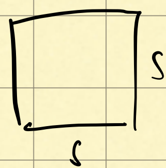
6.1

1. $A(x) = s^2$



$s = y$

$y = \sqrt{x} - (-\sqrt{x})$

$= 2\sqrt{x}$

$A(x) = \frac{1}{2}(2\sqrt{x})^2$

$= 2x$

$V = \int_0^4 2x \, dx \rightarrow = x^2 \Big|_0^4$

$= 16$

5. a. $A(x) = \frac{\sqrt{3}}{4} a^2 \rightarrow a = y$

$A(x) = \frac{\sqrt{3}}{4} (2\sqrt{\sin x})^2$

$= \frac{\sqrt{3}}{\cancel{4}} \cancel{4} \sin x$

$= \sqrt{3} \sin x$

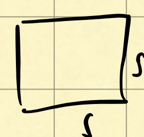
$V = \int_0^{\pi} \sqrt{3} \sin x \, dx$

$= \sqrt{3} \int_0^{\pi} \sin x \, dx$

$= -\cos x \Big]_0^{\pi} (\sqrt{3}) \rightarrow (1 - (-1))(\sqrt{3})$

$= 2\sqrt{3}$

b. $A(x) = s^2$



$s = y$

$A(x) = (2\sqrt{\sin x})^2$

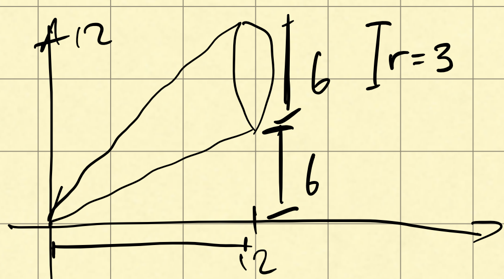
$= 4 \sin x$

$V = \int_0^{\pi} 4 \sin x \, dx$

$-\cos x \Big]_0^{\pi} (4)$

$1 - (-1) \cdot 4 = 8$

14. In the picture, we know that the height of the solid is 12 since $y=x$, which means when $x=12$, then $y=12$. In the line $y=\frac{x}{2}$, at $x=12$ then $y=6$. From there!



from the graph, $r=3$, and height is 12.

which means with the same $A(x)$, and interval $[0,12]$, which represents the height 12, we know that the V will be the same.

19. $y=x^2$, $y=0$, $x=2$

$$V=\pi\int_0^2 (x^2)^2\,dx$$

$$=\pi\int_0^2 x^4\,dx$$

$$=\frac{x^5}{5}\Big|_0^2\pi$$

$$V=\frac{32\pi}{5}$$

24. $y = \sec x,\ y=0,\ x = -\pi/4,\ x = \pi/4$

$$V = \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x \, dx$$

$$= \tan x \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \pi$$

$$= (1+1)\pi$$

$$= 2\pi$$

30. $x = \sqrt{\cos(\pi y/4)},\ -2 \le y \le 0,\ x=0$

$$V = \pi \int_{-2}^{0} \left(\sqrt{\cos(\pi y/4)}\right)^2 dy$$

$$= \pi \int_{-2}^{0} \cos \frac{\pi y}{4} \, dy$$

$$= \frac{\sin \pi y}{4\pi}\Big|_{-2}^{0} \pi$$

$$= \frac{\pi \sin \pi y}{4\pi}\Big|_{2}^{0}$$

$$= \frac{0 - \sin \pi(-2)}{4-2}$$

$$= 0 + 1 = 1$$

30. $y = 4 - x^2$, $y = 2 - x$

$y_1 = y_2$

$4 - x^2 = 2 - x$

$-x^2 + x + 4 - 2 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1)$

$x = 2$

$x = -1$

$$= \pi \int_{-1}^{2} \left(f(x)^2 - g(x)^2\right)\, dx$$

$$= \pi \int_{-1}^{2} (4-x^2)^2 - (2-x)^2\, dx$$

$$= \pi \int_{-1}^{2} x^4 - 8x^2 + 16 - x^2 + 4x - 4\, dx$$

$$= \pi \int_{-1}^{2} x^4 - 9x^2 + 4x + 12\, dx$$

$$= \frac{x^5}{5} - 3x^3 + 2x^2 + 12x \Big|_{-1}^{2} \pi$$

$$= \frac{32}{5} - 24 + 8 + 24 - \left(\frac{-1}{5} + 3 + 2 - 12\right) \Big| \pi$$

