

- Multiply the followings 2's-complement operands using add and shift implementation. Only the multiplier implementation is acceptable.
- (i)
- | | | | | | | | | | | | | | | | | |
|--|-----|---|---|---|---|---|-------|--------|--|------|---|---|---|---|---|--------|
| | (i) | 0 | 0 | 1 | 1 | 1 | 1 | (Mult) | | (ii) | 0 | 0 | 0 | 1 | 1 | (Mult) |
| | x | 0 | 1 | 1 | 0 | 1 | (Mpy) | | | x | 1 | 1 | 1 | 0 | 1 | (Mpy) |
- (1)
- | | | | | |
|---------|-----------|-----------|---------|--------------------|
| Mult | E | Acc | Mpy | |
| 1 1 1 1 | 0 | 0 0 0 0 0 | 1 1 0 | 1 |
| + 0 | 1 1 1 1 | | | Add & shift |
| 0 | 1 1 1 1 | | | |
| 0 | 0 1 1 1 | | 1 1 1 | 0 |
| | | | | Shift |
| 0 | 0 0 1 1 | | 1 1 1 | 1 |
| + 0 | 1 1 1 1 | | | Add & shift |
| 1 | 0 0 0 1 0 | | | |
| 0 | 1 0 0 0 1 | | 0 1 1 | 1 |
| + 0 | 1 1 1 1 | | | Add & shift |
| 1 | 1 0 0 0 0 | | | |
| 0 | 1 1 0 0 0 | | 0 0 1 1 | |
| | | | | Final Result = 195 |
- (ii) Since the multiplier is negative, we convert it to positive first before proceeding.
- | | | | | |
|---------|-----------|-----------|---------|-------------|
| Mult | E | ACC | Mpy | |
| 0 0 1 1 | 0 | 0 0 0 0 0 | 0 0 1 | 1 |
| + 0 | 0 0 0 1 1 | | | Add & shift |
| 0 | 0 0 0 1 1 | | | |
| 0 | 0 0 0 1 1 | | 1 0 0 | 1 |
| + 0 | 0 0 0 1 1 | | | Add & shift |
| 0 | 0 0 1 0 0 | | | |
| 0 | 0 0 0 1 0 | | 0 1 0 | 0 |
| | | | | Shift |
| 0 | 0 0 0 0 1 | | 0 0 1 | 0 |
| | | | | Shift |
| 0 | 0 0 0 0 0 | | 1 0 0 1 | |
| | | | | Result = 9 |
- This result must be then converted to negative. Therefore, the Final Result is -9.

Non-restoring:							Dividend				Divisor								
E	ACC																		
0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	1	0	1	1
0	0	0	0	0	0	1	0	0	0	0	1	1	X	Shift					
+ 1	1	1	1	1	1	0								Subtract					
	1	1	1	1	1	0	0	0	0	1	1	0	(set q=0)						
	1	1	0	1	1	0	0	0	1	1	0	X	Shift						
+ 0	0	0	0	1	0	1							Add						
	1	1	1	0	1	1	0	0	1	1	0	0	(set q=0)						
	1	1	0	1	1	0	0	1	1	0	0	X	Shift						
+ 0	0	0	0	1	0	1							Add						
	1	1	1	1	0	0	0	1	1	0	0	0	(set q=0)						
	1	1	1	0	1	0	1	1	0	0	0	X	Shift						
+ 0	0	0	0	1	0	1							Add						
	1	1	1	1	1	0	1	1	0	0	0	0	(set q=0)						
	1	1	1	1	0	1	1	0	0	0	0	X	Shift						
+ 0	0	0	0	1	0	1							Add						
	0	0	0	1	1	0	1	0	0	0	0	1	(set q=1)						
	0	0	0	1	1	0	0	0	0	0	1	X	Shift						
+ 1	1	1	1	1	0	1							Subtract						
	0	0	0	0	1	0	0	0	0	0	1	1	(set q=1)						
	0	0	0	0	1	0	0	0	0	0	1	1	Final result						
remainder >> 2							quotient >> 3												

No registers are accessed!

Two-operand format

----	--dd	dddd	rrrr
------	------	------	------

(e.g., ADD, CP, MOV, etc.)

I/O format

----	--AA	dddd	AAAA
------	------	------	------

(e.g., IN, OUT)

Displacement format

--q-	qq-d	dddd	qqq
------	------	------	-----

(e.g., LDD, STD)

Perform SP-1

Immediate format

----	KKKK	dddd	KKKK
------	------	------	------

(e.g., LDI, ANDI, ORI, etc.)

One-operand format

----	--dd	dddd	----
------	------	------	------

(e.g., INC, DEC, COMP, etc.)

