2.6

11. $$\lim_{t \to \infty} \frac{2-t+\sin t}{t+\cos t} = \frac{2-t+\sin t\left(\frac{1}{t}\right)}{t+\cos t\left(\frac{1}{t}\right)} = \frac{\frac{2}{t}-1+\frac{\sin t}{t}}{1+\frac{\cos t}{t}}$$

$$\lim_{t \to \infty} \frac{0-1+0}{1+0} = \frac{-1}{1}$$

$$= -1$$

16. $$f(x) = \frac{3x+7}{x^2-2}$$

a) $$\lim_{x \to \infty} \frac{3x+7}{x^2-2} \frac{\left(\frac{1}{x^2}\right)}{\left(\frac{1}{x^2}\right)} = \frac{\frac{3}{x}+\frac{7}{x^2}}{1-\frac{2}{x^2}} = \frac{0}{1} = 0$$

b) $$\lim_{x \to -\infty} \frac{3x+7\left(\frac{1}{x^2}\right)}{x^2-2\left(\frac{1}{x^2}\right)} = \frac{\frac{3}{x}+\frac{7}{x^2}}{1-\frac{2}{x^2}} = \frac{0}{1} = 0$$

36. $$\lim_{x \to -\infty} \frac{4-3x^3}{\sqrt{x^6+9}} = \frac{4-3x^3\left(\frac{1}{x^3}\right)}{\sqrt{x^6+9}\left(\frac{1}{x^3}\right)} = \frac{\frac{4}{x^3}-3}{\sqrt{\frac{\cancel{x^6}}{\cancel{x^6}}+\frac{9}{x^6}}} = \frac{-3}{\sqrt{1}}$$

$$\lim_{x \to -\infty} \frac{-3}{-|1|} = 3$$

48. $$\lim_{x \to 0} \frac{1}{x^{2/3}} = \frac{1}{0^{2/3}} = \infty$$

82. $$\lim_{x \to -\infty} \left(\sqrt{x^2+3}+x\right) = \left(\sqrt{x^2+3}+x\right)\left(\frac{1}{x}\right) = \sqrt{\frac{x^2}{x^2}+\frac{3}{x^2}}+1$$

$$= \sqrt{1}+1$$

$$\lim_{x \to -\infty} -|1|+1$$

$$= 0$$

86. $\lim_{x\to\infty}\left(\sqrt{x^2+x}-\sqrt{x^2-x}\right)\cdot\frac{\sqrt{x^2+x}+\sqrt{x^2-x}}{\sqrt{x^2+x}+\sqrt{x^2-x}}$

$$=\frac{x^2+x-(x^2-x)}{\sqrt{x^2+x}+\sqrt{x^2-x}}=\frac{\cancel{x^2}+x-\cancel{x^2}+x}{\sqrt{x^2+x}+\sqrt{x^2-x}}=\frac{2x\left(\frac{1}{x}\right)}{\sqrt{x^2+x}+\sqrt{x^2-x}\left(\frac{1}{x}\right)}$$

$$=\frac{\frac{2x}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}}+\sqrt{\frac{x^2}{x^2}-\frac{x}{x^2}}}$$

$$=\frac{2}{\sqrt{1}+\sqrt{1}}=1$$

100. $y=\frac{x^{\boxed{2}}+1}{x^{\boxed{1}}-1}$ = numerator degree > denominator degree by 1, then it is a slant asymptote

