8.5

6. $\dfrac{z}{z^3-z^2-6z} = \dfrac{1}{z^2-z-6} = \dfrac{1}{(z-3)(z+2)} = \dfrac{A}{z-3} + \dfrac{B}{z+2}$

$$1 = A(z+2) + B(z-3)$$

$$= z(A+B) + (2A-3B)$$

$$= A+B=0$$

$$2A-3B=1$$

$$B=\frac{-1}{5}, \quad A=\frac{1}{5}$$

$$= \frac{\frac{1}{5}}{z+2} - \frac{\frac{1}{5}}{z-3}$$

7. $\dfrac{t^2+8}{t^2-5t+6} = 1+\dfrac{5t+2}{t^2-5t+6} = \dfrac{5t+2}{(t-3)(t-2)} = \dfrac{A}{t-3} \dfrac{+B}{t-2}$

$$5t+2 = A(t-2) + B(t-3)$$

$$= t(A+B) + (-2A-3B)$$

$= A + B = 5$

$-2A - 3B = 2$

$= 1 + \frac{17}{t-3} - \frac{12}{t-2}$

$-B = (10 + 2) = 12$

$B = -12$

$A = 17$

19. $\int \frac{dx}{(x^2-1)^2}$

$= \frac{1}{(x^2-1)^2} = \frac{1}{(x+1)^2(x-1)^2} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x-1)} + \frac{D}{(x-1)^2}$

$= A(x+1)(x-1)^2 + B(x-1)^2 + C(x-1)(x+1)^2$
$+ D(x+1)^2$

$x = -1 \rightarrow B(-2)^2$

$x = 1 \rightarrow D(4)$

$B(4) = 1$

$B = \frac{1}{4}$

$\frac{1}{4} = D$

$$1 = A(x+1)(x-1)^2 + \frac{1}{4}(x-1)^2 + C(x-1)(x+1)^2$$

$$+ \frac{1}{4}(x+1)^2$$

$$x=0 \longrightarrow 1 = A + \frac{1}{4} - C + \frac{1}{4}$$

$$A - C = \frac{1}{2}$$

$$x=2 \longrightarrow A(3) + \frac{1}{4} + C(9) + \frac{9}{4}$$

$$3A + 9C = 1 - \frac{1}{4} - \frac{9}{4}$$

$$3A + 9C = -\frac{3}{2}$$

$A - C = \frac{1}{2}$ $\quad\begin{vmatrix} 3 \\ 1 \end{vmatrix}$

$3A + 9C = -\frac{3}{2}$

$0 - 12C = 3$

$C = -\frac{3}{12}$

$= -\frac{1}{4}$

$A + \frac{1}{4} = \frac{1}{2}$

$A = \frac{1}{2} - \frac{1}{4}$

$= \frac{1}{4}$

$$\int \frac{1/4\,dx}{(x+1)} + \int \frac{1/4\,dx}{(x+1)^2} + \int \frac{-1/4\,dx}{(x-1)} + \int \frac{1/4\,dx}{(x-1)^2}$$

$$= \frac{1}{4}\ln(x+1) - \frac{1}{4(x+1)} - \frac{1}{4}\ln(x-1) - \frac{1}{4(x-1)} + C$$

$$= \frac{1}{4}\left|\frac{x+1}{x-1}\right| - \frac{1}{4(x+1)} - \frac{1}{4(x-1)} + C$$

20. $\displaystyle\int \frac{x^2\,dx}{(x-1)(x^2+2x+1)}$

$$\frac{x^2}{(x-1)(x^2+2x+1)} = \frac{A}{(x-1)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2} = x^2$$

$$= A(x+1)^2 + B(x-1)(x+1) + C(x-1)$$

$x = -1 \longrightarrow C(-2)$

$1 = -2C$

$C = -\frac{1}{2}$

$x = 1 \longrightarrow A(4)$

$1 = 4A$

$A = \frac{1}{4}$

$x = 0 \longrightarrow \frac{1}{4}(1) + B(-1) - \frac{1}{2}(-1)$

$0 = \frac{1}{4} - B + \frac{1}{2}$

$$\frac{1}{4}+\frac{1}{2}=B$$

$$B = \frac{3}{4}$$

$$\int \frac{x^2\,dx}{(x-1)(x^2+2x+1)} = \frac{1}{4}\int\frac{dx}{(x-1)} + \frac{3}{4}\int\frac{dx}{(x+1)} - \frac{1}{2}\int\frac{dx}{(x+1)^2}$$

$$= \frac{1}{4}\ln(x-1) + \frac{3}{4}\ln(x+1) - \frac{1}{2(x+1)} + C$$

$$= \frac{\ln|(x-1)(x+1)^3|}{4} + \frac{1}{2(x+1)} + C$$

24. $\int \frac{8x^2+8x+2}{(4x^2+1)^2}\,dx$

$$\frac{8x^2+8x+2}{(4x^2+1)^2} = \frac{Ax+B}{(4x^2+1)} + \frac{Cx+D}{(4x^2+1)^2}$$

