

3.6

$$12. \quad y = \left(\frac{\sqrt{x}}{2} - 1 \right)^{-10} \quad v = \left(\frac{\sqrt{x}}{2} - 1 \right)$$

$$y = f(v)$$

$$v = g(x)$$

$$y' = -10 \left(\frac{\sqrt{x}}{2} - 1 \right)^{-11}$$

$$v' = \frac{1}{2}(x)^{\frac{1}{2}}$$

$$v' = \frac{1}{4}x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = -10 \left(\frac{\sqrt{x}}{2} - 1 \right)^{-11} \cdot \frac{1}{4}x^{-\frac{1}{2}}$$

$$39. \quad q = \sin \left(\frac{t}{\sqrt{t+1}} \right) \quad \text{derivative of } \sin x = \cos x$$

$$v = \frac{t}{\sqrt{t+1}}$$

$$v' = \frac{1 \cdot (t+1)^{\frac{1}{2}} - t \cdot \frac{1}{2}(t+1)^{-\frac{1}{2}} \cdot 1}{(t+1)^{\frac{1}{2}}} : \left((t+1)^{\frac{1}{2}} - \frac{t}{2\sqrt{t+1}} \right) \frac{1}{t+1}$$

↓

$$\frac{2t+2-t}{2(t+1)^{\frac{3}{2}}} = \frac{t+2}{2(t+1)^{\frac{3}{2}}}$$

$$q' = \cos \left(\frac{t}{\sqrt{t+1}} \right) \left(\frac{t+2}{2(t+1)^{\frac{3}{2}}} \right)$$

$$= \frac{\sqrt{t+1}}{t+1} - \frac{t}{2\sqrt{t+1} \cdot t+1}$$

$$= \frac{1}{\sqrt{t+1}} - \frac{t}{2\sqrt{t+1} \cdot \sqrt{t+1} \cdot \sqrt{t+1}}$$

$$= \frac{2\sqrt{t+1} \cdot \sqrt{t+1} - t}{2(\sqrt{t+1})^3}$$

$$= \frac{2(t+1) - t}{2(t+1)^{\frac{3}{2}}}$$

58. Find dy/dt

$$y = \sqrt{3t + \sqrt{2+t}}$$
$$= (3t + (2 + (1-t)^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}$$

$$y' = \frac{1}{2} (3t + (2 + (1-t)^{\frac{1}{2}})^{\frac{1}{2}})^{-\frac{1}{2}} \cdot (3 + \frac{1}{2}(2 + (1-t)^{\frac{1}{2}})^{-\frac{1}{2}}) \cdot (\frac{1}{2}(1-t)^{-\frac{1}{2}}).$$

(-1)

$$= \left(\frac{1}{2 \sqrt{3t + \sqrt{2+t}}} \right) \cdot \left(3 + \frac{1}{2 \sqrt{2+t}} \right) \cdot \left(-\frac{1}{2 \sqrt{1-t}} \right)$$