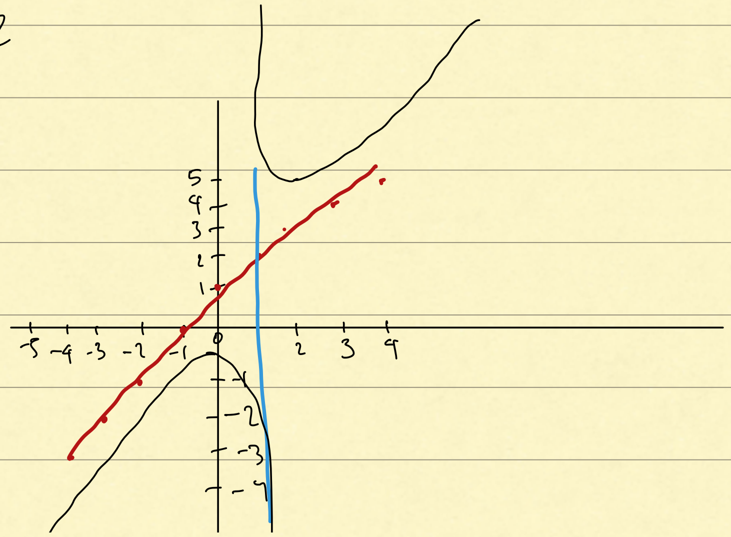
$$=\begin{array}{r} x+1 \\ x-1\overline{)x^2+0x+1} \\ \underline{x^2-x}\ - \\ x+1 \\ \underline{x-1}\ - \\ 2 \end{array}$$

$\boxed{y=x+1}$

$x-1=0$

$\boxed{x=1}$

graph:



3.1

23. a) $P'(5)$ means the rate of change of the amount of Cells at $t=5$. The units is cells/hour

b) $P'(3)$ is bigger since the rate of range of $P'(3)$ is bigger

c) $P(5) = 6.10t^2 - 9.28t + 16.43$

$P'(5) = 12.20t - 9.28$

$= 12.20(5) - 9.28 = 51.72$

35. Does the graph of

$$f(x) = \begin{cases} x^2 \sin(1/x), & x \neq 0 \\ 0 & x = 0 \end{cases}$$

have a tangent at the origin?

tangent → $\lim_{x \to 0} \frac{f(a+x) - f(a)}{x}$

$= \frac{f(0 + x^2 \sin(1/x) - f(0)}{x}$

$= \frac{\cancel{x}^2 \sin\frac{1}{x}}{\cancel{x}} = x \sin\frac{1}{x} = \lim_{x \to 0} \quad 0$

Yes it does have a tangent at the origin since the derivative exists when we input $x=0$.

36. Does the graph of

$$g(x) = \begin{cases} x \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

have a tangent at the origin?

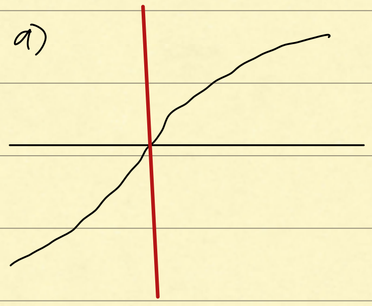
tangent → $\lim\limits_{x \to 0} \dfrac{f(a+x) - f(a)}{x}$

$$= \frac{f(0 + x\sin(\frac{1}{x})) - f(0)}{x} = \frac{x \sin \frac{1}{x}}{\cancel{x}} = \lim_{x \to 0} \sin \frac{1}{0} \quad \text{DNE}$$

does not have tangent since it DNE

42. $y = x^{3/5}$

a)



b) $\lim\limits_{x \to 0} \dfrac{f(a+x) - f(a)}{x}$

$$\lim_{x \to 0} \frac{x^{\frac{3}{5}}}{x} = x^{\frac{3}{5}} x^{-1}$$

$$= x^{-\frac{2}{5}}$$

$$= \frac{1}{x^{\frac{2}{5}}} = \infty$$

3.2

20. $y = 1 - \frac{1}{x}$

$y' = -\left(-\frac{1}{x^2}\right)$

$= \frac{1}{(\sqrt{3})^2} = \frac{1}{3}$

27. b 28. a 29. d 30. c

46. a) $-2 \le x < -1$, $-1 < x < 0$, $0 < x < 2$, $2 < x \le 3$ b) $x = -1$ c) $x = 0$

