CS 372: Introduction to Computer Networks

Extra Credit Homework - Solution

(based on Chapters 5 and 6 of *Kurose & Ross, 6th Ed.*)

REVIEW QUESTIONS

Answer the following review questions from Chapter 6 of the textbook:

- R3 [5 pts]

  The three types of impairments experienced by wireless links are:

  - *Path loss*, where the transmitted electromagnetic signal attenuates (decreases in strength) as it propagates through air and other matter along its path to the receiver. The longer it takes for a signal to propagate to a receiver, the harder it is to distinguish that attenuated signal from any noise which may be present on the channel.
  - *Multipath propagation*, where portions of the transmitted signal reflect off of objects and the ground, effectively taking multiple paths of different lengths to reach the receiver. These reflections can (and often do) reduce the quality of the received signal.
  - *Interference from other sources*, where other nodes transmit at the same time (leading to collisions), or where non-network devices (such as industrial machines) emit electromagnetic noise in the same frequency range as the wireless channel.

- R7 [5 pts]

  Acknowledgments are used in 802.11 because, unlike nodes using wired Ethernet links, it is difficult for wireless nodes to detect other transmissions while also transmitting themselves (in other words, it is hard to detect collisions). Wireless nodes can instead use the lack of an expected ACK to indicate that a collision has possibly occurred.

PROBLEMS

Answer the following homework problems from Chapter 5 of the textbook:

- P2 [5 pts]

  The following two-dimensional matrix has been configured with *even parity*; redundant data in the form of parity bits have been added, so that every row and column has an even number of 1s:

$$\begin{array}{cccc} 0 & 0 & 0 & \underline{0} \\ 1 & 1 & 1 & \underline{1} \\ 0 & 1 & 0 & \underline{1} \\ \underline{1} & \underline{0} & \underline{1} & \underline{0} \end{array}$$

  Suppose the above matrix is transmitted from some sender to some receiver, and that during transmission a single bit error occurs in row 2, column 3 of the matrix:

$$\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{array}$$

  When the matrix arrives at the receiver and a parity check is performed, the receiver will observe that the parity of row 2 and column 3 is wrong (there are an odd number of 1s). Since row 2 and column 3 uniquely identify a single bit, this means the two-dimensional parity check is able to detect *and* correct this single bit error.

Now suppose instead that two bit errors occur during transmission: a bit error in row 2, column 2, and a bit error in row 2, column 3:

$$\begin{matrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{matrix}$$

After these bit errors occur, the parity of row 2 is still correct! The parity of columns 2 and 3 is wrong, but the receiver can't uniquely identify the row where errors occurred. This is an example of a double bit error that can be detected *but not* corrected.

- P17 [10 pts]

  When $K = 100$, and the link's bitrate is 10 Mbps, the adapter will wait for:

  $$\frac{100 * 512}{10 \text{ Mbps}} = 5.12 \text{ milliseconds}$$

  When $K = 100$, and the link's bitrate is 100 Mbps, the adapter will wait for:

  $$\frac{100 * 512}{100 \text{ Mbps}} = 512 \text{ microseconds}$$

Answer the following homework problems from Chapter 6 of the textbook:

- P5 [10 pts]

  a. When the two stations attempt to transmit at the same time, a collision will occur because they share the same wireless channel (even though the two APs have different MAC addresses and SSIDs). They will both use the CSMA/CA protocol to recover from this collision, and eventually take turns transmitting, effectively sharing the channel and splitting its bandwidth.

  b. Now that each AP is operating on a different channel, they can proceed to transmit simultaneously without having any collisions.

ADDITIONAL QUESTIONS

From Chapter 5:

- [25 pts] Consider two nodes, A and B, that use the slotted ALOHA protocol to contend for a channel. Suppose node A has more data to transmit than node B, and node A's retransmission probability, p_A, is greater than node B's retransmission probability, p_B.

  a. Demonstrate how to choose p_A and p_B so that node A's average throughput is three times as large as that of node B. (In other words, figure out expressions for A's throughput and B's throughput, and then solve for p_A in terms of p_B.)

  Nodes are successful whenever they choose to transmit and all other nodes decide not to transmit. Therefore, we can calculate the average throughput of A and B as follows:

  $$\text{Average throughput for A} = p_A(1 - p_B) = 3 \cdot (\text{Average throughput for B})$$
  $$\text{Average throughput for B} = p_B(1 - p_A)$$

  Next, we can use these two equations to solve for p_A in terms of p_B:

$$p_A(1-p_B) = 3p_B(1-p_A)$$
$$p_A(1-p_B) + p_A(3p_B) = 3p_B$$
$$p_A(1-p_B+3p_B) = 3p_B$$
$$p_A(1+2p_B) = 3p_B$$
$$p_A = \frac{3p_B}{1+2p_B}$$

b. In general, suppose there are N nodes, among which node A has retransmission probability $3p$ and all other nodes have retransmission probability p. Provide an expression (in terms of N and p) for the total (combined) average throughput.

The average throughput for node A will be $3p(1-p)^{N-1}$ (probability that A transmits $\times$ probability that all other nodes do not transmit).

The average throughput for each of the other nodes will be $p(1-p)^{N-2}(1-3p)$ (probability that one non-A node transmits $\times$ probability that all other non-A nodes do not transmit $\times$ probability that A does not transmit). Since each of these $N-1$ non-A nodes will have the same average throughput, we can say that the total average throughput will be:

$$3p(1-p)^{N-1} + [N-1]p(1-p)^{N-2}(1-3p)$$

From Chapter 6:

- [15 pts] Suppose an 802.11b station (with a transmission rate of 11 Mbps) is configured to always reserve the channel using the RTS/CTS sequence. Suppose this station suddenly wants to transmit 1000 bytes of data, and all other stations are idle. As a function of SIFS and DIFS, and ignoring propagation delay and assuming there are no bit errors, calculate the total time required to transmit the frame and receive the acknowledgment.

