ECE 271 Homework 1

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1) Number System Conversion

1.11 Smallest (most negative) 16-bit binary number

a) unsigned numbers? $\boxed{0}$ since unsigned numbers are all positive

b) twos complement numbers?
$-2^{N-1} \rightarrow -2^{16-1} = \boxed{-2^{15} = -32768}$

c) sign/magnitude numbers?
$-2^{N-1}+1 \rightarrow -2^{16-1}+1 = \boxed{-2^{15}+1 = -32767}$

1.13 Unsigned Binary Numbers → Decimal

a) 1010_2 $\quad \frac{1}{8}\,\frac{0}{4}\,\frac{1}{2}\,\frac{0}{1}$ $\quad 8+2 = \boxed{10}_{10}$

b) 110110_2 $\quad \frac{1}{32}\,\frac{1}{16}\,\frac{0}{8}\,\frac{1}{4}\,\frac{1}{2}\,\frac{0}{1}$ $\quad 32+16+4+2 = \boxed{54}_{10}$

c) 11110000_2 $\quad \frac{1}{128}\,\frac{1}{64}\,\frac{1}{32}\,\frac{1}{16}\,\frac{0}{8}\,\frac{0}{4}\,\frac{0}{2}\,\frac{0}{1}$ $\quad 128+64+32+16 = \boxed{240}_{10}$

d) 000100010100111_2
$\frac{0}{16384}\,\frac{0}{8192}\,\frac{0}{4096}\,\frac{1}{2048}\,\frac{0}{1024}\,\frac{0}{512}\,\frac{0}{256}\,\frac{1}{128}\,\frac{0}{64}\,\frac{1}{32}\,\frac{0}{16}\,\frac{0}{8}\,\frac{1}{4}\,\frac{1}{2}\,\frac{1}{1}$
$2048+128+32+4+2+1 = \boxed{2215}_{10}$

1.17 Hexadecimal Numbers → Decimal

a) $A5_{16}$ $\quad A \times 16^1 = 10 \times 16^1 = 160$ $\quad 5 \times 16^0 = 5$ $\quad 160+5 = \boxed{165}_{10}$

b) $3B_{16}$ $\quad 3 \times 16^1 = 48$ $\quad B \times 16^0 = 11 \times 16^0 = 11$ $\quad 48+11 = \boxed{59}_{10}$

c) $FFFF_{16}$ $\quad F \times 16^3 = 15 \times 16^3 = 61440$ $\quad F \times 16^2 = 15 \times 16^2 = 3840$ $\quad F \times 16^1 = 15 \times 16 = 240$
$F \times 16^0 = 15 \times 16^0 = 15$ $\quad 61440 + 3840 + 240 + 15 = \boxed{65535}_{10}$

d) $D0000000_{16}$ $\quad D \times 16^7 = 13 \times 16^7 = \boxed{3489660928}_{10}$

1.22 Twos Complement Binary Numbers → Decimal

a) $\underset{\text{negative}}{\underset{\uparrow}{1}}110_2$
$$\begin{array}{r} 1110 \\ +\,0010 \\ \hline 10000 \end{array}$$
$0010 \rightarrow \frac{1}{2}$ $\quad = \boxed{-2}_{10}$

b) $\underset{\text{negative}}{\underset{\uparrow}{1}}00011_2$ $\quad \frac{1}{32}\,\frac{0}{16}\,\frac{0}{8}\,\frac{0}{4}\,\frac{1}{2}\,\frac{1}{1}$ $\quad -32+2+1 = \boxed{-29}_{10}$

c) 01001110_2 (positive) $\quad \frac{0}{128}\ \frac{1}{64}\ \frac{0}{32}\ \frac{0}{16}\ \frac{1}{8}\ \frac{1}{4}\ \frac{1}{2}\ \frac{0}{1}$

$$64+8+4+2 = \boxed{78}_{10}$$

d) 10110101_2 (negative) $\quad \frac{1}{128}\ \frac{0}{64}\ \frac{1}{32}\ \frac{1}{16}\ \frac{0}{8}\ \frac{1}{4}\ \frac{0}{2}\ \frac{1}{1}$

$$-128+32+16+4+1 = \boxed{-75}_{10}$$

1.26 Decimal Numbers → Unsigned Binary

a) $14_{10} \quad \frac{1}{8}\ \frac{1}{4}\ \frac{1}{2}\ \frac{0}{1} \rightarrow \boxed{1110}_2$

