2.4

2. a) True   b) False
c) False   d) True
e) True   f) True
g) True   h) True
i) True   j) False
k) True

5. a) DNE, since there is nothing that is approaching from the right when $\lim_{x \to 0^+}$

b) exist, it is 0

c) DNE, It is only approached from the left.

18. a) $\lim_{x \to 1^+} \frac{\sqrt{2x}(x-1)}{|x-1|} = \frac{\sqrt{2x}\cancel{(x-1)}}{\cancel{x-1}} = \sqrt{2x}$

$= \sqrt{2}$

b) $\lim_{x \to 1^-} \frac{\sqrt{2x}(x-1)}{|x-1|} = \frac{\sqrt{2x}\cancel{(x-1)}}{-\cancel{(x-1)}} = \frac{\sqrt{2x}}{-1} = -\sqrt{2}$

20. a) $\lim_{t \to 4^+} (t - \lfloor t \rfloor)$

$= 4 - 4$

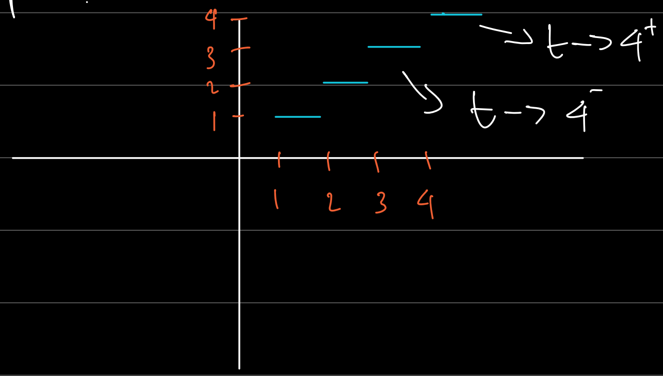
$= 0$

b) $\lim_{t \to 4^-} (t - \lfloor t \rfloor)$

$= 4 - 3$

$= 1$

graph :



32. $\lim\limits_{x\to 0} \dfrac{x - x\cos x}{\sin^2 3x} = \dfrac{x(1-\cos x)}{\sin^2 3x}$

$$= \frac{x\left(1-\left(1-2\sin^2\left(\frac{x}{2}\right)\right)\right)}{\sin^2 3x} = \frac{x\left(1+1\sin^2\left(\frac{x}{2}\right)\right)}{\sin^2 3x} = \frac{x\left(2\sin^2\left(\frac{x}{2}\right)\right)}{\sin^2 3x}$$

$$\frac{2x\,\cancel{\sin \tfrac{1}{2}x}\,\cancel{\sin\tfrac{1}{2}x}}{\cancel{\sin 3x}\,\cancel{\sin 3x}} = \frac{2(0)\,\frac{1}{4}}{9} = \boxed{0}$$

34. $\lim\limits_{h\to 0} \dfrac{\sin(\sin h)}{\sin h}$

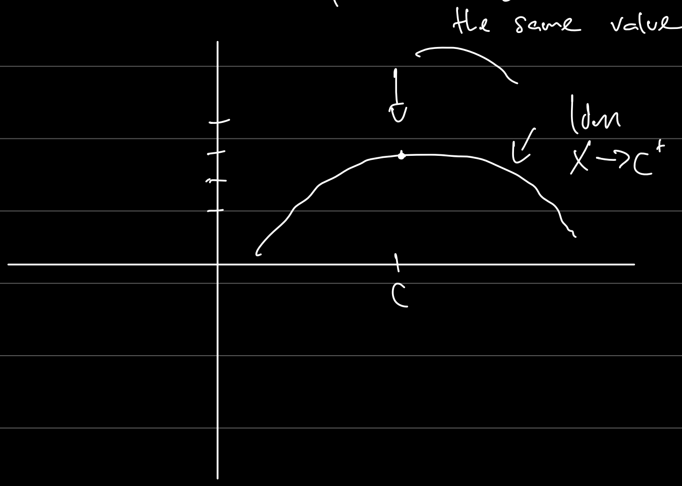
if $\sin h = x$, then $\dfrac{\sin x}{x} = \boxed{1}$

38. $\lim\limits_{\theta\to 0} \sin\theta \cot 2\theta$

$$\sin\theta \cdot \frac{\cos 2\theta}{\sin 2\theta} = \frac{\sin\theta \cdot \cos 2\theta \cdot \frac{\theta}{\theta}\cdot\frac{2\theta}{2\theta}}{\sin 2\theta \cdot \frac{\theta}{\theta}\cdot\frac{2\theta}{2\theta}}$$

$$= \frac{\cancel{\theta}\cdot\cos 2\theta}{2\cancel{\theta}} = \frac{\cos 2\theta}{2} = \frac{\cos 0}{2} = \boxed{\frac{1}{2}}$$

49. If $\lim_{x\to c} f(x)$ exists, then it is approached from both left and right. By looking from the $\lim_{x\to c^+} f(x)$ which is from the right, the value is the same as $\lim_{x\to c} f(x)$. Such that the following graph:

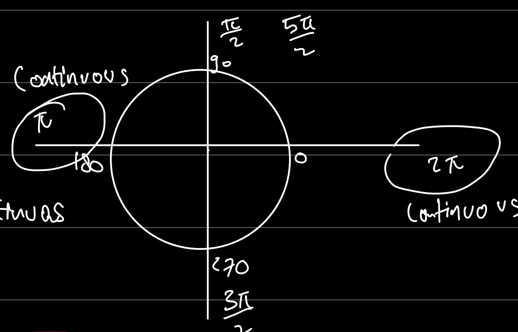


46. If f is an even function of x, then f of x is equal to f of $-x$ for all the values of x. Which means that $\lim_{x\to 2^-}$ is equal to $\lim_{x\to -2^-} f(x)$ and $\lim_{x\to -2^+}$.