  When the station wants to transmit, it will listen to the idle channel for a fixed amount of time (DIFS), and then send out a 32 byte RTS control frame (just an 802.11 frame with a 0 byte payload). When the station's destination receives the RTS frame, it will also wait for a fixed amount of time (SIFS), and then send out a 32 byte CTS control frame.

  Once the original station receives the CTS frame, it will wait for SIFS, and then begin sending its entire 1000 + 32 byte frame of data. When the station's destination receives the data frame, it will again wait for SIFS, and then send a 32 byte ACK control frame back to the station.

  Therefore, the total amount of time required for the station to send its 1000 bytes of data will be: DIFS + RTS + SIFS + CTS + SIFS + DATA + SIFS + ACK. Although DIFS and SIFS are not given for this problem, we can calculate the rest of the terms as follows:

  DATA = time required to send 1000 bytes of data and 32 bytes of 802.11 MAC frame header
  $= \dfrac{1032 \text{ bytes} \times 8 \text{ bits/byte}}{11 \text{ Mbps}} = 750.5\ \mu s$

  RTS, CTS, ACK = time required to send a 32 byte control frame with no payload (802.11 MAC header only)
  $= \dfrac{32 \text{ bytes} \times 8 \text{ bits/byte}}{11 \text{ Mbps}} = 23.3\ \mu s$

  The final result is:

  DIFS + 23.3 μs + SIFS + 23.3 μs + SIFS + 750.5 μs + SIFS + 23.3 μs = DIFS + 3*SIFS + 820.4 μs

- [25 pts] Consider the following idealized LTE scenario, with a downstream channel that is slotted in time (similar to Figure 6.20 in the textbook). Data sent downstream from the base station to four nodes (A, B, C, and D) is sent at rates 12 Mbps, 6 Mbps, 3 Mbps, and 1 Mbps, respectively. Assume the base station has an infinite amount of data to send to each of the nodes, and can send to any one of these four nodes during any time slot.

  a. What is the maximum rate at which the base station can send to the nodes, assuming it can send to any node it chooses during each time slot? Is your solution fair? Explain and define what you mean by "fair".

The maximum rate is 12 Mbps, which occurs when the base station sends data exclusively (i.e., in every time slot) to node A. This solution, however, would not be "fair" because none of the other nodes ever receive any data.

b. Suppose there is a fairness requirement that each node must receive an equal amount of data from the base station during each one second interval. What is the average transmission rate by the base station (to all nodes)? Explain how you arrived at your answer. (If necessary for your solution, you may assume that the fractions of one second that each node receives fit into integer multiples of time slots.)

To start with, we can observe that all four nodes are going to receive some fraction of the 1 second interval, and the time spent by all four nodes will fill each interval. Therefore, we can say that: $t_A + t_B + t_C + t_D = 1$.

Next, we can write out the relationship between these times, based on the relative differences between each node's downstream rate (for example, node A only needs 1/12 as much time as node D to receive the same amount of data): $12t_A = 6t_B = 3t_C = t_D$. At this point, we can solve for t_D, which is node D's fraction of the interval:

$$\frac{t_D}{12} + \frac{t_D}{6} + \frac{t_D}{3} + t_D = 1$$

$$\frac{t_D + 2t_D + 4t_D + 12t_D}{12} = 1$$

$$t_D = \frac{12}{19}$$

Now that we've solved for t_D, we can see that $t_A = \frac{1}{19}$, $t_B = \frac{2}{19}$, and $t_C = \frac{4}{19}$. Finally, the average transmission rate can be calculated as follows:

$$\frac{1}{19}(12 \text{ Mbps}) + \frac{2}{19}(6 \text{ Mbps}) + \frac{4}{19}(3 \text{ Mbps}) + \frac{12}{19}(1 \text{ Mbps}) = 2.53 \text{ Mbps}$$

c. Suppose that the fairness criterion is that any node can receive at most twice as much data as any other node during each one second interval. What is the maximum average transmission rate by the base station (to all nodes)? Explain how you arrived at your answer. (Again, if needed you may assume that the fractions of one second that each node receives fit into integer multiples of time slots.)

In this case, it turns out that we can maximize the total average transmission rate by having nodes A and B receive twice as much data as nodes C and D. As before, all four nodes will share some portion of the 1 second interval, such that $t_A + t_B + t_C + t_D = 1$. Let's solve for t_D again, followed by t_A, t_B, and t_C:

$$6t_A = 3t_B = 3t_C = t_D$$

$$\frac{t_D}{6} + \frac{t_D}{3} + \frac{t_D}{3} + t_D = 1$$

$$\frac{t_D + 2t_D + 2t_D + 6t_D}{6} = 1$$

$$t_D = \frac{6}{11}, \text{ and therefore } t_A = \frac{1}{11},\ t_B = \frac{2}{11}, \text{ and } t_C = \frac{2}{11}$$

Finally, this maximum average transmission rate can be calculated as follows:

$$\frac{1}{11}(12 \text{ Mbps}) + \frac{2}{11}(6 \text{ Mbps}) + \frac{2}{11}(3 \text{ Mbps}) + \frac{6}{11}(1 \text{ Mbps}) = 3.27 \text{ Mbps}$$