Rhea Mae Edwards

CS 321: Homework #1

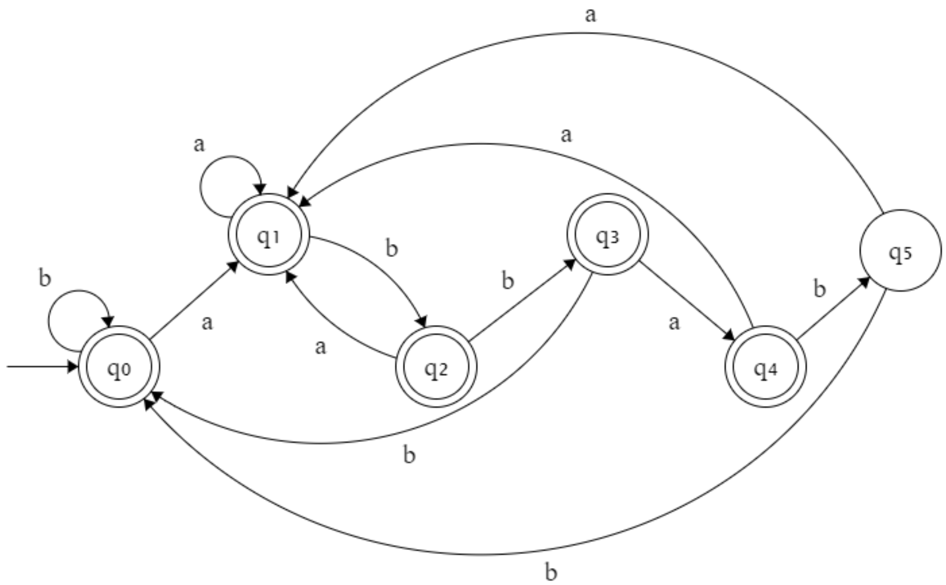
1. $\{x \in \{a, b\}^* \mid$ last 5 characters of x are **not** *abbab*$\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$

$\Sigma = \{a, b\}$

δ:

(q, c)	δ (q, c)
(q_0, a)	q_1
(q_0, b)	q_0
(q_1, a)	q_1
(q_1, b)	q_2
(q_2, a)	q_1
(q_2, b)	q_3
(q_3, a)	q_4
(q_3, b)	q_0
(q_4, a)	q_1
(q_4, b)	q_5
(q_5, a)	q_1
(q_5, b)	q_0



$s = q_0$

$F = \{q_0, q_1, q_2, q_3, q_4\}$

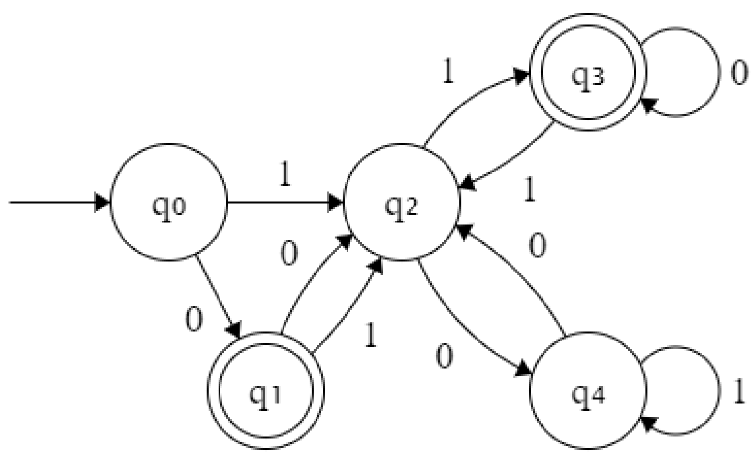
2. $\{x \in \{0, 1\}^* \mid x$ is a binary encoding of a multiple of 3, with **no unnecessary leading zeroes**$\}$

$Q = \{q_0, q_1, q_2, q_3, q_4\}$

$\Sigma = \{0, 1\}$

δ:

(q, c)	δ (q, c)
$(q_0, 0)$	q_1
$(q_0, 1)$	q_2
$(q_1, 0)$	q_2
$(q_1, 1)$	q_2
$(q_2, 0)$	q_4
$(q_2, 1)$	q_3
$(q_3, 0)$	q_3
$(q_3, 1)$	q_2
$(q_4, 0)$	q_2
$(q_4, 1)$	q_4