PC-relative format

----	kkkk	kkkk	kkkk
------	------	------	------

k12

----	--kk	kkkk	k----
------	------	------	-------

k7

(e.g., RJMP, RCALL, BRcc, etc.)

Direct format

----	--kk	kkkk	----
kkkk	kkkk	kkkk	kkkk

(e.g., JMP, CALL)

- 1- Consider the implementation of the SUBI Rd, K (*Subtract Immediate*) instruction on the enhanced AVR datapath.
- List and explain the sequence of microoperations required to implement SUBI Rd, K.
 - List and explain the control signals and the Register Address Logic (RAL) output for the SUBI Rd, K instruction.
- Note that this instruction takes one execute cycle (EX). Control signals for the Fetch cycle are given below. Clearly explain your reasoning.

Solution:

- (a) Only one execute cycle is needed. Similar to CPI discussed in class, subtraction is performed, however the destination register stores the result.

Stage	Micro-operations
EX	Rd ← Rd - K

(b)

Control Signals	IF	SUBI
MJ	00	xx
MK	0	x
ML	0	x
IR_en	1	x
PC_en	1	0
PCh_en	0	0
PCL_en	0	0
NPC_en	1	x
SP_en	0	0
DEMUX	x	x
MA	x	0
MB	x	0
ALU_f	xxxx	0010
MC	xx	00
RF_wA	0	0
RF_wB	0	1
MD	x	0
ME	x	x
DM_r	x	x
DM_w	0	0
MF	x	x
MG	xx	xx
Adder_f	xx	xx
MH	x	x
MI	x	x

- 2- Consider the implementation of the ST+ R, R (*Store Indirect and Post-Increment*) instruction on the enhanced AVR datapath.

- List and explain the sequence of microoperations required to implement ST+ R, R.
 - List and explain the control signals and the Register Address Logic (RAL) output for the ST+ R, R instruction.
- Note that this instruction takes two execute cycles (EX1 and EX2). Control signals for the Fetch cycle are given below. Clearly explain your reasoning.

Solution:

- (a) As with all load and store operations we require 2 cycles. This instruction performs a store with post-increment where the first cycle uses the data path as outlined in Slide 27 of Lecture 4. Note that as we set the Data Memory pointer through DMAR, we are also incrementing the R pointer and writing the modified value back to the register file. In the second cycle we transfer the contents of the source register Rr to the location pointed to by X, which was already set in the DMAR register on the first cycle.

Stage	Micro-operations
EX1	DMAR ← X[X], Xh[X] ← Xh[X] + 1
EX2	M[DMAR] ← Rr

(b)

Control Signals	IF	ST+ R, R
MJ	00	xx
MK	0	xx
ML	0	x
IR_en	1	0
PC_en	1	0
PCh_en	0	0
PCL_en	0	0
NPC_en	1	x
SP_en	0	0
DEMUX	x	x
MA	x	x
MB	x	x
ALU_f	xxxx	xxxx
MC	xx	01
RF_wA	0	1
RF_wB	0	1
MD	x	x
ME	x	x
DM_r	x	0
DM_w	0	0
MF	x	x
MG	xx	xx
Adder_f	xx	01
MH	x	0
MI	x	x

to prevent the PC register and SP register, respectively, from being overwritten. Note that IR_en can be "don't care" since this is the last execute cycle and the IR register will be overwritten in the Fetch (i.e. next) cycle.

1- Consider the implementation of the RCALL (Relative Call to Subroutine) instruction on the enhanced AVR datapath shown below.

- List and explain the sequence of microoperations required to implement RCALL.
 - List and explain the control signals and the Register Address Logic (RAL) output for the RCALL instruction.
- Note that this instruction takes three execute cycles (EX1, EX2 and EX3). Control signals for the Fetch cycle are given below. Clearly explain your reasoning.

Solution:

- (a) RCALL involves overwriting the PC with a target address computed relative to PC+1 (the address of the next instruction). Furthermore, the return address must be stored onto the stack.

Stage	Micro-operations
EX1	M[SP] ← RARI, SP ← SP - 1
EX2	M[SP] ← RARB, SP ← SP - 1
EX3	PC ← NPC + ΔK

(b)

Control Signals	IF	RCALL
MJ	00	xx
MK	0	xx
ML	0	x
IR_en	1	0
PC_en	1	0
PCh_en	0	0
PCL_en	0	0
NPC_en	1	0
SP_en	0	1
DEMUX	x	x
MA	x	x
MB	x	x
ALU_f	xxxx	xxxx
MC	xx	xx
RF_wA	0	0
RF_wB	0	0
MD	x	0
ME	x	0
DM_r	x	0
DM_w	0	1
MF	x	x
MG	xx	xx
Adder_f	xx	10
MH	x	x
MI	x	0

- 1- Consider the implementation of the CPI Rd, K (*Compare Register with Immediate*) instruction on the enhanced AVR datapath.