$$= \frac{32}{5} + 8 + \frac{1}{5} - 3 - 2 + 12 \Big| \pi$$

$$= \left(\frac{32 + 40 + 1 - 15 - 10 + 60}{5}\right)\pi = \frac{108\pi}{5}$$

57. $x^2+y^2=16^2$

$x^2=16^2-y^2$

$V=\int_{16}^{-7}\pi(16^2-y^2)$

$=16^2y-\frac{y^3}{3}\Big|_{-16}^{-7}\pi$

$=\pi\left((16^2)(-7)-\frac{(-7)^3}{3}-((16^2)(-16)-\frac{(-16)^3}{3}\right)$

$=\pi\cdot 1053$

$=3308\ cm^3$

59. $y=f(x)>0,\ x=a>0,\ x=b>a$, and $y=0$

$V=\int_a^b \pi(f(x))^2\,dx=4\pi$

$y=-1$

$V=\int_a^b \pi(f(x))^2\,dx=8\pi$

so, $8\pi-4\pi$

$=\pi\int_a^b ((f(x))^2+2f(x)+1-[f(x)]^2)\,dx=4\pi$

$=\int_a^b(2f(x)+1)\,dx=4$

$=2\int_a^b f(x)\,dx+\int_a^b dx=4\to\int_a^b f(x)\,dx=\frac{4-b+a}{2}$

6.2

2. $y = 2 - \frac{x^2}{4}$, $x = 2$, $y = 2$

$$V = \int_a^b 2\pi \begin{pmatrix} \text{shell} \\ \text{radius} \end{pmatrix} \begin{pmatrix} \text{shell} \\ \text{height} \end{pmatrix} dx$$

$$= 2\pi \int_0^2 x \left(2 - \frac{x^2}{4}\right) dx$$

$$= 2\pi \int_0^2 2x - \frac{x^3}{4} \, dx$$

$$= x^2 - \frac{x^4}{16} \Big|_0^2 2\pi$$

$$= (4 - 1) 2\pi$$

$$= 6\pi$$

6. $y = \frac{9x}{\sqrt{x^3+9}}$

$$V = \int_a^b 2\pi \begin{pmatrix}\text{shell}\\\text{radius}\end{pmatrix}\begin{pmatrix}\text{shell}\\\text{height}\end{pmatrix} dx$$

$$= 2\pi \int_0^3 x \left(\frac{9x}{(x^3+9)^{\frac{1}{2}}} \right) dx$$

let $x^3+9 = u$

$$= 2\pi \int_0^3 \frac{9x^2}{u^{\frac{1}{2}}} dx$$

$$= 2\pi \int_0^3 \cancel{9x^2} \cdot u^{-\frac{1}{2}} \frac{du}{\cancel{3x^2}}$$

$$= 6\pi \int_0^3 u^{-\frac{1}{2}} du$$

$$= 2u^{\frac{1}{2}} \Big|_0^3 6\pi$$

$$= 2(x^3+9)^{\frac{1}{2}} \Big|_0^3 6\pi$$

$$= (12 - 6)6\pi$$

$$= 36\pi$$

14 b. $y = \frac{\tan^2 x}{x}$, $x = \frac{\pi}{4}$

$$V = \int_a^b 2\pi \begin{pmatrix}\text{shell}\\ \text{radius}\end{pmatrix}\begin{pmatrix}\text{shell}\\ \text{height}\end{pmatrix}$$

$$= 2\pi \int_0^{\frac{\pi}{4}} \cancel{x}\left(\frac{\tan^2 x}{\cancel{x}}\right) dx$$

