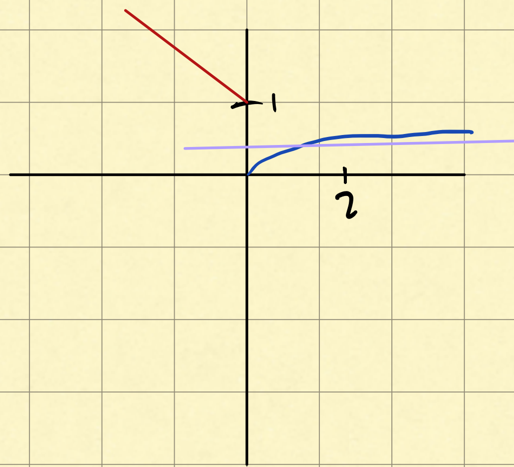
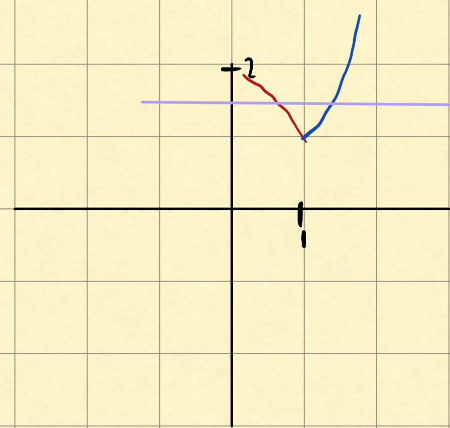
7.1

9. $f(x) = \begin{cases} 1 - \frac{x}{2}, & x \le 0 \\ \frac{x}{x+2}, & x > 0 \end{cases}$



it is one to one

10. $f(x) = \begin{cases} 2 - x^2, & x \le 1 \\ x^2, & x > 1 \end{cases}$



not one to one

32. $f(x) = \dfrac{\sqrt{x}}{\sqrt{x} - 3}$

Domain: $(-\infty, 0] \cup (1, +\infty)$

Range: $[0, 9) \cup (9, \infty)$

$$x = \frac{\sqrt{y}}{\sqrt{y} - 3}$$

$$x(\sqrt{y} - 3) = \sqrt{y}$$

$$x\sqrt{y} - 3x = \sqrt{y}$$

$$x \cdot \sqrt{y} - 3x = \sqrt{y}$$

$$\sqrt{y}(x-1) = 3x$$

$$\sqrt{y} = \frac{3x}{x-1} \longrightarrow y = \left(\frac{3x}{x-1}\right)^2$$

33. $f(x) = x^2 - 2x \ , \ x \leq 1$

let $f^{-1}(y) = x$

Domain: $[-1, \infty)$

Range : $(-\infty, 1]$

$y = x^2 - 2x$

$y = x^2 - 2x + 1 - 1$

$y = (x-1)^2 - 1$

$y+1 = (x-1)^2$

$\sqrt{y+1} = x-1$

$\sqrt{y+1} + 1 = x$

$f^{-1}(y) = 1 + \sqrt{y+1}$

$f^{-1}(x) = 1 + \sqrt{x+1}$

41. $y = x^3 - 3x^2 - 1 \ , \ x \geq 2$

$x = y^3 - 3y^2 - 1 \ , \ y \geq 2$

at $x = -1 \rightarrow y = 0$ or 3 but y should be 3

$1 = 3y^2 y' - 6yy'$

$y' = \frac{1}{9}$ at $x = -1$

for given function $y(3) = -9$, therefore df^{-1}/dx

$= \frac{1}{f'(x)}$ at $x = -1$

42. $y = x^2 - 4x - 5$, $x > 2$

$x^2 - 4x - 5 - y = 0$

$x^2 - 4x - (5+y) = 0$

$x = \frac{4 \pm \sqrt{16 + 4(5+y)}}{2}$

Because $x > 2$, we take the positive root.