66. Find the value of $(f \circ g)'$ at the given value of x .

$$f(v) = 1 - \frac{1}{v}, \quad v = g(x) = \frac{1}{1-x}, \quad x = -1$$

$$(f \circ g)' = f'(g(x))$$
$$= f'(g(x)) \cdot g'(x)$$

$$f(g(x)) = 1 - \frac{1}{\frac{1}{1-x}}$$

$$f'(g(x)) = 1 - (1-x)$$

$$= x$$

$$= 1$$

$$g'(x) = \frac{1}{1-x}$$

$$= (1-x)^{-1}$$

$$= -1(1-x)^{-2} \cdot -1$$

$$= \frac{1}{(1-x)^2}$$

$$f'(g(x)) \cdot g'(x) = 1 \cdot \frac{1}{(1-x)^2}$$

$$= \frac{1}{x^2 - 2x + 1} = \frac{1}{(-1)^2 - 2(-1) + 1} = \frac{1}{9}$$

$$74. \ a) 5f(x) - g(x), x=1$$

$$= 5 \cdot f'(x) - g'(x)$$

$$= 5 \cdot -\frac{1}{3} - \left(-\frac{8}{3}\right)$$

$$= -\frac{5}{3} + \frac{8}{3} = \frac{3}{3} = 1$$

$$b) f(x) g^3(x), x=0$$

$$= f'(x) (g(x))^3 + f(x) 3(g(x))^2 g'(x)$$

$$= 5 \cdot (1)^3 + 1 \cdot 3 \cdot (1)^2 \cdot \frac{1}{3}$$

$$= 5 + 1$$

$$= 6$$

$$c) \frac{f(x)}{g(x)+1}, x=1$$

$$= \frac{f'(x) \cdot (g(x)+1) - f(x) \cdot g'(x)}{(g(x)+1)^2}$$

$$= \frac{\left(-\frac{1}{3} \cdot -3\right) - \left(3 \cdot -\frac{8}{3}\right)}{(-3)^2} = \frac{\frac{3}{3} + \frac{24}{3}}{9}$$

$$= \frac{9}{9} = 1$$

$$d) f(g(x)), x=0$$

$$= f'(g(x)) g'(x)$$

$$= f'(1) \cdot \frac{1}{3}$$

$$= -\frac{1}{3} \cdot \frac{1}{3} = -\frac{1}{9}$$

$$e) g(f(x)), x=0$$

$$= g'(f(x)) \cdot f'(x)$$

$$= g'(1) \cdot 5$$

$$= -\frac{8}{3} \cdot 5 = -\frac{40}{3}$$

$$f) \quad (x'' + f(x))^{-2}, \quad x=1$$

$$= -2(x'' + f(x))^{-3} \cdot (11x^{10} + f'(x))$$

$$= -2(1'' + 3)^{-3} \cdot (11(1)^{10} - \frac{1}{3})$$

$$= -\frac{2}{9^3} \cdot (11 - \frac{1}{3})$$

$$= -\frac{2}{9} \cdot \frac{32}{3} = \frac{-1}{3}$$

$$g) \quad f(x+g(x)), \quad x=0$$

$$= f'(x+g(x)) \cdot 1 + g'(x)$$

$$= f'(1) \cdot \frac{3+1}{3}$$

$$= -\frac{1}{3} \cdot \frac{4}{3} = -\frac{4}{9}$$

$$83. \quad S = A \cos(2\pi b t)$$

$$S' = -A \sin(2\pi b t) (2\pi b) \quad \text{Velocity}$$

$$= -2A\pi b \sin(2\pi b t)$$

$$S'' = -A \cos(2\pi b t) (2\pi b) (2\pi b) \quad \text{Acceleration}$$

$$= -4A(\pi b)^2 \cos(2\pi b t)$$

$$S''' = A \sin(2\pi b t) (2\pi b) (2\pi b) (2\pi b) \quad \text{Jerk}$$

$$= 8A(\pi b)^3 \sin(2\pi b t)$$

Doubling frequency = $b = 2b$

$$\text{Velocity} = -4\pi vb A \sin(4\pi vb t)$$

$$\text{Acceleration} = -16(\pi vb)^2 A \cos(4\pi vb t)$$

$$\text{Jerk} = 64(\pi vb)^3 A \sin(4\pi vb t)$$

∴ So, doubling the frequency, multiplies the velocity, acceleration, and jerk by 2, 4, & respectively.

3.7

$$14. x \cos(2x+3y) = y \sin x$$

$$1 \cdot (\cos(2x+3) + x \cdot -\sin(2x+3y)(2+3 \cdot y')) = y' \cdot \sin x + y \cdot \cos x \cdot 1$$

$$\cos(2x+3) - 2x \sin(2x+3y) - y'(3x \sin(2x+3y)) - y' \cdot \sin x - y \cos x$$

$$y'(3x \sin(2x+3y) + \sin x) = \cos(2x+3) - 2x \sin(2x+3y) - y \cos x$$

$$y' = \frac{\cos(2x+3) - 2x \sin(2x+3y) - y \cos x}{(3x \sin(2x+3y) + \sin x)}$$

