4.6

4. $f(x) = 2x - x^2 + 1$

$f'(x) = -2x + 2$

$x_0 = 0$, $x_0 = 2$

Newton's method: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

LHS

when $x_0 = 0$, $f(0) = 2(0) - (0)^2 + 1 = 1$, $f'(0) = -2(0) + 2$

$x_1 = 0 - \frac{f(x_0)}{f'(x_0)}$ $= 2$

$x_1 = 0 - \frac{1}{2}$

$x_1 = -\frac{1}{2}$

when $x = -\frac{1}{2}$ $f(\frac{1}{2}) = 2(-\frac{1}{2}) - (-\frac{1}{2})^2 + 1$, $f'(-\frac{1}{2}) = -2(-\frac{1}{2}) + 2$

$x_2 = -\frac{1}{2} - \frac{f(x_1)}{f'(x_1)}$ $= -1 - \frac{1}{4} + 1 = -\frac{1}{4}$ $= 3$

$x_2 = -\frac{1}{2} + \frac{\frac{1}{4}}{3} = -\frac{1}{2} + \frac{1}{3} \cdot \frac{1}{4} = \frac{-6+1}{12} = \frac{-5}{12}$

RHS

when $x_0 = 2$ $f(2) = 2(2) - (2)^2 + 1 = 1$, $f'(2) = -2(2) + 2$

$x_1 = 2 - \frac{f(x_0)}{f'(x_0)}$ $= -2$

$x_2 = \frac{5}{2} - \frac{f(x_1)}{f'(x_1)}$ $f(\frac{5}{2}) = 2(\frac{5}{2}) - (\frac{5}{2})^2 + 1 = -\frac{1}{4}$

$x_1 = 2 + \frac{1}{2}$

$= \frac{5}{2}$

$x_2 = \frac{5}{2} - \frac{(-\frac{1}{4})}{(-3)}$ $f'(\frac{5}{2}) = -2(\frac{5}{2}) + 2$

$= \frac{29}{12}$ $= -3$

7. Note that:

x_0 = root of $f(x)=0$

Assume that $f'(x_0)$ is defined and not 0

Q.

what happens to x_1 and later approximations?

Ans:

$$X_{n+1} = X_n - \frac{f(x_n)}{f'(x_n)}$$

Since $f(x_0)=0$, then $X_1 = 0$. It applies to $\forall n \geq 0$

So, $X_n, \forall n \geq 0$ will equal to x_0.

9. Note that:

if $h>0$, applying Newton's method to

$$f(x) = \begin{cases} \sqrt{x}, & x \geq 0 \\ \sqrt{-x}, & x < 0 \end{cases}$$

leads to $x_1 = -h$ if $x_0 = h$ and to $x_1 = h$ if $x_0 = -h$.

Q.

Draw a picture that shows what is going on.

Ans:

When $x_0 = h > 0$:

$$X_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= h - \frac{f(h)}{f'(h)} = h - \frac{\sqrt{h}}{\frac{1}{2\sqrt{h}}}$$

$= h - (\sqrt{h})(2\sqrt{h})$

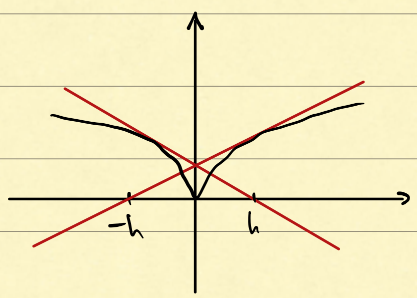
$= h - 2h = -h$

When $x_0 = -h < 0$

$x_1 = -h - \frac{f(h)}{f'(h)} = -h - \frac{\sqrt{-h}}{\frac{1}{2\sqrt{h}}}$

$= -h + 2h$

$= h$



10. Note that:

Apply Newton's method to $f(x) = x^{\frac{1}{3}}$ with $x_0 = 1$ and calculate x_1, x_2, x_3 and x_4.