$$x = \frac{4 + \sqrt{16 + 4(5+y)}}{2}$$

$$= \frac{4 + \sqrt{4(4+(5+y))}}{2}$$

$$= \frac{\cancel{4} + \cancel{2}\sqrt{4+(5+y)}}{\cancel{2}}$$

replace x with y

$$y = 2 + \sqrt{9+y}$$

$$f^{-1}(x) = 2 + \sqrt{9+x}$$

$$\frac{df^{-1}}{dx} = \frac{1}{2+\sqrt{9+x}}$$

$$x = 0 = f(5)$$

$$\frac{df^{-1}}{dx} = \frac{1}{2\sqrt{9+5}} = \frac{1}{2\sqrt{14}}$$

43. $y = f(x)$, f passes through point $(2,4)$, slope $= \frac{1}{3}$, $x = 9$

$$y - 4 = \frac{1}{3}(x - 2)$$

$$\left(y = \frac{1}{3}x - \frac{2}{3} + 4\right)3$$

$$3y = x - 2 + 12$$

$$3y = x + 10$$

$$3y - 10 = x$$

$$f^{-1}(x) = 3x - 10$$

$$\frac{d}{dx} = 3x - 10$$

$$= 3$$

57. range of g lies in the domain of f.

f is one to one, g is one to one

$x_1 \neq x_2$, $g(x_1) \neq g(x_2)$

$$f(g(x_1)) \neq f(g(x_2))$$

So, $f \circ g$ is also one to one

58. Yes, g must be one to one since, if we say that g is not one to one, and we say that x_1 & x_2 belongs to the domain of $f \circ g$:

Note that $x_1 \neq x_2$, $g(x_1) = g(x_2) = y$

$\to (f \circ g)(x_1) = f(g(x_1)) = f(y)$

$\to (f \circ g)(x_2) = f(g(x_2)) = f(y)$

$\to (f \circ g)(x_1) = (f \circ g)(x_2)$

we can see that both of both of them are the same. Therefore the fact that g is not one to one is wrong

7-2

2. a) $\ln \frac{1}{125} = \ln 5^{-3} = -3\ln 5$

b) $\ln 9.8 = \ln \frac{49}{5}$

$= \ln 49 - \ln 5$

$= \ln 7^2 - \ln 5$

$= 2\ln 7 - \ln 5$

c) $\ln 7\sqrt{7} = \ln 7(7)^{\frac{1}{2}} = \ln 7^{\frac{3}{2}}$

$= \frac{3}{2}\ln 7$

d) $\ln 1225 = \ln 25 \times 49$

$= \ln 5^2 + \ln 7^2$

$= 2\ln 5 + 2\ln 7$

e) $\ln 0.056 = \ln 7 \times 0.008$

$= \ln 7 + \ln 5^{-3}$

$= \ln 7 - 3\ln 5$

f) $(\ln 35 + \ln \frac{1}{7}) / \ln 25 = \frac{\ln 7 \times 5 + \ln 7^{-1}}{\ln 5^2} = \frac{\ln 7 + \ln 5 - \ln 7}{2\ln 5}$

$= \frac{\ln 5}{2\ln 5} = \frac{1}{2}$

29. $y = \ln(\ln(\ln x))$

$$\frac{dy}{dx} = \frac{1}{\ln(\ln x)} \cdot \frac{1}{\ln x} \cdot \frac{1}{x}$$

$$= \frac{1}{\ln(\ln x) \cdot \ln x \cdot x}$$

35. $y = \int_{x^2/2}^{x^2} \ln \sqrt{t}\, dt$

$$= \int_0^{x^2} \ln \sqrt{t}\, dt - \int_0^{\frac{x^2}{2}} \ln \sqrt{t}\, dt$$

$$\frac{dy}{dx} = 2x \ln|x| - x \ln \frac{|x|}{\sqrt{2}}$$

45. $\int_2^4 \frac{dx}{x(\ln x)^2}$

let $\ln x = u$

$$\frac{du}{dx} = \frac{1}{x}$$

$$dx = x \cdot du$$

$$= \int_2^4 \frac{1}{x(\ln x)^2}\, dx$$

$$= \int_2^4 \frac{\cancel{x}}{\cancel{x}(u)^2}\, du$$

$$= \int_2^4 \frac{1}{u^2}\, du \longrightarrow = -u^{-1}\Big|_2^4 = -\ln x^{-1}\Big|_2^4 = -\frac{1}{\ln 4} + \frac{1}{\ln 2}$$