b) $52_{10} \quad \frac{1}{32}\ \frac{1}{16}\ \frac{0}{8}\ \frac{1}{4}\ \frac{0}{2}\ \frac{0}{1} \rightarrow \boxed{110100}_2$

c) $339_{10} \quad \frac{1}{256}\ \frac{0}{128}\ \frac{1}{64}\ \frac{0}{32}\ \frac{1}{16}\ \frac{0}{8}\ \frac{0}{4}\ \frac{1}{2}\ \frac{1}{1} \rightarrow \boxed{101010011}_2$

d) $711_{10} \quad \frac{1}{512}\ \frac{0}{256}\ \frac{1}{128}\ \frac{1}{64}\ \frac{0}{32}\ \frac{0}{16}\ \frac{0}{8}\ \frac{1}{4}\ \frac{1}{2}\ \frac{1}{1} \rightarrow \boxed{1011000111}_2$

1.28 Decimal Numbers → Hexadecimal

a) $14_{10} = \frac{E}{1} \rightarrow \boxed{E}_{16}$

b) $52_{10} \quad \frac{3}{16}\ \frac{4}{1} \rightarrow \boxed{34}_{16}$

c) $339_{10} \quad \frac{1}{256}\ \frac{5}{16}\ \frac{3}{1} \rightarrow \boxed{153}_{16}$

d) $711_{10} \quad \frac{2}{256}\ \frac{C}{16}\ \frac{7}{1} \rightarrow \boxed{2C7}_{16}$

1.34 4-Bit Two's Complement Numbers → 8-Bit Two's Complement

a) 0111_2 (positive) $= \boxed{00000111}_2$

b) 1001_2 (negative) $= \boxed{11111001}_2$

1.36 4-Bit Unsigned Binary Numbers → 8-Bit Unsigned Binary

a) $0111_2 = \boxed{00000111}_2$ - All Numbers Positive (Unsigned)

b) $1001_2 = \boxed{00001001}_2$ - All Numbers Positive (Unsigned)

1.38 Decimal Numbers → Octal (Base 8)

a) $14_{10} \quad \frac{1}{8}\ \frac{6}{1} \rightarrow \boxed{16}_8$

b) $52_{10} \quad \frac{6}{8}\ \frac{4}{1} \rightarrow \boxed{64}_8$

c) $339_{10} \quad \frac{5}{64}\ \frac{2}{8}\ \frac{3}{1} \rightarrow \boxed{523}_8$

d) 711_{10} $\frac{1}{512}$ $\frac{3}{64}$ $\frac{0}{8}$ $\frac{7}{1}$ → $\boxed{1307}_8$

2) Arithmetic

1.56 Decimal Numbers → 6-Bit Two's Complement Binary
Add Them
Sum Overflows a 6-Bit Result?

a) $16_{10} + 9_{10}$ $16_{10} = \frac{1}{16}\ \frac{0}{8}\ \frac{0}{4}\ \frac{0}{2}\ \frac{0}{1} \rightarrow 010000$

$9_{10} = \frac{1}{8}\ \frac{0}{4}\ \frac{0}{2}\ \frac{1}{1} \rightarrow 001001$

$$\begin{array}{r} 010000 \\ +\ 001001 \\ \hline \boxed{011001}_2 \end{array} = 25\ \boxed{\text{Doesn't overflow}}$$

b) $27_{10} + 31_{10}$ $27_{10} = \frac{1}{16}\ \frac{1}{8}\ \frac{0}{4}\ \frac{1}{2}\ \frac{1}{1} \rightarrow 011011$

$31_{10} = \frac{1}{16}\ \frac{1}{8}\ \frac{1}{4}\ \frac{1}{2}\ \frac{1}{1} \rightarrow 011111$

$$\begin{array}{r} 011011 \\ +\ 011111 \\ \hline \boxed{111010}_2 \end{array} = 58\ \boxed{\text{Overflows}}$$

c) $-4_{10} + 19_{10}$ $-4_{10} = \frac{1}{4}\ \frac{0}{2}\ \frac{0}{1} \rightarrow 111100$