$$22. y^2 - 2x = (-2y)$$

$$2y \cdot y' - 2 = -2 \cdot y'$$

$$2y \cdot y' + 2 \cdot y' = 2$$

$$y'(2y+2) = 2$$

$$y' = \frac{2}{(2y+2)}$$

$$y' = \frac{1}{y+1}$$

$$y'' = \underbrace{0 \cdot (y+1) - 1 \cdot y'}_{(y+1)^2} = \frac{-y'}{(y+1)^2}$$

$$42. \quad y^2(2-x) = x^3$$

$$2y \cdot y'(2-x) + y^2(-1) = 3x^2$$

$$y'(2y)(2-x) - y^2 = 3x^2$$

$$y' = \frac{3x^2 + y^2}{(2y)(2-x)} = \frac{3x^2 + y^2}{4y - 2yx}$$

$$m = \frac{3x^2 + y^2}{4y - 2yx}$$

$$M = \frac{3(1)^2 + (1)^2}{4(1) - 2(1)(1)} = \frac{4}{2} = 2$$

tangent line equation

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 2(x - 1)$$

$$y - 1 = 2x - 2$$

$$y = 2x - 1$$

normal line equation:

$$m(\text{tangent}) \cdot m(\text{normal}) = -1$$

$$2 \cdot -\frac{1}{2} = -1$$

$$m(\text{normal}) = -\frac{1}{2}$$

$$y - 1 = -\frac{1}{2}(x - 1)$$

$$y - 1 = -\frac{1}{2}x + \frac{1}{2}$$

$$y = -\frac{1}{2}x + \frac{3}{2}$$

$$96. \text{ if } y = x^{p/q}$$

$$y^q = x^p$$

$$qy^{q-1}y' = px^{p-1}$$

$$y' = \frac{px^{p-1}}{qy^{q-1}}$$

$$= \frac{px}{qy^{q-1}}$$

$$= \frac{px}{q(x^{p/q})^{q-1}}$$

$$= \frac{p}{q} x^{(p-1) - (p/q)(q-1)}$$

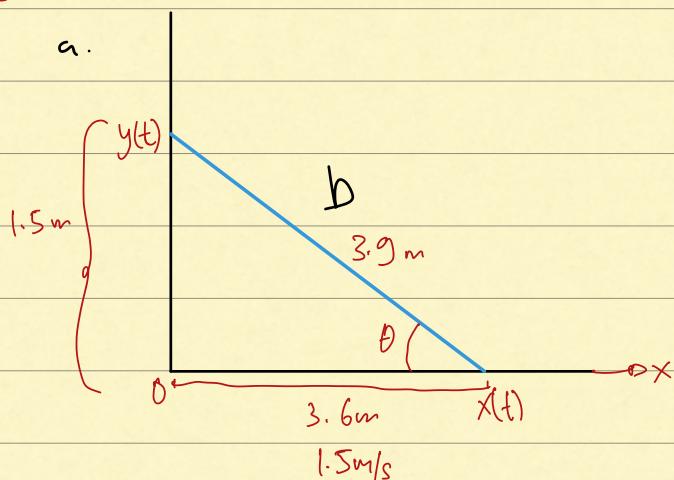
$$y' = \frac{p}{q} x^{(\frac{p}{q})-1}$$

or

$$\frac{d}{dx} x^{p/q} = \frac{p}{q} x^{(\frac{p}{q})-1}$$

3.8

23. a.



