7.8

2. a) slower e) slower
b) slower f) faster
c) slower g) slower
d) slower h) same rate

10. a) T e) T
b) F f) F
c) F g) T
d) T h) T

18. $\sqrt{x^4+x}$ and $\sqrt{x^4-x^3}$

$$\lim_{x\to\infty} \frac{x^2}{\sqrt{x^4+x}} = \frac{x^2\left(\frac{1}{x^2}\right)}{\sqrt{x^4+x}\left(\frac{1}{x^2}\right)} = \frac{1}{1} = 1$$

$$\lim_{x\to\infty} \frac{x^2}{\sqrt{x^4-x^3}} = \frac{x^2\left(\frac{1}{x^2}\right)}{\sqrt{x^4-x^3}\left(\frac{1}{x^2}\right)} = \frac{1}{1} = 1$$

same rate

21. a) $\lim_{x\to\infty} \frac{\ln x}{x^{1/n}} = \frac{\frac{1}{x}}{\frac{1}{n}x^{\frac{1}{n}-1}} = \frac{n}{x^{1/n}} = 0$ slower

23. a) $n\log_2 n,\ n^{3/2},\ n(\log_2 n)^2$

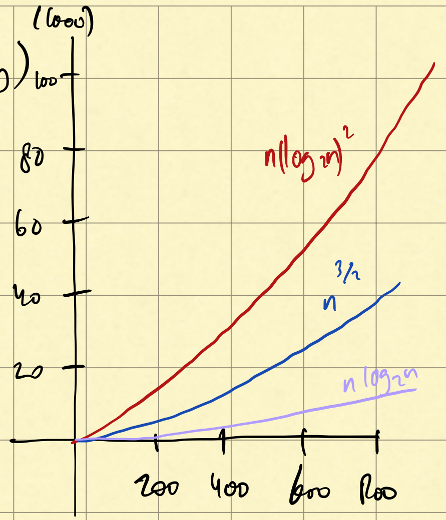
$$\lim_{n\to\infty} \frac{\cancel{n}\log_2 n}{n^{\cancel{3/2}}} = \frac{\log_2 n}{n^{1/2}} = \frac{\ln n}{\ln 2\cdot n^{1/2}}$$

$$= \frac{1}{\ln 2}\lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{2}n^{-\frac{1}{2}}}$$

$$= \frac{2}{\ln 2}\lim_{n\to\infty} \frac{1}{\frac{1}{n^{\frac{1}{2}}}} = 0$$

$$\lim_{n\to\infty} \frac{\cancel{n\log_2 n}}{\cancel{n}(\log_2 n)^{\cancel{2}}} = \frac{1}{\log_2 n} = 0$$

$\therefore n\log_2 n$ is most efficient b)



8.2

6. $\int_1^e x^3 \ln x \, dx$

$u = \ln x \qquad dv = x^3 \, dx$

$du = \frac{1}{x} \qquad v = \frac{1}{4}x^4$

$$\int u \, dv = uv - \int v \, du \longrightarrow = \frac{1}{4}x^4 \ln x - \int \frac{1}{4}x^4 \frac{1}{x} \, dx$$

$$= \frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^3 \, dx$$

$$= \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 \Big|_1^e$$

$$= \frac{e^4}{4} - \frac{e^4}{16} - \left(-\frac{1}{16}\right)$$

$$= \frac{e^4}{4} - \frac{e^4}{16} + \frac{1}{16} = \frac{4e^4 - e^4 + 1}{16}$$

16. $\int (x^2-2x+1)\, e^{2x}\, dx$

	D	I
+	x^2-2x+1	e^{2x}
−	$-2x-2$	$\frac{1}{2}e^{2x}$
+	-2	$\frac{1}{4}e^{2x}$
−	0	$\frac{1}{8}e^{2x}$

$$\int (x^2-2x+1)\, e^{2x}\, dx = \frac{1}{2}e^{2x}(x^2-2x+1) - \frac{1}{4}e^{2x}(-2x-2) - \frac{1}{4}e^{2x} + C$$

