

Week 7 Problem Set:

Section 6.1: 2, 3, 4, 8, 9, 11, 14, 21, 34, 35, 41, 47, 65

#2

$$27 \cdot 37 = 999 \text{ offices}$$

#3

- a) $4^{10} = 1048576$ ways
- b) $5^{10} = 9765625$ ways

#4

$$12 \cdot 2 \cdot 3 = 72 \text{ types}$$

#8

$$26 \cdot 25 \cdot 24 = 15600 \text{ combinations}$$

#9

$$1 \cdot 26 \cdot 26 = 676 \text{ combinations}$$

#11

$$1 \cdot 2^8 \cdot 1 = 256 \text{ bit strings}$$

#14

$$1 \cdot 2^n \cdot 1 = 2^n$$

#21

- a) (Floor Function) $50 / 7 = 7$ integers {56, 63, 70, 77, 84, 91, 98}
- b) (Ceiling Function) $50/11 = 5$ integers {55, 66, 77, 88, 99}
- c) 1 integer {77}

#34

- a) $2^{10} = 1024$ functions
- b) $3^{10} = 59049$ functions
- c) $4^{10} = 1048576$ functions
- d) $5^{10} = 9765625$ functions

#35

- a) $P(4, 5) = 4! / (4 - 5)! =$ Since the codomain is smaller than the domain, there is no one-to-one function possible.
- b) $P(5, 5) = 5! / (5 - 5)! = 120$ one-to-one functions
- c) $P(6, 5) = 6! / (6 - 5)! = 720$ one-to-one functions
- d) $P(7, 5) = 7! / (7 - 5)! = 2520$ one-to-one functions

#41

If n is even $= 2^{n/2}$ bit strings

If n is odd $= 2^{(n+1)/2}$ bit strings

#47

- a) $2 \cdot 5! = 240$ ways
- b) $6! - 5! = 480$ ways
- c) $5! + (4 \cdot 4!) + (3 \cdot 4!) + (2 \cdot 4!) + (1 \cdot 4!) = 360$ ways

#65

$$4! - (3 \cdot 2) = 18 \text{ ways}$$

Section 6.3: 1, 3, 6, 7, 11, 17, 19, 24, 28, 30, 39, 40, 41

#1

abc, acb, bac, bca, cab, cba

#3

$$6! = 720 \text{ permutations}$$

#6

- a) $C(5, 1) = P(5, 1) / P(1, 1) = 5! / 1! (5 - 1)! = 5$
- b) $C(5, 3) = P(5, 3) / P(3, 3) = 5! / 3! (5 - 3)! = 10$
- c) $C(8, 4) = P(8, 4) / P(4, 4) = 8! / 4! (8 - 4)! = 70$
- d) $C(8, 5) = P(8, 5) / P(5, 5) = 8! / 5! (8 - 5)! = 56$
- e) $C(8, 0) = P(8, 0) / P(0, 0) = 8! / 0! (8 - 0)! = 1$
- f) $C(12, 6) = P(12, 6) / P(6, 6) = 12! / 6! (12 - 6)! = 924$

#7

$$P(9, 5) = 9! / (9 - 5)! = 15120$$

#11

- a) $C(10, 4) = 210$ bit strings
- b) $C(10, 0) + C(10, 1) + C(10, 2) + C(10, 3) + C(10, 4) = 386$ bit strings
- c) $C(10, 4) + C(10, 5) + C(10, 6) + C(10, 7) + C(10, 8) + C(10, 9) + C(10, 10) = 848$ bit strings
- d) $C(10, 5) = 252$ bit strings

#17

$$(2^{100} - 1) - (C(100, 1) + C(100, 2)) = 2^{100} - 5051 \text{ subsets}$$

#19

- a) $2^{10} = 1024$ outcomes
- b) $(1/2)^{10} \cdot P(10, 2) = 45$ outcomes
- c) $((1/2)^{10} \cdot 10!)((3! \cdot 7!) + (2! \cdot 8!) + (1! \cdot 9!) + (0! \cdot 10!)) = 176$ outcomes
- d) $(1/2)^{10} \cdot P(10, 5) = 252$ outcomes

#24

$$P(10, 10) \cdot P(11, 6) \text{ ways}$$

#28

$$C(40, 17) = 40! / 17! (40 - 17)! = 88732378800 \text{ answer keys}$$

#30

- a) $C(16, 7) \cdot C(11, 5) = (16! / 7! (16 - 7)!) \cdot (11! / 5! (11 - 5)!) = 5285280$ ways
- b) $C(16, 7) \cdot C(16, 9) \cdot C(11, 5) = (16! / 7! (16 - 7)!) \cdot (16! / 9! (16 - 9)!) \cdot (11! / 5! (11 - 5)!) = 423245222400$ ways

#39

$$10^3 \cdot C(26, 3) + 26^3 \cdot C(10, 3) - C(26, 3) \cdot C(10, 3) = 17022720 \text{ license plates}$$

#40

$$5! / (3 (2!)) = 20 \text{ circular 3-permutations}$$

#41
 $n! / (r (n - r)!)$

Section 6.5: 7, 8, 13, 15, 17, 18, 25, 35, 53

#7
 $(5 + 3 - 1)! / 3! (5 - 1)! = 35$ ways

#8

#13
 $(3000 + 3 - 1)! / 3! (3000 - 1)! = 4504501$ ways

#15
a) 10626 solutions
b) 1365 solutions
c) 11649 solutions
d) 106 solutions

#17
2520 strings

#18

#25
30492 positive integers

#35
19635 strings

#53
65 ways