- List and explain the sequence of microoperations required to implement CPI Rd, K.
 - List and explain the control signals and the Register Address Logic (RAL) output for the CPI Rd, K instruction.
- Note that this instruction takes one execute cycle (EX). Control signals for the Fetch cycle are given below. Clearly explain your reasoning.

Stage	Micro-operations
EX	Rd ← K

(b)

Control Signals	IF	CPI
MJ	00	xx
MK	0	x
ML	0	x
IR_en	1	x
PC_en	1	0
PCh_en	0	0
PCL_en	0	0
NPC_en	1	x
SP_en	0	0
DEMUX	x	0
MA	x	0
MB	x	x
ALU_f	xxxx	0010
MC	xx	xx
RF_wA	0	0
RF_wB	0	0
MD	x	x
ME	x	x
DM_r	x	x
DM_w	0	0
MF	x	x
MG	xx	xx
Adder_f	xx	xx
MH	x	x
MI	x	x

EX1: Contents of Rd is read from the register file by providing Rd to rA. The immediate value K is then subtracted from input-A of the ALU by setting MA to 0. Then subtraction is performed but nothing is written back to the register file. The condition flags, N,Z,V,C ect will be set based on the outcome of the subtraction operation. All other control signals can be don't cares except DM_w, SP_en, and PC_en which need to be 0 to prevent overwrite. IR_en can be don't care since it's the last execute cycle

- 4- Consider the multi-cycle implementation of the RETI (*Return from Interrupt*) instruction on the enhanced AVR datapath.

- List the sequence of microoperations required to implement RETI.
 - List and explain the control signals and the Register Address Logic (RAL) output for the RETI instruction.
- Note that this instruction takes three execute cycles (EX1, EX2, and EX3) and must modify the status register (I flag), however you may ignore any required modifications to the status register for this problem.

Control Signals	IF	RETI
MJ	00	xx
MK	0	x
ML	0	x
IR_en	1	0
PC_en	1	0
PCh_en	0	0
PCL_en	0	0
NPC_en	1	x
SP_en	0	1
DEMUX	x	x
MA	x	x
MB	x	x
ALU_f	xxxx	xxxx
MC	x	x
RF_wA	0	0
RF_wB	0	0
MD	x	x
ME	x	x
DM_r	x	1
DM_w	0	0
MF	x	x
MG	xx	xx
Adder_f	xx	01
MH	x	x
MI	x	x

- Consider the implementation of the Rjmp (*Relative jump*) instruction on the enhanced AVR datapath.
 - List and explain the sequence of microoperations required to implement Rjmp.
 - List and explain the control signals and the Register Address Logic (RAL) output for the Rjmp instruction.
- Note that this instruction takes one execute cycle (EX1). The Fetch cycle is shown below.

Stage	Micro-operations
IF	IR ← M[PC], PC ← PC + 1, NPC ← PC + 1, RAR ← PC + 1

(b) EX1: PC ← NPC + ΔK

Control Signals	IF	Rjmp
MJ	00	01
MK	0	x
ML	0	x
IR_en	1	x
PC_en	0	1
PCh_en	0	0
PCL_en	0	0
NPC_en	0	0
SP_en	0	0
DEMUX	x	x
MA	x	x
MB	x	x
ALU_f	xxxx	xxxx
MC	xx	xx
RF_wA	0	0
RF_wB	0	0
MD	x	x
ME	x	x
DM_r	x	x
DM_w	0	0
MF	x	0
Adder_f	xx	01
MH	x	x
MI	x	x

- 2- Consider the implementation of the LD Rd, Y+ (*Load Indirect and Post-Increment*) instruction on the enhanced AVR datapath.

- List and explain the sequence of microoperations required to implement LD Rd, Y+.
- List and explain the control signals and the Register Address Logic (RAL) output for the LD Rd, Y+ instruction.