53. $\int \frac{dx}{2\sqrt{x}+2x}$

$= \int \frac{1}{2\sqrt{x}+2x}\, dx$

$= \int \frac{1}{2\sqrt{x}(1+\sqrt{x})}\, dx$ Let $u = 1+\sqrt{x}$

$\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

$dx = \frac{du}{\frac{1}{2}x^{-\frac{1}{2}}}$

$= \int \frac{1}{\cancel{2}\cancel{\sqrt{x}}(u)\cdot\cancel{\frac{1}{2}x^{-\frac{1}{2}}}}\, du$

$= \int \frac{1}{u}\, du \longrightarrow = \ln u + C$

$= \ln(1+\sqrt{x}) + C$

54. $\int \frac{\sec x\, dx}{\sqrt{\ln(\sec x+\tan x)}}$

$= \int \frac{\cancel{\sec x}}{\sqrt{u}\cdot\cancel{\sec x}}\, du$ Let $u = \ln(\sec x + \tan x)$

$\frac{du}{dx} = \sec x$

$= \int \frac{1}{\sqrt{u}}\, du$

$dx = \frac{du}{\sec x}$

$$= 2u^{\frac{1}{2}} = 2(\ln(\sec x + \tan x))^{\frac{1}{2}} + C$$

86. $y = \sqrt{(x^2+1)(x-1)^2}$

$$\ln y = \ln((x^2+1)(x-1)^2)^{\frac{1}{2}}$$

$$\frac{d}{dx}\ln y = \ln(x^2+1)^{\frac{1}{2}} + \ln(x-1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\ \frac{1}{2}\ln(x^2+1) + \ln(x-1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{\cancel{2}x}{\cancel{2}(x^2+1)} + \frac{1}{x-1}$$

$$\frac{dy}{dx} = y\left(\frac{x}{x^2+1} + \frac{1}{x-1}\right)$$

$$= \sqrt{(x^2+1)(x-1)^2}\left(\frac{x}{x^2+1} + \frac{1}{x-1}\right)$$

65. $y = \dfrac{x\sqrt{x^2+1}}{(x+1)^{2/3}}$

$$\ln y = \ln \frac{x\sqrt{x^2+1}}{(x+1)^{2/3}}$$

$$= \ln x\sqrt{x^2+1} - \ln(x+1)^{2/3}$$

$$= \ln x + \ln\sqrt{x^2+1} - \ln(x+1)^{2/3}$$

$$\frac{1}{y}\frac{dy}{dx} = \ln x + \frac{1}{2}\ln(x^2+1) - \frac{2}{3}\ln(x+1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\ln x + \frac{1}{2}\ln(x^2+1) - \frac{2}{3}\ln(x+1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{\cancel{2}x}{\cancel{2}(x^2+1)} - \frac{2}{3(x+1)}$$

$$\frac{dy}{dx} = y\left(\frac{1}{x} + \frac{x}{(x^2+1)} - \frac{2}{3(x+1)}\right)$$

$$\frac{dy}{dx} = \frac{x\sqrt{x^2+1}}{(x+1)^{2/3}}\left(\frac{1}{x} + \frac{x}{(x^2+1)} - \frac{2}{3(x+1)}\right)$$