$$= 2\pi \int_0^{\pi/4} \tan^2 x \quad dx$$

$$= \tan x - x \Big|_0^{\frac{\pi}{4}} \; 2\pi$$

$$= 2\pi\left(1 - \frac{\pi}{4}\right)$$

$$= 2\pi - \frac{\cancel{2}\pi^2}{\cancel{4}}$$

$$= 2\pi - \frac{\pi^2}{2}$$

$$= \frac{4\pi - \pi^2}{2}$$

$$= 1.35$$

6-3

2. $y = x^{3/2}$ from $x=0$ to $x=4$

$$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$$

$$= \int_0^4 \sqrt{1+\left(\frac{3}{2}x^{\frac{1}{2}}\right)^2}\,dx$$

$$= \int_0^4 \sqrt{1+\frac{9x}{4}}\,dx$$

$$= \int_0^4 \sqrt{\frac{4+9x}{4}}\,dx$$

$$= \frac{1}{2}\int_0^4 \sqrt{4+9x}\,dx$$

Let $4+9x = u$

$$= \frac{1}{2}\int_0^4 u^{\frac{1}{2}}\,\frac{du}{9}$$

$$= \frac{1}{18}\int_0^4 u^{\frac{1}{2}}\,du$$

$$= \frac{\cancel{2}}{3}u^{\frac{3}{2}}\Big|_0^4 \frac{1}{\cancel{18}}$$

$$= u^{\frac{3}{2}}\Big|_0^4 \frac{1}{27}$$

$$= (4+9x)^{\frac{3}{2}}\Big|_0^4 \frac{1}{27}$$

$$= \left(\sqrt{40^3} - \sqrt{64}\right)\frac{1}{27}$$

$$= (252,98 - 8)\frac{1}{27}$$

$$= \frac{244,98}{27}$$

3. $x = (y^3/3) + \frac{1}{4y}$ from $y=1$ to $y=3$

$$\frac{dx}{dy} = y^2 - \frac{1}{4y^2}$$

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_1^3 \sqrt{1 + \left(y^2 - \frac{1}{4y^2}\right)^2}\, dy$$

$$= \int_1^3 \sqrt{1 + \left(y^4 - \frac{1}{2} + \frac{1}{16y^4}\right)}\, dy$$

$$= \int_1^3 \sqrt{y^4 - \frac{1}{2} + \frac{1}{16y^4}}\, dy$$

$$= \int_1^3 \sqrt{\left(y^2 - \frac{1}{4y^2}\right)^2}\, dy$$

$$= \int_1^3 y^2 - \frac{1}{4y^2}\, dy$$

$$= \frac{y^3}{3} + \frac{1}{4y} \Big|_1^3$$

$$\frac{27}{3} + \frac{1}{12} - \left(\frac{1}{3} + \frac{1}{4}\right)$$

$$= \frac{108 + 1}{12} - \frac{1}{3} - \frac{1}{4}$$

$$= \frac{109 - 4 - 3}{12} = \frac{102}{12}$$

$$= 8\frac{6}{12}$$

$$= 8\frac{1}{2}$$

11. $x = \int_0^y \sqrt{\sec^4 t - 1}\, dt$, $-\pi/4 \le y \le \pi/4$

$$L = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{\sec^4 y}\; dy$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 y \; dy$$