Note that this instruction takes two execute cycles (EX1 and EX2). The Fetch cycle is shown below:

Stage	Micro-operations
IF	IR ← M[PC], PC ← PC + 1, NPC ← PC + 1, RAR ← PC + 1

(a) EX1: DMAR ← Yh,Yl, Yh,Yl ← Yh,Yl + 1

EX2: Rd ← M[DMAR]

Control Signals	IF	LD Rd, Y+
MJ	00	xx
MK	0	x
ML	0	x
IR_en	1	0
PC_en	0	0
PCh_en	0	0
PCL_en	0	0
NPC_en	0	x
SP_en	0	0
DEMUX	x	x
MA	x	x
MB	x	x
ALU_f	xxxx	xxxx
MC	xx	01
RF_wA	0	1
RF_wB	0	1
MD	x	x
ME	x	x
DM_r	x	1
DM_w	0	0
MF	x	0
Adder_f	xx	01
MH	x	0
MI	x	x

- 3- Consider the multi-cycle implementation of the RET (Return from subroutine) instruction on the enhanced AVR datapath.

- List and explain the sequence of microoperations required to implement RET.
 - List and explain the control signals and the Register Address Logic (RAL) output for the RET instruction.
- Note that this instruction takes three execute cycles (EX1, EX2, and EX3). The Fetch cycle is shown below.

Stage	Micro-operations
IF	IR ← M[PC], PC ← PC + 1, NPC ← PC + 1, RAR ← PC + 1

(b) EX1: SP ← SP + 1

EX2: PC ← M[SP], SP ← SP + 1

EX3: PC ← M[SP]

Control Signals	IF	RET
MJ	00	xx
MK	0	x
ML	0	x
IR_en	1	0
PC_en	1	0
PCh_en	0	0
PCL_en	0	0
NPC_en	1	x
SP_en	0	1
DEMUX	x	x
MA	x	x
MB	x	x
ALU_f	xxxx	xxxx
MC	xx	01
RF_wA	0	0
RF_wB	0	0
MD	x	x
ME	x	x
DM_r	x	1
DM_w	0	0
MF	x	0
Adder_f	xx	01
MH	x	x
MI	x	x

- 4- Consider the implementation of the PUSH (*Push Register on Stack*) instruction on the enhanced AVR datapath.

- List and explain the sequence of microoperations required to implement PUSH.
- List and explain the control signals and the Register Address Logic (RAL) output for the PUSH instruction

Stage	Micro-operations
EX1	SP ← SP - 1
EX2	PC ← M[SP], SP ← SP + 1
EX3	PC ← M[SP]

RAL Output	EX1	EX2	EX3
wA	x	x	x
wB	x	x	x
rA	x	x	x
rB	x	x	x

Control Signals	IF	PUSH
MJ	00	x
ML	0	x
IR_en	1	0
NPC_en	1	0
MG	xx	xx
MA	x	0
MB	xx	xx
ALU_f	xxxx	0101
MC	x	x
MD	xx	01
RF_wA	0	1
RF_wB	0	1
ME	x	0
MF	x	x
DM_r	x	x
DM_w	0	1
Adder_f	xx	xx
MH	x	x
MI	x	0
MI	00	xx
DEMUX	x	x
PCh_en	0	0
PCL_en	0	0

[35 pts]

- 3- Consider the implementation of the LPM (*Load Program Memory*) instruction on the enhanced AVR datapath.

- List and explain the sequence of microoperations required to implement LPM.
 - List and explain the control signals and the Register Address Logic (RAL) output for the LPM instruction.
- Note that this instruction takes three execute cycles (EX1, EX2, and EX3). Control signals for the Fetch cycle are given below. Clearly explain your reasoning.

(a) EX1: PMAR ← Zh,Zl

EX2: MDR ← M[PMAR]

EX3: R0 ← MDR

(b)

Control Signals	IF	LPM
MJ	00	xx
MK	0	x
ML	0	x
IR_en	0	x
PC_en	1	0
PCh_en	0	0
PCL_en	0	0
NPC_en	0	x
SP_en	0	0
DEMUX	x	x
NPA	x	x
MB	x	x
ALU_f	xxxx	xxxx
MC	xx	xx
RF_wA	0	0
RF_wB	0	0
MD	x	x
ME	x	x
DM_r	x	x
DM_w	0	0
MF	x	x
MG	xx	xx
Adder_f	xx	xx
MH	x	0
MI	x	x

- 2- Consider the implementation of the ICALL (*Indirect Call to Subroutine*) instruction on the enhanced AVR datapath shown below. ICALL is similar to the RCALL (Relative Call to Subroutine) instruction, except that the Z register points to the target address.

- List and explain the sequence of microoperations required to implement ICALL.
- List and explain the control signals and the Register Address Logic (RAL) output for the ICALL instruction.

Note that this instruction takes two execute cycles (EX1 and EX2). Control signals for the Fetch cycle are given below. Clearly explain your reasoning.

