Introduction to Statistics for Engineers
Homework 4

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Instructions

- The homework is due on Thursday, May. 26th and must be submitted on Canvas before midnight. (Please read the policies on late homeworks on the syllabus)
- Homeworks must be submitted to Canvas as a Word or PDF document. Any other format (including JPEG) will not be accepted and your homework will be considered late. (Please read the policies on late homeworks on the syllabus)
- You must show your work and provide complete answers in order to receive full credit. Solutions restricted only the final numerical values that do not reflect your statistical reasoning will not receive full credit. The homework is worth 25 points.
- If not using the space assigned for each question, you must clearly indicate the problem that you are working.
- You must include your name and OSU-ID number in your homework document.
- Failing to follow any of these instructions may result in a delay in the grading or a penalization in your final score.

$H_0: \mu = 16oz$

$H_1: \mu \neq 16oz$

$n = 35$

$\bar{x} = 16.32oz$

$\sigma = 0.8oz$ (Known)

1. A production line is supposed to operate with a mean filling weight of 16 ounces per container. Since over/under-filling can be dangerous, a quality control inspector samples 35 items to determine whether or not the filling weight needs to be adjusted. The observed sample mean is 16.32 ounces. Assume, the standard deviation is known to be 0.8 ounces.

   (a) Construct a 95% confidence interval for the mean filling weight of the production line.

   $\bar{x} \pm z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}}$

   95% confidence interval

   $z_{\alpha/2} = z_{0.025} = 1.96$

   $16.32 \pm (1.96) \times \frac{0.8}{\sqrt{35}}$

   ± 0.265

   $\boxed{(16.05, 16.59)}$

   (b) Based on the confidence interval, is it reasonable to believe that the mean filling weight is 16 ounces? Or the production line needs to be adjusted? Explain

   Based on the confidence interval, it is not reasonable to believe the mean filling weight is 16 ounces, because 16 is not a value contained with the interval of (16.05, 16.59). It seems to be that the production line has been over-filling based off the 95% confidence interval calculated above.

   (c) Suppose we want to construct a 95% confidence interval with a margin of error no greater than 0.1. How big the sample size should be to obtain such interval?

   $n = \frac{\sigma^2 z_{\alpha/2}^2}{m^2}$

   $n = \frac{(0.8)^2 (1.96)^2}{(0.1)^2} = 245.86$

   $m = 0.1$

   $z_{\alpha/2} = 1.96$

   $\sigma = 0.8oz$

   $\boxed{n = 246 \text{ items}}$

2. Data on stress limits (measured in Mpa) for specimens constructed using two different types of wood are summarized in the following table:

Type of wood	Sample size	Sample mean (Mpa)	Sample St. Dev.
Red Oak	14	8.50	0.80
Douglas Fir	10	6.65	1.28

(a) Construct a 95% confidence interval for the difference of the mean stress limits between Red Oak and Douglas Fir. $\sqrt{S_p^2} = \sqrt{\dfrac{(14-1)(0.8)^2 + (10-1)(1.28)^2}{14+10-2}} = 1.024$

$\sigma_1 = \sigma_2 ?$

$\frac{1}{2} < \frac{S_1}{S_2} < 2$,

considered $\sigma_1 = \sigma_2$

$\frac{S_1}{S_2} = \frac{0.80}{1.28} = 0.625$

since $\frac{1}{2} < 0.625 < 2$,

then $\sigma_1 = \sigma_2$

$1.85 \pm (2.07) \times (1.02) \sqrt{\frac{1}{14} + \frac{1}{10}}$

± 0.879

$\boxed{(0.971, 2.73)}$

so...

$\bar{x} - \bar{y} \pm t_{n_1+n_2-2, \alpha/2} \times S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$, $S_p^2 = \dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$

↳ 1.85 ↳ $t_{22, 0.025} = 2.074$

(b) Based on the confidence interval can you conclude that either type of wood has a greater mean stress limit? Explain.

Based on the confidence interval, we can conclude that either/a type of wood has a greater mean stress limit, because a mean difference of the value zero is not contained within the 95% confidence interval calculated above of (0.971, 2.73), which indicates there is difference of some sort based off the interval, saying that Red Oak has a greater mean stress limit than Douglas Fir based off this process with their calculated difference.