57. DNE, since when $g(0) = h(0) = 0$, we get

$\lim_{t \to 0} \frac{g(0)}{h(0)}$ = which is $\frac{0}{0}$ /DNE

58. $|f(x)| \le x^2$ for $-1 \le x \le 1$

$|f'(x)| \le 2x$

$|f'(0)| \le 2(0) = 0$

proof: In this case, $\lim_{x \to 0} \frac{|f(a+x) - f(a)|}{x}$

$\lim_{x \to 0} \frac{|f(0+x) - f(0)|}{x} = \lim_{x \to 0} \frac{|f(x)|}{x}$

$\lim_{x \to 0} \frac{x^{\cancel{2}}}{\cancel{x}} = \lim_{x \to 0} 0 \le 0 \longrightarrow f'(0)$

∴ So, at $x = 0$, $f'(0)$ exists, cts which makes it differentiable

3.3

12. $r = \frac{12}{\theta} - \frac{4}{\theta^3} + \frac{1}{\theta^4}$

$r' = \frac{-12}{\theta^2} + \frac{12}{\theta^4} - \frac{4}{\theta^5}$

$r'' = \frac{24}{\theta^3} - \frac{48}{\theta^5} + \frac{20}{\theta^6}$

18. $z = \frac{4-3x}{3x^2+x} \longrightarrow \frac{d}{dx}\left[\frac{f}{g}\right] = \frac{gf' - fg'}{g^2}$

$f = 4-3x \quad f' = -3$

$g = 3x^2+x \quad g' = 6x+1$

$= \frac{(3x^2+x)(-3) - (4-3x)(6x+1)}{(3x^2+x)^2} = \frac{-9x^2 - 3x - 24x - 4 + 18x^2 + 3x}{(3x^2+x)^2}$

$= \frac{9x^2 - 24x - 4}{(3x^2+x)^2}$

32. $y = (4x^2+3)(2-x)x$

$(4x^2+3)(2-x) \quad x$

$= (8x^2 - 4x^3 + 6 - 3x)x$

$= 8x^3 - 4x^4 + 6x - 3x^2$

$y' = 24x^2 - 16x^3 + 6 - 6x$

$y'' = 48x - 48x^2 - 6$

$y''' = 48 - 96x$

$y^{(''''} = -96$, $y^{(n)} = 0$ when $n \geq 5$

48. Curves $y = x^2 + ax + b$ and $y = cx - x^2$ have a common tangent line at point $(1,0)$. Find a, b, c.

at point $(1,0)$, $y = x^2 + ax + b \longrightarrow 0 = 1^2 + a(1) + b$

$0 = 1 + a + b$

$y = cx - x^2 \longrightarrow 0 = c(1) - (1)^2$

$0 = c - 1$

$c = 1$

to find a,b , we use derivative of both y's

$y' = 2x + a$

$y' = c - 2x$

because they have a common tangent line,

$2x + a = c - 2x$

$2(1) + a = c - 2(1) \rightarrow 2 + a = c - 2$

$a = 1 - 2 - 2$

$a = -3$

$1 + a + b = 0$

$a + b = -1$

$b = -1 + 3$

$b = 2$

56. $$\lim_{x \to -1} \frac{x^{2/9} - 1}{x+1} = \frac{\frac{2}{9}x^{-\frac{7}{9}}}{1} = \frac{2}{9}(1)^{-\frac{7}{9}}$$