Q.

Find a formula for $|x_n|$. What happens to $|x_n|$ as $n \to \infty$? Draw a picture that shows what is going on.

Ans:

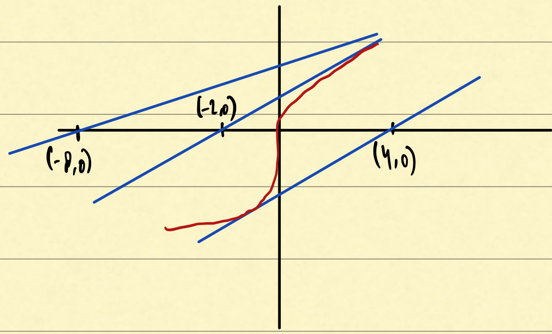
$f(x) = x^{\frac{1}{3}}$, $f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

$x_{n+1} = x_n - \frac{\cancel{x^{\frac{1}{3}}}}{\frac{1}{3}\cancel{x^{-\frac{2}{3}}}}$

$= -2x_n$

$x_1 = -2(1) = -2$, $x_2 = -2(-2) = 4$, $x_3 = -2(4) = -8$, $x_4 = -2(-8) = 16$

Since $|X_n| = 2|X_{n-1}|$, $n \to \infty = |X_n| \to \infty$



4.7

12. a) $\pi \cos \pi x$

$$\int \pi \cos \pi x \, dx$$

$$= \sin(\pi x) + C$$

b) $\frac{\pi}{2} \cos \frac{\pi x}{2}$

$$\int \frac{\pi}{2} \cos \frac{\pi x}{2} \, dx$$

$$\frac{\pi}{2} \frac{2}{\pi} \left(\sin \frac{\pi x}{2}\right) + C$$

$$= \sin \frac{\pi x}{2} + C$$

c) $\frac{\cos \pi x}{2} + \pi \cos x$

$$\int \frac{\cos \pi x}{2} + \pi \cos x \, dx$$

$$= \frac{2 \sin \frac{\pi x}{2}}{\pi} + \pi \sin x + C$$

28. $$\int \left(\frac{\sqrt{x}}{2} + \frac{2}{\sqrt{x}}\right) dx$$

$$= \frac{1}{2}\int \sqrt{x} + 2\int \frac{1}{\sqrt{x}}$$

$$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} x^{\frac{3}{2}} + 2 \cdot 2\sqrt{x}$$

$$= \frac{x^{\frac{3}{2}}}{3} + 4\sqrt{x} + C$$

32. $$\int x^{-3}(x+1) \, dx$$

$$\int \frac{x}{x^3} + \frac{1}{x^3}$$

$$\int x^{-2} + x^{-3}$$

$$= -x^{-1} - 2x^{-2} + C$$

$$= -\frac{1}{x} - \frac{1}{2x^2} + C$$

34. $\int \frac{4+\sqrt{t}}{t^3}\,dt$

$\int \frac{4}{t^3} + \frac{t^{\frac{1}{2}}}{t^3}$

$4\int \frac{1}{t^3} + \frac{1}{t^{\frac{5}{2}}} = -\frac{4}{2t^2} - \frac{2}{3t^{\frac{3}{2}}}$

$= -\frac{2}{t^2} - \frac{2}{3t^{\frac{3}{2}}} + C$

43. $\int (4\sec x \tan x - 2\sec^2 x)\,dx$

$= 4\sec x - 2\tan x + C$

56. $\int \frac{\csc\theta}{\csc\theta - \sin\theta}\,d\theta$

$= \int \frac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta} - \sin\theta}\,d\theta$

$= \int \frac{\frac{1}{\cancel{\sin\theta}}}{\frac{1-\sin^2\theta}{\cancel{\sin\theta}}} = \int \frac{1}{\cos^2\theta}$

$= \tan\theta + C$

86. $\frac{d^2s}{dt^2} = \frac{3t}{8}$; $\frac{ds}{dt}\Big|_{t=4} = 3$, $s(4) = 4$