$8x^2+8x+2 = Ax+B(4x^2+1) + Cx+D$

$= 4Ax^3 + Ax + 4Bx^2 + B + Cx + D$

$= 4Ax^3 + (A+C)x + 4Bx^2 + B+D$

$A=0, \quad B=2, \quad C=8, \quad D=0$

$= \int \frac{2\,dx}{4x^2+1} + \int \frac{8x\,dx}{(4x^2+1)^2}$

$2\int \frac{dx}{4x^2+1} \qquad x \to \frac{1}{2}\tan\theta \qquad dx = \frac{1}{2}\sec^2\theta$

$2\int \frac{1}{\tan^2\theta+1} \cdot \frac{1}{2}\sec^2\theta\, d\theta$

$2\int \frac{1}{2} d\theta$

$\tan^{-1}\theta + C$

$\tan^{-1}(2x + C$

$$\int \frac{8x\,dx}{(4x^2+1)^2} \qquad \text{Let } u = 4x^2+1 \qquad du = 8x\,dx$$

$$\int \frac{\cancel{8x}}{u^2} \cdot \frac{du}{\cancel{8x}} \qquad \Big| \; -\frac{1}{4x^2+1} + C$$

$$-u^{-1} + C \qquad \Big| \; \tan^{-1}(2x) - \frac{1}{4x^2+1} + C$$

37. $$\int \frac{y^4+y^2-1}{y^3+y}$$

$$\frac{y^4+y^2-1}{y^3+y} = y - \frac{1}{y(y^2+1)} \qquad \frac{1}{y(y^2+1)} = \frac{A}{y} + \frac{By+C}{y^2+1}$$

$$1 = A(y^2+1) + By+C(y)$$

$$1 = Ay^2 + A + By^2 + Cy$$

$$1 = y^2(A+B) + A + Cy$$

$A=1$ $\qquad y=0 \to C(0)$

$A+B=0$ $\qquad C=0$

$1+B=0$

$B=-1$

$$\int \frac{y^4+y^2-1}{y^3+y}\,dy = \int y\,dy - \int \frac{dy}{y} + \int \frac{y\,dy}{y^2+1}$$

$$= \frac{y^2}{2} - \ln y + \frac{1}{2}\ln(y^2+1) + C$$

42. $\displaystyle\int \frac{\sin\theta\,d\theta}{\cos^2\theta+\cos\theta-2}$

Let $u=\cos\theta$ $\qquad du = -\sin\theta\,d\theta$

$\qquad d\theta = \dfrac{du}{-\sin\theta}$

$$= \int \frac{\cancel{\sin\theta}}{u^2+u-2}\,\frac{du}{-\cancel{\sin\theta}}$$

$$= \int \frac{-du}{u^2+u-2} = -\int \frac{du}{u^2+u-2} = \frac{A}{(u+2)} + \frac{B}{(u-1)}$$

$$1 = A(u-1) + B(u+2)$$

$u=-2 \rightarrow A(-3)$ | $u=1 \rightarrow B(3)$

$A=-\frac{1}{3}$ | $B=\frac{1}{3}$

$$= \int \frac{-\frac{1}{3}}{(u+2)} + \int \frac{\frac{1}{3}}{(u-1)}$$

$$= \frac{1}{3} \ln \left| \frac{u+2}{u-1} \right| + C = \frac{1}{3} \ln \left| \frac{\cos\theta + 2}{\cos\theta - 1} \right| + C$$

