Introduction to Statistics for Engineers
Homework 1

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Instructions

- The homework is due on Friday Apr. 15th and must be submitted on Canvas before midnight. (Please read the policies on late homeworks on the syllabus)
- Homeworks must be submitted to Canvas as a Word or PDF document. Any other format (including JPEG) will not be accepted and your homework will be considered late. (Please read the policies on late homeworks on the syllabus)
- You must show your work and provide complete answers in order to receive full credit. Solutions restricted only the final numerical values that do not reflect your statistical reasoning **WILL NOT** receive full credit. The homework is worth 25 points.
- If not using the space assigned for each question, you must clearly indicate the problem that you are working.
- You must include your name and OSU-ID number in your homework document.
- Failing to follow any of these instructions may result in a delay in the grading or a penalization in your final score.

1. A certain sports car comes equipped with either an automatic or a manual transmission, and the car is available in one of four colors. Relevant probabilities for various combinations of transmission type and color are given in the following table:

	Color			
Transmission	White	Blue	Black	Red
A	0.13	0.10	0.11	0.11
M	0.15	0.07	0.15	0.18

Define the events $A = \{\text{the car has an automatic transmission}\}$ and $R = \{\text{the car is red}\}$.

(a) Compute $P(A)$, $P(R)$ and $P(A \cap R)$.

$$P(A) = \frac{0.13 + 0.10 + 0.11 + 0.11}{1} = 0.45$$

$$P(R) = \frac{0.11 + 0.18}{1} = 0.29$$

$$P(A \cap R) = \frac{0.11}{1} = 0.11$$

$$\boxed{\begin{aligned} P(A) &= 0.45 \\ P(R) &= 0.29 \\ P(A \cap R) &= 0.11 \end{aligned}}$$

Computing Probabilities (Definition)

$$P(A) = \frac{\text{Size of event } A}{\text{Size of the Sample Space } S}$$

(b) Given that a car has an automatic transmission, what is the probability that it is red?

$$P(R|A) = \frac{P(R \cap A)}{P(A)} = \frac{0.11}{0.45} = 0.24$$

$$\boxed{P(R|A) = 0.24}$$

Conditional Probability (Definition)

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

(c) Given that a car is red, what is the probability that it has an automatic transmission?

$$P(A|R) = \frac{P(A \cap R)}{P(R)} = \frac{0.11}{0.29} = 0.38$$

$$\boxed{P(A|R) = 0.38}$$

Conditional Probability

Independent Events

$P(A|B) = P(A)$

Equivalent to

$P(B|A) = P(B)$

(d) Are A and R independent events? Justify your answer.

A and R are not independent events, because $P(R|A)$ is not equal to $P(R)$ being $0.24 \neq 0.29$, and also with $P(A|R)$ is not equal to $P(A)$ being $0.38 \neq 0.45$, which both individual would define A and R not being independent events.

2. The number of tickets issued by a meter reader for parking-meter violations can be modeled by a Poisson process with a rate parameter of five per hour.

(a) What is the probability that exactly three tickets are given out during a particular hour?

Let X be the number of tickets given out in an hour.
The expected rate in one hour is $\lambda = 5 \times 1 = 5$.

$X \sim \text{Poisson}(\lambda)$

$$p(x) = \frac{\lambda^x}{x!} e^{-\lambda}, \quad x = 1, 2, \cdots$$

Poisson Distribution

$$P(X=3) = \frac{5^3}{3!} e^{-5} = 0.14$$

$$\boxed{P(X=3) = 0.14}$$

(b) What is the probability that at least three tickets are given out during a particular hour?

$$P(X \geq 3) = P(X=3) + P(X=4) + P(X=5) + \ldots$$

$$P(X \geq 3) = 1 - P(X < 3) = 1 - (P(X=2) + P(X=1) + P(X=0))$$

$$P(X \geq 3) = 1 - \left(\frac{5^2}{2!} e^{-5} + \frac{5^1}{1!} e^{-5} + \frac{5^0}{0!} e^{-5}\right)$$

$$= 1 - (0.12)$$

$$= 0.88$$

Poisson Distribution

$$\boxed{P(X \geq 3) = 0.88}$$

(c) How many tickets do you expect to be given during a 45-min period?