$$= \tan y \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$

$$= 1 + 1 = 2$$

23. $y = \int_0^x \sqrt{\cos 2t}\, dt$, from $x=0$ to $x = \pi/4$

$$L = \int_0^{\frac{\pi}{4}} \sqrt{1 + \cos 2x}\; dx$$

$$= \sqrt{2} \;\sin x \Big|_0^{\pi/4}$$

$$= \sqrt{2} \sin 45$$

$$= 1$$

$30.$ a) $m = f'(x_{k-1})$

$$AC^2 = \Delta x^2 k + BC^2$$

$$AC^2 = \Delta x^2 k + (\Delta x_k \tan\theta)^2$$

$$AC = \sqrt{(\Delta x^2 k)^2 + (f'(x_{k-1})\Delta x_k)^2}$$

b) $$AC = \sqrt{(\Delta x^2 k)^2 + (f'(x_{k-1})\Delta x_k)^2}$$

$$\Delta x_k = \frac{b-a}{n}$$

$$f'(x_{k-1}) = f'(x)$$

$$= \sum \sqrt{1 + f'(x)^2}\, \Delta x_k$$

$$= \sum \sqrt{1 + (f'(x))^2}\, \frac{b-a}{n}$$

$$= \int_a^b \sqrt{1 + (f'(x))^2}$$

6.4

13. $y=\frac{x^3}{9}$, $0 \le x \le 2$; x axis

$$f'(x)=\frac{x^2}{3}$$

$$S=\int_a^b 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx$$

$$=\int_0^2 2\pi\left(\frac{x^3}{9}\right)\sqrt{1+\left(\frac{x^2}{3}\right)^2}\,dx$$

$$=2\pi\left(\frac{x^3}{9}\right)\int_0^2\sqrt{1+\frac{x^4}{9}}\,dx$$

$$=2\pi\left(\frac{x^3}{9}\right)\int_0^2\sqrt{\frac{9+x^4}{9}}\,dx$$

$$=\frac{2\pi}{3}\left(\frac{x^3}{9}\right)\int_0^2\sqrt{9+x^4}\,dx$$

let $9+x^4=u$

$$=\frac{2\pi}{3}\left(\frac{\cancel{x^3}}{9}\right)\int_0^2 u^{\frac{1}{2}}\,\frac{du}{4\cancel{x^3}}$$

$$=\frac{\cancel{2}\pi}{\cancel{108}\ 54}\int_0^2 u^{\frac{1}{2}}\,du$$

$$=\frac{2}{3}u^{\frac{3}{2}}\Big|_0^2\ \frac{\pi}{54}$$

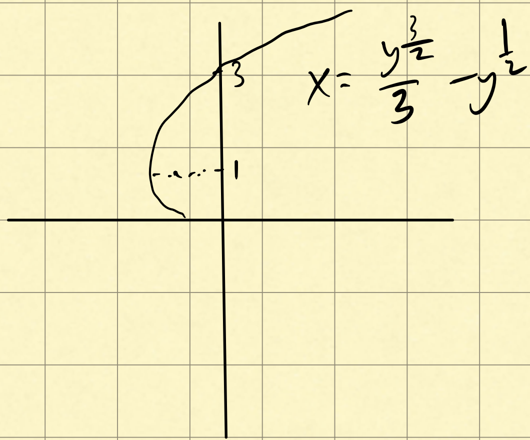
$$=\frac{2}{3}(9+x^4)^{\frac{3}{2}}\Big|_0^2\ \frac{\pi}{54}$$

$$=\left(\frac{250}{3}-\frac{54}{3}\right)\frac{\pi}{54}$$

$$=\frac{\cancel{196}\pi}{\cancel{162}}$$

$$=\frac{98\pi}{81}$$

18. $X = \left(\frac{1}{3}\right)y^{\frac{3}{2}} - y^{\frac{1}{2}}$, $1 \leq y \leq 3$; y-axis