$$y = \sqrt{(3.6)^2 - (3.6)^2}$$

$$= \sqrt{15.21 - 12.96}$$

$$= 1.5 \text{ m}$$

$$b^2 = x^2 + y^2$$

$$2b \frac{db}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$$

$$2b(0) = 2(3.6)(1.5 \text{ m/s}) + 2(1.5) \frac{dy}{dt}$$

$$0 = 10.8 + 3 \frac{dy}{dt}$$

$$\frac{-10.8}{3} = \frac{dy}{dt}$$

$$-3.6 \text{ m/s} = \frac{dy}{dt}$$

b) triangle area = $\frac{1}{2} \times y$

product rule $u'v + uv'$

$$\frac{1}{2} \frac{dx}{dt} \cdot y + \frac{1}{2} x \cdot \frac{dy}{dt}$$

$$= \frac{1}{2}(1.5)(1.5) + \frac{1}{2}(3.6) \cdot -3.6$$

$$= 1.125 - 6.48 = -5.355 \text{ m}^2/\text{s}$$

$$C) \sin \theta = \frac{y}{b}$$

$$= \cos \theta \frac{d\theta}{dt} = \frac{1}{b} \cdot (1) \frac{dy}{dt}$$

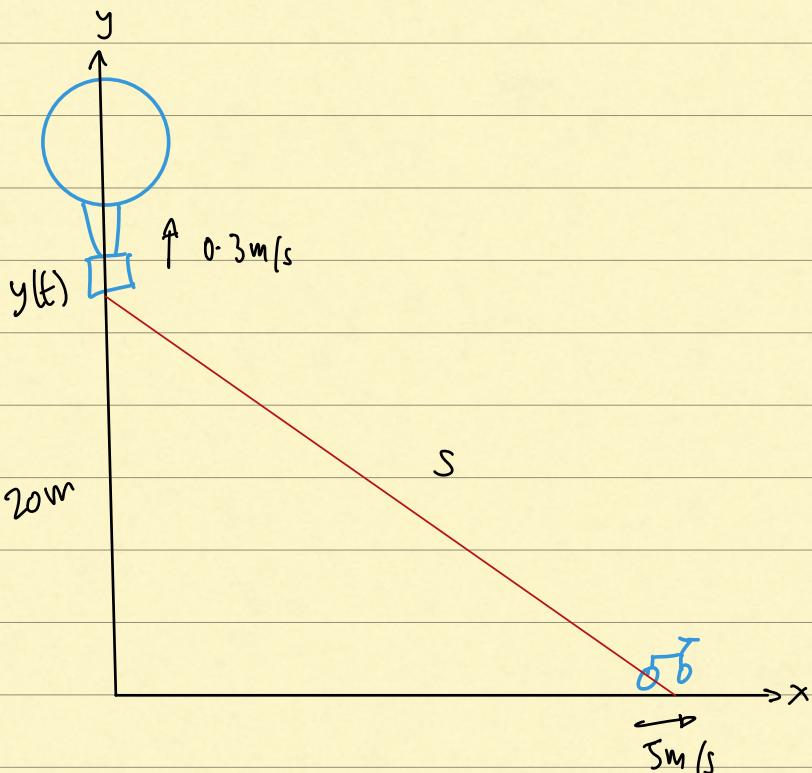
$$= \frac{3.6}{3g} \frac{d\theta}{dt} = \frac{1}{3g} \frac{dy}{dt}$$

$$= 3.6 \times \frac{d\theta}{dt} = \frac{dy}{dt}$$

$$\frac{d\theta}{dt} = -\frac{3.6}{3.6}$$

$$\frac{d\theta}{dt} = -1 \text{ rad/s}$$

33.



$$\frac{dy}{dt} = 0.3 \text{ m/s}$$

$$x = 5 \text{ m/s} \cdot 3 \text{ s} = 15 \text{ m}$$

$$\frac{dx}{dt} = 5 \text{ m/s}$$

$$y = 0.3 \text{ m/s} \cdot 3 \text{ s} = 0.9 \text{ m} + 20 \text{ m}$$

$$s = \sqrt{x^2 + y^2} \text{ or } (x^2 + y^2)^{\frac{1}{2}}$$

$$s = \sqrt{(15)^2 + (20.9)^2}$$

$$= \sqrt{225 + 436.81}$$

$$= 25.72 \text{ m}$$

$$\frac{ds}{dt} = \frac{1}{2} (x^2 + y^2)^{-\frac{1}{2}} \left(2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} \right)$$