(a) EX1: M[SP] ← RARI, SP ← SP - 1

EX2: M[SP] ← RARB, SP ← SP - 1, PC ← Z

(b)

Control Signals	IF	ICALL
MJ	00	xx
MK	0	x
ML	0	x
IR_en	1	0
PC_en	1	x
PCh_en	0	0
PCL_en	0	0
NPC_en	0	1
SP_en	0	1
DEMUX	x	x
MA	x	x
MB	x	x
ALU_f	xxxx	xxxx
MC	xx	xx
RF_wA	0	0
RF_wB	0	0
MD	x	0
ME	x	0
DM_r	x	0
DM_w	0	1
MF	x	x
MG	xx	xx
Adder_f	xx	10
MH	x	0
MI	x	0

[30 pts]

- 1- Consider the implementation of the LD Rd, -X (*Load Indirect and Pre-decrement*) instruction on the enhanced AVR datapath.

- List and explain the sequence of microoperations required to implement LD Rd, -X.
- List and explain the control signals and the Register Address Logic (RAL) output for the LD Rd, -X instruction.

Note that this instruction takes two execute cycles (EX1 and EX2). Control signals for the Fetch cycle are given below. Clearly explain your reasoning.

(a) EX1: DMAR ← Zh,Xl - 1, Xh,Xl ← Xh,Xl - 1

EX2: Rd ← M[DMAR]

(b)

Control Signals	IF	LD rd, -X	
		EX1	EX2
MJ	00	xx	xx
MK	0	x	x
ML	0	x	x
IR_en	1	0	x
PC_en	1	0	x
PCh_en	0	0	0
PCL_en	0	0	0
NPC_en	1	x	x
SP_en	0	0	0
DEMUX	x	x	x
MA	x	x	x
MB	x	x	1
LIU_f	xxxxx	xxxxx	xxxxx
MC	xxx	01	000
RF_wA	0	1	1
RF_wB	0	1	1
MD	x	x	x
ME	x	x	1
DM_f	x	x	1
DM_w	0	0	0
MF	x	x	x
MG	xxx	10	xxx
Addr_f	xxx	10	xxx
MI	x	x	x
MI	x	x	x

Consider the implementation of the LDD Rd, Y+q (Load Indirect with Displacement Using Index Y) instruction on the enhanced AVR datapath shown on the following page.

- List and explain the sequence of microoperations required to implement LDD.
- List and explain the control signals and the Register Address Logic (RAL) output for the LDD instruction. Some of the control signals are given below.

Note that this instruction takes two execute cycles (EX1 and EX2). The Fetch cycle is shown below.

IF: $IR \leftarrow [PC]$, $PC \leftarrow PC + 1$, $NPC \leftarrow PC + 1$, $RA[R] \leftarrow PC + 1$

- EX1: $DMAR \leftarrow Y+q$ Setting the pointer, i.e. placing the address computed by $Y+q$ into the DMAR
EX2: $Rd \leftarrow M[DMAR]$ Loading the value pointed to by $Y+q$ into the destination register

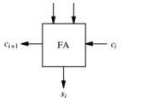
Control Signals	IF	EX1	EX2
MR	0	x	x
IR_en	1	0	0
PC_en	1	0	0
NPC_en	1	0	0
MG	00	xx	xx
MA	x	0	x
MB	xx	00	xx
ALU_f	xxxx	0000	xxxx
MC	x	x	1
MD	xx	xx	0
RF_wA	0	0	0
RF_wB	0	0	1
ME	x	x	x
MF	x	x	1
DM_f	x	x	1
DM_w	0	0	0
Adder_f	xx	xx	xx
MH	x	x	x
MI	x	1	x
MJ	00	xx	xx
DEMUX	x	x	x
PCL_en	0	0	0
PCL_n	0	0	0

Explanation of control signals

EX1: To add the q offset to address register Y, we must provide both values to the 16-bit ALU given in the datapath. The only way Y can be used as an operand to the ALU is through MUXA coming from the concatenation unit. First the high (Yh) and low (Yl) addresses of the Y register are generated by the RAL for rA and rB respectively. Then Y is chosen by setting MA=0. We also notice q must pass through MUXB via input 0, by setting MB=00. The addition operation is already set for w (ALU_f=0000). The output of the ALU ($Y+q$) must be routed to the DMAR through MUXF by setting MD=1. To prevent changes to registers and memory, RF_wA, RF_wB, and DM_w must all be set to zero. All other control signals to be filled out can be don't cares.

EX2: Since we are loading a value from Data Memory, we select the appropriate address coming from DMAR through MUXF by setting MF=0. Also, to read from Data Memory, DM_r=1 and DM_w=0. To route the 8-bit value that is read from Data Memory to the appropriate register Rd, only one path exists leading to inB of the register file. So RAL generates the address for Rd on wB. The two multiplexers MUXC and MUXD then route the value to be loaded into the destination register by setting MC=1 and MD=0. To allow for the input on inB to be written to the destination register, RF_wB=1. Since we do not modify any other registers, RF_wB=0. All other control signals can be don't cares.