12. $\int \sin^{-1} y\, dy$

$u = \sin^{-1} y \qquad dv = 1$

$du = \frac{1}{\sqrt{1-y^2}} \qquad v = y$

$$\int \sin^{-1} y\, dy = y \sin^{-1} y - \int \frac{y}{\sqrt{1-y^2}}$$

$$= y \sin^{-1} y + \sqrt{1-y^2} + C$$

24. $\int e^{-2x} \sin 2x\, dx = A$

$u = \sin 2x \qquad dv = e^{-2x}$

$du = 2\cos 2x \qquad v = -\frac{1}{2} e^{-2x}$

$$\int e^{-2x} \sin 2x\, dx = -\frac{1}{2} e^{-2x} \sin 2x - \int -e^{-2x} \cos 2x\, dx$$

$u = \cos 2x \qquad dv = -e^{-2x}$

$du = -2 \sin 2x \qquad V = \frac{1}{2} e^{-2x}$

$$= -\frac{1}{2}e^{-2x}\sin 2x - \left(\frac{1}{2}e^{-2x}\cos 2x - \int -e^{-2x}\sin 2x\,dx\right)$$

$$= -\frac{1}{2}e^{-2x}\sin 2x - \frac{1}{2}e^{-2x}\cos 2x - A$$

$$2A = -\frac{1}{2}e^{-2x}\sin 2x - \frac{1}{2}e^{-2x}\cos 2x$$

$$\frac{1}{2}2A = \frac{1}{2}\left(-\frac{1}{2}e^{-2x}\sin 2x - \frac{1}{2}e^{-2x}\cos 2x\right)$$

$$A = -\frac{1}{4}e^{-2x}\sin 2x - \frac{1}{4}e^{-2x}\cos 2x + C$$

30. $\int z(\ln z)^2\,dz$

$u = (\ln z)^2$ $\qquad dv = z\,dz$

$du = 2(\ln z)\frac{1}{z}$ $\qquad v = \frac{1}{2}z^2$

$$\int z((\ln z)^2 = \frac{1}{2}z^2(\ln z)^2 - \int (\ln z)\,z$$

$$= \frac{1}{2}z^2(\ln z)^2 - \int \frac{1}{2}z$$

$$= \frac{1}{2}z^2(\ln z)^2 - \frac{1}{4}z^2 + C$$

31. $\int x \sec x^2 \, dx$

Let $u = x^2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int x \sec x^2 \, dx = \int \cancel{x} \sec u \, \frac{du}{\cancel{2}x}$

$= \frac{1}{2} \int \sec u \, du$

$= \frac{1}{2} \int \frac{\sec u + \tan u + \sec^2 u}{\tan u + \sec u} \, du$

let $v = \tan u + \sec u$

$dv = \frac{du}{\sec^2 u + \sec u \tan u}$

$= \frac{1}{2} \int \frac{\cancel{\sec u \tan u + \sec^2 u}}{v} \, \frac{dv}{\cancel{\sec^2 u + \sec u \tan u}}$

$= \frac{1}{2} \ln v$

$= \frac{1}{2} \ln |\tan x^2 + \sec x^2| + C$

43. $\int \sqrt{x} \ln x \, dx$

$u = \ln x \qquad dv = \sqrt{x}$

$du = \frac{1}{x} \qquad v = \frac{2}{3} x^{3/2}$

$\int \sqrt{x} \ln x \, dx = \frac{2}{3} x^{3/2} \ln x - \int \frac{2}{3} x^{3/2} \frac{1}{\cancel{x}}$

$= \frac{2}{3} x^{3/2} \ln x - \int \frac{2}{3} x^{1/2} = \frac{2}{3} x^{3/2} \ln x - \frac{4}{9} x^{3/2}$

