Exercise 2.1

Average Rates of Change

5. $P(\theta) = \sqrt{4\theta + 1}$; $[0, 2]$

$$\frac{f(b) - f(a)}{b - a} = \frac{\sqrt{4(2)+1} - \sqrt{4(0)+1}}{2 - 0}$$

$$= \frac{\sqrt{9} - \sqrt{1}}{2 - 0} = \frac{3 - 1}{2} = \frac{2}{2} = 1$$

8. $y = 7 - x^2$, $P(2,3)$, $Q(2+h, 7-(2+h)^2)$

a) $$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x_1+h) - f(x_1)}{h}$$

$$\frac{\Delta y}{\Delta x} = \frac{7 - (2+h)^2 - 3}{h}$$

$$= \frac{7 - (h^2 + 4h + 4) - 3}{h}$$

$$= \frac{4 - (h^2 + 4h + 4)}{h} = \frac{\cancel{4} - h^{\cancel{2}} - 4\cancel{h} - \cancel{4}}{\cancel{h}}$$

$$= -h - 4$$

$= -4$

b) $y - y_1 = m(x - x_1)$

$y - 3 = -4(x - 2)$

$y - 3 = -4x + 8$

$y = -4x + 11$

10. $y = x^2 - 4x$, $P(1, -3)$, $Q(1+h, (1+h)^2 - 4(1+h))$

a) $\frac{dy}{dx} = \frac{f(x_1 + h) - f(x_1)}{h}$

$= \frac{((1+h)^2 - 4(1+h)) + 3}{h}$

$= \frac{h^2 + 2h + 1 - 4 - 4h + 3}{\cancel{h}}$

$= h - 2$

$= -2$

b) $y - y_1 = m(x - x_1)$

$y + 3 = m(x - 1)$

$y + 3 = -2(x - 1)$

$y + 3 = -2x + 2$

$y = -2x + 2 - 3$

$y = -2x - 1$

19. Let $g(x) = \sqrt{x}$ for $x \geq 0$

a) $(1,2)$, $(1,1.5)$, $(1, 1+h)$

$$\boxed{\frac{f(b) - f(a)}{b - a}}$$

* $\frac{\sqrt{2} - \sqrt{1}}{2 - 1} = \frac{\sqrt{2} - 1}{1} = 1{,}414 - 1$

$= 0{,}414$

* $\frac{\sqrt{1.5} - \sqrt{1}}{1.5 - 1} = \frac{1.224 - 1}{0.5} = \frac{0.224}{0.5} = 0{,}448$

* $\frac{\sqrt{1+h} - \sqrt{1}}{1+h - 1} = \frac{\sqrt{1+h} - 1}{h}$

b.

$(1, 1+h)$	$g(x) = \sqrt{x}$
1. $(1, 1.1)$	0.488
2. $(1, 1.01)$	0.498
3. $(1, 1.001)$	0.499
4. $(1, 1.0001)$	0.4999
5. $(1, 1.00001)$	0.49999
6. $(1, 1.000001)$	0.499999

1. $$\frac{\sqrt{1.1} - \sqrt{1}}{1.1 - 1} = \frac{\sqrt{1.1} - 1}{0.1} = \frac{1.04 - 1}{0.1} = 0.488$$

2. $$\frac{\sqrt{1.01} - \sqrt{1}}{1.01 - 1} = \frac{\sqrt{1.01} - 1}{0.01} = 0.498$$

3. $$\frac{\sqrt{1.001} - \sqrt{1}}{1.001 - 1} = \frac{\sqrt{1.001} - 1}{0.001} = 0.499$$

4. $$\frac{\sqrt{.0001} - \sqrt{1}}{1.0001 - 1} = \frac{\sqrt{1.0001} - 1}{0.0001} = 0.4999$$

5. $$\frac{\sqrt{1.00001} - \sqrt{1}}{1.00001 - 1} = \frac{\sqrt{1.00001} - 1}{0.00001} = 0.49999$$

6. $$\frac{\sqrt{1.000001} - \sqrt{1}}{1.000001 - 1} = \frac{\sqrt{1.000001} - 1}{0.000001} = 0.499999$$

c. it approaches 0.5

d. $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$\frac{\sqrt{1+h}-\sqrt{1}}{h} = \frac{\sqrt{1+h}-\sqrt{1}}{h}$$

$$= \frac{\sqrt{1+h}-1}{h} \cdot \frac{\sqrt{1+h}+1}{\sqrt{1+h}+1}$$

$$= \frac{1+h-1}{h(\sqrt{1+h}+1)} = \frac{\cancel{h}}{\cancel{h}(\sqrt{1+h}+1)}$$

$$= \frac{1}{\sqrt{1+h}+1}$$

$$= \lim_{h\to 0} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$$

Exercise 2.2

2. a) $\lim_{t\to -2} f(t) = 0$ Exist

b) $\lim_{t \to -1} f(t) = -1$ Exist

c) $\lim_{t \to 0} f(t) =$ Does Not Exist since it is only approached from the left.