$s = q_0$

$F = \{q_1, q_3\}$

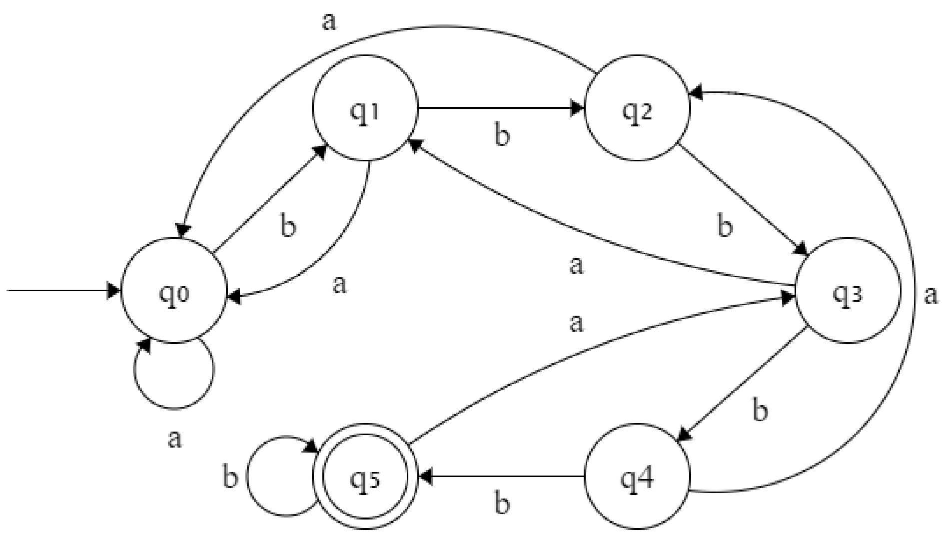
3. $\{x \in \{a, b\}^* \mid x$ contains at least 3 occurrences of the substring $bbb\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$

$\Sigma = \{a, b\}$

δ:

(q, c)	δ (q, c)
(q_0, a)	q_0
(q_0, b)	q_1
(q_1, a)	q_0
(q_1, b)	q_2
(q_2, a)	q_0
(q_2, b)	q_3
(q_3, a)	q_1
(q_3, b)	q_4
(q_4, a)	q_2
(q_4, b)	q_5
(q_5, a)	q_3
(q_5, b)	q_5



$s = q_0$

$F = \{q_5\}$

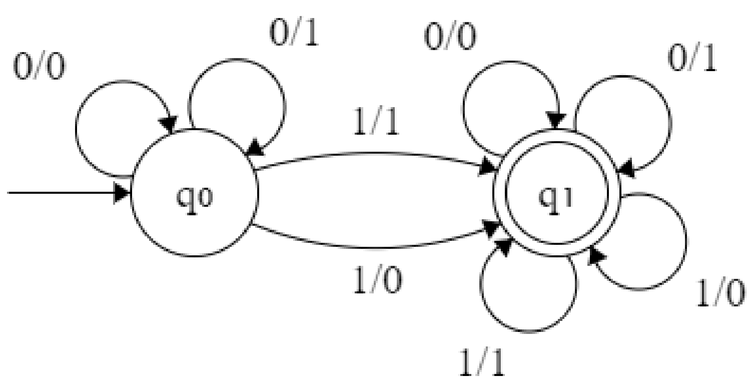
4. $\{x \in \Sigma^* \mid$ the top row of x encodes a larger binary number than the bottom row of $x\}$

$Q = \{q_0, q_1\}$

$\Sigma = \{\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}\}$

δ:

(q, c)	δ (q, c)
$(q_0, \begin{bmatrix}0\\0\end{bmatrix})$	q_0
$(q_0, \begin{bmatrix}0\\1\end{bmatrix})$	q_0
$(q_1, \begin{bmatrix}1\\0\end{bmatrix})$	q_1
$(q_1, \begin{bmatrix}1\\1\end{bmatrix})$	q_1
$(q_2, \begin{bmatrix}0\\0\end{bmatrix})$	q_1
$(q_2, \begin{bmatrix}0\\1\end{bmatrix})$	q_1
$(q_3, \begin{bmatrix}1\\0\end{bmatrix})$	q_1
$(q_3, \begin{bmatrix}1\\1\end{bmatrix})$	q_1



$s = q_0$

$F = \{q_1\}$

Rhea Mae Edwards

5. Let w be a string, and define rev(w) to be its **reversal**.

   We can define the reversal operation rev : $\Sigma^* \rightarrow \Sigma^*$ formally and recursively as:

   rev(ε) = ε

   rev(wb) = $b$rev(w), for $w \in \Sigma^*$ and $b \in \Sigma$

   Using this definition, prove that rev(xy) = rev(y) rev(x), for all x, $y \in \Sigma^*$.

   Hint: Use induction on the length of y.

   <u>Claim</u>:

   rev(xy) = rev(y) rev(x), for all x, $y \in \Sigma^*$

   <u>Proof</u>:

   By induction on the length of y.

   Base Case:

   rev(ε) = ε

   Induction Step:

   rev(wb) = $b$rev(w), for $w \in \Sigma^*$ and $b \in \Sigma$

   rev($x\varepsilon$) = εrev(x) = rev(ε) rec(x) //Define by base case

   rev(y) = $a$rev(z) = rev(az) // Define by inductive step

   rev(azx) = rev(az) rev(x) = a(rev(z)) rev(x) = rec(z) rev(x)

   rev(xy) = rev(y) rev(x) □