Full Adder



$$S_i = X_i \oplus Y_i \oplus C_i$$

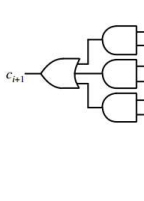
$$C_{i+1} = X_i Y_i + (X_i \oplus Y_i) C_i$$

or

$$C_{i+1} = X_i Y_i + (X_i + Y_i) C_i$$

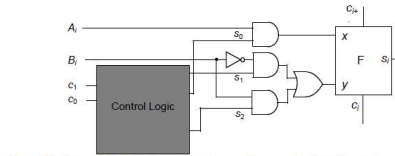
X_i	Y_i	C_i	S_i	C_{i+1}
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	0	1
1	1	1	1	1

X_i	Y_i	C_i	S_i	C_{i+1}
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	0	1
1	1	1	1	1



3- Consider P bit-slice of a n -bit arithmetic unit with A and B as inputs and three control signals c_1 , c_2 , and c_3 where c_3 is the carry-in to the n -bit arithmetic unit (not shown). Design the control logic required implement the following set of arithmetic operations. That is, define the truth table, K-map, and realization for the control logic to implement n -bit arithmetic unit. Your design must result in minimum number of logic gates.

c_3	c_2	c_1	Operation
0	0	0	$F = A + B$
0	0	1	$F = A + B + 1$
0	1	0	$F = A$
0	1	1	$F = B$
1	0	0	$F = B'$
1	0	1	$F = B' + 1$
1	1	0	$F = A + B$
1	1	1	$F = A + B' + 1$



We can define the truth table by looking at the requirement for s_i , s_{i+1} , and s_{i+2} depending on the operation. Thus, we have the following truth table

Input	Output
c_3	s_2
c_2	s_1
c_1	s_0
c_0	s_{-1}

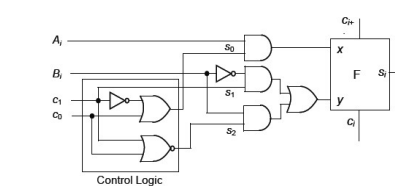
Note that control signals s_2 , s_1 , and s_0 are the same regardless whether s_{i+1} is 0 or 1. Therefore, the truth table can be simplified to

Input	Output
c_3	s_2
c_2	s_1
c_1	s_0
c_0	s_{-1}

This can be minimized using the following three K-maps

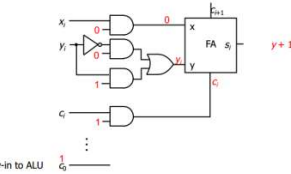
c_3	c_2	c_1	s_2
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Thus, we have the following implementation for each stage or bit slice.



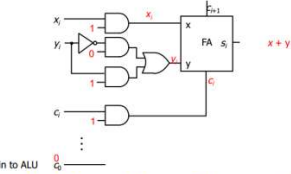
Arithmetic - Example 1

ARITHMETIC	M	S ₂	S ₁	S ₀	C ₀	Micro Operation	Description
1	0	0	1	1	1	$s = x + y + 1$	Increment y



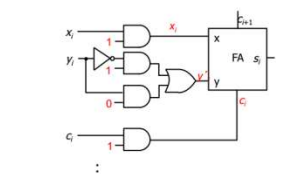
Arithmetic - Example 4

ARITHMETIC	M	S ₂	S ₁	S ₀	C ₀	Micro Operation	Description
1	1	0	1	0	0	$s = x + y$	Add x and y



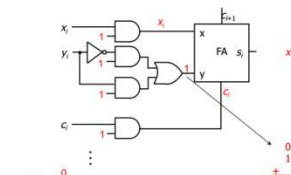
Arithmetic - Example 6

ARITHMETIC	M	S ₂	S ₁	S ₀	C ₀	Micro Operation	Description
1	1	1	0	1	1	$s = x + y' + 1$	x plus 2's complement of y



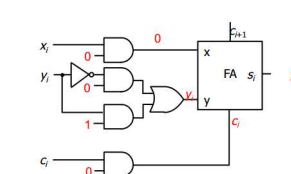
Arithmetic - Example 7

ARITHMETIC	M	S ₂	S ₁	S ₀	C ₀	Micro Operation	Description
1	1	1	1	0	0	$s = x - 1$	Decrement x