$$A = \int_a^b x \, dy$$

$$= \int_1^3 \left(\frac{y^{\frac{3}{2}}}{3} - y^{\frac{1}{2}}\right) dy$$

$$= \frac{1}{3}\int_1^3 y^{\frac{3}{2}} - 3y^{\frac{1}{2}} \, dy$$

$$= \frac{2}{5}y^{\frac{5}{2}} - 2y^{\frac{3}{2}} \Big|_1^3 \frac{1}{3}$$

$$= \frac{2(y-5)\,y^{3/2}}{5} \Big|_1^3 \frac{1}{3}$$

$$= \frac{2}{15}\left(-2\sqrt{27} + 4\right)$$

$$= 0.852$$

22. $y = \frac{1}{3}(x^2+2)^{\frac{3}{2}}$, $0 \le x \le \sqrt{2}$

$$f'(x) = \frac{1}{\cancel{2}}(x^2+2)^{\frac{1}{2}} \cdot \cancel{2}x$$

$$= x\sqrt{x^2+2}$$

$$L = \int_0^{\sqrt{2}} \sqrt{1+\left(x\sqrt{x^2+2}\right)^2}\, dx$$

$$= \int_0^{\sqrt{2}} \sqrt{1+x^2(x^2+2)}\, dx$$

$$= \int_0^{\sqrt{2}} \sqrt{1+x^4+2x^2}\, dx$$

$$= \int_0^{\sqrt{2}} \sqrt{(x^2+1)^2}\, dx$$

$$= \int_0^{\sqrt{2}} x^2+1\, dx$$

$$= \frac{x^3}{3}+x \Big|_0^{\sqrt{2}}$$

$$= \frac{2\sqrt{2}}{3}+\sqrt{2} = \frac{5\sqrt{2}}{3}$$

26.

$$y = \left(\frac{r}{h}\right)x$$

$$f'(x) = \frac{r}{h}$$

$$(f'(x))^2 + 1$$

$$= \frac{r^2}{h^2} + 1$$

$$S = 2\pi \int_0^h \frac{rx}{h} \sqrt{\frac{r^2+h^2}{h^2}}\, dx$$

$$= 2\pi \sqrt{\frac{r^2+h^2}{h^2}} \int_0^h \frac{rx}{h}\, dx$$

$$= \frac{2\pi r\sqrt{r^2+h^2}}{h^2} \int_0^h x\, dx$$

$$= \frac{x^2}{\cancel{2}}\Big|_0^h \frac{\cancel{2}\pi r\sqrt{r^2+h^2}}{h^2}$$

$$= \cancel{h^2}\left(\frac{\pi r\sqrt{r^2+h^2}}{\cancel{h^2}}\right)$$

$$= \pi r\sqrt{r^2+h^2}$$

28. $y = \sqrt{r^2 - x^2}$

$= (r^2 - x^2)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(r^2 - x^2)^{-\frac{1}{2}} \cdot -2x$

$= \frac{-\cancel{2}x}{\cancel{2}}(r^2 - x^2)^{-\frac{1}{2}}$

$= \frac{-x}{\sqrt{r^2 - x^2}}$

$(f'(x))^2 = \frac{x^2}{r^2 - x^2}$

$$S = 2\pi \int_a^{a+h} \sqrt{r^2 - x^2} \sqrt{1 + \frac{x^2}{r^2 - x^2}} \, dx$$

$$= 2\pi \int_a^{a+h} \frac{\sqrt{\cancel{r^2 - x^2}}\,(r^2)}{\cancel{r^2 - x^2}} \, dx$$

$$= 2\pi \int_a^{a+h} r \, dx$$

$= 2\pi r (a + h - a)$

$= 2\pi r h$

30. a) $x^2+y^2=15^2$

$x^2=15^2-y^2$

$x=\sqrt{15^2-y^2}$

$\frac{dx}{dy}=\frac{-y}{\sqrt{15^2-y^2}}$

$\left(\frac{dx}{dy}\right)^2=\frac{y^2}{15^2-y^2}$

$S=2\pi\int_{-7.5}^{15}\sqrt{15^2-y^2}\sqrt{1+\frac{y^2}{15^2-y^2}}\,dy$

$=2\pi\int_{-7.5}^{15}\sqrt{\frac{\cancel{15^2-y^2}(15^2)}{\cancel{15^2-y^2}}}\,dy$