$$= \frac{\frac{2}{9}}{1^{\frac{7}{9}}} = \frac{2}{9}$$

65. $$P = \frac{nRT}{V-nb} - \frac{an^2}{V^2}$$

$$= nRT(V-nb)^{-1} - an^2(V)^{-2}$$

$$= -nRT(V-nb)^{-2} + 2an^2(V)^{-3}$$

$$= -\frac{nRT}{V-nb^2} + \frac{2an^2}{V^3}$$

3.4

8. a) $v = t^2 - 4t + 3$

$= (t-3)(t-1)$

$= t_1 = 3$ $t_2 = 1$, when t is 3, 1 then $v = 0$

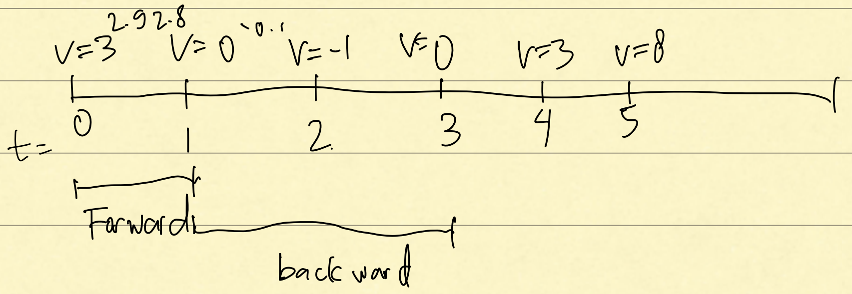
$v' = 2t - 4$, we insert t when t is 3 and 1

$a_1 = 2(3) - 4$ $a_2 = 2(1) - 4$

$= 2$ m/s $= -2$ m/s

b) when $t=0$ to $t=1$, the body is moving forward. When $t=1$ to $t=3$ the body is moving backwards. And when $t \geq 3$ the body is moving forward.

The explanation below tells why.



c) when $t=0$ to $t=1$, the body's velocity is decreasing. when $t=1$ to $t=2$ the velocity is increasing. when $t=2$ to $t=3$ the velocity is decreasing. And from $t \geqslant 3$ the body is increasing.

19. $s = 490t^2$

a) $* \; 160 = 490t^2$

$$\frac{160}{490} = t^2$$

$$\sqrt{\frac{16\cancel{0}}{49\cancel{0}}} = t \longrightarrow \boxed{\frac{4}{7}\ s}$$

it takes $\frac{4}{7}$ seconds for balls to fall the first 160 cm.

$$* \quad V = \frac{s}{t} \longrightarrow V = \frac{160}{\frac{4}{7}} = \frac{1120}{4} = 280 \text{ cm/s}$$

b) * how fast?

We use derivative to $490t^2 = 980t$

$= 9\cancel{8}0\left(\frac{4}{\cancel{7}}\right) = 560$ cm/s

* acceleration uses the third derivative

$980t = 980$

So, the acceleration when they reached 160-cm mark is $980 \text{ cm}/s^2$

c) number of flashes = 17

time to 160 cm = $\frac{4}{7}$ second

How fast = $\frac{17}{\frac{4}{7}} = \frac{119}{4}$

= 29,75 flashes/s

24. $r(x) = 20{,}000\left(1 - \frac{1}{x}\right)$

a) $20000 - \frac{20000}{x}$

$r'(x) = \frac{20000}{x^2}$

$= \frac{20000}{100^2} \rightarrow \frac{2\cancel{0000}}{1\cancel{0000}} = 2$

b) $\frac{20000}{101^2} = \frac{20000}{10201} = 1{,}96$

c) $\lim_{x \to \infty} \frac{20000}{x^2} = \frac{20000}{\infty}$

$= 0$

∴ As the number of items increases, the revenue will approach 0

3.5

30. $p = \dfrac{\tan q \quad (f)}{1+\tan q \; (g)}$ use $= \dfrac{d}{dx}\left[\dfrac{f}{g}\right] = \dfrac{gf' - fg'}{g^2}$