44. $\displaystyle \int \frac{(x+1)^2 \tan^{-1}(3x) + 9x^3 + x}{(9x^2+1)(x+1)^2} \, dx$

$$\int \frac{\cancel{(x+1)^2} \tan^{-1}(3x)}{(9x^2+1)\cancel{(x+1)^2}} \, dx + \int \frac{x\cancel{(9x^2+1)}}{\cancel{(9x^2+1)}(x+1)^2} \, dx$$

$$= \int \frac{\tan^{-1}(3x)}{9x^2+1}\,dx + \int \frac{x}{(x+1)^2}\,dx$$

$$\int \frac{\tan^{-1}(3x)}{9x^2+1}\,dx$$

let $u = \tan^{-1}(3x)$

$$du = \frac{3}{9x^2+1}\,dx$$

$$dx = \frac{9x^2+1}{3}\,du$$

$$\int \frac{u}{\cancel{9x^2+1}} \frac{\cancel{9x^2+1}}{3}\,du$$

$$\int \frac{1}{3} u \, du \longrightarrow \frac{1}{6} u^2 + C$$

$$\frac{1}{6} (\tan^{-1}(3x))^2 + C$$

$$\int \frac{x}{(x+1)^2}\,dx = \frac{A}{x+1} + \frac{B}{(x+1)^2}$$

$$x = A(x+1) + B$$

$$x = Ax + A + B$$

$x=1 \rightarrow 1=2A+B$

$x=0 \rightarrow 0=A+B$

$$\begin{array}{r|c|} 2A+B=1 & 1 \\ A+B=0 & 2 \\ \hline -B=1 & \end{array} \quad -$$

$B=-1$

$A-1=0$

$A=1$

$$\int \frac{1}{x+1} - \int \frac{1}{(x+1)^2} = \ln|x+1| - \frac{1}{x+1} + C$$

$$\therefore \frac{1}{6}\left(\tan^{-1}(3x)\right)^2 + \ln|x+1| - \frac{1}{x+1} + C$$

8.7

d. $\int_2^4 \frac{1}{(s-1)^2}\, ds$

$n=4$

I. a) $T = \frac{\Delta x}{2}\left(y_0 + 2y_1 + 2y_2 + 2y_3 + y_4\right)$

$= \frac{0.5}{2}\left(1 + 2\left(\frac{4}{9}\right) + 2\left(\frac{1}{4}\right) + 2\left(\frac{2}{5}\right) + \frac{1}{9}\right)$

$= \frac{1}{4}\left(\frac{141}{50}\right) = \frac{141}{200} = 0{,}705$

$M = f'(s) = \frac{-2}{(s-1)^3}$

$f''(s) = \frac{6}{(s-1)^4}$

$f''(2) = \frac{6}{1} = 6$

b) $\int_2^4 \frac{1}{(s-1)^2}\,ds$

$= \left.\frac{-1}{(s-1)}\right|_2^4$

$= \frac{-1}{3} + 1 = \frac{2}{3}$

$|E_T| = \int_2^4 \frac{1}{(s-1)^2}\,ds - T$

$= \frac{2}{3} - 0.705$

$= -0.03833$

c) $\frac{|E_T|}{\text{True Value}} \times 100 = \frac{0.03833}{\frac{2}{3}} \times 100 = 6\%$

II a) $n = 4$

$$\Delta x = \frac{b-a}{n}$$

$$= \frac{4-2}{4} = \frac{1}{2} \Rightarrow \frac{\Delta x}{3} = \frac{1}{6}$$

$$\sum m f(s_i) = \frac{1813}{450} \Rightarrow S = \frac{1}{6}\left(\frac{1813}{450}\right) \approx 0.671$$

$$f^{(3)}(s) = \frac{-24}{(s-1)^5}$$

$$\Rightarrow f^{(4)}(s) = \frac{120}{(s-1)^5} \Rightarrow M = 120$$

$$\Rightarrow |E_S| \leq \frac{4-2}{180}\left(\frac{1}{2}\right)^4 (120) = \frac{1}{12} \approx 0.083$$

b) $$\int_2^4 \frac{1}{(s-1)^2}\,ds = \frac{2}{3}$$

$$|E_S| = \int_2^4 \frac{1}{(s-1)^2}\,ds - S \approx \frac{2}{3} - 0.67148 = -0.00481$$

$$\approx 0.00481$$

c) $\frac{|E_S|}{\text{True Value}} \times 100 = \frac{0.00481}{\frac{2}{3}} \times 100 \approx 1\%$