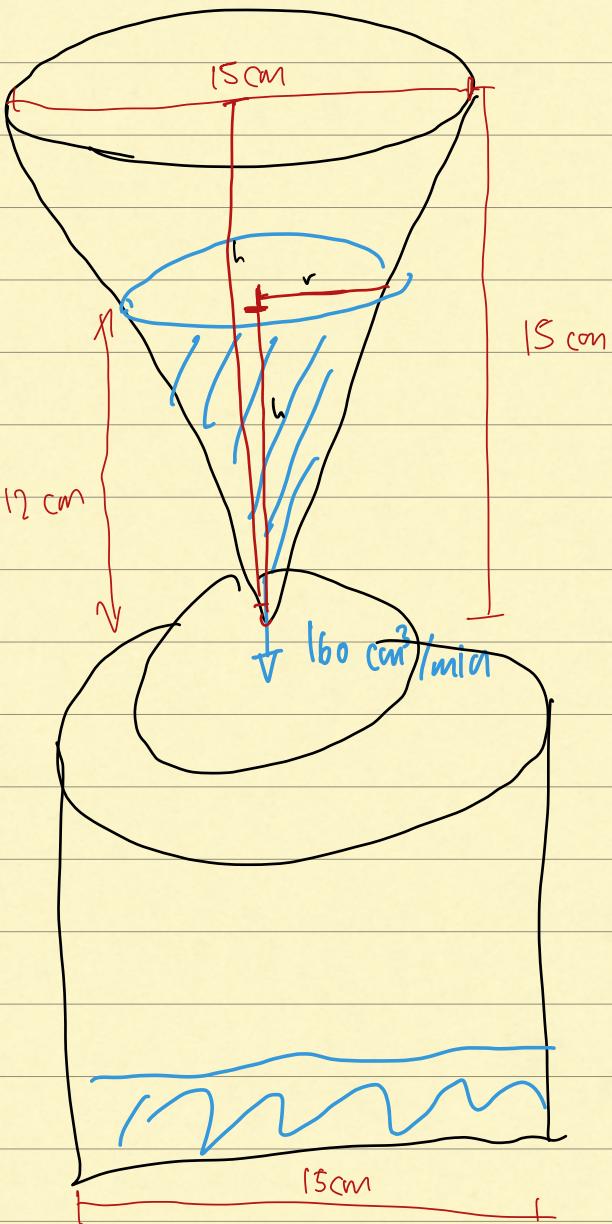
$$= \frac{1}{2 \sqrt{x^2 + y^2}} \cdot \left(2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} \right)$$

$$= \frac{1}{2 \times 25.72} \cdot \left(2(15) \cdot 5 + 2(20.9) \cdot 0.3 \right)$$

$$= \frac{1}{51.44} \cdot (150 + 12.54)$$

$$= \frac{162.54}{51.44} = 3.16 \text{ m/s}$$

34.



$$a) \frac{dv}{dt} : 160 \text{ cm}^3/\text{min}$$

$$V_{\text{pot}} = \pi r^2 h$$

$$\frac{dv}{dt} = \pi r^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{dv}{dt} \cdot \frac{1}{\pi r^2}$$

$$= 160 \cdot \frac{1}{\pi (7.5)^2} = 0.9 \text{ cm/min}$$

$$b) \frac{r}{h} = \frac{7.5}{15} = \frac{1}{2}$$

$$r = \frac{1}{2}h$$

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi \left(\frac{1}{2}h\right)^2 h$$

$$V_{\text{cone}} = \frac{1}{3}\pi \frac{1}{4}h^3 \quad \frac{dV}{dt} = -160 \text{ cm}^3/\text{min} \quad (- \text{ Since it is decreasing})$$

$$= \frac{\pi}{12}h^3$$

$$\frac{dV}{dt} = \frac{\pi}{12} 3h^2 \frac{dh}{dt}$$

$$-160 = \frac{\pi}{12} \cdot 3(12)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{-160}{113.04} = -1.41 \text{ cm/min}$$

