Introduction to Statistics for Engineers
Homework 2

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Instructions

- The homework is due on Friday Apr. 22nd and must be submitted on Canvas before midnight. (Please read the policies on late homeworks on the syllabus)
- Homeworks must be submitted to Canvas as a Word or PDF document. Any other format (including JPEG) will not be accepted and your homework will be considered late. (Please read the policies on late homeworks on the syllabus)
- You must show your work and provide complete answers in order to receive full credit. Solutions restricted only the final numerical values that do not reflect your statistical reasoning will not receive full credit. The homework is worth 25 points.
- If not using the space assigned for each question, you must clearly indicate the problem that you are working.
- You must include your name and OSU-ID number in your homework document.
- Failing to follow any of these instructions may result in a delay in the grading or a penalization in your final score.

1. A manufacturer of water filters for refrigerators monitors the process for defective filters. Historically, this process averages 5% defective filters. Suppose ten filters are randomly selected for testing. $D = \{\text{Defective Filters}\}$ $P(D) = 0.05$

   (a) Find the probability that all ten filters are not defective.

   <u>Binomial Distribution</u>

   $X \sim \text{Binomial}(10, 0.05)$

   $X =$ "The Number of Defective Filters in the Sample" $p(x) = \frac{n!}{x!(n-x)!} p^x (1-p)$

   $p(x=0) = \frac{10!}{\cancel{0!}(10-0)!} \cancel{(0.05)}^{0} (1-0.05) = 0.95$

   $\boxed{p(x=0) = 0.95}$

   (b) Find the probability that at least three filters are defective. <u>Binomial Distribution</u>

   $X \sim \text{Binomial}(10, 0.05)$

   $X =$ "The Number of Defective Filters in the Sample"

   $p(x \geq 3) = 1 - p(x < 3) = 1 - \left(\frac{10!}{2!(10-2)!}(0.05)^2(1-0.05)\right)$

   $= 1 - (0.107) = 0.893$ <u>DeMorgan's Law</u> AND <u>Independence</u>

   $\boxed{P(x \geq 3) = 0.893}$

   (c) Find the expected number of defective filters, the variance and the standard deviation.

   <u>Expected Number</u>: $\mu = np$ <u>Variance</u>: $\sigma^2 = np(1-p)$

   <u>Standard Deviation</u>: $\sigma = \sqrt{np(1-p)}$ for <u>Binomial Distribution</u>

   $\mu = 10(0.05) = 0.50$

   $\sigma^2 = 10(0.05)(1-0.05) = 0.475$

   $\sigma = \sqrt{10(0.05)(1-0.05)} = 0.69$

   $\boxed{\begin{aligned} \mu &= 0.50 \\ \sigma^2 &= 0.48 \\ \sigma &= 0.69 \end{aligned}}$

2. The EverLast Battery Company claims that the lifetimes of its AA batteries follow a normal distribution with a mean of 250 hours and a standard deviation of 25 hours.

$\mu = 250$ hr

$\sigma = 25$ hr

(a) Find the probability that a randomly selected EverLast AA battery will fail in less than 210 hours.

Normal Distribution

$X \sim \text{Normal}(\mu, \sigma^2)$

$X \sim \text{Normal}(250, 625)$

$X = 210$ hours

$z = \frac{210-250}{25} = -1.6$

$\to 0.0548$

Z-Scores

$z\text{-score} = \frac{x-\mu}{\sigma}$

$\boxed{P(x < 210) = 0.0548}$

(b) What percentage of the Everlast Batteries have a lifetime between 200 and 300 hours?

$P(200 < x < 300) = ?$

Z-Scores

$z = \frac{200-250}{25} = -2$

$\to 0.0228$

$z = \frac{300-250}{25} = 2$

$\to 0.9772$

$P(200 < x < 300) = 0.9772 - 0.0228 = 0.9544$

$\boxed{P(200 < x < 300) = 0.9544}$

(c) What battery lifetime corresponds to the 95th percentile?

0.9500

$\to z = 1.645$

Z-Scores

$25 \cdot 1.645 = \frac{x - \cancel{250}}{\cancel{25}} \; {\cancel{+250}} \; \cancel{\cdot 25}$

$+250$

$x = 291.125$

$\boxed{95^{th} \text{ Percentile} = 291 \text{ hours}}$

3. The safety manager at a chemical facility knows that the time between accidents (in days) can be modeled by an exponential distribution. From historical data he calculated that the mean time between accidents is 25 days.

$\mu = 25$ days

(a) Write down the pdf for the time between accidents.

$X \sim \text{Exponential}(\lambda)$

$\lambda = \frac{1}{\mu} = \frac{1}{25}$

$$\boxed{f(x) \begin{cases} \frac{1}{25} e^{-(1/25)x} & , x > 0 \\ 0 & , x \le 0 \end{cases}}$$

Exponential Distribution

(pdf)

$f(x) \begin{cases} \lambda e^{-\lambda x} & , x > 0 \\ 0 & , x \le 0 \end{cases}$

(mean)

$\mu = \frac{1}{\lambda}$

(b) Find the probability that the time between accidents is more than 25 days.