$\frac{ds}{dt} = \frac{3}{8}\int t$

$= \frac{3}{16}t^2 + C_1$

$s'(4) = \frac{3}{\cancel{16}}\cancel{4^2} + C_1 = 3 + C$

$3 = 3 + C_1$

$C_1 = 0$

$\int \frac{3}{16}t^2$

$= \frac{t^3}{16} + C_2$

$s(4) = \frac{4^3}{16} + C_2 = 4$

$C_2 = 0$

$s(t) = \frac{t^3}{16}$

96. $\frac{dy}{dx} = \frac{1}{2\sqrt{x}} + \pi \sin \pi x$

point: $(1,2)$

$\int \frac{1}{2\sqrt{x}}\,dx + \int \pi \sin \pi x \, dx$

$y = \sqrt{x} + \cos \pi x + C$

$2 = \sqrt{1} + \cos x(1) + C$

$2 = 1 - 1 + C$

$2 = C$

$y = \sqrt{x} + \cos \pi x + 2$

100. Note that:

require to brake from 48 km/h (13.3 m/s) to 0 in 13.7 m

Q.

What constant deceleration does it take to do that?

Ans:

$\frac{d^2s}{dt^2} = -k$

$\frac{ds}{dt} = \int -k\,dt$

$= -kt + C$

$t_0 = 13.3\,m/s$

$13.3\,m/s = -k(0) + C$

$C = 13.3$

$\frac{ds}{dt} = -kt + 13.3$

$s = \frac{-kt^2}{2} + 13.3t + C_1 \longrightarrow s = 0, t = 0$

$s = \frac{-kt^2}{2} + 13.3t + 0$ $\qquad 0 = -\frac{k(0)^2}{2} + 13.3(0) + C_1$

$C_1 = 0$

therefore, $\frac{ds}{dt} = 0$

$-kt + 13.3 = 0$

$t = \frac{13.3}{k}$

$$S = \left(\frac{13.3}{k}\right) = \frac{-k\left(\frac{13.3}{k}\right)^2}{2} + 13.3\left(\frac{13.3}{k}\right) = 13.7$$

$$= \frac{-88.445}{k} + \frac{176.89}{k} = 13.7$$

$$k = \frac{88.445}{13.7} = 6.445\,m/s^2$$

103. Note that:

$$S = \frac{a}{2}t^2 + V_0 t + S_0$$

$$\underbrace{V_0 = \text{body's Velocity}, \quad S_0 = \text{body's position}}_{\text{at } t=0}$$

Ans:

$$\frac{d^2 s}{dt^2} = a$$

$$\frac{ds}{dt} = at + c$$

$C = V_0$ since $t = 0$, $\frac{ds}{dt} = V_0$

$$\frac{ds}{dt} = at + V_0$$

$S = \frac{at^2}{2} + V_0 t + C_1$ is $= S_0$

If $t = 0$, $S_0 = \frac{a(0)^2}{2} + V_0(0) + C_1$

$C_1 = S_0 \rightarrow s = \frac{at^2}{2} + V_0 t + S_0$

5.1

4. $f(x) = 4 - x^2$ between $x = -2$ and $x = 2$

$m = 2$, $\Delta x = \frac{2+2}{2} = 2$	$m = 4$, $\Delta x = \frac{4}{4} = 1$
left end point $= -2, 0$	left endpoint $= -2, -1, 0, 1$
right end point $= 0, 2$	right endpoint $= -1, 0, 1, 2$