95. $\int \cos\sqrt{x}\, dx$

Let $u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2} x^{-\frac{1}{2}}$

$dx = 2\sqrt{x}\, du$

$\int \cos u \; 2\sqrt{x}\, du$

$2\int u \cos u \, du$

$2(u \cdot \sin u + \cos u)$

$= 2\sqrt{x} \cdot \sin\sqrt{x} + 2\cos\sqrt{x} + C$

66. $\int (\ln x)^n dx = x(\ln x)^n - n\int (\ln x)^{n-1} dx$

$u = (\ln x)^n$ $\qquad dv = dx$

$du = \frac{n}{x}(\ln x)^{n-1}$ $\qquad v = x$

$x(\ln x)^n - \int \frac{n}{x}(\ln x)^{n-1} \cdot x \, dx$

$= x(\ln x)^n - n\int (\ln x)^{n-1} dx$

69. $\int_a^b \left(\int_x^b f(t)\, dt\right) dx$

$u = \int_x^b f(t)\, dt$ $\qquad dv = dx$

$du = -f(x)\, dx$ $\qquad v = x$

$= x\int_x^b f(t)\, dt \Big|_a^b - \int_a^b - xf(x)\, dx$

$= b\int_b^b f(t)\, dt - a\int_a^b f(t)\, dt + \int_a^b xf(x) dx$

$= 0 - \int_a^b a f(x)\, dx + \int_a^b x\, f(x)\, dx$

$= \int_a^b (x-a)\, f(x)\, dx$

8.3

3. $\int \cos^3 x \sin x \, dx$

$u = \cos x$

$du = -\sin x \, dx$

$= -\int u^3 \, du$

$= -\frac{1}{4} u^4 + C$

$= -\frac{1}{4} \cos^4 x + C$

8. $\int_0^{\pi} \sin^5 \frac{x}{2} \, dx$

$u = \cos \frac{x}{2}$

$du = -\frac{1}{2} \sin \frac{x}{2} dx$

when $x=0$, $u=1$

when $x=\pi$, $u=0$

$= \int_0^{\pi} \left(1 - \cos^2 \frac{x}{2}\right)^2 \sin \frac{x}{2} \, dx$

$= -2 \int_1^0 (1-u^2)^2 \, du$

$= 2 \int_0^1 1 - 2u^2 + u^4 \, du$

$= u - \frac{2}{3} u^3 + \frac{1}{5} u^5 \Big|_0^1 2$

$= 2\left(1 - \frac{2}{3} + \frac{1}{5}\right)$

$= 2\left(\frac{15 - 10 + 3}{15}\right)$

$= 2\left(\frac{8}{15}\right)$

$= \frac{16}{15}$

20. $\int_0^{\pi} 8 \sin^4 y \cos^2 y \, dy$

$= \int_0^{\pi} 8 \left(\frac{1 - \cos 2y}{2}\right)^2 \left(\frac{1 + \cos 2y}{2}\right) dy$

$= \int_0^{\pi} (1 - \cos 2y)^2 (1 + \cos 2y) \, dy$

$= \int_0^{\pi} 1 + \cos 2y - \cos^2 2y - \cos^3 2y \, dy$

$= y + \frac{1}{2} \sin 2y \Big|_0^{\pi} - \int_0^{\pi} (\cos^2 2y + \cos^3 2y) \, dy$

$$\int_0^{\pi} \cos^2 2y \, dy = \int_0^{\pi} \frac{1}{2}(1+\cos 4y)\, dy = \frac{1}{2}y + \frac{1}{8}\sin 4y \Big|_0^{\pi} = \frac{\pi}{2}$$

$$\int_0^{\pi} \cos^3 2y \, dy = \int_0^{\pi} (1-\sin^2 2y)\cos 2y \, dy = \frac{1}{2}\int_0^{\pi} (1-\sin^2 2y)(\sin 2y)$$

$$= \sin 2y - \frac{1}{3}\sin^3 2y \Big|_0^{\pi} \frac{1}{2} = 0$$

$$= \int_0^{\pi} 8\sin^4 y \cos^2 y \, dy = y + \frac{1}{2}\sin 2y \Big|_0^{\pi} - \frac{\pi}{2} - 0$$

$$= \pi - \frac{\pi}{2}$$

$$= \frac{\pi}{2}$$

29. $\displaystyle\int_{5\pi/6}^{\pi} \frac{\cos^4 x}{\sqrt{1-\sin x}} \, dx = \frac{4}{5}\left(\frac{3}{2}\right)^{5/2} - \frac{2}{7}\left(\frac{3}{2}\right)^{7/2} - \frac{18}{35}$

