

## Week 6 Problem Set:

### Section 9.3: 1, 5, 6, 7, 9, 16, 17

#1

a. 
$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

b. 
$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

c. 
$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

d. 
$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

#5

The relation is irreflexive if and only if the main diagonal of the matrix contains only 0s.

#6

The relation is asymmetric if and only if  $m_{ij} = 1$  with  $i \neq j$ , then  $m_{ji} = 0$ , or vice versa.

#7

- a. Reflexive, Symmetric, Transitive
- b. Antisymmetric, Transitive
- c. Symmetric

#9

- a. 4950
- b. 9900
- c. 99
- d. 100
- e. 1

#16

$k$  nonzero entries

#17

$$n^2 - k$$

**Section 2.3: 7, 12, 13, 14, 21, 22, 33, 42**

#7

- a. Domain: Positive Integers  
Range:  $[0, \infty)$
- b. Domain: Positive Integers  
Range:  $[0, 9]$
- c. Domain: Set of Bit Strings  
Range:  $\mathbb{N}$
- d. Domain: Set of Bit Strings  
Range:  $\mathbb{N}$

#12

- a. One-To-One
- b. Not One-To-One
- c. One-To-One
- d. One-To-One

#13

(a) and (d)

#14

- a. Onto
- b. Not Onto
- c. Onto
- d. Onto
- e. Not Onto

#21

- a.  $f(x) = 3x + 1$  when  $x \geq 0$
- b.  $f(x) = |x| + 1$
- c.  $f(x) = 2x + 1$  when  $x \geq 0$
- d.  $f(x) = x^2 + 1$

#22

- a. Bijection
- b. Not Bijection
- c. Bijection
- d. Not Bijection

#33

- a. Let  $x$  and  $y$  be distinct elements of  $A$ . Because  $g$  is one-to-one,  $g(x)$  and  $g(y)$  are distinct elements of  $B$ . Because  $f$  is one-to-one,  $f(g(x)) = (f \circ g)(x)$  and  $f(g(y)) = (f \circ g)(y)$  are distinct elements of  $C$ . Hence,  $f \circ g$  is one-to-one.
- b. Let  $y \in C$ . Because  $f$  is onto,  $y = f(b)$  for some  $b \in B$ . Not because  $g$  is onto,  $b = g(x)$  for some  $x \in A$ . Hence,  $y = f(b) = f(g(x)) = (f \circ g)(x)$ . It follows that  $f \circ g$  is onto.

#42

- a.  $\{1\}$
- b.  $0 < x < 1$
- c.  $x > 16$

**Section 2.6: 3, 4 (Matrix Multiplication Problems)**

#3

a.  $\begin{pmatrix} 1 & 11 \\ 2 & 18 \end{pmatrix}$

b.  $\begin{pmatrix} 2 & -2 & -3 \\ 1 & 0 & 2 \\ 9 & -4 & 4 \end{pmatrix}$

c.  $\begin{pmatrix} -4 & 15 & -4 & 1 \\ -3 & 10 & 2 & -3 \\ 0 & 2 & -8 & 6 \\ 1 & -8 & 18 & -13 \end{pmatrix}$

#4

a.  $\begin{pmatrix} -1 & 1 & -1 \\ 0 & 1 & -1 \\ 1 & -2 & 1 \end{pmatrix}$

b.  $\begin{pmatrix} 4 & -1 & -7 & 6 \\ -7 & -5 & 8 & 5 \\ 4 & 0 & 7 & 3 \end{pmatrix}$

c.  $\begin{pmatrix} 2 & 0 & -3 & -4 & -1 \\ 26 & -7 & 20 & 29 & 2 \\ -10 & 4 & -17 & -24 & -3 \end{pmatrix}$