X = number of tickets

$$X = \left(\frac{45}{60}\right) \cdot 5 = 3.75$$

- Round down for the number of tickets within a 45-minute period

$$\boxed{X = 3 \text{ tickets}}$$

Cumulative Distribution Function

(Definition)

$$F(x) = P(X < x)$$

$$= \sum_{t \leq x} P(X = t)$$

3. A manufacturer of silicon wafers has encountered an operating problem where too many of the chips made are unacceptable. An inspector selects a sample of five wafers from each shift. The following table describes the distribution of the number of unacceptable wafers in the sample.

y_i	0	1	2	3	4	5
$p(y_i)$	0.3106	0.4313	0.2098	0.0442	0.0040	0.0001

(a) Find the probability that at most two wafers in the sample are unacceptable.

$$P(y_i \leq 2) = P(0) + P(1) + P(2)$$
$$= 0.3106 + 0.4313 + 0.2098 = 0.9517$$

$$\boxed{P(y_i \leq 2) = 0.9517}$$

(b) Find the probability that less than 4 are unacceptable.

$$P(y_i < 4) = P(0) + P(1) + P(2) + P(3)$$
$$= 0.3106 + 0.4313 + 0.2098 + 0.0442 = 0.9959$$

$$\boxed{P(y_i < 4) = 0.9959}$$

(c) Find the expected number of unacceptable wafers.

(mean)

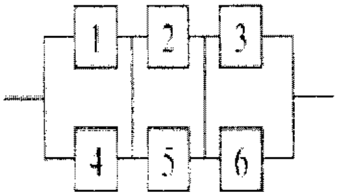
Population Mean

$$\mu = E(x) = \sum x \cdot p(x)$$

$$\mu = (0 \cdot 0.3106) + (1 \cdot 0.4313) + (2 \cdot 0.2098) + (3 \cdot 0.0442) +$$
$$(4 \cdot 0.0040) + (5 \cdot 0.0001) = 1$$

$$\boxed{E(x) = 1 \text{ unacceptale wafers}}$$

4. Consider the total-cross-tied system shown below obtained from the series-parallel array of cells by connecting ties across each column of junctions. The system works correctly if its lifetime exceeds 100 hours. The system fails as soon as an entire a column fails and column fails if both cells fail.



Let A_i denote the event that the lifetime of cell i exceeds 100 hours ($i = 1, 2, \ldots, 6$). Assume that the cells are independent and that $P(A_i) = 0.9$ for every cells. What is the probability that the whole system lifetime exceeds 100 hours?

$A_i = \{$lifetime of cell i exceeds 100 hours$\}$

$i = 1, 2, 3, 4, 5, 6$ $\qquad P(A_i) = 0.9$

cells are independent

$B = \{$whole system lifetime exceeds 100 hours$\}$

$$P(B) = P(1 \cup 4)P(2 \cup 5)P(3 \cup 6)$$

$$P(1 \cup 4) = P(2 \cup 5) = P(3 \cup 6)$$

$$P(B) = [P(1 \cup 4)]^3 = [P(1) + P(4) - P(1 \cap 4)]^3$$

$$= [P(1) + P(4) - P(1)P(4)]^3, \text{ \underline{By Independence}}$$

$$= [0.9 + 0.9 - (0.9)(0.9)]^3 = (0.99)^3 = 0.97$$

$$\boxed{P(B) = 0.97}$$

Extra Credit (1 Point)
Using the definition, compute the mean and variance of a Poisson distiribution.

$X \sim \text{Poisson}(\lambda)$

$$p(x) = \frac{\lambda^x}{x!} e^{-\lambda}, \quad x = 0, 1, 2, \ldots \quad \text{where } \lambda > 0$$

mean $= \mu = \lambda$

variance $= \sigma^2 = \lambda$

$\mu = E(X) = \sum x \cdot p(x)$

$\sigma^2 = \sum (x-\mu)^2 \cdot p(x)$

$E(X) = \sum x \cdot p(x)$

$E(X^2) = \sum x^2 \cdot p(x)$

$\sigma^2 = E(X^2) - [E(X)]^2$

$\sum p(x) = 1$

<u>Mean</u>

$$\mu = \sum_{x=0}^{\infty} x \cdot p(x) = \sum_{x=0}^{\infty} x \cdot \frac{\lambda^x}{x!} e^{-\lambda} = \lambda \sum_{x=0}^{\infty} x \cdot \frac{\lambda^{x-1}}{x!} e^{-\lambda}$$

$$= \lambda \sum_{x=0}^{\infty} \underbrace{\frac{\lambda^{x-1}}{(x-1)!} e^{-\lambda}}_{\text{Same Form as } p(x) = \frac{\lambda^x}{x!} e^{-\lambda}}$$

$$= \lambda \cdot \sum_{x=0}^{\infty} p(x) = \lambda \cdot (1) = \lambda$$

Variance

$$\sigma^2 = \sum_{x=0}^{\infty} (x-\mu)^2 \cdot p(x) = \sum (x^2 - 2x\mu + \mu^2) \cdot p(x)$$

$$= \sum p(x)x^2 - p(x)(-2x\mu + \mu^2)$$

[Ran Out of Thinking-Power to Continue/Finish...]