$19_{10} = \frac{1}{16}\ \frac{0}{8}\ \frac{0}{4}\ \frac{1}{2}\ \frac{1}{1} \rightarrow 010011$

$$\begin{array}{r} 111100 \\ +\ 010011 \\ \hline \boxed{001111}_2 \end{array} = 15\ \boxed{\text{Doesn't overflow}}$$

d) $3_{10} + -32_{10}$ $3_{10} = \frac{1}{2}\ \frac{1}{1} \rightarrow 000011$

$-32_{10} = \frac{1}{32}\ \frac{0}{16}\ \frac{0}{8}\ \frac{0}{4}\ \frac{0}{2}\ \frac{0}{1} \rightarrow 100000$

$$\begin{array}{r} 000011 \\ +\ 100000 \\ \hline \boxed{100011}_2 \end{array} = 29\ \boxed{\text{Doesn't Overflow}}$$

e) $-16_{10} + -9_{10}$ $-16_{10} = \frac{1}{16}\ \frac{0}{8}\ \frac{0}{4}\ \frac{0}{2}\ \frac{0}{1} \rightarrow 110000$

$-9_{10} = \frac{1}{8}\ \frac{0}{4}\ \frac{0}{2}\ \frac{1}{1} \rightarrow 111001$

$$\begin{array}{r} 110000 \\ +\ 111001 \\ \hline \boxed{101001}_2 \end{array} = -25\ \boxed{\text{Doesn't Overflow}}$$

f) $-27_{10} + -31_{10}$ $-27_{10} = \frac{1}{16}\ \frac{1}{8}\ \frac{0}{4}\ \frac{1}{2}\ \frac{1}{1} \rightarrow 111011$

$-31_{10} = \frac{1}{16}\ \frac{1}{8}\ \frac{1}{4}\ \frac{1}{2}\ \frac{1}{1} \rightarrow 111111$

```
   1 1 1 0 1 1
 + 1 1 1 1 1 1
 -------------
 1 1 1 1 0 1 0
```

$\boxed{1111010}_2 = -58$ [Overflows]

1.58 Add Unsigned Hexadecimal Numbers
Sum Overflows an 8-Bit (Two Hex Digit) Result?

a) $7_{16} + 9_{16}$ $7 + 9 = 16 \rightarrow \frac{1}{16}\ \frac{0}{1} \rightarrow \boxed{10}$ [Doesn't Overflow]

b) $13_{16} + 28_{16}$ $13 + 28 = 41 \rightarrow \frac{2}{16}\ \frac{9}{1} \rightarrow \boxed{29}$ [Doesn't Overflow]

c) $AB_{16} + 3E_{16}$ $AB_{16} = \frac{10}{16}\ \frac{11}{1} = 160 + 11 = 171$

$3E_{16} = \frac{3}{16}\ \frac{14}{1} = 48 + 14 = 62$

$171 + 62 = 233 \rightarrow \frac{E}{16}\ \frac{9}{1} \rightarrow \boxed{E9}$ [Doesn't Overflow]

d) $8F_{16} + AD_{16}$ $8F_{16} = \frac{8}{16}\ \frac{15}{1} = 128 + 15 = 143$

$AD_{16} = \frac{10}{16}\ \frac{13}{1} = 160 + 13 = 173$

$143 + 173 = 316 \rightarrow \frac{1}{256}\ \frac{3}{16}\ \frac{C}{1} \rightarrow \boxed{13C}$ [Overflows]

1.60 Decimal Numbers → 5-Bit Two's Complement Binary
Subtract Them
Difference Overflows a 5-Bit Result

a) $9_{10} - 7_{10}$ $9_{10} = \frac{1}{8}\ \frac{0}{4}\ \frac{0}{2}\ \frac{1}{1} \rightarrow 01001$

$7_{10} = \frac{1}{4}\ \frac{1}{2}\ \frac{1}{1} \rightarrow 00111$

```
   0 0 1 1 1        0 1 0 0 1
 + 1 1 0 0 1      + 1 1 0 0 1
 -----------      -----------
 1 0 0 0 0 0      1 0 0 0 1 0
```

$\boxed{100010} = 2$ [Doesn't Overflow]

b) $12_{10} - 15_{10}$ $12_{10} = \frac{1}{8}\ \frac{1}{4}\ \frac{0}{2}\ \frac{0}{1} \rightarrow 01100$

$15_{10} = \frac{1}{8}\ \frac{1}{4}\ \frac{1}{2}\ \frac{1}{1} \rightarrow 01111$

```
   0 1 1 1 1        0 1 1 0 0
 + 1 0 0 0 1      + 1 0 0 0 1
 -----------      -----------
 1 0 0 0 0 0        1 1 1 0 1
```

$\boxed{11101} = -29$ [Overflows]

— I'm unsure how this result matches the answer key, but doesn't make sense overall...