67. $y = \sqrt[3]{\frac{x(x-2)}{x^2+1}}$

$$= \frac{(x(x-2))^{\frac{1}{3}}}{(x^2+1)^{\frac{1}{3}}}$$

$$= (x(x-2))^{\frac{1}{3}} \cdot (x^2+1)^{-\frac{1}{3}}$$

$$\ln y = \ln (x(x-2))^{\frac{1}{3}} \cdot (x^2+1)^{-\frac{1}{3}}$$

$$= \frac{1}{3} \ln (x(x-2)) - \frac{1}{3} \ln (x^2+1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx} \ \frac{1}{3} \ln (x^2-2x) - \frac{1}{3} \ln (x^2+1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2x-2}{3(x^2-2x)} - \frac{2x}{3(x^2+1)}$$

$$\frac{dy}{dx} = y\left(\frac{2x-2}{3(x^2-2x)} - \frac{2x}{3(x^2+1)}\right)$$

$$= y\left(\frac{2}{3}\left(\frac{x-1}{x^2-2x}\right) - \frac{2}{3}\left(\frac{x}{x^2+1}\right)\right)$$

$$\frac{dy}{dx} = \frac{1}{3}\sqrt[3]{\frac{x(x-2)}{x^2+1}}\left(\frac{1}{x} + \frac{1}{x-2} - \frac{2}{3(x+1)}\right)$$

70. a) $f(x) = x - \ln x$

$f'(x) > 0$

$1 - \frac{1}{x} > 0$

$\frac{x-1}{x} > 0$

$x > 1$

b) $\ln x < x$ if $x > 1$

$\ln x < x$

$(\ln x - x) < 0$

$\frac{d}{dx}(\ln x - x) < 0$

$\frac{1}{x} - 1 < 0$

$\frac{1}{x} < 1$

$x > 1$

77. a) $y = (x^2/8) - \ln x \quad , \quad 4 \le x \le 8$

$$\int_4^8 \sqrt{1 + (f'(x))^2}$$

$$\frac{dy}{dx} = \frac{x^2 - 4}{4x}$$

$$= 1 + \left(\frac{x^2 - 4}{4x} \right)^2$$

$$= 1 + \frac{x^4 - 8x^2 + 16}{16x^2}$$

$$= \frac{8x^2 + x^4 + 16}{16x^2}$$

$$L = \int_4^8 \sqrt{\frac{8x^2 + x^4 + 16}{16x^2}}$$

$$= 6 + \ln 2$$

b. $x = (y/4)^2 - 2\ln(y/4)$, $4 \leq y \leq 12$

$$\frac{d}{dy} = \frac{y^2 - 16}{8y}$$

$$= 1 + \left(\frac{y^2 - 16}{8y}\right)^2$$

$$= 1 + \frac{y^4 - 32y^2 + 256}{64y^2}$$

$$= \frac{32y^2 + y^4 + 256}{64y^2}$$

$$= \int_4^{12} \sqrt{\frac{32y^2 + y^4 + 256}{64y^2}}$$

$$= 8 + \ln 9$$

7.6

13. $\lim_{x \to 1^-} \sin^{-1} x = \sin^{-1}(1^-)$

$= \frac{\pi}{2}$

22. $y = \cos^{-1}(1/x)$

$= \sec^{-1} x$

$\frac{d \cdot \sec^{-1}(x)}{dx} = \frac{1}{|x| \cdot \sqrt{x^2-1}}$

41. $\int \frac{dx}{x\sqrt{5x^2-4}}$

Let $u = \sqrt{5x^2}$

$u = x\sqrt{5}$

$\frac{u}{\sqrt{5}} = x$

$\frac{du}{dx} = \sqrt{5}$

$\frac{du}{\sqrt{5}} = dx$

let $a^2 = 4$

$a = 2$

$= \int \frac{\frac{du}{\cancel{\sqrt{5}}}}{\frac{u}{\cancel{\sqrt{5}}}\sqrt{u^2-a^2}}$

