

CS 321: Homework #4

$$1. \overline{L(R)} = L((ab + aab)^* b)$$

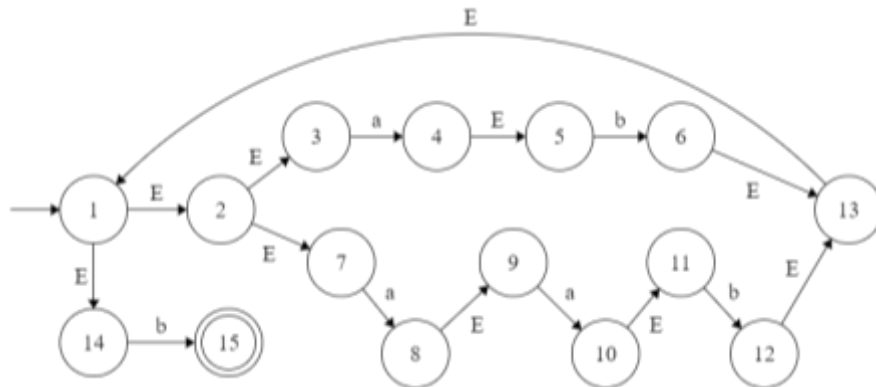
Process:

Regular Expression \rightarrow NFA \rightarrow DFA \rightarrow Modify DFA for Complement \rightarrow NFA \rightarrow Regular Expression

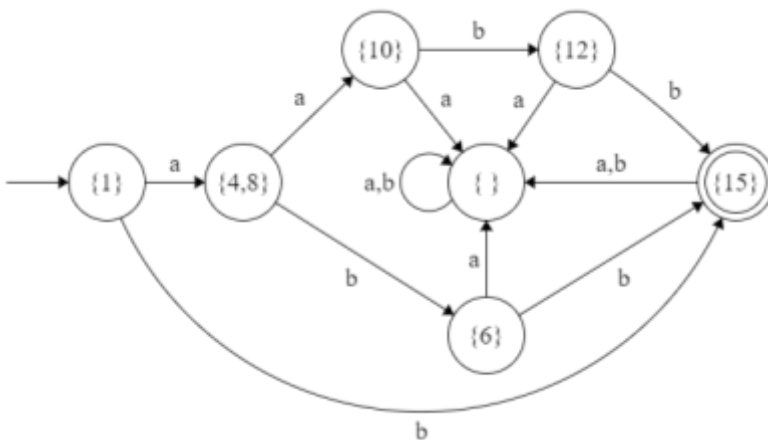
Regular Expression:

$$L((ab + aab)^* b)$$

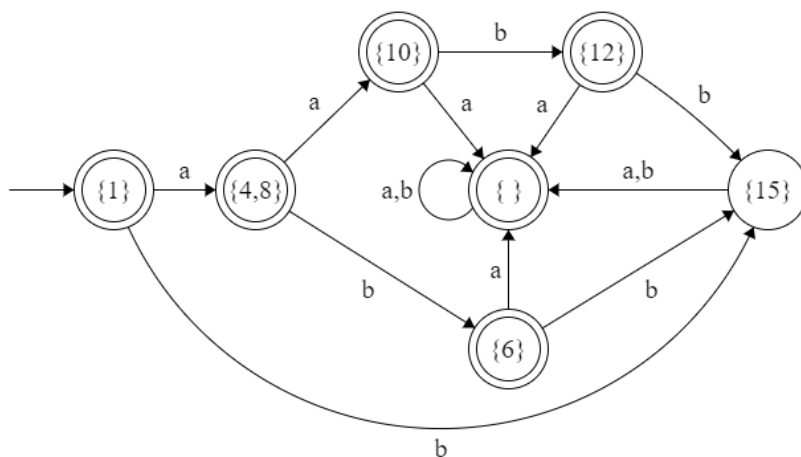
NFA:



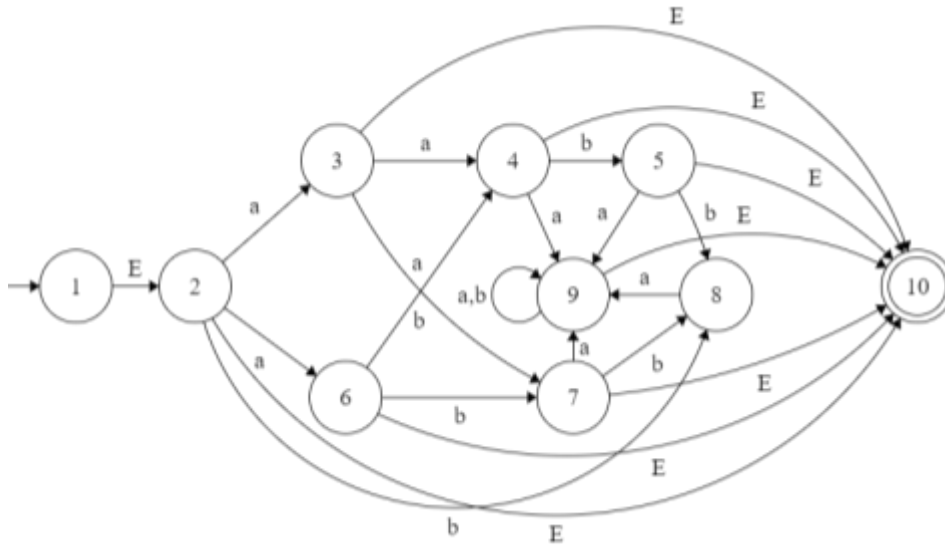
DFA:



DFA's Complement:



NFA:



Regular Expression:

$$L(R) = L(\epsilon + a + aa + ab + aab + ba(a+b)^* + aaa(a+b)^* + aba(a+b)^* + aaba(a+b)^* + abba(a+b)^* + aabba(a+b)^*)$$

$$2. \epsilon + a(ba^*b + ba)^* b$$

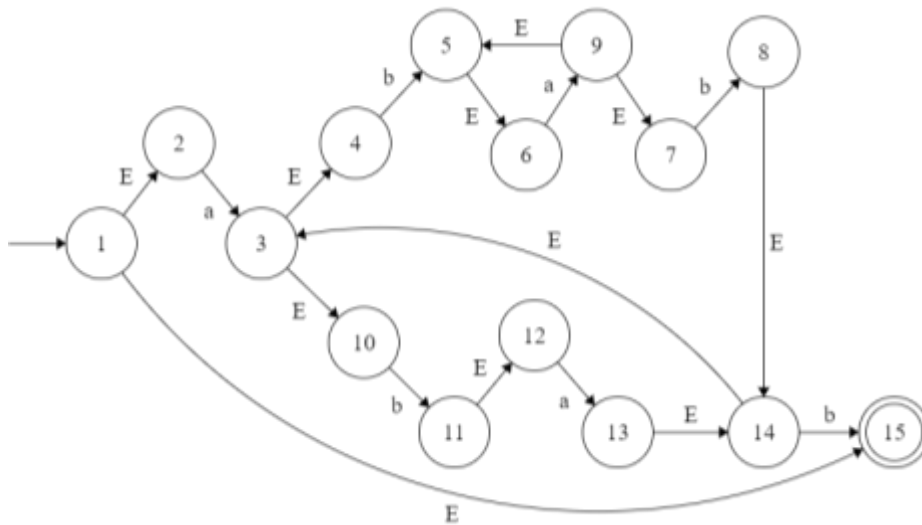
Process:

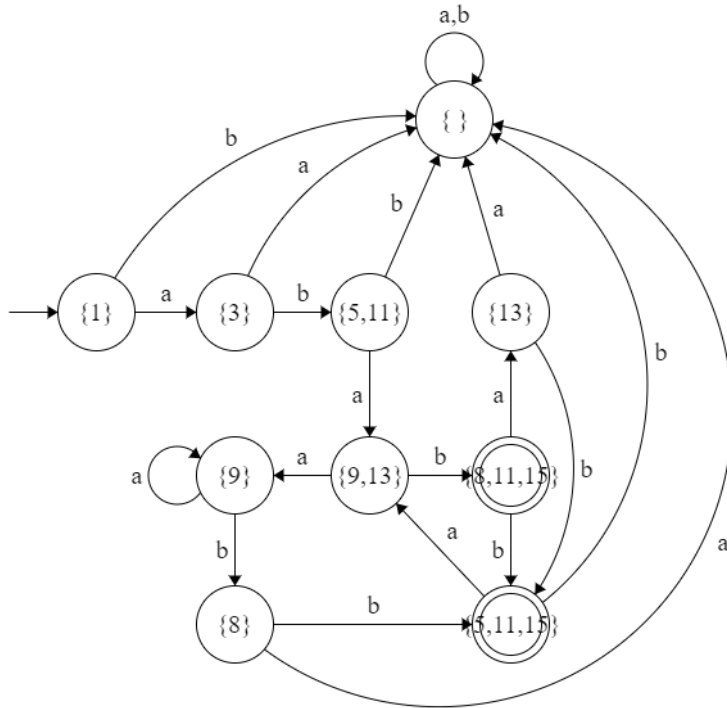
Regular Expression \rightarrow NFA \rightarrow DFA

Regular Expression:

$$\epsilon + a(ba^*b + ba)^* b$$

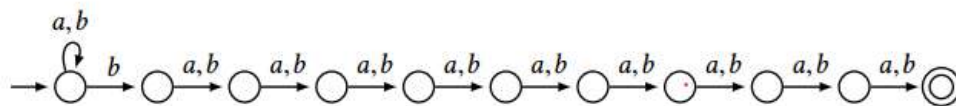
NFA:



DFA:

3.

$$A = \{w \in \{a,b\}^* \mid \text{10th character from the end of } w \text{ is } b\}$$



Prove that DFA M has $L(M) = A$ has at least 1024 states.

Hint: Prove the contrapositive: Suppose M has fewer than 1024 states, then show that $L(M)$ disagrees with A (show a string in A that M rejects, or a string not in A that M accepts).

Consider $\delta^*(s, w)$ for all strings w of length 10.

Proof:

[Unable to fully complete this problem before the homework deadline...]

4. Prove $\{a^n b^m c^{n-m} \mid n, m \in \mathbb{N} \text{ and } n \geq m\}$ is not regular.

Proof: By the Pumping Lemma game...

1) Adversary picks $p \geq 0$.

2) I choose $w = a^p b^{p-1} c^{p-(p-1)} = a^p b^{p-1} c$

Satisfies: $w \in A$ and $|w| = 2p - 1 \geq p$

3) Adversary chops w into xyz .

Since $|xy| \leq p$, y contains only a 's.

4) I choose $i = 0$.

$x y^i z = xz$ has $p - |y|$ a 's

and $p - 1$ b 's

and 1 c

since $p - |y| \leq p - 1$

$x y^i z$ is not an element in A , which shows that $\{a^n b^m c^{n-m} \mid n, m \in \mathbb{N} \text{ and } n \geq m\}$ is not regular.