$=2\pi\int_{-7.5}^{15}15$

$=30\pi(15+7.5)$

$=675\pi\ m^2$

6.5

2. natural length = 10 cm, $F = 800$ N, stretch to 14 cm

a) $F = kx$

$800 = k(0.04)$

$k = 20000\ N/m$

b) $\Delta x = 12 - 10$
$= 2$

$W = \frac{k\Delta x^2}{2}$

$= \frac{20000 \cdot \cancel{2}^2}{\cancel{2}}$

$= 40000\ Nm$

c) $F = 1600$ N

$F = k\Delta x$

$1600 = 20000\ \Delta x$

$\frac{16}{200} = \Delta x$

$\Delta x = 0.08\ m$

10. $f(x) = \frac{k}{x^2}$

$$W = \int_b^a \frac{k}{x^2}\, dx$$

$$= \left.\frac{-k}{x}\right|_b^a$$

$$= \frac{-k}{a} + \frac{k}{b}$$

$$= \frac{-ka + kb}{ab} = \frac{k(-a+b)}{ab}$$

23. $x = \frac{dx}{dt}$

$$F = m\left(\frac{dv}{dt}\right)$$

$$= m\, v\, \frac{dv}{dx}$$

$$W = \int_{v_1}^{v_2} m v \frac{dv}{\cancel{dx}} \cancel{dx}$$

$$= \int_{v_1}^{v_2} m v \; dv$$

$$= \left.\frac{mv^2}{2}\right|_{v_1}^{v_2}$$

$$= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

29. $\Delta V = \pi (r)^2 (h)$

$$= \pi \left(\frac{y+36}{12}\right)^2 \Delta y \text{ cm}^3$$

$$f(y) = \frac{4}{9} \Delta V$$

$$= \frac{4\pi}{9} \left(\frac{y+36}{12}\right)^2 \Delta y \text{ cm}^3$$

$$\Delta W = \left(\frac{4}{9}\pi\right) \frac{(y+36)^2}{12^2} (21-y)$$

$$W = \sum_{0}^{18} \frac{4}{9}\pi \frac{(y+36)^2}{12^2} (21-y)$$

$$= \int_0^{18} \frac{4\pi}{9 \cdot 12^2} (y+36)^2 (21-y) \; dy$$

$$\frac{4\pi}{9 \cdot 12^2} \int_0^{18} (-51y^2 - y^3 + 216y + 27216) \; dy$$

$$= 3873.5$$

36. $x^2+y^2=5^2$

$x=\sqrt{25-y^2}$

$=2x$

$=2\sqrt{25-y^2}$

$F = 9800\int_0^5 (6-y)(2\sqrt{25-y^2})\,dy$

$= 14600\int_0^5 6\sqrt{25-y^2} - y\sqrt{25-y^2}\,dy$

Area of quarter circle:

$\frac{\pi r^2}{4} = \frac{25\pi}{4}$

$\int_0^5 \sqrt{25-y^2}\,dy = \frac{25\pi}{4}$

let $u = 25-y^2$

$\int_0^5 \sqrt{u}\,\frac{du}{-2}$

$= \frac{\cancel{2}}{3} u^{\frac{3}{2}}\Big|_0^5 \cdot -\frac{1}{\cancel{2}}$

$= -\frac{1}{3}\sqrt{u^3}\Big|_0^5$

$= -1\left(\frac{\sqrt{0}}{3} - \frac{\sqrt{25^3}}{3}\right)$

$= \frac{125}{3}$

$F = 9800\left(\overset{3}{\cancel{6}}\,\frac{25\pi}{\underset{2}{\cancel{4}}} - \frac{125}{3}\right)$

$= 9800\left(\frac{225\pi - 250}{6}\right)$

$= 746202$ N

47. $x^2+y^2=1$

$x=\sqrt{1-y^2}$

$F = 9800\int_{-1}^{0}\left(\sqrt{1-y^2}\right)(-2y)\,dy$

upper limit $=1$

lower limit $=0$

Let $1-y^2=u$

$= 9800\int_0^1 (\sqrt{u})\cancel{(-2y)}\ \dfrac{du}{\cancel{-2y}}$

$= 9800\int_0^1 \sqrt{u}\,du$

$= \frac{2}{3}u^{\frac{3}{2}}\Big|_0^1 9800$

$= 9800\ \frac{2}{3}$

$= 2178$

So, the tank will overflow since the movable end moves 2.18m before the tank overflow