

# Homework #6

Rheal Mae Edwards  
Student ID #  
932-389-303

## 1) Arithmetic Circuits

### 5.1 Delay Times for 64-Bit Adders

Two-Input Gate Delay = 150 ps  
Full Adder Delay = 450 ps

#### a) A Ripple Carry Adder

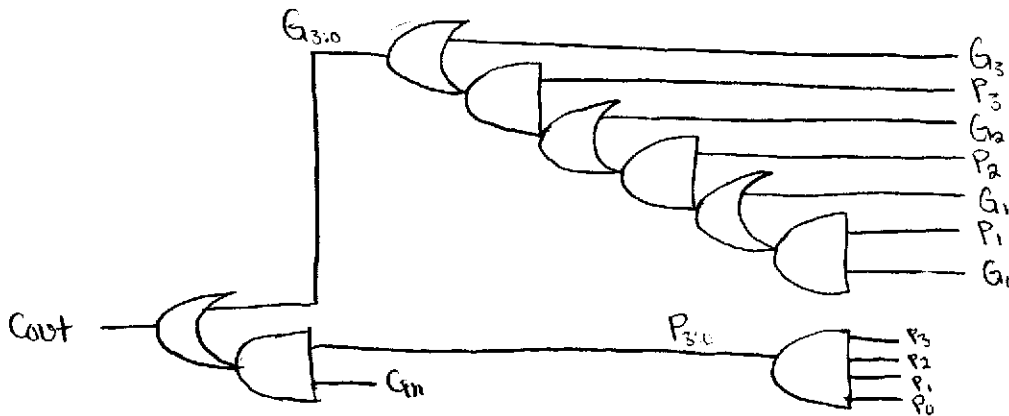
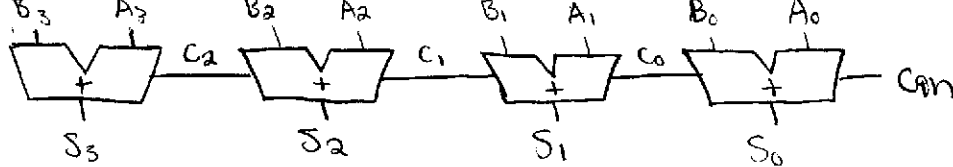
$$t_{ripple} = N t_{FA}$$

$$t_{ripple} = 64(450)$$

$$t_{ripple} = 28800 \text{ ps}$$

#### b) A Carry-lookahead Adder with 4-Bit Blocks

$$t_{CLA} = t_{pg} + t_{pg-blk} + (N/K - 1) t_{AND-OR} + K t_{FA}$$



$$t_{CLA} = 150 + 6(150) + (64/4 - 1)(2 \cdot 150) + 4(450)$$

$$t_{CLA} = 7350 \text{ ps}$$

#### c) A Prefix Adder

$$t_{PA} = t_{pg} + \log_2 N (t_{pg} - t_{prefix}) + t_{xor}$$

$\uparrow$  rows       $\uparrow$  2-Gate delay Each Row

$\leftarrow$  only have to wait for one of the XOR gates

$$t_{PA} = 150 + 6(2 \cdot 150) + 150$$

$$t_{PA} = 2100 \text{ ps}$$

## 5.2 Comparing a 64-Bit Ripple-Carry Adder and a 64-Bit Carry-Lookahead Adder

- only Two Input Gates Used

Static Power Negligible

$$\begin{aligned}\text{Two-Input Gate} &= 15 \mu\text{m}^2 \\ &= 50 \text{ ps delay} \\ &= 20 \text{ fF total gate capacitance}\end{aligned}$$