(c) Does your answer in part (b) changes if you construct a 90% confidence interval?

$t_{22, 0.05} = \text{invT}(0.95, 22) = 1.717$

↳ Calculated with TI-84 Plus Calculator

$1.85 \pm (1.717) \times (1.02) \sqrt{\frac{1}{14} + \frac{1}{10}}$

± 0.728

$(1.12, 2.58)$

My answer in part (b) doesn't change, because the value zero is still not contained within the interval, and by decreasing the confidence level, the interval just narrows its range from the 95% confidence interval, which didn't contained zero initially.

3. A manufacturer of shock absorbers is comparing the durability of one of his models with those of his competitors. To conduct the experiment he installs one of his shock absorbers and one of the competitors absorbers on each one of ten pairs of cars selected at random. Each car was then driven for 20,000 miles. After that, each shock absorber was measured for strength. The results are summarized in the table below.

Car	1	2	3	4	5	6	7	8	9	10
Manufacturer	10.0	11.7	13.7	9.9	9.8	14.4	15.1	10.6	9.8	12.1
Competitor	9.6	11.9	13.1	9.4	10.0	14.0	14.6	10.8	9.4	12.3

(a) What type of confidence interval would you use to compare the strengths of the shock absorbers in this situation? Explain.

Would use a confidence interval for the mean difference in a matching pair case, because all conditions of the experiment done by the manufacturer were the same for each paired shock absorbers, one of each shock absorber was placed in the same car being able to compare each pair and all pairs overall sample mean difference with this sample/situation.

(b) Construct a 99% confidence interval following the procedure you chose in part (a)

$\bar{d} \pm t_{n-1,\alpha/2} \frac{S_d}{\sqrt{n}}$

$0.20 \pm (3.25)\left(\frac{0.67}{\sqrt{10}}\right)$

± 0.685

$\boxed{(-0.485, 0.885)}$

$\bar{x}_1 = 11.71$

$\bar{x}_2 = 11.51$

$t_{9,0.005} = \text{invT}(0.995, 9) = 3.25$

↳ calculated with TI-89 Plus Calculator

$\bar{d} = (\bar{x}_1 - \bar{x}_2) = 0.20$

$s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}} = \sqrt{\frac{(-2)^2}{9}} = 0.67$

(c) Based on your confidence interval, can we conclude that the manufacturer's shock absorbers are better than the competitors'? Explain

Based on the 99% confidence interval calculated above of (-0.485, 0.885), we can conclude that the manufacturers shock absorbers are not better than the competitors, because there's the possibility that the mean difference is zero, meaning there's no difference between the two shock absorbers types tested, since the value zero is contained within the interval calculated above.

$H_1: \mu > 1$
$H_0: \mu = 1$
$\sigma = 0.11$
$n = 10$
$\bar{x} = 1.07$
$\alpha = 0.05$

4. A company wishes to detect an increase in the thickness of the silicon oxide layers because thicker layers require longer etching times. Process specifications state a target value of 1 micron for the true mean thickness. Historically, the layer thickness has a standard deviation of 0.11 micron. A recent random sample of 10 wafers yielded a sample mean of 1.07 micron. Conduct a hypothesis test to determine whether the true mean thickness has increased. Use a significance level $\alpha = 0.05$.

(a) State the null and alternative hypothesis for this test.

Null Hypothesis $H_0: \mu = 1$ micron

Alternative Hypothesis $H_1: \mu > 1$ micron

(b) Compute the appropriate test statistic for this test.

Test Statistic for testing against the population mean $H_0: \mu = \mu_0$

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{1.07 - 1}{0.11/\sqrt{10}}$$

· Population standard deviation known

$$\boxed{z = 2.012}$$

(c) Obtain the P-value for this test. What are your conclusions based on the significance level of the test?

$z = 2.012 \rightarrow 0.9778 \rightarrow$ P-value $= 1 - 0.9778 = \underline{0.0222}$
using the z-table

Since the P-value is less than $\alpha = 0.05$ being $0.0222 < 0.05$, there is evidence that we can reject $H_0: \mu = 1$, and conclude (based on the data) that there is a detected increase in the thickness of the silicon oxide layers.