c) $-6_{10} - 11_{10}$ $\quad -6_{10} = \frac{1}{4}\ \frac{1}{2}\ \frac{0}{1} \to 11110$

$11_{10} = \frac{1}{8}\ \frac{0}{4}\ \frac{1}{2}\ \frac{1}{1} \to 01011$

$$\begin{array}{r} 01011 \\ +\ 10101 \\ \hline 100000 \end{array} \qquad \begin{array}{r} 11110 \\ +\ 10101 \\ \hline \boxed{110011} \end{array} = -19\ \boxed{\text{overflow}}$$

- unsure why..

d) $4_{10} - -8_{10}$ $\quad 4_{10} = \frac{1}{4}\ \frac{0}{2}\ \frac{0}{1} \to 00100$

$-8_{10} = \frac{1}{8}\ \frac{0}{4}\ \frac{0}{2}\ \frac{0}{1} \to 11000$

$$\begin{array}{r} 11000 \\ +\ 01000 \\ \hline 100000 \end{array} \qquad \begin{array}{r} 00100 \\ +\ 01000 \\ \hline \boxed{01100} \end{array} = 12\ \boxed{\text{Doesn't overflow}}$$

3) Extraterrestrial

1.66 How many fingers would you expect Martians to have?

Martian Number System Equation: $325 + 42 = 411$

Highest Number 5
- At least 6 fingers

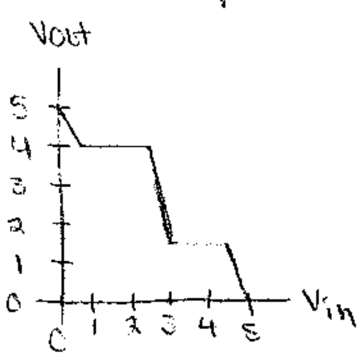
$$\begin{array}{r} 325 \\ +\ 42 \\ \hline 411 \end{array}$$

- 6 being the highest, one leftover

Martians would have at least $\boxed{6 \text{ Fingers}}$

4) Threshold Voltages

1.78 Is it possible?



$V_{OL} < V_{IL}$ $\quad NM_L = V_{IL} - V_{OL}$

$V_{OH} > V_{IH}$ $\quad NM_H = V_{OH} - V_{IH}$

$$\boxed{\begin{array}{ll} V_{IL} = 2.5V & V_{OL} = 1.5V \\ V_{IH} = 3V & V_{OH} = 4V \end{array}}$$

$NM_L = 2.5 - 1.5 = 1V$

$NM_H = 4 - 3 = 1V$

$$\boxed{\begin{array}{l} NM_L = 1V \\ NM_H = 1V \end{array}}$$

1.82 Two-Input Gate

a) What kind of logic gate?

Since the represented characteristics displays only holding given outputs of only a single portion of the figure, the gate is an $\boxed{\text{AND Gate.}}$

b) Logic Levels

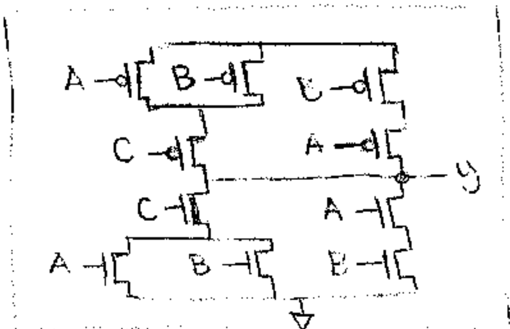
$$\boxed{\begin{array}{ll} V_{IL} = 1.5\,V & V_{OL} = 0\,V \\ V_{IH} = 2.25\,V & V_{OH} = 3\,V \end{array}}$$