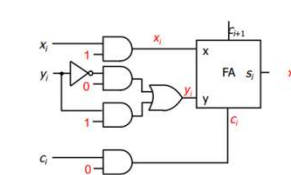
Logic - Example 2

ARITHMETIC	M	S ₂	S ₁	S ₀	C ₀	Micro Operation	Description
0	0	0	1	1	1	$s = y$	Transfer y



Logic - Example 6

ARITHMETIC	M	S ₂	S ₁	S ₀	C ₀	Micro Operation	Description
0	1	0	1	1	0	$s = x \oplus y$	EOR



Design a 4-bit arithmetic circuit with three control signals S_2 , S_1 , and S_0 (carry-in) using a 4-bit ripple-carry adder and logic gates, which performs the following arithmetic operations:

S_2	S_1	S_0	C_0	Operation
0	0	0	0	$F = A + B$
0	0	1	1	$F = A + B + 1$
0	1	0	0	$F = A$
0	1	1	0	$F = A + 1$
1	0	0	0	$F = B'$
1	0	1	0	$F = B' + 1$
1	1	0	0	$F = A + B'$
1	1	1	0	$F = A + B' + 1$

based on the design we saw in class, we can define the truth table by looking at the requirement for X_i and inputs to each FA. For X_i , we need an AND gate with A_i as one input and some control signal, say E_i , then control whether $X_i = A_i$. This is shown below



This can be defined by the following truth table and realization:

Input	Output
S_2	E_i
S_1	E_i
S_0	E_i
C_0	E_i

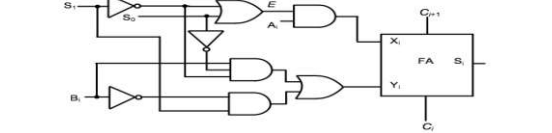
For Y_i , we need a MUX that determines whether $Y_i = B_i$, $Y_i = B' + 1$, or $Y_i = 0$. This is done by a logic shown below:



Based on this, we can define the following truth table for c_i and c_{i+1} :

Input	Output
S_2	c_i
S_1	c_i
S_0	c_i
C_0	c_i

Finally, we can implement "1" by setting $C_{i+1} = 1$ (i.e., carry into the adder). Thus, we have the following implementation for each stage or bit slice.



Carry Propagation

- 2's complement is the best.
- 1's complement twice as long.
- Significant delay reduction using carry look ahead concept.

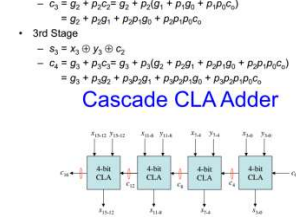
Example: 64 bit adder - reduced from 130 gate delays to 14, or improved by a factor of 8

CLA Equations

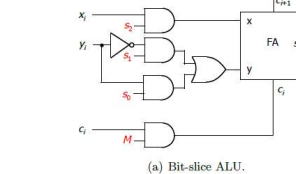
- 0th Stage:
 - $s_0 = x_0 \oplus y_0 \oplus c_0$
 - $c_1 = g_0 + p_0 c_0$
 - $c_2 = g_2 + p_2 g_1 + p_2 p_1 c_0$
 - $c_4 = g_4 + p_4 g_3 + p_4 p_3 g_2 + p_4 p_3 p_2 c_0$
- 1st Stage:
 - $s_1 = x_1 \oplus y_1 \oplus c_1$
 - $c_2 = g_1 + p_1 c_1 = g_1 + p_1 (g_0 + p_0 c_0) = g_1 + p_1 g_0 + p_1 p_0 c_0$
- 2nd Stage:
 - $s_2 = x_2 \oplus y_2 \oplus c_2$
 - $c_3 = g_2 + p_2 c_2 = g_2 + p_2 (g_1 + p_1 g_0 + p_1 p_0 c_0) = g_2 + p_2 g_1 + p_2 p_2 g_0 + p_2 p_2 p_0 c_0$
- 3rd Stage:
 - $s_3 = x_3 \oplus y_3 \oplus c_3$
 - $c_4 = g_3 + p_3 c_3 = g_3 + p_3 (g_2 + p_2 g_1 + p_2 p_2 g_0 + p_2 p_2 p_0 c_0) = g_3 + p_3 g_2 + p_3 p_3 g_1 + p_3 p_3 p_2 g_0 + p_3 p_3 p_2 p_0 c_0$

Can be implemented with two-level logic! (2gds)

Cascade CLA Adder



- Total delay:
 - 1 gd for g_i 's and p_i 's for $i = 0, 1, \dots, n-1$
 - 2 gds x 3 for c_0, c_1, c_2
 - 2 gds for c_{13}, c_{14}, c_{15} (after c_{12} is available)
 - 2 gds for $s_{15,12}$
- For m CLA stages, total delay = $3 + 2m$. For $m=4$, 11 gds (compared to 32 gds for RCA).