3.9

$$15 \cdot (1+x)^k \approx 1 + kx$$

$$a) (1.0002)^{50}$$

$$(1+0.0002)^{50} \approx 1 + 50 \cdot 0.0002$$

$$1.01004916 \approx 1.01$$

$$b) \sqrt[3]{1.009}$$

$$(1+0.009)^{\frac{1}{3}} \approx 1 + \frac{1}{3} \cdot 0.009$$

$$1.0299 \approx 1.003$$

30. $f(x) = 2x^2 + 4x - 3$, $x_0 = -1$, $dx = 0.1$

$$f(x_0) = 2(-1)^2 + 4(-1) - 3$$

$$= 2 - 4 - 3 = -5$$

$$f'(x_0) = 4x + 4$$

$$= 4(-1) + 4 = 0$$

$$\begin{aligned} f(x_0 + dx) &= 2(-1+0.1)^2 + 4(-1+0.1) - 3 \\ &= 2(-0.9)^2 + 4(-0.9) - 3 \\ &= 1.62 - 3.6 - 3 \\ &= -4.98 \end{aligned}$$

$$a) \Delta f = f(x_0 + dx) - f(x_0)$$

$$\Delta f = -4.98 + 5$$

$$= 0.02$$

$$b) df = f'(x_0) dx$$

$$= 0 \times 0.1$$

$$\approx 0$$

$$c) |\Delta f - df| = |0.02 - 0| = 0.02$$

39. Write a differential formula that estimates the given change
in volume or surface area

$$V = \pi r^2 h$$

We can derive by using $\frac{dv}{dr} = \lim_{dr \rightarrow 0} \frac{f(r+dr) - f(r)}{dr}$

$$\lim_{dr \rightarrow 0} \frac{\pi(r+dr)^2 h - \pi r^2 h}{dr}$$

$$\lim_{dr \rightarrow 0} \frac{\pi h(r^2 + 2rdr + dr^2) - \pi r^2 h}{dr} = \lim_{dr \rightarrow 0} \frac{\cancel{\pi r^2 h} + \pi h(2rdr) + \pi h(dr^2) - \cancel{\pi r^2 h}}{dr}$$

$$\lim_{dr \rightarrow 0} \frac{\cancel{\pi h(2rdr) + \pi h(dr^2)}}{dr} = \cancel{dr} \frac{\pi h(2r + dr)}{\cancel{dr}}$$

$$\lim_{dr \rightarrow 0} \frac{\pi h(2r + dr)}{dr} = \pi h(2r + dr) \rightarrow \frac{dv}{dr}$$

$$\text{So, } dv = \pi h(2r + dr)$$

$$51. W = PV + \frac{V\delta v^2}{2g},$$

When P V δ and v remain constant, W becomes a function of g,

$$W = a + \frac{b}{g} \quad (a, b \text{ constant})$$

$$g_{\text{Moon}} = 1.6 \text{ m/s}^2$$

$$g_{\text{Earth}} = 9.8 \text{ m/s}^2$$

$$\frac{dw}{dg} = 0 + \frac{0 \cdot g - bg'}{g^2} = \left(\frac{-b}{g^2}\right) \cdot g'$$

$$\frac{-b}{g^2} : \frac{-b}{g^2} = \frac{\cancel{-b}}{(1.b)^2} \times \frac{(9.8)^2}{\cancel{-b}} = \frac{1}{256} \times \frac{96.04}{1}$$

Moon Earth

$$= 37.51$$

55. a) $Q(x) = b_0 + b_1(x-a) + b_2(x-a)^2$ be a quadratic approximation to $f(x)$ at $x=a$ with the properties:

i) $Q(a) = f(a)$

ii) $Q'(a) = f'(a)$

iii) $Q''(a) = f''(a)$

Determine the coefficients b_0, b_1 , and b_2

i) $Q(a) = b_0 + b_1(a-a) + b_2(a-a)^2$

$f(a) = b_0$

ii) $Q'(x) = 0 + b_1(1) + 2b_2(x-a)(1)$

$Q'(a) = 0 + b_1(1) + 2b_2(a-a)(1)$

$f'(a) = b_1$

iii) $Q''(x) = 0 + 2b_2$

$Q''(a) = 2b_2$

$b_2 = \frac{f''(a)}{2}$

$$b) f(x) = \frac{1}{(1-x)} \quad \text{at} \quad x > 0$$

$$f(0) = \frac{1}{(1-0)} = 1$$

$$\begin{aligned} f'(x) &= -1(1-x)^{-2}(-1) \rightarrow f'(0) = \frac{1}{(1-0)^2} = 1 \\ &= \frac{1}{(1-x)^2} \end{aligned}$$