20. $\int_0^3 \frac{1}{\sqrt{x+1}}\,dx$

a) $f(x) = \frac{1}{\sqrt{x+1}} \Rightarrow f'(x) = -\frac{1}{2}(x+1)^{-3/2}$

$f''(x) = \frac{3}{4}(x+1)^{-5/2}$

$= \frac{3}{4(\sqrt{x+1})^5} \Rightarrow M = \frac{3}{4(\sqrt{1})^5} = \frac{3}{4}$

$\Delta x = \frac{3}{n} \Rightarrow |E_T| \le \frac{3}{12}\left(\frac{3}{n}\right)^2\left(\frac{3}{4}\right) = \frac{3^4}{48n^2} < 10^{-4}$

$\Rightarrow n > \sqrt{\frac{3^4(10^4)}{48}} \Rightarrow n > 129.9$

$n = 130$

b) $f^{(3)}(x) = -\frac{15}{8}(x+1)^{-7/2} \Rightarrow f^{(4)}(x) = \frac{105}{16}(x+1)^{-9/2}$

$= \frac{105}{16(\sqrt{x+1})^9} \Rightarrow M = \frac{105}{16(\sqrt{1})^9} = \frac{105}{16}$

$\Delta x = \frac{3}{n} \Rightarrow |E_S| \le \frac{3}{180}\left(\frac{3}{n}\right)^4\left(\frac{105}{16}\right) = \frac{3^5(105)}{16(180)n^4} < 10^{-4}$

$$n^4 > \frac{3^5(105)(10^4)}{16(180)} \Rightarrow n > \sqrt[4]{\frac{3^5(105)(10^4)}{16(180)}}$$

$$n > 17.25$$

$$\boxed{n = 18}$$

25. $\Delta x = 1$

$$\frac{\Delta x}{3} = \frac{1}{3}$$

$$\sum M_{yi} = 0.5(1) + 0.55(4) + 0.6(2) + 0.65(4) + 0.7(2) + 0.75(4)$$
$$+ 0.75(1) = 11.65$$

Cross section area $= \frac{1}{3} \times (11.65) = 3.883$

Volume $\vartheta = x = 3.883x$

mow: 2000 kg at 679 kg/m^3

$$\vartheta : \frac{2000}{679} = 2.94$$

$$2.94 = 3.8x$$

$$\boxed{x = 0.77}$$

27. a) $|E_S| \leq \frac{b-a}{180}(\Delta x^4) M$; $n=4$

$\Delta x = \frac{\frac{\pi}{2}-0}{4} = \frac{\pi}{8}$; $|f^{(4)}| \leq 1 \to M=1$

$|E_S| \leq \frac{(\frac{\pi}{2}-0)}{180}\left(\frac{\pi}{8}\right)^4 (1) \approx 0.00021$

b) $\Delta x = \frac{\pi}{8} \to \frac{\Delta x}{3} = \frac{\pi}{24}$

$\Sigma m f(x_i) = 10.472$

$S = \frac{\pi}{24}(10.472) \approx 1.37$

c) $\approx \left(\frac{0.00021}{1.37079}\right) \times 100 \approx 0.015\%$

28. a) $\Delta x = \frac{b-a}{n} = \frac{1-0}{10} = 0.1$

$erf(1) = \frac{2}{\sqrt{3}}\left(\frac{0.1}{3}\right)(y_0 + 4y_1 + 2y_2 + 4y_3 + \ldots + 4y_9 + y_{10})$

$= \frac{2}{30\sqrt{\pi}}(e^0 + 4e^{-0.01} + 2e^{-0.04} + 4e^{-0.09} + \ldots + 2/e^{-0.81} + e^{-1}) \approx 0.843$

b) $|E_S| \leq \frac{1-0}{180}(0.1)^4(12) \approx 6.7\times10^{-6}$

3) a) $a=1,\ e=\frac{1}{2}$

$$\text{length} = 4\int_0^{\pi/2}\sqrt{1-\frac{1}{4}\cos^2 t}\,dt$$

$$= 2\int_0^{\pi/2}\sqrt{4-\cos^2 t}\,dt$$

$$= \int_0^{\pi/2} f(t)\,dt$$

$n=10$

$\Delta t = \frac{b-a}{n} = \frac{\pi}{20}$

$$\int_0^{\pi/2}\sqrt{4-\cos^2 t}\,dt \approx \sum_{n=0}^{10} m f(x_n) = 37.36$$

$T = \frac{\Delta t}{2}(37.36)$

$= 2.94$

length ≈ 5.870

8.8

9. $\int_0^4 \frac{dx}{\sqrt{4-x}} = \lim_{b\to 4^-} \int_0^b (4-x)^{-1/2}\, dx = \lim_{b\to 4^-} \left[-2\sqrt{4-b} - (-2\sqrt{4})\right]$

