 + 22 + \cdots + k^2) + (k + 1)^2 = [k(k + 1)(2k + 1)/6] + (k + 1)^2 = [(k + 1)/6][k(2k + 1) + 6(k + 1)] =$ $[(k + 1)/6](2k^2 + 7k + 6) = [(k + 1)/6](k + 2)(2k +3) = (k +1)(k +2)(2k +3)/6$
f. We have completed both the basis step and the inductive step, so by the principle of mathematical induction, the statement is true for every positive integer n.

#5

$P(n) = 12 + 32 + \cdots + (2n + 1)^2 = (n + 1)(2n + 1)(2n + 3)/3$
<u>Basis Step:</u>
P(0) is true because $12 = 1 = (0 + 1)(2(0) + 1)(2(0) + 3)/3$
<u>Inductive Step:</u>
Assume that P (k) is true. Then…
$12 + 32 + \cdots + (2k + 1)^2 + [2(k + 1) + 1]^2 = (k + 1)(2k + 1)(2k + 3)/3 + (2k + 3)^2 = (2k + 3)[(k + 1)(2k + 1)/3 + (2k + 3)] =$ $(2k + 3)(2k^2 + 9k + 10)/3 = (2k + 3)(2k + 5)(k + 2)/3 = [(k + 1) + 1][2(k + 1) + 1][2(k + 1) + 3]/3$ □

#10

a. $1/(1(2)) + 1/(2(3)) + \ldots + 1/(n(n + 1))$
b. Prove

#14

#20

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#25
$P(n) = 1 + nh \leq (1 + h)^n$, $h > -1$
<u>Basis Step:</u>
P(0) is true because $1 + 0(h) = 1 \leq 1 = (1 + h)0$
<u>Inductive Step:</u>
Assume $1 + kh \leq (1 + h)^k$
Then because $(1 + h) > 0$, $(1 + h)^k + 1 = (1 + h)(1 + h)^k \geq (1 + h)(1 + kh) = 1 + (k + 1)^h + kh^2 \geq 1 + (k + 1)h$ □

#33
$P(n) = n^5 - n$ is divisible by 5
<u>Basis Step:</u>
P(0) is true because $0^5 - 0 = 0$ is divisible by 5
<u>Inductive Step:</u>
Assume that P(k) is true, that is, $k^5 - 5$ is divisible by 5
Then $(k + 1)^5 - (k + 1) = (k^5 + 5k^4 + 10k^3 + 10k^2 + 5k + 1) - (k + 1) = (k^5 - k) + 5(k^4 + 2k^3 + 2k^2 + k)$ is also divisible by 5, because both terms in this sum are divisible by 5 □

#35
P(n) be the proposition that $(2n - 1)2 - 1$ is divisible by 8
<u>Basis Case:</u> P(1) is true because $8 \mid 0$
Now assume that P (k) is true
Because $[(2(k + 1) - 1]\ 2 - 1 = [(2k - 1)2 - 1] + 8k$, P (k + 1) is true because both terms on the right-hand side are divisible by 8
This shows that P (n) is true for all positive integers n, so $m2 - 1$ is divisible by 8 whenever m is an odd positive integer □

#37
<u>Basis Step:</u>
$11^{1 + 1} + 12^{2(1) - 1} = 121 + 12 = 133$
<u>Inductive Step:</u>
Assume the inductive hypothesis, that $11^{n + 1} + 12^{2n - 1}$ is divisible by 133
Then $11^{(n + 1) + 1} + 12^{2(n + 1) - 1} = 11(11^{n + 1}) + 144(12^{2n - 1}) = 11(11^{n + 1}) + (11 + 133)(12^{2n - 1}) =$
$11(11^{n + 1} + 12^{2n - 1}) + 133(12^{2n - 1})$
The expression in parentheses is divisible by 133 by the inductive hypothesis, and obviously the second term is divisible by 133, so the entire quantity is divisible by 133, as desired □

#50

#51
The mistake is in applying the inductive hypothesis to look at $\max(x - 1, y - 1)$, because even though x and y are positive integers, $x - 1$ and $y - 1$ need not be (one or both could be 0).