$$\begin{aligned} f''(x) &= -2(1-x)^{-3}(-1) \rightarrow f''(0) = \frac{2}{(1-0)^3} = 2 \\ &= \frac{2}{(1-x)^3} \end{aligned}$$

$$so, b_0 = 1, b_1 = 1, b_2 = \frac{2}{2} = 1$$

$$\therefore Q(x) = 1 + x + x^2$$

$$56 \quad 1. \quad E(a) = 0 \quad y = f(x)$$

$$g(x) = m(x-a) + c$$

$$E(x) = f(x) - g(x)$$

$$E(a) = f(a) - m(a-a) - c = 0$$

$$f(a) = c$$

$$2. \lim_{x \rightarrow a} \frac{E(x)}{x-a} = 0$$

$$\lim_{x \rightarrow a} \frac{f(x) - m(x-a) - c}{x-a} = 0$$

$$\lim_{x \rightarrow a} \frac{f(x)}{x-a} - \frac{m(x-a)}{x-a} - \frac{c}{x-a} = 0$$

$$\lim_{x \rightarrow a} \frac{f(x) - c}{x-a} - m = 0$$

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x-a} - m = 0$$

$$= f'(a) = m$$

4.1

3. absolute maximum at point C, but there is no absolute minimum

4. It is discontinuous in point a, c, b so there is no absolute maximum and minimum

$$30. g(x) = -\sqrt{5-x^2}, -\sqrt{5} \leq x \leq 0$$

$$g(x) = -(5-x^2)^{\frac{1}{2}}$$

$$g'(x) = -\frac{1}{2}(5-x^2)^{\frac{1}{2}}(-2x)$$

$$= \frac{x}{\sqrt{5-x^2}}$$

Endpoints at domain $-\sqrt{5}, 0$

$$\frac{x}{\sqrt{5-x^2}} = 0$$

$$x = 0$$

$$\sqrt{5x^2} = 0$$

$$5-x^2 = 0$$

$$x^2 = 5$$

$$x = \pm\sqrt{5} \rightarrow \text{Critical points}$$

$$46. f(x) = \frac{x^2}{x-2}$$

$$f'(x) = \frac{2x(x-2) - x^2(1)}{(x-2)^2} = 0$$

$$= \frac{2x^2 - 4x - x^2}{(x-2)(x-2)} = \frac{x^2 - 4x}{(x-2)(x-2)} = \frac{x(x-4)}{(x-2)(x-2)}$$

$$\frac{x(x-4)}{(x-2)(x-2)} > 0 \rightarrow x(x-4)$$

$$x=0 \quad x=4$$

$$g'(0) = -\sqrt{5-0^2}$$

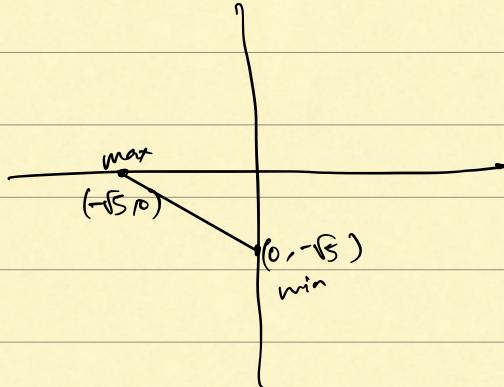
$= -\sqrt{5} \rightarrow \text{Absolute Min}$

$$g'(-\sqrt{5}) = -\sqrt{5-(\sqrt{5})^2}$$

$= 0 \rightarrow \text{Absolute Max}$

Absolute max at $x = -\sqrt{5}$ is 0 $(-\sqrt{5}, 0)$

Absolute min at $x = 0$ is $-\sqrt{5} (0, -\sqrt{5})$



$$TS. \quad y = \frac{1}{\sqrt[3]{1-x^2}}$$

Domain

$$y' = -\frac{1}{3}(1-x^2)^{-\frac{2}{3}}(-2x) \quad (-x^2 > 0)$$

$$-x^2 > -1$$

$$0 = \frac{2x}{3(1-x^2)^{\frac{2}{3}}} \quad x < 1$$

$$2x = 0 \quad x > -1$$

$$x = 0 \quad -1 < x < 1$$

$$y(0) = \frac{1}{\sqrt[3]{1-0}} = \frac{1}{1} = 1$$

So, there is a local minimum at $(0, 1)$

$$66. \quad y = \begin{cases} -\frac{1}{9}x^2 - \frac{1}{2}x + \frac{15}{4}, & x \leq 1 \\ x^3 - 6x^2 + 8x, & x > 1 \end{cases}$$