### 64-Bit Ripple-Carry Adder

$$64(7 \text{ gates}) = 448 \text{ Gates}$$

### 64-Bit Carry-Lookahead Adder

$$16(47 \text{ gates}) = 752 \text{ Gates}$$

a) Operating at 100 MHz and 1.2V, Area, Delays, Power

$$\text{Area: } A_{\text{ripple}} = 448(15)$$

$$A_{\text{CLA}} = 752(15)$$

$$A_{\text{ripple}} = 6720 \mu\text{m}^2$$

$$A_{\text{CLA}} = 11280 \mu\text{m}^2$$

$$A_{\text{ripple}} < A_{\text{CLA}}$$

$$\text{Delay: } t_{\text{FA}} = 3(50) = 150 \text{ ps}$$

$$t_{\text{ripple}} = 64(150)$$

$$t_{\text{ripple}} = 9600 \text{ ps}$$

$$t_{\text{CLA}} = 50 + 6(50) + (64/4 - 1)(2 \times 50) + 4(150)$$

$$t_{\text{CLA}} = 2450 \text{ ps}$$

$$t_{\text{ripple}} > t_{\text{CLA}}$$

$$\text{Power: } P = \frac{1}{2} C V^2 f$$

$$P_{\text{ripple}} = \frac{1}{2} ((448 \times 20 \times 10^{-15}) (1.2)^2 (100 \times 10^6))$$

$$P_{\text{ripple}} = 0.00064512 \text{ W}$$

$$P_{\text{CLA}} = \frac{1}{2} ((752 \times 20 \times 10^{-15}) (1.2)^2 (100 \times 10^6))$$

$$P_{\text{CLA}} = 0.00108288 \text{ W}$$

$$P_{\text{ripple}} < P_{\text{CLA}}$$

6) The Ripple-Carry Adder is smaller and requires less power when compared to the Carry-Lookahead Adder but it does take longer to process due to its longer delays. The Carry-Lookahead Adder may be bigger and requires more power, but it is also faster when compared to the Ripple-Carry Adder.

### 5.3 Designer Decisions

A designer might choose a ripple-carry adder instead of a carry-lookahead adder because it requires less space and less power; even though it may take a little bit longer to process due to its longer delay time. Maybe the designer is designing a calculator where physical space and less usage of power consumption may seem more important over the length of time it takes for the device to calculate.

## 2) Interview Questions

### 5.1 Largest Possible Result

Multiplying Two Unsigned N-Bit Numbers

$$\begin{array}{r} \phantom{x} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ x \phantom{1} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \hline 110001 \\ (3\text{-bit}) \end{array}$$

$$3 \rightarrow 49 (7^2)$$

$$\begin{array}{r} \phantom{x} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ x \phantom{1} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \hline 11100001 \\ (4\text{-bit}) \end{array}$$

$$4 \rightarrow 225 (15^2)$$

$$\begin{array}{r} \phantom{x} \phantom{1} \phantom{1} \\ x \phantom{1} \phantom{1} \phantom{1} \\ \hline 1001 \\ (2\text{-bit}) \end{array}$$

$$2 \rightarrow 9 (3^2)$$

$$\begin{array}{r} \phantom{x} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ x \phantom{1} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \hline 1111000001 \\ (5\text{-bit}) \end{array}$$

$$5 \rightarrow 961 (31^2)$$

$$\boxed{\text{Largest Possible Result} = (2^n - 1)^2}$$

What is the largest possible result when adding two unsigned N-bit numbers?

$$\begin{array}{r} \phantom{+} \phantom{1} \phantom{1} \\ + \phantom{1} \phantom{1} \phantom{1} \\ \hline 110 \\ (2\text{-bit}) \end{array}$$

$$2 \rightarrow 6 (2^3)$$

$$\begin{array}{r} \phantom{+} \phantom{1} \phantom{1} \phantom{1} \\ + \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \hline 1110 \\ (3\text{-bit}) \end{array}$$

$$3 \rightarrow 14 (2^4)$$

$$\begin{array}{r} \phantom{+} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ + \phantom{1} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \hline 11110 \\ (4\text{-bit}) \end{array}$$

$$4 \rightarrow 30 (2^5)$$

$$\begin{array}{r} \phantom{+} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ + \phantom{1} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \hline 111110 \\ (5\text{-bit}) \end{array}$$

$$5 \rightarrow 62 (2^6)$$

$$\boxed{\text{Largest Possible Result} = 2(2^n - 1)}$$