$= \int \frac{du}{u\sqrt{u^2-a^2}}$

$= \frac{1}{a} \sec^{-1} \left|\frac{u}{a}\right| + C$

$= \frac{1}{2} \sec^{-1} \left|\frac{\sqrt{5}x}{2}\right| + C$

51. $\int_0^2 \frac{dt}{8+2t^2}$

Let $a^2=8$

$a=2\sqrt{2}$

Let $u^2=2t^2$

$u=\sqrt{2}\,t$

$\frac{du}{dt}=\sqrt{2}$

$du=\sqrt{2}\,dt$

$\frac{du}{\sqrt{2}}=dt$

$$=\int_0^2 \frac{dt}{8+2t^2}$$

$$=\int \frac{\frac{dy}{\sqrt{2}}}{a^2+u^2}$$

$$=\frac{1}{a}\cdot\frac{1}{\sqrt{2}}\cdot\frac{du}{a^2+u^2}$$

$$=\tan^{-1}(1)=\frac{\pi}{4}$$

$$=\frac{1}{2\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\cancel{\sqrt{2}}t}{2\cancel{\sqrt{2}}}\right)\Bigg|_0^2$$

$$=\frac{1}{2\cdot 2}\tan^{-1}\left(\frac{t}{2}\right)\Bigg|_0^2$$

$$=\frac{1}{4}\cdot\frac{\pi}{4}-\frac{1}{4}\cdot 0$$

$$=\frac{\pi}{16}$$

65. $\displaystyle\int \frac{y\,dy}{\sqrt{1-y^4}}$

Let $u^2 = y^4$

$u = y^2$

$\frac{du}{dy} = 2y$

$\frac{\frac{du}{2}}{dy} = y\,dy$

Let $a^2 = 1$

$a = 1$

$= \displaystyle\int \frac{\frac{du}{2}}{\sqrt{a^2-u^2}}$

$= \frac{1}{2}\displaystyle\int \frac{du}{\sqrt{a^2-u^2}}$

$= \frac{1}{2} \cdot \sin^{-1} y^2 + C$

70. $\displaystyle\int_{1/2}^{1} \frac{6\,dt}{\sqrt{3+4t-4t^2}}$

$C = \left(-\frac{1}{2}\right)^2 = \frac{1}{4}$

$4t - 4t^2 = 4(t - t^2)$

$= 4\left(t - t^2 + \frac{1}{4} - \frac{1}{4}\right)$

$= 4t - 4t^2 + 1 - 1$

$= -(4t^2 - 4t + 1) + 1$

$$= \int_{1/2}^{1} \frac{6\,dt}{\sqrt{3-(4t^2-4t+1)+1}}$$

$$= \int_{1/2}^{1} \frac{6\,dt}{\sqrt{4-(2t-1)^2}}$$

$$= \int_{1/2}^{1} \frac{6\,dt}{\sqrt{2^2(2t-1)^2}}$$

Let $a^2 = 4$ Let $u = 2t-1$

$a = 2$ $du = 2\,dt$

$\frac{du}{2} = dt$

$$= \int_{1/2}^{1} \frac{{}^{3}\cancel{6} \cdot \frac{du}{\cancel{2}}}{\sqrt{a^2-u^2}}$$

$$= 3\int \frac{du}{\sqrt{a^2-u^2}}$$

$$= 3 \cdot \sin^{-1}\left(\frac{u}{a}\right)$$

$$= 3 \cdot \sin^{-1}\left(\frac{2t-1}{2}\right)\Big|_{1/2}^{1}$$

$$= 3 \cdot \sin^{-1}\left(\frac{1}{2}\right) - 3\sin^{-1}\left(\frac{\cancel{2} \cdot \frac{1}{\cancel{2}} - 1}{2}\right)$$

$$= 3 \cdot \sin^{-1}\left(\frac{1}{2}\right) - 3\sin^{-1}(0) - 0 = \cancel{3} \cdot \frac{\pi}{\cancel{6}} = \frac{\pi}{2}$$