a) lower sum two rec: $\Delta x\,(f(-2) + f(0)) = 2(4) = 8$

b) lower sum four rec: $\Delta x\,(f(-2) + f(-1) + f(0) + f(1)) = 10$

c) upper sum two rec: $\Delta x\,(f(0) + f(2)) = 8$

d) upper sum four rec: $\Delta x\,(f(-1) + f(0) + f(1) + f(2)) = 10$

11. a) $10(0 + 15 + 5 + 12 + 10 + 15 + 12 + 5 + 7 + 12 + 15 + 10)$

$= 10(118) = 1180\text{ m}$

b) $10(15 + 5 + 12 + 10 + 15 + 12 + 5 + 7 + 12 + 15 + 10 + 12)$

$= 10(130) = 1300\text{ m}$

5.2

30. a. $\sum_{k=9}^{36} k = \sum_{k=1}^{36} - \sum_{k=1}^{8} = \frac{36(37)}{2} - \frac{8(9)}{2}$

$= 666 - 36$

$= 630$

b. $\sum_{k=3}^{17} k^2 = \sum_{k=1}^{17} k^2 - \sum_{k=1}^{2} k^2 = \frac{17(\cancel{18}^{9})(35)}{\cancel{2}} - \frac{\cancel{2(3)}(5)}{\cancel{6}}$

$= 1780$

c. $\sum_{k=18}^{71} k(k-1) = \sum_{k=18}^{71} k^2 - k = \left(\sum_{k=1}^{71} k^2 - \sum_{k=1}^{17} k^2\right) - \left(\sum_{k=1}^{71} k - \sum_{k=1}^{17} k\right)$

$= \left(\frac{71(72)(143)}{6} - \frac{17(18)(35)}{6}\right) - \left(\frac{71(72)}{2} - \frac{17(18)}{2}\right)$

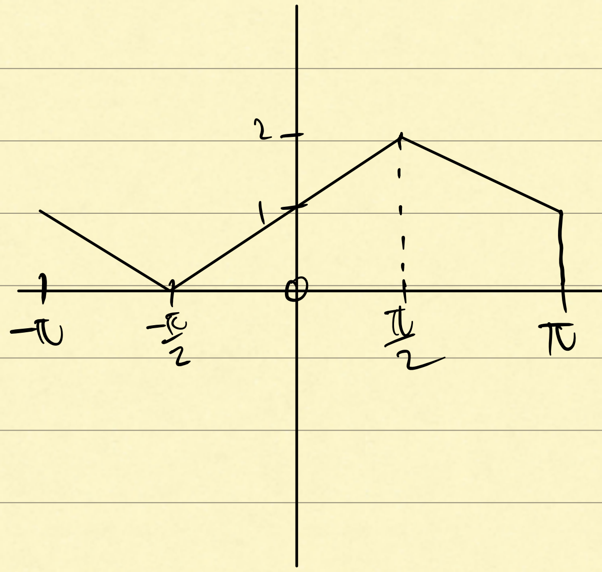
$= 120{,}051 - 2403 = 117.648$

32. a. $\sum_{k=1}^{n}\left(\frac{1}{n} + 2n\right) = \sum_{k=1}^{n} \frac{1}{n} + \sum_{k=1}^{n} 2n = 1 + 2n^2$

b. $\sum_{k=1}^{n} \frac{c}{n} = c\sum_{k=1}^{\cancel{n}} \frac{1}{\cancel{n}} = C$

c. $\sum_{k=1}^{n} \frac{k}{n^2} = k\sum_{k=1}^{\cancel{n}} \frac{1}{n^{\cancel{2}}} = \frac{k}{n}$

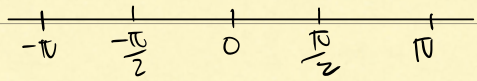
36. $f(x) = \sin x + 1$, $[-\pi, \pi]$



$\Delta x = \frac{b-a}{n}$

$= \frac{\pi + \pi}{4}$ → we use 4 rectangles

$= \frac{\cancel{2}\pi}{\cancel{4}} = \frac{\pi}{2}$



a) $\frac{\pi}{2}\left(f(-\pi) + f\left(-\frac{\pi}{2}\right) + f(0) + f\left(\frac{\pi}{2}\right)\right)$