$P(X > 25) = \int_{25}^{\infty} \lambda e^{-\lambda t} dt$ $\quad u = -\lambda t \; dt$, $du = -\lambda$

$= -\int_{25}^{\infty} e^{u} du = -e^{u} \Big/_{25}^{\infty}$

$= -e^{-\lambda t} \Big/_{25}^{\infty} = -\frac{1}{e^{\lambda t}} \Big/_{25}^{\infty} = 0 + \left(+\frac{1}{e^{\lambda(25)}}\right) = \frac{1}{e^{\frac{1}{25}(25)}} = \frac{1}{e}$

Exponential Distribution

(cdf)

$F(x) = P(X \le x) = \int_0^x \lambda e^{-\lambda t} dt = 1 - e^{-\lambda x}$

$$\boxed{P(X > 25) = \frac{1}{e}} = 0.368$$

(c) Find the probability that the time between accidents is between 20 and 30 days.

$P(20 < X < 30) = \int_{20}^{30} \lambda e^{-\lambda t} dt$ $\quad u = -\lambda t \, dt$, $du = -\lambda$

Exponential Distribution

(cdf)

$= -\int_{20}^{30} e^{u} du = -e^{u} \Big/_{20}^{30}$

$= -e^{-\lambda t} \Big/_{20}^{30} = -\frac{1}{e^{\lambda t}} \Big/_{20}^{30} = -\left(\frac{1}{e^{\lambda(30)}}\right) + \left(+\frac{1}{e^{\lambda(20)}}\right)$

$= \frac{1}{e^{20/25}} - \frac{1}{e^{30/25}} = \frac{1}{e^{4/5}} - \frac{1}{e^{6/5}} = 0.148$

$$\boxed{P(20 < X < 30) = 0.148}$$

4. Suppose the probability density function (pdf) of weekly gravel sales Y (in 100's of tons) is given by

$$f(x) = \begin{cases} \frac{1}{8}(4-x) & ,0 \le x \le 4 \\ 0 & ,\text{otherwise} \end{cases}$$

(a) Derive the cumulative distribution function $F(x)$ for this distribution.

$F(x) = P(0 \le x \le 4) = \int_0^4 \frac{1}{8}(4-t)\,dt$

Uniform Distribution

(pdf) $f(x) = \begin{cases} \frac{1}{b-a} & , a \le x \le b \\ 0 & , \text{otherwise} \end{cases}$

$X \sim \text{Uniform}(a,b)$

$P(0 \le x \le 4) = \frac{1}{8}\int_0^4 4-t\,dt$

$= \frac{1}{8}\left(4t - \frac{t^2}{2}\right)\Big|_0^4$

$= \frac{1}{8}\left(\left(4(4) - \frac{4^2}{2}\right) - \cancel{\left(4(0) - \frac{0^2}{2}\right)}^{0}\right)$

$= \frac{1}{8}(16-8) = \frac{1}{\cancel{8}}\cancel{(8)}^{1} = 1$

$$\boxed{F(x) = P(0 \le x \le 4) = \int_0^4 \frac{1}{8}(4-t)\,dt = 1}$$

(b) What is the probability that less than 100 tons of gravel is sold in a given week?

$P(X<1) = \int_0^1 \frac{1}{8}(4-t)\,dt = \frac{1}{8}\int_0^1 4-t\,dt$

$= \frac{1}{8}\left(4t - \frac{t^2}{2}\right)\Big|_0^1 = \frac{1}{8}\left(\left(4(1) - \frac{1^2}{2}\right) - \cancel{\left(4(0) - \frac{0^2}{2}\right)}^{0}\right)$

$\boxed{P(X<1) = 0.4375}$

$= \frac{1}{8}\left(4 - \frac{1}{2}\right) = \frac{1}{8}\left(\frac{7}{2}\right) = \frac{7}{16} = 0.4375$

Uniform Distribution

(c) What is the probability that between 200 and 300 tons of gravel are sold in a given week?

$P(2 < x < 3) = \int_2^3 \frac{1}{8}(4-t)\,dt = \frac{1}{8}\int_2^3 4-t\,dt$

$= \frac{1}{8}\left(4t - \frac{t^2}{2}\right)\Big|_2^3 = \frac{1}{8}\left(\left(4(3)-\frac{3^2}{2}\right) - \left(4(2) - \frac{2^2}{2}\right)\right)$

$\boxed{P(2 < x < 3) = 0.1875}$

$= \frac{1}{8}\left(\left(12 - \frac{9}{2}\right) - (8-2)\right) = \frac{1}{8}\left(\frac{15}{2} - 6\right)$

$= \frac{1}{8}\left(\frac{3}{2}\right) = \frac{3}{16} = 0.1875$ Uniform Distribution

(d) Determine the expected (mean) amount of gravel sold per week.

$\mu = \frac{0+4}{2}$

$= \frac{4}{2}$

$= 2$

$a \le x \le b$

$0 \le x \le 4$

Uniform Distribution

$\mu = \frac{a+b}{2}$

$\boxed{\mu = 200 \text{ tons of gravel}}$

(e) The profit P for weekly gravel sales is given by $P = 2500X - 1250$. Find the expected profit for one week.

$P = 2500x - 1250$

$P = 2500(2) - 1250 = \$3750$

$\boxed{\text{Expected Profit} = \$3750}$

Application of Mean of Uniform Distribution and Algebra