74. $\displaystyle\int_2^4 \frac{2\,dx}{x^2-6x+10}$

$$c = \left(\frac{\cancel{-6}}{\cancel{2}}\right)^2 = 9$$

$$x^2-6x = x^2-6x+9-9$$
$$= (x-3)^2-9$$

$$= \int_2^4 \frac{2\,dx}{(x-3)^2+1}$$

$$= \int_2^4 \frac{2\,dx}{(x+3)^2+1}$$

let $a=1$ Let $u=x-3$ $\longrightarrow \displaystyle\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)+c$

$$= 2\cdot 1\tan^{-1}\left(\frac{x-3}{1}\right)\Big|_2^4$$

$$= 2\tan^{-1}(1) - 2\tan^{-1}(-1)$$

$$= 2\cdot\frac{\pi}{4} - 2\left(-\frac{\pi}{4}\right)$$

$$= \pi$$

80. $\int \frac{dx}{(x-2)\sqrt{x^2-4x+3}}$

$b = \left(-\frac{4}{2}\right)^2$

$= (-2)^2$

$= 4$

$x^2 - 4x = x^2 - 4x + 4 - 4$

$= (x-2)^2 - 4$

$\int \frac{dx}{(x-2)\sqrt{(x-2)^2-4+3}}$

$\int \frac{dx}{(x-2)\sqrt{(x-2)^2-1^2}}$

let $u = x-2$ let $a = 1$

$= \int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a} \sec^{-1}\left|\frac{u}{a}\right| + C$

$= \sec^{-1}\left|\frac{x-2}{1}\right| + C$

$= \sec^{-1}|x-2| + C$

84. $\int \frac{\sqrt{\tan^{-1} x}}{1+x^2}\,dx$

Let $u = \tan^{-1} x$

$\frac{du}{dx} = \frac{1}{1+u^2}$

$du = \frac{dx}{1+u^2}$

$= \int u^{\frac{1}{2}}\,du = \frac{1}{\frac{1}{2}+1} u^{\frac{1}{2}+1} + C$

$= 1 \cdot \frac{2}{3} u^{3/2} + C$

$= \frac{2}{3} u^{3/2} + C$

$= \frac{2}{3}(\tan^{-1} x)^{3/2} + C$

$= \frac{2}{3}\sqrt{(\tan^{-1} x)^3} + C$

88. $\int_{\frac{2}{\sqrt{3}}}^{2} \frac{\cos(\sec^{-1} x)\,dx}{x\sqrt{x^2-1}}$

Let $u = \sec^{-1} x$

$du = \frac{1}{|x|\sqrt{x^2-1}}\,dx$

$\int \cos u\,du = \sin u$

$= \sin(\sec^{-1}(2)) - \sin\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

$= \sin\frac{\pi}{3} - \sin\frac{\pi}{6}$

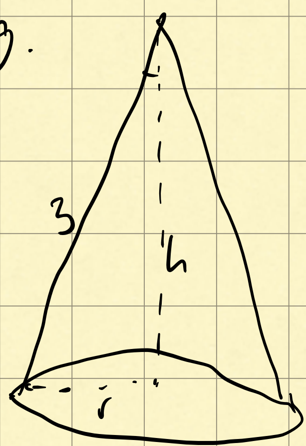
$= \sin 60 - \sin 30$

$= \frac{1}{2}\sqrt{3} - \frac{1}{2}$

$= \frac{\sqrt{3}-1}{2}$

107.

109.



$V = \frac{1}{3}\pi r^2 h$

$r = 3\sin\theta$

$h = 3\cos\theta$

$$\frac{1}{3}\pi(3\sin\theta)^2(3\cos\theta) = 9\pi(\cos\theta - \cos^3\theta)$$

$$\frac{dv}{d\theta} = -9\pi\left((\sin\theta) - \sin\theta\, 3\cos^2\theta\right)$$

$$= -9\pi\sin\theta(1-3\cos^2\theta)$$

$\theta = 0$ when $\sin\theta = 0$

or

$\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

local max $= \cos^{-1}\frac{1}{\sqrt{3}}$

local min $= 0$