d) $\lim_{t \to -0.5} f(t) = -1$ Exist

9. a) False, $\lim_{x \to 2} f(x) = 1$

b) False

c) True

d) True

e) True

10. Yes, since as $f(1) = 5$ then $\lim_{x \to 1} f(x) = 5$, so we can depend on $\lim_{x \to 1} f(x)$

18. $\lim\limits_{y \to 2} \dfrac{y+2}{y^2+5y+6}$

$$= \frac{2+2}{2^2+5(2)+6} = \frac{4}{4+10+6} = \frac{4}{20} = \frac{1}{5}$$

32. $\lim\limits_{x \to 0} \dfrac{\frac{1}{x-1} + \frac{1}{x+1}}{x}$

$$\frac{\left(\frac{1}{x-1} + \frac{1}{x+1}\right)}{x} = \frac{\left(\frac{(x+1)+(x-1)}{(x-1)(x+1)}\right)}{x}$$

$$= \frac{\frac{2x}{(x-1)(x+1)}}{x} = \frac{2\cancel{x}}{(x-1)(x+1)} \cdot \frac{1}{\cancel{x}}$$

$$= \frac{2}{(x-1)(x+1)}$$

$$\lim_{x \to 0} = \frac{2}{x^2-1}$$

$$= \frac{2}{-1} = -2$$

36. $\lim_{x \to 4} \frac{4x - x^2}{2 - \sqrt{x}} = \frac{x(4-x)}{2-\sqrt{x}} \cdot \frac{2+\sqrt{x}}{2+\sqrt{x}}$

$$= \frac{x\cancel{(4-x)}\,2+\sqrt{x}}{\cancel{4-x}} = x\,(2+\sqrt{x})$$

$$= 2x + x\sqrt{x}$$

$$\lim_{x \to 4} = 2(4) + 4 \cdot 2$$

$$= 8 + 8$$

$$= 16$$

91. $\lim_{x \to -3} \frac{2 - \sqrt{x^2-5}}{x+3}$

$$= \frac{2-\sqrt{x^2-5}}{x+3} \cdot \frac{2+\sqrt{x^2-5}}{2+\sqrt{x^2-5}}$$

$$= \frac{4-(x^2-5)}{(x+3)(2+\sqrt{x^2-5})} = \frac{4-x^2+5}{(x+3)(2+\sqrt{x^2-5})} = \frac{-x^2+9}{(x+3)(2+\sqrt{x^2-5})}$$

$$= \frac{(-x+3)\cancel{(x+3)}}{\cancel{(x+3)}(2+\sqrt{x^2-5})} = \frac{-x+3}{2+\sqrt{x^2-5}}$$

$$\lim_{x \to 3} \frac{-x+3}{2+\sqrt{x^2-5}} = \frac{3+3}{2+\sqrt{9-5}} = \frac{6}{2+2} = \frac{6}{4} = \frac{3}{2}$$

50. $\lim_{x \to 0} \sqrt{7 + \sec^2 x}$

$$= \sqrt{7 + \frac{1}{\cos^2 x}} = \sqrt{7 + \frac{1}{\cos x \cdot x}}$$

$$= \sqrt{7 + \frac{1}{\cos 0}}$$

$$= \sqrt{7 + \frac{1}{1}}$$

$$= \sqrt{8} = \sqrt{4 \cdot 2}$$

$$= 2\sqrt{2}$$

54. Suppose $\lim_{x \to 4} f(x) = 0$ and $\lim_{x \to 4} g(x) = -3$, Find:

a) $\lim_{x \to 4} (g(x) + 3)$

$-3 + 3 = 0$

b) $\lim_{x \to 4} x f(x)$

$4 \cdot 0 = 0$

c) $\lim_{x \to 4} (g(x))^2$

$(-3)^2 = 9$

d) $\lim_{x \to 4} \frac{g(x)}{f(x)-1}$

$\frac{-3}{0-1} = 3$

66. a) If we use the Sandwich theorem where we use

$\lim_{x \to 0} \frac{1}{2} - \frac{x^2}{24}$, $\frac{1}{2} - \frac{0^2}{24} = \frac{12}{24} = \frac{1}{2}$

for $g(x) = h(x) = L$

then $f(x) = L$, so the theorem is true where

$\frac{1 - \cos x}{x^2} = \frac{1}{2}$

75. $C = -1, 0, 1$, when $c = 0$ then

$f(x) = 0$, when c is $-1, 1$ then

$f(x) = 1$

76. No, $\lim_{x \to 2} f(x)$ should not be 0.

Since according to the sandwich theorem,

$g(x) = h(x) = L$, then $f(x) = L$.

So, $\lim_{x \to 2} f(x) = -5$

For $f(x)$, $g(x)$, $h(x)$ or $f(2)$, $g(2)$, $h(2)$, the value can be any number.

And, Yes $f(2)$ can be 0 since it is not the same as $\lim_{x \to 2} f(x)$

77. If $\lim_{x \to 4} \frac{f(x)-5}{x-2} = 1$, find $\lim_{x \to 4} f(x)$.

$f(x) - 5 = x - 2$

$f(x) = x - 2 + 5$

$f(x) = 4 - 2 + 5$

$f(x) = 7$

78. If $\lim_{x \to -2} \frac{f(x)}{x^2} = 1$ find, $\rightarrow \lim_{x \to -2} f(x) = x^2$

a) $\lim_{x \to -2} f(x) = -2^2 = 4$

b) $\lim_{x \to -2} \frac{f(x)}{x} = \frac{-2^2}{-2} = \frac{4}{-2} = -2$