$= \frac{\pi}{\cancel{2}}(\cancel{4}^{2})$

$= 2\pi$

b) $\frac{\pi}{2}\left(f\left(-\frac{\pi}{2}\right) + f(0) + f\left(\frac{\pi}{2}\right) + f(\pi)\right)$

$= 2\pi$

c) $\frac{\pi}{2}\left(f\left(-\frac{3\pi}{4}\right) + f\left(-\frac{\pi}{4}\right) + f\left(\frac{\pi}{4}\right) + f\left(\frac{3\pi}{4}\right)\right)$

$= \frac{\pi}{2}\left(4 - \frac{4}{\sqrt{2}}\right)$

$= \pi(2 - \sqrt{2})$

38. Find the norm of the partition $P = \{-2, -1.6, -0.5, 0, 0.8, 1\}$.

$|x_1 - x_0| = |-1.6 + 2| = 0.4$
$|x_2 - x_1| = |-0.5 + 1.6| = 1.1$
$|x_3 - x_2| = |0 + 0.5| = 0.5$
$|x_4 - x_3| = |0.8 - 0| = 0.8$
$|x_5 - x_4| = |1 - 0.8| = 0.2$

the largest is $\|P\| = 1.1$

40. $f(x) = 2x$ over the interval $[0, 3]$

$\Delta x = \frac{b-a}{n}$ $\qquad c_k = 0 + k\Delta_x = \frac{3k}{n}$

$= \frac{3-0}{n}$ $\qquad \sum_{k=1}^{n} f(c_k)\,\Delta x$

$= \frac{3}{n}$ $\qquad = \sum_{k=1}^{n} \left(\frac{3k}{n}\right)\frac{3}{n}$

$= \left(\frac{6}{n}\right)\left(\frac{3}{n}\right)\sum_{k=1}^{n} k$

$= \frac{18}{n^2}\,\frac{n(n+1)}{2}$

$= \frac{\cancel{18}^{9}\cancel{n}(n+1)}{\cancel{2}n^2}$

$= \frac{9(n+1)}{n}$

Area:
$\lim_{n\to\infty} \frac{9(n+1)}{n}$

$$\lim_{n\to\infty} \frac{9\cancel{n}+9}{\cancel{n}}$$

$$\lim_{n\to\infty} 9 + \cancel{\frac{9}{n}}$$

$$= 9$$

44. $f(x) = 3x + 2x^2$ over the interval $[0,1]$

$$\Delta x = \frac{b-a}{n}$$

$$= \frac{1-0}{n}$$

$$= \frac{1}{n}$$

$$c_k = \frac{k}{n}$$

$$\sum_{k=1}^{n} f(c_k)\,\Delta x$$

$$\sum_{k=1}^{n} f\left(\frac{k}{n}\right)\frac{1}{n}$$

$$\sum_{k=1}^{n}\left(3\left(\frac{k}{n}\right)+2\left(\frac{k}{n}\right)^2\right)\frac{1}{n}$$

$$= \left(\frac{3k}{n} + \frac{2k^2}{n}\right)\frac{1}{n}$$

$$= \frac{3}{n^2}\cdot\frac{n(n+1)}{2} + \frac{\cancel{2}}{n^2}\cdot\frac{n(n+1)(2n+1)}{\cancel{6}3}$$

$$= \frac{3\cancel{n}(n+1))}{2n\cancel{^2}} + \frac{n(n+1)(2n+1)}{3n^2}$$

$$= \frac{3(n+1)}{2n} + \frac{n(n+1)(2n+1)}{3n^2}$$

$$\text{Area}: \lim_{n\to\infty} \frac{3\cancel{n}(1+\frac{1}{n})}{2\cancel{n}} + \frac{\cancel{n}(1+\frac{1}{n})(2+\frac{1}{n})}{3n\cancel{^2}}$$

$$= \frac{3}{2} + \frac{2}{3} = \frac{9+4}{6} = \frac{13}{6}$$