$f = \tan q \qquad f' = \sec^2 q$

$g = 1 + \tan q \qquad g' = \sec^2 q$

$$\frac{(1+\tan q)(\sec^2 q) - (\tan q)(\sec^2 q)}{(1+\tan q)^2}$$

$$\frac{\sec^2 q + \cancel{(\tan q)(\sec^2 q)} - \cancel{(\tan q)} \sec^2 q)}{(1+\tan q)^2}$$

$$= \frac{\sec^2 q}{(1+\tan q)^2}$$

34. Find $y^{(4)} = \dfrac{d^4 y}{dx^4}$ if

a) $y = -2\sin x$

$y' = -2\cos x$

$y'' = 2\sin x$

$y''' = 2\cos x$

$y'''' = -2\sin x$

b) $y = 9\cos x$

$y' = -9\sin x$

$y'' = -9\cos x$

$y''' = 9\sin x$

$y'''' = 9\cos x$

46. a) the tangent to the curve at P

$y = 1 + \sqrt{2}\csc x + \cot x$, $P(\frac{\pi}{4}, 4)$

find the slope:

$y' = -\sqrt{2}\cot x\csc x - \csc^2 x$

$f'(\frac{\pi}{4}) = -\sqrt{2}\cot(\frac{\pi}{4})\csc(\frac{\pi}{4}) - \csc^2(\frac{\pi}{4})$

$= \csc(\frac{\pi}{4})(-\sqrt{2}\cot(\frac{\pi}{4}) - \csc(\frac{\pi}{4}))$

$f'(\frac{\pi}{4}) = \frac{1}{\sin\frac{\pi}{4}}(-\sqrt{2} - \frac{1}{\sin\frac{\pi}{4}})$

$f'(\frac{\pi}{4}) = -4$

$m = -4$

tangent line at $P(\frac{\pi}{4}, 4)$ $\quad y - y_1 = m(x - x_1)$

$y - 4 = -4(x - \frac{\pi}{4})$

$y - 4 = -4x + \pi$

$y = -4x + \pi + 4$

b) the horizontal tangent to the curve at Q.

Since it is horizontal, $m = 0$

$y' = -\sqrt{2}\cot x\csc x - \csc^2 x$

$-\sqrt{2}\cot x\csc x - \csc^2 x = 0$

$-\sqrt{2}\cot x\csc x = \csc^2 x$

$$-\sqrt{2}\frac{\cos x}{\sin x}\cdot\frac{1}{\cancel{\sin x}} = \frac{1}{\sin^{\cancel{2}} x}$$

$$-\sqrt{2}\frac{\cos x}{\cancel{\sin x}} = \frac{1}{\cancel{\sin x}}$$

$-\sqrt{2}\cos x = 1 \rightarrow \cos x = \frac{1}{-\sqrt{2}}$

$x = \frac{3}{4}\pi$

$y = 1 + \sqrt{2}\ \csc x + \cot x$

$y = 1 + \sqrt{2}\ \csc\left(\frac{3}{4}\pi\right) + \cot\left(\frac{3}{4}\pi\right)$

$= 1 + \sqrt{2}\ \frac{1}{\sin\frac{3}{4}\pi} + \frac{1}{\tan\frac{3}{4}\pi}$

$= 1 + 2 - 1$

$= 2$

$y - y_1 = m(x - x_1)$

$y - 2 = 0$

$y = 2$

54. $\lim_{\theta \to 0} \cos\left(\frac{\pi\theta}{\sin\theta}\right)$

$\cos\left(\frac{\pi\cancel{\theta}}{\cancel{\sin\theta}}\right) = \cos\left(\frac{\pi}{1}\right)$

$= -1$

58. $g(x) = \begin{cases} x+b, & x < 0 \\ \cos x, & x \geq 0 \end{cases}$

$x + b = \cos x$

$0 + b = 1$

$b = 1$

$\therefore$ at $b = 1$ it is continuous at $x = 0$

$g'(x) = \begin{cases} 1, & x < 0 \\ -\sin x, & x \geq 0 \end{cases}$

$\downarrow$

$1 = -\sin x$

$1 = -\sin(0)$

$\therefore$ there is no b when we use derivative, so there is no differetiable at $x=0$