34. $\displaystyle\int \sec x \tan^2 x \, dx$

$$\int \sec x (\sec^2 x - 1)$$

$$\int \sec^3 x - \sec x$$

$$= \frac{\sec x \tan x}{2} + \frac{1}{2}\int \sec x \, dx$$

$$\int \sec x \, dx$$

$$= \ln(\tan x + \sec x) = \frac{\sec x \tan x - \ln(\tan x + \sec x)}{2} + C$$

37. $\int \sec^2 x \tan x \, dx$

Let $u = \tan x$

$dx = \frac{1}{\sec^2 x} du$

$\int \cancel{\sec^2 x} \, u \, \frac{du}{\cancel{\sec^2 x}}$

$= \int u^2 \, du$

$= \frac{1}{3} u^3 \to \frac{1}{3} \tan^3 x + C$

38. $\int \sec^4 x \tan^2 x \, dx$

$\int \sec^2 x \tan^2 x (\tan^2 x + 1) \, dx$

$u = \tan^2 x$

$dx = \frac{1}{\sec^2 x} du$

$= \int u^2(u^2+1) \, du$

$= \int u^4 + u^2 \, du$

$= \frac{1}{5} u^5 + \frac{2u^3}{3} \to \frac{1}{5} \tan^5 x + \frac{2}{3} \tan^3 x + \tan x$