5) CMOS Gates Arrays

1.86 Minority Gate

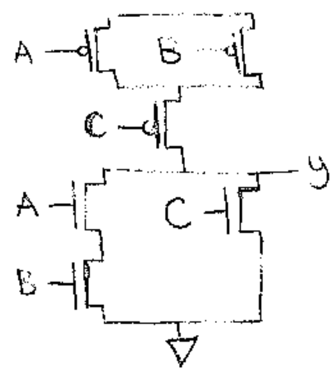
$x < n$	1
$x \geq n$	0

Transistor-Level Circuit:
Three-Input CMOS
Minority Gate



1.88 Truth table of the Mystery Schematic

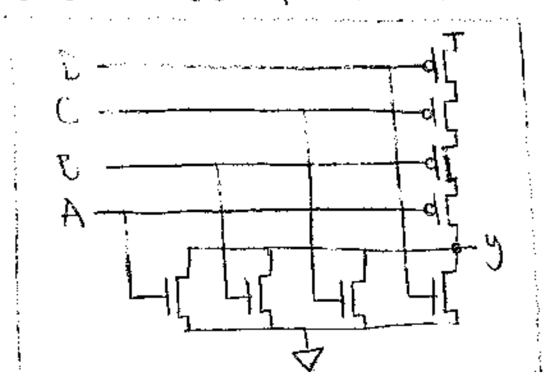
A	B	C	y
0	0	0	
0	0	1	0
0	1	1	0
1	0	1	0
0	1	0	1
1	0	0	0
1	1	0	1
1	1	1	0



1.90 Resistor-Transistor Logic (RTL)

- nMOS transistors pull gate output LOW
- weak resistor pull output HIGH
- RTL NOT Gate

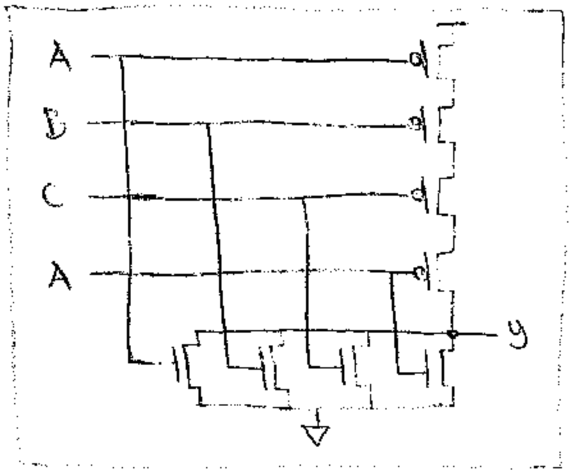
- Sketch three-input RTL NOR Gate



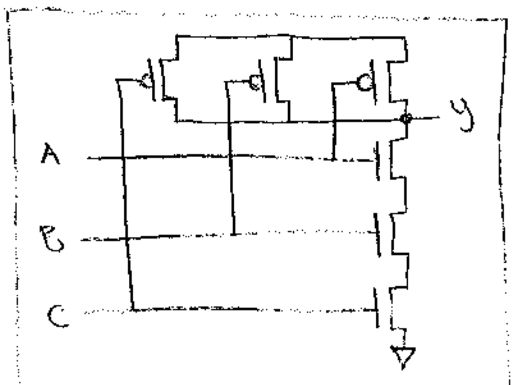
6) Interview Questions

1.1 Transistor-Level Circuit

a) CMOS Four-Input NOR Gate

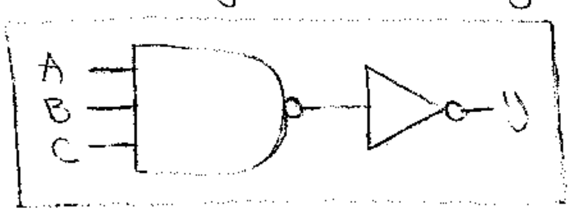


b) CMOS Three-Input NAND Gate



c) CMOS Three-Input AND Gate

- It's impossible to build an AND gate with a single CMOS gate. The best way to build an AND gate using CMOS transistors is to use a NAND gate followed by a NOT gate.



1.2 The Weight of the Fake

$\frac{64 \text{ coins}}{2}$

① $\frac{32 \text{ coins}}{2}$

② $\frac{16 \text{ coins}}{2}$

③ $\frac{8 \text{ coins}}{2}$

④ $\frac{4 \text{ coins}}{2}$

⑤ $\frac{2 \text{ coins}}{2}$

⑥ 1 coin

6 times. Based on this method of dividing the group in half.

13 The Speed to Conquer the Snake

Freshman Track Star = 1 minute (FTS)
Digital Design Student = 2 minutes (DDS)
Teaching Assistant = 5 minutes (TA)
Professor = 10 minutes (P)

FTS + P ⟶ 10 minutes
1 minute ⟵ FTS
FTS + TA ⟶ 5 minutes
1 minute ⟵ FTS
FTS + DDS ⟶ 2 minutes

Total Time: 19 minutes

DDS + FTS ⟶ 2 minutes
2 minutes ⟵ DDS
TA + P ⟶ 10 minutes
1 minute ⟵ FTS
DDS + FTS ⟶ 2 minutes

$\boxed{\text{Total Time: 17 minutes}}$