$= 4$

10. $\int_{-\infty}^{2} \frac{2dx}{x^2+4} = \lim_{b\to -\infty} \tan^{-1}\frac{x}{2}\Big|_b^2 = \lim_{b\to -\infty}\left(\tan^{-1} 1 - \tan^{-1}\frac{b}{2}\right)$

$= \frac{\pi}{4} - \left(-\frac{\pi}{2}\right) = \frac{3\pi}{4}$

14. $\int_{-\infty}^{\infty} \frac{x\,dx}{(x^2+4)^{3/2}} = \int_{-\infty}^{0} \frac{x\,dx}{(x^2+4)^{3/2}} + \int_0^{\infty} \frac{x\,dx}{(x^2+4)^{3/2}}$

$\begin{bmatrix} u = x^2+4 \\ du = 2x\,dx \end{bmatrix} \to \int_{\infty}^{4} \frac{du}{2u^{3/2}} + \int_4^{\infty} \frac{du}{2u^{3/2}} = \lim_{b\to\infty} -\frac{1}{\sqrt{u}}\Big|_b^4$

$+ \lim_{c\to\infty} -\frac{1}{\sqrt{u}}\Big|_4^c$

$= \lim_{b\to\infty}\left(-\frac{1}{2} + \frac{1}{\sqrt{b}}\right) + \lim_{c\to\infty}\left(-\frac{1}{\sqrt{c}} + \frac{1}{2}\right) = \left(-\frac{1}{2} + 0\right) + \left(0 + \frac{1}{2}\right) = 0$

19. $\int_0^\infty \frac{dv}{(1+v^2)(1+\tan^{-1}v)} = \lim_{b\to\infty} \ln|1+\tan^{-1}v|\Big|_0^b$

$= \lim_{b\to\infty} \left[\ln|1+\tan^{-1}b| - \ln|1+\tan^{-1}0|\right]$

$= \ln\left(1+\frac{\pi}{2}\right) - \ln(1+0) = \ln\left(1+\frac{\pi}{2}\right)$

28. $\int_0^1 \frac{4r\,dr}{\sqrt{1-r^4}} = \lim_{b\to 1^-} 2\sin^{-1}(r^2)\Big|_0^b$

$= \lim_{b\to 1^-}\left[2\sin^{-1}(b^2) - 2\sin^{-1}0\right] = 2\cdot\frac{\pi}{2} - 0 = \pi$

32. $\int_0^2 \frac{dx}{\sqrt{|x-1|}} = \int_0^1 \frac{dx}{\sqrt{1-x}} + \int_1^2 \frac{dx}{\sqrt{x-1}}$

$= \lim_{b\to 1^-} -2\sqrt{1-x}\Big|_0^b + \lim_{c\to 1^+} 2\sqrt{x-1}\Big|_c^2$

$= \lim_{b\to 1^-}\left[(-2\sqrt{1-b}) - (-2\sqrt{1-0})\right] + \lim_{c\to 1^+}\left[2\sqrt{2-1} - (2\sqrt{c-1})\right]$

$= 0+2+2-0 = 4$

35. $\int_0^{\pi/2} \tan\theta \, d\theta = \lim_{b\to(\frac{\pi}{2})^-} -\ln|\cos\theta| \Big|_0^b$

$= \lim_{b\to(\frac{\pi}{2})^-} \left[-\ln|\cos b| + \ln 1\right] = \lim_{b\to(\frac{\pi}{2})^-} \left[-\ln|\cos b|\right]$

$= +\infty$

Diverges

39. $\int_0^{\ln 2} x^{-2} e^{-1/x} \, dx$

$\left[\frac{1}{x} = y\right] \to \int_{\infty}^{1/\ln 2} \frac{y^2 e^{-y} \, dy}{-y^3} = \int_{1/\ln 2}^{\infty} e^{-y} \, dy$

$= \lim_{b\to\infty} -e^{-y} \Big|_{1/\ln 2}^b = \lim_{b\to\infty} \left[-e^{-b} - \left(-e^{-1/\ln 2}\right)\right]$

$= 0 + e^{-1/\ln 2} = e^{-1/\ln 2}$

Converges

41. $\int_0^{\pi} \frac{dt}{\sqrt{t}+\sin t}$

Since $0 \le t \le \pi$, $0 \le \frac{1}{\sqrt{t}+\sin t} \le \frac{1}{\sqrt{t}}$ and $\int_0^{\pi} \frac{dt}{\sqrt{t}}$

converges, so the original integral converges as well by the Direct Comparison Test.