2.5

22. $y = \tan \frac{\pi x}{2}$

$= \dfrac{\sin \frac{\pi x}{2}}{\cos \frac{\pi x}{2}} \neq 0$ to be continuous



Continuous at all integers except when $\frac{x\pi}{2}$, where x is an odd integer

26. $y = \sqrt[4]{3x-1} =$ x has to be $> \frac{1}{3}$ to be continuous since when $x \leq \frac{1}{3}$ the function will be undefined

30. $f(x) = \begin{cases} \frac{x^3-8}{x^2-4}, & x \neq 2,\ x \neq -2 \\ 3, & x = 2 \\ 4, & x = -2 \end{cases}$

when $x = 2$, $\lim_{x \to 2} \frac{x^3-8}{x^2-4} = \frac{\cancel{(x-2)}(x^2+2x+2^2)}{\cancel{(x-2)}(x+2)}$

$= \frac{4+4+4}{4} = \frac{12}{4} = 3$

when $x = -2$, $\lim_{x \to -2} \frac{x^3-8}{x^2-4} = \frac{\cancel{(x-2)}(x^2+2x+2^2)}{\cancel{(x-2)}(x+2)}$

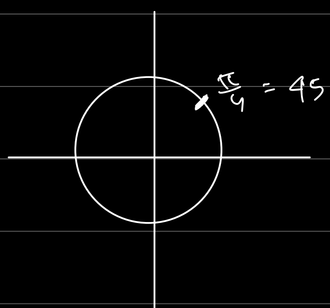
$= \frac{4-4+4}{0} = \text{DNE}$

$\therefore$ therefore $f(x)$ when $x=2$ is continous

34. $\lim_{x \to 0} \tan\left(\frac{\pi}{4} \cos\left(\sin x^{\frac{1}{3}}\right)\right)$

$$= \frac{\sin\left(\frac{\pi}{4}\cos\left(\sin x^{\frac{1}{3}}\right)\right)}{\cos\left(\frac{\pi}{4}\cos\left(\sin x^{\frac{1}{3}}\right)\right)}$$

$$= \frac{\sin\left(\frac{\pi}{4}\cos 0\right)}{\cos\left(\frac{\pi}{4}\cos 0\right)} = \frac{\sin\left(\frac{\pi}{4}\cdot 1\right)}{\cos\left(\frac{\pi}{4}\cdot 1\right)}$$



$$= \frac{\sin\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)} = \frac{\sin 45}{\cos 45}$$

$$= 1$$

∴ It is continuous since it is not 0

46. $g(x) \begin{cases} \frac{x-b}{b+1} & x < 0 \\ x^2 + b & x > 0 \end{cases}$

$$\frac{x-b}{b+1} = x^2 + b$$

$$x - b = x^2 + b(b+1)$$

$$0 - b = b(b+1)$$

$$-b = b^2 + b$$

$$b^2 + b + b = 0$$

$$b^2 + 2b = 0$$

$b(b+2)=0$

$b=0 \quad b=-2$

48. $g(x)\begin{cases} 9x+2b & x\leq 0 \\ x^2+3a-b & 0<x\leq 2 \\ 3x-5 & x>2 \end{cases}$

$\lim_{x\to 0^-} g(x) = \lim_{x\to 0^+} g(x)$

$9x+2b = x^2+3a-b$

$9(0)+2b = 0^2+3a-b$

$2b = 3a-b$

$3b = 3a$

$b-a=0$

$\lim_{x\to 2^-} g(x) = \lim_{x\to 2^+} g(x)$

$x^2+3a-b = 3x-5$

$4+3a-b = 6-5$

$3a-b = -3$

$$\begin{array}{r} 3a-b=-3 \\ a-b=0 \\ \hline 2a=-3 \end{array} \quad -$$

$a = -\frac{3}{2}$

$b+\frac{3}{2}=0$

$b=-\frac{3}{2}$

56. $f(x) = (x-a)^2 (x-b)^2 + x$ takes the value $\frac{a+b}{2}$ for some x

Using the intermediate Value theorem where $a < c < b$

input a $\to f(a) = (a-a)^2 (a-b)^2 + a$

$= a$

input b $\to f(b) = (b-a)^2 (b-b)^2 + b$

$= b$

} the interval is $[a, b]$

According to the theorem, $\frac{a+b}{2}$ is between the interval $[a,b]$, and $a < b$

Proof: for $a < b$, we can say that,

$$a < \frac{a+b}{2} < b$$

$$\downarrow$$

$\frac{a+b}{2}$ here is $f(c)$

65. Yes, it is true since using the intermediate value theorem, a continuous function will change signs when it is 0 at some point. e.g. $-2 < f(x) < -1$, $f(x)$ will never change signs since it is never zero

67. function f is continuous on the closed interval $[0,1]$
and $0 \leq f(x) \leq 1$.

when $x=1 \to f(1)=1$, when $x=0 \to f(0)=0$

, using the intermediate value theorem, c must exist
in the interval $[0,1]$ since $f(c)=c$ or
$0 \leq f(c) \leq 1$