(a) Bit-slice ALU.

ARITHMETIC	M	S ₂	S ₁	S ₀	C ₀	Micro Operation	Description
1	0	0	1	1	1	$s = y + 1$	Increment y
1	0	1	0	1	1	$s = y + 1$	2's complement y
1	1	0	0	1	1	$s = x + y$	Increment x
1	1	0	1	0	0	$s = x + y$	Add x and y
1	1	1	0	0	0	$s = x + y'$	x plus 1's complement of y
1	1	1	0	1	0	$s = x + y' + 1$	x plus 2's complement of y
1	1	1	1	0	0	$s = x - 1$	Decrement x

(b) Arithmetic operations.

LOGIC	M	S ₂	S ₁	S ₀	C ₀	Micro Operation	Description
0	0	0	0	0	0	$s = 0$	Clear
0	0	0	1	1	1	$s = y$	Transfer y
0	0	1	0	0	0	$s = y'$	Comp. y
0	0	1	1	1	1	$s = 1$	Set
0	1	0	0	0	0	$s = x$	Transfer x
0	1	0	1	0	0	$s = x \oplus y$	EOR
0	1	1	0	0	0	$s = (x \oplus y)'$	ENOR
0	1	1	1	1	0	$s = x'$	Comp. x

(c) Logic operations.

2- Consider the following AVR instruction code sequence for the 16-bit multiplier from Lab 3:

```

MUL16_LOOP:  ldi  A, X+      ; Get byte of A operand
              ldi  B, Y      ; Get byte of B operand
              mul  A, B      ; Multiply A and B
              ldi  A, Z+      ; Get a result byte from memory
              ldi  B, Z+      ; Get the next result byte from memory
              add  r10, A      ; r10 <= r10 + A
              adc  r11, B      ; r11 <= r11 + B + carry
              ldi  A, Z      ; Get a third byte from the result
              adc  A, zero     ; Add carry to A
              st   Z, A       ; Store third byte to memory
              st   -2, r11    ; Store second byte to memory
              st   -2, r10    ; Store third byte to memory
              adiw ZH:ZL, 1    ; Z <= Z + 1
              dec  loop       ; Decrement counter
              brne MUL16_LOOP ; End inner for loop
  
```

Solution:

(a) Since all loads and stores are 2 cycles each, we only list the remaining instructions:

mul, addw: 2 cycles
add, adc, dec: 1 cycle
brne: 1 cycle when condition is false, 2 cycles when condition is true (for pipelining)

brne is similar to breq; whether one or two execute cycles take place is specifically related to pipelining. As discussed on Slide 64 of the class notes, a one-cycle branch penalty is incurred if the branch is taken (i.e., condition is true) or not.

(b) Non-pipelined:
The fetch and execute cycles for each instruction are listed below. Note for the branch instruction one execute cycle is used regardless if the branch is taken or not. This is because pipelining is not employed so any given instruction cannot be fetched until the instruction preceding it has completed the execute cycle.

Instruction	Fetch	EX	Total
ld A, X+	1	2	3
ld B, Y	1	2	3
mul A, B	1	2	3
ld A, Z+	1	2	3
ld B, Z+	1	2	3
add rlo, A	1	1	2
adc rhi, B	1	1	2
ld A, Z	1	2	3
adc A, zero	1	1	2
st Z, A	1	2	3
st -Z, rhi	1	2	3
st -Z, rlo	1	2	3
addw RHILL, 1	1	2	3
dec iloop	1	1	2
brne MUL16_ILOOP	1	1	2

The total number of cycles amounts to 40.

(c) Pipelined:
For the pipelined version, after the first fetch, each Fetch cycle is overlapped with the last execute cycle of the preceding instruction. Also, for the brne instruction, since the condition is true, we will have a one-cycle branch penalty resulting in two execute cycles.

Instruction	Fetch	EX	Total
ld A, X+	1	2	3
ld B, Y	-	2	2
mul A, B	-	2	2
ld A, Z+	-	2	2
ld B, Z+	-	2	2
add rlo, A	-	1	1
adc rhi, B	-	1	1
ld A, Z	-	2	2
adc A, zero	-	1	1
st Z, A	-	2	2
st -Z, rhi	-	2	2
st -Z, rlo	-	2	2
addw RHILL, 1	-	2	2
dec iloop	-	1	1
brne MUL16_ILOOP	-	2	2

With pipelining, the total number of cycles is now 27.

(c) The performance improvement is $(40-27)/40 = .325 = 32.5\%$.