52. $\int_2^{\infty} \frac{dx}{\sqrt{x^2-1}}$

$$\lim_{x\to\infty} \frac{\left(\frac{1}{\sqrt{x^2-1}}\right)}{\frac{1}{x}} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2-1}} = \lim_{x\to\infty} \frac{1}{\sqrt{1-\frac{1}{x^2}}} = 1$$

$$\int_2^{\infty} \frac{1}{x}\,dx = \lim_{b\to\infty} \left[\ln b\right]_2^b = \infty$$

diverges.

61. $\int_1^{\infty} \frac{1}{\sqrt{e^x - x}}\, dx$

$$\lim_{x\to\infty} \frac{\frac{1}{\sqrt{e^x - x}}}{\frac{1}{\sqrt{e^x}}} = \lim_{x\to\infty} \frac{\sqrt{e^x}}{\sqrt{e^x - x}} = \lim_{x\to\infty} \frac{1}{\sqrt{1-\frac{x}{e^x}}} = \frac{1}{\sqrt{1-0}} = 1$$

$$\int_1^{\infty} \frac{dx}{\sqrt{e^x}} = \int_1^{\infty} e^{-x/2}\, dx = \lim_{b\to\infty} -2e^{-x/2}\Big|_1^b$$

$$= \lim_{b\to\infty} \left(-2e^{-b/2} + 2e^{-1/2}\right) = \frac{2}{\sqrt{e}}$$

$$\int_1^{\infty} e^{-x/2}\, dx$$

Converges

64. $\int_{-\infty}^{\infty} \frac{dx}{e^x + e^{-x}} = 2\int_0^{\infty} \frac{dx}{e^x + e^{-x}}; \quad 0 < \frac{1}{e^x + e^{-x}} < \frac{1}{e^x}$

for $x > 0$

$$\int_0^\infty \frac{dx}{e^x}$$ Converges

65. a) $$\int_1^2 \frac{dx}{x(\ln x)^p}$$

$$[t = \ln x] \to \int_0^{\ln 2} \frac{dt}{t^p} = \lim_{b\to 0^+} \frac{1}{-p+1} t^{1-p} \Big|_b^{\ln 2}$$

$$= \lim_{b\to 0^+} \left[\frac{b^{1-p}}{p-1} + \frac{1}{1-p}(\ln 2)^{1-p}\right]$$

Converges for $p<1$ and diverges for $p\geq 1$

b) $$\int_2^\infty \frac{dx}{x(\ln x)^p}$$

$$[t = \ln x] \to \int_{\ln 2}^\infty \frac{dt}{t^p}$$

converges for $p>1$ and diverges for $p\leq 1$

66. $\int_0^\infty \frac{2x\,dx}{x^2+1} = \lim_{b\to\infty} \ln(x^2+1)\Big|_0^b = \lim_{b\to\infty} [\ln(b^2+1) - 0]$

$= \lim_{b\to\infty} \ln(b^2+1) = \infty$

Integral $\int_{-\infty}^{\infty} \frac{2x}{x^2+1}\,dx$ diverges.

But $\lim_{b\to\infty} \int_{-\infty}^{b} \frac{2x\,dx}{x^2+1} = \lim_{b\to\infty} \ln(x^2+1)\Big|_{-b}^{b} = \lim_{b\to\infty}[\ln(b^2+1) - \ln(b^2+1)]$

$= \lim_{b\to\infty} \ln\left(\frac{b^2+1}{b^2+1}\right) = \lim_{b\to\infty} (\ln 1) = 0$

76. a) $V = \int_1^\infty \pi\left(\frac{1}{x}\right)^2 dx$

$= \pi \lim_{b\to\infty} -\frac{1}{x}\Big|_1^b = \pi \lim_{b\to\infty} \left[\left(-\frac{1}{b}\right) - \left(-\frac{1}{1}\right)\right]$

$= \pi(1) = \pi$

b) limit ∞ means that you are not modeling the real world (finite).