44. $\int \sec^6 x \, dx$

$\int \sec^2 x (\tan^2 x + 1)^2 \, dx$

$u = \tan x$

$dx = \frac{1}{\sec^2 x} du$

$= \int (u^2+1)^2 \, du$

$= \int u^4 + 2u^2 + 1$

$= \frac{1}{5} u^5 + \frac{2}{3} u^3 + u$

$= \frac{1}{5} \tan^5 x + \frac{2}{3} \tan^3 x + \tan x$

52. $\int \sin 2x \cos 3x \, dx$

$= \frac{1}{2} \int 2 \sin 2x \cos 3x \, dx$

$= \frac{1}{2} \int \sin 5x - \sin x$

$= \frac{1}{2} - \frac{1}{5} \cos 5x - C \cos x$

$= -\frac{1}{10} \cos 5x - \frac{1}{2} \cos x + C$

64. $\int \frac{\sin^3 x}{\cos^4 x} \, dx$

$\int \sec^2 x - 1 \cdot \sec x \tan x$

$u = \sec x$

$du = \frac{1}{\sec x \tan x}$

$\int u^2 - 1$

$= \frac{u^3}{3} - u$

$= \frac{1}{3} \sec^3 x - \sec x + C$

8.4

2. $\int \frac{3\,dx}{\sqrt{1+9x^2}}$

$3x = \tan\theta$

$3dx = \sec^2\theta\, d\theta$

$= \int \frac{\sec^2\theta\, d\theta}{\sec\theta}$

$= \int \sec\theta\, d\theta$

$= \ln|\sec\theta + \tan\theta| + C$

$= \ln|\sqrt{1+9x^2} + 3x| + C$

4. $\int_0^2 \frac{dx}{8+2x^2}$

$= \frac{1}{2}\int_0^2 \frac{dx}{4+x^2}$

$= \frac{1}{2} \tan^{-1}\frac{x}{2}\Big|_0^2$

$= \frac{1}{4}\left(\frac{\pi}{4} - 0\right)$

$= \frac{\pi}{16}$

6. $\int_0^{1/2\sqrt{2}} \frac{2dx}{\sqrt{1-4x^2}}$

$2x = \sin\theta$

$2dx = \cos\theta\, d\theta$

$= \int \frac{\cos\theta\, d\theta}{\cos\theta}$

$= \theta$

$= \sin^{-1} 2x \Big|_0^{\frac{1}{2\sqrt{2}}}$

$= \frac{\pi}{4}$

9. $\int \frac{dx}{\sqrt{4x^2-49}}, \quad x > \frac{7}{2}$

$2x = 7\sec\theta$

$dx = \frac{7}{2}\sec\theta\tan\theta\, d\theta$

$= \frac{\frac{7}{2}\sec\theta \tan\theta\, d\theta}{7\tan\theta}$

$= \frac{1}{2}\int \sec\, d\theta$

$= \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$

$= \frac{1}{2}\ln\left|\frac{2}{7}x + \frac{\sqrt{4x^2-49}}{7}\right| + C$

12. $\int \frac{\sqrt{y^2-25}}{y^3}\,dy \quad , y>5$

$y = 5\sec\theta$

$dy = 5\sec\theta\tan\theta\,d\theta$

$\int \frac{5\tan\theta}{125\sec^3\theta} 5\sec\theta\tan\theta\,d\theta$

$= \frac{1}{5}\int \sin^2\theta\,d\theta$

$= \frac{1}{10}\int (1-\cos 2\theta)\,d\theta$

$= \frac{1}{10}\theta - \frac{1}{20}\sin 2\theta + C$

$= \frac{1}{10}\theta - \frac{1}{10}\sin\theta\cos\theta + C$

$= \frac{1}{10}\sec^{-1}\theta - \frac{1}{10}\frac{5\sqrt{y^2-25}}{y^2} + C$

$= \frac{1}{10}\sec^{-1}\left(\frac{y}{5}\right) - \frac{\sqrt{y^2-25}}{2y^2} + C$

57. $\int x^3\sqrt{1-x^2}\,dx$

a) $u = x^2 \qquad dv = x\sqrt{1-x^2}\,dx$

$du = 2x \qquad v = -\frac{1}{3}(1-x^2)^{3/2}$

$= -\frac{1}{3}x^2(1-x^2)^{3/2} + \frac{1}{3}\int 2x(1-x^2)^{3/2}\,dx$

$= -\frac{1}{3}x^2(1-x^2)^{3/2} - \frac{2}{15}(1-x^2)^{5/2} + C$

b) $\int x^2 \cdot x\sqrt{1-x^2}\,dx$

$u = 1-x^2$

$du = -2x\,dx$

$= -\frac{1}{2}\int (1-u)\sqrt{u}\,du$

$= -\frac{1}{2}\int u^{1/2} - u^{3/2}\,du$

$= -\frac{1}{3}u^{3/2} + \frac{1}{5}u^{5/2} + C$

$= -\frac{1}{3}(1-x^2)^{3/2} + \frac{1}{5}(1-x^2)^{5/2} + C$

c) $x = \sin\theta$ $\quad \sqrt{1-x^2} = \cos\theta$

$dx = \cos\theta\,d\theta$

$= \int \sin^3\theta \cos^2\theta\,d\theta$

$= \int (1-\cos^2\theta)\cos^2\theta \sin\theta\,d\theta$

$= \int \cos^2\theta \sin\theta\,d\theta - \int \cos^4\theta \sin\theta\,d\theta$

$= -\frac{1}{3}\cos^3\theta + \frac{1}{5}\cos^5\theta + C$

$= -\frac{1}{3}(1-x^2)^{3/2} + \frac{1}{5}(1-x^2)^{5/2} + C$

58. a) $AB^2 = 100 - x^2$

$AB = \sqrt{100-x^2}$

$\tan\theta = \frac{AB}{AC}$

$= \frac{\sqrt{100-x^2}}{x}$

$f'(x) = \tan(\pi - \theta)$

since $\tan\pi = 0$

$\theta - \tan\theta$

$= -\frac{\sqrt{100-x^2}}{x}$

b) $f'(x) = -\frac{\sqrt{100-x^2}}{x}$

$x = 10\cos\theta$

$dx = -10\sin d\theta$

$\int -\frac{\sqrt{100-(10\cos\theta)^2}}{10\cos\theta} - 10\sin d\theta$

$\int \frac{\sqrt{100(1-\cos^2\theta)}}{\cos\theta} \cdot \sin d\theta$

$\int \frac{\sqrt{100\sin^2\theta}}{\cos\theta} \sin\theta\,d\theta$

$\int \frac{10\sin^2\theta}{\cos\theta}\,d\theta$

$\int \frac{10(1-\cos^2\theta)}{\cos\theta}\,d\theta$

$y = 10\ln|\sec\theta + \tan\theta| - 10\sin\theta + C$