Critical point:

$$\text{for: } -\frac{1}{9}x^2 - \frac{1}{2}x + \frac{15}{4}, \quad x \leq 1$$

$$-\frac{1}{2}x - \frac{1}{2} = 0$$

$$-\frac{1}{2}x = \frac{1}{2}$$

$$x = -1$$

$$\left. \begin{aligned} 0 &= 12 + \frac{\sqrt{48}}{6} = 3,15 \vee \\ &= \frac{12 - \sqrt{48}}{6} = 0,85 \times \end{aligned} \right\}$$

$$\text{for: } x^3 - 6x^2 + 8x, \quad x > 1$$

$$3x^2 - 12x + 8$$

$$\frac{12 \pm \sqrt{(12)^2 - 4(3)(8)}}{6} = \frac{12 \pm \sqrt{48}}{6}$$

Domain Endpoints

$$\text{for: } -\frac{1}{4}(-1)^2 - \frac{1}{2}(1) + \frac{15}{4}$$

$$= -\frac{1}{4} + \frac{1}{2} + \frac{15}{4}$$

$$= \frac{-1 + 2 + 15}{4} = \frac{16}{4} = 4$$

$$\text{for: } (3.15)^3 - 6(3.15)^2 + 8(3.15)$$

$$= 31.26 - 59.535 + 25.2$$

$$= -3.075$$

Extreme Value:

local max at $x = -1$ is 4

local min at $x = 3.15$ is -3.075

$$68. f(x) = |x^3 - 9x|$$

a) Does $f'(0)$ exist?

$$x^3 - 9x > 0 \rightarrow f'(x) = 3x^2 - 9 \\ = -9$$

$$-x^3 + 9x < 0 \rightarrow f'(x) = -3x^2 + 9 \\ = 9$$

-9 ≠ 9 DNE

b. Does $f'(3)$ exist?

$$3x^2 - 9 \longrightarrow f'(3) = 3(3)^2 - 9 \\ = 18$$

$$-3x^2 + 9 \longrightarrow f'(3) = -3(3)^2 + 9 \\ = -18$$

$18 \neq -18$ DNE

c. Does $f'(-3)$

$$3x^2 - 9 \longrightarrow f'(-3) = 3(-3)^2 - 9 \\ = 18$$

$$-3x^2 + 9 \longrightarrow f'(-3) = -3(-3)^2 + 9 \\ = -18$$

$18 \neq -18$ DNE

d. Determine all extrema of f .

Critical Points

$$f'(x) = 0$$

$$3x^2 - 9$$

$$= 3(x^2 - 3) = 0$$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$

$f'(x)$ DNE on $x = 0$ and $x = \pm 3$

Local Min

all $f'(x)$ change sign. So - becomes +

at $x = 0$

$$(0)^3 - g(0) \approx 0$$

at $x = 3$

$$(3)^3 - g(3) = 0$$

at $x = -3$

$$(-3)^3 - g(-3) = 0$$

Local Min at $x = 1, 3, -3$ is 0

Local Max

$$x = \pm\sqrt{3}$$

$$x^3 - g(x)$$

$$\text{for } x = \sqrt{3} \quad (\sqrt{3})^3 - g(\sqrt{3})$$

$$= -6\sqrt{3}$$

$$\text{for } x = -\sqrt{3} \quad (-\sqrt{3})^3 - g(-\sqrt{3})$$

$$= 6\sqrt{3}$$

Local Max at $x = \sqrt{3}$ is $-6\sqrt{3}$

at $x = -\sqrt{3}$ is $6\sqrt{3}$